Synaptic Fluctuations: Questions

• Q1. Temporal Correlations

In the limit of fast synapses the fluctuating voltage can be written as

$$V = E_0 + \sigma_V \sqrt{2\tau_0} \int_{-\infty}^{t} \frac{dt'}{\tau_0} e^{-(t-t')/\tau_0} \xi_s(t'). \tag{1}$$

A measure of the temporal correlations is the autocorrelation in which the similarity of the trace to itself is compared at a time interval T. In an unnormalised form we can consider the function

$$A(T) = \langle (V(t+T) - E_0)(V(t) - E_0) \rangle \tag{2}$$

[Q] Demonstrate that

$$A(T) = \sigma_V^2 e^{-|T|/\tau_0}. \tag{3}$$

[A] Assume first $T > 0$, we have

$$A(T) = 2\sigma_V^2 \tau_0 \int_{-\infty}^{t+T} \frac{dt'}{\tau_0} \int_{-\infty}^{t} \frac{dt''}{\tau_0} e^{-(t+T-t')/\tau_0} e^{-(t-t')/\tau_0} \langle \xi(t')\xi(t'') \rangle \tag{4}$$

$$A(T) = 2\sigma_V^2 e^{-T/\tau_0} \int_{-\infty}^{t} \frac{dt'}{\tau_0} e^{-2(t-t')/\tau_0} \tag{5}$$

$$A(T) = \sigma_V^2 e^{-T/\tau_0}. \tag{6}$$

If T was negative, this would be equivalent to $\langle (V(t) - E_0)(V(t+|T|) - E_0) \rangle$ and so we arrive at the general result above. NB A less simple way to get the negative T case is to redo the integrals, with the upper limit on the resulting integral after the delta function has been used being $t + T < t$.

• Q2. Correlations Between Excitation and Inhibition

In the calculations given in the class and lecture notes we assumed that excitation and inhibition were uncorrelated $\langle \xi_e(t)\xi_i(t') \rangle = 0$. We now relax this condition. Let us assume that ξ_e and ξ_i are linear combinations of the uncorrelated, delta-autocorrelated, unit variance Gaussian white noise processes ξ_1 and ξ_2

$$\xi_e = a\xi_1 + b\xi_2 \quad \text{and} \quad \xi_i = c\xi_1 + d\xi_2. \tag{7}$$

[Q] What are the restrictions on a, b, c, d if $\langle \xi_e(t)\xi_e(t') \rangle = \delta(t-t')$ and $\langle \xi_i(t)\xi_i(t') \rangle = \delta(t-t')$?
[A] Because ξ_1 and ξ_2 are uncorrelated

$$a^2 + b^2 = 1 \quad \text{and} \quad c^2 + d^2 = 1 \tag{8}$$

[Q] Show that the cross-correlation satisfies

$$\langle \xi_e(t)\xi_i(t') \rangle = \kappa\delta(t-t') \quad \text{where} \quad \kappa = ac + bd. \tag{9}$$

[A] Substituting in the forms

$$\langle \xi_e(t)\xi_i(t') \rangle = ac\langle \xi_1(t)\xi_1(t') \rangle + bd\langle \xi_2(t)\xi_2(t') \rangle \tag{10}$$

from which the result follows.
[Q] By considering a trigonometric interpretation, or otherwise, show that $|\kappa| \leq 1$.
[A] Call $a = \cos(\theta_e), b = \sin(\theta_e)$ and $c = \cos(\theta_i), d = \sin(\theta_i)$, then

$$\kappa = ac + bd = \cos(\theta_e - \theta_i) \leq 1. \tag{11}$$

• Q3. EFFECTS OF CORRELATED EXCITATION AND INHIBITION

We now consider the voltage variance for the case of correlated excitation and inhibitory noise $\langle \xi_e(t)\xi_i(t') \rangle = \kappa\delta(t-t')$:

$$\tau_0 \frac{dV}{dt} = E_0 - V + a_e\tau_0\sqrt{R_e}\xi_e + \tau_0 a_i\sqrt{R_i}\xi_i \tag{12}$$

[Q] What are the interpretations, and what are the signs of a_e and a_i?
[A] They are the size of the voltage jumps when excitation or inhibition arrives. Excitatory jumps are positive $a_e > 0$ and inhibitory negative $a_i < 0$.
[Q] Use the rule for the summation of random numbers to replace the fluctuating parts of the above equation with a single gaussian white noise process ξ taking into account the correlation strength κ.
[A] We write

$$a_e\tau_0\sqrt{R_e}\xi_e + \tau_0 a_i\sqrt{R_i}\xi_i = B\xi \tag{13}$$

and calculate the autocorrelation at times t and t' to yield

$$\tau_0^2 \left(a_e^2 R_e + a_i^2 R_i + 2\kappa a_e a_i\sqrt{R_e R_i} \right) \delta(t-t') = B^2\delta(t-t') \tag{14}$$

Using this result and the standard form for an Ornstein-Uhlenbeck process

$$\tau_0 \frac{dV}{dt} = E_0 - V + \sqrt{2\tau_0}\sigma_V\xi(t) \tag{15}$$

[Q] show that

$$\sigma_V^2 = \frac{\tau_0}{2} \left(a_e^2 R_e + a_i^2 R_i + 2\kappa a_e a_i\sqrt{R_e R_i} \right). \tag{16}$$

[A] This comes from setting

$$B\xi(t) = \sqrt{2\tau_0}\sigma_V\xi(t). \tag{17}$$

Assume that a particular balance of drive exists such that $a_e^2 R_e = a_i^2 R_i$.
[Q] How are the voltage fluctuations extremised for fully positive $\kappa = 1$ and negative correlations $\kappa = -1$.
[A] For positive correlations the variance is minimised (it vanishes in this balanced case). For negative correlations the variance is maximised.