

POPULATIONS OF NEURONS: QUESTIONS

• Q1. STEADY-STATE DISTRIBUTION FOR THE SUBTHRESHOLD VOLTAGE

The current equation for a leaky IF model

$$J = \frac{(E_0 - V)}{\tau_0} P - \frac{\sigma_V^2}{\tau_0} \frac{\partial P}{\partial V}. \quad (1)$$

Consider a population of neurons that do not have spike-generating currents, and so there is no threshold for action potential generation. The population is in the steady state.

[Q] What form does the function for the current J take?

[Q] Show that in this case the probability distribution P is Gaussian.

• Q2. FIRING-RATE OF THE NON-LEAKY IF NEURON

The Non-Leaky IF model has a voltage dynamics

$$\tau_0 \frac{\partial V}{\partial t} = E_0 + \sqrt{2\tau_0} \sigma_V \xi(t) \quad (2)$$

where $E_0 > 0$ and so has a corresponding Fokker-Planck equation

$$\tau_0 \frac{\partial P}{\partial t} = \sigma_V^2 \frac{\partial^2 P}{\partial V^2} - E_0 \frac{\partial P}{\partial V} \quad (3)$$

[Q] Use the continuity equation to show that the current J obeys

$$\tau_0 J = E_0 P - \sigma_V^2 \frac{\partial P}{\partial V} \quad (4)$$

We now impose a threshold V_{th} and reset V_{re} such that the current obeys $J = r\theta(V - V_{re})$ for $V < V_{th}$ where r is the firing rate. The density vanishes at threshold $P(V_{th}) = 0$. Rescale the voltage by $x = V/\sigma_V$ and define $\mu = E_0/\sigma_V$, $x_{re} = V_{re}/\sigma_V$ and $x_{th} = V_{th}/\sigma_V$.

[Q] Show that this rescaling yields:

$$-r\tau_0\theta(x - x_{re}) = \frac{\partial p}{\partial x} - \mu p \quad (5)$$

where $p(x)dx = P(V)dV$.

[Q] Solve this equation to give the probability density as a function of the unknown firing rate. You may write the answer in terms of an integral - the full solution is required in the next question.

[Q] Use the fact that the probability density integrates to one to show that

$$1 = r\tau_0 \int_{-\infty}^{x_{th}} dx e^{\mu x} \int_x^{x_{th}} dy e^{-\mu y} \theta(y - x_{re}). \quad (6)$$

[Q] Reverse the order of integrals to drop the θ function and then perform the integrations to show that

$$r = \frac{1}{\tau_0} \frac{\mu}{x_{th} - x_{re}} \quad (7)$$

and therefore that for this simple model the firing rate is independent of the noise strength σ_V .

• Q3. PROBABILITY DENSITY FOR THE NON-LEAKY IF NEURON

We continue the analysis of the NLIF model now by calculating the form of the distribution.

[Q] Using the form of the probability density and the current calculated for the previous question show that

$$p(x) = \frac{1}{x_{th} - x_{re}} \left(1 - e^{-\mu(x_{th}-x)}\right) \quad \text{for } x > x_{re} \quad (8)$$

$$p(x) = \frac{1}{x_{th} - x_{re}} \left(e^{-\mu(x_{re}-x)} - e^{-\mu(x_{th}-x)}\right) \quad \text{for } x < x_{re} \quad (9)$$

The current J given in equation (4) evaluated at threshold gives the firing rate r .

[Q] Verify this using equation (8).

[Q] By considering the currents at the reset, show that the following is true:

$$J(V_{re-}) + r = J(V_{re+}) \quad (10)$$

[Q] Given that $P(V_{re-}) = P(V_{re+})$ show that the gradient discontinuity in $P(V)$ is proportional to the firing rate.

[Q] Sketch the probability density. What happens when the noise goes to zero? (i.e. $\mu \rightarrow \infty$).