A Bayesian and Frequentist Multiverse Pipeline for MPT models

Applications to Recognition Memory

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Multiverse Approach

• Statistical analysis usually requires several (more or less) arbitrary decisions between reasonable alternatives:
  • Data processing and preparation (e.g., exclusion criteria, aggregation levels)
  • Analysis framework (e.g., statistical vs. cognitive model, frequentist vs. Bayes)
  • Statistical analysis (e.g., testing vs. estimation, fixed vs. random effects, pooling)

• Combination of decisions spans multiverse of data and results (Steegen, Tuerlinckx, Gelman, & Vanpaemel, 2016):
  • Usually one path through multiverse (or 'garden of forking paths', Gelman & Loken, 2013) is reported
  • Valid conclusions cannot be contingent on arbitrary decisions

• Multiverse analysis attempts exploration of possible results space:
  • Reduces problem of selective reporting by making fragility or robustness of results transparent.
  • Conclusions arising from many paths are more credible than conclusions arising from few paths.
  • Helps identification of most consequential choices.

• Limits of multiverse approach:
  • Implementation not trivial.
  • Results must be commensurable across multiverse (e.g., estimation versus hypothesis testing).
Current Project

• DFG *Scientific Network* grant to Julia Groß and Beatrice Kuhlmann
  • "Hierarchical MPT Modeling – Methodological Comparisons and Application Guidelines"
  • 6 meetings over 3 years with 15 people plus external experts
  • Multinomial processing tree (MPT) models: class of discrete-state cognitive models for multinomial data (Riefer & Batchelder, 1988)
  • MPT models traditionally analysed with frequentist methods (i.e., $\chi^2/G^2$) and aggregated data
  • Several hierarchical-Bayesian approaches exist. Do we need those?

• *Today*: First results from model for recognition memory
Our Multiverse

• Statistical framework:
  • Frequentist (i.e., maximum-likelihood)
  • Bayesian (i.e., MCMC)

• Pooling:
  • Complete pooling (aggregated data)
  • No pooling (individual-level data)
  • Partial pooling (hierarchical-modelling): Individual-level parameters with group-level distribution

• Results:
  1. **Parameter point estimates**: MLE and posterior mean
  2. **Parameter uncertainty**: ML-SE and MCMC-SE
  3. **Model adequacy**: $G^2$ p-value and posterior predictive p-value (Klauer, 2010)
Our Multiverse Pipeline (in R)

• Frequentist (uses MPTinR; Singmann & Kellen, 2013):
  • (1) Traditional approach: Frequentist asymptotic complete pooling
  • (2) Frequentist asymptotic no pooling
  • (3) Frequentist no-pooling with parametric bootstrap
  • (4) Frequentist no-pooling with non-parametric bootstrap

• Bayesian (uses TreeBUGS; Heck, Arnold, & Arnold, 2018):
  • (5) Bayesian complete pooling (custom C++ sampler)
  • (6) Bayesian no pooling (unique method, custom C++ sampler)
  • (7) Bayesian partial pooling I a, Jags: Beta-MPT Jags (Smith & Batchelder, 2010)
  • (8) Bayesian partial pooling I b, C++: Beta-MPT C++
  • (9) Bayesian partial pooling II: Latent trait MPT (Klauer, 2010; Jags)
  • (10) Bayesian partial pooling III: Latent trait MPT w/o correlation parameters (Jags)

• (11) latent-class approach (Klauer, 2006)

• All implemented in R packages
  • MPTmultiverse: (1)-(10): [https://cran.r-project.org/package=MPTmultiverse](https://cran.r-project.org/package=MPTmultiverse)
  • hmpt (for latent-class only): [https://github.com/mpt-network/hmpt](https://github.com/mpt-network/hmpt)
Example Application: Recognition Memory

1. Learning Phase

   HOUSE
   BOAT
   FOOD
   SEAL
   RAIN

2. Test Phase:

   RAIN
   CAT
   KING
   FOOD
   SEAL

old? new?
6-point ROCs

<table>
<thead>
<tr>
<th>Data set</th>
<th>Sample N</th>
<th>Mean No. of Trials</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dube &amp; Rotello (2012, E1, Pictures (P))</td>
<td>27</td>
<td>400</td>
</tr>
<tr>
<td>Dube &amp; Rotello (2012, E1, Words (W))</td>
<td>22</td>
<td>400</td>
</tr>
<tr>
<td>Heathcote et al. (2006, Exp. 1)</td>
<td>16</td>
<td>560</td>
</tr>
<tr>
<td>Heathcote et al. (2006, Exp. 2)</td>
<td>23</td>
<td>560</td>
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<tr>
<td>Jaeger et al. (2012, Exp. 1, no cue)</td>
<td>63</td>
<td>120</td>
</tr>
<tr>
<td>Jang et al. (2009)</td>
<td>33</td>
<td>140</td>
</tr>
<tr>
<td>Koen &amp; Yonelinas (2010, pure study)</td>
<td>32</td>
<td>320</td>
</tr>
<tr>
<td>Koen &amp; Yonelinas (2011)</td>
<td>20</td>
<td>600</td>
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<tr>
<td>Koen et al. (2013, Exp. 2, full attention)</td>
<td>48</td>
<td>200</td>
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<tr>
<td>Koen et al. (2013, Exp. 4, immediate test)</td>
<td>48</td>
<td>300</td>
</tr>
<tr>
<td>Pratte et al. (2010)</td>
<td>97</td>
<td>480</td>
</tr>
<tr>
<td>Smith &amp; Duncan (2004, Exp. 2)</td>
<td>30</td>
<td>140</td>
</tr>
</tbody>
</table>

- Total $N = 459$
- Mean trials = 350
- Still missing:
  - 8-point ROCs
  - 4 studies
  - Total $N$: 459
  - reasons: more model variants possible, takes longer
2-high threshold model (2HTM) for 6-point confidence-rating data (e.g., Bröder, et al., 2013)

- 3 core parameters:
  - \( Do \): Probability to detect old item as old
  - \( Dn \): Probability to detect new item as new
  - \( g \): Probability to guess item is old (conditional on non-detection)

- 8 response mapping parameters (at least one needs to be equated for identifiability)
• Each data point is one study
• Value corner: CCC = concordance correlation coefficient (measure of absolute agreement)
Parameter trade-off/fungibility (based on Latent-trait group-level posteriors)
Conclusions

• Overall, methods appear to agree with each other (maximal difference ≈ .25)

• Agreement between estimation method, depends on parameter
  • Even for structurally very similar core parameters (Do versus Dn) we see differences
  • Compared to latent trait methods, Dn and g are slightly underestimated by other methods
  • Effect of over/under-estimation does not appear to be related to sample size

• Parameters that show imprecision in estimation, seem to show larger parameter trade-offs

• Recommendation: Use multiverse approach to take uncertainty of modeling framework into account
2-high threshold model (2HTM) for 6-point confidence-rating data (e.g., Bröder, et al., 2013)

\[ r_1 = r_1 \]
\[ r_2 = (1 - r_1)r_2 \]
\[ r_3 = (1 - r_1)(1 - r_2) \]

same for \( q \)

\[ r_6 = r_6 \]
\[ r_5 = (1 - r_6)r_5 \]
\[ r_4 = (1 - r_6)(1 - r_5) \]

same for \( q \)

- For \( r, r_1 \) and \( r_6 \) expected to have most data
- For \( q, q_5 \) and \( q_2 \) expected to have most data

- Data provides 10 independent data points, full model has 11 free parameters:
  - 3 core parameters: \( Dn, Do, \) and \( g \).
  - 8 response mapping parameters
  - For parameter identifiability: at least one parameter needs to be equated
Identifiability Restrictions

• original Bröder et al. (2013) variant:
  • $q_5 = q_2$
  • $r_5 = r_2$

• only $q$-restricted:
  • $q_5 = q_2$

• only $r$-restricted:
  • $r_5 = r_2$

\begin{align*}
r1 &= r_1 \\
r2 &= (1 - r_1)r_2 \\
r3 &= (1 - r_1)(1 - r_2) \text{ same for } q \\
r6 &= r_6 \\
r5 &= (1 - r_6)r_5 \\
r4 &= (1 - r_6)(1 - r_5) & \text{ same for } q
\end{align*}