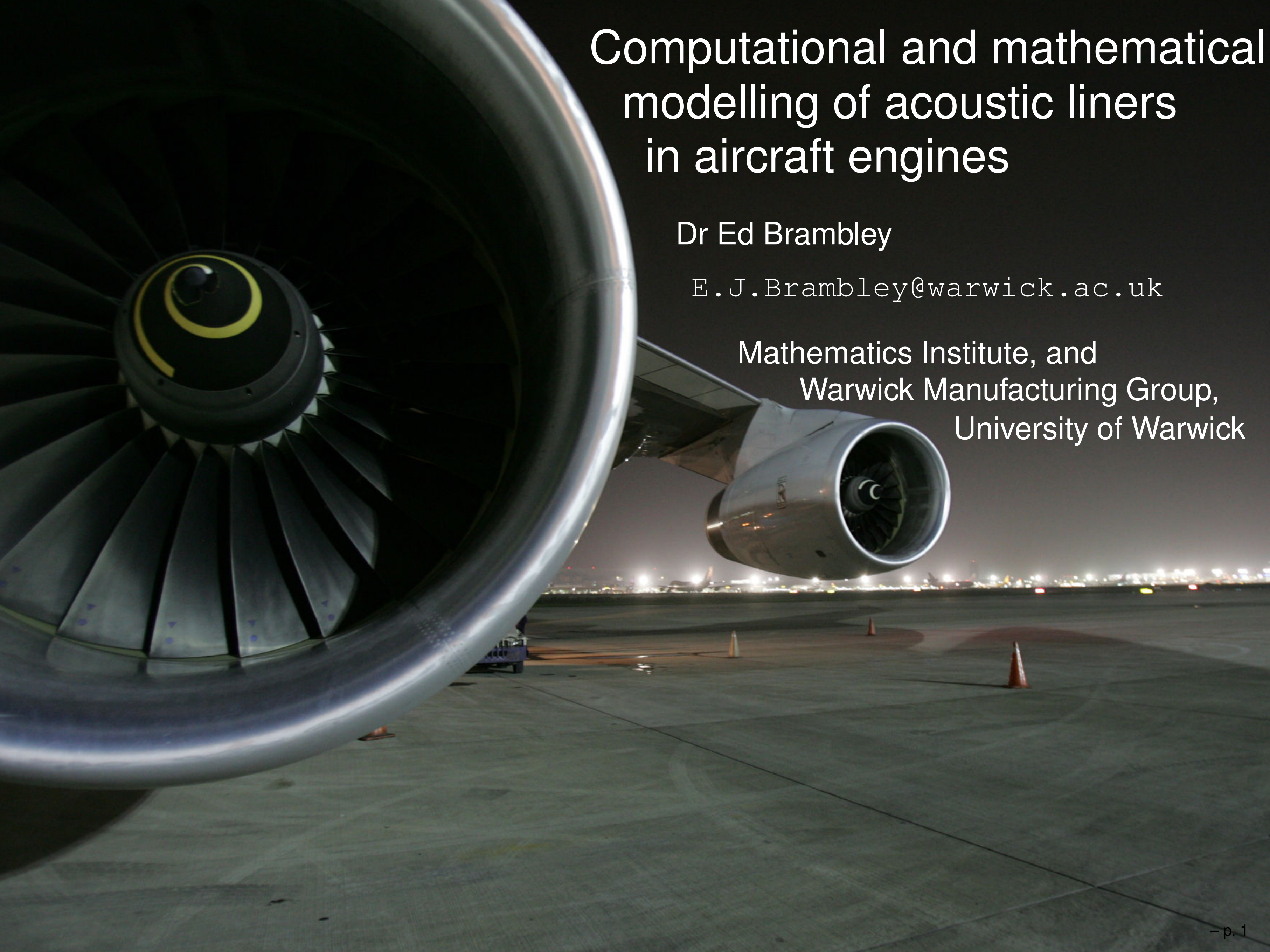

Computational and mathematical modelling of acoustic liners in aircraft engines

Dr Ed Brambley

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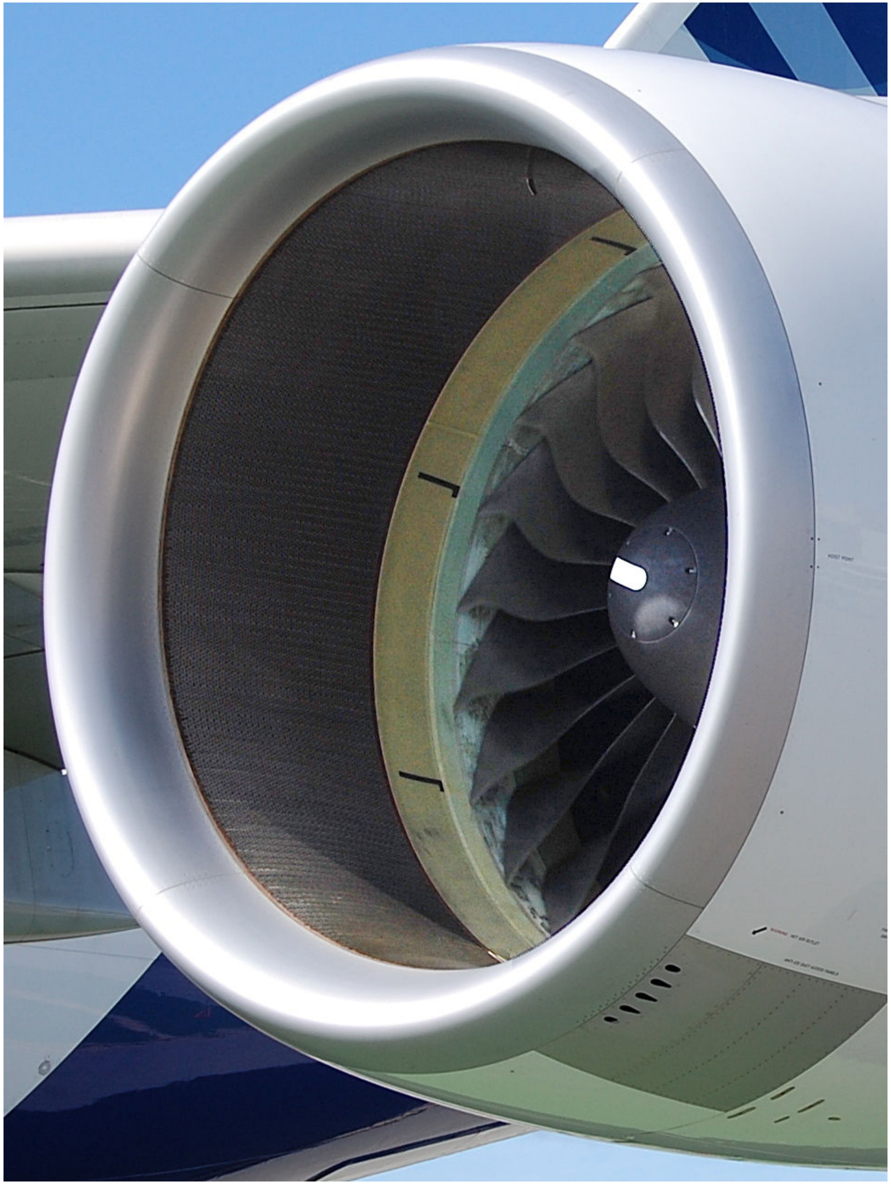
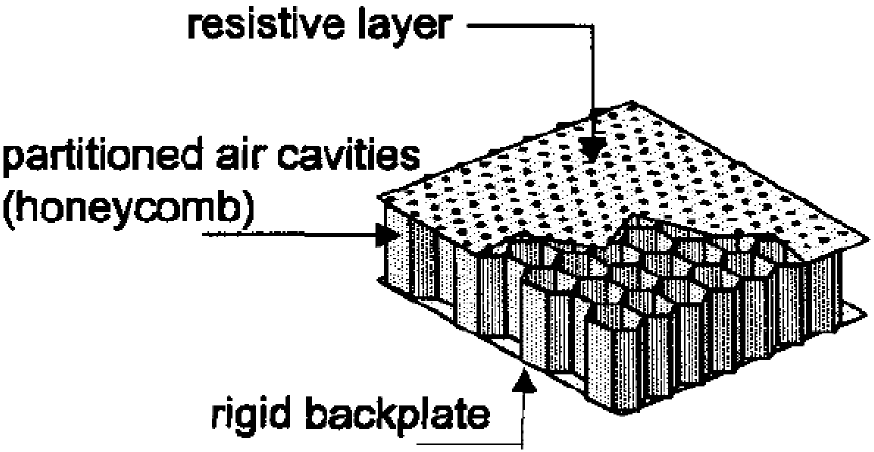
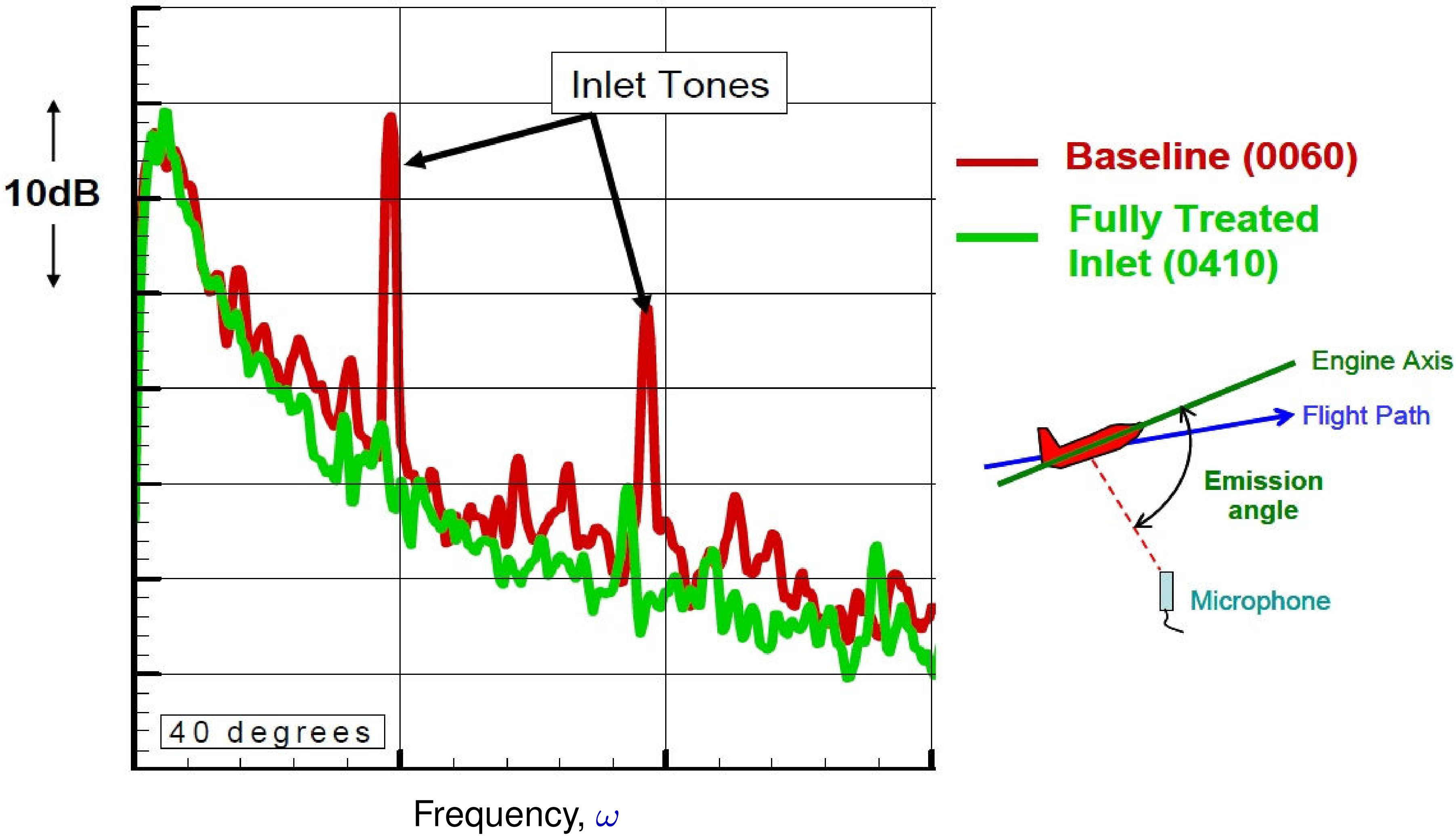
Mathematics Institute, and
Warwick Manufacturing Group,
University of Warwick

Acoustic linings





Acoustic lining effectiveness



Aeroengine noise sources

R.J. Astley et al. / Journal of Sound and Vibration 330 (2011) 3832–3845

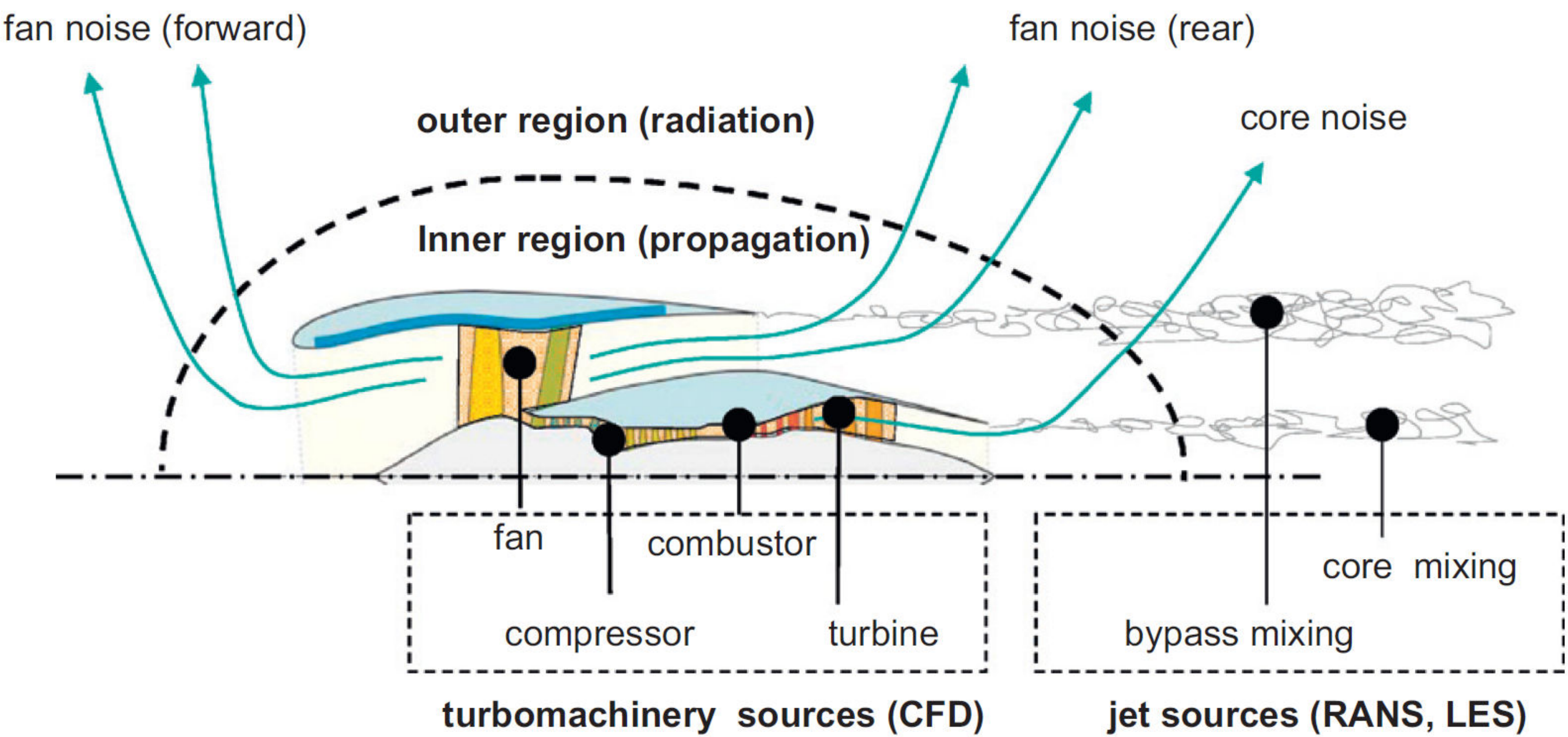
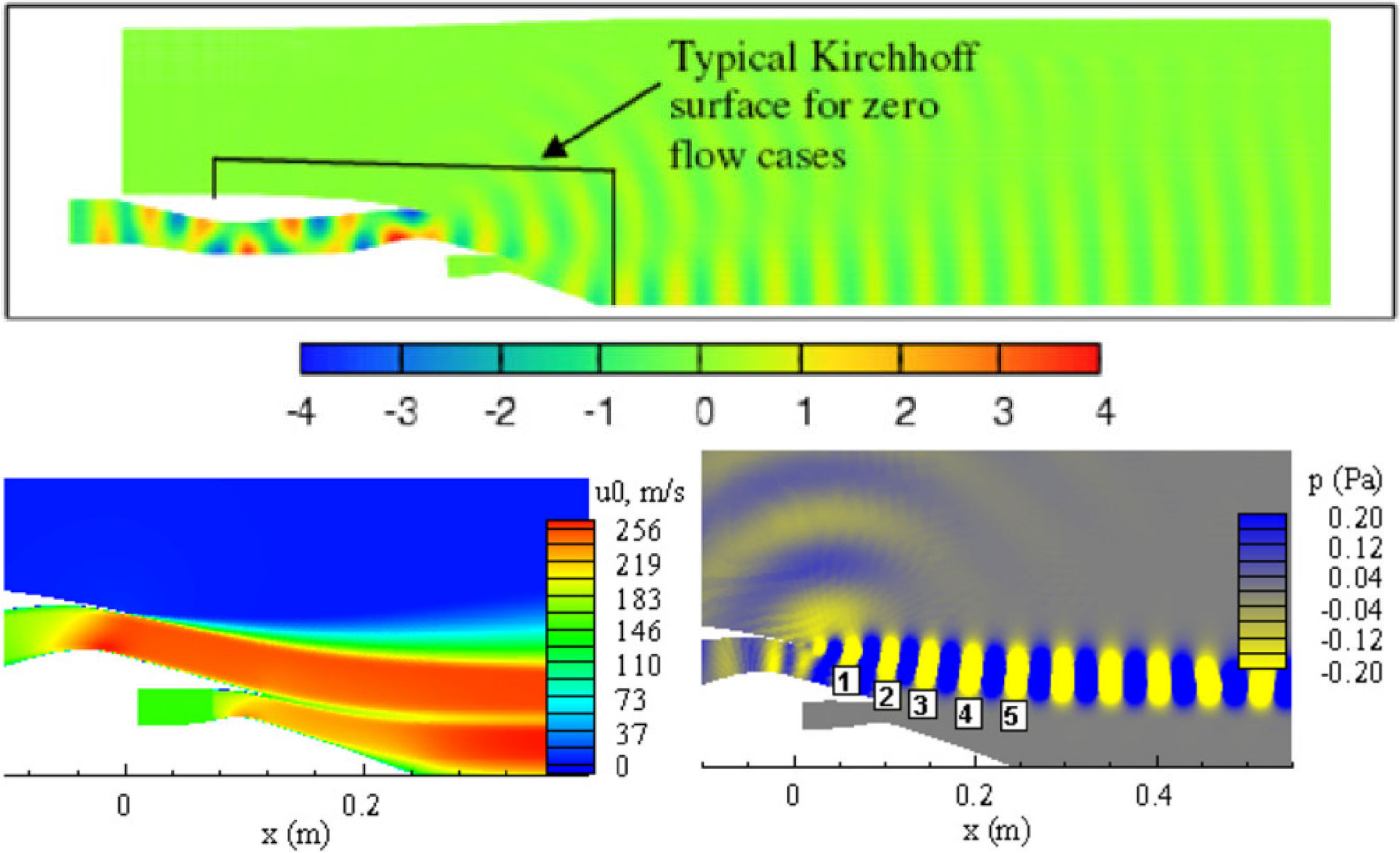


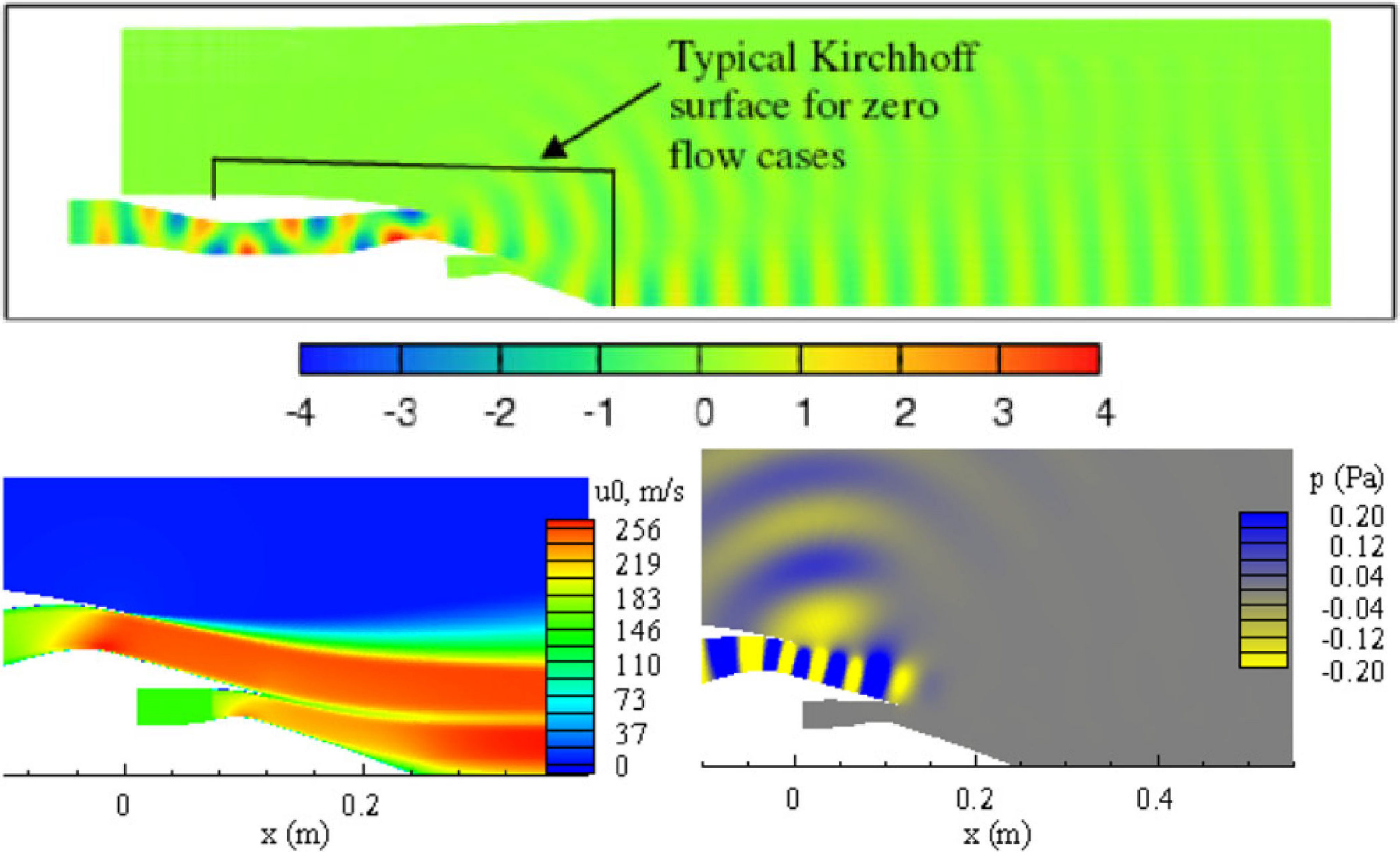
Fig. 1. Noise sources and transmission paths in a turbofan engine.

Computational AeroAcoustics (CAA)



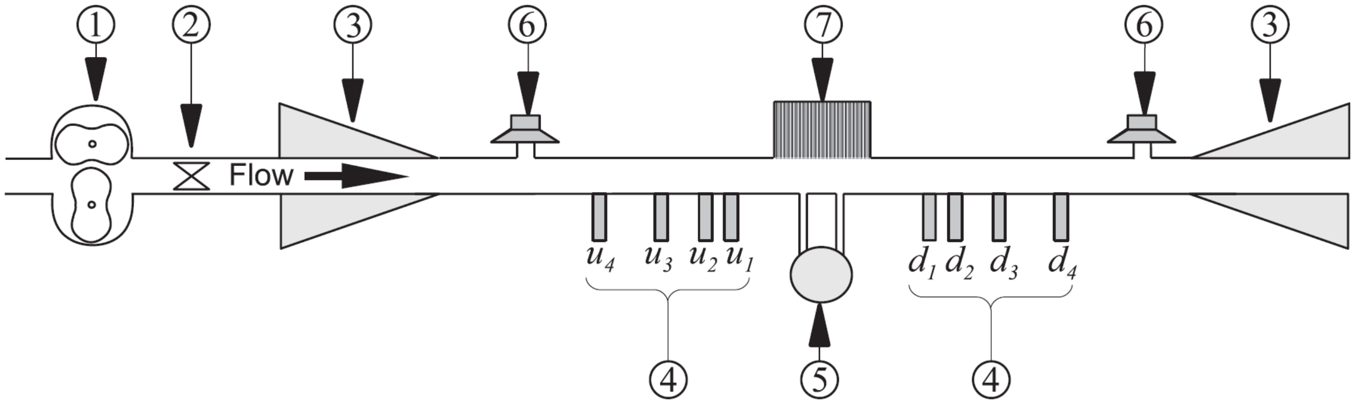
Taken from Özyörük & Tester (2011, JSV).

Computational AeroAcoustics (CAA)

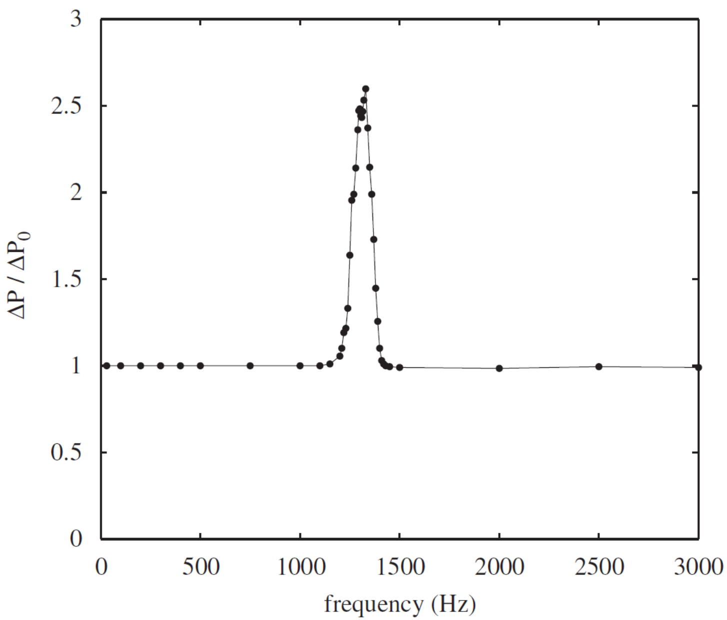


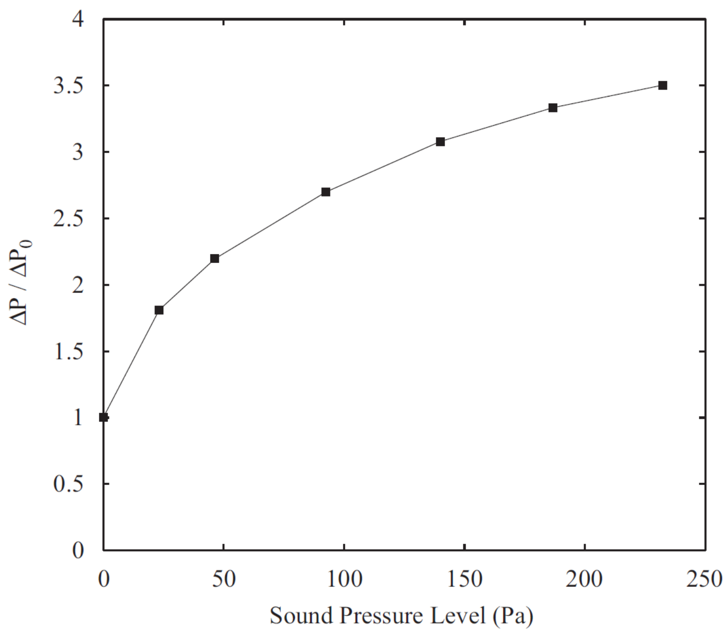
Taken from Özyörük & Tester (2011, JSV).

Experimental evidence of instability



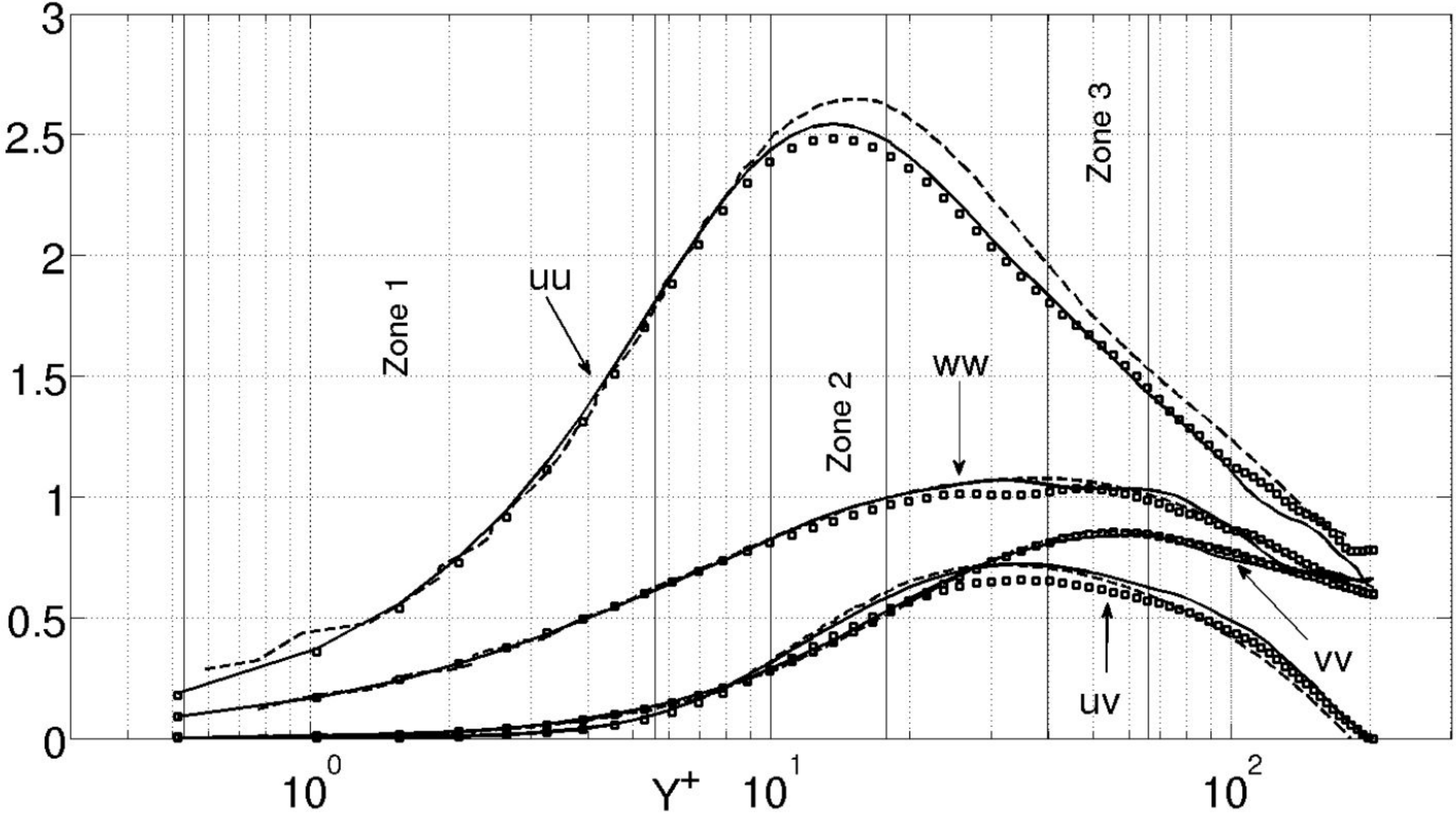
1: compressor, 2: flowmeter, 3: anechoic terminations, 4: microphones, 5: static pressure measurement, 6: acoustical source, 7: lined wall.





Taken from Aurégan & Leroux (2008, JSV).

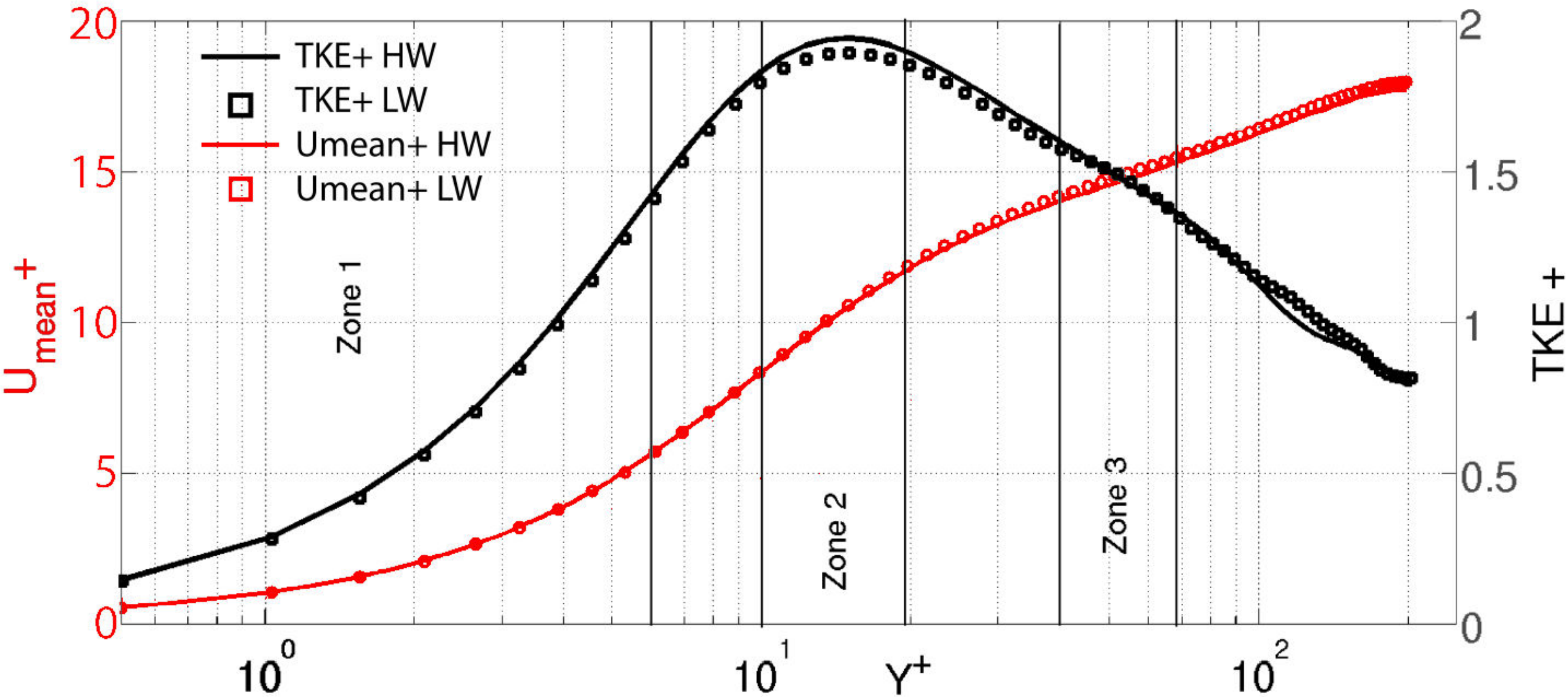
Numerical (lack of) evidence of instability



– – – reference (unlined); — unlined; □ lined

Taken from Olivetti, Sandberg & Tester (2014, JSV).

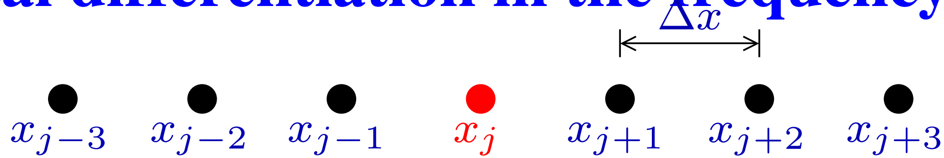
Numerical (lack of) evidence of instability



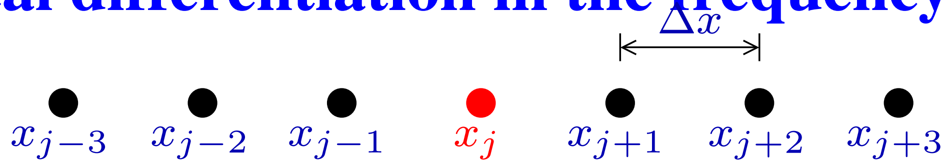
Taken from Olivetti, Sandberg & Tester (2014, JSV).

Computational AeroAcoustics (CAA)

Numerical differentiation in the frequency domain



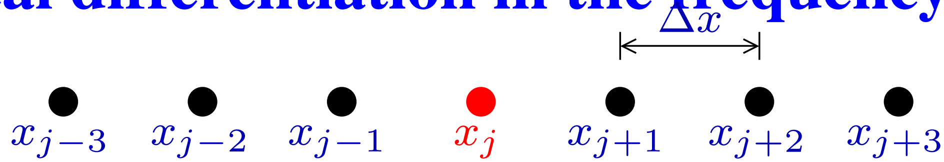
Numerical differentiation in the frequency domain



- If $f_j = f(x_j)$, then $f_{j+n} = f(x_j) + n\Delta x f'(x_j) + \frac{1}{2}(n\Delta x)^2 f''(x_j) + \cdots$. Then

$$\frac{f_{j+1} - f_{j-1}}{2\Delta x} = f'(x_j) + O\big((\Delta x)^2\big)$$

Numerical differentiation in the frequency domain



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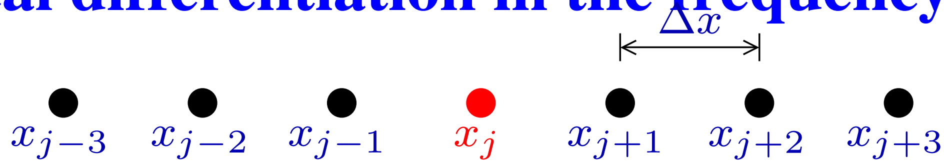
$$\frac{1}{\Delta x}\left[\tfrac{2}{3}\big(f_{j+1} - f_{j-1}\big) - \tfrac{1}{12}\big(f_{j+2} - f_{j-2}\big)\right] = f'(x_j) + O\big((\Delta x)^4\big)$$

Numerical differentiation in the frequency domain



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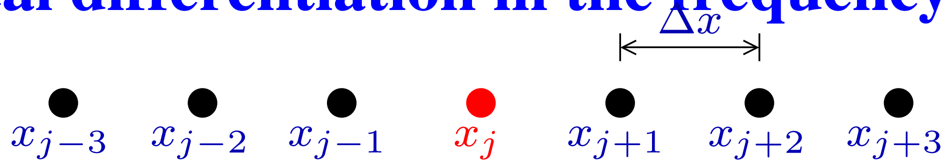
- Approximate $f'(x_j)$ by $f'_j = \dfrac{1}{\Delta x}\displaystyle\sum_{n=1}^{N} d_n\big(f_{j+n} - f_{j-n}\big)$.

Numerical differentiation in the frequency domain



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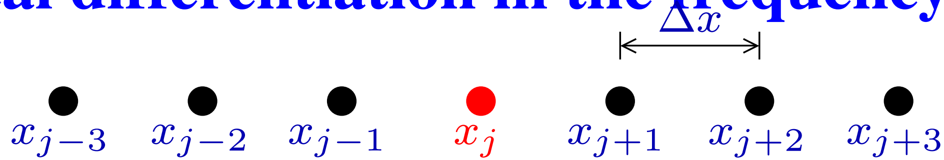
- For a harmonic wave $f(x_j) = \exp\{-\mathrm{i}kx_j\}$,

$$f'_j = \exp\{-\mathrm{i}kx_j\}\frac{1}{\Delta x}\sum_{n=1}^{N} d_n\big(\exp\{-\mathrm{i}kn\Delta x\} - \exp\{\mathrm{i}kn\Delta x\}\big)$$

Numerical differentiation in the frequency domain



- If $f_j = f(x_j)$, then $f_{j+n} = f(x_j) + n\Delta x f'(x_j) + \frac{1}{2}(n\Delta x)^2 f''(x_j) + \cdots$. Then

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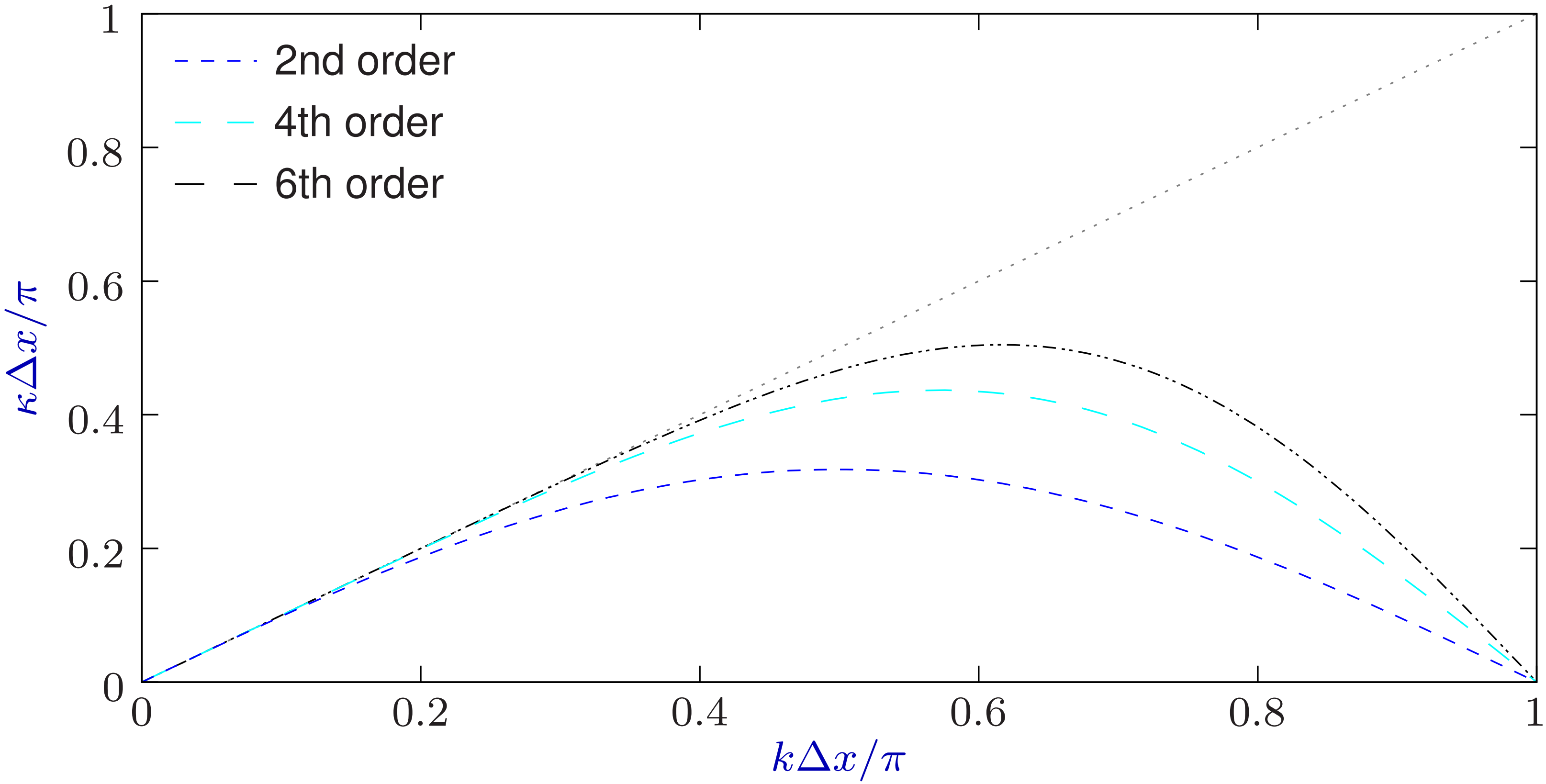
$$= -\mathrm{i}\kappa\exp\{-\mathrm{i}kx_j\} \qquad \text{where} \qquad \kappa\Delta x = \sum_{n=1}^{N} 2d_n\sin(nk\Delta x)$$

Numerical differentiation in the frequency domain (DRP schemes)

$$\kappa\Delta x = \sum_{n=1}^{N} 2d_n \sin(nk\Delta x)$$

- Could use d_n to get $O\big((\Delta x)^{2N}\big)$ accuracy.



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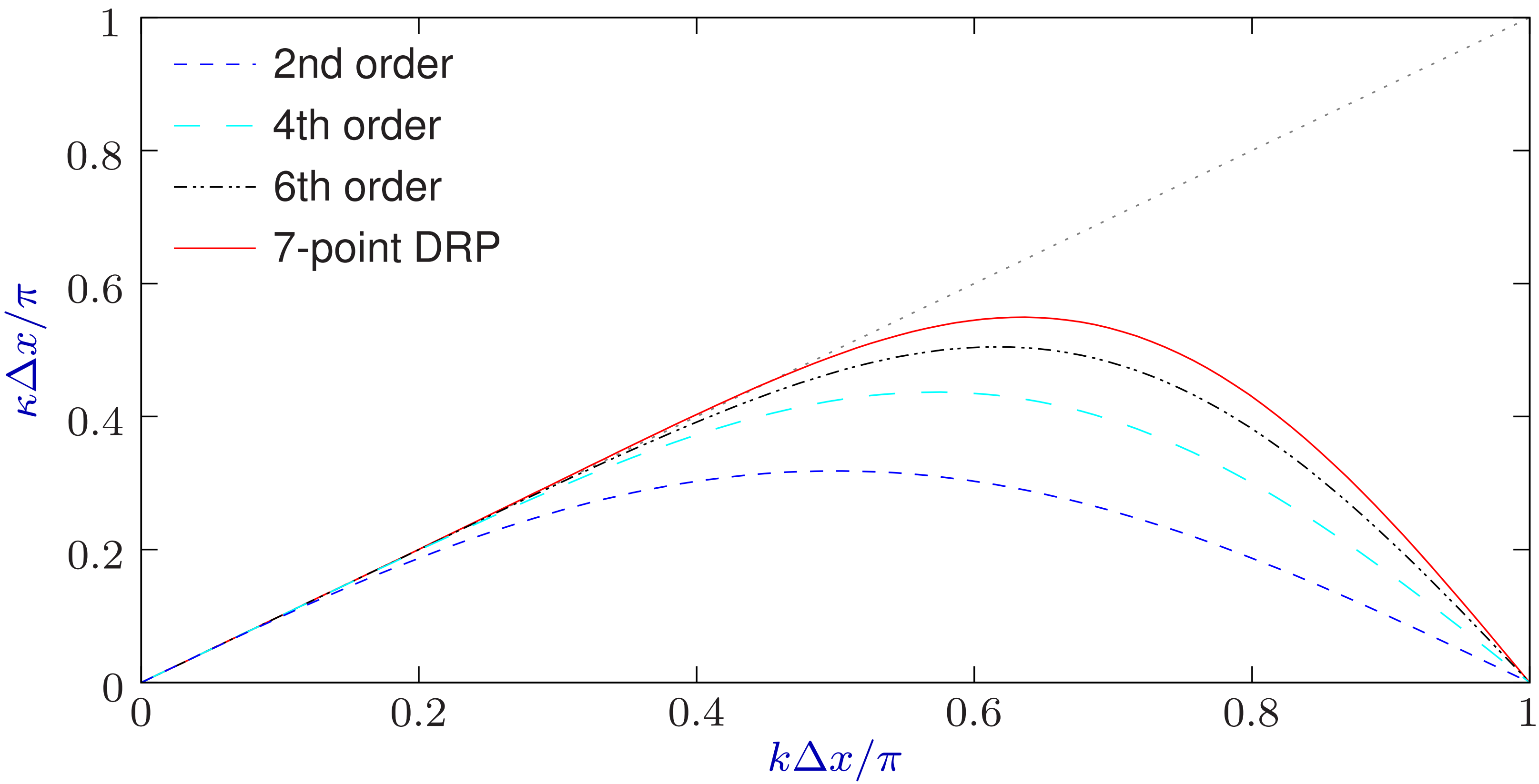


- Tam & Webb (1993, JCP) used $N = 3$ but only $O\big((\Delta x)^4\big)$ accuracy. Remaining degree of freedom optimized to get a “Dispersion Relation Preserving” scheme.

DRP scheme optimization

$$f'_j = \frac{1}{\Delta x} \sum_{n=1}^{N} d_n \left(f_{j+n} - f_{j-n}\right) \qquad \Rightarrow \qquad \kappa\Delta x = \sum_{n=1}^{N} 2d_n \sin(nk\Delta x)$$

- Tam & Webb (1993, JCP) took $N = 3$, required $O\big((\Delta x)^4\big)$ accuracy, and optimized

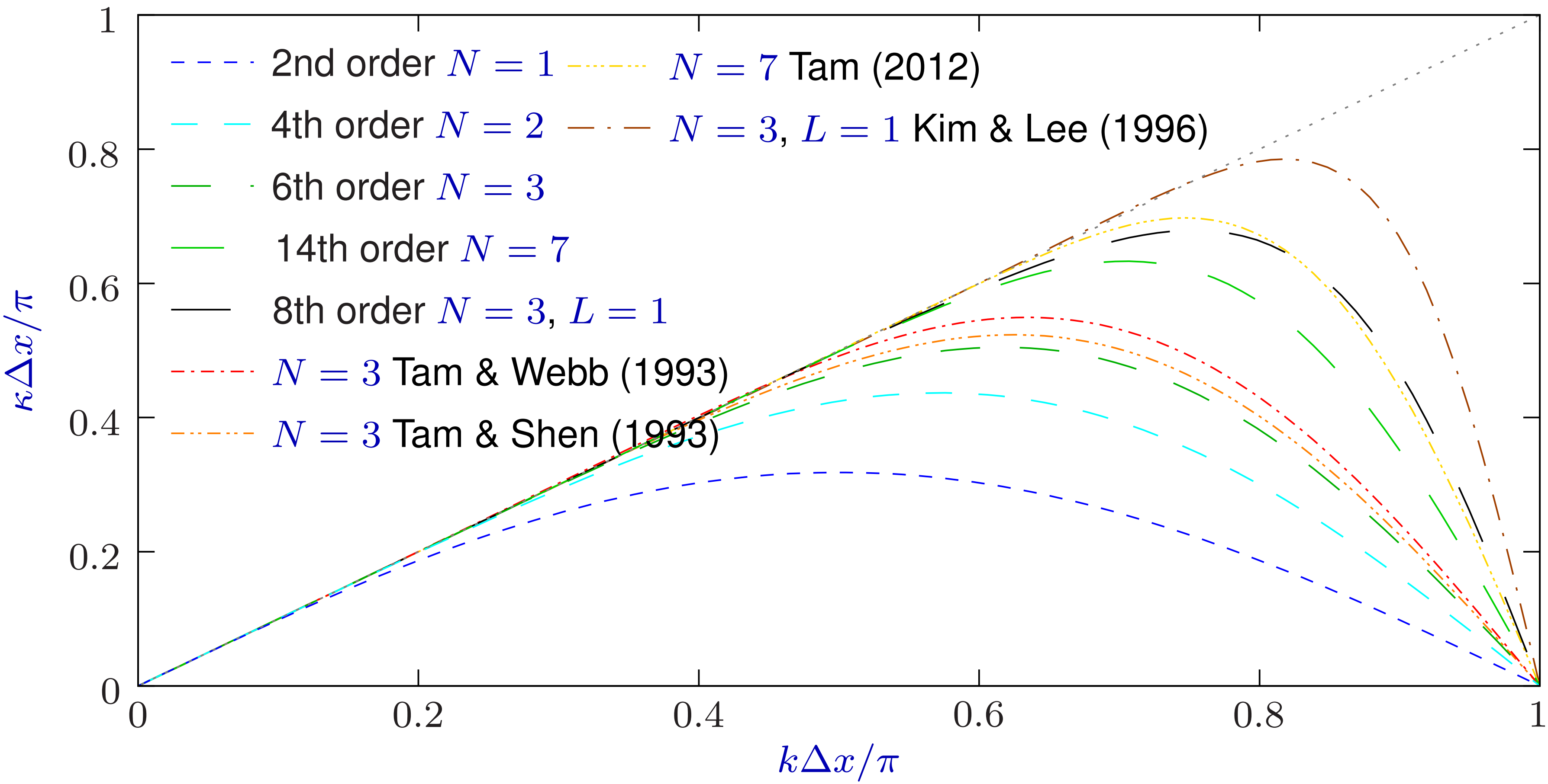
$$\int_0^{\pi} \left(\kappa\Delta x - k\Delta x\right)^2 \mathrm{d}(k\Delta x)$$

- Others have:
  - reoptimized for $N > 3$
  - optimized over other intervals than $[0, \pi]$
  - optimized $\left|\frac{\kappa\Delta x}{k\Delta x} - 1\right|$
  - added weighting functions
  - optimized $\|\kappa\Delta x - k\Delta x\|_\infty$
  - optimized $\left|\frac{\mathrm{d}\kappa\Delta x}{\mathrm{d}k\Delta x} - 1\right|$

- Can also consider *implicit* or *compact* schemes:

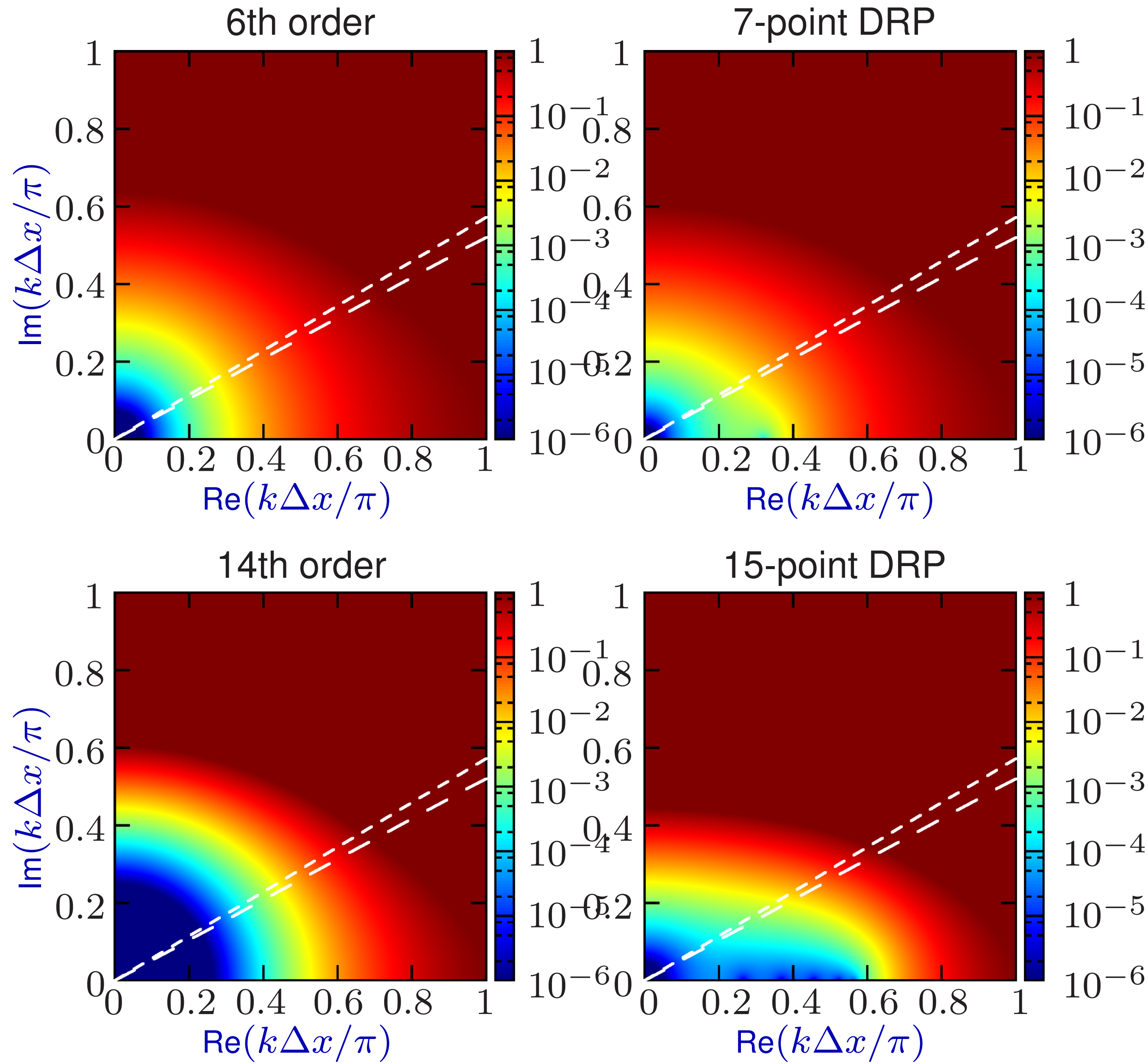
$$f'_j + \sum_{q=1}^{L} \beta_q \left(f'_{j+q} + f'_{j-q}\right) = \frac{1}{\Delta x} \sum_{n=1}^{N} d_n \left(f_{j+n} - f_{j-n}\right) \qquad \Rightarrow \qquad \kappa\Delta x = \frac{\sum_{n=1}^{N} 2d_n \sin(nk\Delta x)}{1 + \sum_{q=1}^{L} 2\beta_q \cos(qk\Delta x)}$$

DRP schemes

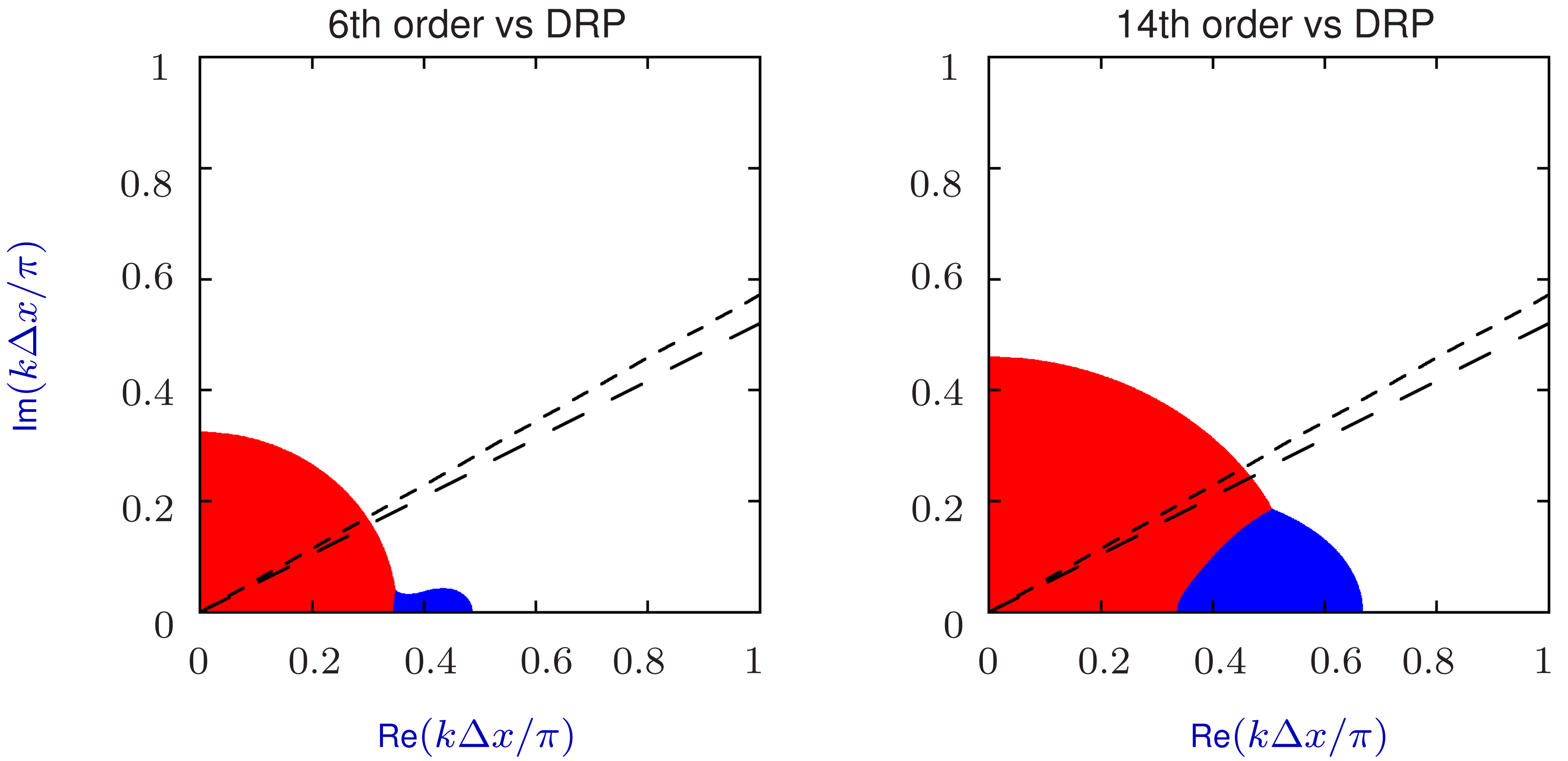


- But k is always real . . .

Accuracy of derivatives for complex k

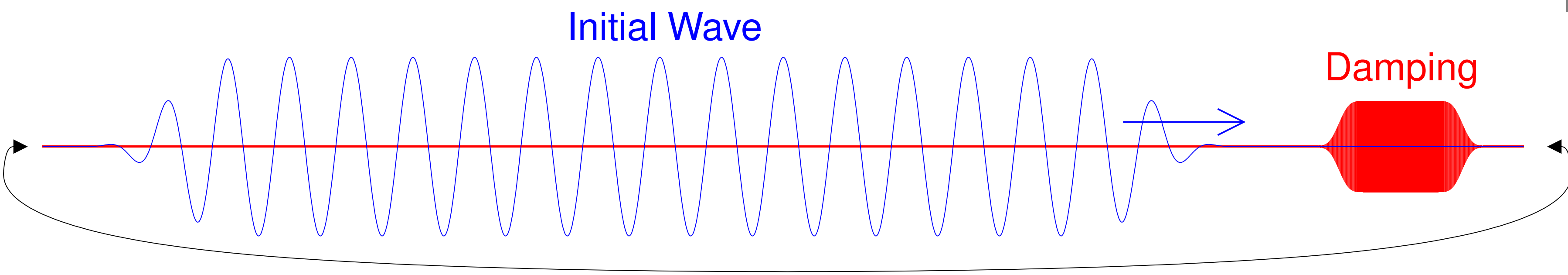


Accuracy of derivatives for complex k

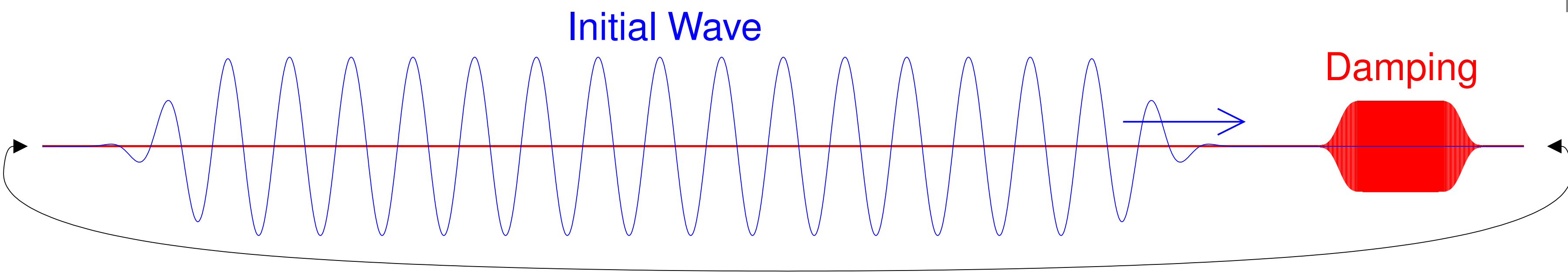


- Red: maximum order is most accurate.
- Blue: DRP is most accurate.
- White: Neither gives $\left|\dfrac{\kappa\Delta x}{k\Delta x} - 1\right| < 10^{-2}$.
- For details, see Brambley (AIAA Paper 2015–2540).

1D damped wave example



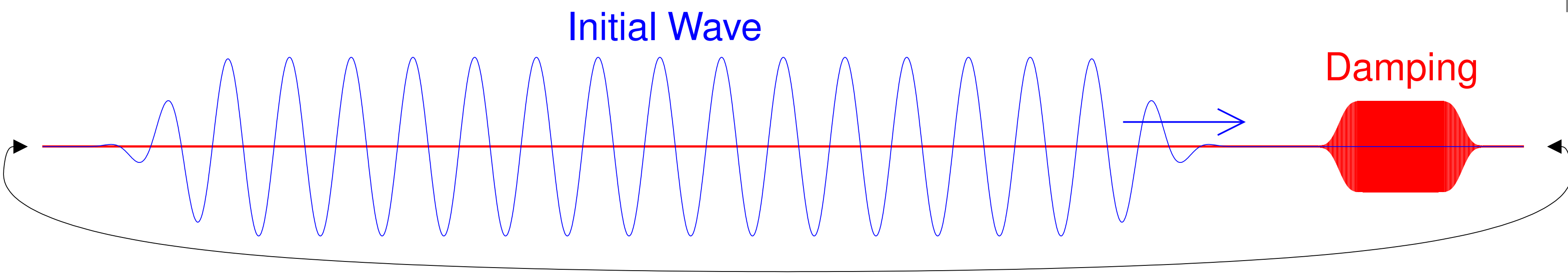
1D damped wave example



$$\frac{\partial p}{\partial t} + \frac{\partial v}{\partial x} = -k_p(x)p$$

$$\frac{\partial v}{\partial t} + \frac{\partial p}{\partial x} = -k_v(x)v$$

1D damped wave example



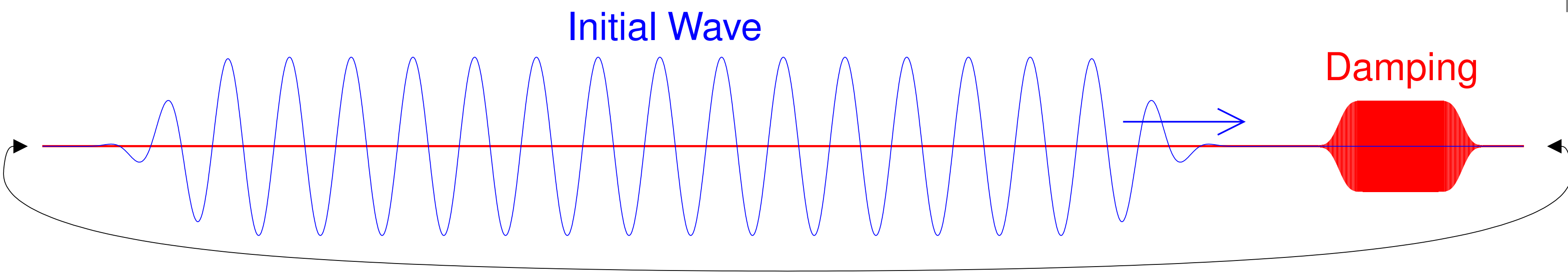
$$\frac{\partial p}{\partial t} + \frac{\partial v}{\partial x} = -k_p(x)p$$

$$\frac{\partial v}{\partial t} + \frac{\partial p}{\partial x} = -k_v(x)v$$

If $k_p \equiv k_v$, this has an exact travelling wave solution

$$p(x,t) = v(x,t) = f(x-t)\exp\left\{-\int_{x-t}^{x} k_p(X)\,\mathrm{d}X\right\}.$$

1D damped wave example



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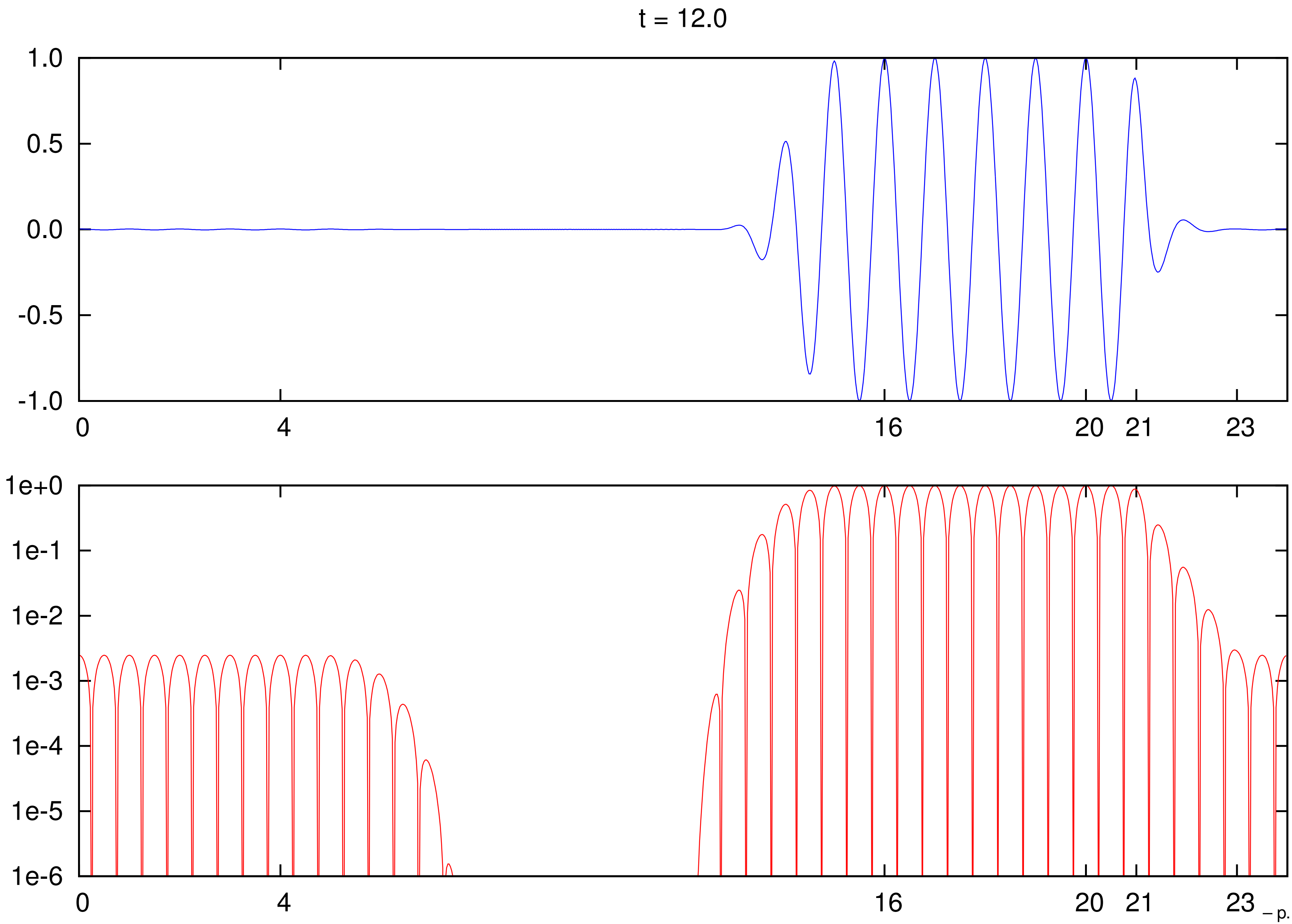
$$p(x,t) = v(x,t) = f(x-t)\exp\left\{-\int_{x-t}^{x} k_p(X)\,\mathrm{d}X\right\}.$$

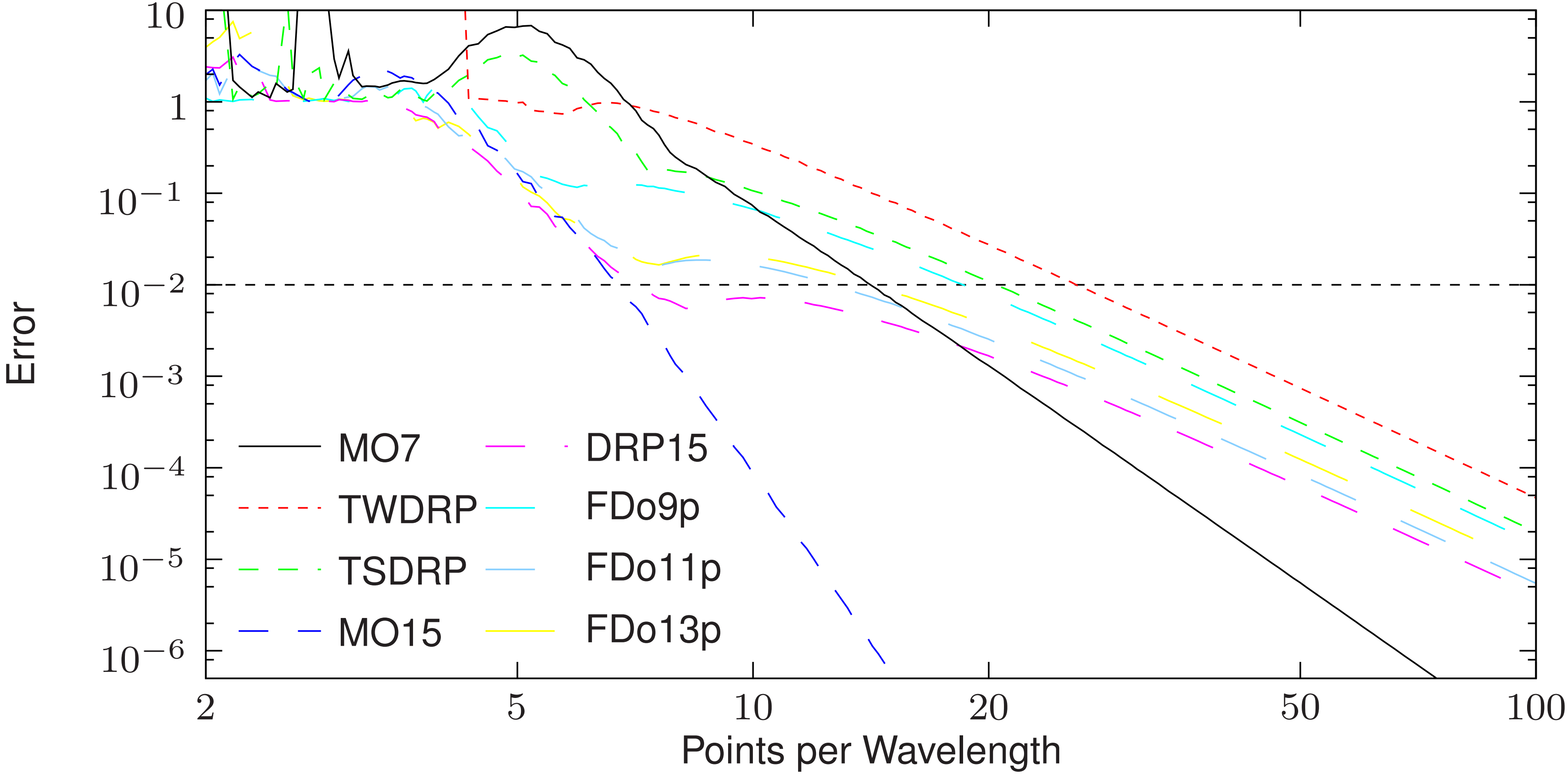
For a periodic domain of length L, therefore,

$$\left|p(x,L)\exp\left\{\int_0^L k_p(X)\,\mathrm{d}x\right\} - p(x,0)\right| = \text{Error} = 0.$$

1D damped wave example movie



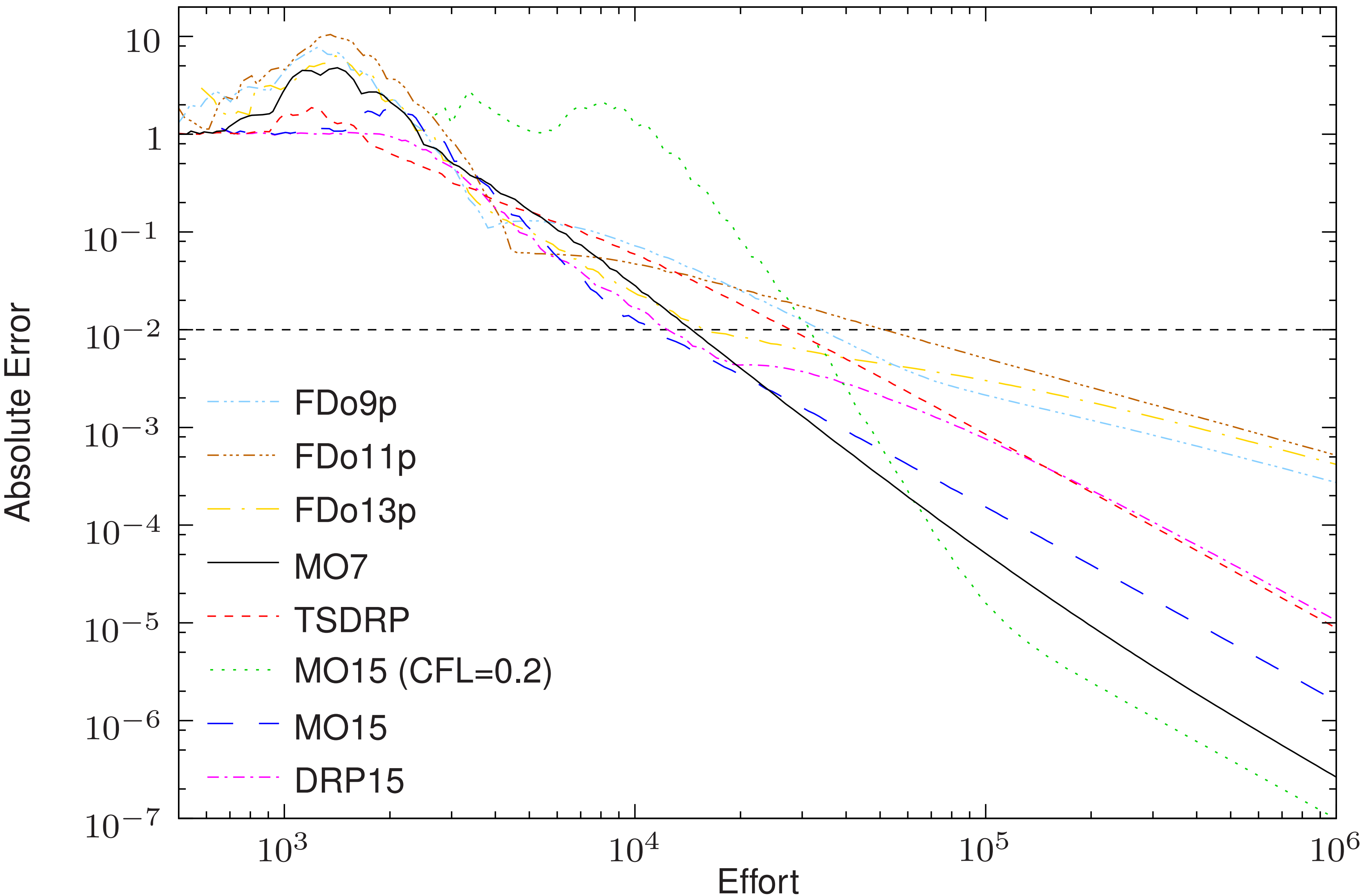
Numerical errors for 1D damped wave example



Parameters taken from Tam, Ju & Chien (2008) as realistic for an aircraft engine intake (1% accuracy is needed to resolve scattered waves that dominate in the far field).

See Brambley (JCP 2016) for details.

Numerical errors for 1D damped wave example



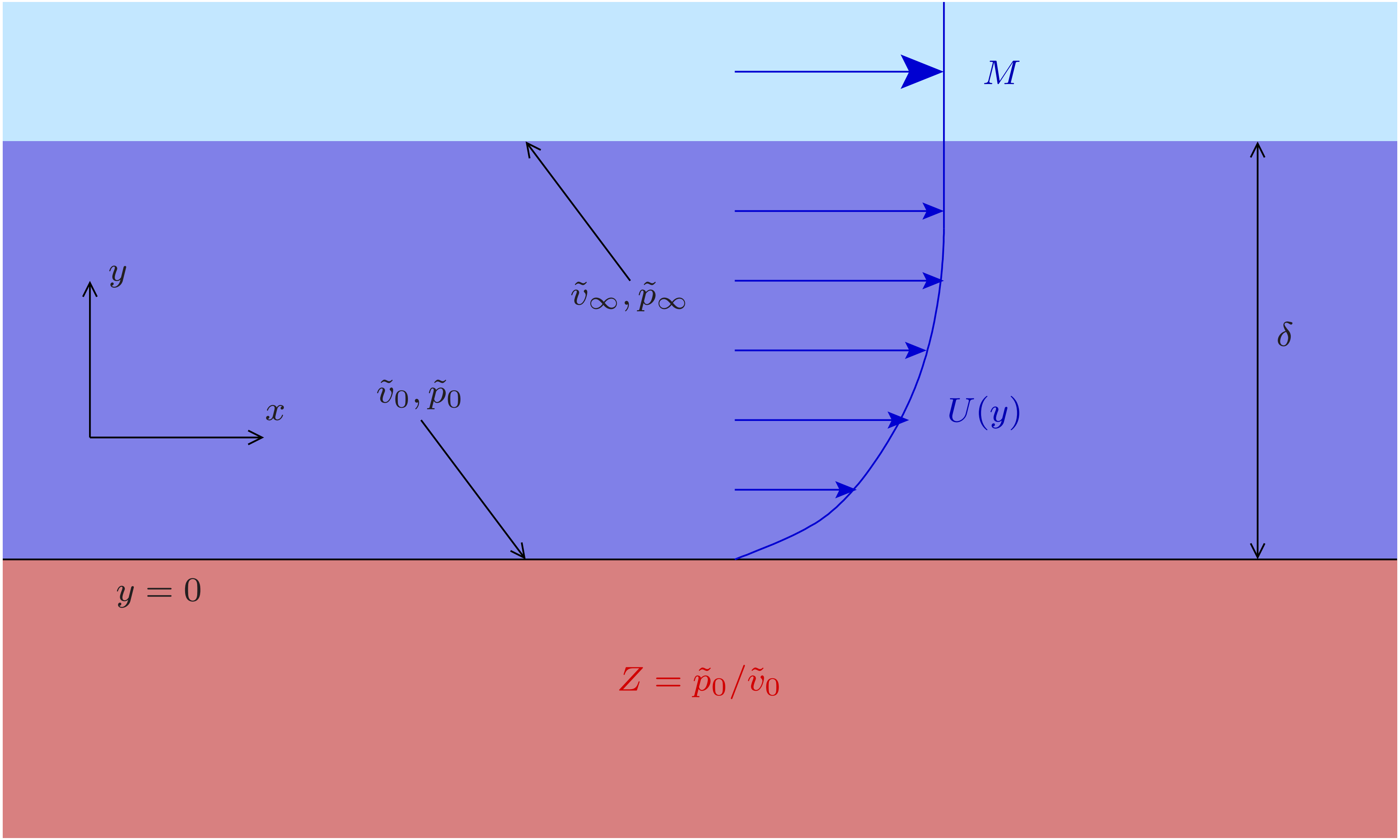
See Brambley (JCP 2016) for details.

Current research

- Can we do any better (reoptimize)?
- What about filtering?
- What about time-stepping?
- What about combined derivative/filtering/time-stepping?

Modelling Flow Instability over Acoustic Linings

Flow over an impedance surface



Viscous compressible acoustics in a cylinder

- Governing equations:

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{u}) = 0$$

$$\rho \frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} = -\boldsymbol{\nabla} p + \boldsymbol{\nabla} \cdot \sigma$$

$$\sigma_{ij} = \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \left(\mu^{\mathrm{B}} - \frac{2}{3}\mu \right) \delta_{ij} \boldsymbol{\nabla} \cdot \boldsymbol{u}$$

$$\rho \frac{\mathrm{D}T}{\mathrm{D}t} = \frac{\mathrm{D}p}{\mathrm{D}t} + \sigma_{ij} \frac{\partial u_i}{\partial x_j} + \boldsymbol{\nabla} \cdot (\kappa \boldsymbol{\nabla} T)$$

$$T = \frac{\gamma}{\gamma - 1} \frac{p}{\rho}$$

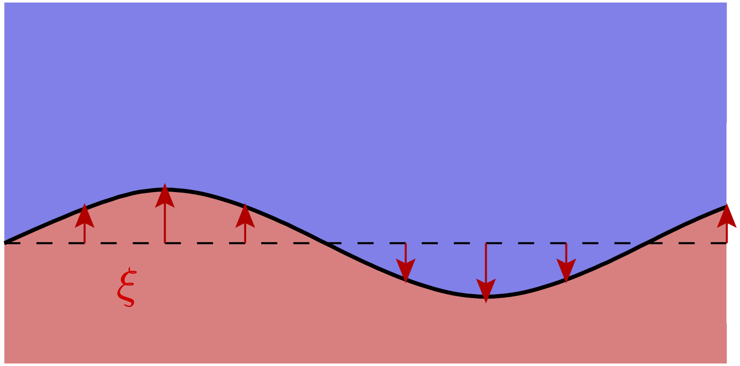
$\mu, \mu^{\mathrm{B}}, \kappa$ linear in T and independent of p.

- Expand as a steady parallel baseflow plus an acoustic perturbation. E.g.

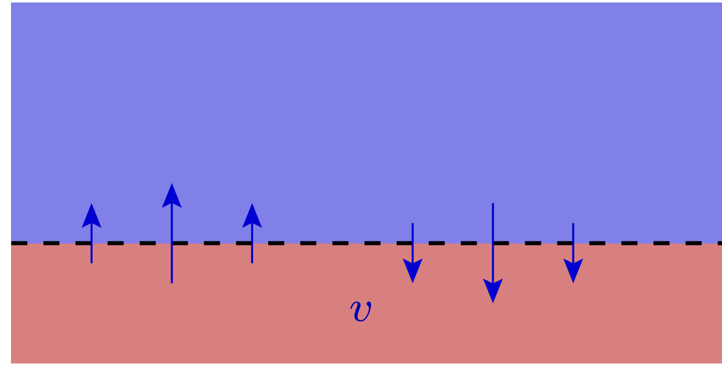
$$p(x, r, \theta, t) = p_0(r) + \tilde{p}(r) \exp\{\mathrm{i}\omega t - \mathrm{i}kx - \mathrm{i}m\theta\}$$

The impedance of a surface

Compliant



Permeable



- Suppose a boundary with velocity $v = \partial\xi/\partial t$ obeys

$$d\frac{\partial^2\xi}{\partial t^2} = -K\xi - R\frac{\partial\xi}{\partial t} + T\frac{\partial^2\xi}{\partial x^2} - B\frac{\partial^4\xi}{\partial x^4} + p.$$

- If $p = \tilde{p}\exp\{\mathrm{i}\omega t - \mathrm{i}kx\}$ and $v = \tilde{v}\exp\{\mathrm{i}\omega t - \mathrm{i}kx\}$,

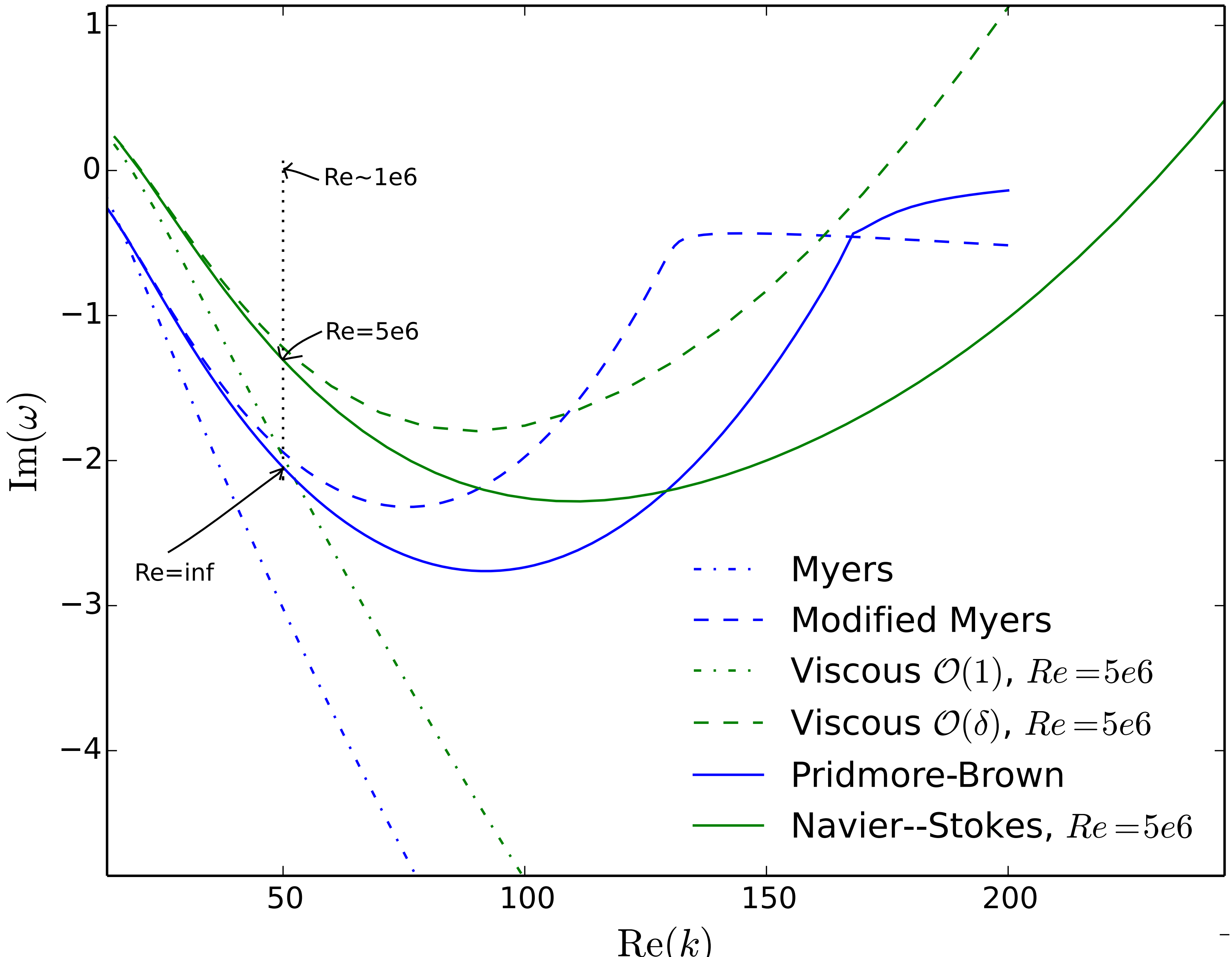
$$\frac{\tilde{p}}{\tilde{v}} = Z = R + \mathrm{i}\left(d\omega - \frac{K}{\omega} - \frac{Tk^2}{\omega} - \frac{Bk^4}{\omega}\right)$$

- Setting bending stiffness B and tension T to zero gives a mass–spring–damper model.
  - No k dependence: “locally reacting”.

- For the Extended Helmholtz Resonator (EHR) model (Rienstra, 2006 AIAA Paper),

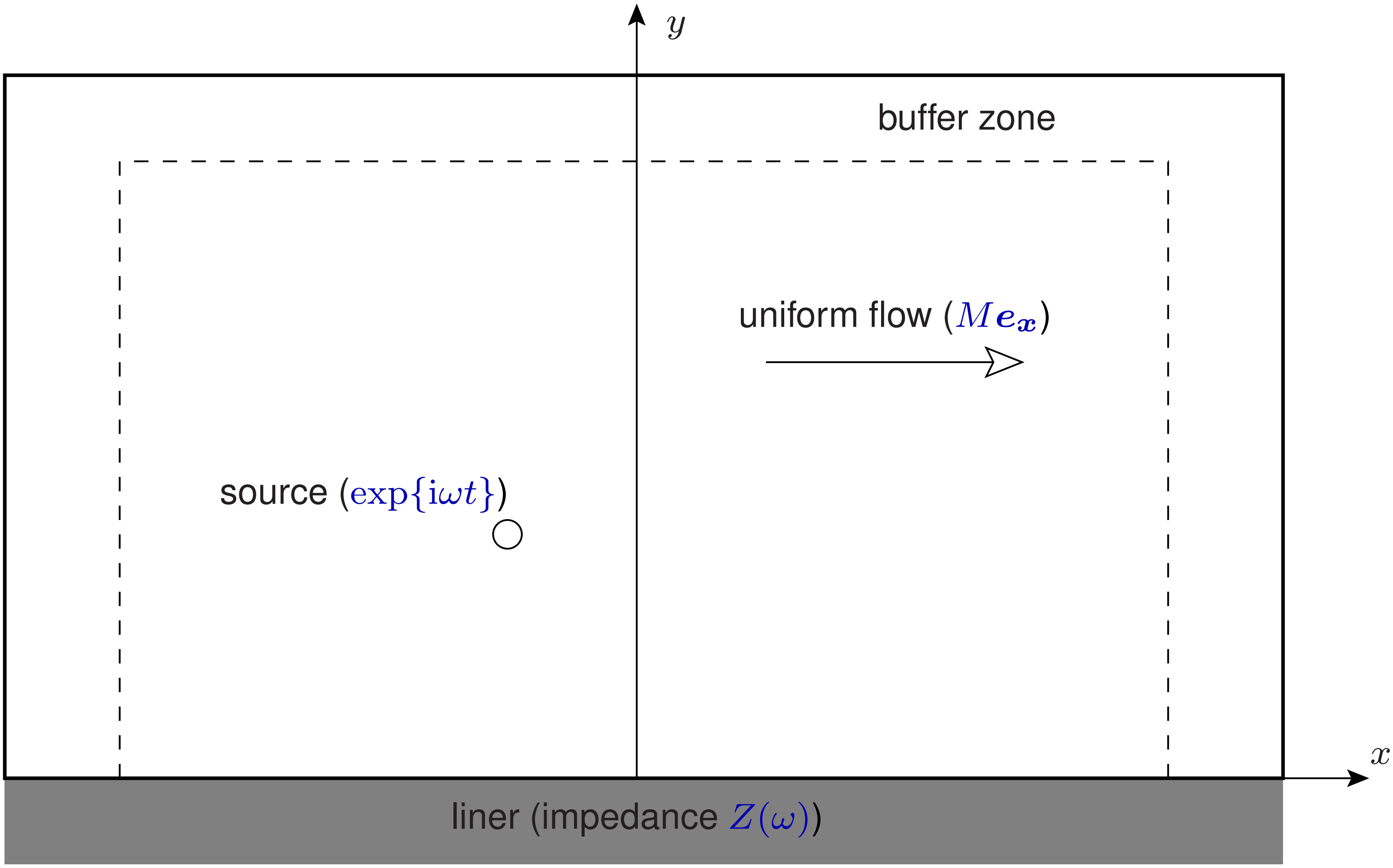
$$Z = R + \mathrm{i}d\omega - \mathrm{i}\nu\cot(\omega L - \mathrm{i}\varepsilon/2).$$

Initial value problem (given k, find ω)



Simulations of inviscid lining instability

2D test case



y
buffer zone
uniform flow ($M\boldsymbol{e_x}$)
source ($\exp\{i\omega t\}$)
x
liner (impedance $Z(\omega)$)

Analytic solution (Brambley & Gabard, 2014 JSV)

$$p = p_0 + p_{\mathrm{dir}} + p_{\mathrm{refl}}$$

$$p_{\mathrm{dir}}(x,y,t;y_s) = \mathrm{e}^{\mathrm{i}\omega t}\int_{-\infty}^{\infty}\frac{(\omega - Mk)}{4\pi\alpha}\mathrm{e}^{-\mathrm{i}kx-\mathrm{i}\alpha|y-y_s|}\,\mathrm{d}k,$$

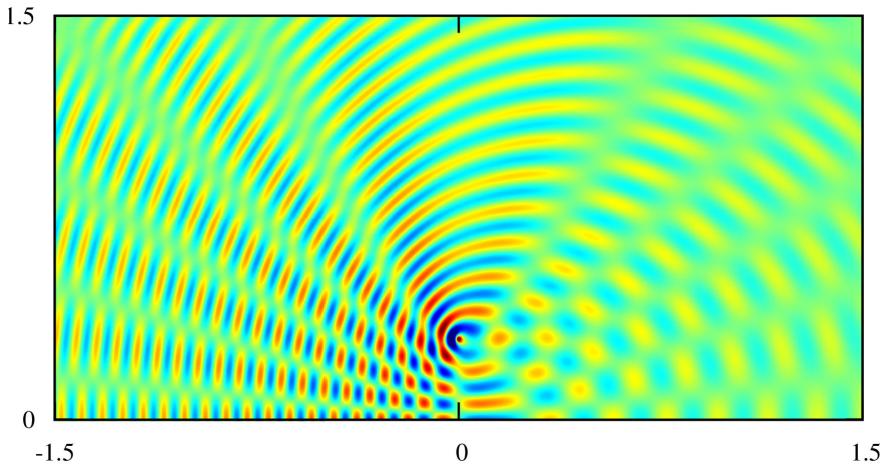
$$= \frac{\omega}{4\beta^3}\exp\left\{\mathrm{i}\omega\left(t + Mx/\beta^2\right)\right\}\left[H_0^{(2)}\left(\omega r/\beta^2\right) + \frac{\mathrm{i}Mx}{r}H_1^{(2)}\left(\omega r/\beta^2\right)\right]$$

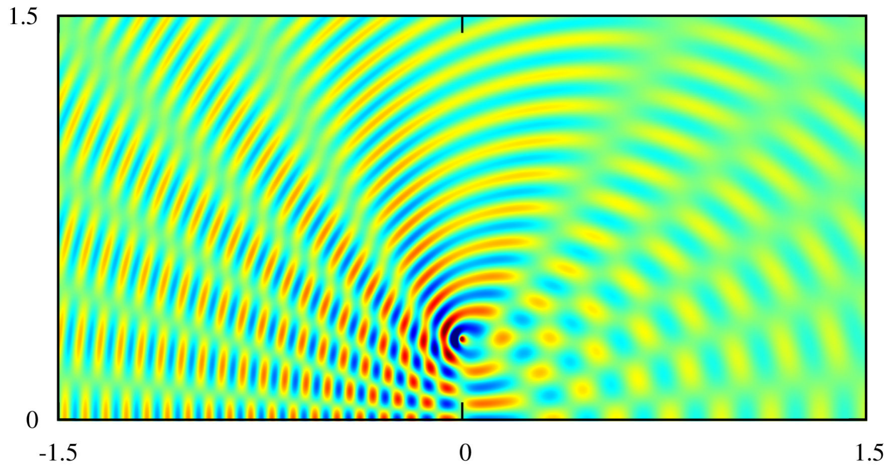
$$p_{\mathrm{refl}}(x,y,t;y_s) = \mathrm{e}^{\mathrm{i}\omega t}\int_{-\infty}^{\infty}\frac{\alpha\omega Z - (\omega - Mk)^2}{\alpha\omega Z + (\omega - Mk)^2}\frac{(\omega - Mk)}{4\pi\alpha}\mathrm{e}^{-\mathrm{i}kx-\mathrm{i}\alpha|y+y_s|}\,\mathrm{d}k$$

Where

$$\beta^2 = 1 - M^2 \qquad\qquad r^2 = x^2 + \beta^2(y-y_s)^2.$$

Numerical vs Analytical: Hard wall





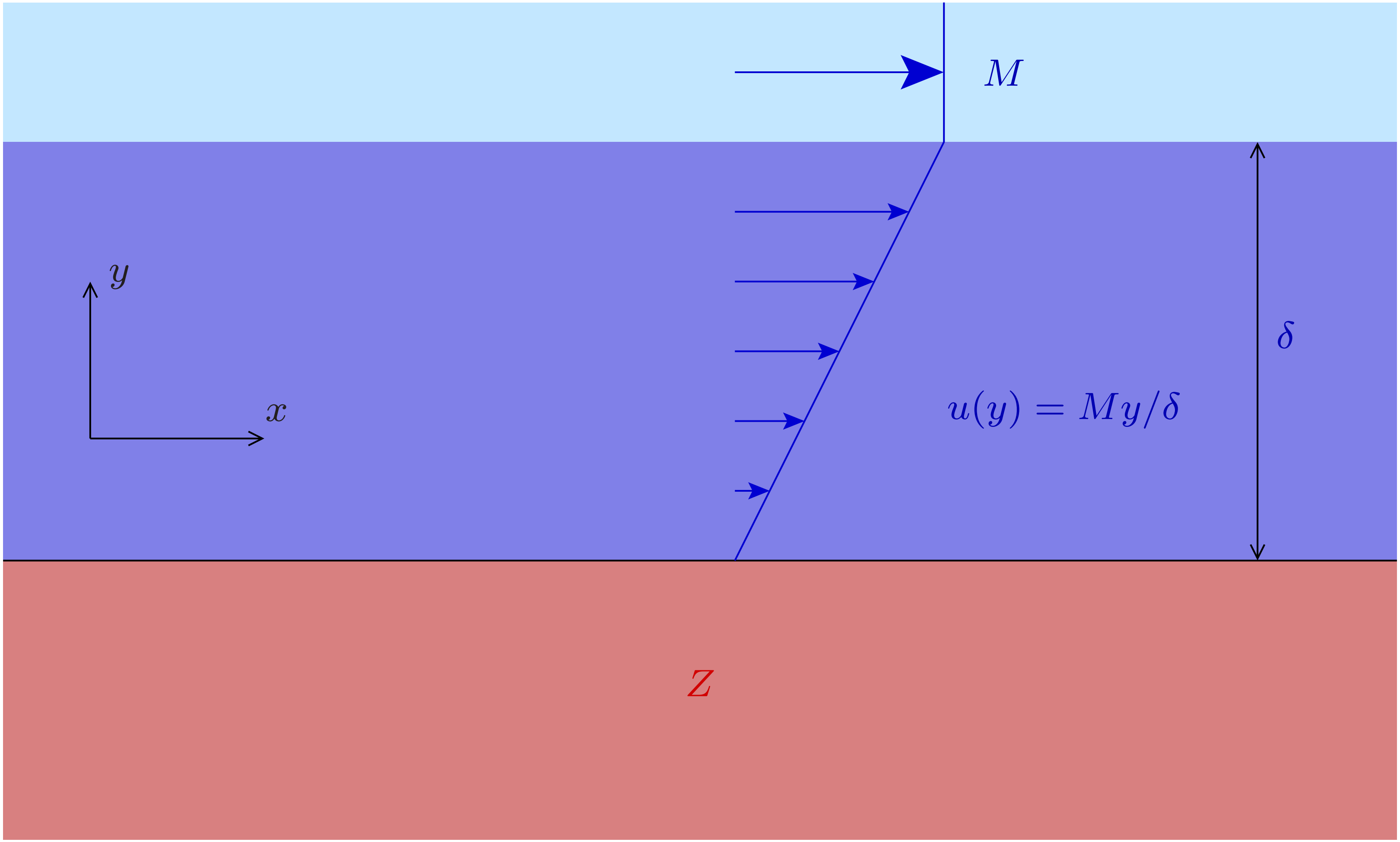
Both figures use the same colour scale:

- Left: Numerics (*entire* domain) for p at time $t = 32$ (64,000 time steps).
- Right: Analytic result for $t = \infty$.

Inviscid boundary condition in the time-domain

- For a linear-velocity constant-density boundary layer,

$$\mathrm{i}\omega v = \mathrm{i}(\omega - Mk)\frac{p}{Z} + \delta Mk(\omega - \tfrac{2}{3}Mk)\frac{v}{Z} - \delta\frac{Mk^3 p}{\omega - Mk} + O(\delta^2)$$



Inviscid boundary condition in the time-domain

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- Similarly, $v/Z = \nu$ satisfies the same equation but forced by v not p. E.g.

$$\frac{\partial \hat{\nu}}{\partial t} = \frac{1}{d}\big[-K\hat{\eta} - R\hat{\nu} - \hat{v} \big] \qquad \frac{\partial \hat{\eta}}{\partial t} = \hat{\nu}$$

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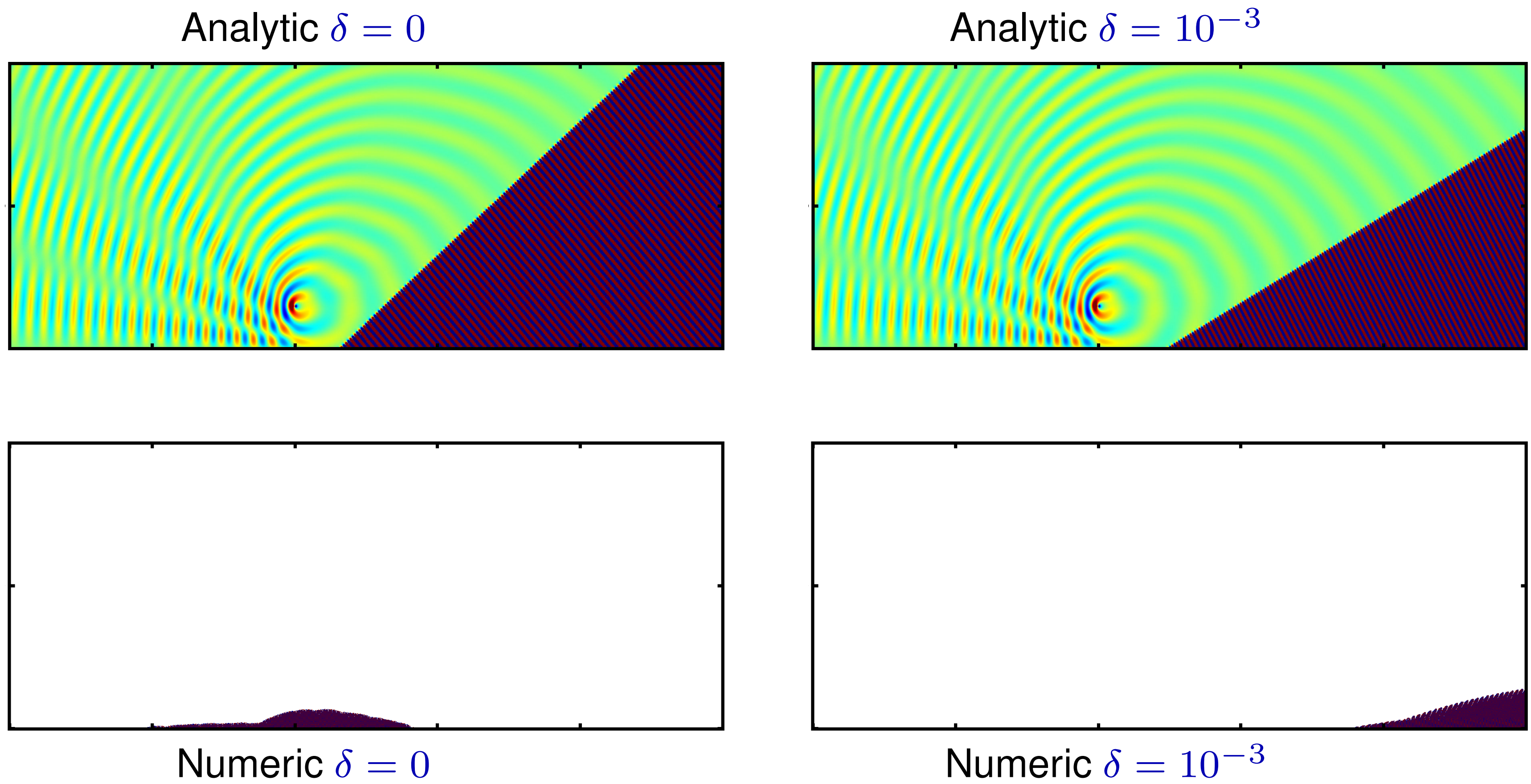
- Finally, the time-domain boundary condition becomes

$$\frac{\partial \hat{v}}{\partial t} = \left(\frac{\partial}{\partial t} + M\frac{\partial}{\partial x}\right)\hat{v_s} + \delta M\left[\left(\frac{\partial}{\partial t} + \tfrac{2}{3}M\frac{\partial}{\partial x}\right)\frac{\partial \hat{\nu}}{\partial x} + \frac{\partial^2 \hat{u}}{\partial x^2}\right]$$

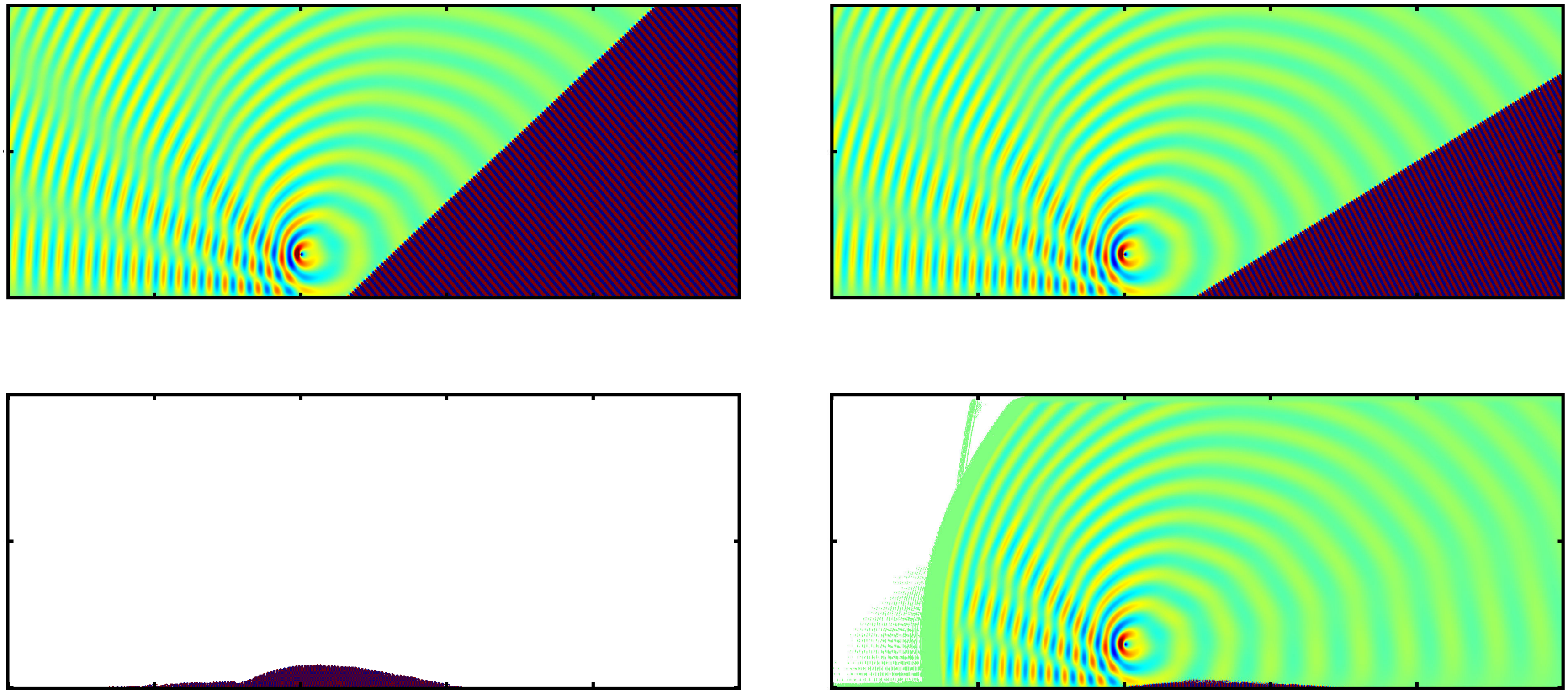
Comparison



- $\omega = 31$, $M = 0.5$, mass–spring–damper impedance with $d = 0.01$, $K = 10$ and $R = 0.75$.
- Numerics has $\Delta x = 2.5 \times 10^{-3}$ and $\Delta t = 1.5 \times 10^{-3}$.

Comparison with filtering

Analytic $\delta = 0$ | Analytic $\delta = 10^{-3}$



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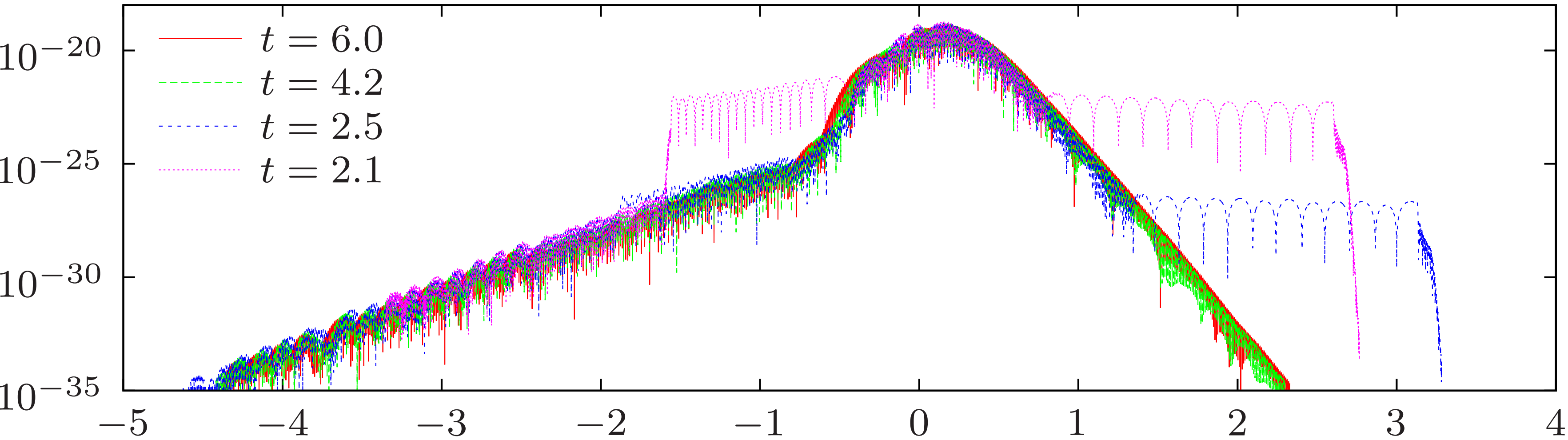
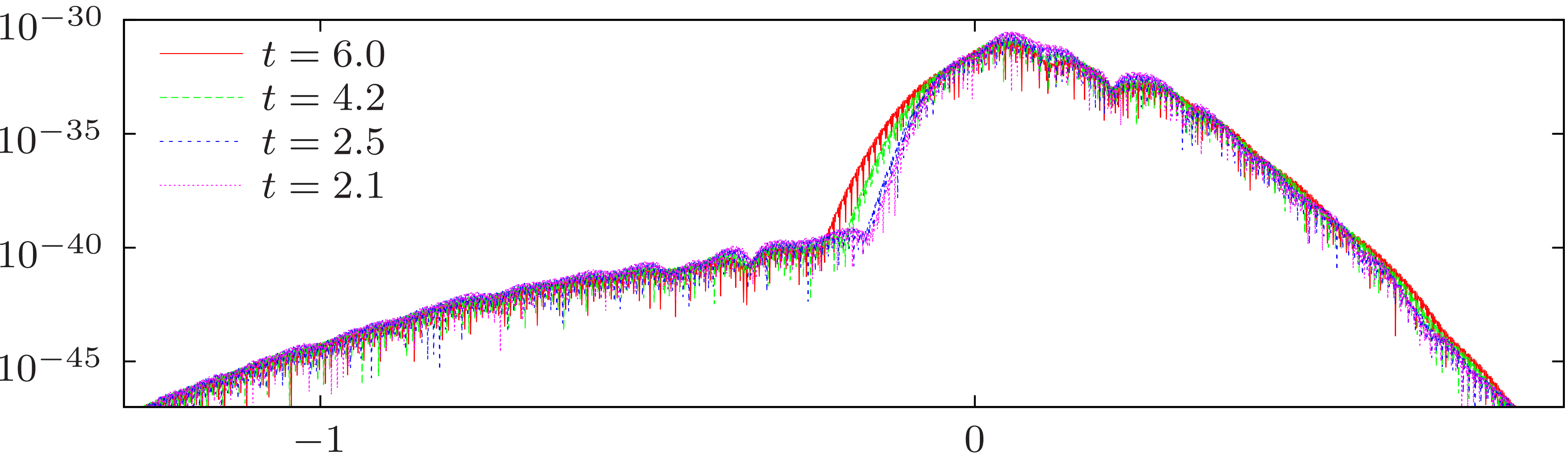
Temporal evolution of a convecting instability

(a) $\delta = 10^{-3}$, $U = 0.25$, $G = 23.5$

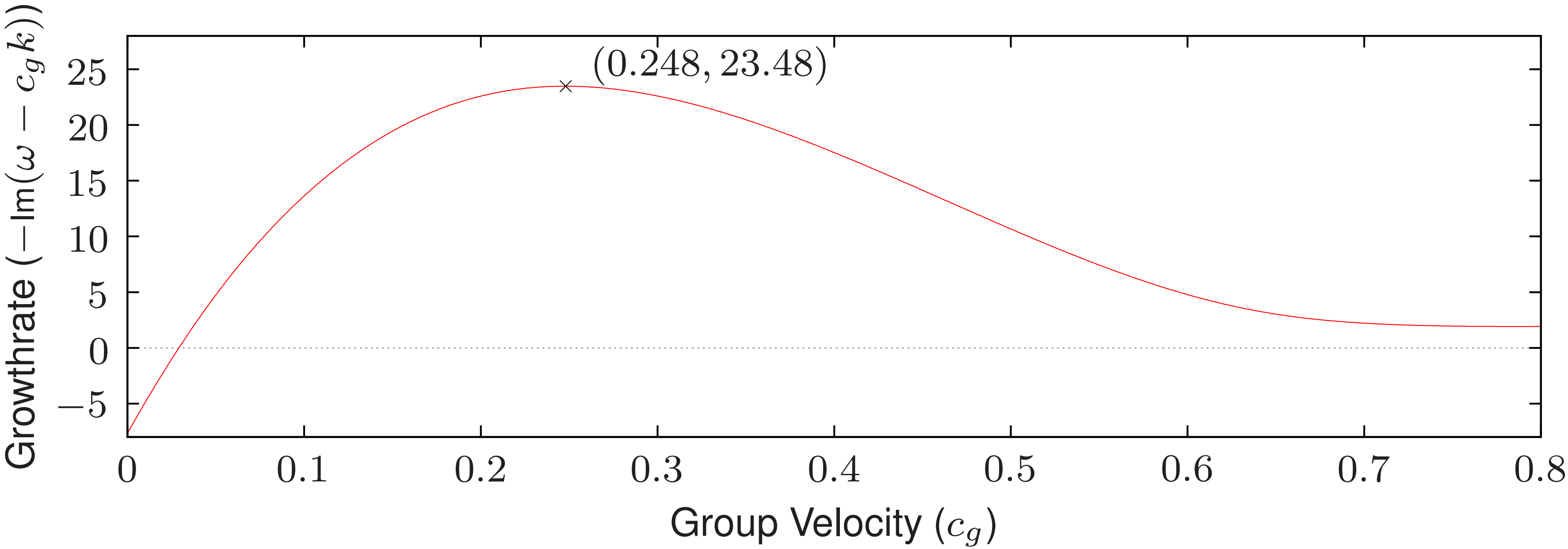


(b) $\delta = 0$, $U = -0.01$, $G = 121$



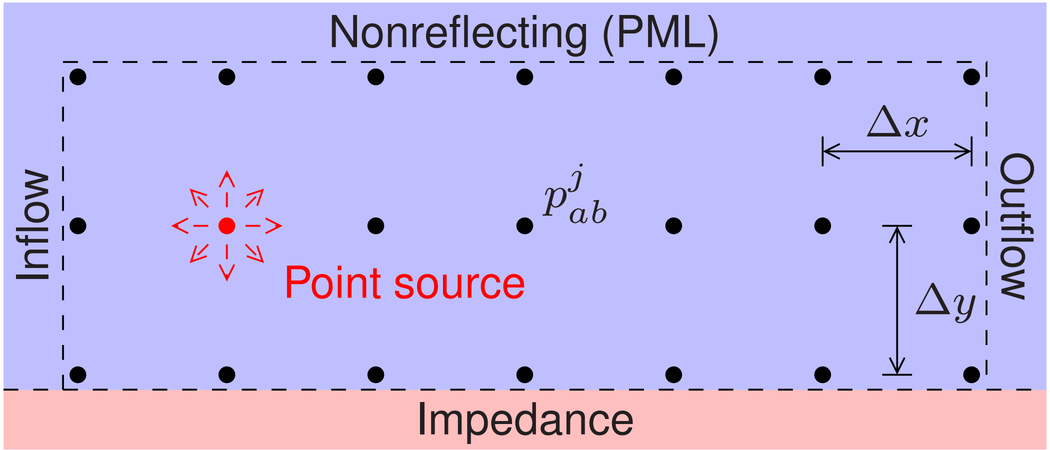
- Plots of $p(x + Ut, 0, t)e^{-Gt}$.

Theoretical predictions of temporal convective instabilities



Discrete dispersion analysis

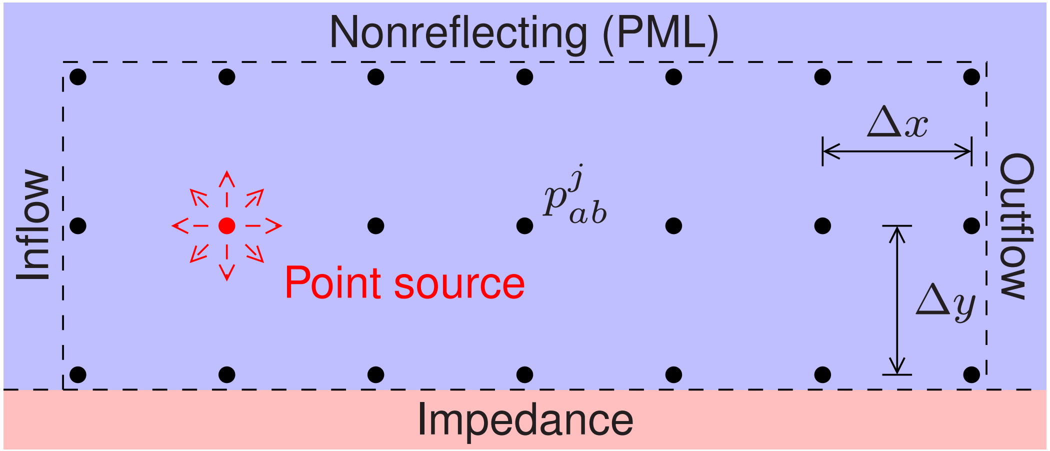
Time-domain numerics



- Discretize in x, y and t: $p^j_{ab} = p(a\Delta x, b\Delta y, j\Delta t)$.
- Given solution at $t = j\Delta t$, timestep forward to $j + 1$.
- Solve Linearized Euler Equations (LEE) in conservative form with a point mass source.
- Uses 7-point 4th order centered spatial derivatives.
- Uses 6-stage 4th order Runge–Kutta timestepping.
- Uses 11-point spatial filtering.
- Uses non-reflecting top boundary condition (Perfectly Matched Layers, PML).
- Uses nonreflecting or periodic inflow and outflow.

Discrete dispersion analysis

Time-domain numerics



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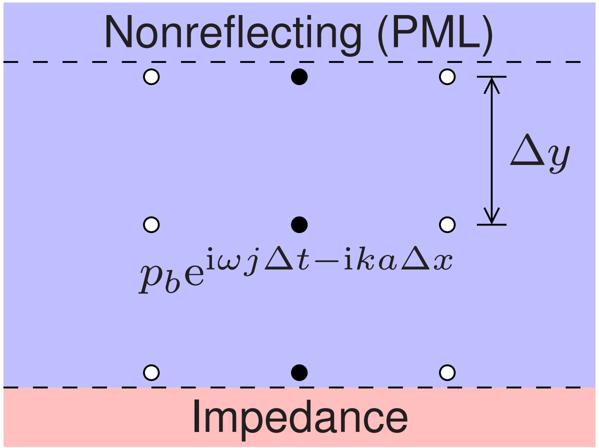
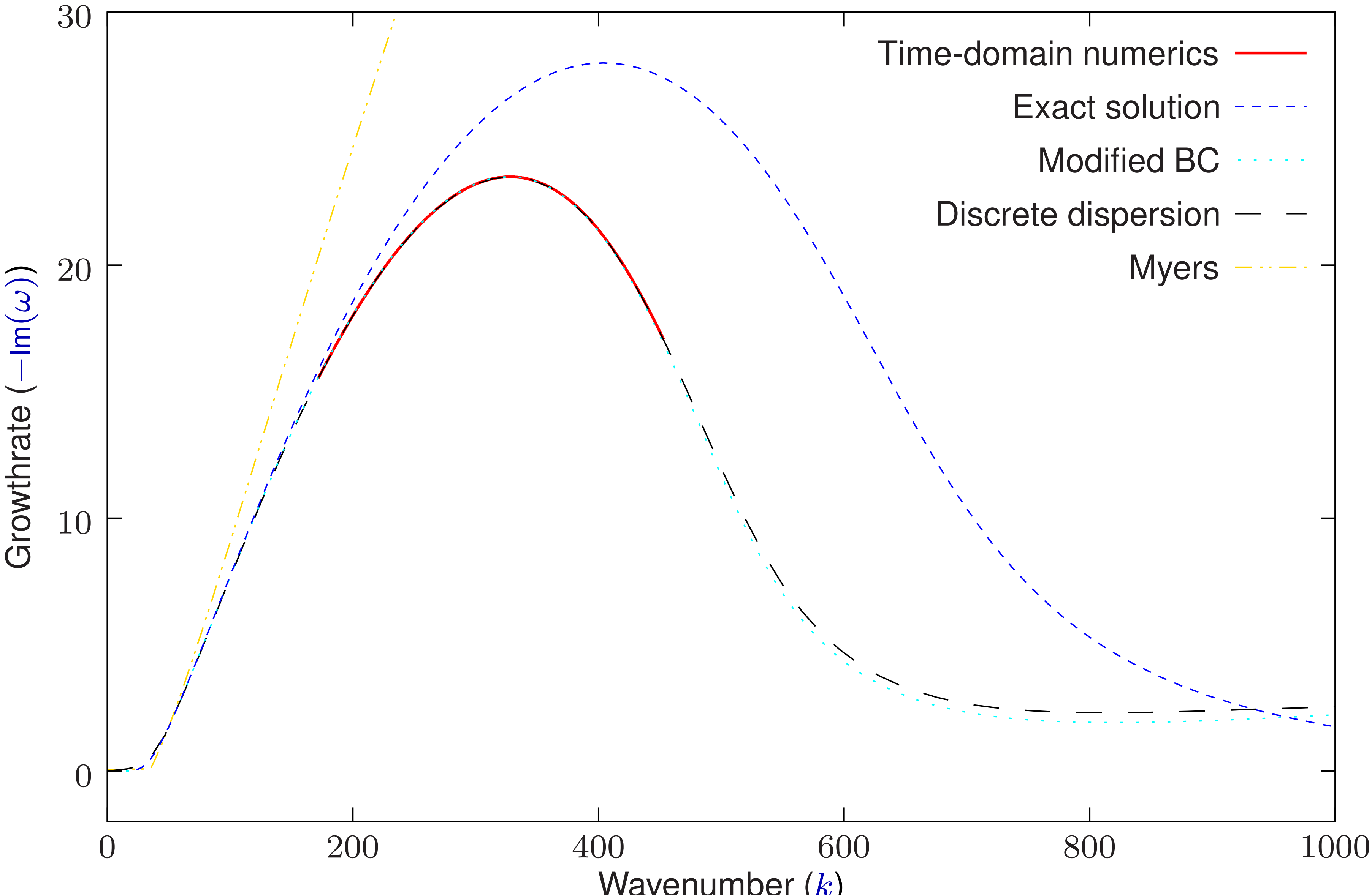


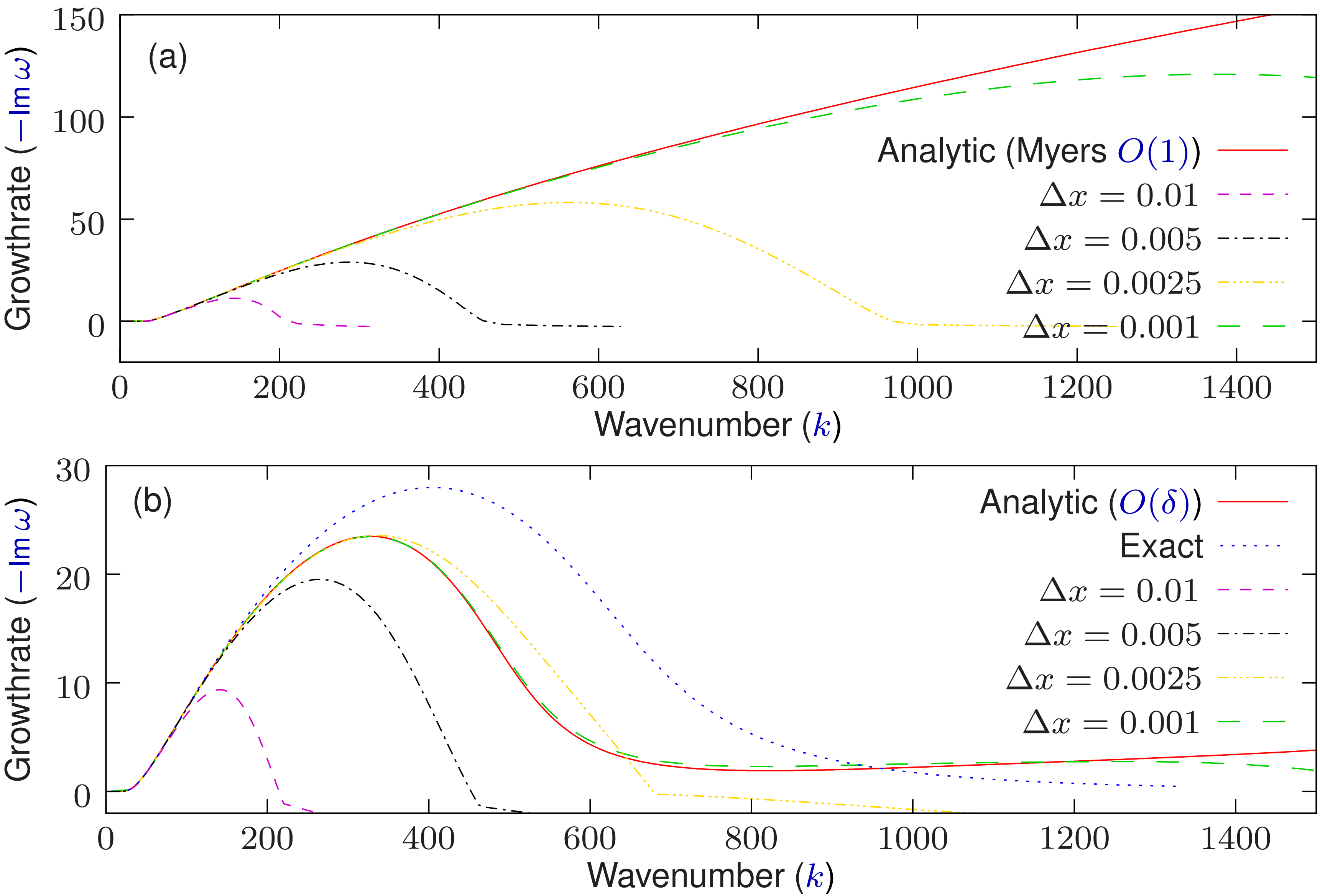
- Discretize in y only.
- Solve an eigenvalue problem for $\omega(k)$ or $k(\omega)$.
- Same governing equation.
- Same spatial derivatives.
- Same temporal evolution.
- Same filtering.
- Same top and bottom boundary conditions.
- No equivalent of inflow or outflow.

Comparison

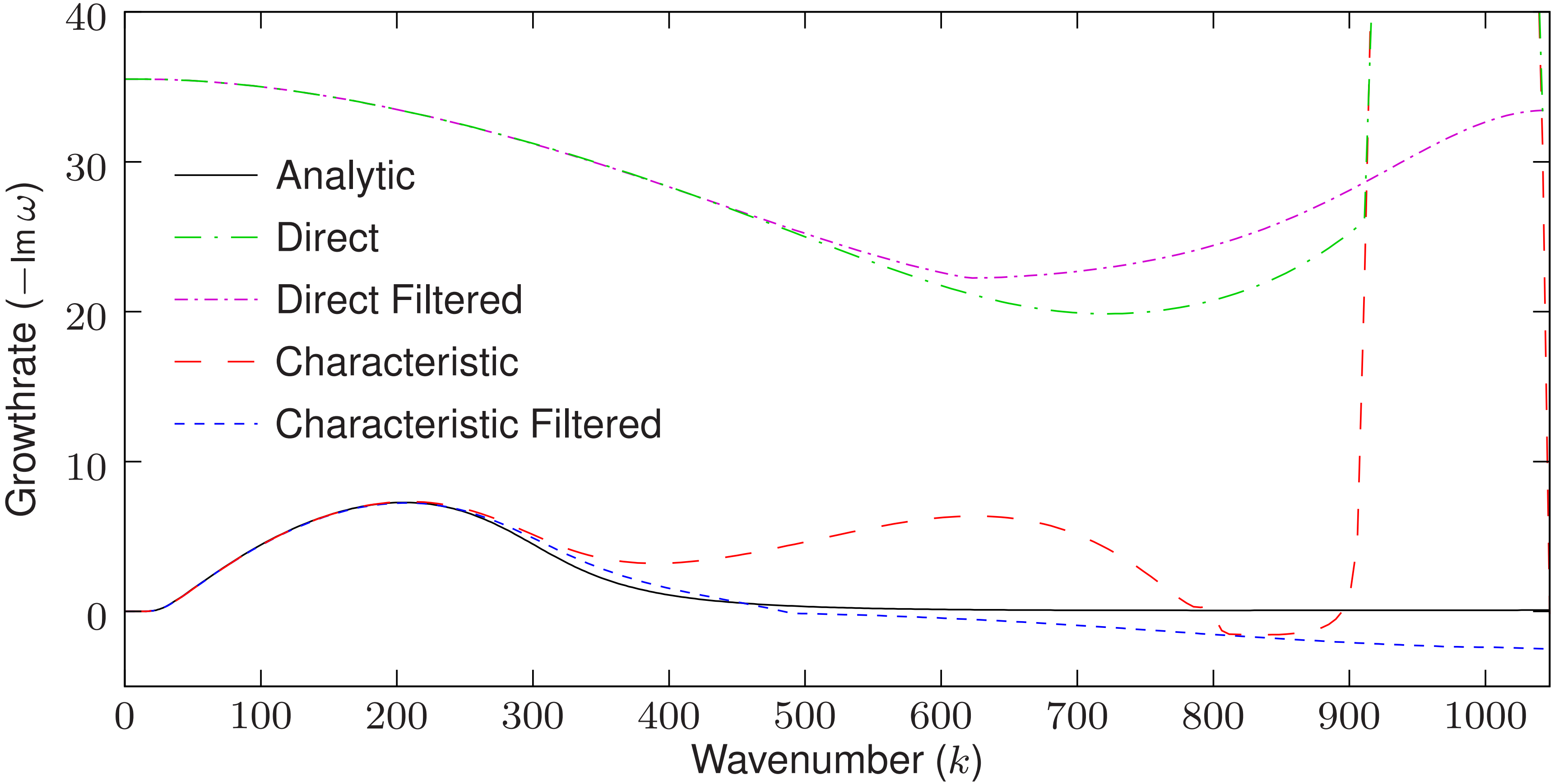


Time-domain numerics
Exact solution
Modified BC
Discrete dispersion
Myers
Growthrate ($-\text{Im}(\omega)$)
Wavenumber (k)
0
10
20
30
0
200
400
600
800
1000

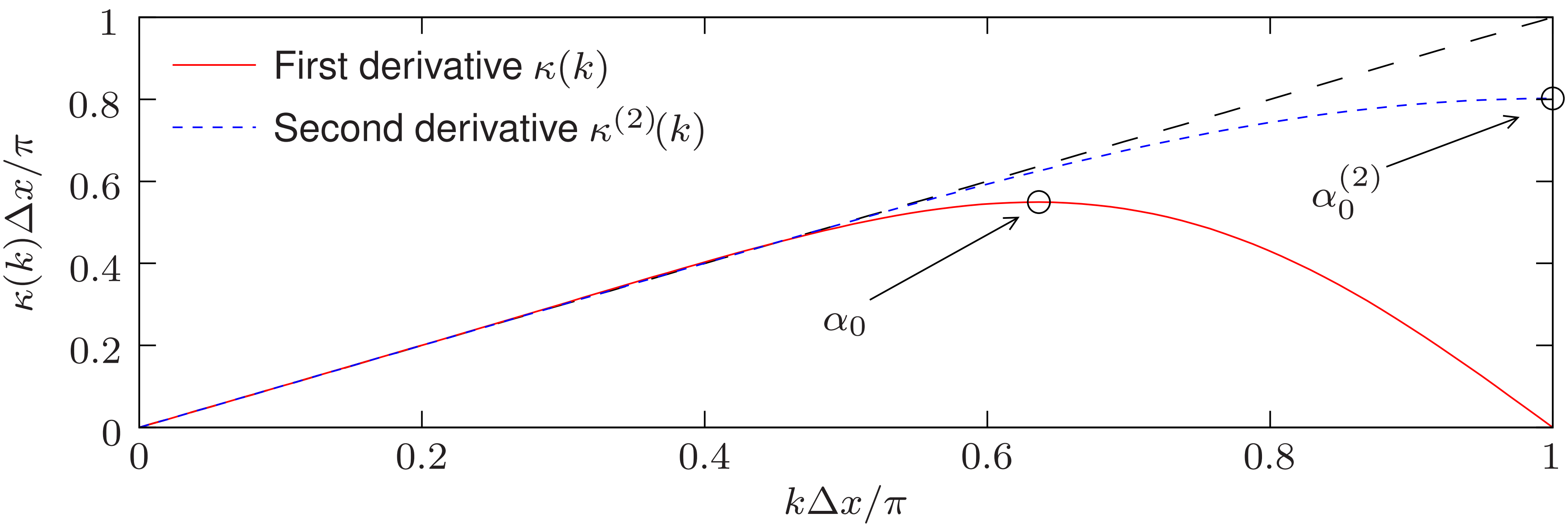
Discrete dispersion analysis for varying grid spacing



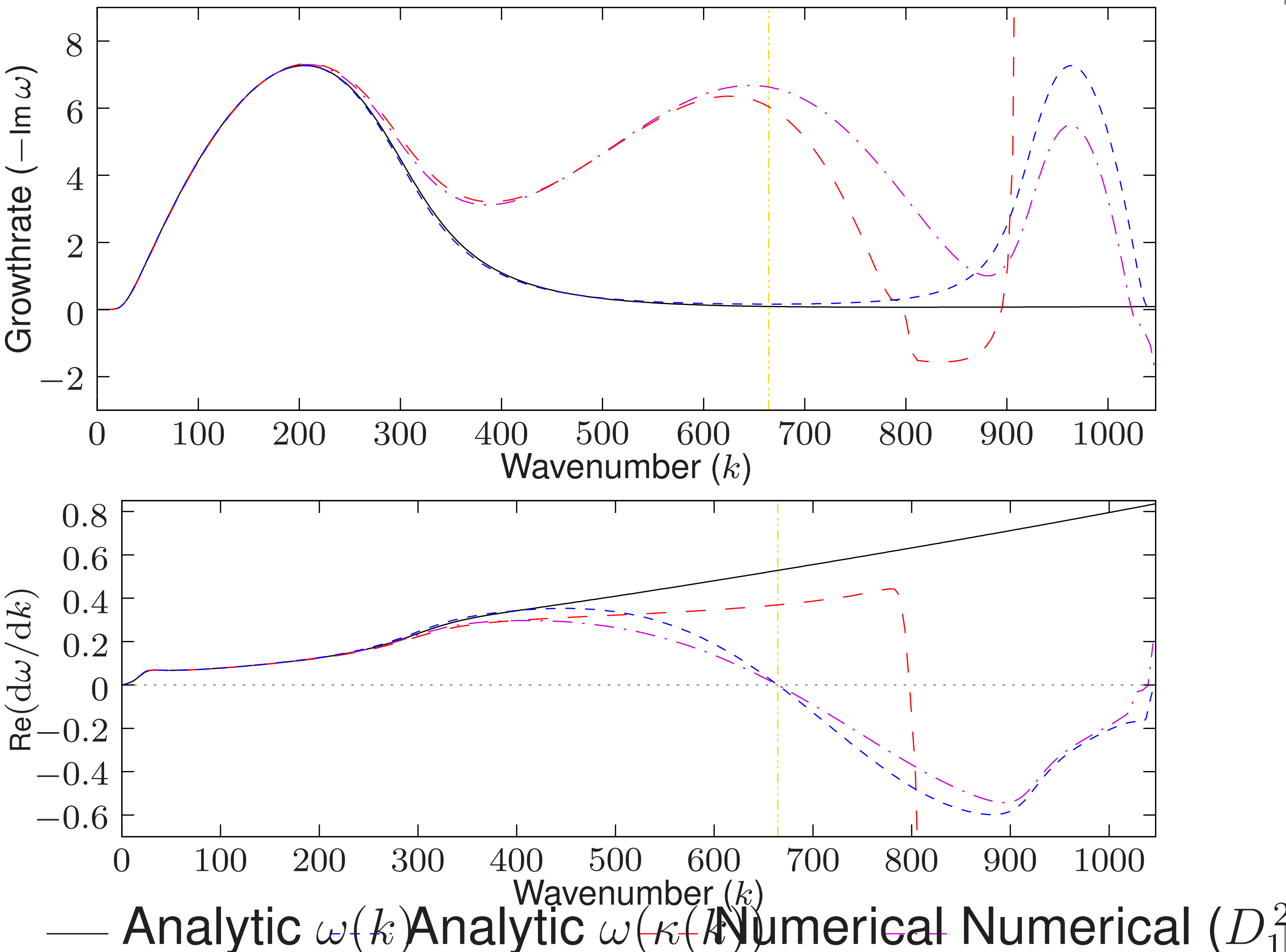
Discrete dispersion and filtering



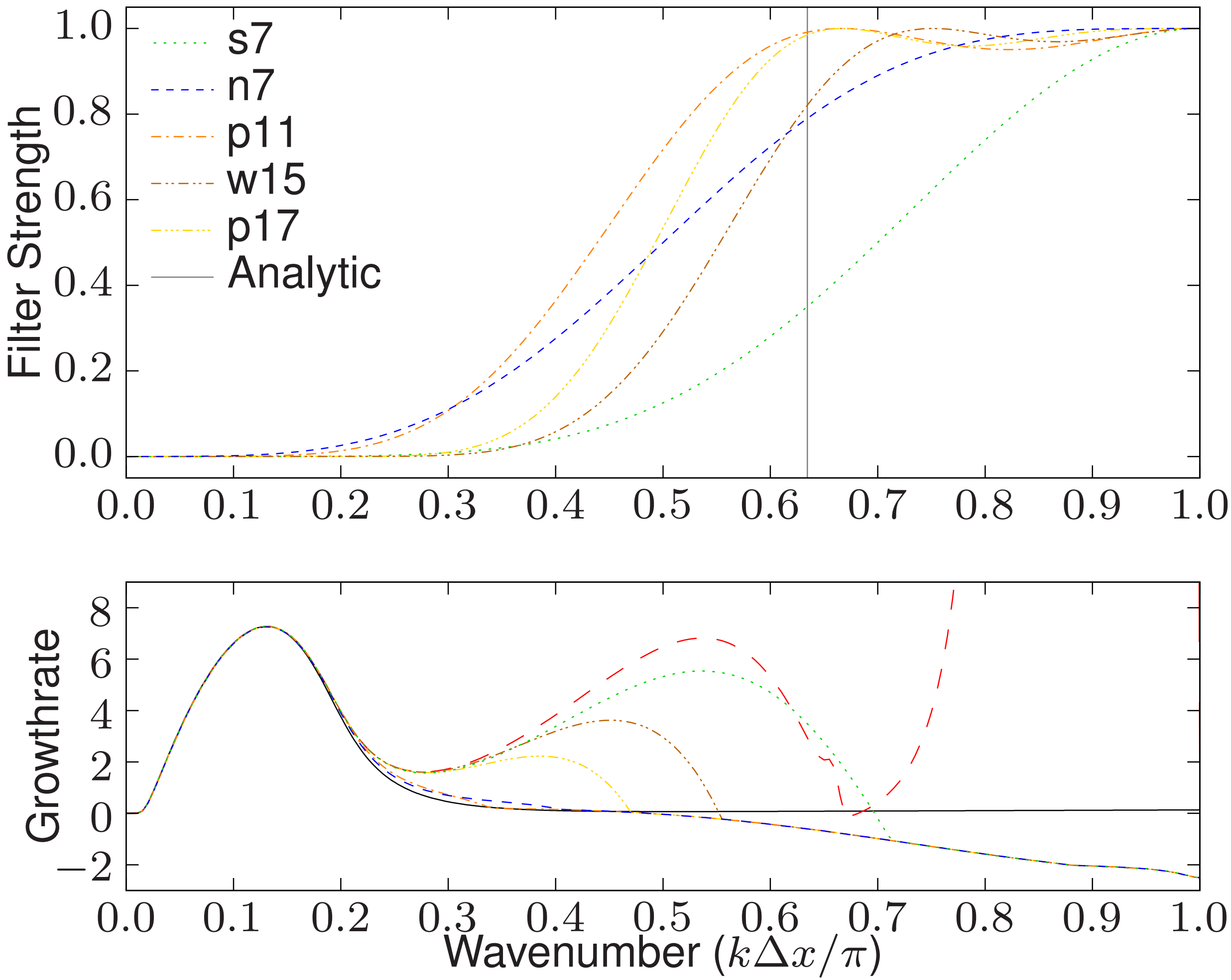
Artificial zero numerical group velocity



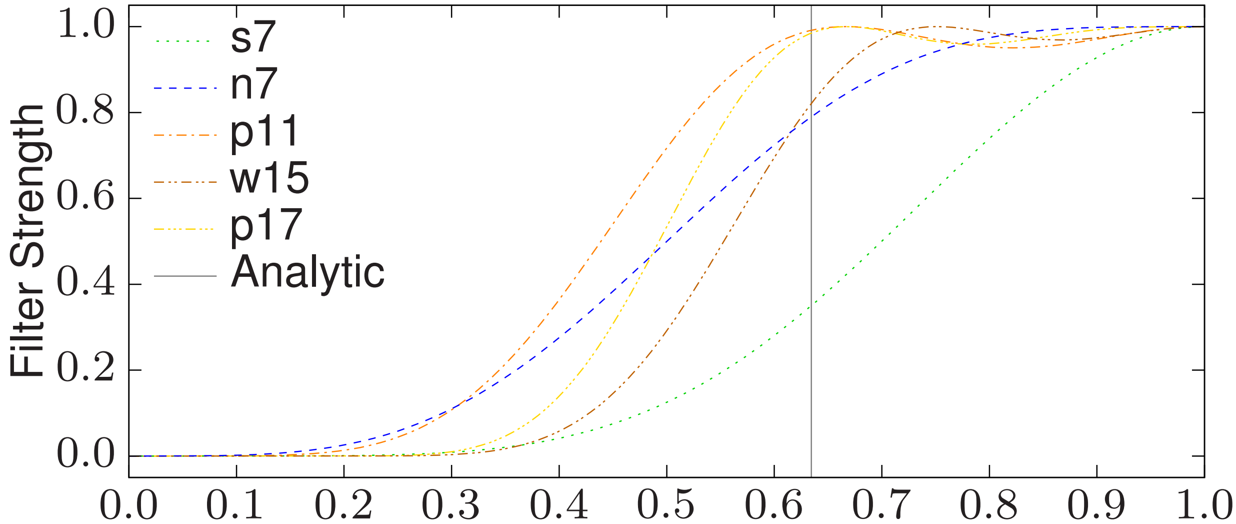
Effect of zero group velocity

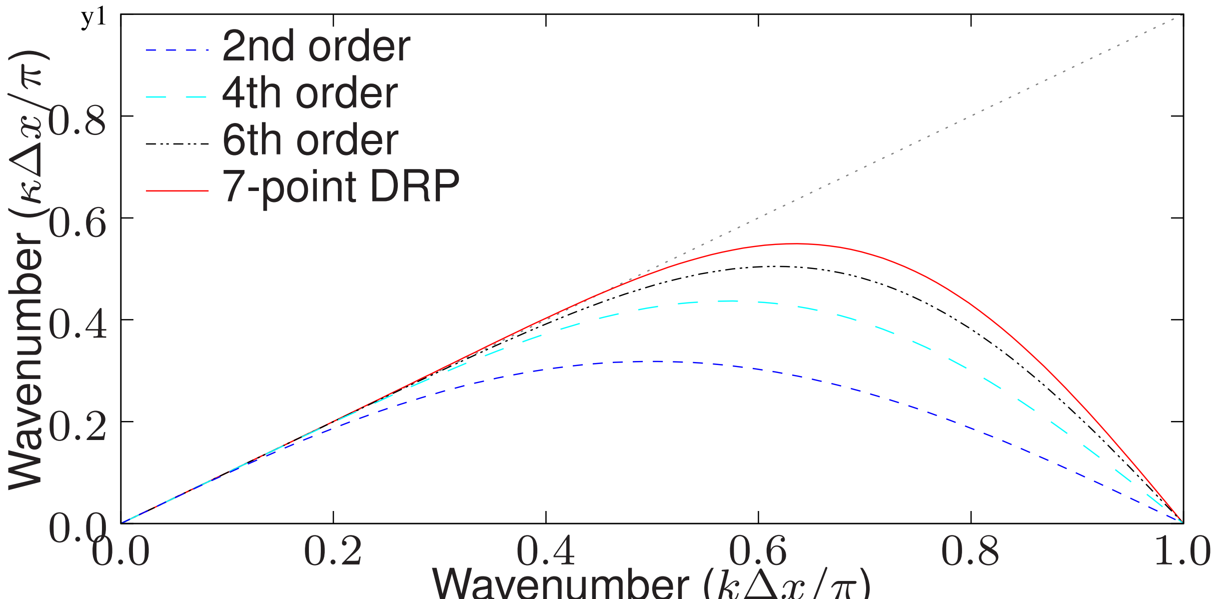


Effect of filtering



Effect of filtering





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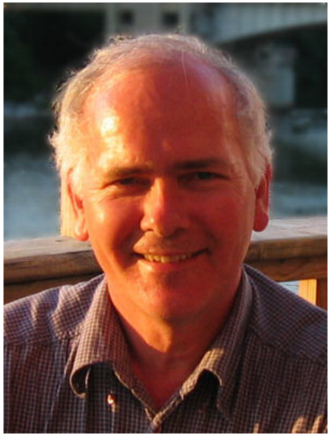

Prof. Sjoerd Rienstra
Maths Department
Eindhoven University of Technology

Conclusions

- Optimization of finite differences for constant-amplitude waves gives poor results for non-constant-amplitude waves.
  - Open question: can we do better for non-constant-amplitude waves than maximal order?
- Getting the numerics correct for unstable linear systems takes great care.
  - The artificial zero group velocity combines with convective instability to give absolute instability.
  - We can now justify the correct level of filtering necessary to get correct results.
  - Attempts at capping growth by including nonlinear terms have so far failed. Why?
- Other effects:
  - Can we do better by simultaneously designing derivatives and filters?
  - What is the effect of the time-stepping used?
- Why does this matter?
  - Simulation of an actual aircraft engine's acoustics might have regions where instabilities are present, even if overall the solution is bounded.
  - Designers would like to use computations to optimize liner effectiveness, correctly avoiding (or including) instability.
  - Other simulation techniques (e.g. LES) are too dissipative at present to correctly predict acoustics. (Perturbations of order 10^{-6} are loud!)