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# Uncertainty Quantification and Aircraft Engines

*F Montomoli*
*UQLab, Dept of Aeronautics*

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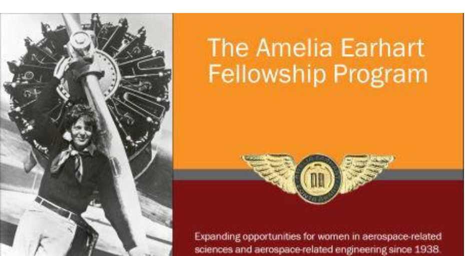
# UQLab

- **People**: 1 RAEng Fellow, 2+1 Post Docs (1 to be hired), 6+1 PhD students, 1 Academic

**Prizes:**

- **Audrey**: Amelia Earhart Fellowship, worldwide prize, one of the best 32 females worldwide in aviation
- **Marco**: STEM for Britain selected at UK Parliament as one of best UK researches, Take AIM second place
- **Richard**: Francis Prize as best PhD student of Imperial College
  EPSRC Fellowship Award best research, RAEng Fellowship




STEM for BRITAIN

EPSRC
Engineering and Physical Sciences Research Council

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# Facilities


**One of Largest Wind Tunnels in EU**

**Computing Facilities ☺**
**(below P Vincent, Aero)**

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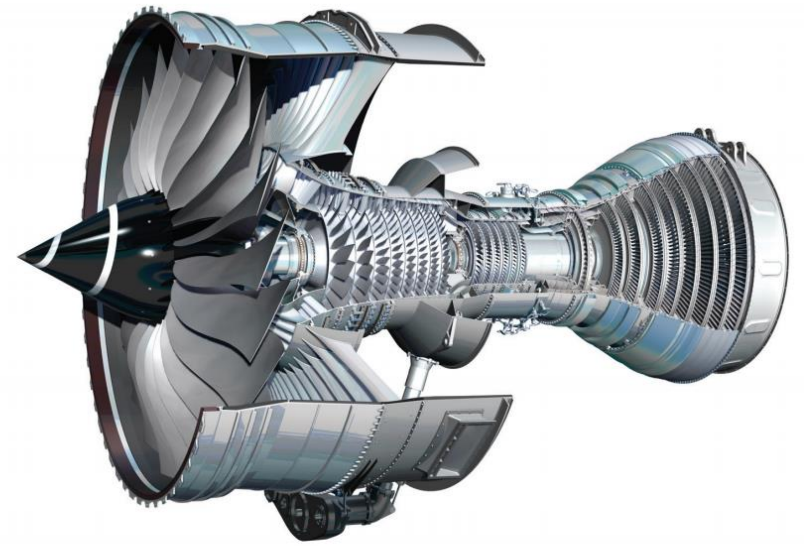
# Aircraft Engines



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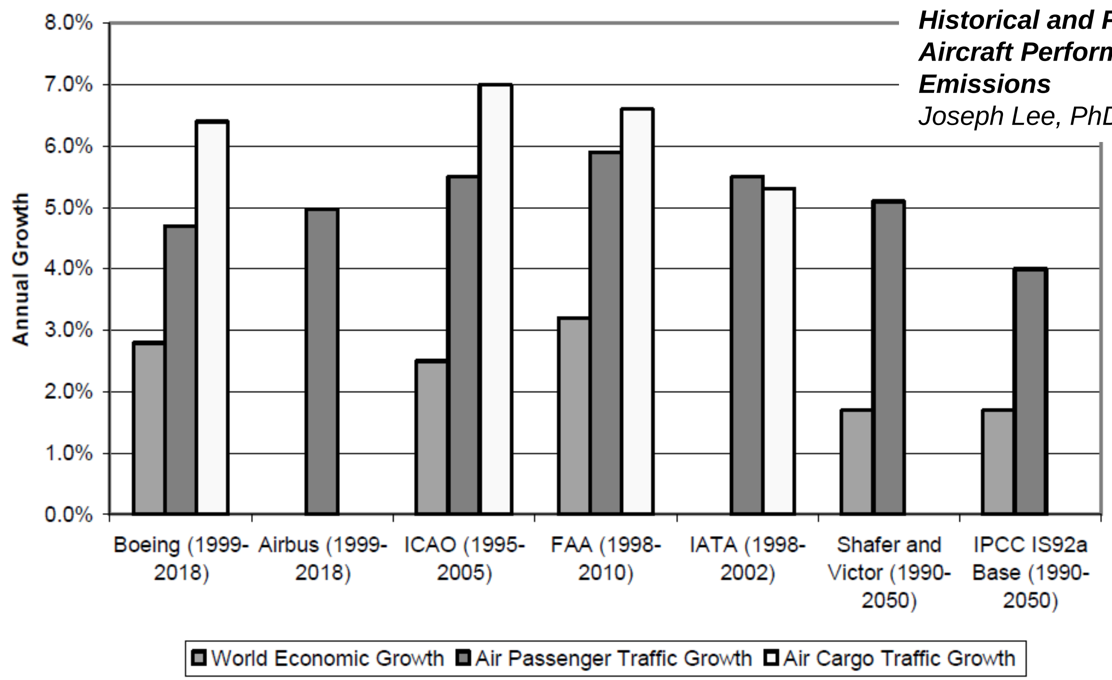
# Problem

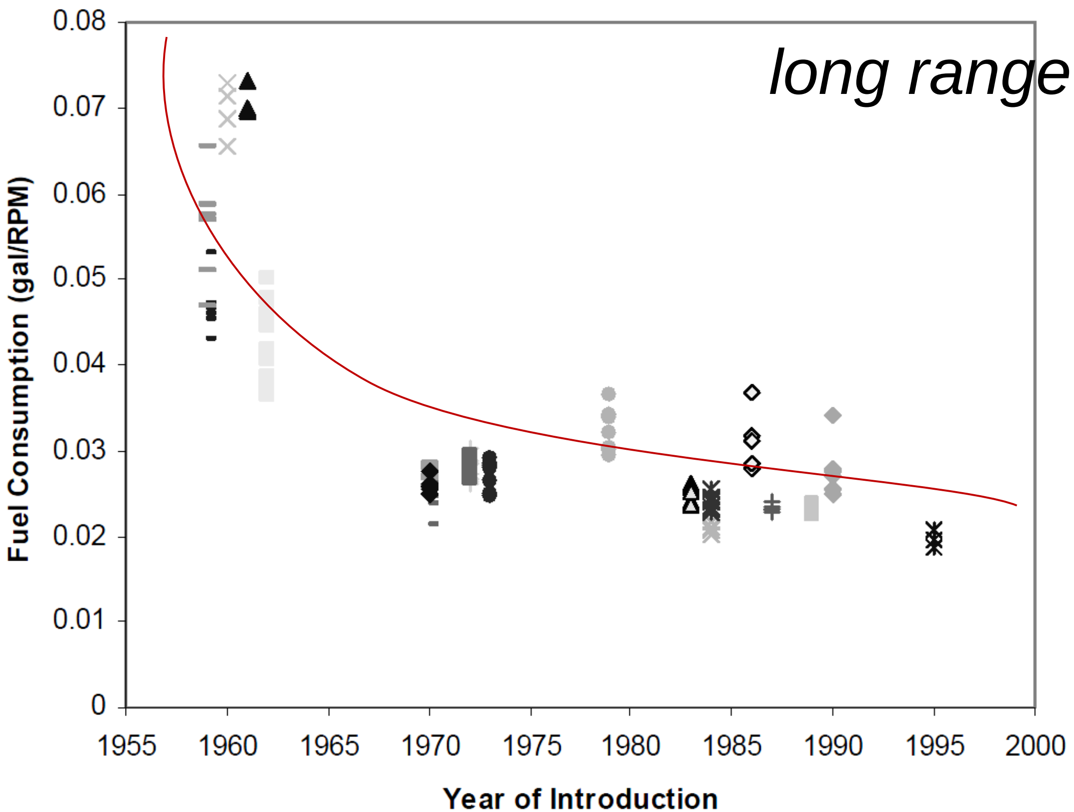
- Civil Aviation: 2% $CO_2$ overall emissions (ACARE 2050)
- Civil Aviation EU emissions: +87% from 1990 to 2006
- How to improve the performance of the engines?




***Historical and Future Trends in Aircraft Performance, Cost, and Emissions***
*Joseph Lee, PhD thesis*

# History

- in 50 Years reduction of 70% emissions per passenger
- In 50 years 70% quieter



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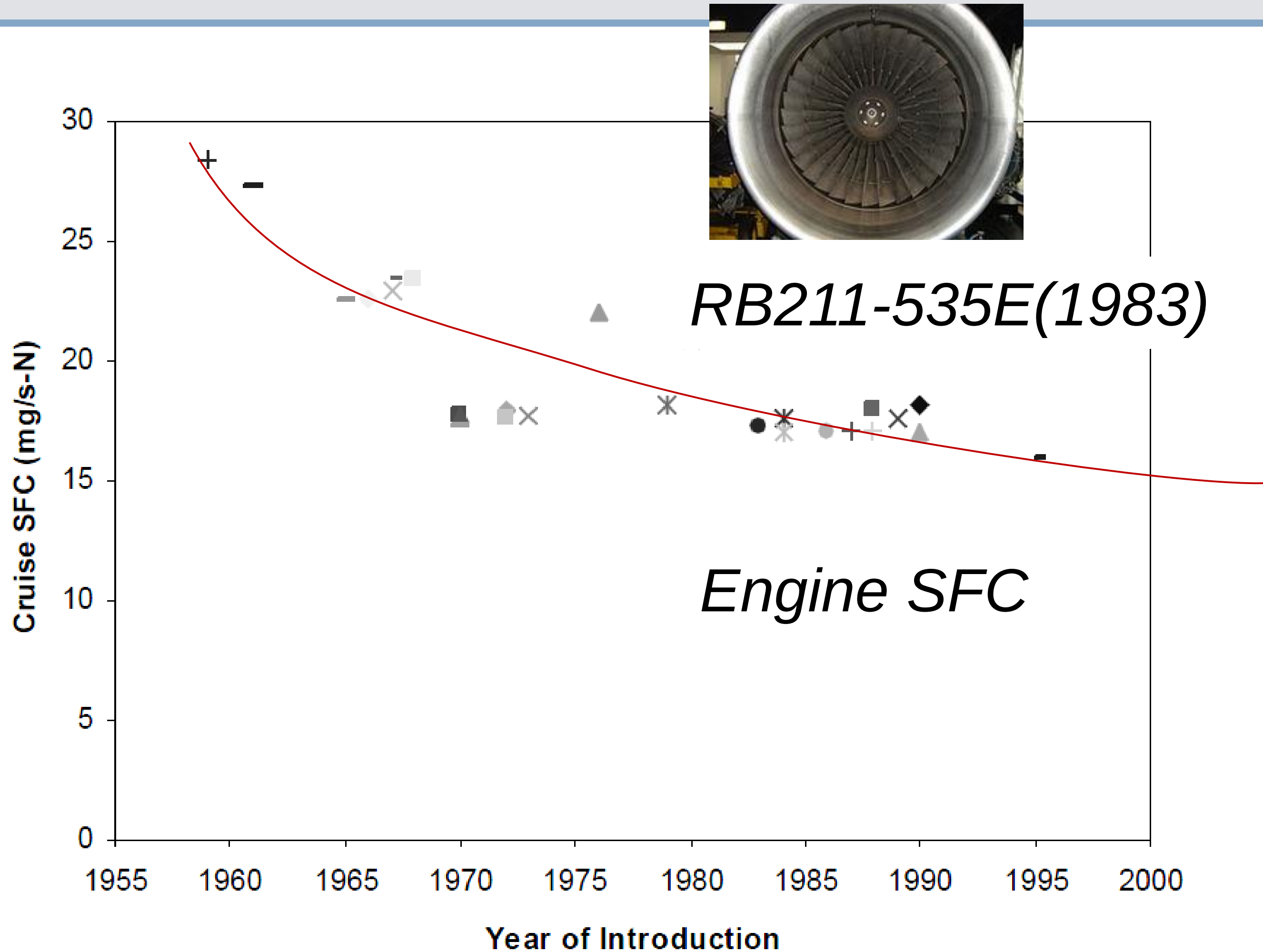
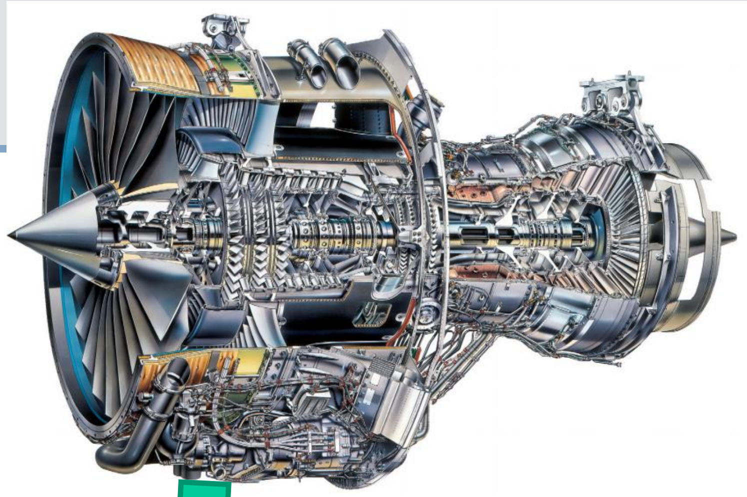
# Specific Fuel Consumption

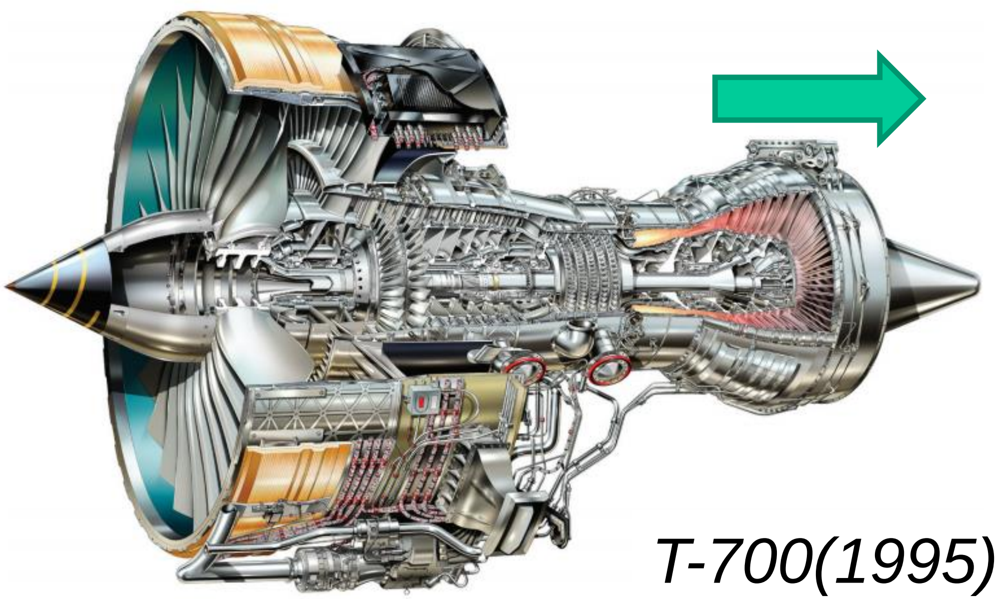



RB211-535E(1983)

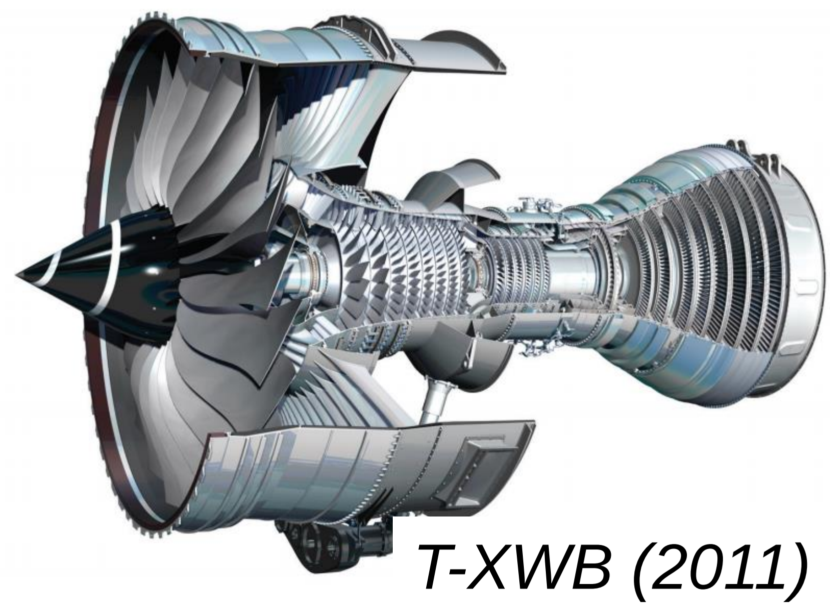
T-XWB (2011)



*RB211-535E(1983)*



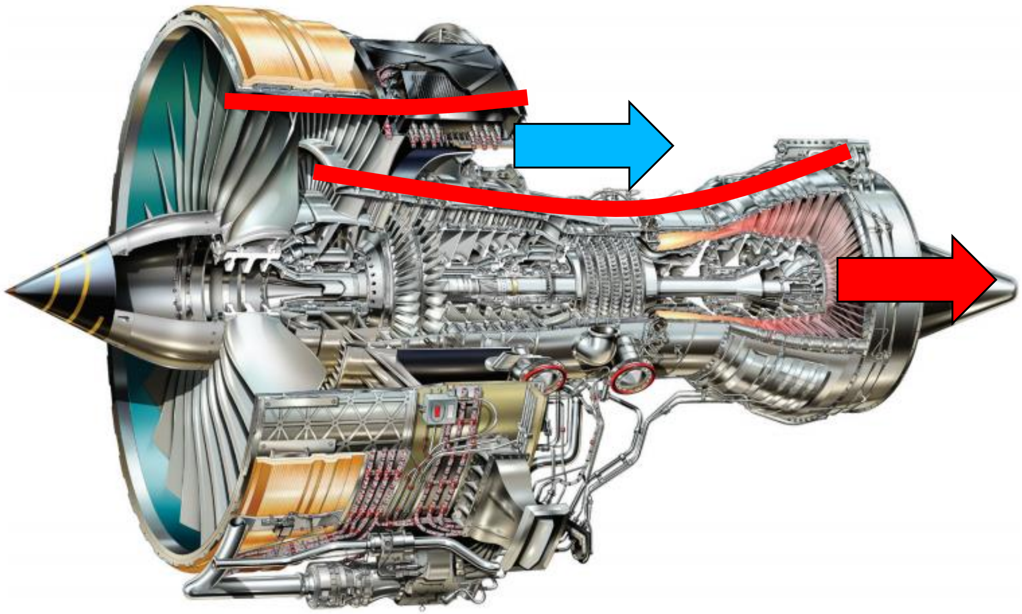
*T-700(1995)*



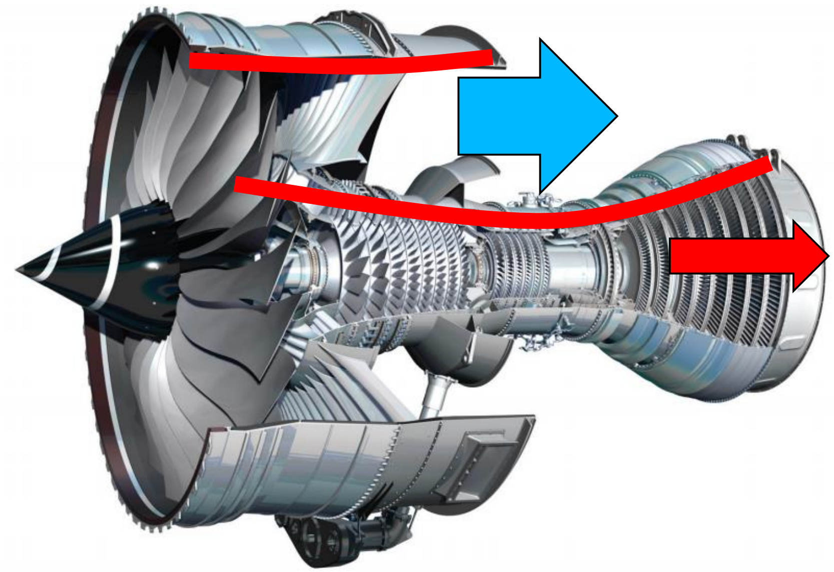
*T-XWB (2011)*

# Evolution

- to reduce losses, lower air velocity
- to have same thrust increase mass flow in the bypass
- higher bypass ratio (from 5 to 10)
- the core is becoming smaller



*T-700(1995)*



*T-XWB (2011)*

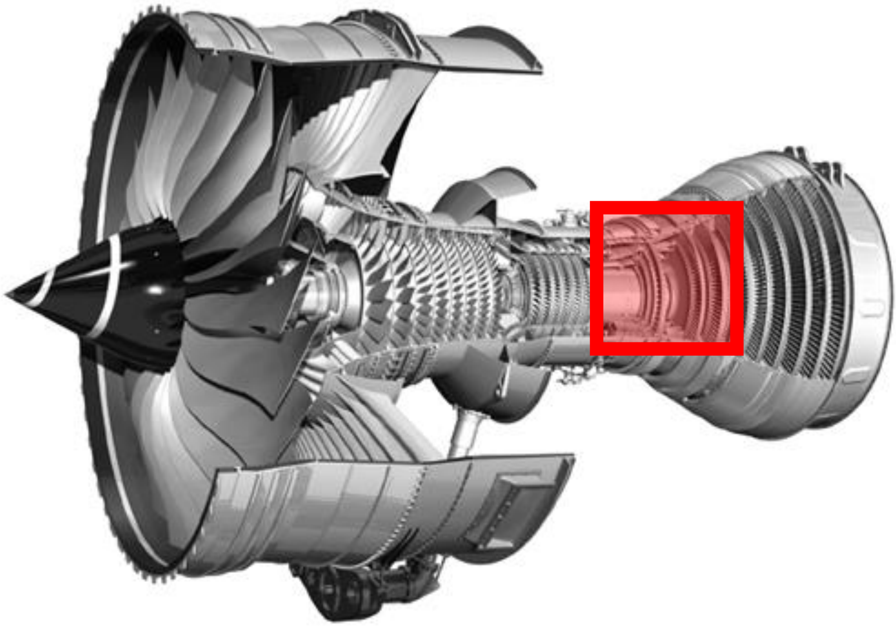
# High Pressure Turbine

- To increase efficiency, we increased the engine temperature

⇩

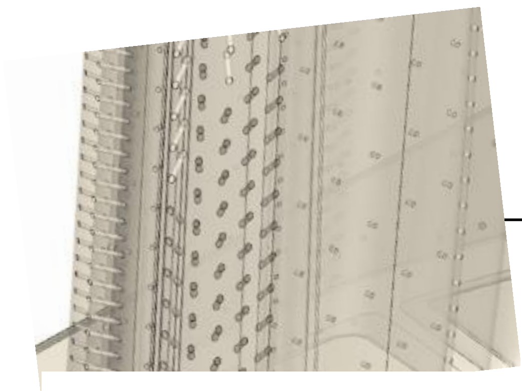
- nowadays gas temperature is ~ 2000C
- as reference melting temperature of steel is ~ 1500C
- the Sun temperature is ~ 5000C

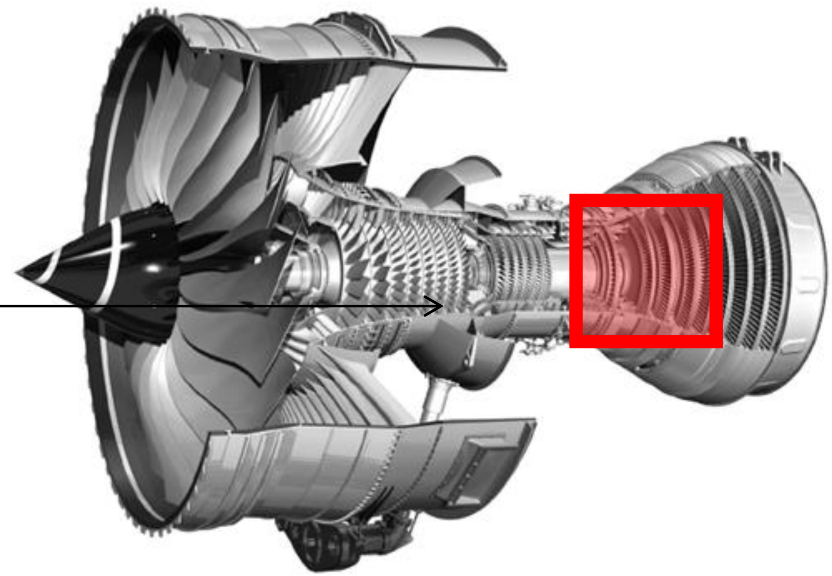
**Why the engine**
**does not melt down?**
**How does it work?**



# The components have thousands of holes

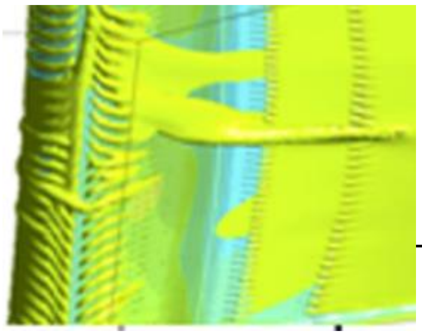
- The components are heavily cooled, like a shower head (the “cold” air is at 600C ….)

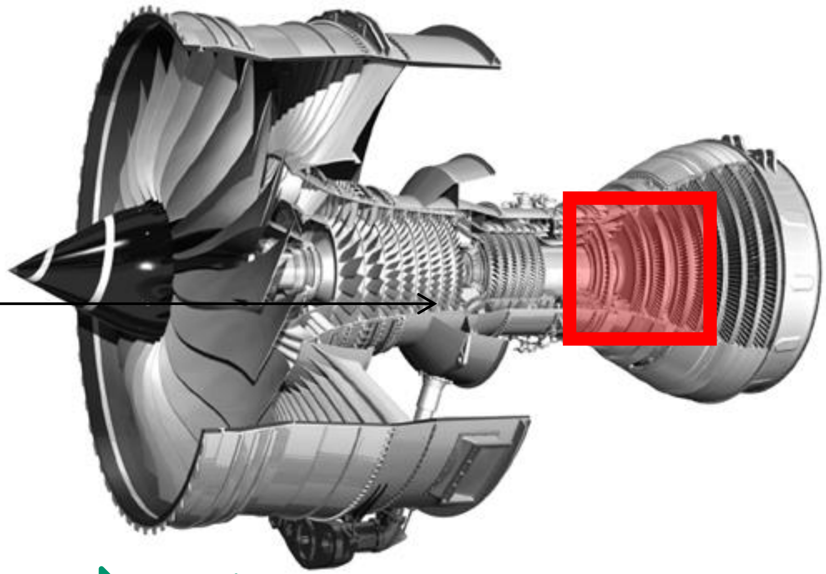


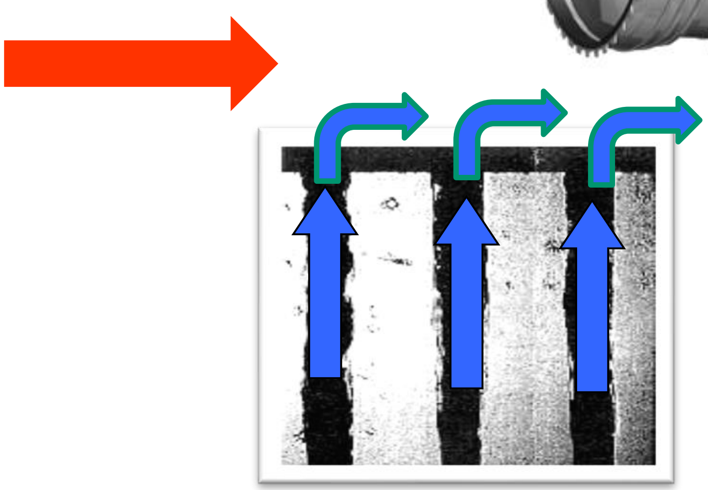


# With coolant .... (air at 600C)

- The components are heavily cooled, like a shower head
  (the “cold” air is at 600C ….)

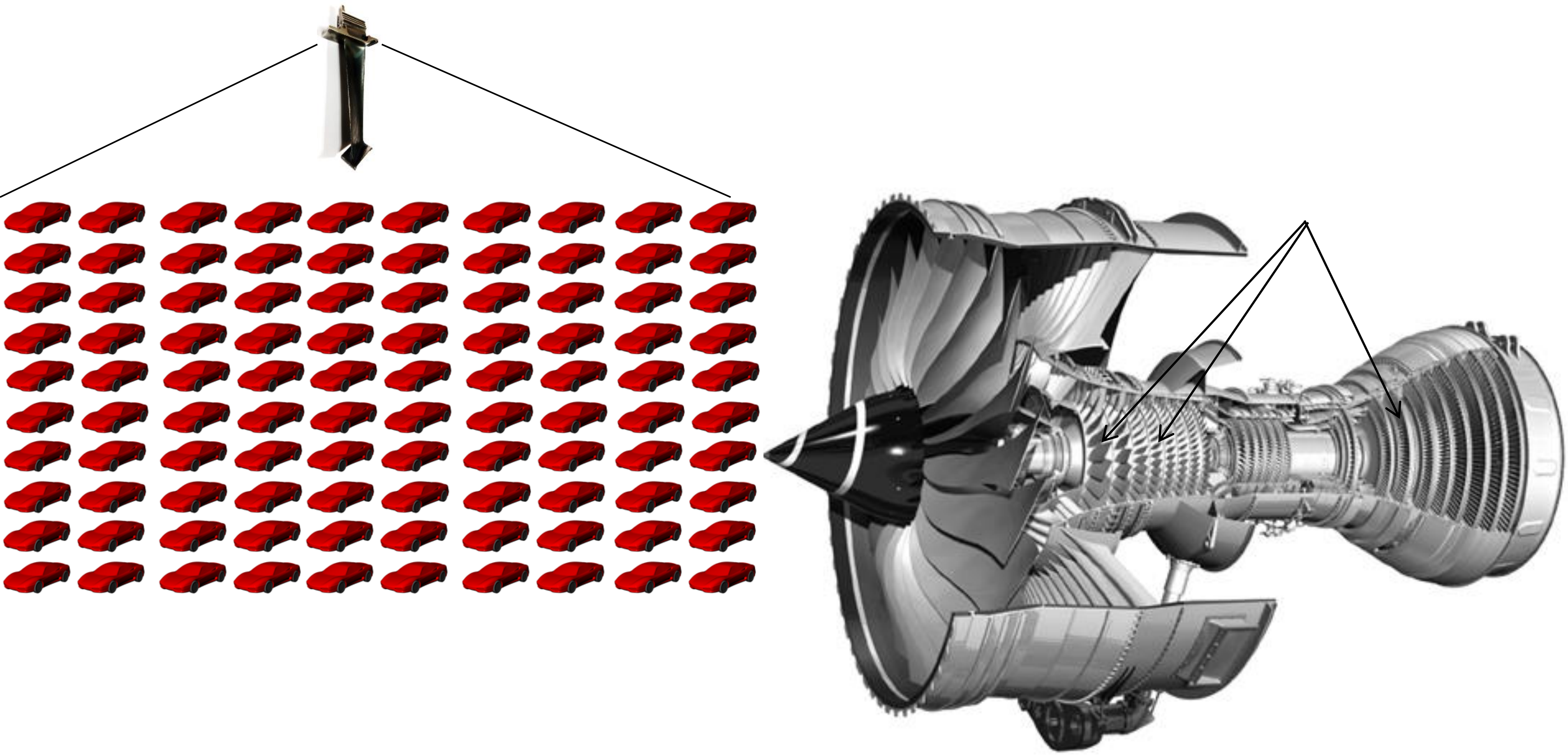






# High Stresses

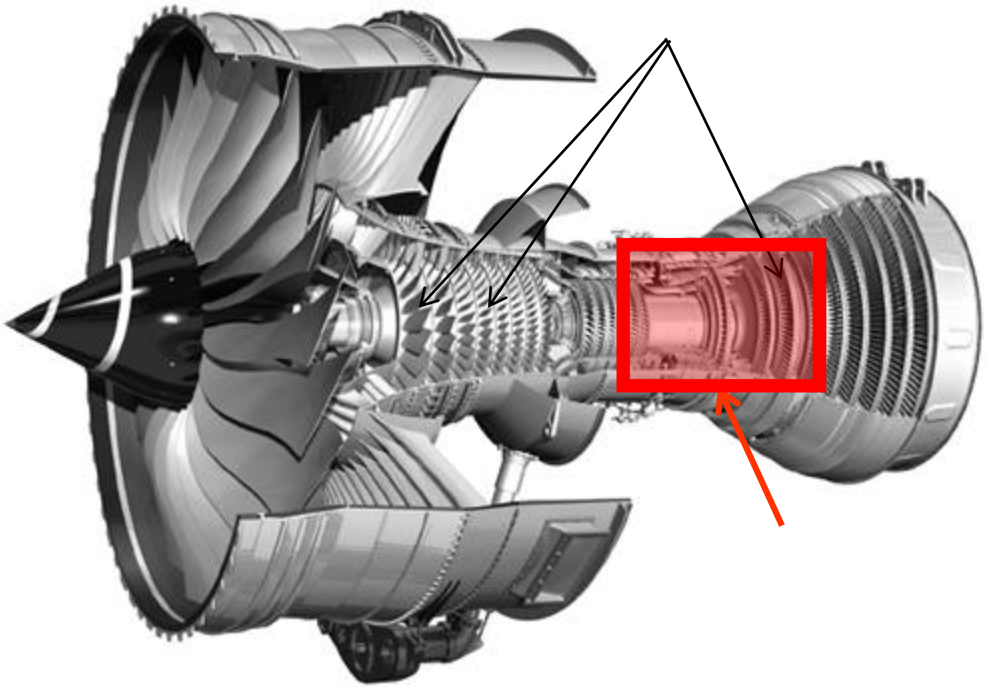
- Equivalent to hanging 100 cars on each blade (~1000 blades)



- Temperatures: about 0.5T of the sun

- Forces: 100 cars on a single blade

- a small error becomes crucial



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# Aircraft Engine Errors

- *State of the art: Laser Percussion Drilling*

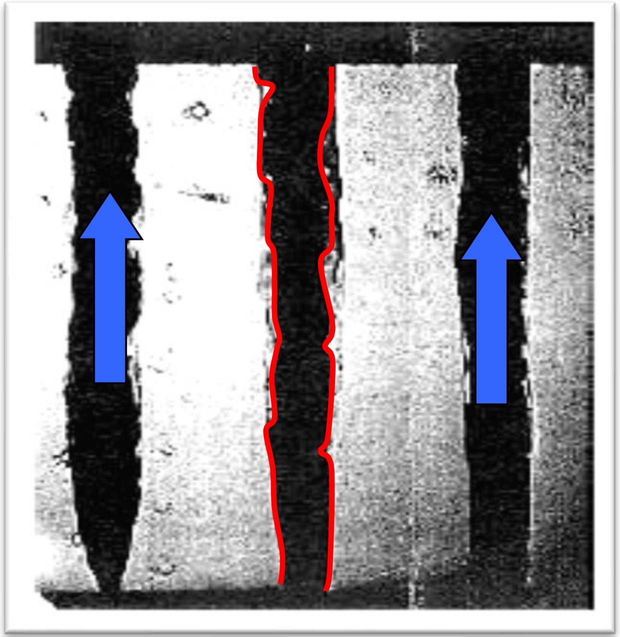
*(Smith W. R., "Models for solidification and splashing in laser percussion drilling", Journal on Applied Mathematics, Vol. 62, No. 6, 2002, pp. 1899-1923)*

- *General Electric: hole accuracy 10% of diameter*
- *variation +20°C metal temperature about -33% component life*

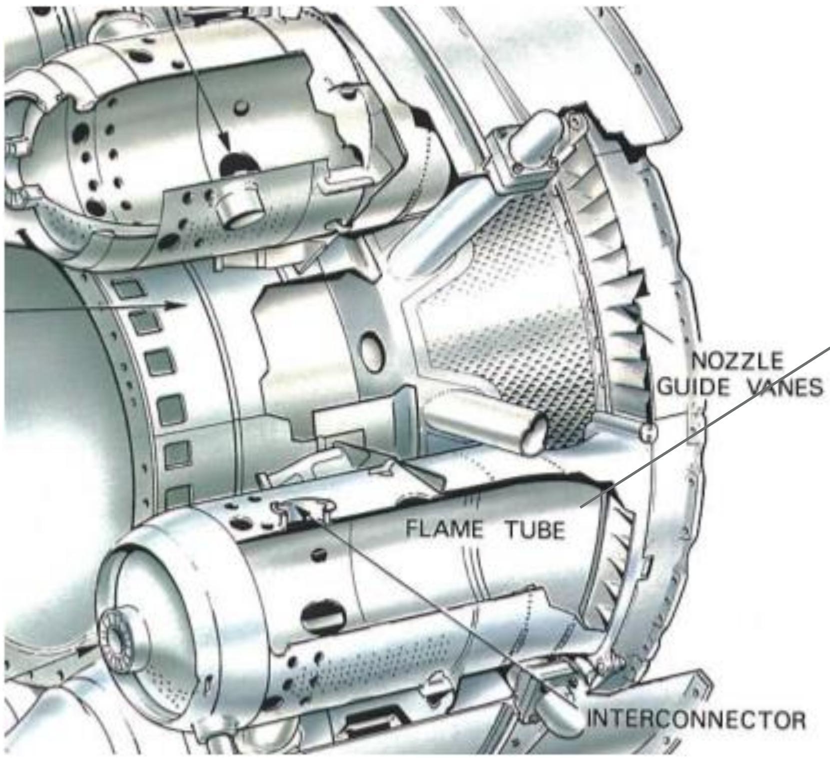
*(Bunker R.: GT2008-50124)*

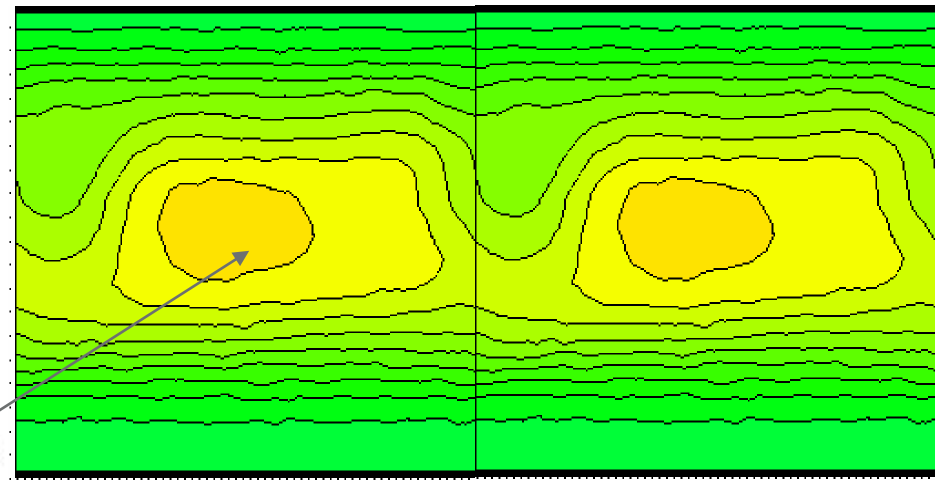
*manufacturing uncertainty*

*without in service variations*



# Some Data Cannot Be Measured



Salvadori S, **Montomoli** F, Martelli F, Adami P, Chana K., Castillon L.: “Aero-thermal study of the unsteady flow field in a transonic gas turbine with inlet temperature distortions”, **J. of Turbomachinery**, 2010

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# Sand Ingestion

Air contains and carries a large number of particles/contaminants

Sizes ranging from 0.1μm to 50 μm or even larger

Volcanic hashes, sand etc

# New Manufacturing Methods

Good control on Leading Edge even with composites

GE and RR are using titanium

The rest of the geometry is not perfect

Composites have less control than metal parts

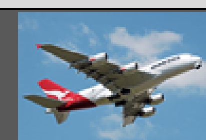
# The Importance of Rare Events/Black Swans

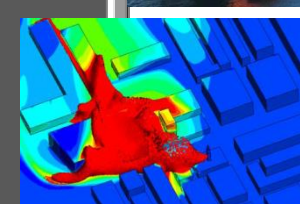
- *Designed for 1:1.000.000 accidents*
- *Estimated 1:100.000*
- ***2:135** flights were accidents*

*(1.481:100.000, 3 orders of magnitude higher than estimated…..)*

# Matrix of Knowledge

| | | Epistemic: Known | Epistemic: Unknown |
|---|---|---|---|
| Aleatoric | Known | Deterministic CFD<br>1st | Turbulence Closures<br>2nd |
| Aleatoric | Unknown | Uncertainty Quantification<br>3rd | Black Swans<br>4th |





# Why do we need UQ in Turbomachinery?

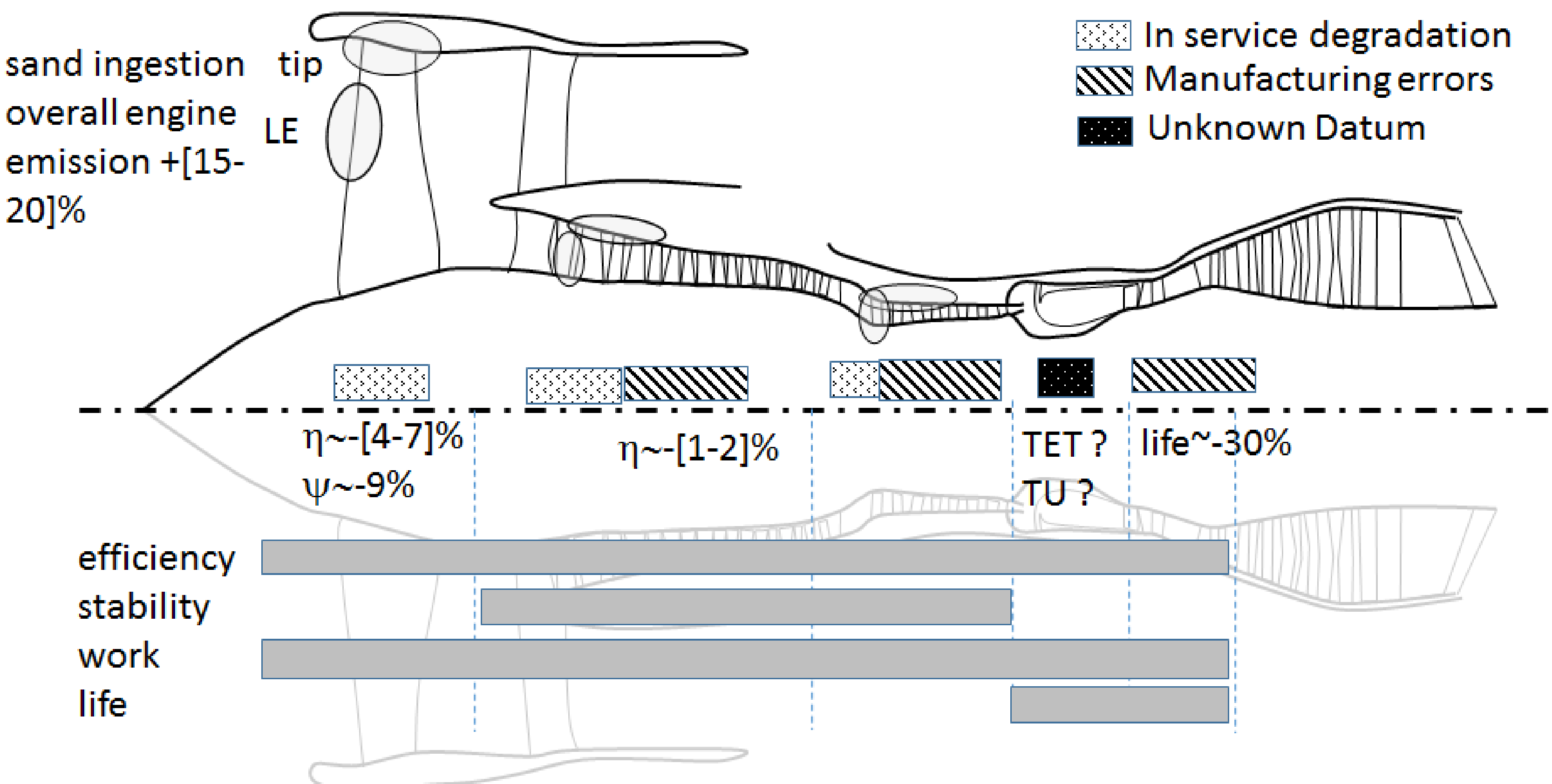
**Geometrical Variations:**

- Manufacturing Errors
- In service degradation
- Engine movements

**Operational variations**

**Unknown data**

# Different Impact in Different Components



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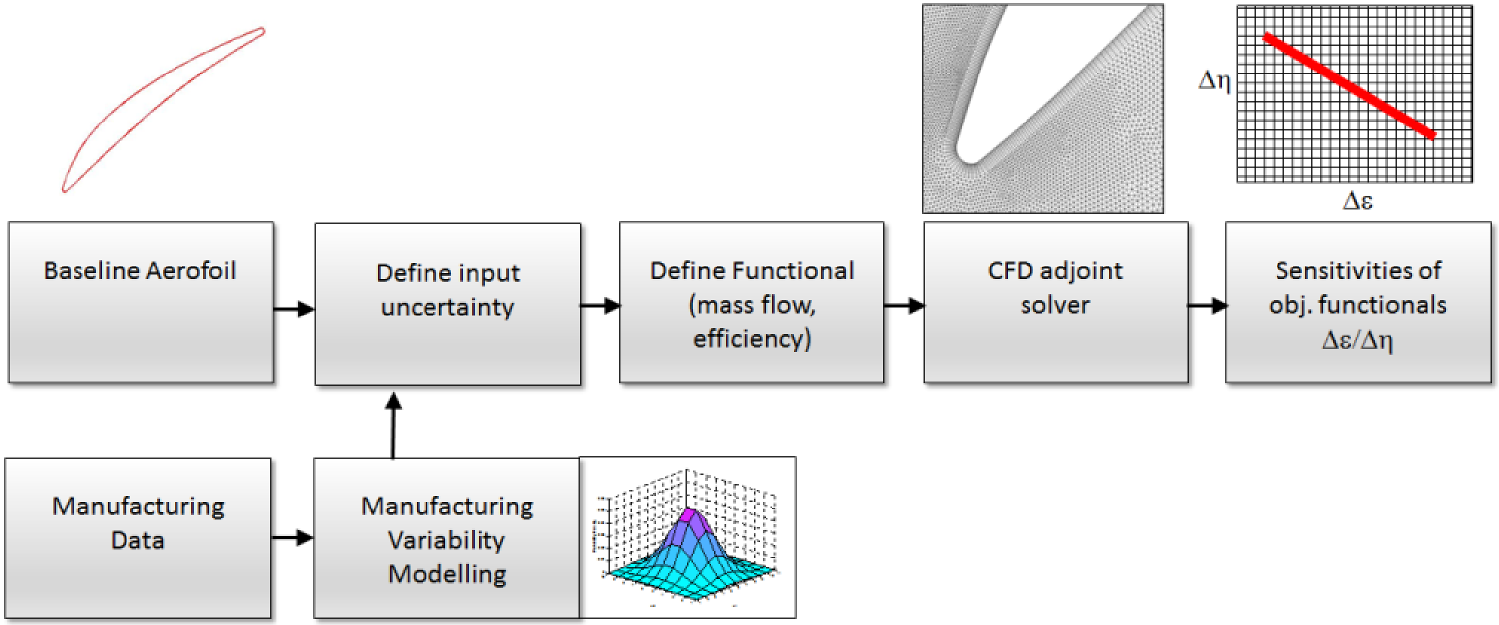
# *Methods*

# Adjoints
# Monte Carlo
# Non-Intrusive Polynomial Chaos

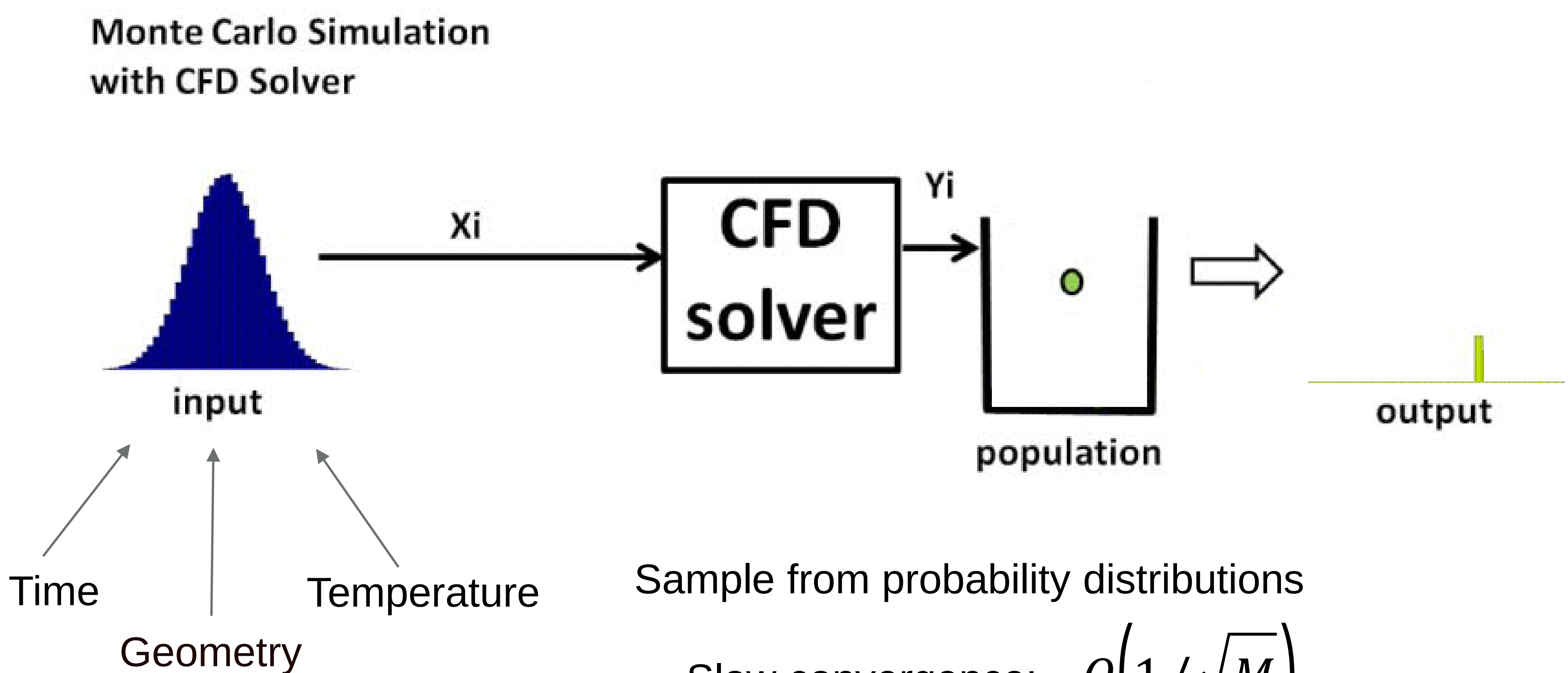
# Adjoints

- Calculate the sensitivities of objective functionals wrt a high number of variations in geometric parameters.
- Valid mainly when the solution variation is (almost) linear.
- Valid for small variations of compressor geometry.




Giebmanns, A., Backhaus, J., Frey, C., 2013, "Compressor leading edge sensitivities and analysis with an adjoint flow solver," Proceedings of the ASME Turbo Expo, 6 A, .

# Monte Carlo Methods




Sample from probability distributions

- Slow convergence: $O\left(1/\sqrt{M}\right)$
- **Monte Carlo needs too many CFD runs**

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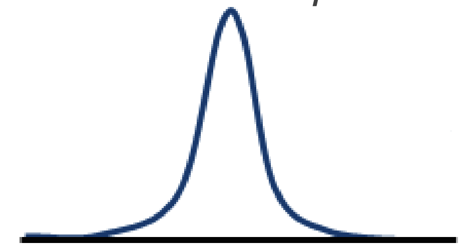
# Idea Behind Polynomial Chaos

1. Find a series of basis functions ψ(ξ) for the input random variable ξ

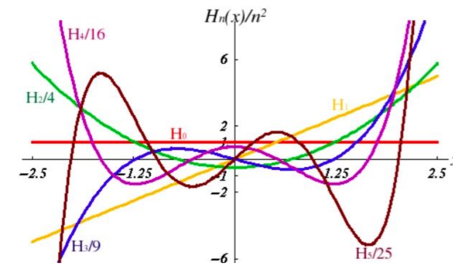
1. Make the assumption that the solution y(ξ) can be approximated through a linear combination of these basis functions

2. Determine the coefficients α of the basis function expansion with fewer model runs than by random sampling

*Random Input*



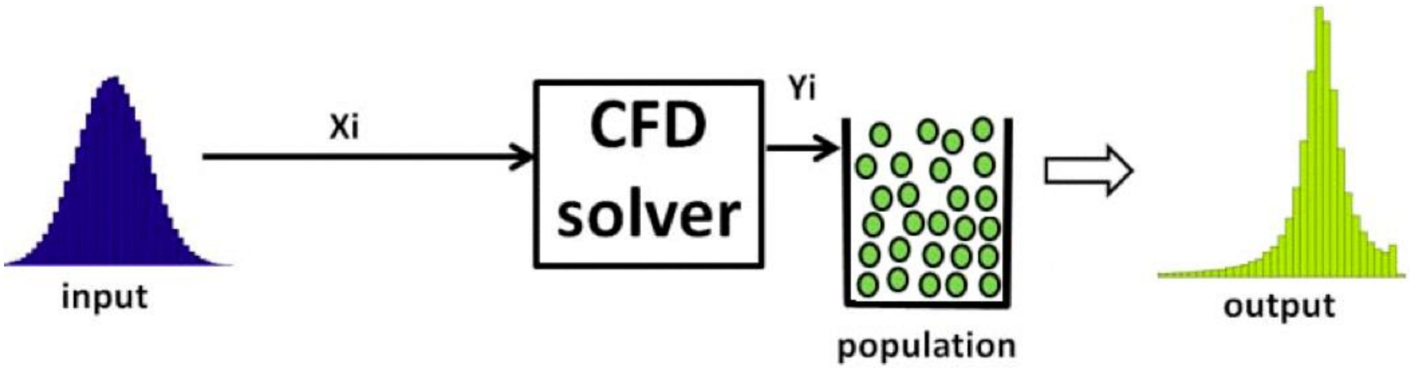
*Polynomial Series*



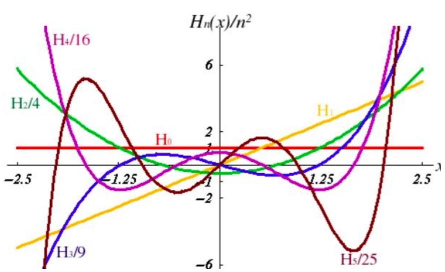

$$Y(x,\xi) \approx \sum_{i=0}^{N} \alpha_i(x)\,\psi_i(\xi)$$

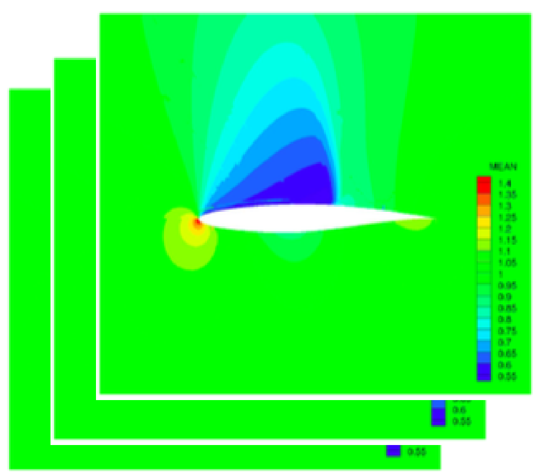
# Non-Intrusive Polynomial Chaos

CFD simulations are used as a black box (no need to modify codes)




Polynomial coefficients are calculated based on the response evaluations

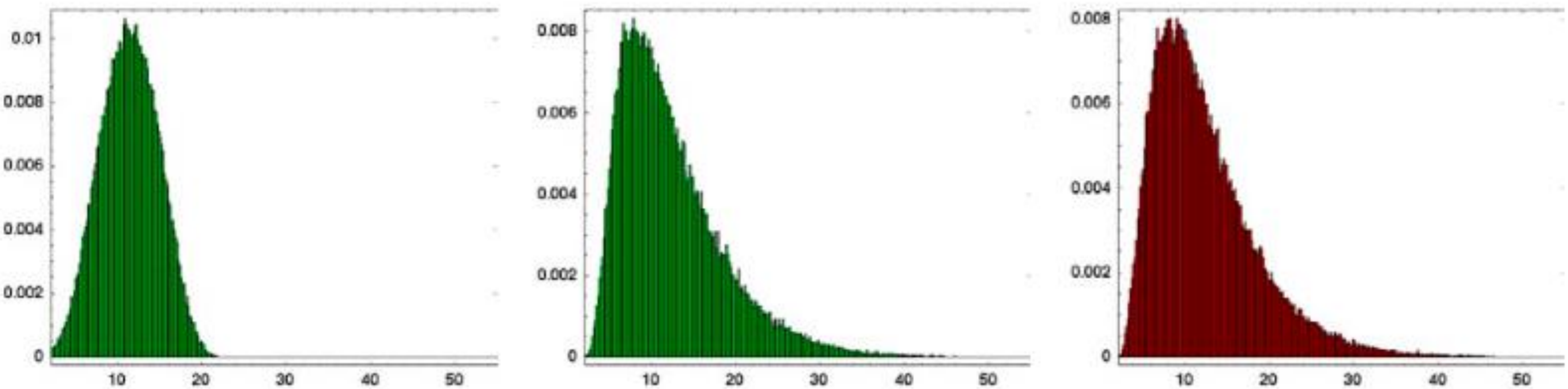


$$Y(\xi,x) \approx \sum_{i=0}^{N} a_i(x)\psi_i(\xi)$$

# Advantage of Polynomial Chaos for CFD

**2 PC runs** | **4 PC runs** | **100 000 MC runs**




(a) PCol NIPC, polynomial degree=1 (b) PCol NIPC, polynomial degree=3 (c) Monte Carlo with 100,000 samples

**Hosder**, 2007, "Efficient Sampling for Non-Intrusive Polynomial Chaos Applications with Multiple Uncertain Input Variables," 48th AIAA Structures, Structural Dynamics, and Materials Conference

# The Mathematics behind Polynomial Chaos

Assume that the solution Y can be decomposed into separable deterministic and stochastic components:

$$Y(x,\xi) \approx \sum_{i=0}^{N} \alpha_i(x)\,\psi_i(\xi)$$

α are deterministic coefficients and ψ(ξ) are random basis functions (optimal orthogonal polynomials) chosen in accordance with the probability distribution w

For example, for N = 2 the expansion becomes:

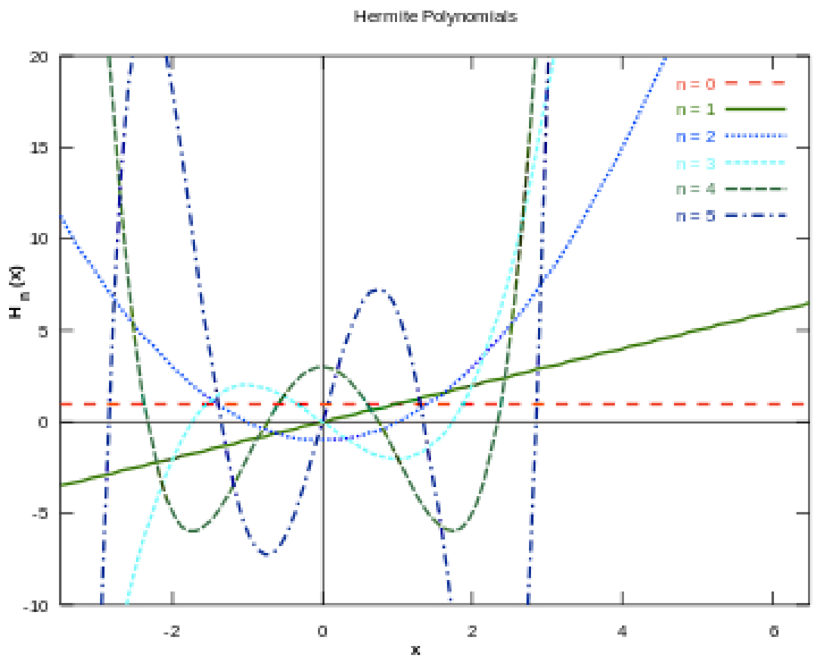
$$Y(x,\xi) \approx \alpha_0(x)\psi_0(\xi) + \alpha_1(x)\psi_1(\xi) + \alpha_2(x)\psi_2(\xi)$$

# Optimal Orthogonal Polynomials as Basis Functions

$$\int_{x\in\Omega} \psi^k(x)\psi^l(x)w(x)dx = \delta_{kl} \quad \forall\ k,l=\overline{0,N}$$

The Probabilistic Hermite Polynomials (Gaussian distribution)

$$\psi_0(x)=1$$
$$\psi_1(x)=x$$
$$\psi_2(x)=x^2-1$$
$$\psi_3(x)=x^3-3x$$
$$\psi_4(x)=x^4-6x^2+3$$




Wavelets are also possible, but less well researched

# Generalised Polynomial Chaos and the Askey Scheme

Certain orthogonal polynomials are optimal with respect to the inner product weight function and corresponding support range of a specific random variable

Askey Scheme table for most common PDFs:

| Distribution | Density Function | Polynomial Basis | Orthogonality Weight | Support |
|---|---|---|---|---|
| Normal | $\frac{1}{\sqrt{2\pi}}e^{\frac{-x^2}{2}}$ | Hermite $He_n(x)$ | $e^{\frac{-x^2}{2}}$ | $[-\infty, \infty]$ |
| Uniform | $\frac{1}{2}$ | Legendre $P_n(x)$ | $1$ | $[-1, 1]$ |
| Beta | $\frac{(1-x)^\alpha(1+x)^\beta}{2^{\alpha+\beta+1}B(\alpha+1,\beta+1)}$ | Jacobi $P_n^{(\alpha,\beta)}(x)$ | $(1-x)^\alpha(1+x)^\beta$ | $[-1, 1]$ |
| Exponential | $e^{-x}$ | Laguerre $L_n(x)$ | $e^{-x}$ | $[0, \infty]$ |
| Gamma | $\frac{x^\alpha e^{-x}}{\Gamma(\alpha+1)}$ | Gen. Laguerre $L_n^{(\alpha)}(x)$ | $x^\alpha e^{-x}$ | $[0, \infty]$ |

# Practical Problem: Curse of Dimensionality

1 CFD Simulation to determine each polynomial coefficient in 1D

For multiple input random variables, the tensor product of the individual evaluations has to be formed

This leads to a rapidly increasing number of evaluations, called the curse of dimensionality

| Number of Random Variables | Needed CFD Runs $4^{th}$ Order |
|---|---|
| 1 | 5 |
| 2 | 25 |
| … | … |
| 10 | **1 Million** |

# Curse of Dimensionality (Computational Cost)

## Sparse Methods
## Active Subspaces
## Multifidelity Models

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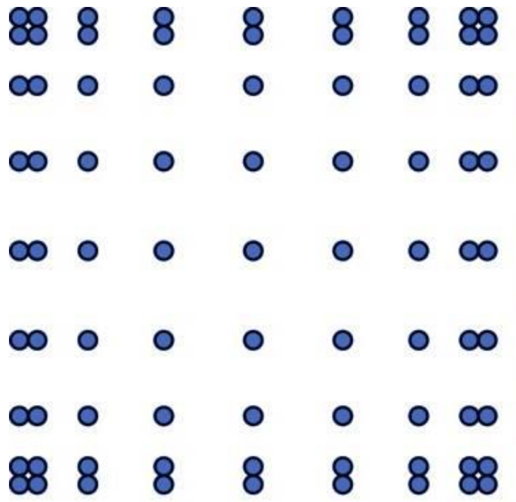
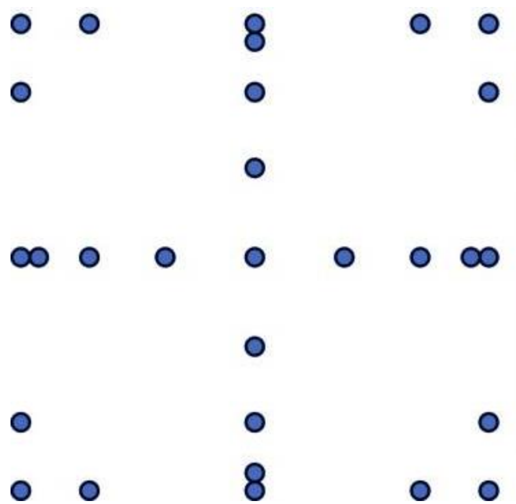
# Sparse Grid Methods

The number of function evaluations can be reduced by focussing on low order connections between random variables

Tensor: 81 nodes



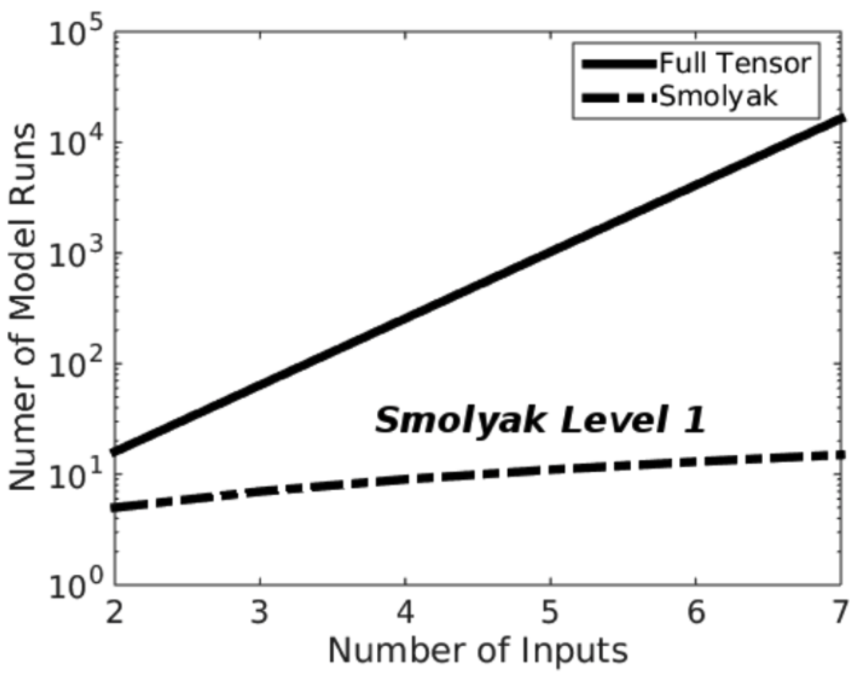
Sparse Grid: 29 nodes



# Lower Computational Effort of Sparse Grids

The most commonly used sparse grid rule is Smolyak

It works well for moderately high number of inputs (less than 20)




More than 100 variables is still not feasible with PC

# Checking Convergence

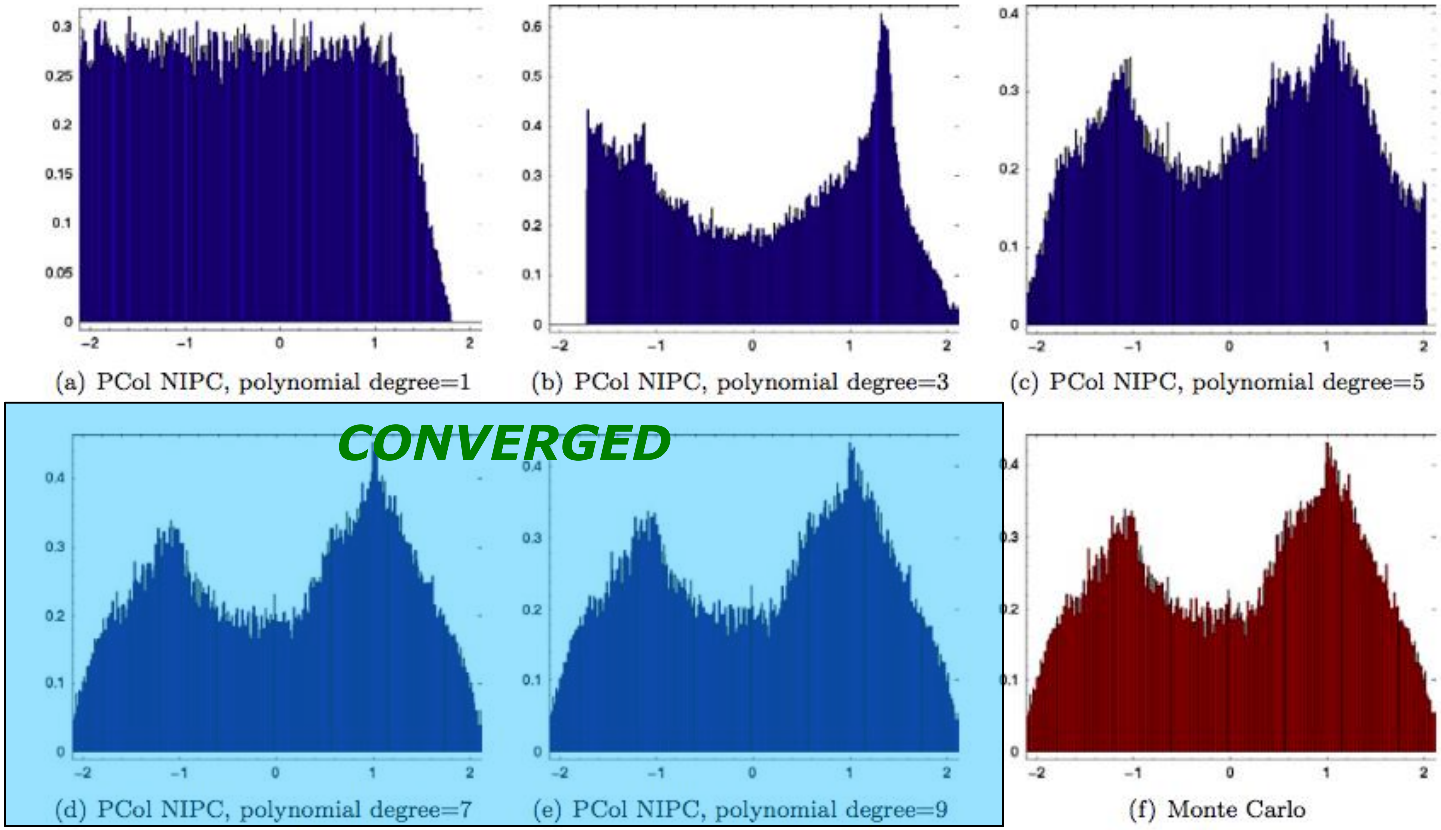



**Figure 6. The histogram of $f(x_1, x_2)$ obtained with the Point-Collocation (PCol) NIPC (HS and $n_p = 2$) for various polynomial degrees. Monte Carlo histogram is included for comparison.**

**Hosder**, 2007, "Efficient Sampling for Non-Intrusive Polynomial Chaos Applications with Multiple Uncertain Input Variables," 48th AIAA Structures, Structural Dynamics, and Materials Conference

# Reduction of Dimensionality (Active Subspace, Constantine)

- $f(\boldsymbol{x}) = \sin(0.9x_1 + 0.2x_2)$

- $f$ varies only along direction

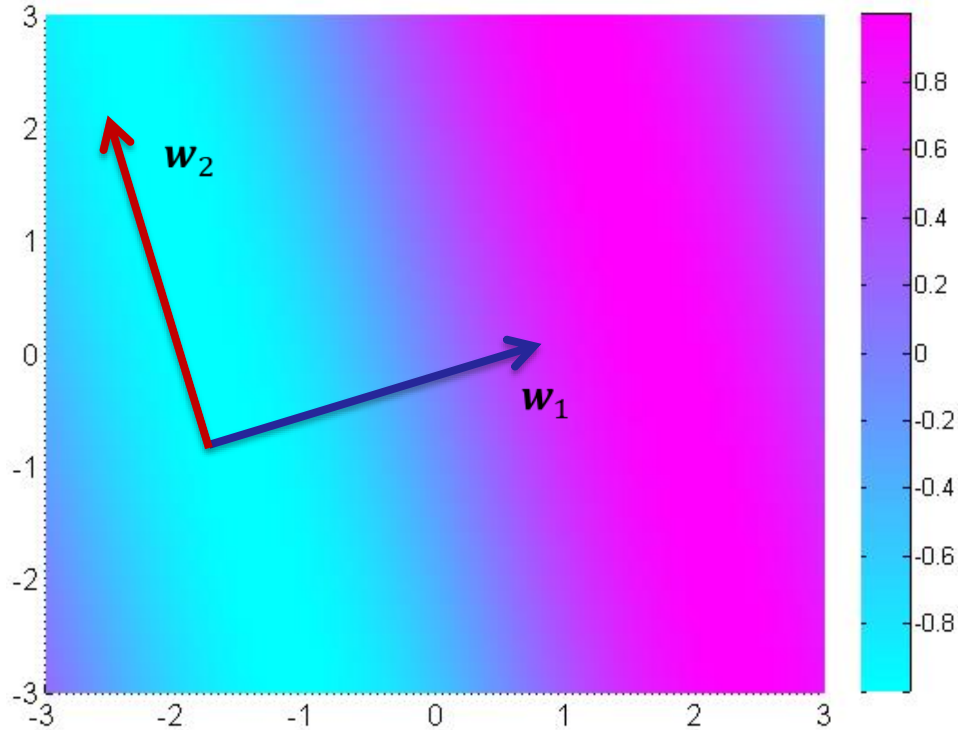
$$\boldsymbol{w}_1 = \left(\frac{0.9}{\sqrt{0.9^2 + 0.2^2}}, \frac{0.2}{\sqrt{0.9^2 + 0.2^2}}\right)$$

while it's constant along

$$\boldsymbol{w}_2 = \left(-\frac{0.2}{\sqrt{0.9^2 + 0.2^2}}, \frac{0.9}{\sqrt{0.9^2 + 0.2^2}}\right)$$

- $f(\boldsymbol{x}) = \sin(c\boldsymbol{w}_1 \cdot \boldsymbol{x}) = g(y)$

Adapted from Constantine et al. 2014




By looking for appropriate rotations of the **input** space, along directions which maximize the variation of the **output**, we may manage to reduce the dimensionality of the problem

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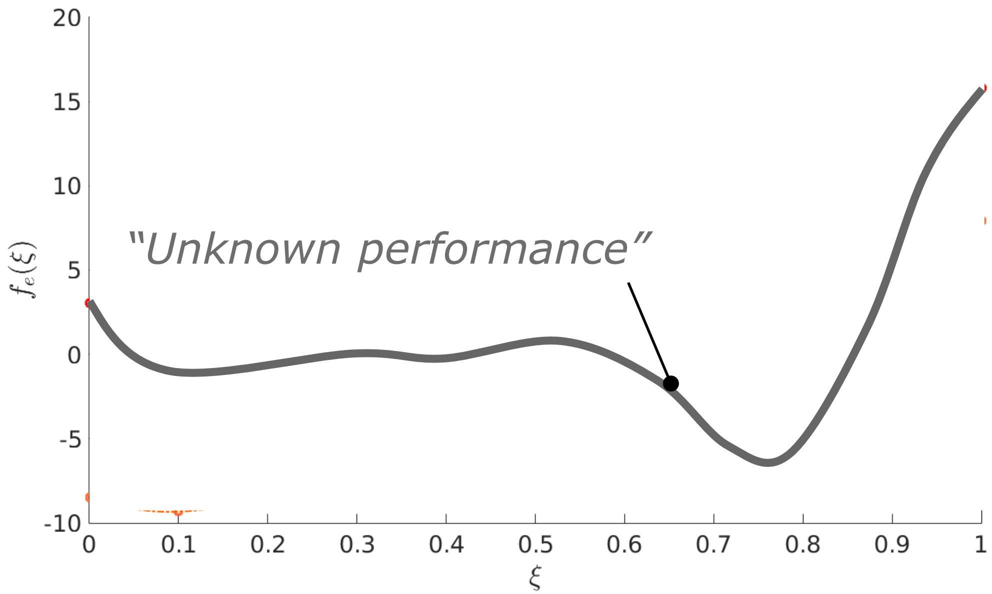
# Mutifidelity Co-kriging, example

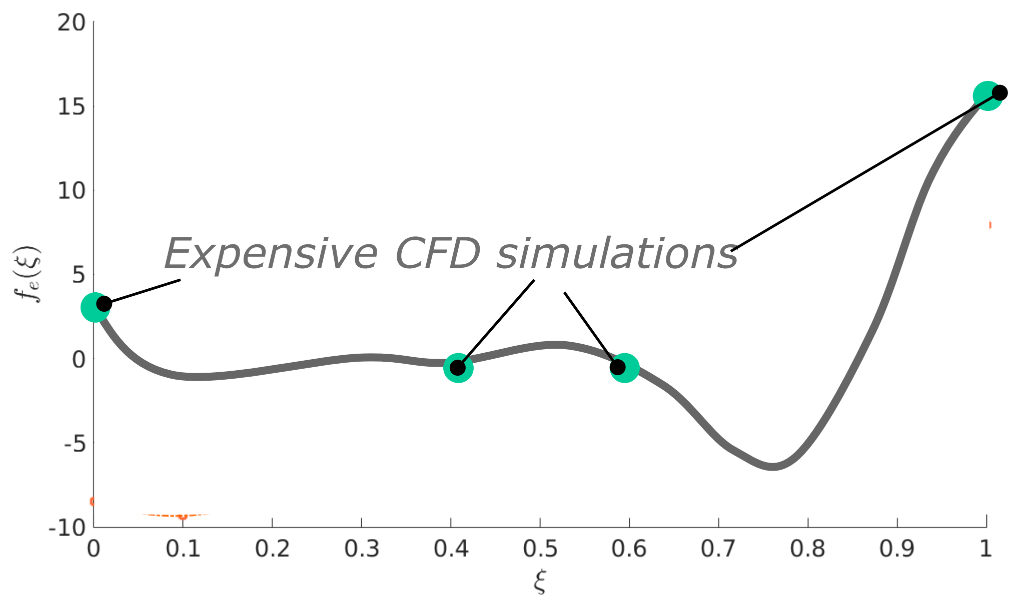
- Extension of Kriging
- Uses multiple data sets of varying fidelity (low fidelity and high fidelity CFD simulations)
- Cheaper data used to fill the “gaps” between expensive data points




Forrester et al 2007, it is a test function

# Expensive CFD simulations, DNS

- Let's say I can run 4 expensive CFD simulations ●
- What kind of information do I have?



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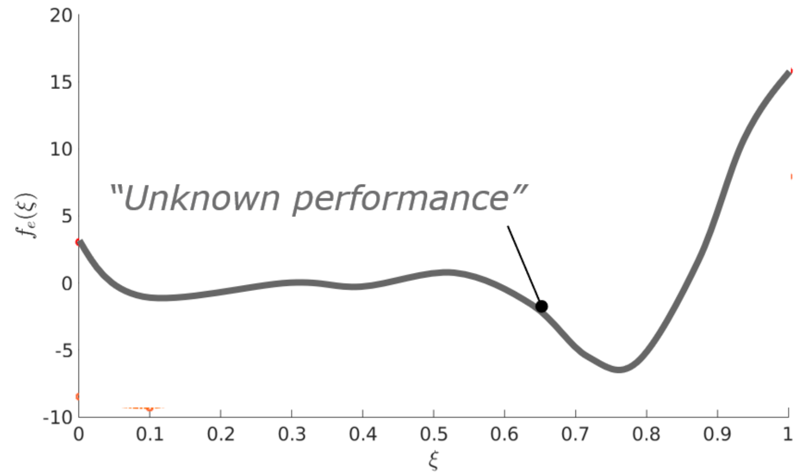
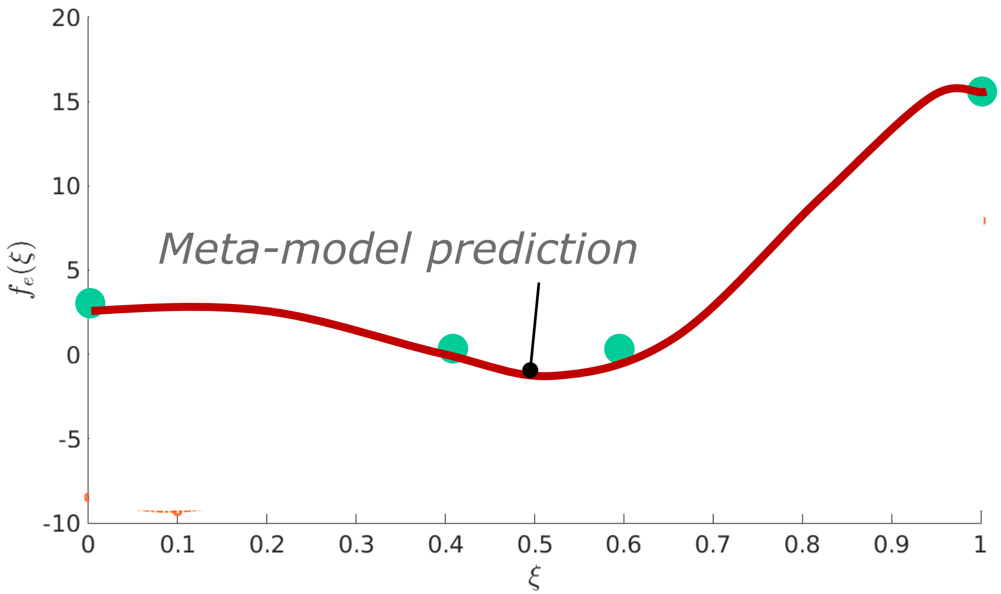
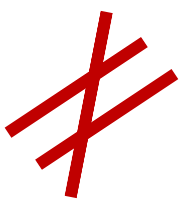
# Metamodel prediction

- A meta-model based on 4 simulations
- It does not capture the “trend”



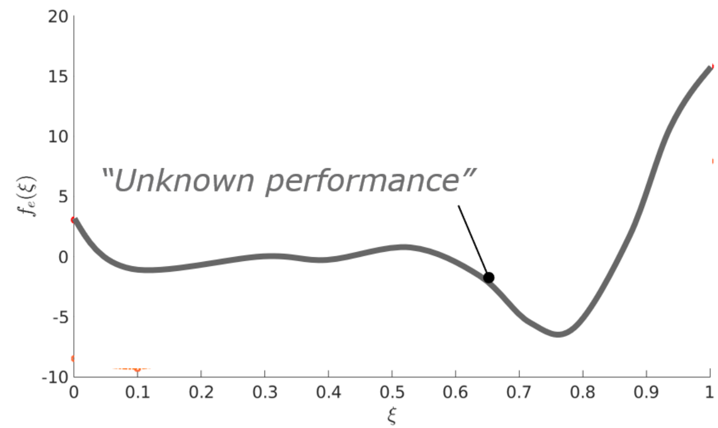

Forrester et al 2007, it is a test function



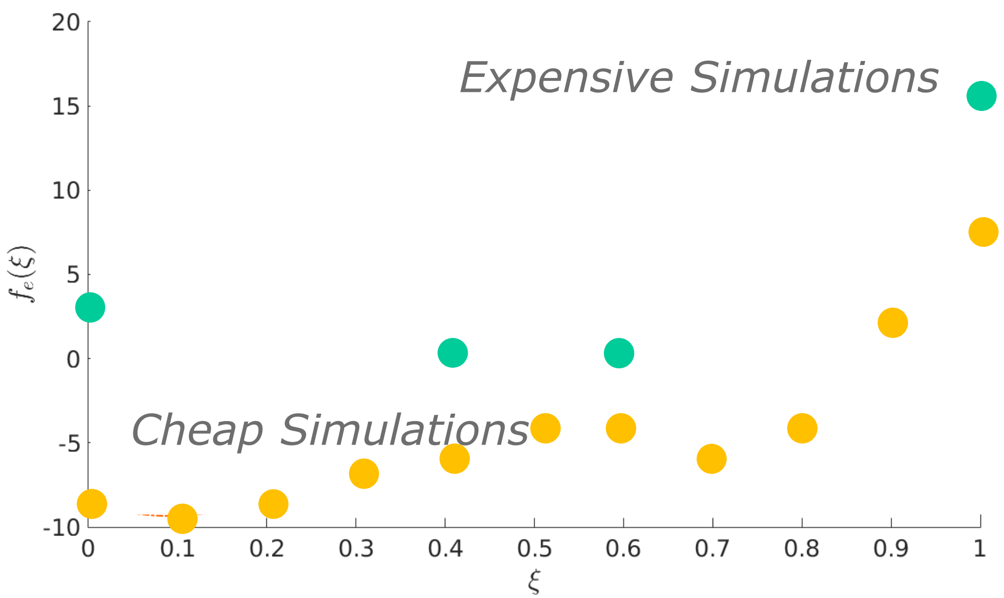



# Possible to use cheap simulations

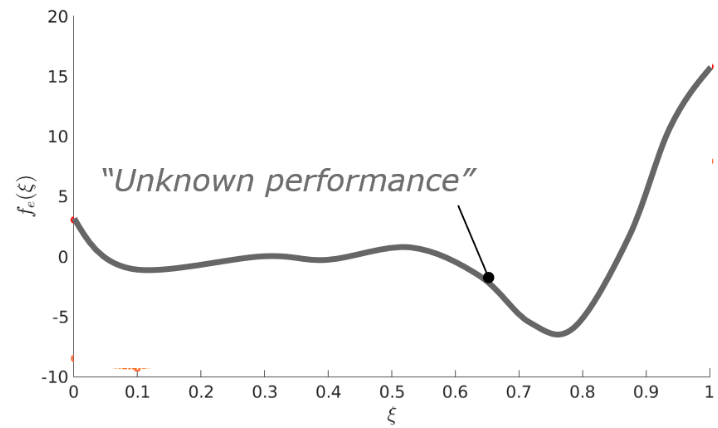
- Cheap, fast CFD simulations
- To improve the "trend"




Forrester et al 2007, it is a test function

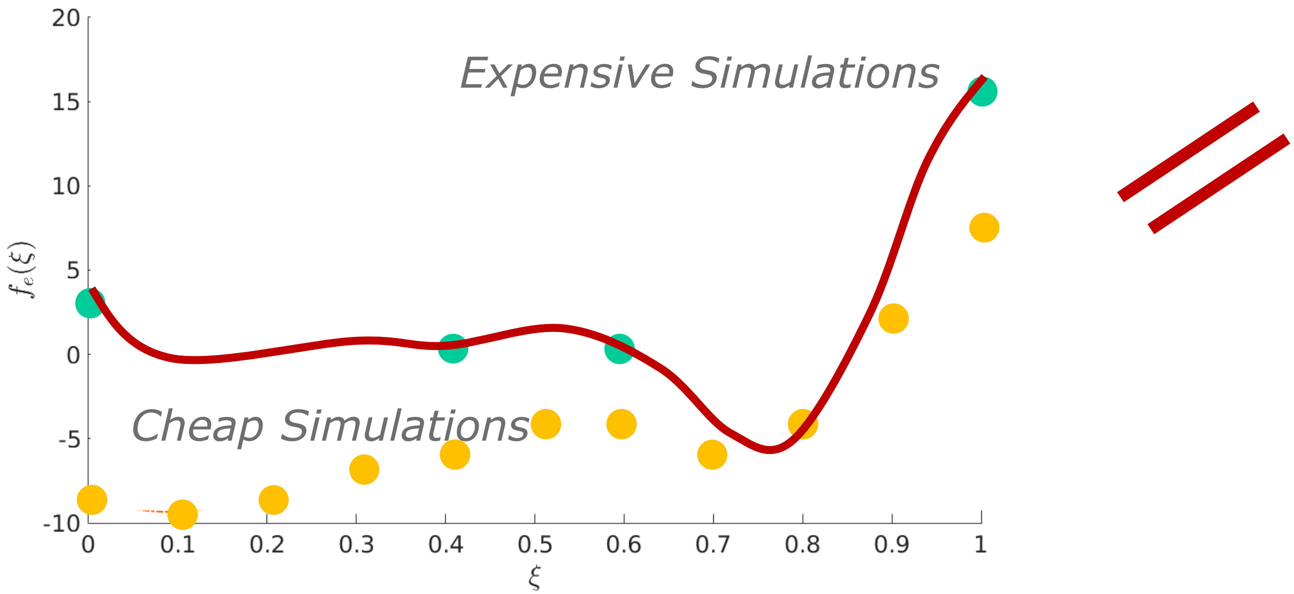


# Possible to use cheap simulations



Forrester et al 2007, it is a test function

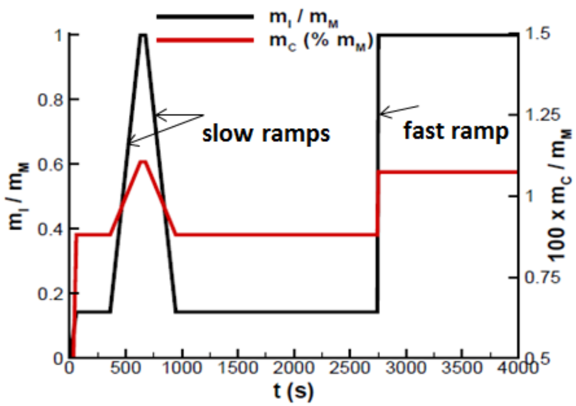
- New metamodel combination of high and low fidelity CFD

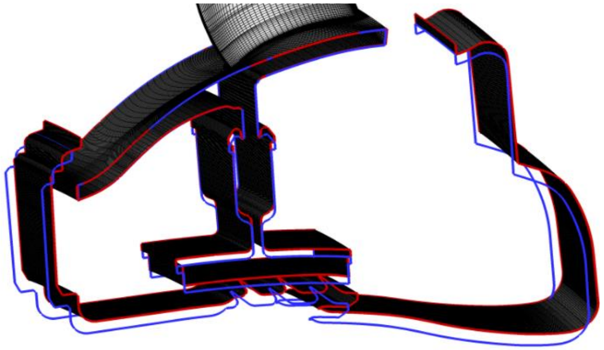


# *Examples*

# CFD-Structural simulation

- Real Geometry
- Transient CFD simulation, Hydra
- Thermo-mechanical analysis SCO3
- Components displacement prediction
- Robust mesh reconstruction

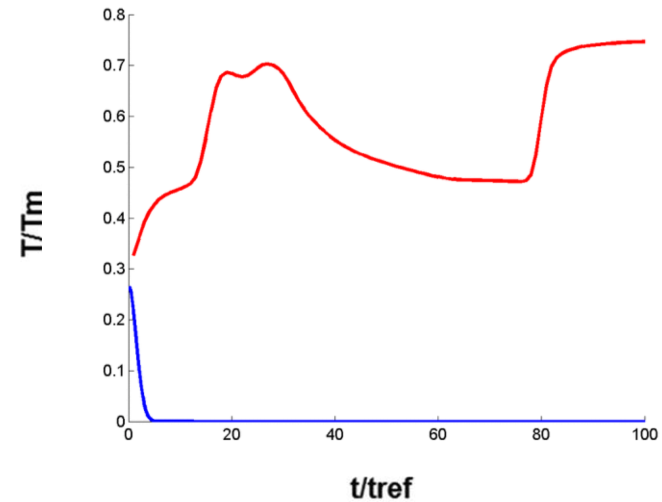


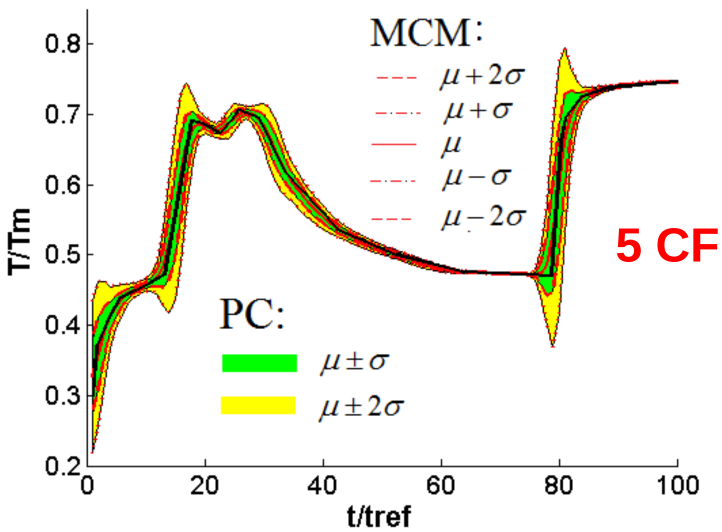
*Amirante D, Hills NJ, Barnes CJ. "A moving mesh algorithm for aero-thermo-mechanical modelling in turbomachinery", Int. J. for Numerical Methods in Fluids 2012*

# Example 1: Results for Temperature Gradients

FEM

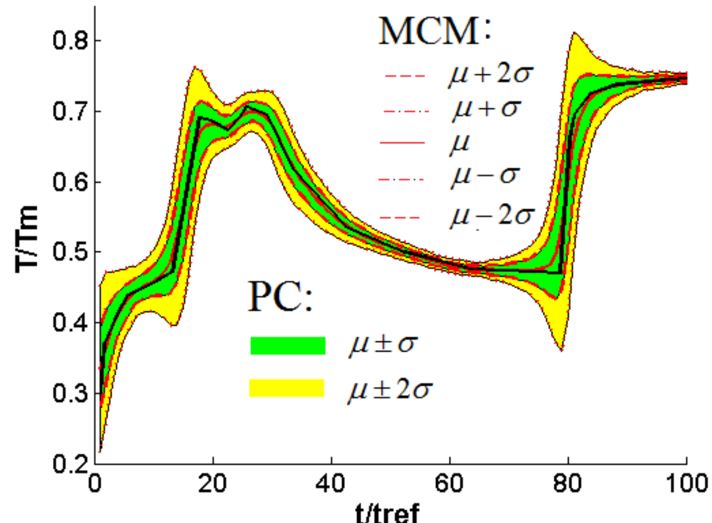



Gaussian PC:




**5 CFD runs**

Fat-Tailed PC:



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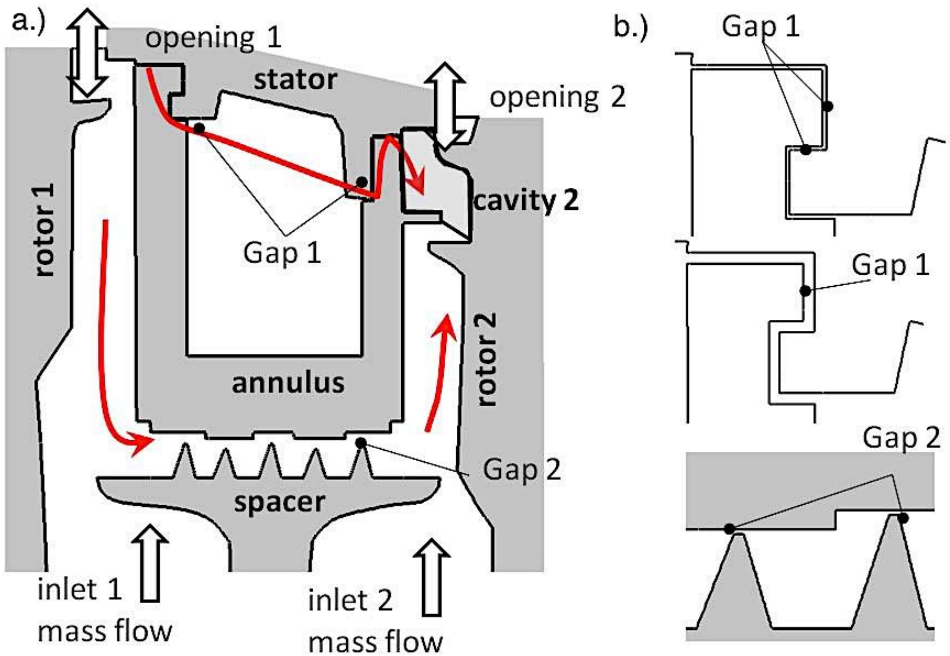
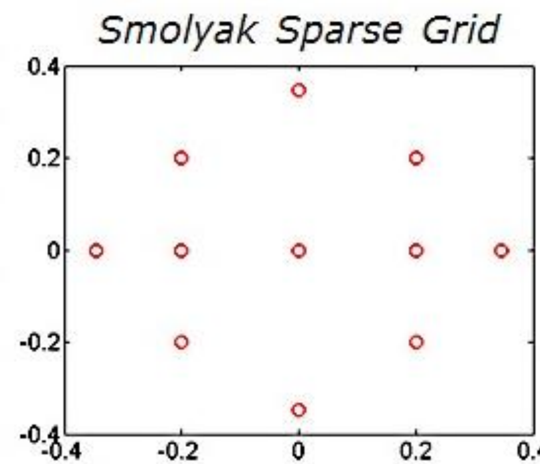
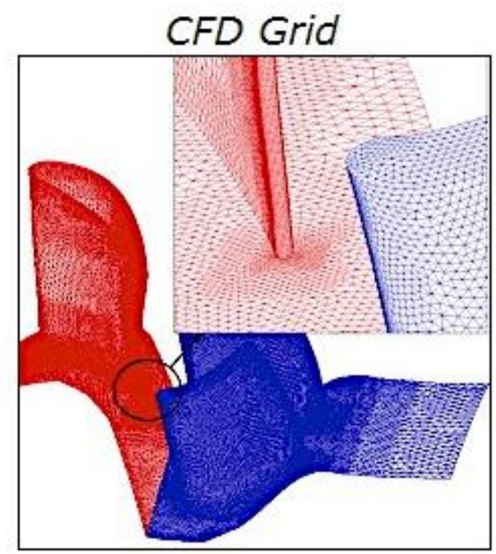
# Example 2: Hot Gas Ingestion



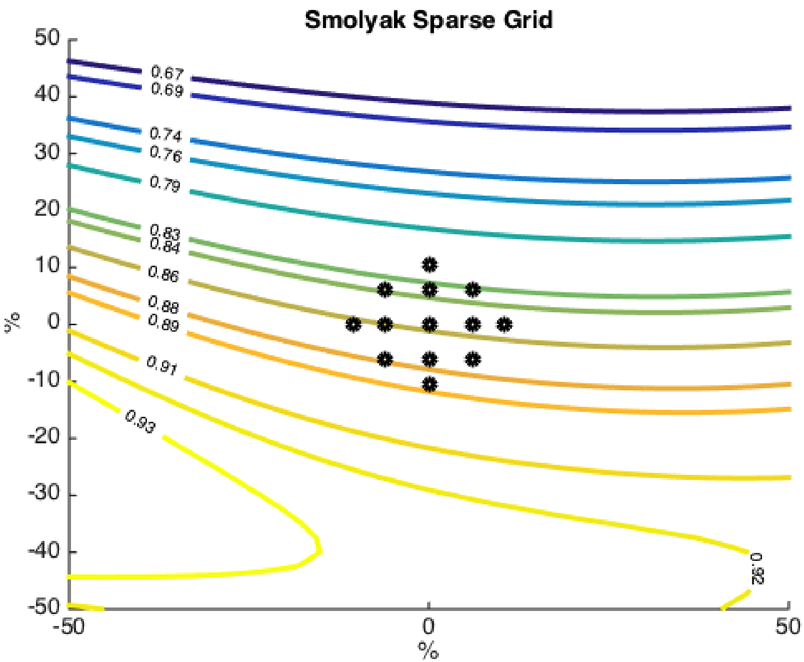



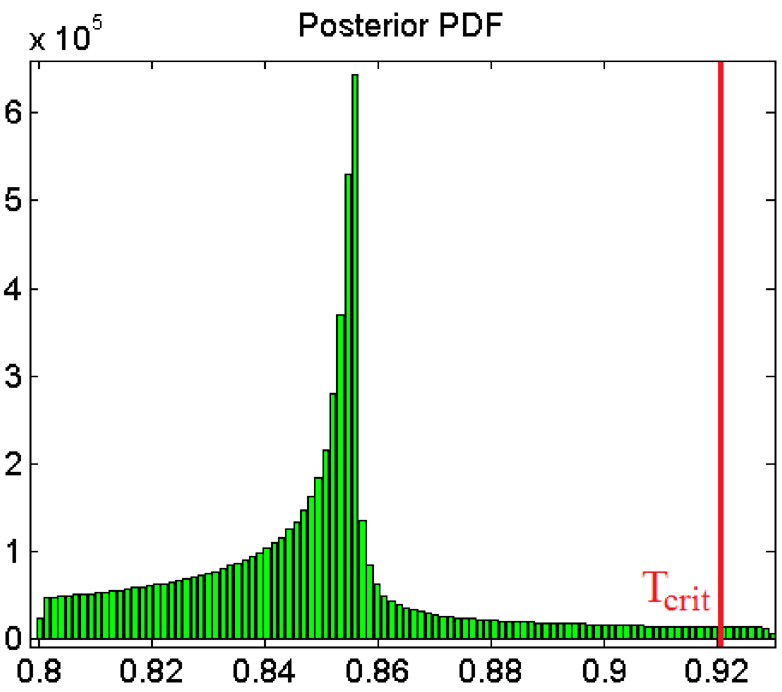

Ingestion of hot gas into inter-wheel region between rotors and spacers can reduce component life

*(Gap diameter size is essential)*

# Example 2: Results for Hot Gas Ingestion

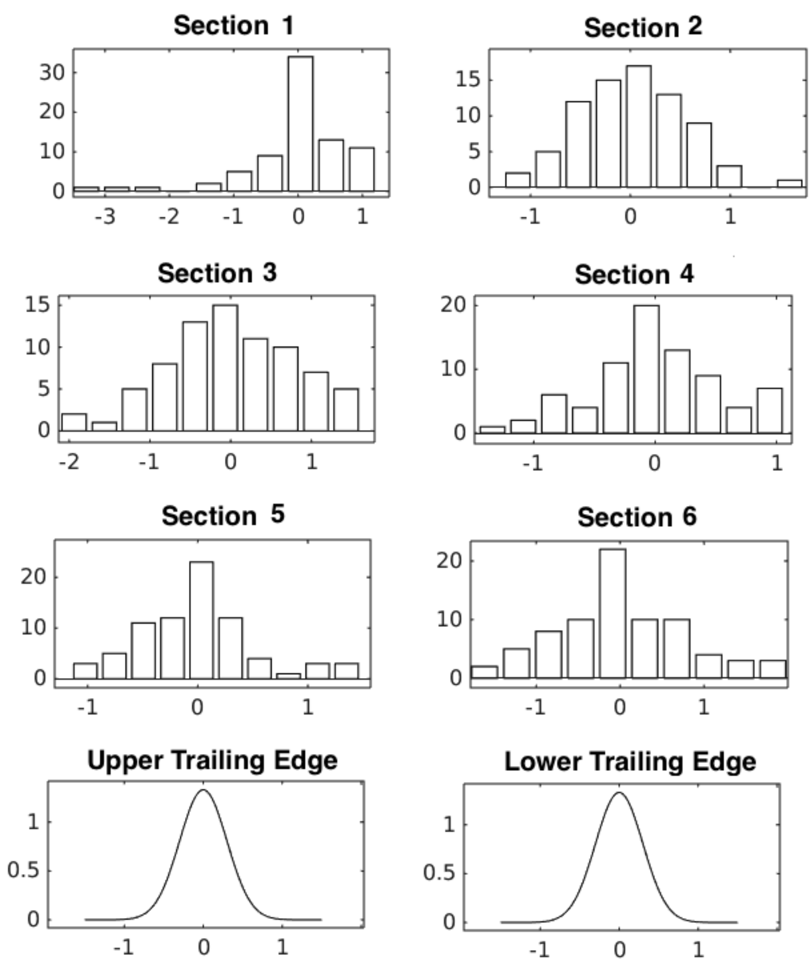
The input optimal collocation points indicate the relevant domain in the output



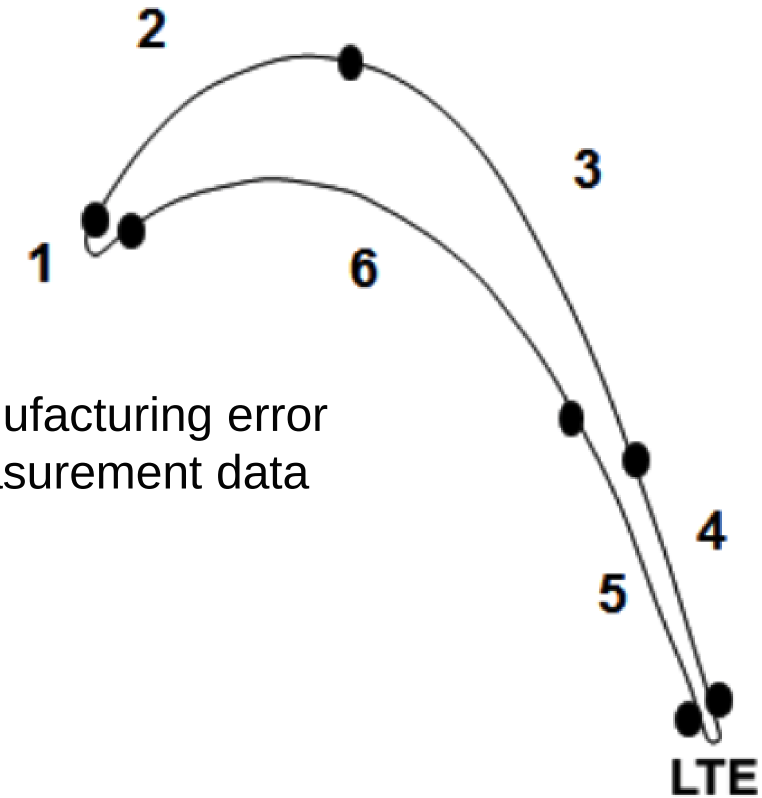

The output PDF was obtained by sampling the PCE with 1 billion samples

# Example 3: Manufacturing Uncertainty




LPT manufacturing error measurement data

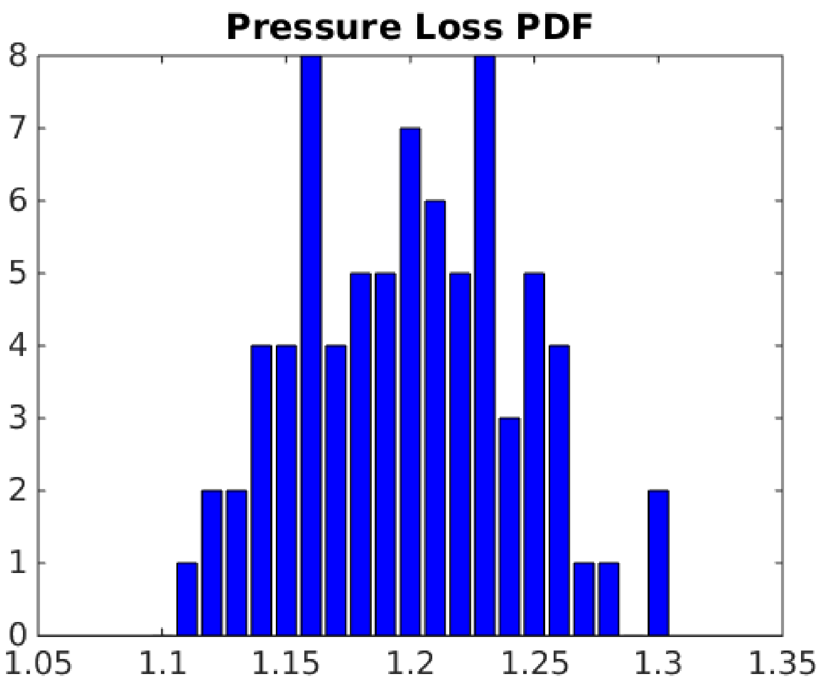
Two assumed PDF for trailing edge variation




Profile pressure losses are effected by local manufacturing uncertainty

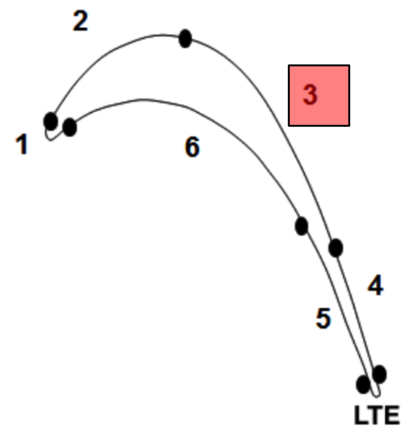
# Example 3: Results for Manufacturing Uncertainty

Only 17 model runs were needed for PDF and sensitivity evaluation




**Sensitivity Analysis**

| Section | Sobol index |
|---|---|
| 1 | 0.0205 |
| 2 | 0.0204 |
| 3 | 0.8587 |
| 4 | 0.0365 |
| 5 | 0.0020 |
| 6 | 0.0615 |
| UTE | 0.0003 |
| LTE | 0.0001 |

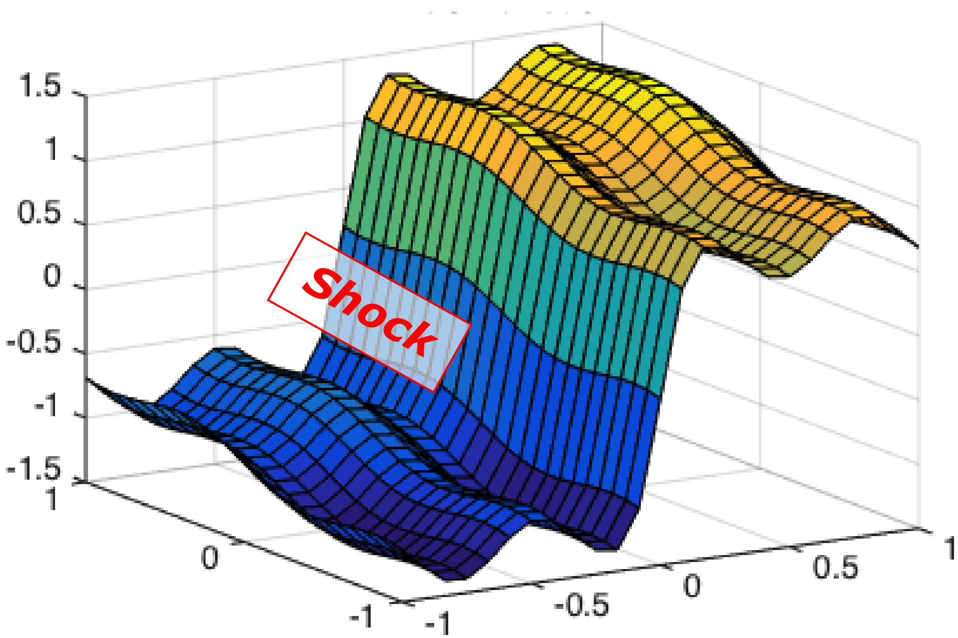



Highly efficient way to perform sensitivity analysis for random inputs

# Example 4: Discontinuities

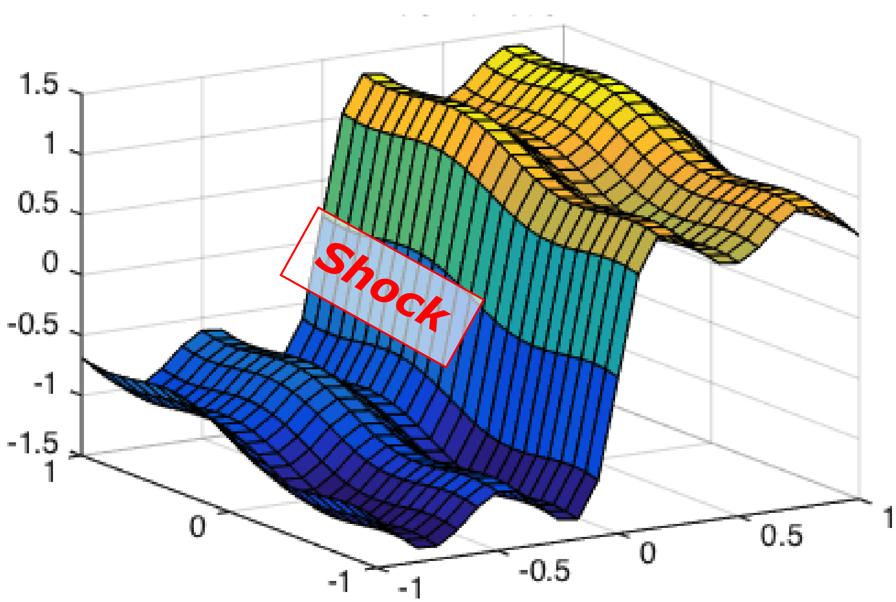
3D film-cooling
and shock interaction

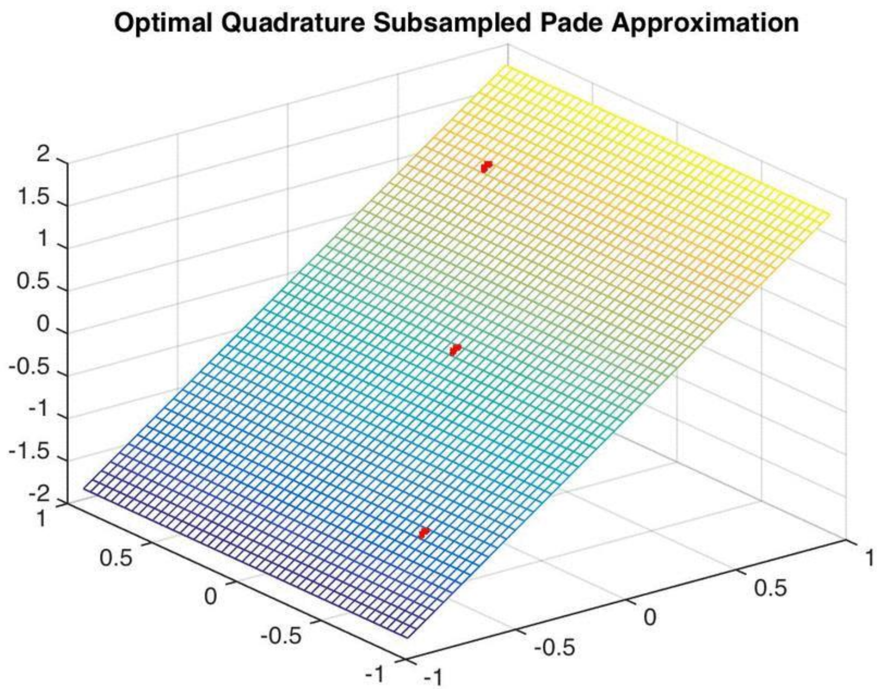
$$f(x_1,x_2)=\tanh(10x_1)+0.2\sin(10x_1)+0.3x_2+0.1\sin(5x_2)$$



# Example 4: Active Methods

$$f(x_1, x_2) = \tanh(10x_1) + 0.2\sin(10x_1) + 0.3x_2 + 0.1\sin(5x_2)$$



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# Conclusions

UQ is important in Aviation, this is why we are working on this

There are several models that can be developed, not all of them are applicable

We are moving towards numerical certification of Aircraft Engines performance and these variations need to be included

More than happy to collaborate