Feedback control of falling liquid films using a hierarchical model approach

Susana Gomes

Mathematics Institute, University of Warwick

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Who am I

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Falling liquid films

Control development using a hierarchy of models. *(today's talk)*
Developing models with efficient control as a goal.

Pedestrian dynamics

Inference and control considering the interplay between agent based models (experimental data) and PDE models crowds

Outline of this talk

Motivation and hierarchy of models

Control of weakly nonlinear models

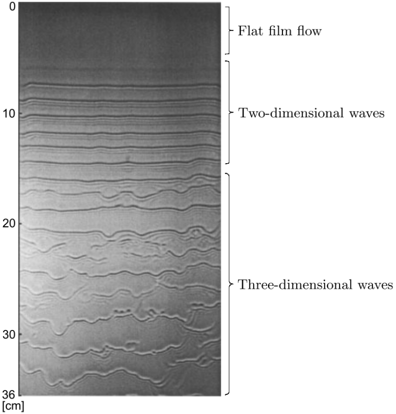
Controlling long-wave models

Application of control strategies to the full model

Discussion

Falling liquid films

Thin films flowing down an inclined plane are an everyday phenomenon



Industrial applications:

- Coating: prefer smooth interface
- Heat transfer: enhanced by waves

Images from Falling Liquid Films, Kalliadasis et al. 2012, Springer

Falling liquid films

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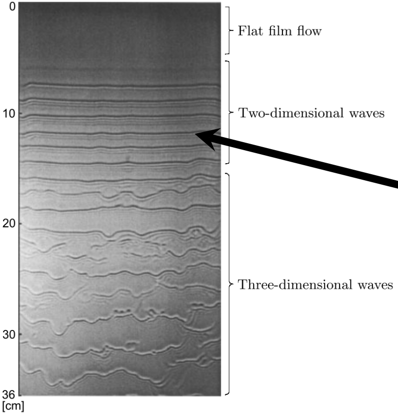
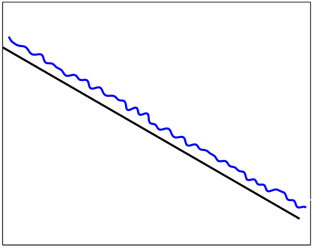


Under certain circumstances, the flat solution is unstable and travelling waves appear:



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Falling liquid films

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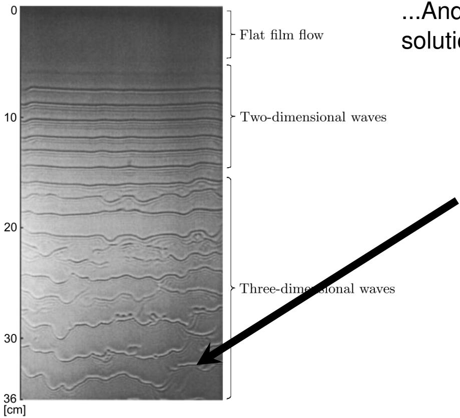
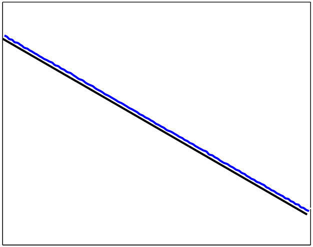


...And the film can evolve into a chaotic solution:



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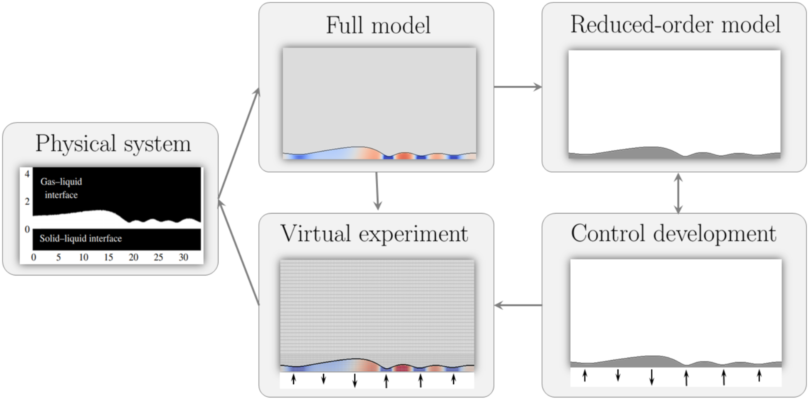
Our goal

Use **reduced order models** to develop controls that **suppress chaotic behaviour** and drive the system to any desired state.



Control methodology and hierarchy of models

Models and control methodology

Thin film flows are modelled using the **Navier–Stokes equations:**

$$
\begin{aligned}
R\left(u_t + uu_x + vu_y\right) &= -p_x + 2 + \Delta u, \\
R\left(v_t + uv_x + vv_y\right) &= -p_y - 2\cot\theta + \Delta v, \\
u_x + v_y &= 0,
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with appropriate boundary conditions at the interface $y = h(x, t)$.

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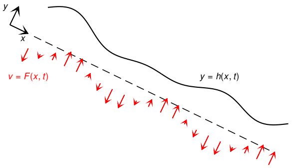
The controls are applied via same-fluid suction and injection at the wall $y = 0$

$$u = 0 \quad \text{and} \quad v = F(x,t).$$



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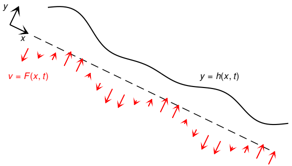

Problem:

Computationally expensive: parameter exploration and/or control design is prohibitively time consuming.
$\Rightarrow$ Explore reduced order models!

Exploring the long-wave nature of the problem

The thickness h_0^* of a falling liquid film is usually small when compared to the domain length L.

[1] For details on the derivation of the long-wave models, see [Thompson *et al*, JFM, 2016].

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- Galerkin expansion of u (**weighted-residual system**).

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Long-wave models are *highly nonlinear PDEs*

- complex dynamics
- impossible to obtain analytical results
- very stiff PDEs in certain parameter regimes

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Weakly nonlinear analysis

Further asymptotic analysis leads to *weakly nonlinear models*. These describe the evolution of a small perturbation u to a flat interface, $h(x,t) = 1$, close to the critical value for instabilities:

$$h(x,t) = 1 + \epsilon u(x,t), \qquad F(x,t) = \epsilon^2 f(x,t).$$

After changing to a moving frame and rescaling to a periodic $x \in [0, 2\pi]$ domain, we obtain the **controlled Kuramoto-Sivashinsky** (KS) equation:

$$u_t + \nu u_{xxxx} + u_{xx} + u u_x = f(x,t),$$

which lives in a periodic domain $x \in [0, 2\pi]$ and where $\nu = \left(\frac{2\pi}{L}\right)^2$.

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The KS equation is a “simple" PDE

Perfect environment for analytical results: I develop the control methodology as best as possible here, and propagate it back through the hierarchy.

Summary

Navier Stokes equations — $h(x,t)$, $\mathbf{u}(x,y,t)$, $F(x,t)$

↓ asymptotic expansions

Weighted residual model — $h(x,t)$, $q(x,t)$, $F(x,t)$

↓

Benney equation — $h(x,t)$, $F(x,t)$

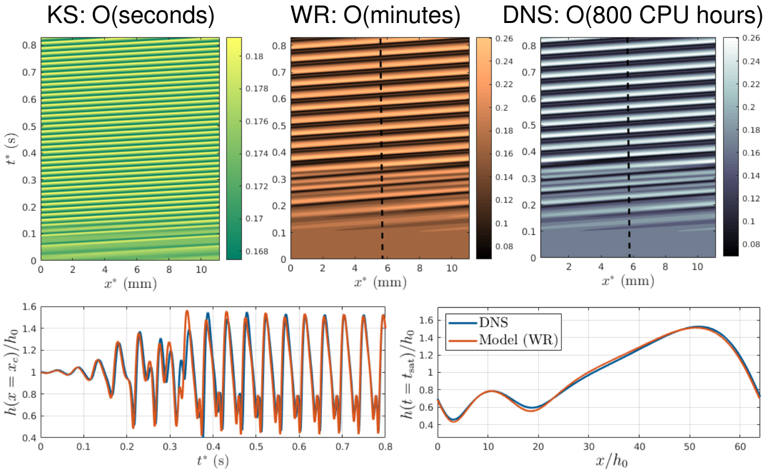
↓ weakly nonlinear analysis

Kuramoto–Sivashinsky equation — $u(x,t)$, $f(x,t)$,
$h(x,t) = 1 + \epsilon u(x,t)$

Question:

Can we design controls for the simplest models and build up on these to create efficient controls for the full system?

Summary



See R. Cimpeanu, SNG, D.T. Papageorgiou, arXiv:2008.12746 (2020)

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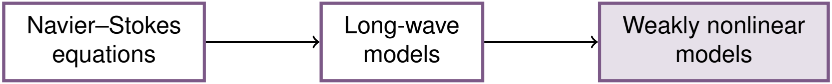
Control of the KS equation



SNG, M. Pradas, S. Kalliadasis, D.T. Papageorgiou, G.A. Pavliotis, Phys Rev E 92:022912 (2015)
SNG, D.T. Papageorgiou, G.A. Pavliotis, IMA J. Appl. Math. 82, 158-194 (2017)

The Kuramoto-Sivashinsky equation

The uncontrolled KS equation in a periodic domain $x \in [0, 2\pi]$ is

$$u_t + uu_x + \nu u_{xxxx} + u_{xx} = 0, \qquad \nu = \frac{2\pi}{L}.$$

[2] Tadmor 1986, Kevrekidis *et al* 1990, Otto 2009

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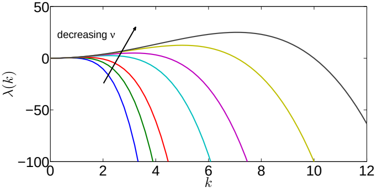
The zero solution is linearly unstable:

$$u_t + uu_x + \nu u_{xxxx} + u_{xx} = 0$$
$$u(x,t) \sim e^{ikx+\lambda t} \Rightarrow \lambda(k) = -\nu k^4 + k^2$$



So, without the nonlinear term, the solutions would grow exponentially!

[2] Tadmor 1986, Kevrekidis *et al* 1990, Otto 2009

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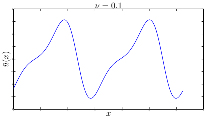
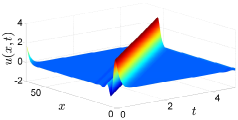
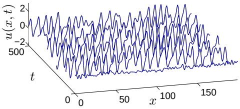
Nonlinear term guarantees bounds on the energy of solutions[2]
$\Rightarrow$ existence of steady states and travelling wave solutions



Steady state
$\nu = 0.1$, $\mu = \delta = 0$



Travelling wave
$\nu = 0.01$, $\mu = \delta = 0$



Chaotic solution
$\nu \approx 9 \times 10^{-4}$

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The Kuramoto-Sivashinsky equation

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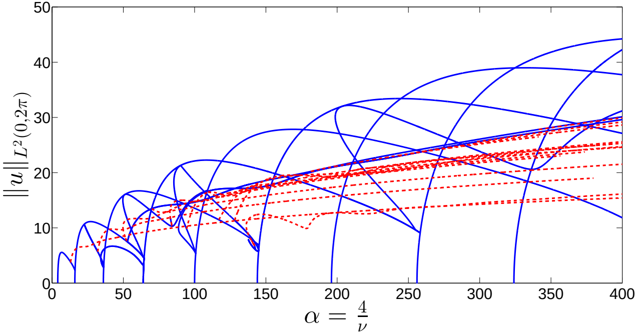


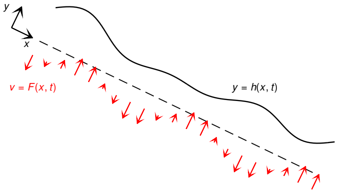
Red: Travelling waves, Blue: Steady states

[2] Tadmor 1986, Kevrekidis *et al* 1990, Otto 2009

Controlled KS equation

We use point actuated controls:

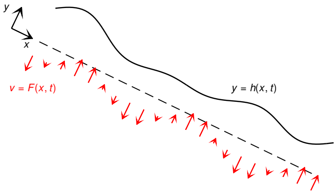
$$u_t + \nu \partial_x^4 u + \partial_x^2 u + u u_x = \sum_{i=1}^{\mathbf{m}} b_i(x) f_i(t)$$



where $b_i(x) = \delta(x - x_i)$, $f_i(t)$ are the control rules to be determined and **m** is the number of control actuators.

Controlled KS equation

And discretise using spectral methods:



$$u_t - Au + N(u, u) = BF$$

where $A = \mathrm{diag}(-\nu k^4 + k^2)$, $B_{kl} = \int_0^{2\pi} b_l(x) e^{ikx}\, dx$, and where N and F discretise the nonlinear term and the controls.

Controlled KS equation

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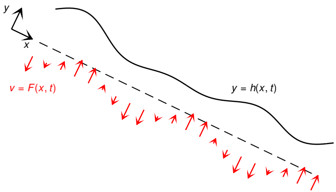

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Proposition [Gomes *et al*, IMA J. Appl. Math., 2016]

Let $\bar{u}$ be a linearly unstable steady state or travelling wave solution of the KS equation and let $2l + 1$ be the number of unstable eigenvalues of the system $u_t = \mathcal{A}u$, *i.e.*,

$$l + 1 \geq \frac{1}{\sqrt{\nu}} > l.$$

If $m = 2l + 1$ and there exists a matrix $K \in \mathbb{R}^{m \times m}$ such that all of the eigenvalues of the matrix $A + BK$ have negative real part, then the state feedback controls $F = K(u - \bar{u})$ stabilise $\bar{u}$.

Sketch of the proof

- Write $u = \bar{u} - v$ and obtain controlled equation for v:

$$v_t + \nu v_{xxxx} + v_{xx} + v v_x + (\bar{u} v)_x = \sum_{i=1}^{m} b_i(x) f_i(t)$$

- Stabilise zero solution of equation for v:
  - ⋆ Discretise this equation

$$\frac{dv}{dt} = Av + N(v, v) + G(\bar{u}, v) + BF,$$

  - ⋆ Stabilise the linear operator: choose $F = Kv = K(u - \bar{u})$ where K is such that the eigenvalues of $A + BK$ have negative real part. *in fact, eigenvalues need to be smaller than* $-\inf \frac{|\bar{u}_x|}{2}$
  - ⋆ Use a Lyapunov argument and bounds of the solutions to show these controls stabilise the full equation.

Results and discussion

The KS equation is solved using *spectral methods in space* and a second order *BDF* (IMEX) scheme to time-step.

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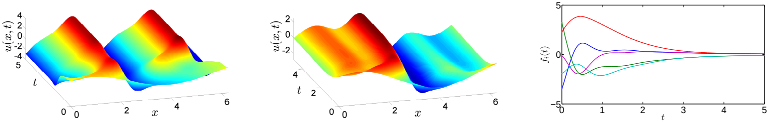
- Steady state for $\nu = 0.1115$: uncontrolled solution (left), controlled to a chosen steady state (middle) and controls applied (right).



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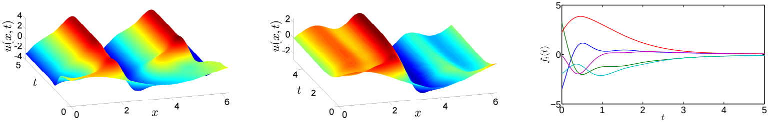

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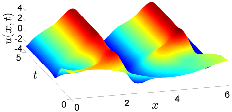
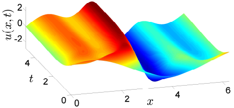
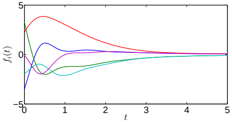




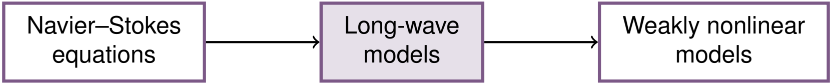


- Travelling wave for $\nu = 0.01$ Travelling wave

- The controls are robust to uncertainty in the parameters
  - ⋆ wrong number of unstable modes/controls Video
  - ⋆ wrong value of ν Video

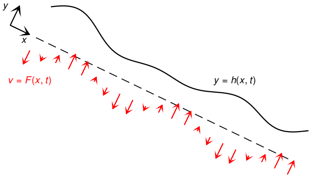
For the videos in this slide please see ***this link****.*

Control of long-wave models



A.B. Thompson, SNG, G.A. Pavliotis, D.T. Papageorgiou, Phys Fluids 28:012107 (2016)
For videos in this section please contact me

Long-wave models



Long-wave models consist of a **mass conservation equation**

$$h_t + q_x = F(x, t),$$

coupled to an equation for $q(x, t)$:

- **Benney equation**

$$q(x, t) = \frac{h^3}{3}\left(2 - 2h_x \cot\theta - \frac{h_{xxx}}{C}\right) + R\left(\frac{8h^6 h_x}{15} - \frac{2h^4 F}{3}\right)$$

- **weighted-residual model**

$$\frac{2}{5} R h^2 q_t + q = \frac{h^3}{3}\left(2 - 2h_x \cot\theta - \frac{h_{xxx}}{C}\right) + R\left(\frac{18q^2 h_x - 34hqq_x}{35} + \frac{hqF}{5}\right)$$

From KS to long-wave models

From the previous results, controls should be of the form

$$F(x,t) = \mathcal{F}(h(x,t) - H(x,t))$$

where H is the desired state and $\mathcal{F}$ is some function.

From KS to long-wave models

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I consider 3 types of controls:

Case 1: Observe h everywhere and apply controls everywhere

Case 2: Observe h everywhere and apply controls at finite number of points

- Note that in the weighted residuals model, this requires observation of h *and* q everywhere.

Case 3: Observe h and apply controls at a finite number of points

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Numerical considerations

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Cases 2 and 3 involve *approximations of δ-functions* and the numerical discretisation is no longer sparse.

Numerical interlude

Both long-wave models are **highly nonlinear** and **stiff**.

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Both long-wave models are **highly nonlinear** and **stiff**. The numerical solutions presented here were solved by

- either spectral discretisation or finite differences in space
- 2^{nd} order BDF scheme (IMEX) in time (this section) or adaptive time-stepping using MATLAB's *ode15s* (next section)

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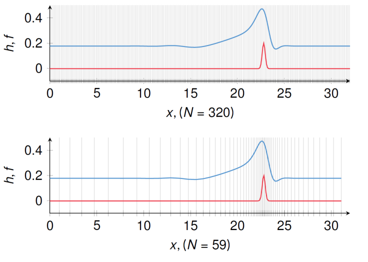
Using adaptive grids (O. Holroyd's URSS project)

Realistic test-cases make the above numerical schemes prohibitively slow.
$\Rightarrow$ Developed adaptive grid methods to overcome this:



Location of control actuators requires small spatial grid.

Similar level of accuracy with computational work focused *only* on important regions.

Numerical interlude

Both long-wave models are **highly nonlinear** and **stiff**. The numerical solutions presented here were solved by

- either spectral discretisation or finite differences in space
- 2^{nd} order BDF scheme (IMEX) in time (this section) or adaptive time-stepping using MATLAB's *ode15s* (next section)

Using adaptive grids (O. Holroyd's URSS project)

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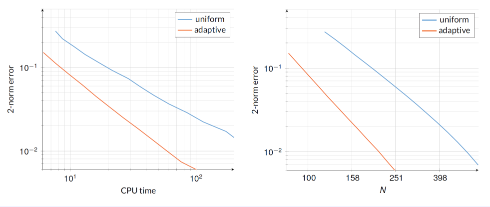

2.5 times fewer gridpoints to achieve error of 2×10^{-2}.

5-fold improvement in CPU time

Stability of the uniform state - Case 1

We use proportional controls

$$F(x,t) = -\alpha\,[h(x,t) - 1]\,, \quad \alpha \geq 0,$$

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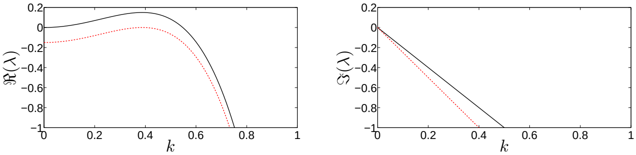

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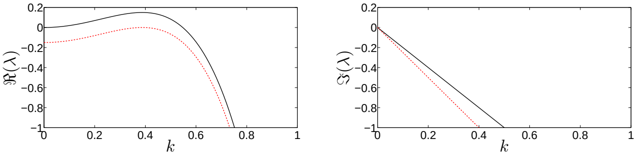
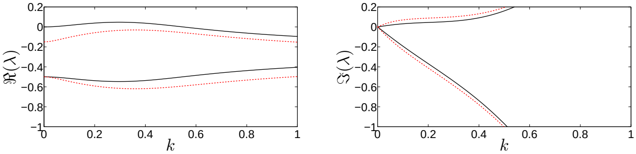


The **weighted-residual model** has two eigenvalues:



$\alpha = 0$, $\alpha = \alpha_B$ Video

Case 2 - Pole placement with full observations

This is equivalent to the controls employed in the KS equation. We use

$$F(x,t) = \sum_{m=1}^{M} b_m(x) f_m(t),$$

with the f_m proportional to $h-1$: $F = BK(h-1)$.

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(Controls **sometimes** work in the weighted-residual model.)

LQR for the weighted-residual model requires observations of both h and q.

Estimate for q	Maximum eigenvalue
Observe q	-5.62×10^{-2}
Estimate $q = \frac{2}{3}$	-5.09×10^{-3}
Estimate $q = \frac{2h^3}{3}$	-5.64×10^{-2}

Stability of the uniform state - Case 3

We now consider controls of the form

$$F(x,t) = \sum_{m=1}^{M} b_m(x) f_m(t),$$

where $f_m(t)$ are to be determined from P observations of $h(x,t)$:

$$y_p(t) = (h(x_p,t) - 1)\,dx, \quad p = 1, \ldots, P.$$

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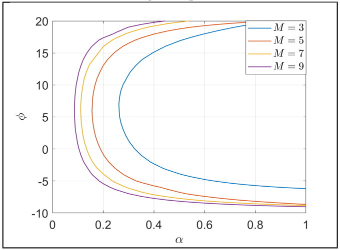
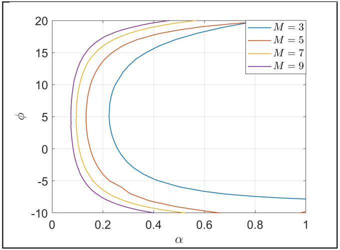
If $P = M$, we choose $x_p = x_m - \phi$, i.e., $f_m(t) = -\alpha(h(x_m - \phi, t) - 1)$. When $b_m(x) = \delta(x - x_m)$, we can write an eigenvalue problem analogous to Case 1.

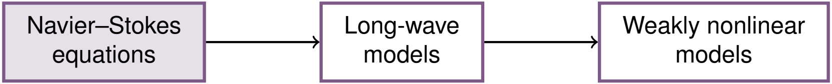
Benney equation



Weighted residual model



Control of the full model



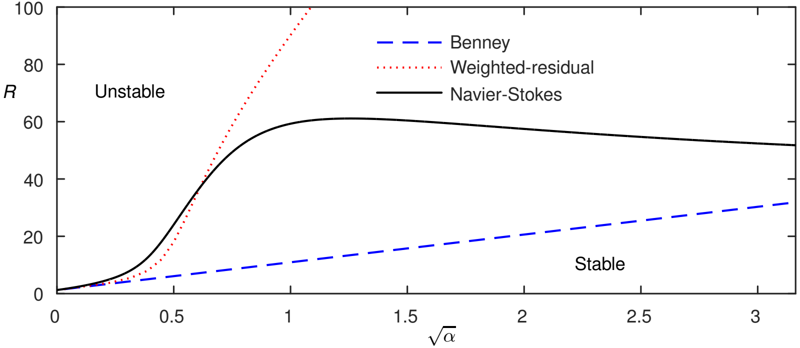
R. Cimpeanu, SNG, D.T. Papageorgiou, Active control of liquid film flows: beyond reduced-order models, arXiv:2008.12746 (2020)

Linear stability of the full system

Linear stability of the flat solution in the 2D Navier–Stokes equations (Orr–Sommerfeld problem) indicates that when

$$F(x,t) = -\alpha[h(x,t) - 1], \quad \alpha \geq 0,$$

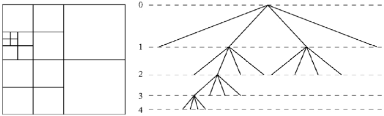
are applied to the full system...

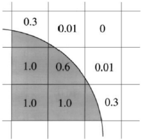


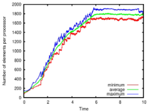
... the values of α that stabilise the flat solution for the long wave models will do so for the full system too.

Direct Numerical Simulations

To validate the theoretical results and for nonlinear computations, we use the **Gerris Flow Solver**, a highly versatile volume-of-fluid package, designed with multiphisics problem solving capabilities.








```
# shift the origin of the reference box to (0,0)
2 1 GfsRiver GfsBox GfsGEdge { x = 0.5 y = 0.5 } {
    # the domain is 3.402 m X 6.804 m
    # units for time are seconds
    PhysicalParams { L = 3.402 g = 9.81 }
    Refine 6

    # maintain the Zb1 variable using the GTS surface
    VariableFunction Zb1 bathy.gts

    # the initial water level is at z = 0, so the depth P is...
    Init {} {
        P = MAX (0., -Zb1)
    }

    # use a Sweby limiter rather than the default minmod which is too
    # dissipative
    AdvectionParams { gradient = gfs_center_sweby_gradient }

    # adapt down to 9 levels based on the slope of the (wet)
    # free-surface and with a tolerance of 1 mm
    AdaptGradient { istep = 1 } {
        cmax = 1e-3
        cfactor = 2
        maxlevel = 9
        minlevel = 6
    } (P < DRY ? 0. : P + Zb)

    # at each timestep
    Init { istep = 1 } {
        # Add a "shelf" to simulate the wall on the right-hand-side boundary
        Zb = (x > 5.448 ? 0.13535 : Zb1)
        # read in the experimental timeseries in variable 'input'
        input = input.cgd
        # implicit quadratic bottom friction with coefficient 1e-3
        U = (P > DRY ? U/(1. + dt*1e-3*Velocity/P) : 0.)
        V = (P > DRY ? V/(1. + dt*1e-3*Velocity/P) : 0.)
        P = (P > DRY ? P : 0.)
    }
```

Effect of controls in the full model

Direct application of the controls developed for the long-wave models in the Navier–Stokes equations is not feasible.

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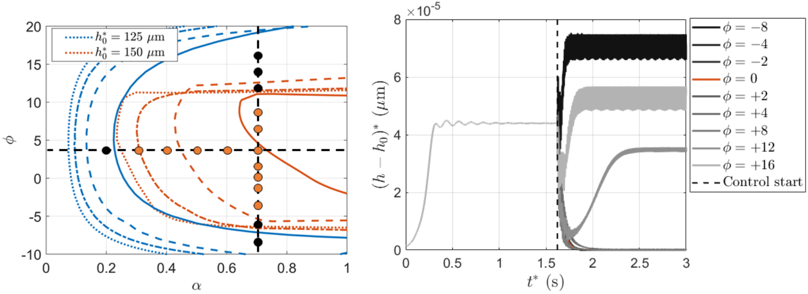
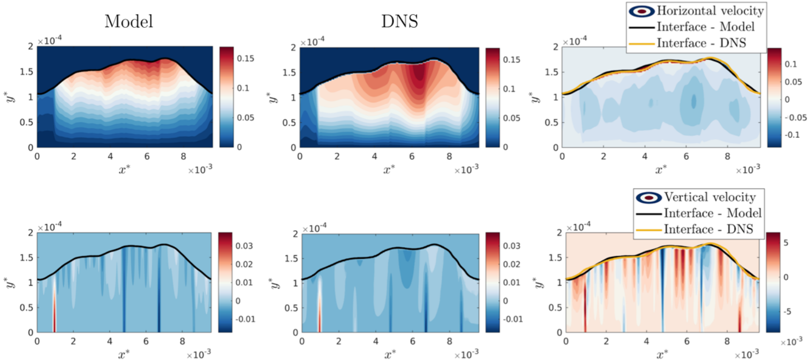
Controls applied to an aqueous-glycerol solution.



Left: stability regions for two values of h_0^* and different number of controls.
Right: Controlled solution for $h_0^* = 150\mu m$: $R \approx 28$, $C = 0.0018$, $\theta = \frac{\pi}{3}$, $M = 5$ and $\alpha = 0.7$.

Effect of controls in the full model

Comparison of the effect of controls in the model and DNS.



Top: Horizontal velocity and bottom: vertical velocity, for Left: model, middle: DNS, and right: difference between the two.

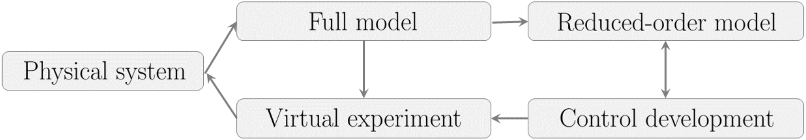
Full control   Point actuated   Steady Pattern   Travelling pattern 1

Final remarks

Summary

I presented a feedback control methodology applicable to a hierarchy of models for falling liquid films.

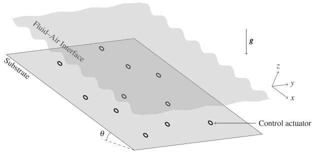
- Theoretical results proved for weakly nonlinear models
- Validity of the results is shown via extensive numerical simulations across the whole hierarchy of models.
- Controls can be applied everywhere or at discrete locations, and can stabilise any desired state.



Some extensions

Similar results can be obtained in the lowest rung of the hierarchy[2]:

- when including other physical effects such as electric fields or dispersion
- for the two-dimensional KS equation



[2] R.J. Tomlin, SNG, et al, SIAM J Appl Dyn Sys 2019, R.J. Tomlin, SNG, IMA J. Appl. Math. 2019

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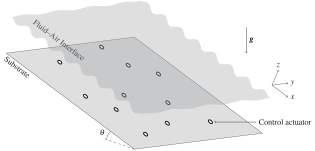

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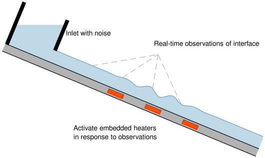

- using temperature as the control[3]
- using electric fields

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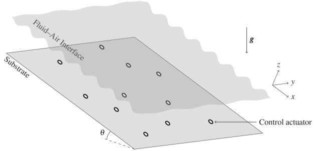

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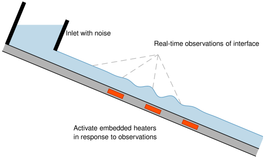

- Incorporating data from *higher order models* into controls developed on lower rungs
- Use the above to incorporate *observations from experiments* into real-time control.

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Thank you for your attention!

Susana N. Gomes

https://warwick.ac.uk/fac/sci/maths/people/staff/gomes

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