Warwick Centre for Predictive Modelling Seminar Series

Sampling constrained probability distributions using Spherical Augmentation

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R0.12, Ramphal Building, Ground Floor

Abstract: Statistical models with constrained probability distributions are abundant in machine learning. Some examples include regression models with norm constraints (e.g., Lasso), probit models, many copula models, and Latent Dirichlet Allocation models. Bayesian inference involving probability distributions confined to constrained domains could be quite challenging for commonly used sampling algorithms.
In this paper, we propose a novel augmentation technique that handles certain type of constraints by mapping the constrained domain to a sphere in the augmented space and defining Monte Carlo dynamics on the sphere. The original boundary corresponds to poles, arcs or the equator of the sphere. This way, the proposed methods handle the constraints implicitly by moving freely on the sphere generating proposals that remain within boundaries when mapped back to the original space.
The proposed Spherical Augmentation technique provides a mathematically natural and computationally efficient framework for sampling probability distributions with constraints defined by vector norm. The application to some typical Markov Chain Monte Carlo methods, such as Hamiltonian Monte Carlo, Lagrangian Monte Carlo shows advantage over some traditional/state-of-the-art methods like exact HMC for truncated multivariate Gaussians, stochastic gradient Riemannian Langevin dynamics on probability simplex, etc. The computational advantage has been evidenced by several examples including truncated Gaussian, Bayesian Lasso, Bayesian bridge regression, reconstruction of quantized stationary Gaussian process and Latent Dirichlet Allocation for topic modeling. We believe such augmentation technique will open a gate for more research on handling constraint problems in statistics and machine learning.
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