




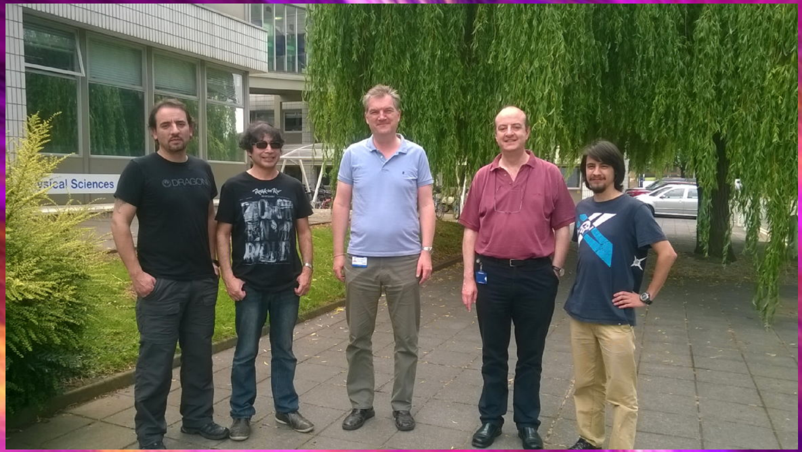

Quantum engineering for electrons and spins

Rudolf A Römer, Physics

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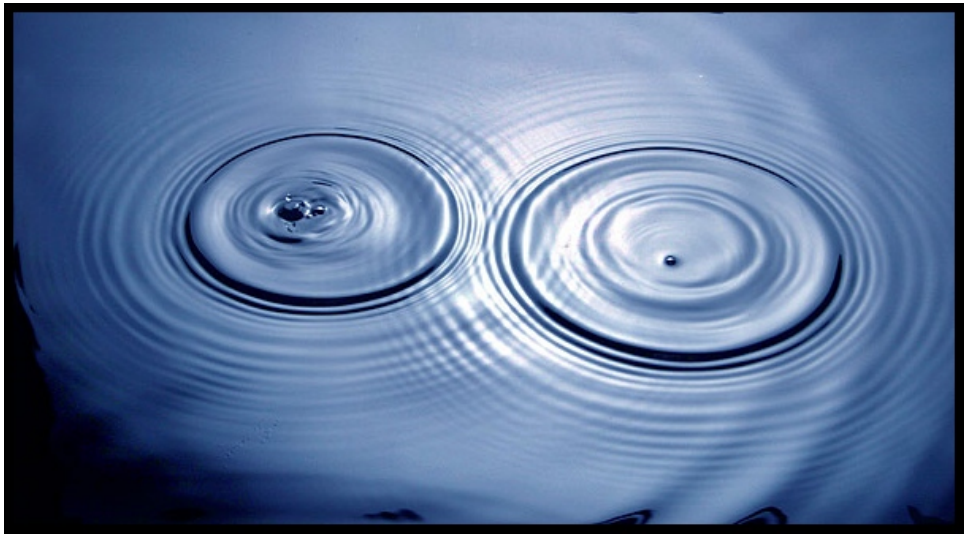
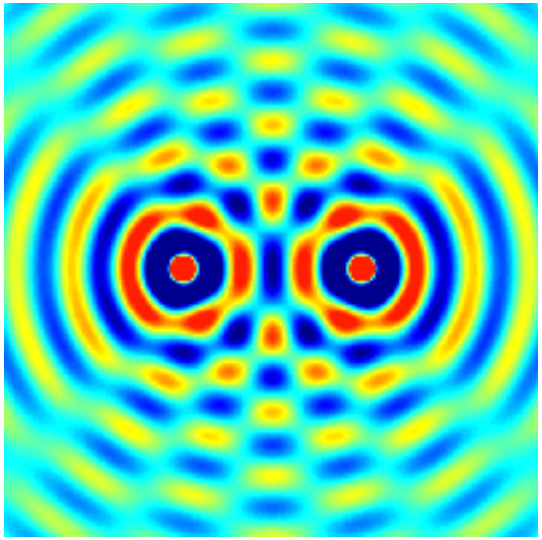
The physics of waves

- Waves interfere due to the superposition principle
- Wave interference can be destructive or constructive

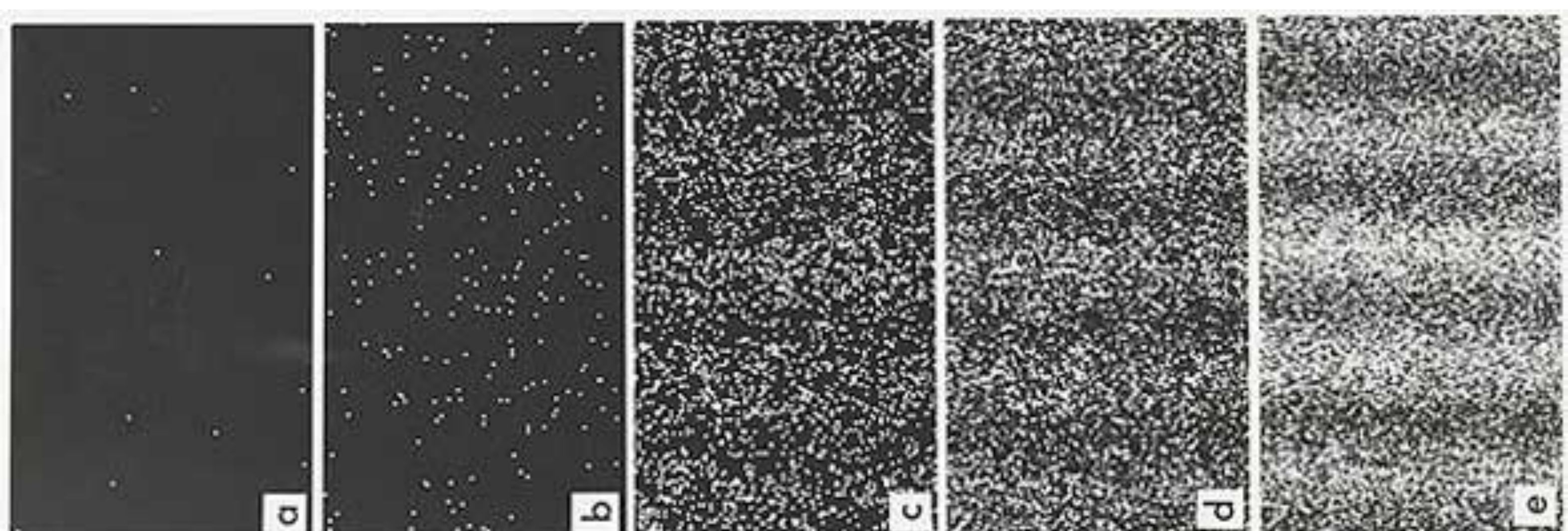




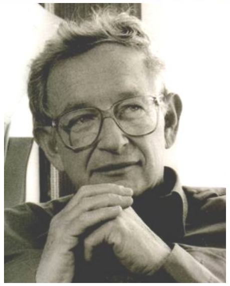
Interference of electrons from a double slit

http://www.hitachi.com/rd/portal/highlight/quantum/index.html#anc04

(Tonomura, 1980’s)



Interference from disorder: Anderson localization



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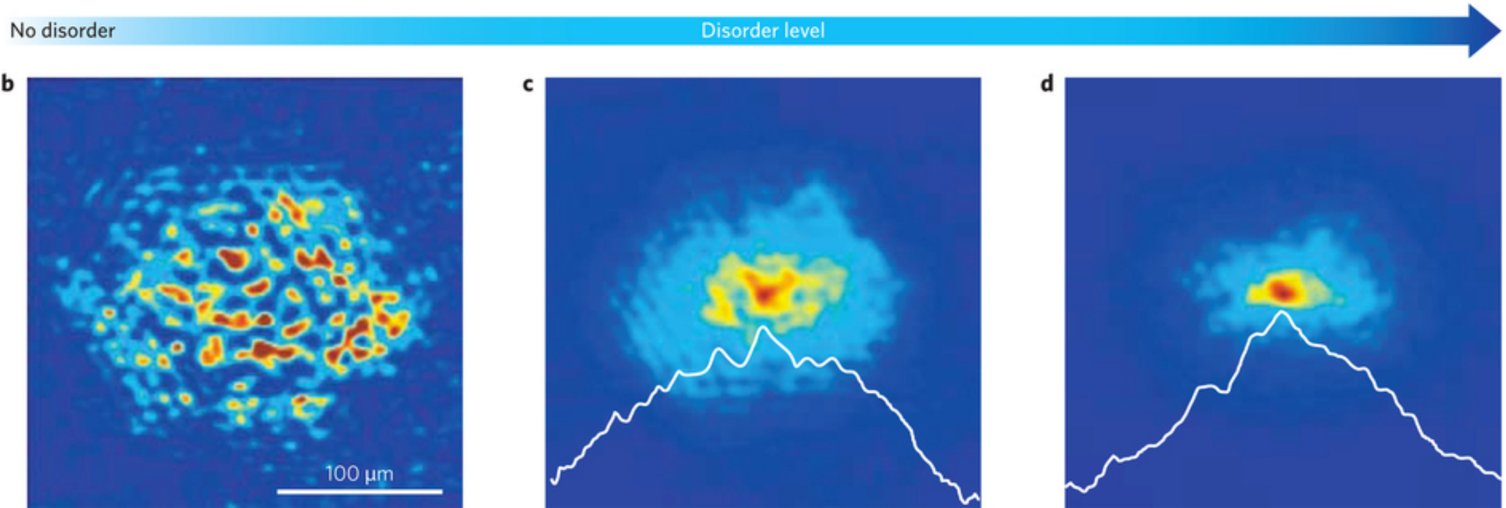
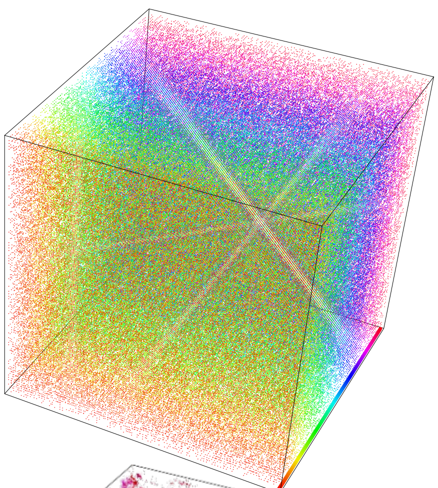
Light, water, sound (ultra), electrons, atoms …

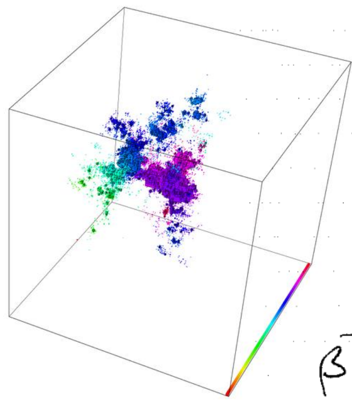
J. Billy *et al.*, Direct observation of Anderson localization of matter waves in a controlled disorder, Nature (2008) 453, 891-894; G. Roati *et al.*, Anderson localization of a non-interacting Bose-Einstein condensate, Nature (2008) 453, 895-898

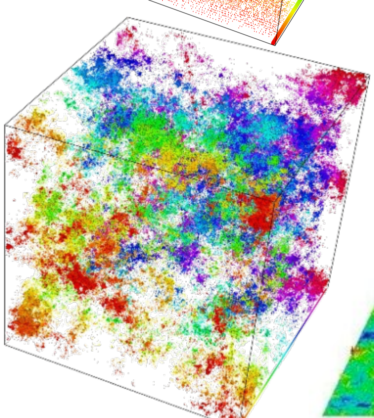


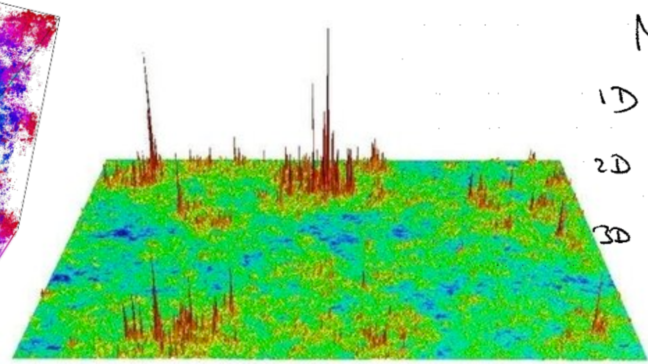
Segev, M., Silberberg, Y., & Christodoulides, D. N. (2013). Anderson localization of light. *Nat Photon*, *7*(3), 197–204.

A transition in $D \geq 3$, none for $D \leq 2$

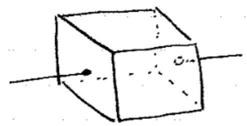








The gang of 4 argument:



$R_3 = \rho \frac{L}{L^2} = \rho L^{-1}$

$R_2 = \rho \frac{L}{L} = \rho$

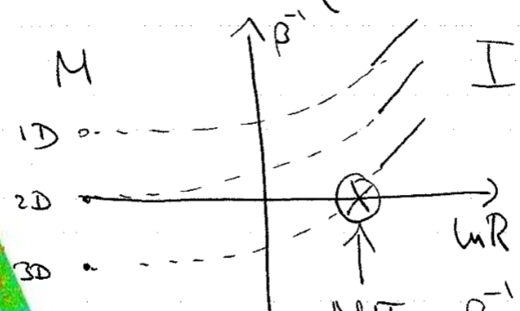
$R_1 = \rho \cdot L$

→ in a metal: $R_d = \rho L^{2-d}$

in an insulator:

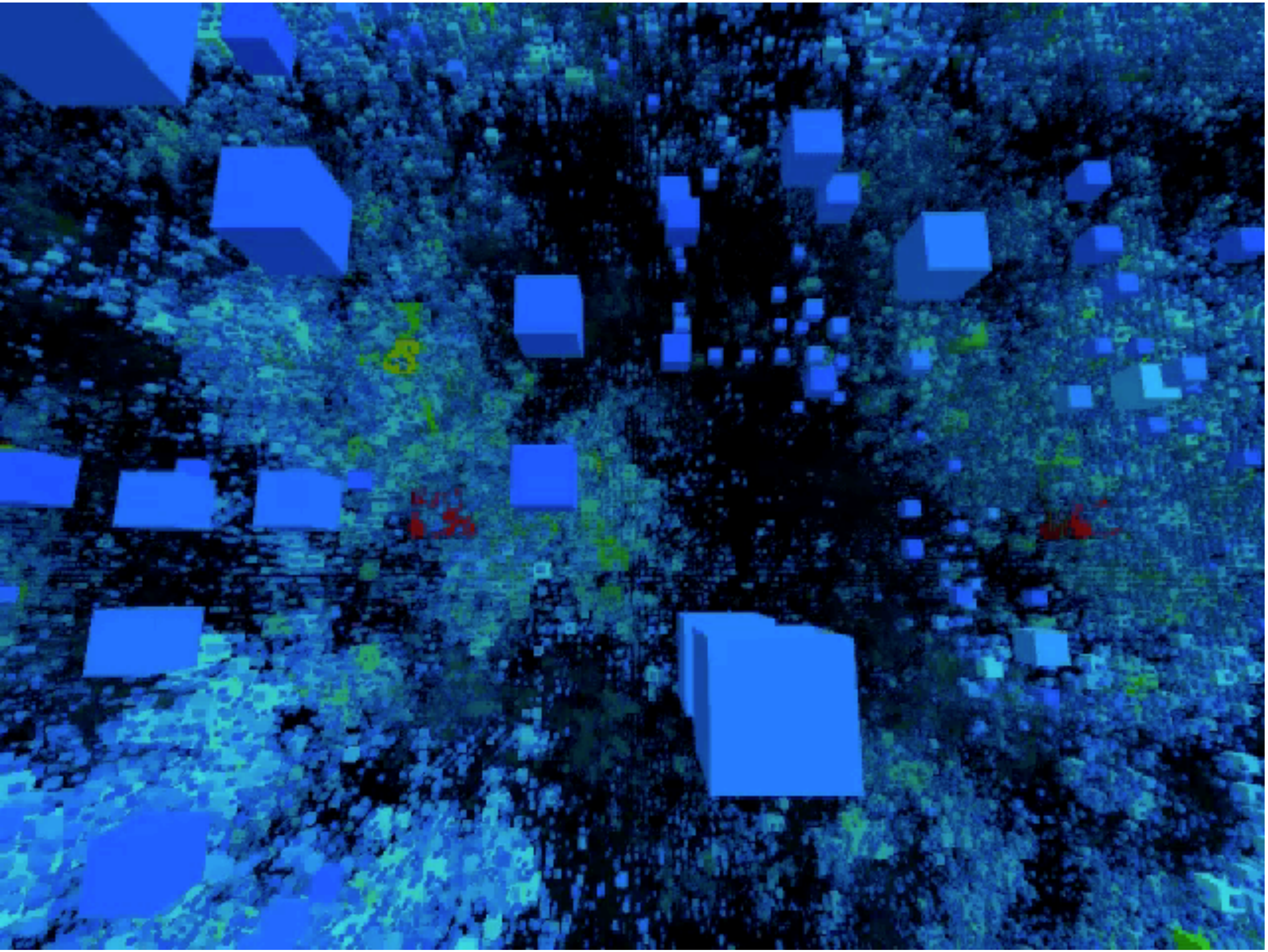
$R = R_0 e^{L/\lambda} \to \infty$ as $L \to \infty$

$$\beta^{-1} = \frac{L}{R}\frac{dR}{dL} = \frac{d \ln R}{d \ln L} = \begin{cases} (2-d) & \text{metal} \\ \ln R & \text{insulator} \end{cases}$$



MIT: $\beta^{-1} = 0$!
$R(L) = \tilde{R}_0$

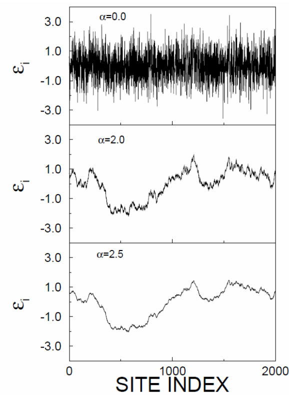
Abrahams E., Anderson P. W., Licciardello D. C. and Ramakrishnan T. V. 1979 *Phys. Rev. Lett.* **42** 673



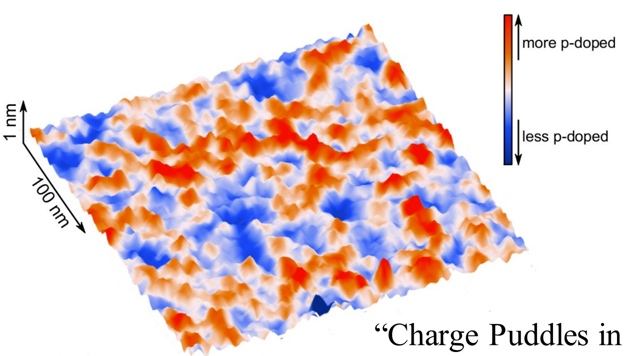
How to “cheat” the gang of 4!

- Many-body physics
  - Magnetism, superconductivity, …
- Correlated randomness
  - Changes of “universality”
  - Perfect correlation = clean system
  - Short-range (irrelevant in large systems) versus long-range correlation (relevant whenever correlation length comparable/larger than system)

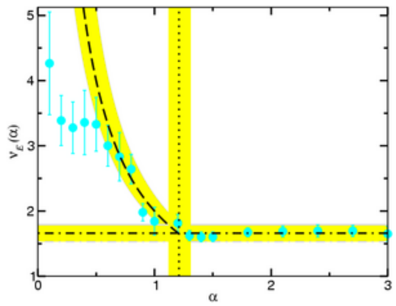
Goldsborough, A. M., Römer, R. A., (2014). Self-assembling tensor networks and holography in disordered spin chains. *Physical Review B*, *89*(21).



de Moura, F. A. B. F., & Lyra, M. L. (1998). Delocalization in the 1D Anderson Model with Long-Range Correlated Disorder. *Physical Review Letters*, *81*(17), 3735–3738.



“Charge Puddles in Graphene Near the Dirac Point”, S. Samaddar, I. Yudhistira, S. Adam, H. Courtois, and C.B. Winkelmann, arXiv:1512.05304



Ndawana, M. L., Römer, R. A., & Schreiber, M. (2004). The Anderson metal-insulator transition in the presence of scale-free disorder. *Epl*, *68*(5), 678–684.

Controlled engineering of extended states in disordered systems

$$\mathcal{H} = \sum_{x=1}^{L_x} \mathbf{c}_x^\dagger \boldsymbol{\epsilon}_x \mathbf{c}_x + \sum_{x=1}^{L_x-1} (\mathbf{c}_x^\dagger \mathbf{t} \mathbf{c}_{x+1} + \mathbf{c}_{x+1}^\dagger \mathbf{t} \mathbf{c}_x)$$

- Transfer-matrix approach:
  - Rewrite $H\psi = E\psi$ as

$$(E\mathbb{1} - \boldsymbol{\epsilon}_x)\boldsymbol{\Psi}_x = t(\boldsymbol{\Psi}_{x+1} + \boldsymbol{\Psi}_{x-1})$$

$\boldsymbol{\epsilon}_x$ is actually disorder matrix $\epsilon_{x,y}$

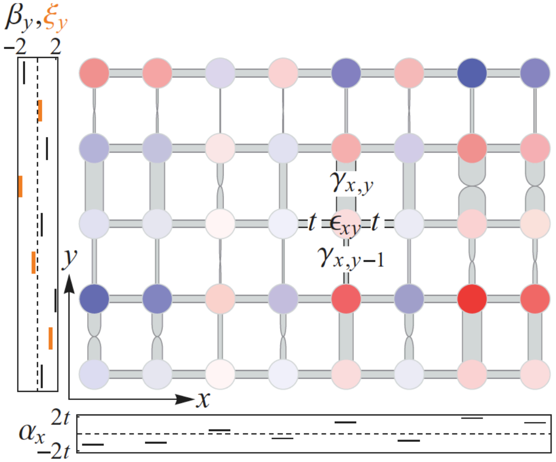
Rodriguez, A., Chakrabarti, A., Römer, R. (2012). Controlled engineering of extended states in disordered systems. *Physical Review B*, *86*(8).

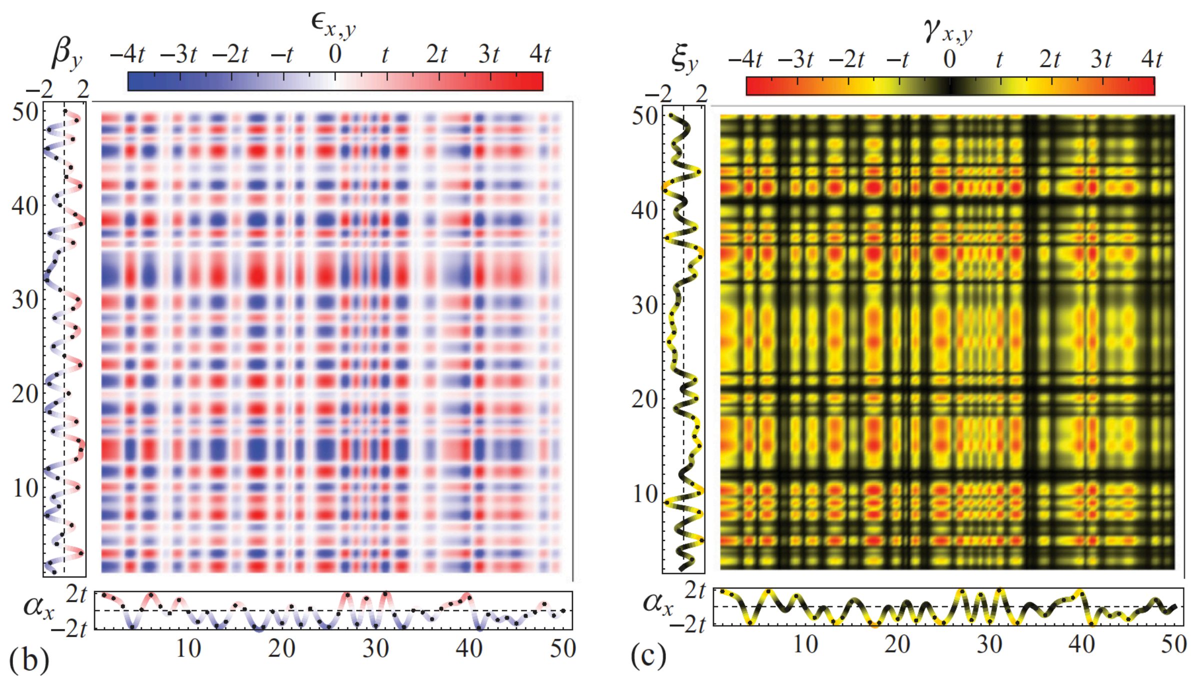
Resonance condition

$\epsilon_{x,y} = \alpha_x \beta_y$ for onsite potentials

$\gamma_{x,y} = \alpha_x \xi_y$ for transverse kinetic energy

3N random numbers for N^2 terms – correlation!





A transformation

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- We can factorize matrix $\epsilon_x = \alpha_x P$, s.t. P does not depend on x.
- $p = U^{-1} P U$ diagonal matrix of eigenvalues of P, U the vector of eigenvectors.
- Then with $\Phi_x = U^{-1} \Psi_x$, we can write

$$(E\mathbb{1} - \alpha_x \boldsymbol{p})\boldsymbol{\Phi}_x = t(\boldsymbol{\Phi}_{x+1} + \boldsymbol{\Phi}_{x-1})$$

Trafo cont'd:

Explicitly, this gives

$$(E - \alpha_x p_1)\phi_x^{(1)} = t\left(\phi_{x+1}^{(1)} + \phi_{x-1}^{(1)}\right)$$

$$\vdots$$

$$(E - \alpha_x p_c)\phi_x^{(c)} = t\left(\phi_{x+1}^{(c)} + \phi_{x-1}^{(c)}\right)$$

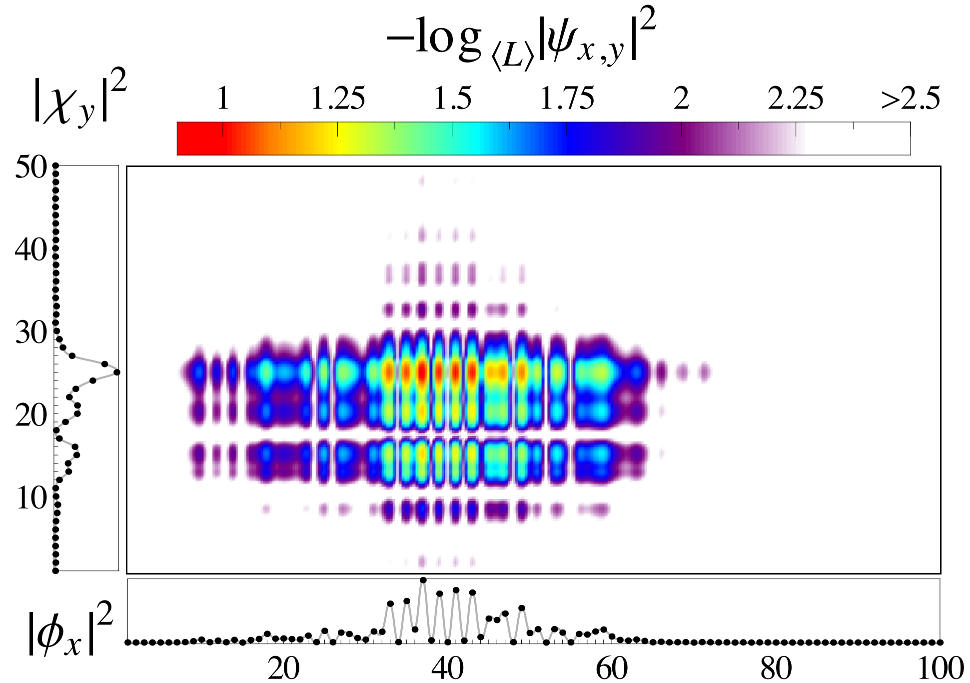
$$\vdots$$

$$(E - \alpha_x p_{L_y})\phi_x^{(L_y)} = t\left(\phi_{x+1}^{(L_y)} + \phi_{x-1}^{(L_y)}\right)$$

Suppose that one of the $p_i \approx 0$, then that equation correspond to a near-perfectly conducting channel!

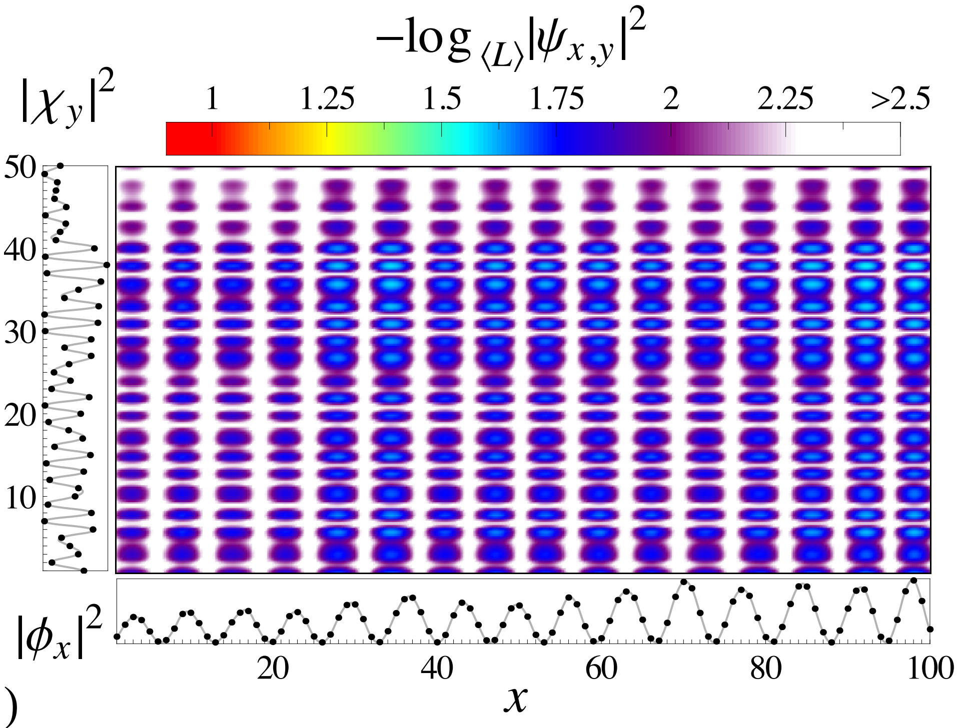
Quantum engineering with electrons

Choosing the distributions of randomness in $\{\alpha\}$, $\{\beta\}$ and $\{\xi\}$, we can then engineer states with any confinement in (*x*,*y*), (*x*) or (*y*) independently or none!

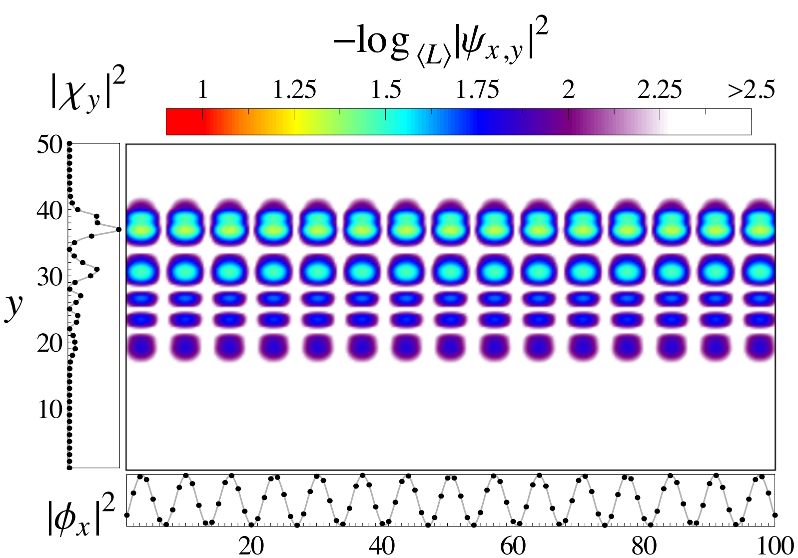


(b)

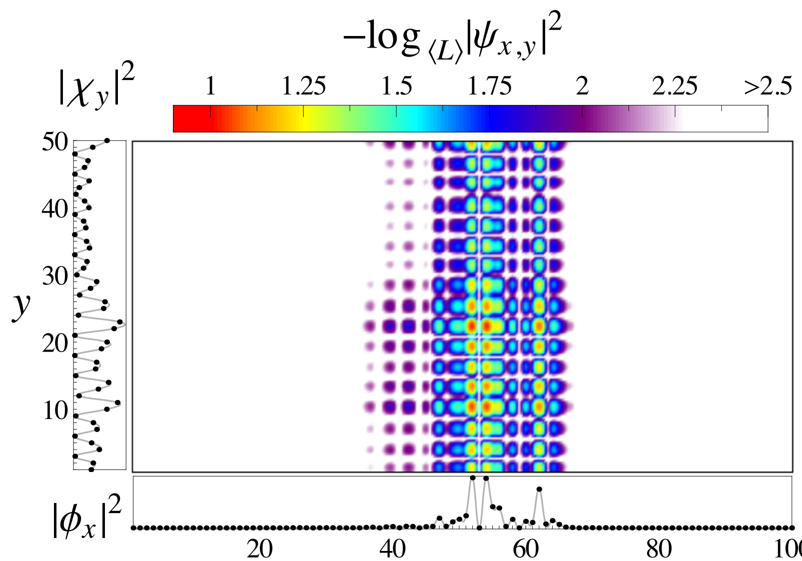
Rodriguez, A., Chakrabarti, A., Römer, R. (2012). Controlled engineering of extended states in disordered systems. *Physical Review B*, *86*(8).


$-\log_{\langle L\rangle}|\psi_{x,y}|^2$
$|\chi_y|^2$
1 1.25 1.5 1.75 2 2.25 >2.5
50 40 30 20 10
y
$|\phi_x|^2$
20 40 60 80 100
x

(a)


$-\log_{\langle L\rangle}|\psi_{x,y}|^2$
$|\chi_y|^2$
1 1.25 1.5 1.75 2 2.25 >2.5
50 40 30 20 10
y
$|\phi_x|^2$
20 40 60 80 100
x

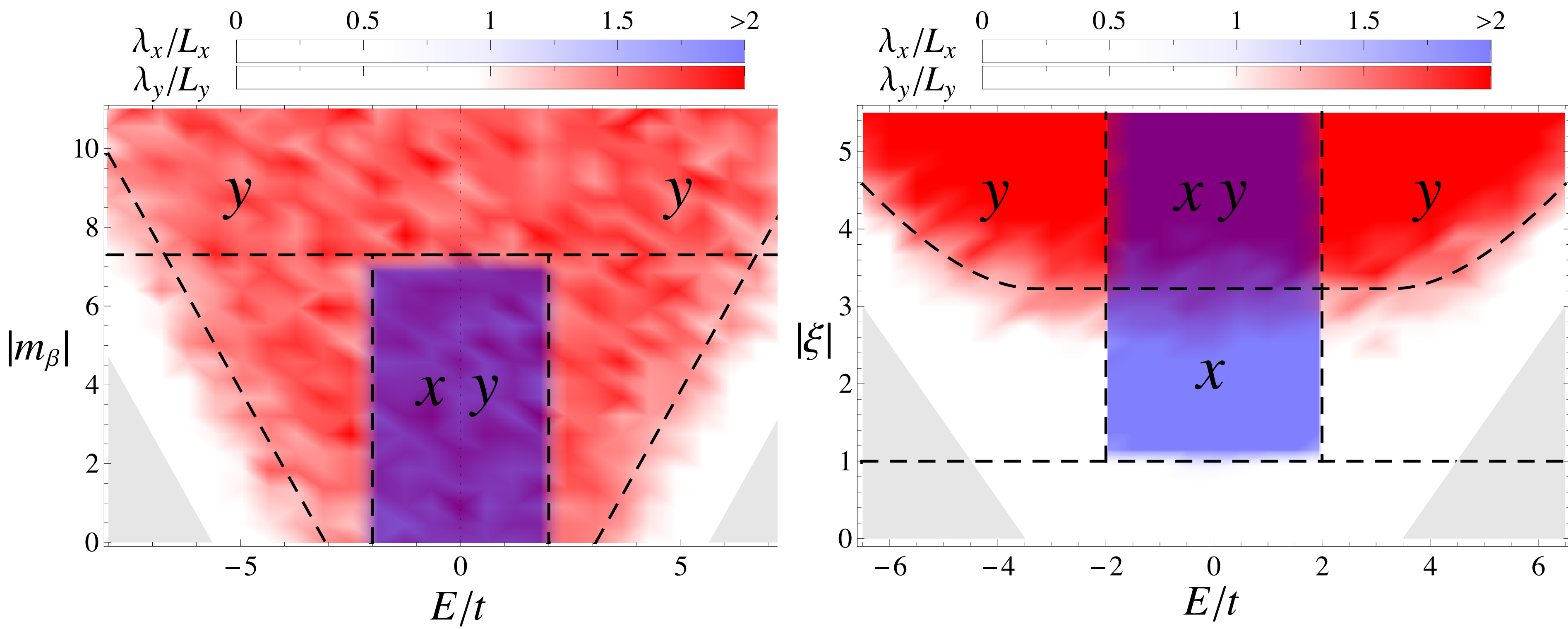
(a)


$-\log_{\langle L\rangle}|\psi_{x,y}|^2$
$|\chi_y|^2$
1 1.25 1.5 1.75 2 2.25 >2.5
50 40 30 20 10
y
$|\phi_x|^2$
20 40 60 80 100
x

(b)

“Phase” diagram of delocalization in 2D

Dashed lines indicate analytic estimates for “phase” boundaries



Quantum engineering with spins

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Spintronics (spin transport electronics), also known as spinelectronics or fluxtronics, is the study of the intrinsic spin of the electron and its associated magnetic moment, in addition to its fundamental electronic charge, in solid-state devices.

https://en.wikipedia.org/wiki/Spintronics

Applications: metal-based (GMR, TMR), doped semiconductor materials with dilute ferromagnetism (ZnO, GaMnAs, …), all for logic/storage devices

Magnetic chain in 1D:

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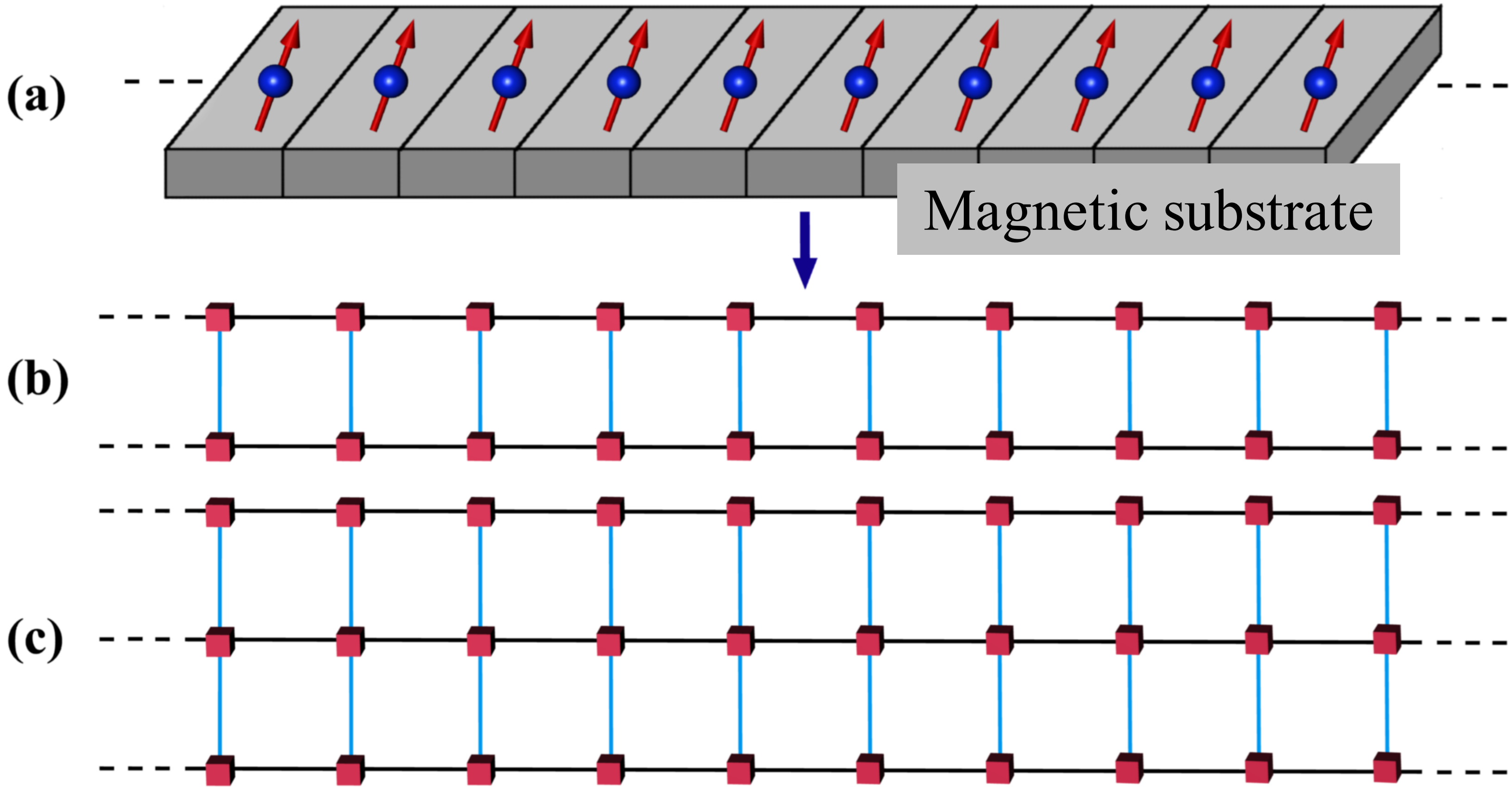


B. Pal, A., Chakrabarti, A., Römer, R. (2016). “Spin filter for arbitrary spins by substrate engineering”, to be submitted to *Physical Review B*, *86*(8).

The model

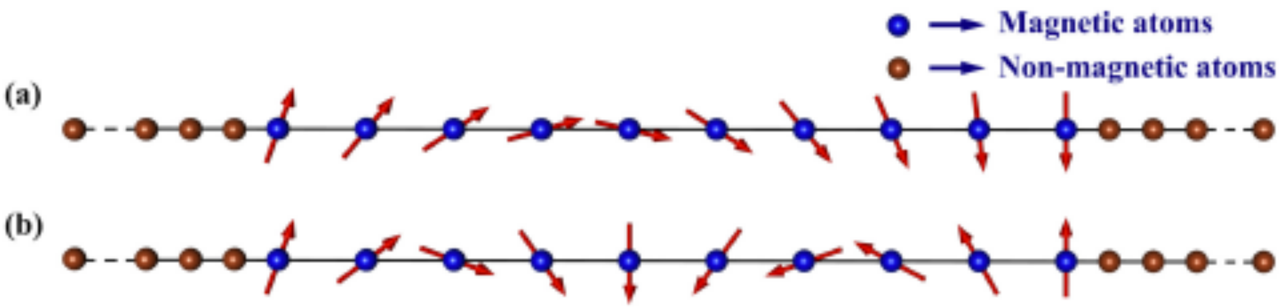
$$\boldsymbol{H} = \sum \mathbf{c}_n^\dagger \left(\boldsymbol{\epsilon}_n - \vec{h}_n \cdot \mathbf{S}_n^{(S)} \right) \mathbf{c}_n + \sum_{\langle n,m \rangle} \mathbf{c}_n^\dagger \mathbf{t}_{n,m} \mathbf{c}_m + \mathbf{c}_m^\dagger \mathbf{t}_{n,m} \mathbf{c}_n,$$

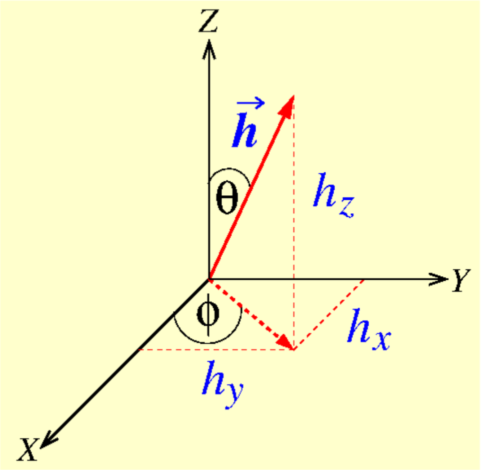
- Additional "Heisenberg term" adds energy if spin is not aligned with magnetic field in substrate and reduces otherwise
- Leads to alignment of spins across system

Variations in substrate magnetization

$$\boldsymbol{h}_n \cdot \boldsymbol{S}_n^{(1/2)} = \boldsymbol{h}_{n,x}\boldsymbol{\sigma}_y + \boldsymbol{h}_{n,x}\boldsymbol{\sigma}_y + \boldsymbol{h}_{n,z}\boldsymbol{\sigma}_z$$

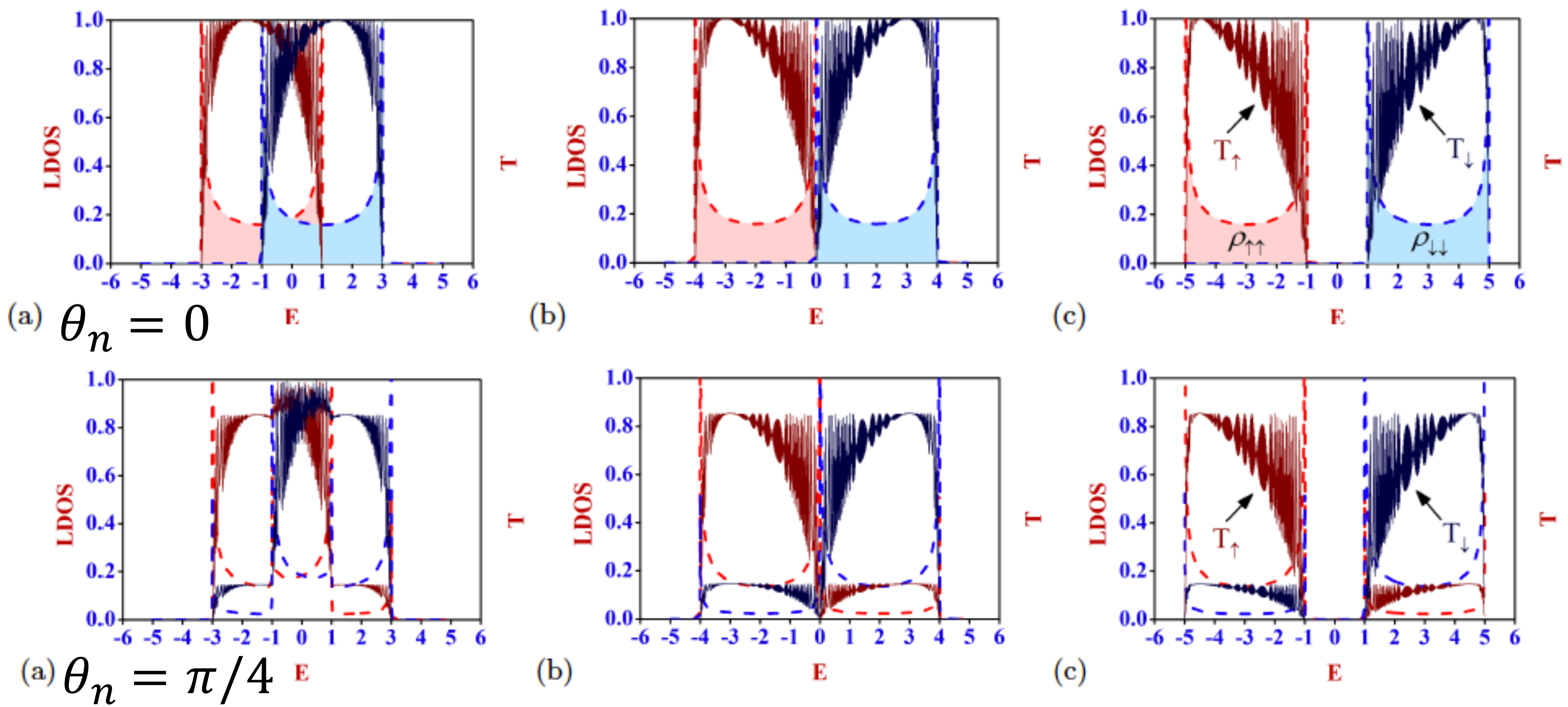
$$= \begin{pmatrix} h_n \cos\theta_n & h_n \sin\theta_n \exp(-i\varphi_n) \\ h_n \sin\theta_n \exp(i\varphi_n) & -h_n \cos\theta_n \end{pmatrix}$$





Transport and density of states

- Heisenberg term splits spin-bands
- Spins transport only in appropriate energies



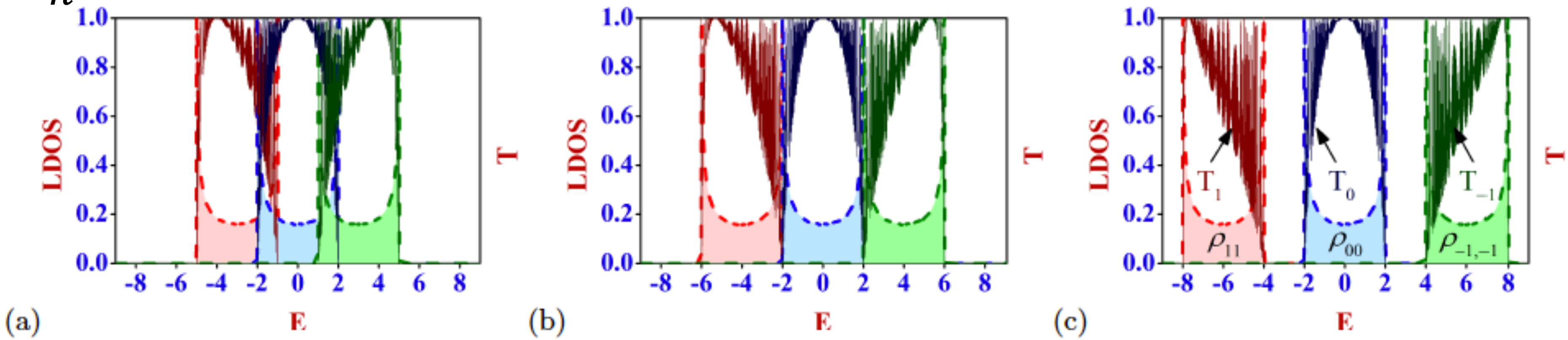
Higher-spin cases

- $S = {}^{1}/_{2}, m_S = -{}^{1}/_{2}, {}^{1}/_{2}$ [2]
- $\boldsymbol{S = 1, m_S = -1, 0, 1}$ **[3]**
- …
- $S = {}^{5}/_{2}, m_S = -{}^{5}/_{2}, -{}^{3}/_{2} \ldots, {}^{3}/_{2}, {}^{5}/_{2}$ [7]

$[2S+1]$

Atomic gases: such higher-spin atoms can be studied

$\theta_n = 0$



Spiral: flipping spins

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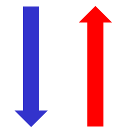
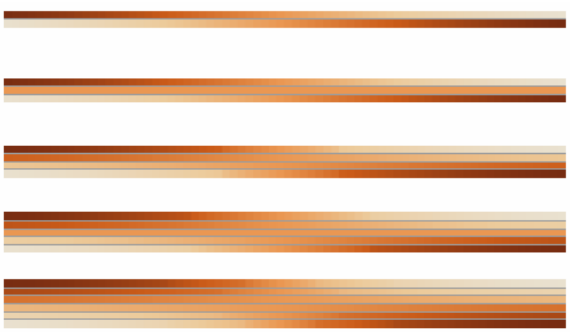
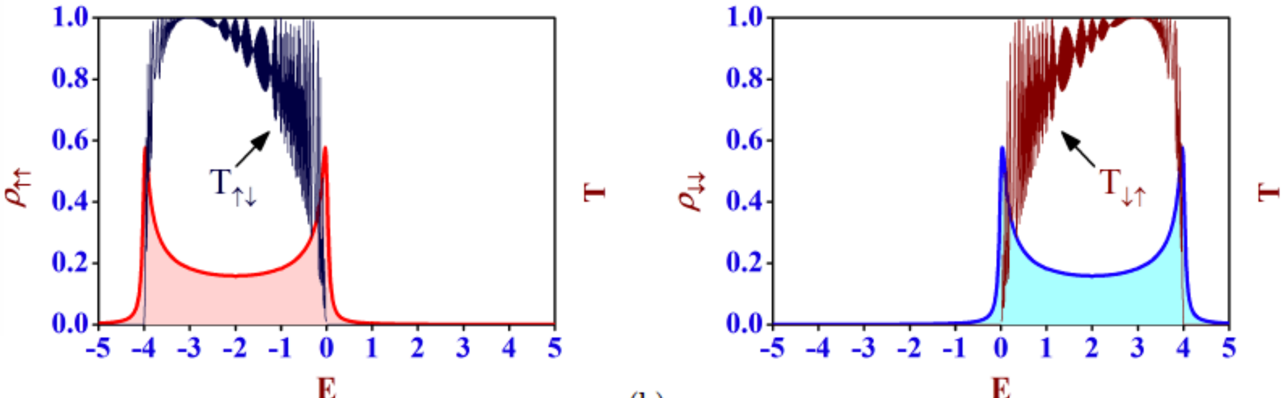
$$\theta_n = {}^{n\pi}/_{L}$$



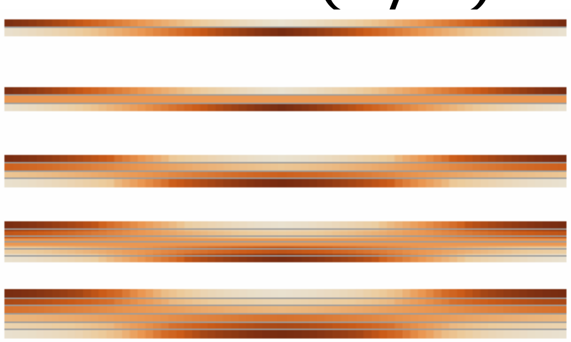
S= 1/2

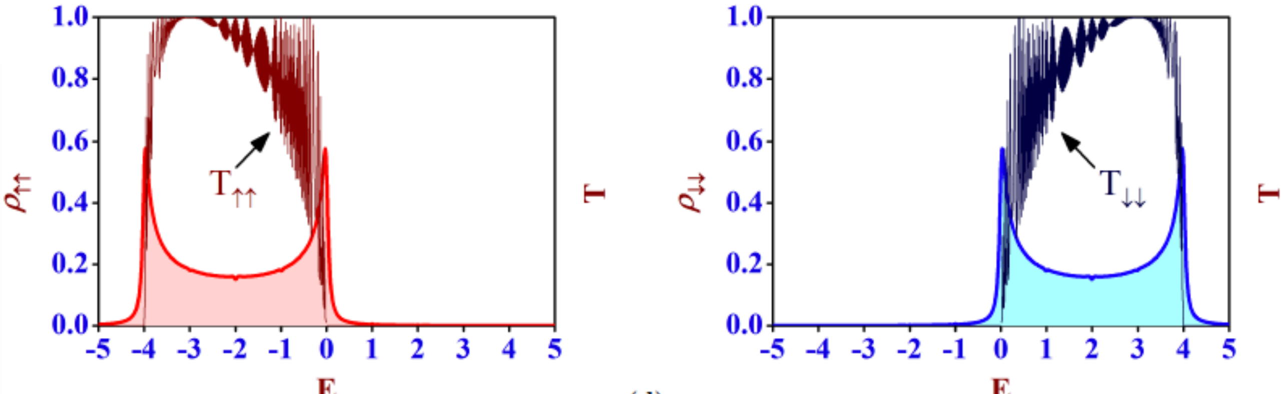




(a) (b)

$$\theta_n = {}^{n\pi}/_{(L/2)}$$





(c) (d)

Spin-spiral for *S*=1

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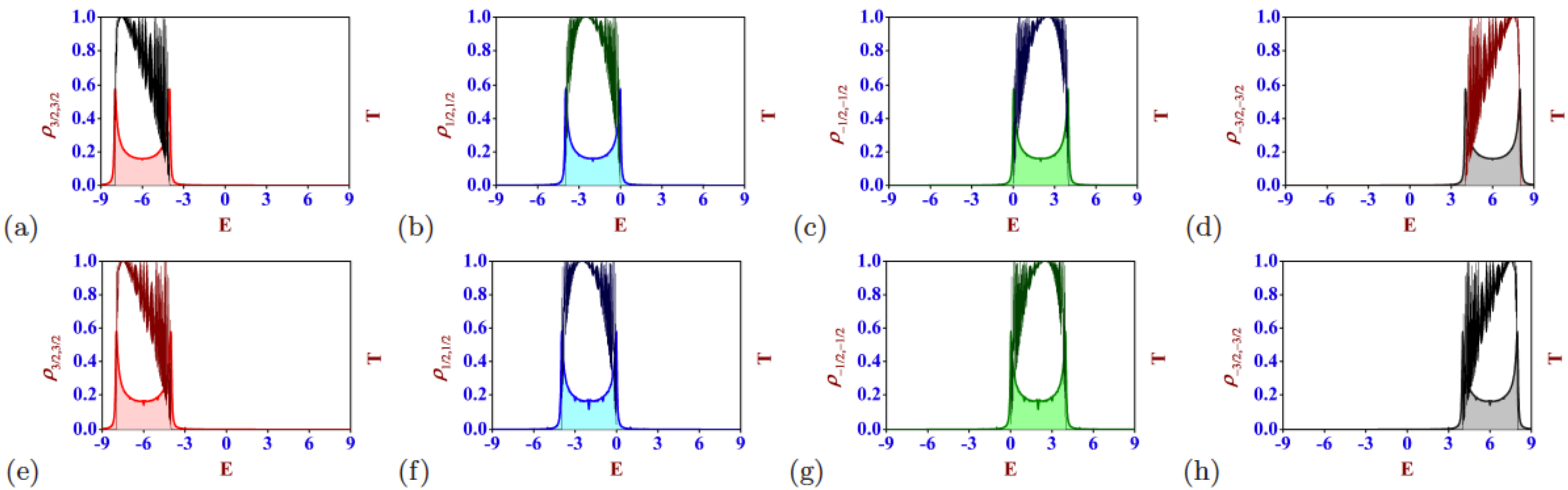
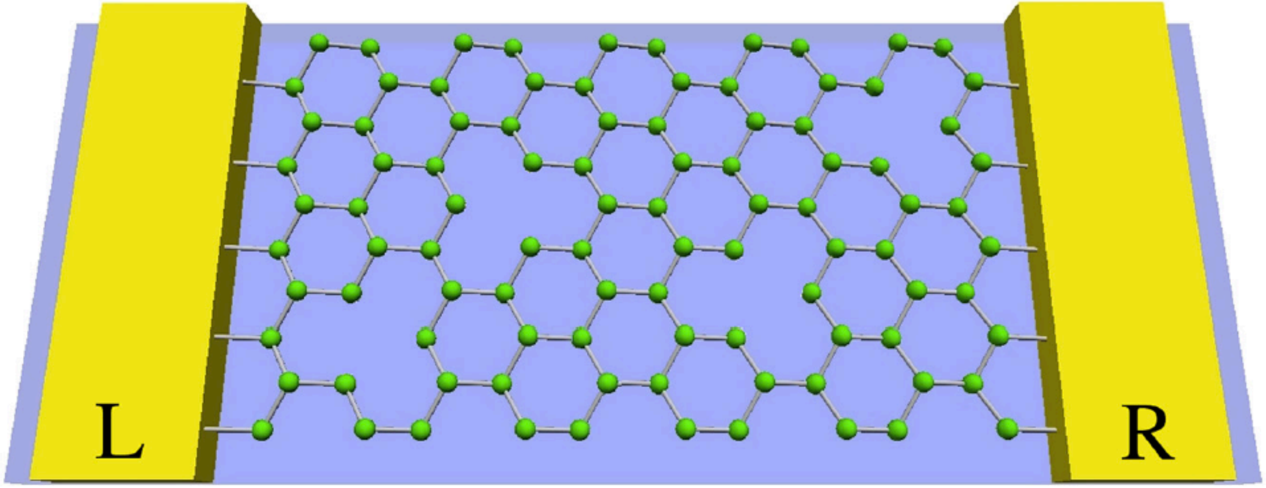
- Perfectly analoguous results for all higher *S*
- A simple model for a spin filter with perfect polarization



Combining localization and spins: using disorder to select the spin

Silicene:

- hexagonal lattice of Si atoms, buckles and has larger spin-orbit (SO) coupling
- Suggestions that gap opens and can be controlled by electric field
- Use SO/substrate to separate spins, and disorder/vacancies to localize spins in different energy ranges



Liu C C, Feng W and Yao Y 2011 Physical Review Letters 107 076802

Transport calculations

$$H = -t \sum_{\langle i,j \rangle \sigma} c^{\dagger}_{i\sigma} c_{j\sigma} + \mathrm{i} \frac{\lambda_{\mathrm{SO}}}{3\sqrt{3}} \sum_{\langle\langle i,j \rangle\rangle \sigma\beta} \nu_{ij} c^{\dagger}_{i\sigma} \sigma_z c_{j\beta} + \sum_{i\sigma} c^{\dagger}_{i\sigma} \left(M\sigma_z + \varepsilon_i \right) c_{i\sigma}$$

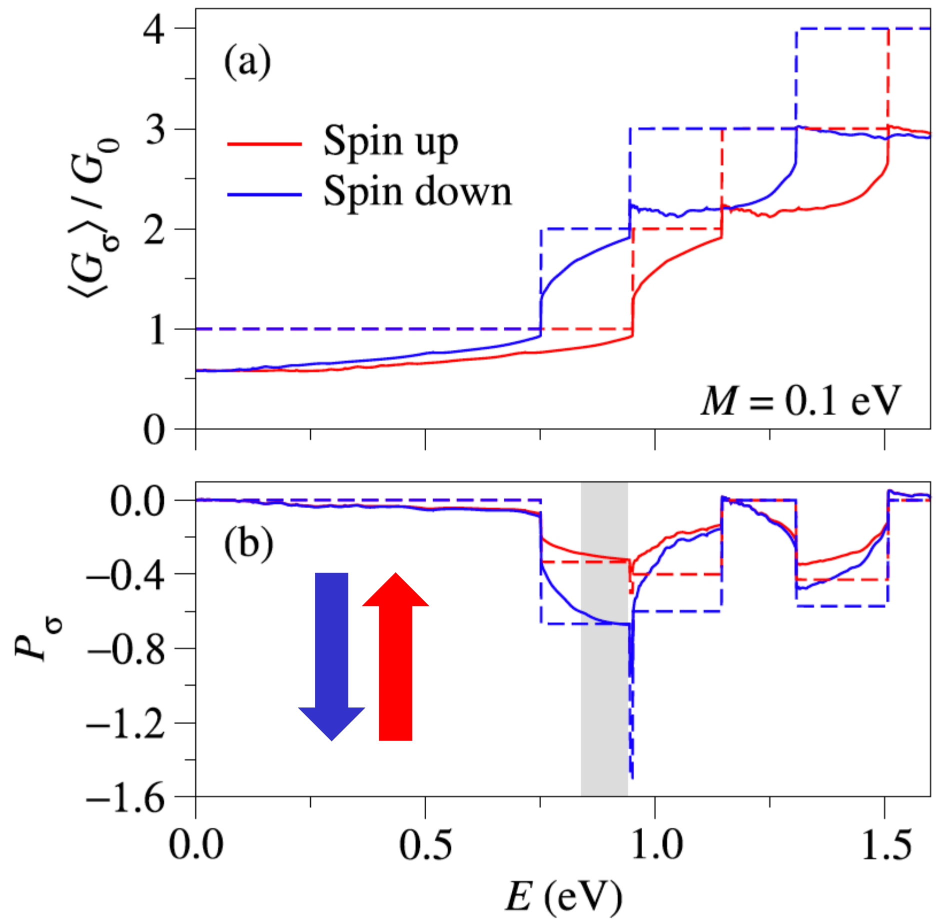
$c = 0.05$

- λ_{SO} spin-orbit coupling (3.9meV)
- M magnetic field splitting (0.1eV, a guess, 0.25eV in G) due to substrate (ferroelectric polymer)

Nano-ribbon:

L= 32.22nm (300 “sites” in transport direction)

W= 2.35nm



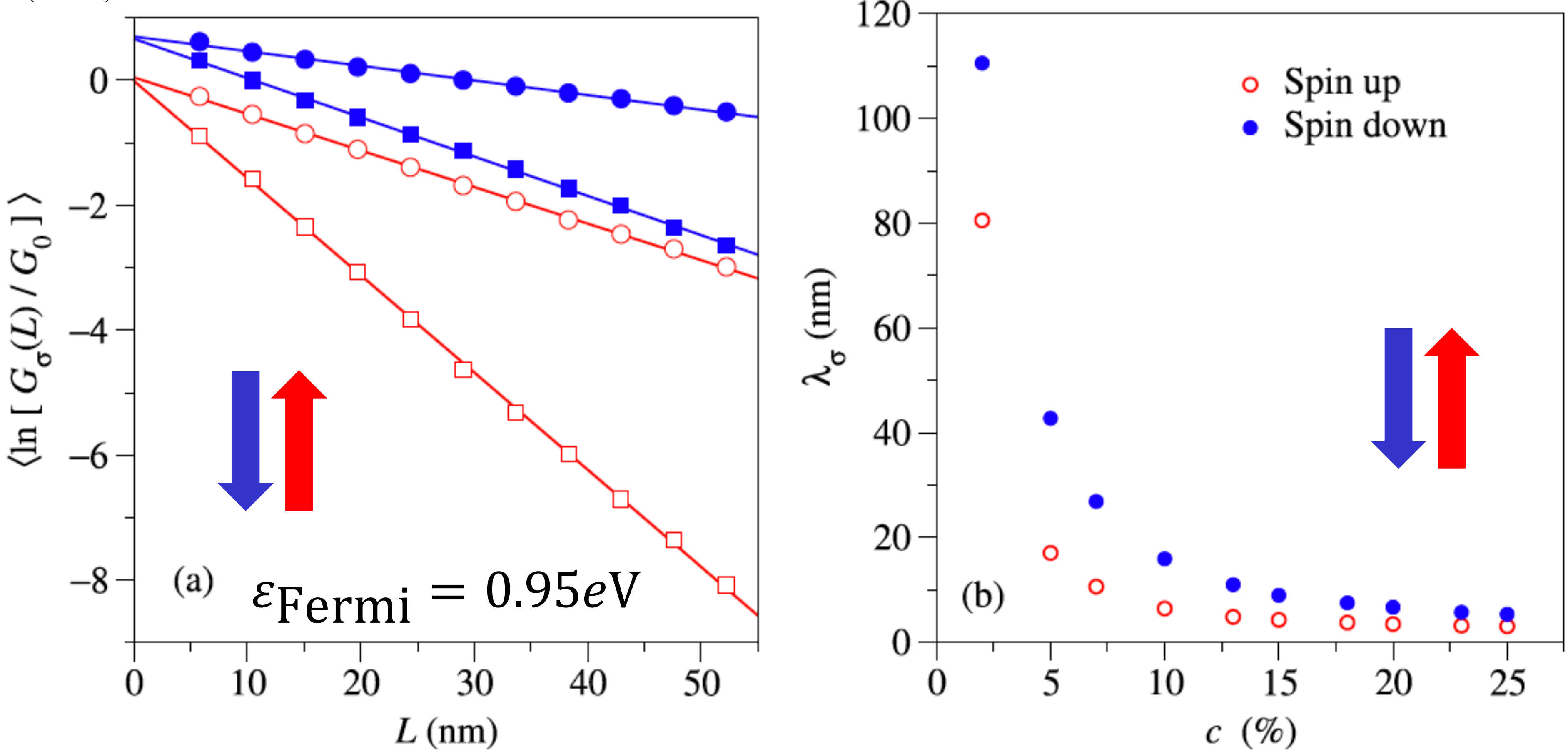
Results

"Silicene-based spin-filter device: Impact of random vacancies“, C. Nunez, F. Dominguez-Adame, P. A. Orellana, L. Rosales, R. A. Römer, accepted for publication in 2D Mat. (2015)

○ $c = 0.05$

□ $c = 0.1$

[errors within symbol size]



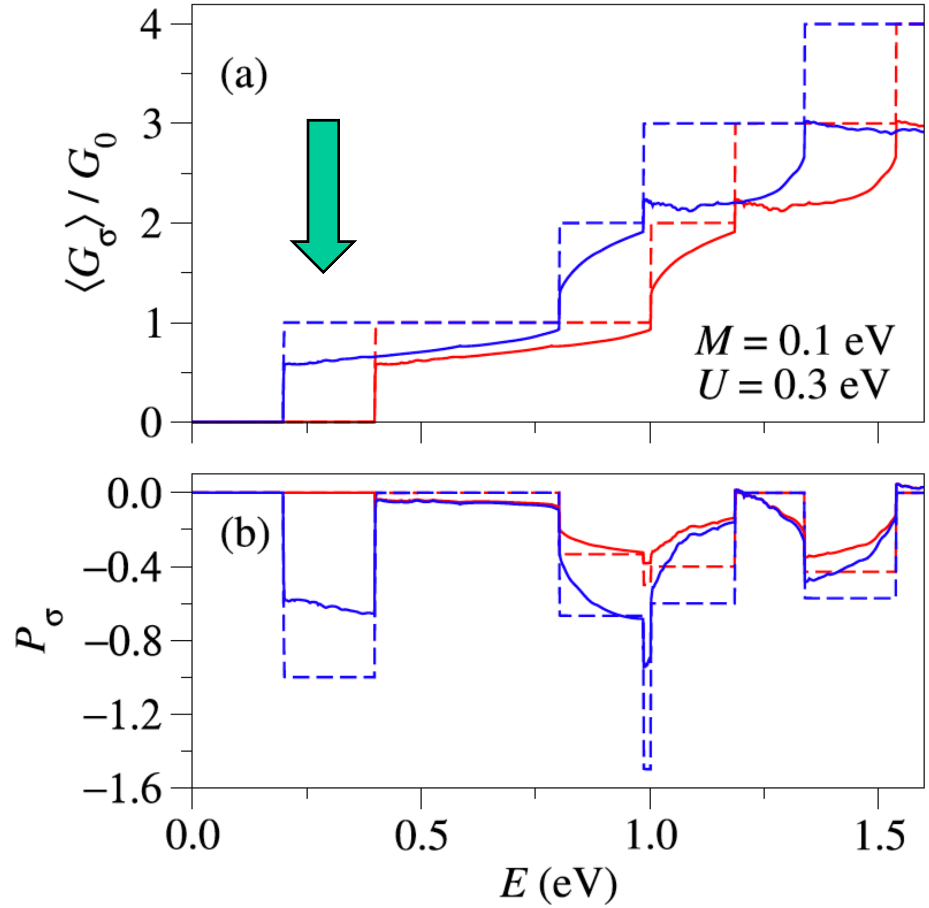
$\varepsilon_{\mathrm{Fermi}} = 0.95eV$

Adding an electric field

$U \sum_{i\sigma} \zeta_i c_{i\sigma}^{\dagger} c_{i\sigma} \quad U \gg \lambda_{\mathrm{SO}}$

$c = 0.05$

- $U = 0.3e\mathrm{V}$
- $\zeta_i = \pm 1$ if $i = \{^A_B\}$
- spin-dependent transport region at low energies
- But large disorder dependence



Conclusions

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- Extended transport channels can open even in low-D disordered systems

- Simple models to show possibilities
- Probably hard to achieve experimentally (but who knows!)

- Thanks for the attention!

Disordered Quantum Systems

- **Localization:** **E. Carnio**, **A Chakrabarti** (Calcut), **N Hine**, **R Lima** (Maceio), A Rodriguez-Gonzales (Freiburg), **E Prodan** (New York), **D Quigley**, **H Schulz-Baldes** (Erlangen)
- **Nano Science**: **F Dominguez-Adame** (Madrid), C Nunez (Chile), **L Rosales** (Chile), **P Orellana** (Chile)

- **Numerical Methods:** M. Bollhoefer (Braunschweig), O Schenk (Basel)
- **Protein Rigidity**: **R Freedman**, E Jimenez, **SA Wells**, J Heal
- **Many-Body Physics:** ME Portnoi, **A Goldsborough**
- **Quantum Hall**: **J Oswald** (Leoben)
- **DNA:** C Paez (Brasil), A Rodriguez (Madrid), C-T Shih (Taichung), SA Wells
- **Rogue Waves**: **A Savojardo**, **M Eberhard** (Aston)

- Funding: DFG, EPSRC, Leverhulme Trust, Nuffield Foundation, Royal Society; HECToR, ARCHER, Hartree Centre