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Towards a multiscale framework for robust simulation of inelastic dynamic processes

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Outline

1 Background

2 Hysteretic Multiscale Finite Elements
- Hysteretic Modeling
- Hysteretic Multiscale FEM
- Applications

3 Future Directions

Ageing Structures in a changing environment

Seismic Hazard

Modena, 2012

Aging Infrastructure

I35W, Minneapolis, 2007

Extreme Environment

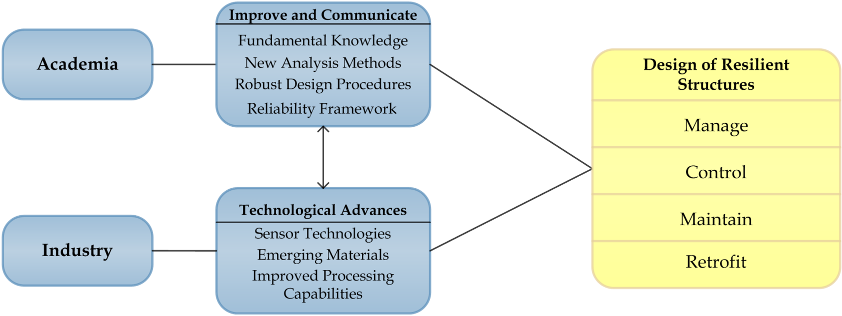
Scapegoat Hill, 2011

Dawlish, 2014

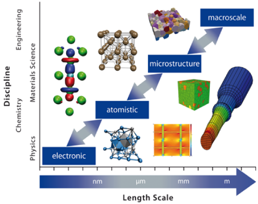
Golf of Mexico, 2005

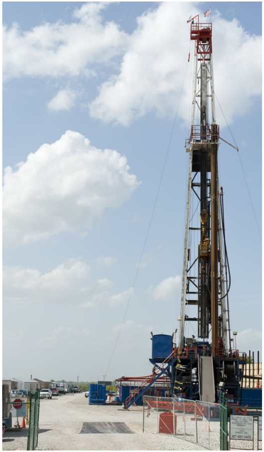
Engineering Resilience

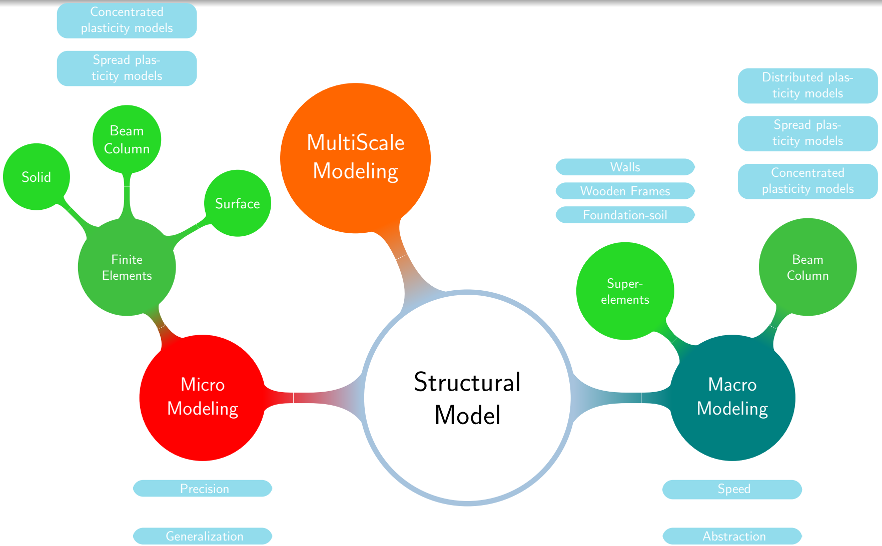
A multi-disciplinary task



Nature is a multiscale process






Concentrated plasticity models
Spread plasticity models
Beam Column
Solid
Surface
Finite Elements
MultiScale Modeling
Micro Modeling
Structural Model
Macro Modeling
Walls
Wooden Frames
Foundation-soil
Distributed plasticity models
Spread plasticity models
Concentrated plasticity models
Super-elements
Beam Column
Precision
Generalization
Speed
Abstraction

Multiscale methods for a multiscale reality

- Heterogeneous material behaviour spans different and not necessary discrete scales
- Mechanical and physical processes span different temporal scales
- Standard modelling tools provide accurate yet not necessarily efficient solutions
- A variety of multiscale methods are currently available, e.g. ,
  - Asymptotic Homogenization
  - Computational Homogenization
  - FE^2 Methods
  - Multiscale Finite Element methods
  - and many more...

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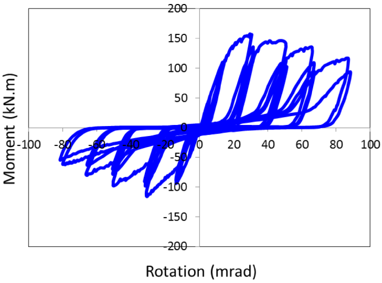
3 Future Directions

Cyclic nonlinear response

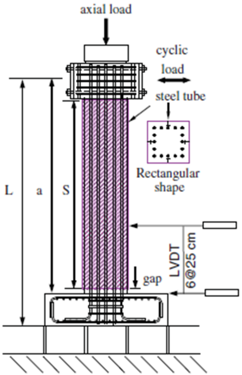
- Hysteresis is a generic property of inelastic material behaviour under cyclic loading
- Hysteretic behaviour is observed both in the micro and the macro scale of structural response
- It sums up nearly every energy dissipating mechanism a material has
- The mathematical representation of hysteresis is a challenging task
- Mechanisms to consider
  - Plasticity
  - Damage (brittle or ductile)
  - Healing
  - Also large strains?

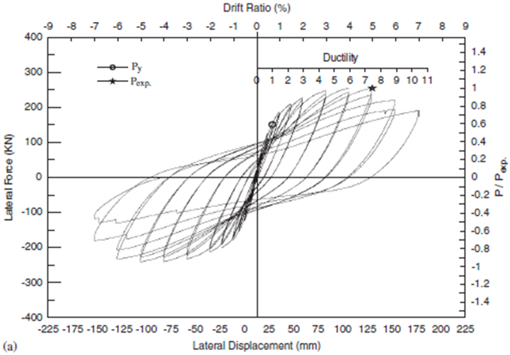
Cyclic nonlinear response



Tizani, W., Wang, Z. Y., & Hajirasouliha, I. (2013). *Engineering Structures*, **46**, 535-546.

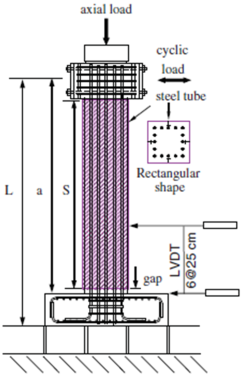
Nonlinear hysteretic behaviour

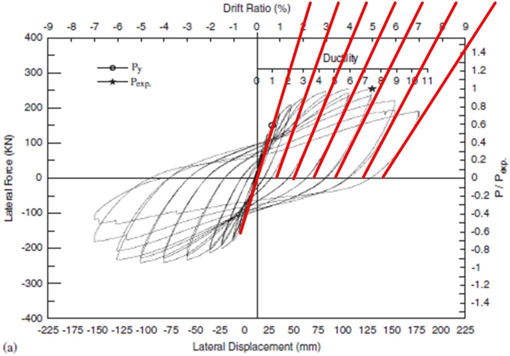




Chen S.J., Yang K.C., Lin K. M., Wang C. D. (2004). *Earthquake Engineering and Structural Dynamics*, **40**, pp. 21-34.

Nonlinear hysteretic behaviour





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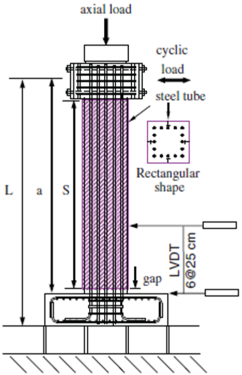
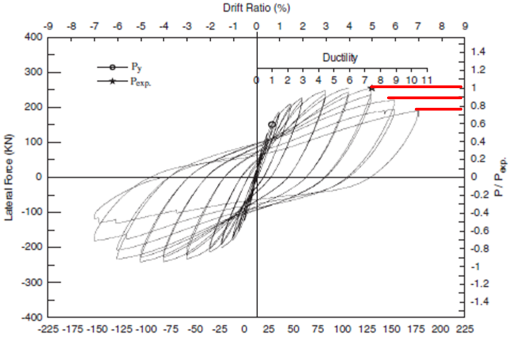


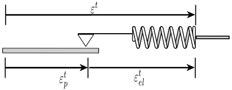


(a)

Chen S.J., Yang K.C., Lin K. M., Wang C. D. (2004). *Earthquake Engineering and Structural Dynamics*, **40**, pp. 21-34.

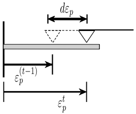
Classical Plasticity in brief

Additive Decomposition of strains



Flow rule

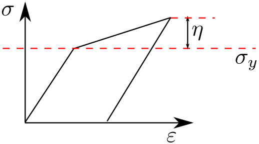
$$\dot{\varepsilon}_p = \dot{\lambda}\frac{\partial \Phi}{\partial \sigma}, \dot{\lambda} \geq 0$$



Yield Criterion and Kinematic Hardening Law

$$\Phi(\sigma, \eta) - \Phi_0 \leq 0$$

$$\dot{\eta} = \dot{\lambda} G(\eta, ...)$$



$$\sigma - \eta - \sigma_y \leq 0$$

$$\dot{\eta} = c\dot{\varepsilon}_p$$

Consistency Condition

$$\dot{\Phi} = 0 \Rightarrow \frac{\partial \Phi}{\partial \sigma}\dot{\sigma} + \frac{\partial \Phi}{\partial \sigma}\dot{\eta} = 0$$

Classical Plasticity

Yield Function

$$\Phi = \sigma - \sigma_y \leq 0$$

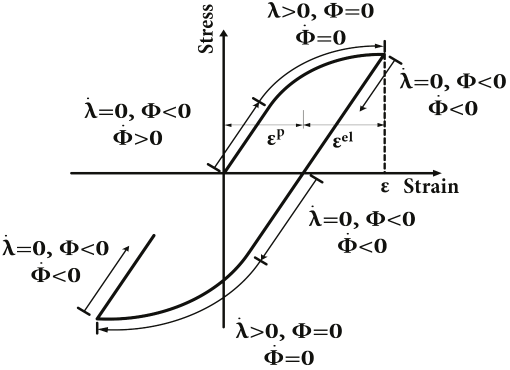
Yield Function Rate

$$\dot{\Phi} = \dot{\sigma} - \dot{\sigma}_y$$

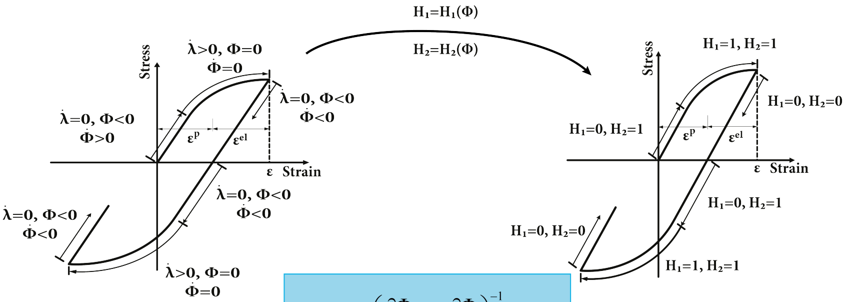
Plastic Multiplier

$$\dot{\lambda} = \left(\frac{\partial \Phi}{\partial \sigma} [E] \frac{\partial \Phi}{\partial \sigma}\right)^{-1} [D] \{\dot{\varepsilon}\}$$

But only as long as $\Phi = 0$



Classical Plasticity



$$\dot{\lambda} = H_1 H_2 \left(\frac{\partial \Phi}{\partial \sigma} [E] \frac{\partial \Phi}{\partial \sigma} \right)^{-1} [D] \{\dot{\varepsilon}\}$$

$$H_1 \approx \left| \frac{\tilde{\Phi}}{\tilde{\Phi}_0} \right|^N \quad \left(\text{e.g., } H_1 \approx \left| \frac{\sigma}{\sigma_y} \right|^N \right) \qquad H_2 \approx \beta + \gamma sgn\left(\dot{\Phi}\right)$$

Smooth evolution equation of plastic strains

- Starting from the derivation of a generic-hysteretic model

A stress-strain relation in rate form

$$\{\dot{\sigma}\} = v_\eta \, [D] \left([I] - v_s \left| \frac{\tilde{\Phi}}{\tilde{\Phi}_0} \right|^N \left(\beta + \gamma \mathrm{sgn} \left(\left(\frac{\partial \Phi}{\partial \{\sigma\}} \right)^T \{\sigma\} \right) \right) [R] \right) \{\dot{\varepsilon}\}$$

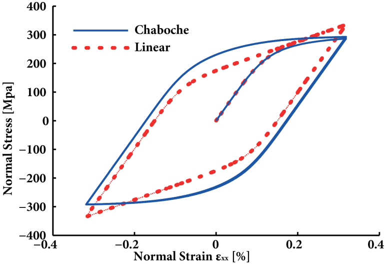
Plastic strains

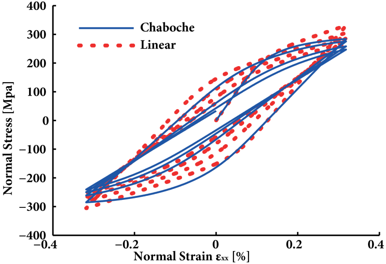
$$\left\{\dot{\varepsilon}^{pl}\right\} = v_s \left| \frac{\tilde{\Phi}}{\tilde{\Phi}_0} \right|^N \left(\beta + \gamma \mathrm{sgn} \left(\{\varepsilon\}^T \{\sigma\} \right) \right) [R] \{\dot{\varepsilon}\}$$

- Isotropic hardening is also accounted for by properly modifying the yield function $\Phi = \Phi(\sigma, \kappa, \eta)$
- Damage is introduced through the evolution of v_η and v_s
  $v_\eta = 1.0 + c_\eta E h \quad v_s = 1.0 + c_s E h$

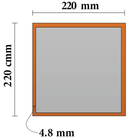
Hysteretic Material Behaviour

As a stress-strain law



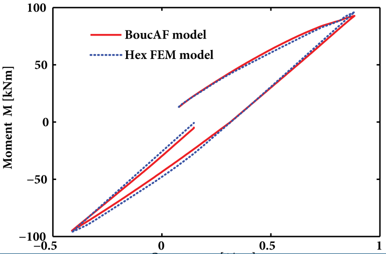


As a Moment-Curvature relation for CFT columns



Concrete
E_c=29 Gpa
v=0.2
fc=25 Mpa
ε_c =3.5 ‰

Steel
E_c=210 Gpa
v=0.3
f_y =235 Mpa
f_u =248 Mpa
ε_u =6%



Visco-elastic plastic cyclic behaviour

The formulation can be further extended to consider visco-elastic response; e.g. the 1D case results in

A stress-strain relation in rate form

$$\dot{\sigma} = E_1 \left(1 - H_1 H_2 \frac{\partial \Phi}{\partial \sigma} [R]\right) (\dot{\varepsilon} - \dot{\alpha}_1)$$

Visco-elastic component (Rate Dependent)

$$\dot{\alpha}_1 = \frac{E_1}{C_1} (\varepsilon - \varepsilon_p - \alpha_1)$$

Plastic Rate

$$\dot{\varepsilon}_p = \frac{\partial \Phi}{\partial \sigma} H_1 H_2 [R] (\dot{\varepsilon} - \dot{\alpha}_1)$$

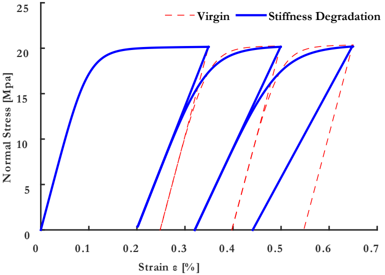
Damage Evolution

- Stiffness Degradation

$$\dot{\sigma} = \frac{\mathbf{1}}{\mathbf{v}_\eta} E_1 \left(1 - \mu H_1 H_2 \frac{\partial \Phi}{\partial \sigma}\right)(\dot{\varepsilon} - \dot{\alpha}_1)$$

$$v_\eta = (1 + c_\eta E_h)$$

$$\dot{E}_h = \sigma \dot{\varepsilon}_p$$

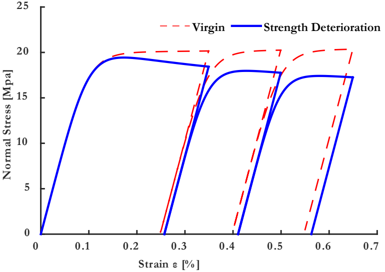


- Strength Deterioration

$$\dot{\sigma} = E_1 \left(1 - \mathbf{v_s} \mu H_1 H_2 \frac{\partial \Phi}{\partial \sigma}\right)(\dot{\varepsilon} - \dot{\alpha}_1)$$

$$v_s = (1 + c_s E_h)$$

$$\dot{E}_h = \sigma \dot{\varepsilon}_p$$

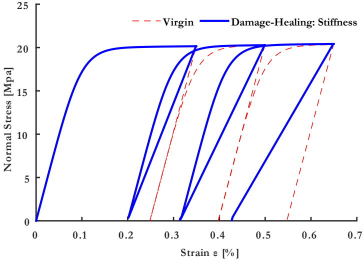


Healing Operators

- Stiffness Retrieval

$$\dot{\sigma} = \frac{\mathbf{h}_\eta}{v_\eta} E_1 \left(1 - \mu H_1 H_2 \frac{\partial \Phi}{\partial \sigma}\right) \left(\dot{\varepsilon} - \dot{\alpha}_1\right)$$

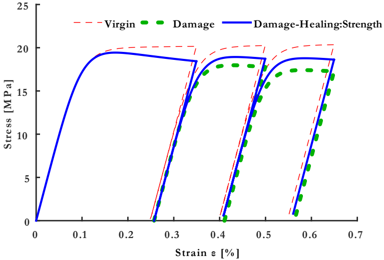
$$h_\eta = 1 + \omega_h \left(v_h - 1\right) \frac{1 + sign\left(\dot{\sigma} \frac{\partial \Phi}{\partial \sigma}\right)}{2}$$



- Strength Retrieval

$$\dot{\sigma} = E_1 \left(1 - \frac{v_s}{\mathbf{h_s}} \mu H_1 H_2 \frac{\partial \Phi}{\partial \sigma}\right) \left(\dot{\varepsilon} - \dot{\alpha}_1\right)$$

$$h_s = 1 + \omega_s \left(v_s^{\max} - 1\right)$$



The hysteretic formulation of finite elements

Starting from the Principle of Virtual Work

$$\int_{V_e} \{\varepsilon\}^T \{\sigma\}\, dV_e = \{d\}^T \{f\}$$

and considering the additive decomposition $\{\sigma\} = [D]\left(\{\varepsilon\} - \{\varepsilon^p\}\right)$

$$\int_{V_e} \{\varepsilon\}^T [D] \{\varepsilon\}\, dV_e - \int_{V_e} \{\varepsilon\}^T [D] \left\{\varepsilon^{pl}\right\} dV_e = \{d\}^T \{f\}$$

Introduce a mixed interpolation scheme where displacements and plastic strains are interpolated

$$\{\varepsilon\} = [B]\{u\} \quad \left\{\varepsilon^{pl}\right\} = [N]_e \left\{\varepsilon^{pl}_{cq}\right\}$$

The hysteretic formulation of finite elements

Then, the following equilibrium equation is derived

$$\left[k^{el}\right]\{d\} - \left[k^{h}\right]\left\{\varepsilon_{cq}^{pl}\right\} = \{P\}$$

Constant State Matrices

$$\left[k^{el}\right] = \int_{V_e} [B]^T [D] [B] dV_e \quad \left[k^{h}\right] = \int_{V_e} [B]^T [D] [N]_e dV_e$$

Additional Hysteretic degrees of freedom

$$\left\{\dot{\varepsilon}_{cq}^{pl}\right\} = \left|\frac{\tilde{\Phi}}{\tilde{\Phi}_0}\right|^N \left(\beta + \gamma \text{sgn}\left(\{\dot{\varepsilon}\}^T \{\sigma\}\right)\right) [R] \{\dot{\varepsilon}\}$$

The hysteretic formulation of finite elements

- Considering the additive decomposition of the strain vector

$$\{\varepsilon\} = \{\varepsilon^e\} + \{\varepsilon^{pl}\}$$

- An evolution equation for the plastic part of total strain is derived

$$\{\dot{\varepsilon}^{pl}\} = \left|\frac{\tilde{\Phi}}{\tilde{\Phi}_0}\right|^N \left(\beta + \gamma \mathrm{sgn}\left(\{\varepsilon\}^T \{\sigma\}\right)\right) [R] \{\varepsilon\}$$

- Element-wise

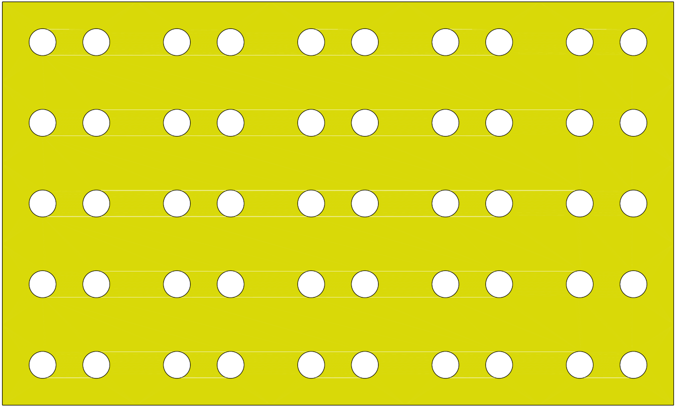
$$\{F\} = [k_e] \{d\} - [k_h] \{\varepsilon_{cq}^{pl}\}$$

- Structural assembly

$$\{P\}_S = [K_{el}]_S \{d\} - [K_H]_S \{\varepsilon_{cq}^{pl}\}_S$$

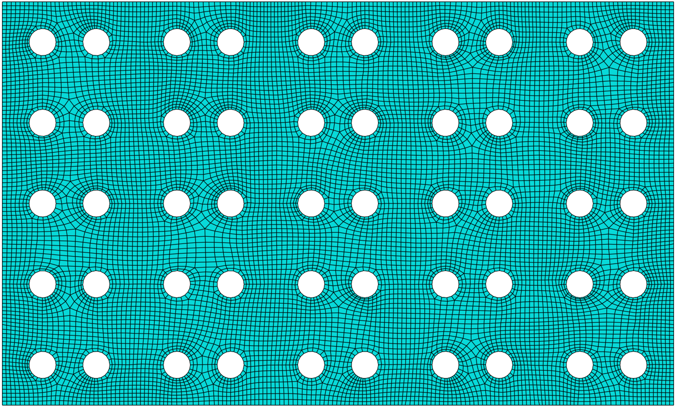
The Multiscale Finite Element scheme

Considering a heterogeneous medium



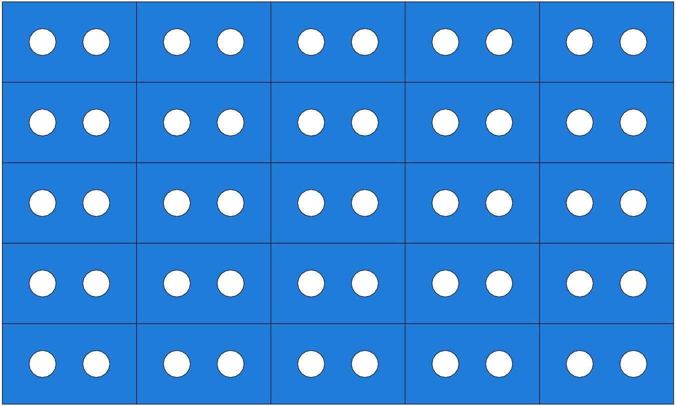
The Multiscale Finite Element scheme

Instead of using this discrete model



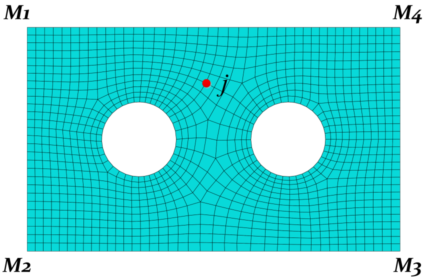
The Multiscale Finite Element scheme

Solve this!



The Multiscale Finite Element scheme

But also establish a relation between the micro and macro displacements at the coarse element level



$$[K]_{RVE}\{d\}_m = \{0\}$$

Assumption

$$\{d\}_S = \{\bar{d}\}$$

$$u_m(x_j, y_j) = \sum_{i=1}^{n_{Macro}} N_{ijxx} u_{M_i} + \sum_{i=1}^{n_{Macro}} N_{ijxy} v_{M_i}$$

$$v_m(x_j, y_j) = \sum_{i=1}^{n_{Macro}} N_{ijxy} u_{M_i} + \sum_{i=1}^{n_{Macro}} N_{ijyy} v_{M_i}$$

$$j = 1 \ldots n_{micro}$$

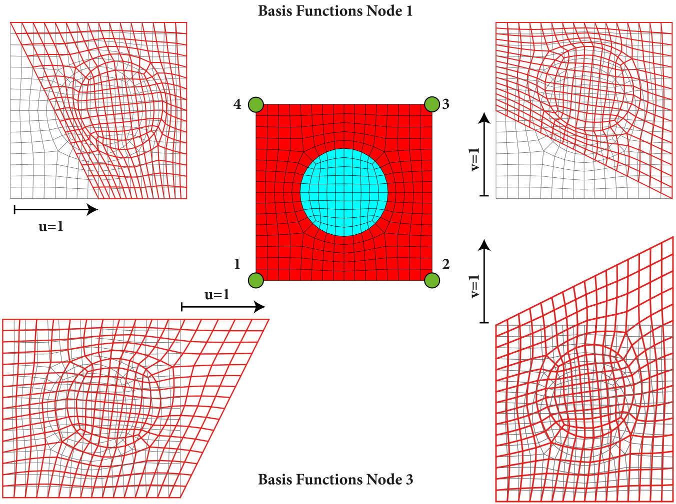
Boundary Conditions

Three main variants

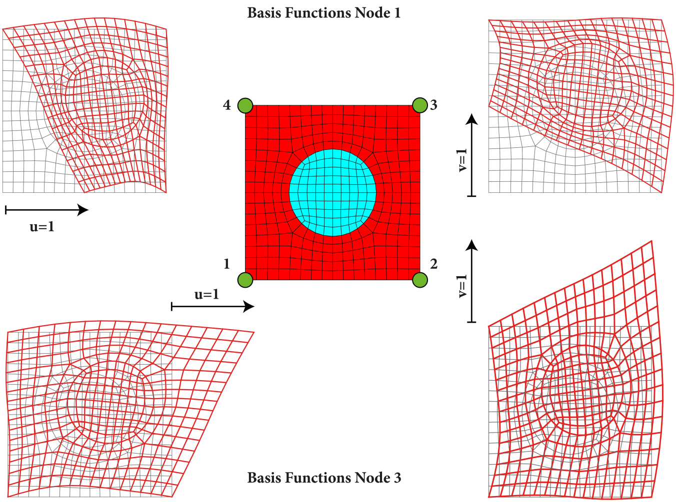
- *Linear Boundary Conditions*
  - Linear variation of displacements along the boundary
  - Applies in every case, however..
  - May lead to overestimated coarse element stiffness
- *Oscillatory Boundary Conditions*
  - "Periodic" boundaries bare oscillating displacements
  - Difficult to implement in non periodically meshed boundaries
- *Linear and Oscillatory Boundary Conditions with Oversampling*
  - Consider also the stiffness of the surrounding medium
  - At the cost of increasing the *offline* computational requirements of the method

1. Efendiev, Y., and Hou, T. Y. (2009). *Multiscale Finite Element Methods, Springer.*
2. Zhang, H. W., Wu, J. K., and Lv, J. (2012). "A new multiscale computational method for elasto-plastic analysis of heterogeneous materials." *Computational Mechanics*, 49**(2)** , 149-169.

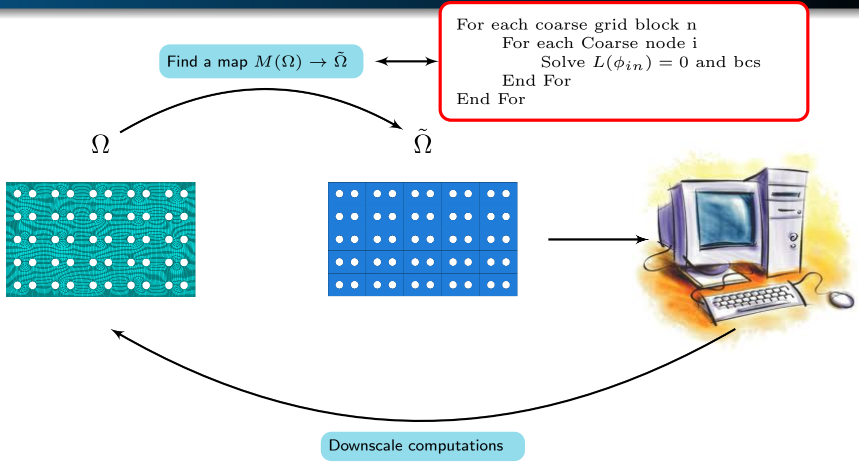
Linear Boundary Conditions



Oscillatory Boundary Conditions



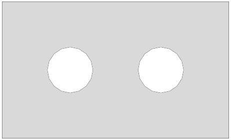
MultiScale FEM



- Re-evaluation of the mapping is required in a nonlinear analysis
- Use the Hysteretic FE in the micro-scale

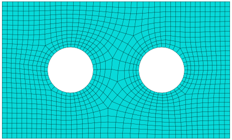
Micro and Macro Energy potential

- The approximation of this field is sought



- “Macro” equivalent energy

$$\mathcal{I}_M = \int_V \{\varepsilon\}_M^T \{\sigma\}_M dV$$



- Micro energy contributors

$$\mathcal{I}_m = \sum_{i=1}^{m_{el}} \int_{V_{mi}} \{\varepsilon\}_{mi}^T \{\sigma\}_{mi} dV_i$$

- HFEM + micro basis approximation

MultiScale FEM

The sum of the micro-energy potentials (Hysteretic FEM approach)

$$\mathcal{I}_m = \sum_{i=1}^{m_{el}} \left(\{d\}_{mi}^T \left[k^{el}\right]_{m(i)} \{d\}_{m(i)} - \{d\}_{mi}^T \left[k^h\right]_{m(i)} \left\{\varepsilon_{cq}^{pl}\right\}_{m(i)} \right)$$

and considering the micro to Macro mapping $\{d\}_m = [N]_{Mi} \{d\}_M$

$$\mathcal{I}_m = \{d\}_M^T \sum_{i=1}^{m_{el}} \left([N]_{Mi}^T \left[k^{el}\right]_{m(i)} [N]_{Mi} \{d\}_M - [N]_{Mi}^T \left[k^h\right]_{m(i)} \left\{\varepsilon_{cq}^{pl}\right\}_{m(i)} \right)$$

Hysteretic MultiScale Formulation

A macro elastic stiffness carrying the micro-elastic information

$$[K]^{M}_{CR(j)} = \sum_{i=1}^{m_{el}} [N]^{T}_{m(i)} \left[k^{el}\right]_{m(i)} [N]_{m(i)}$$

The micro inelastic corrections to the external applied load

$$\left[K^{h}\right]^{M}_{CR(j)} \left\{\varepsilon^{pl}_{cq}\right\} = \sum_{i=1}^{m_{el}} \left[k^{h}\right]^{M}_{m(i)} \left\{\varepsilon^{pl}_{cq}\right\}_{m(i)}$$

The evolution equations of the micro-strain components

$$\left\{\dot{\varepsilon}^{pl}_{cq}\right\}_{m} = H_1 H_2 [R] \left\{\dot{\varepsilon}_{cq}\right\}_{m}$$

Hysteretic MultiScale Formulation

- micro Level (through the hysteretic FE formulation)

$$\{f\}_{m(i)} = \left[k^{el}\right]_{m(i)} \{d\}_{m(i)} - \left[k^{h}\right]_{m(i)} \left\{\varepsilon^{pl}\right\}$$

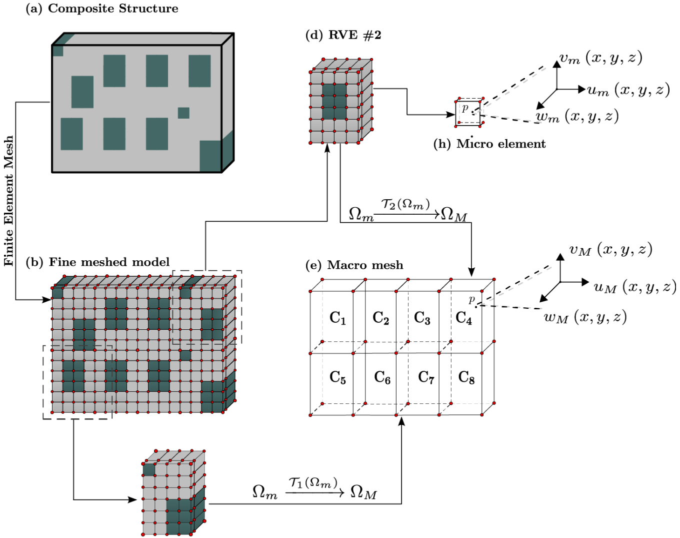
- Macro Level (through the EMsFE mapping)

$$\{f\}_{m(i)}^{M} = [K]_{CR(j)}^{M} \{d\}_{M} - \sum_{i=1}^{m_{el}} \left[k^{h}\right]_{m(i)}^{M} \left\{\varepsilon_{cq}^{pl}\right\}_{m(i)}$$

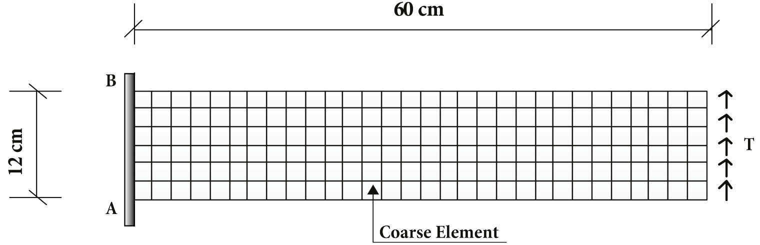
- Structural Level (through compatibility and equilibrium)

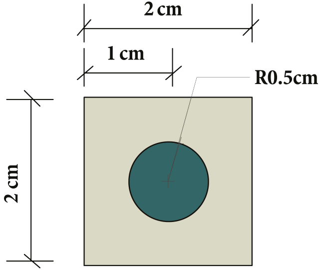
$$\{P\}_{S} = [K_{el}]_{S} \{d\}_{M} - \{P_h\}_{S}$$

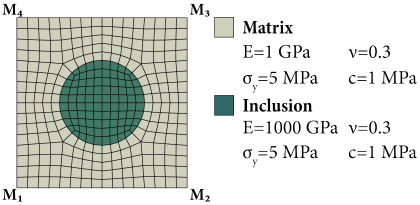
3D Formulation



Composite Cantilever

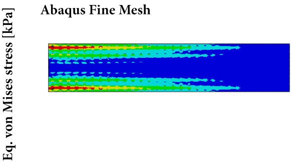


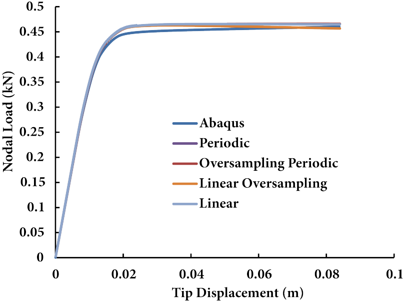


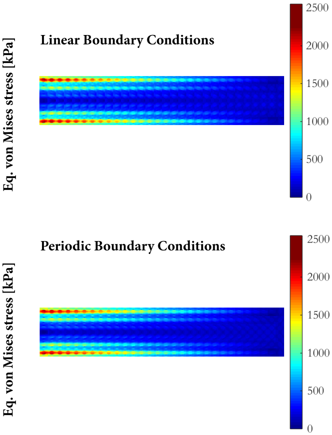


Composite Cantilever

Composite Cantilever

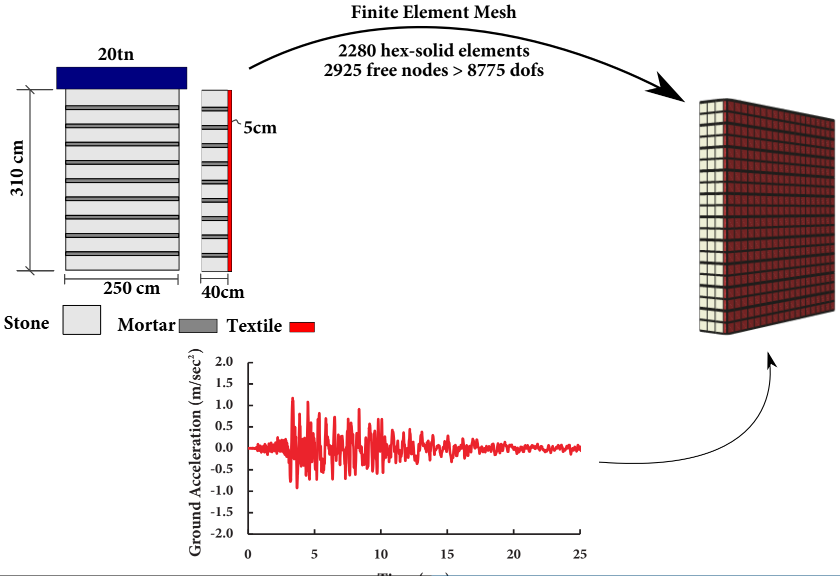






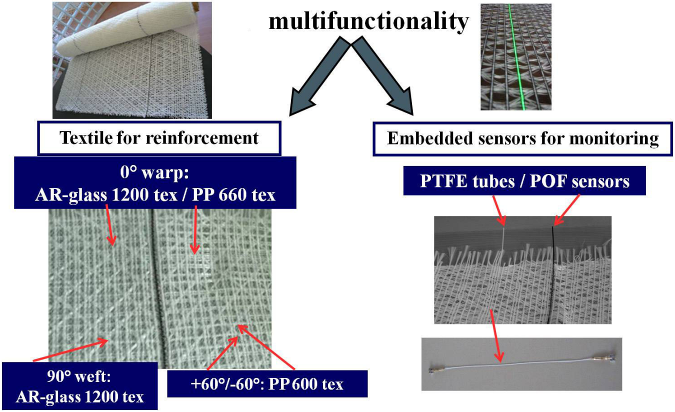
- Practically identical load paths and stress distributions
- 65% reduction on computational time for the same

Textile Reinforced Masonry Wall



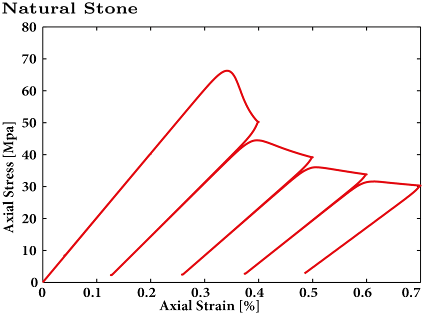
Textile Reinforced Masonry Wall

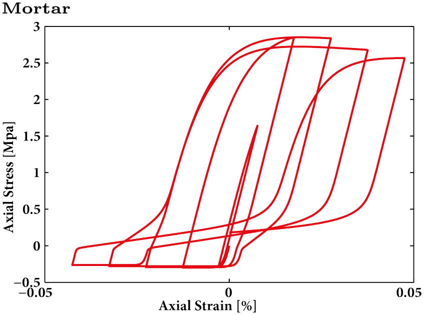
SNF project: Multi-Scale Modeling for the reinforcing of Historical Structures



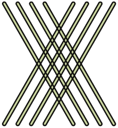
Constituent hysteretic behaviour

Hysteretic law calibrated to coupon tests

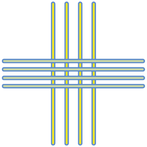




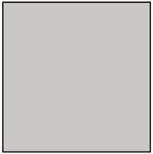
Constituent hysteretic behaviour



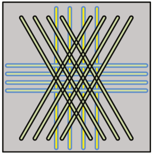
Fibers at -60⁰/60⁰



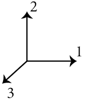
Fibers at 0⁰/90⁰



Resin

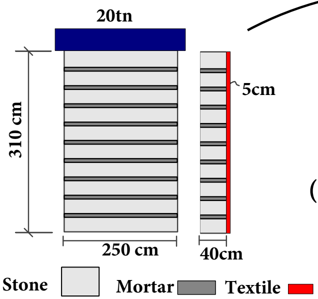


Composite System



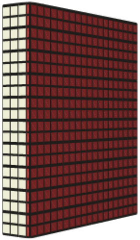
Young's modulus [MPa]					
E_{11}	E_{22}	E_{22}	E_{12}	E_{23}	E_{13}
40000	32000	32000	4500	4500	4500
Poisson's ratio					
ν_{12}		ν_{23}		ν_{13}	
0.14		0.2		0.2	

Textile Reinforced Masonry Wall

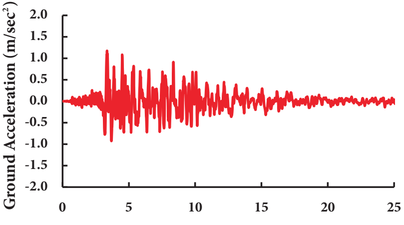


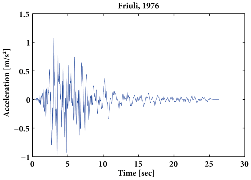
Finite Element Mesh

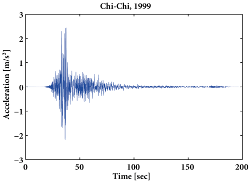
2280 hex-solid elements
2925 free nodes > 8775 dofs

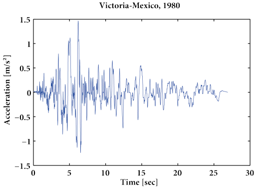


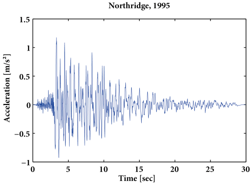
145 minutes and 8 GB of storage for a single run (T=25sec, Incr. Steps=25000)

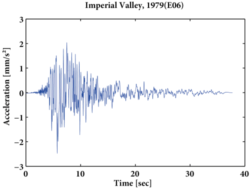


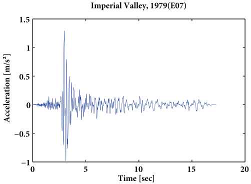

Friuli, 1976
Acceleration [m/s²]
Time [sec]

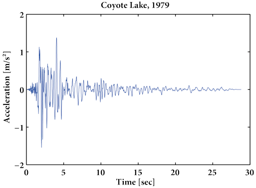

Chi-Chi, 1999
Acceleration [m/s²]
Time [sec]


Victoria-Mexico, 1980
Acceleration [m/s²]
Time [sec]

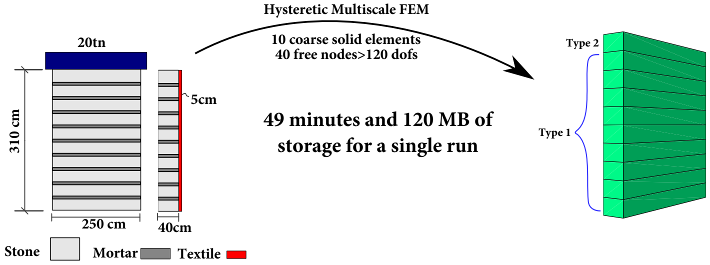

Northridge, 1995
Acceleration [m/s²]
Time [sec]


Imperial Valley, 1979(E06)
Acceleration [mm/s²]
Time [sec]


Imperial Valley, 1979(E07)
Acceleration [m/s²]
Time [sec]

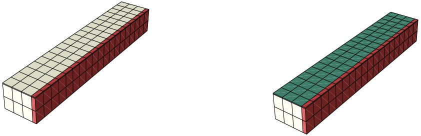

Coyote Lake, 1979
Acceleration [m/s²]
Time [sec]

Textile Reinforced Masonry Wall



49 minutes and 120 MB of storage for a single run

Evaluation of micro to macro basis functions

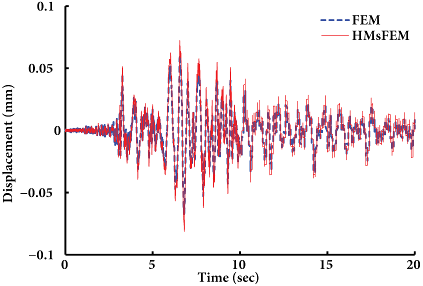


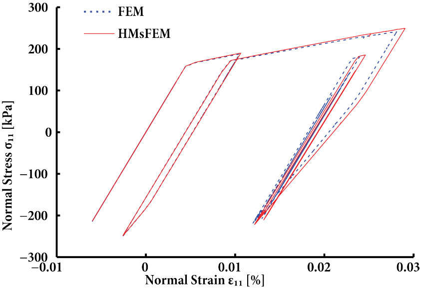
Coarse Type 1
228 elements

Coarse Type 2
228 elements (but different material distribution)

Textile Reinforced Masonry Wall

- Results derived from the HMsFEM formulation are compared to classical FEM

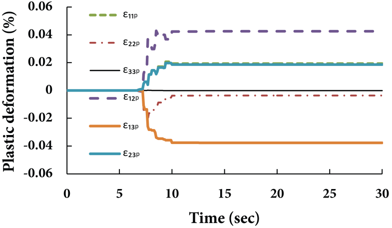


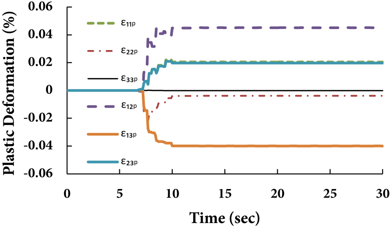


70% Reduction in Computational Time

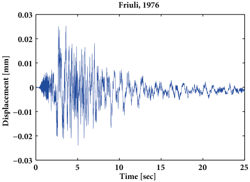
Textile Reinforced Masonry Wall

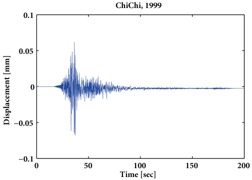
- The plastic strain components are readily derived as part of the solution

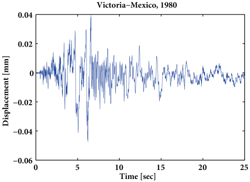


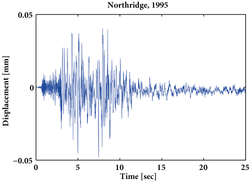


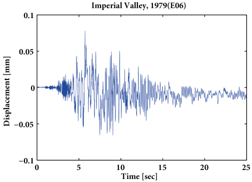
70% Reduction in Computational Time

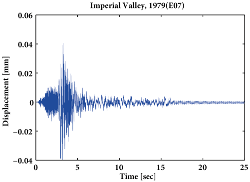

Friuli, 1976
0.03
0.02
0.01
0
−0.01
−0.02
−0.03
Displacement [mm]
0 5 10 15 20 25
Time [sec]

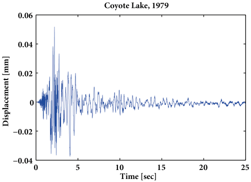

ChiChi, 1999
0.1
0.05
0
−0.05
−0.1
Displacement [mm]
0 50 100 150 200
Time [sec]


Victoria−Mexico, 1980
0.04
0.02
0
−0.02
−0.04
−0.06
Displacement [mm]
0 5 10 15 20 25
Time [sec]


Northridge, 1995
0.05
0
−0.05
Displacement [mm]
0 5 10 15 20 25
Time [sec]


Imperial Valley, 1979(E06)
0.1
0.05
0
−0.05
−0.1
Displacement [mm]
0 5 10 15 20 25
Time [sec]

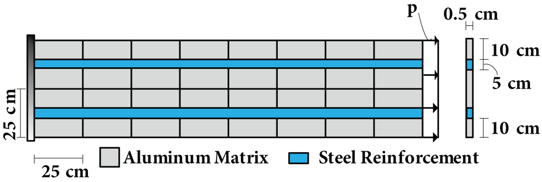

Imperial Valley, 1979(E07)
0.06
0.04
0.02
0
−0.02
−0.04
Displacement [mm]
0 5 10 15 20 25
Time [sec]


Coyote Lake, 1979
0.06
0.04
0.02
0
−0.02
−0.04
Displacement [mm]
0 5 10 15 20 25
Time [sec]

Propagation of Uncertainty through different scales

Strip reinforced aluminum panel

- Structural model



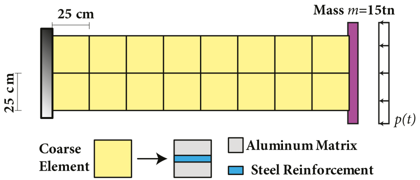
- Material Properties

	Dist.	Aluminum	Steel
Young's modulus [MPa]	LogNormal	70000	200000
Poisson's ratio	-	0.33	0.3
Plasticity	-	von-Mises	von-Mises
Yield Stress [MPa]	LogNormal	214	235

- Latin Hypercube Sampling
- 5000 Monte Carlo iterations

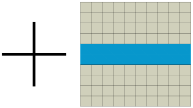
Propagation of Uncertainty through different scales

Multiscale Finite Element Model



16 Quadrilateral Coarse Elements (Q4)

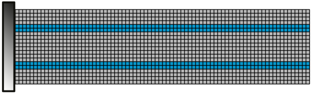
54 Macro Degrees of Freedom



16 - 100 Q4 Elements

121 Micro Degrees of Freedom

Finite Element Model



1600 Quadrilateral Plane Stress Elements

3358 Degrees of Freedom

Propagation of Uncertainty through different scales

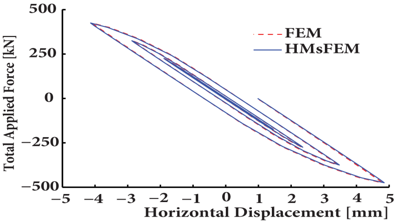
Strip reinforced aluminum panel

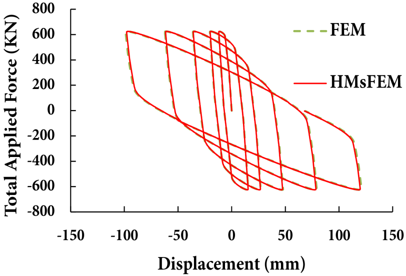
$$p(t) = 20000t \sin(\pi t)\, kPa$$

No Damage effects

$$p(t) = 250000 \sin(\pi t)\, kPa$$

Stiffness Degradation and Strength Deterioration

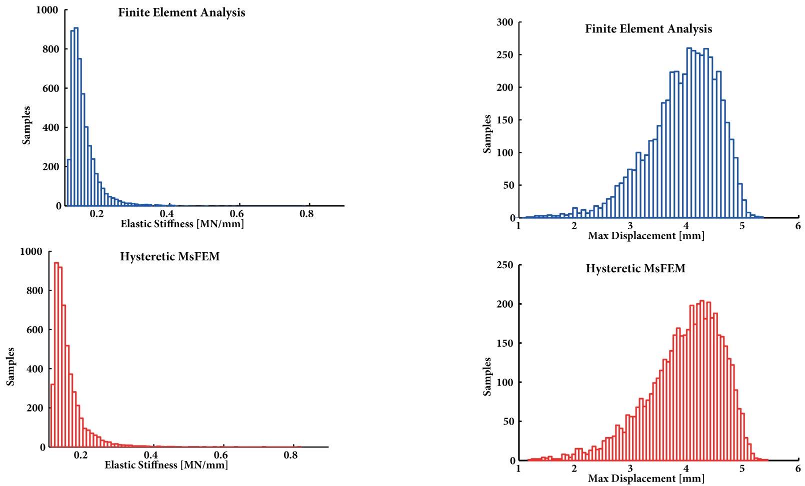




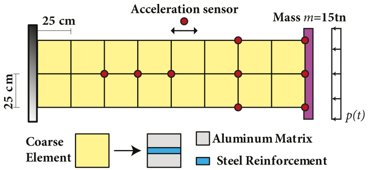
87.5% Reduction in Computational Time for a single simulation

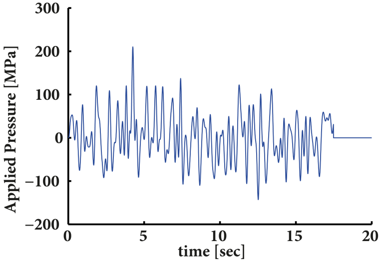
Propagation of Uncertainty through different scales

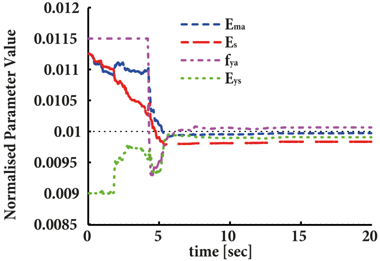
Strip reinforced aluminum panel

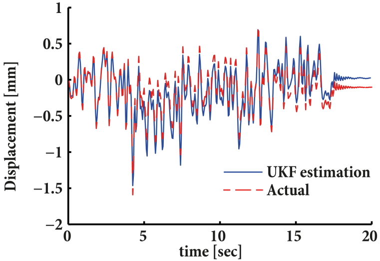


UKF Parameter Identification









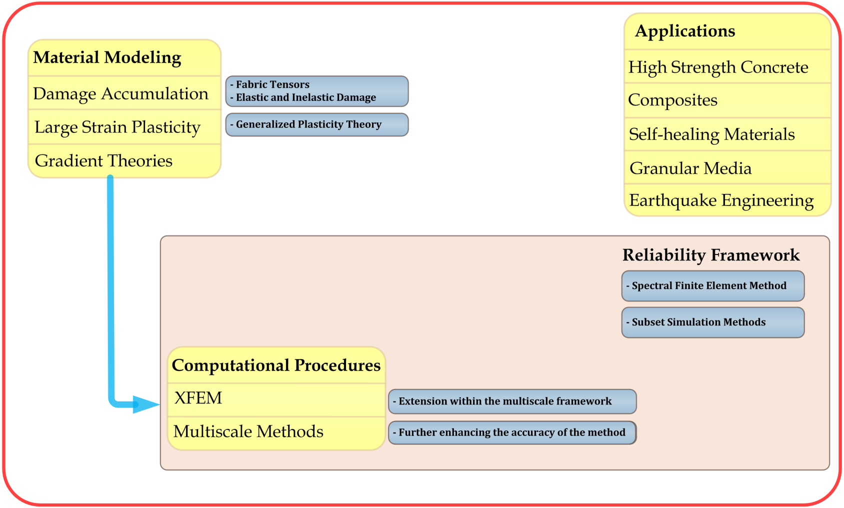
Outline

1 Background

2 Hysteretic Multiscale Finite Elements
- Hysteretic Modeling
- Hysteretic Multiscale FEM
- Applications

3 Future Directions

An open field for research



The Team

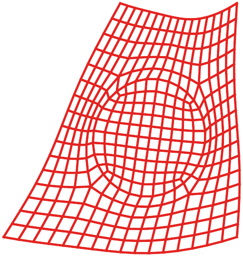
Putting the pieces together

Mr. Richard Evans

Damage Modelling for layered Composites

Mr. Emanouil Kakouris

Multiscale Methods for brittle damage



Mr. Adrian Egger

Multiscale Scaled Boundary FEM
on a joint project with Prof. E. N. Chatzi, ETHZ

Dr. Andreas Kampitsis

Multiscale Poro-Mechanics

Visit us at: http://www.nottingham.ac.uk/~ezzst1/