Quantum engineering for electrons and spins
Rudolf A Römer, Physics
The physics of waves

• Waves interfere due to the superposition principle
• Wave interference can be destructive or constructive
Interference of electrons from a double slit

http://www.hitachi.com/rd/portal/highlight/quantum/index.html#anc04
(Tonomura, 1980’s)
Interference from disorder: Anderson localization

Light, water, sound (ultra), electrons, atoms ...  

Anderson Localization


A transition in $D \geq 3$, none for $D \leq 2$.
How to “cheat” the gang of 4!

- Many-body physics
  - Magnetism, superconductivity, ...

- Correlated randomness
  - Changes of “universality”
  - Perfect correlation = clean system
  - Short-range (irrelevant in large systems) versus long-range correlation (relevant whenever correlation length comparable/larger than system)


Controlled engineering of extended states in disordered systems

\[ H = \sum_{x=1}^{L_x} c_x^\dagger \epsilon_x c_x + \sum_{x=1}^{L_x-1} (c_x^\dagger t c_{x+1} + c_{x+1}^\dagger t c_x) \]

Transfer-matrix approach:

- Rewrite \( H\psi = E\psi \) as

\[ (E \mathbf{1} - \epsilon_x) \Psi_x = t(\Psi_{x+1} + \Psi_{x-1}) \]

\( \epsilon_x \) is actually disorder matrix \( \epsilon_x, y \)

Resonance condition

\[ \varepsilon_{x,y} = \alpha_x \beta_y \] for onsite potentials

\[ \gamma_{x,y} = \alpha_x \xi_y \] for transverse kinetic energy

3N random numbers for \( N^2 \) terms – correlation!
A transformation

- We can factorize matrix $\epsilon_x = \alpha_x P$, s.t. $P$ does not depend on $x$.
- $p = U^{-1}PU$ diagonal matrix of eigenvalues of $P$, $U$ the vector of eigenvectors.
- Then with $\Phi_x = U^{-1}\Psi_x$, we can write

$$(E^1 - \alpha_x p)\Phi_x = t(\Phi_{x+1} + \Phi_{x-1})$$
Trafo cont’d:

Explicitly, this gives

\[(E - \alpha_x p_1)\phi_x^{(1)} = t(\phi_{x+1}^{(1)} + \phi_{x-1}^{(1)})\]

\[
\vdots
\]

\[(E - \alpha_x p_c)\phi_x^{(c)} = t(\phi_{x+1}^{(c)} + \phi_{x-1}^{(c)})\]

\[
\vdots
\]

\[(E - \alpha_x p_{L_y})\phi_x^{(L_y)} = t(\phi_{x+1}^{(L_y)} + \phi_{x-1}^{(L_y)})\]

Suppose that one of the \(p_i \approx 0\), then that equation correspond to a near-perfectly conducting channel!
Quantum engineering with electrons

Choosing the distributions of randomness in \(\{\alpha\}\), \(\{\beta\}\) and \(\{\xi\}\), we can then engineer states with any confinement in \((x,y)\), \((x)\) or \((y)\) independently or none!


[b](b)
“Phase” diagram of delocalization in 2D

Dashed lines indicate analytic estimates for “phase” boundaries
Quantum engineering with spins

Spintronics (spin transport electronics), also known as spinelectronics or fluxtronics, is the study of the intrinsic spin of the electron and its associated magnetic moment, in addition to its fundamental electronic charge, in solid-state devices.

https://en.wikipedia.org/wiki/Spintronics

Applications: metal-based (GMR, TMR), doped semiconductor materials with dilute ferromagnetism (ZnO, GaMnAs, ...), all for logic/storage devices
Magnetic chain in 1D:

The model

\[ H = \sum c_n^\dagger \left( \epsilon_n - \vec{h}_n \cdot \vec{S}^{(S)}_n \right) c_n + \sum_{\langle n,m \rangle} c_n^\dagger t_{n,m} c_m + c_m^\dagger t_{n,m} c_n, \]

- Additional “Heisenberg term” adds energy if spin is not aligned with magnetic field in substrate and reduces otherwise.
- Leads to alignment of spins across system.
Variations in substrate magnetization

\[ h_n \cdot S_{n}^{(1/2)} = h_{n,x} \sigma_y + h_{n,x} \sigma_y + h_{n,z} \sigma_z \]

\[ = \begin{pmatrix} h_n \cos \theta_n & h_n \sin \theta_n \exp(-i \varphi_n) \\ h_n \sin \theta_n \exp(i \varphi_n) & -h_n \cos \theta_n \end{pmatrix} \]
Transport and density of states

- Heisenberg term splits spin-bands
- Spins transport only in appropriate energies

(a) $\theta_n = 0$

(b) $\theta_n = \pi/4$
Higher-spin cases

- \( S = \frac{1}{2}, m_s = -\frac{1}{2}, \frac{1}{2} \) \[2\] \[2S + 1\]
- \( S = 1, m_s = -1, 0, 1 \) \[3\]
- ...
- \( S = \frac{5}{2}, m_s = -\frac{5}{2}, -\frac{3}{2}, \ldots, \frac{3}{2}, \frac{5}{2} \) \[7\]

Atomic gases: such higher-spin atoms can be studied

\( \theta_n = 0 \)
Spiral: flipping spins

\[ \theta_n = \frac{n\pi}{L} \]

\[ \theta_n = \frac{n\pi}{(L/2)} \]
Spin-spiral for $S=1$

- Perfectly analogous results for all higher $S$
- A simple model for a spin filter with perfect polarization
Combining localization and spins: using disorder to select the spin

Silicene:

- hexagonal lattice of Si atoms, buckles and has larger spin-orbit (SO) coupling
- Suggestions that gap opens and can be controlled by electric field
- Use SO/substrate to separate spins, and disorder/vacancies to localize spins in different energy ranges

Liu C C, Feng W and Yao Y
2011 Physical Review Letters 107 076802
Transport calculations

\[
H = -t \sum_{\langle i,j \rangle \sigma} c_{i\sigma}^\dagger c_{j\sigma} + i \frac{\lambda_{SO}}{3\sqrt{3}} \sum_{\langle\langle i,j \rangle\rangle \sigma\beta} \nu_{ij} c_{i\sigma}^\dagger \sigma_z c_{j\beta} + \sum_{i\sigma} c_{i\sigma}^\dagger (M\sigma_z + \varepsilon_i) c_{i\sigma}
\]

- $\lambda_{SO}$ spin-orbit coupling (3.9meV)
- $M$ magnetic field splitting (0.1eV, a guess, 0.25eV in G) due to substrate (ferroelectric polymer)

Nano-ribbon:

$L = 32.22\text{nm} \ (300 \text{ “sites” in transport direction})$

$W = 2.35\text{nm}$
Results


\[ c = 0.05 \]

\[ c = 0.1 \]

[errors within symbol size]
Adding an electric field

\[ U \sum_{i\sigma} \zeta_i c_{i\sigma}^\dagger c_{i\sigma} \quad U \gg \lambda_{SO} \]

- \( U = 0.3 \text{ eV} \)
- \( \zeta_i = \pm 1 \) if \( i = \{ A, B \} \)
- spin-dependent transport region at low energies
- But large disorder dependence

\[ c = 0.05 \]

(a) \[
\frac{\langle G_\sigma \rangle}{G_0}
\]

(b) \[
P_\sigma
\]

\[ M = 0.1 \text{ eV} \]
\[ U = 0.3 \text{ eV} \]
Conclusions

• Extended transport channels can open even in low-D disordered systems

• Simple models to show possibilities

• Probably hard to achieve experimentally (but who knows!)

• Thanks for the attention!
Disordered Quantum Systems

- **Localization:** E. Carnio, A Chakrabarti (Calcut), N Hine, R Lima (Maceio), A Rodriguez-Gonzales (Freiburg), E Prodan (New York), D Quigley, H Schulz-Baldes (Erlangen)
- **Nano Science:** F Dominguez-Adame (Madrid), C Nunez (Chile), L Rosales (Chile), P Orellana (Chile)
- **Numerical Methods:** M. Bollhoefer (Braunschweig), O Schenk (Basel)
- **Protein Rigidity:** R Freedman, E Jimenez, SA Wells, J Heal
- **Many-Body Physics:** ME Portnoi, A Goldsborough
- **Quantum Hall:** J Oswald (Leoben)
- **DNA:** C Paez (Brasil), A Rodriguez (Madrid), C-T Shih (Taichung), SA Wells
- **Rogue Waves:** A Savojardo, M Eberhard (Aston)

- **Funding:** DFG, EPSRC, Leverhulme Trust, Nuffield Foundation, Royal Society; HECToR, ARCHER, Hartree Centre