Journal of Manufacturing Systems
Vol. 25/No. 1
2006

Fixture Workspace Synthesis for Reconfigurable Assembly Using Procrustes-Based Pairwise Configuration Optimization

Zhenyu Kong, School of Industrial Engineering and Management, Oklahoma State University, Stillwater, Oklahoma, USA
Dariusz Ceglarek, Dept. of Industrial and Systems Engineering, The University of Wisconsin-Madison, Madison, Wisconsin, USA

Abstract

In recent years, reconfigurability of assembly systems, which enables a family of products to be produced on a single production line, is becoming of paramount importance in addressing increasing diversification of the market. One of the key issues in reconfigurable assembly systems is to identify the fixture workspace for a family of parts based on the individual configuration of the fixture-locating layout for each part. Aimed at the problems existing in the current research in this area, this paper presents an approach that allocates the best superposition of locating layouts for multiple parts to be assembled on a single reconfigurable fixture. The presented methodology is based on (1) analytical Procrustes analysis, to rapidly eliminate unlikely sets of solutions, and then (2) pairwise optimization of fixture configurations for a given part family. The proposed method is able to provide more accurate solutions for various Engineering Requirement Functions (ERFs) used in industrial practice with much less computational complexity than the existing method presented in the literature. Comprehensive case studies illustrate the proposed methodology as well as its advantages over the existing method.

Keywords: *Reconfigurability, Procrustes Analysis, Pairwise Optimization*

1. Introduction

1.1 Problem Description

In recent years, the manufacturing environment surrounding the automotive industry has been rapidly transforming to reflect a growing diversification of the market with simultaneously increasing dual demands for product quality and durability. These changes, in turn, require shorter production ramp-up/launch and more frequent model changeovers, a scenario which has led to simultaneous manufacturing of a family of products on a single production line.

Major efforts around production ramp-up/launch are focused on identifying the root causes for process and product failures. For example, Hu and Wu (1992), Ceglarek and Shi (1995, 1996), Hu (1997), Ceglarek (1998), Apley and Shi (1998), and Ding, Shi, and Ceglarek (2002) developed methodologies for identifying root causes of product dimensional failures during production and compensating for part dimensional variation. These studies reveal that a majority of the product dimensional failures during ramp-up of assembly lines are related to fixtures. This increases the requirement for accurate fixture fault diagnostics during the production phase, while simultaneously highlighting the critical need for robust fixture workspace synthesis during the design phase.

Yet, to address the ever-increasing demands in product variety, both issues—that of fixture workspace synthesis for production of a family of products on a single assembly line, as well as of product changeover time—must be significantly improved. This emerging trend involves assembling a variety of product models on a single assembly line with the capacity to rapidly reconfigure the assembly line to manufacture a new product. In other words, there is a critical need to develop Reconfigurable Assembly Systems (RASs). Some recent research conducted in the areas of reconfigurability of machining and assembly systems include work done by Koren et al. (1999), Stadzisz and Henrioud (1998); Travaini et al. (2002); and Kong and Ceglarek (2003).

1.2 Related Work

In general, the design and analysis of reconfigurable assembly systems require methods for design of the part fixturing system, which includes (i) fixture design for a single part, that is, fixtures for rigid as well as flexible/compliant parts, and (ii) fixture workspace synthesis for a family of parts.

A number of approaches exist to deal with fixture design for a single rigid part. For example, many algorithmic and heuristic methods were developed to synthesize and analyze setup plan and fixture configurations and to select the locating and clamping positions on a given workpiece (Asada and Bai 1985; Chou, Chandru, and Barash 1989; Sakurai 1992; Roy and Sun 1994; De Meter 1995; Rong and Bai 1996; Park and Mills 2002).

Some research has focused on design and analysis of fixtures for single flexible/compliant parts. Menassa and DeVries (1991) proposed optimization techniques to assist in the design and evaluation of 3-2-1 fixtures for prismatic parts considering deflection of the parts. Cai, Hu, and Yuan (1996) proposed an "*N*-2-1" locating layout for sheet metal part fixturing. They developed an optimal fixture design method, which can reconfigure the *N* locators on the primary datum to minimize total part deformation.

In the area of assembly systems for a family of parts, research mainly focuses on assembly line balancing and assembly sequence. Rekiek, DeLit, and Delchambre (2000) employed the concept of Balance For Ordering (BFO) to solve the problem of assembly line balancing for multi-products. Gupta and Krishnan (1998) presented an approach for product-family-based assembly sequence design, which maximizes the benefit from a commonality of components and assembly operations. However, very little research exists in the area of fixture workspace synthesis for a family of parts used in reconfigurable assembly systems.

Lee, Hu, and Ward (1999) presented a workspace synthesis analysis for fixturing layout of a family of stamping parts using the genetic algorithm. Their approach synthesizes a single-fixture workspace by determining the minimum space required for each locator to produce a family of parts. This approach significantly reduces the necessary fixture workspace for a family of parts. However, this approach utilizes a purely optimization method without analytically eliminating any unlikely set of solutions. This results in a large search domain for the variables that need to be optimized. Furthermore, this method tries to simultaneously optimize all of the variables whose number is proportional to the number of parts in the family. Thus, when dealing with a large number of parts, the iterative process is extremely time consuming and may lead to inaccuracies in the final solution. This further underscores the critical need for an effective fixture workspace synthesis methodology that will overcome the two aforementioned limitations.

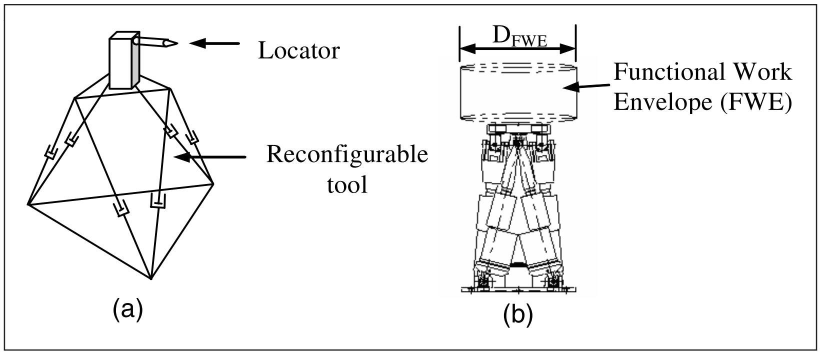

Figure 1
An Example of a Reconfigurable Tool Element of the FANUC Robot

1.3 Proposed Method

The reconfigurable assembly utilizes reconfigurable tools (fixtures) to carry tooling elements (locators and clamps), which, depending on the task, can be repositioned, instead of common fixed locators in traditional dedicated fixtures. *Figure 1* shows an example of a reconfigurable tool with a Stewart platform, where locators and/or clamps are placed on the top plate of the tool and can be freely reconfigured to any position within its Functional Work Envelope (FWE). Thus, the position of a locator is no longer at a single fixed point, but rather at any point within the FWE workspace. The reconfigurable tool shown in *Figure 1b* has an FWE of a cylindrical shape. This type of reconfigurable tool has high flexibility because the top plate of the platform has six degrees of freedom and provides sufficient positioning accuracy and part-holding stability due to its parallel kinematic structure.

On a given part, the points directly in contact with locators of the fixture are called locating points. If all the parts of a given part family are placed in such a way that the corresponding locating points of all the parts are within the FWE of each reconfigurable tool, then the family of parts can be produced using this reconfigurable fixture.

This paper presents an approach that performs fixture workspace synthesis to cope with the aforementioned challenge, that is, allocating the best superposition of locating points for a family of parts to be assembled on a single reconfigurable assembly fixture. The presented work expands on the earlier approach presented by Lee, Hu, and Ward (1999).

The proposed methodology is based on the Procrustes analysis integrated with a pairwise fixture configuration optimization. Procrustes analysis provides least-squares matching of two or more fixture layouts matrices by multi-dimensional transformations such as translation, rotation, and scaling. It has been applied for shape analysis in biology, image analysis, geography, and other fields (Dryden and Mardia 1989; Moon and Sutherland 1992; Crosilla 1999). Then, in the next step, the pairwise optimization of the results obtained from the Procrustes analysis minimizes the locator workspace for a given fixture. The pairwise optimization method is applied with respect to three Engineering Requirement Functions (ERF) formulated based on the constraints used in industrial practice. The proposed methodology is illustrated using an automotive body assembly process.

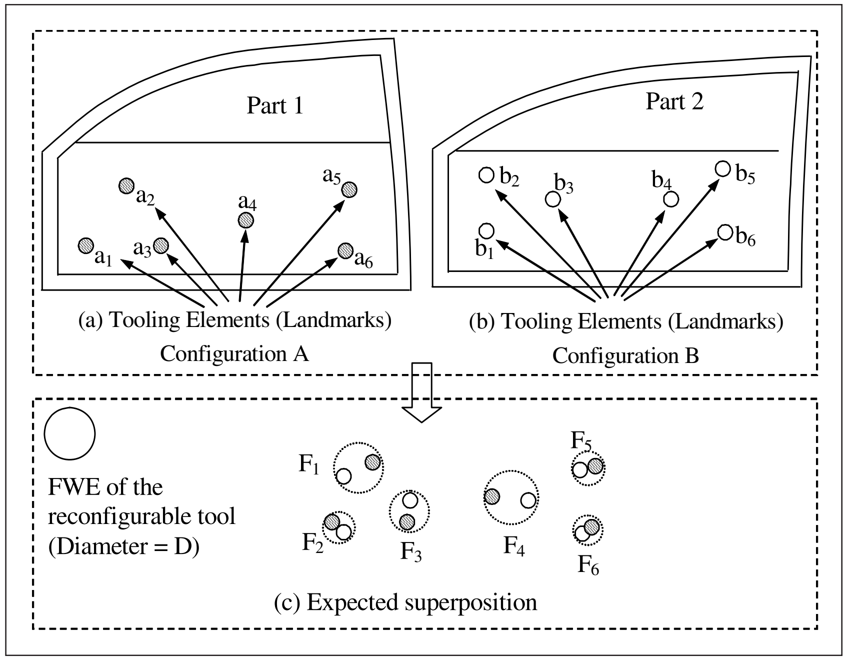

Figure 2
Purpose of Fixture Workspace Synthesis and Associated Terminology

2. Fixture Workspace Synthesis Methodology

2.1 Purpose and Objective of Fixture Workspace Synthesis

Figure 2 shows an example of a part family, which includes two doors that belong to two different vehicle models that are similar but not identical. The fixture workspace synthesis approach focuses on the layout of locating points of the two parts, which have a set of locating points represented by a_i and b_i (i = 1, …, 6), respectively. During the fixture design phase, the best superposition of these two parts on a single fixture needs to be identified; that is, the corresponding locating points of the two parts will coincide or be enclosed within the FWE of a given reconfigurable tool. In the example shown in *Figure 2*, both parts have six locating points; thereby, there are correspondingly six clusters marked as F_1–F_6 in *Figure 2c* that are called Locating Point Clusters (LPCs). Each LPC is held by using one reconfigurable tool. Finally, a total of six reconfigurable tools (locators) are needed to constitute the reconfigurable fixture that can handle the two parts shown in *Figure 2*.

The presented analysis of fixture workspace uses the following basic concepts: tooling elements, landmarks, and fixture (layout) configurations. Each part is represented by a set of locators or clamps called Tooling Elements (TEs). The points used to represent these TEs are called landmarks (*Figures 2a* and *2b*). Two parts classified by the same TEs are recognized as identical within the context of fixture workspace synthesis. Fixture configuration is defined as a set of landmarks on a part that is represented by matrix $k \times m$ in Cartesian coordinates, that is, k TEs each defined in m dimensional space. In the case of fixture workspace synthesis, $k \times 2$ or $k \times 3$ is considered, which corresponds to 2-D or 3-D part fixturing layouts, respectively.

In a general case of fixture synthesis, it is assumed that a given part family has N parts, and each part has k locating points. For each LPC, its workspace is defined as a circle/sphere F_i (i = 1, 2, …, k) that encloses the N corresponding locating points of each part within the part family (*Figure 2* depicts a 2-D case with N = 2 and k = 6). The proposed methodology is presented in the context of three industrial case studies from Lee, Hu, and Ward (1999). The presented three case studies will be used to illustrate not only the proposed methodology but also for comparative analysis of the proposed methodology with the approach presented in Lee, Hu, and Ward (1999). These three case studies focus on addressing the following engineering requirement functions (ERFs):

(i) ERF I: Minimize the FWE of the tooling element with the largest FWE within a given fixture. This engineering requirement function allows for application of the tooling elements (fixture) with the relatively smallest reconfigurability (FWE), which results in relatively cheaper solution. This requirement can be expressed mathematically as minimizing the largest diameter of F_i (i =1, 2, …, k) (see *Figure 2c*).

$$Min\left(Max\left(F_i\right)\right) \quad 1 \le i \le k \tag{1}$$

Let $(F_{max})_{Min(Max)}$ be defined as the largest circle among the k circles obtained based on Eq. (1). Then, during fixture design, the diameter of the FWE for a given reconfigurable tool must be equal to or greater than $(F_{max})_{Min(Max)}$ to ensure that each corresponding locating point cluster can be enclosed within the FWE of the reconfigurable tool.

(ii) ERF II: Minimize the total FWE for all tooling elements within a given fixture. Correspondingly, the engineering requirement function can be expressed as:

$$Min\left(\sum_1^k F_i\right) \tag{2}$$

which is to minimize the overall FWE for all reconfigurable tools.

(iii) ERF III: Minimize FWE of a specified tooling element and simultaneously minimize the total FWE for all tooling elements. The engineering requirement function of this case can be formulated as:

$$Min\left(\sum_1^k F_i\right) \quad \text{under } Min\left(F_s\right) \tag{3}$$

where s indicates a specified tooling element.

Fixture workspace synthesis for a family of parts can be accomplished intuitively by the following approach (2-D case): first, fix one part and then rotate and translate the other $N-1$ parts to meet one of the engineering requirement functions represented by Eqs. (1)–(3). This approach requires simultaneous optimization of $3*(N-1)$ variables because there are two translations and one rotation for each part in 2-D space. This is also the basic approach utilized by Lee, Hu, and Ward (1999). However, as discussed in Section 1.2, this approach has two disadvantages: (1) the search domain for all of the variables is quite large, and (2) there are a large number of variables that need to be simultaneously optimized, especially when dealing with a large number of parts that are arbitrarily placed in space.

To overcome the aforementioned challenges, a fixture workspace synthesis method for a family of parts is developed by applying Procrustes-based pairwise optimization of fixture configurations. Essentially, this method follows two steps shown in *Figure 3*. The first step applies Procrustes analysis to obtain a preliminary workspace layout that reduces the search domain for the variables. The second step is to use the pairwise configuration optimization to obtain the optimized results catering to various ERF functions as shown in Eqs. (1)–(3). *Figure 3* also describes the organization of Sections 2.2 and 2.3.

2.2 Preliminary Fixture Workspace Synthesis Using Procrustes Analysis

The Procrustes analysis uses isotropic scaling, rigid translation, and rotation transformations to best match one configuration with another wherein a configuration consists of set of multidimensional points. It provides an effective modeling tool for fixture workspace synthesis for a family of parts. Procrustes analysis is an analytical method, therefore, it can produce the output instantly. Because it is based on the least sum of squares, which is not exactly the same as the engineering requirement functions represented by Eqs.

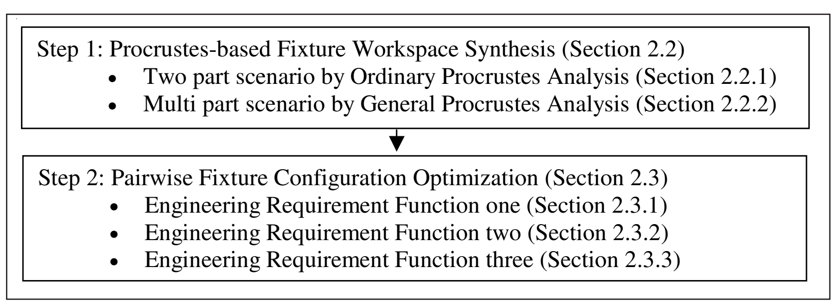

Figure 3
Procedure of Proposed Fixture Synthesis Method

(1)–(3), the Procrustes analysis is further called preliminary fixture workspace synthesis (Step 1 in *Figure 3*). The term "preliminary" means it can shorten the overall optimization process, but an optimization method (Step 2 in *Figure 3*) is still needed to obtain the final required fixture configurations.

In fixture layout design, each tooling element of a part is actually represented as an area instead of a point (Lee, Hu, and Ward 1999). To utilize Procrustes analysis, a circle is identified with a minimum diameter to enclose the locating area. Consequently, a point can be employed that is the center of the circle to represent the locating area, along with the diameter of the circle. The following subsections present the detailed approach.

2.2.1 Fixture Workspace Synthesis for Two-Part Family

Ordinary Procrustes Analysis (OPA) can be used for fixture workspace synthesis involving two parts, where a single configuration is fitted to another one by identifying superposition between two configurations (Crosilla 1999). The Procrustes analysis for fixture synthesis for a family of two parts is presented as follows.

(1) *Translation Transformations for Coordinate Registration*

When two fixture configurations are compared, the first step is to choose a proper coordinate system in which the two configurations are placed. The selected coordinate system should be able to eliminate or significantly reduce the necessary translation transformation and will be further called "coordinate registration."

Let X be a $k \times m$ matrix representing a configuration with k Tooling Elements (TEs) in m dimensions each, and $X_{i,j}$ is the (i, j)-th entry of X. The centered coordinate system is used for the coordinate registration, which can be obtained by the following steps. First, to compute the arithmetic average of the coordinate of each configuration:

$$\bar{X}_j = \frac{1}{k}\sum_{i=1}^{k} X_{i,j}, \quad j = 1,\ldots,m \tag{4}$$

Then, each configuration is translated to make the point with coordinate $\left(\bar{X}_1,\ldots,\bar{X}_m\right)$ coincide with the coordinate origin. *Figure 4* shows an example of 2-D configurations before and after the coordinate registration.

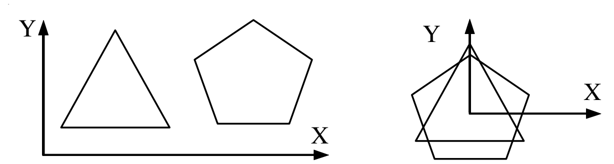

Figure 4
Coordinate Registration of Configurations

(2) *Rotation Transformation*

The distances between the corresponding TEs of the two fixture configurations are minimized by applying similarity transformations, which include translation and rotation of the fixture configurations (the scaling transformation is not used). Thus, after the coordinate registration, some necessary transformations for configurations need to be determined by which the two configurations can reach their best superposition. Assuming C_1 and C_2 are two fixture configurations after the coordinate registration, represented by $k \times m$ matrices, then the following holds:

$$C_1^T L_k = C_2^T L_k = 0$$

where L_k is a $k \times m$ matrix with all elements equal to one. Let R and T be the rotation and translation transformations, respectively. Then, if fitting C_1 to C_2 using R and T transformations, the following is obtained:

$$C_2 = C_1 R + L_k T^T + \varepsilon \tag{5}$$

where ε is the residual error, which indicates the difference between two fixture configurations. By applying the least-squares method, ε can be minimized with the objective function as follows (see *Figure 2c* for notation of F_i):

$$Min\left(\sum_{i=1}^{k} F_i^2\right) \tag{6}$$

Based on Eq. (5), the sum of squares of ε, that is, the Euclidean distance between C_1 and C_2, is:

$$\begin{aligned} D^2\left(C_1, C_2\right) &= \left\|\varepsilon^T \varepsilon\right\| \\ &= trace\ \left(C_2^T C_2 + C_1^T C_1 - C_2^T C_1 R - R^T C_1^T C_2 + kTT^T\right) \end{aligned} \tag{7}$$

Because only the last item (kTT^T) involves "T" in Eq. (7), it can be found that:

$$T = 0 \tag{8}$$

is the condition for minimizing the sum of squares of the residual error. Therefore, Eq. (7) becomes:

$$D^2\left(C_1,C_2\right)= trace\ \left(C_2^TC_2+C_1^TC_1-C_2^TC_1R-R^TC_1^TC_2\right)$$

Consequently, the minimized sum of squares of the residual error is:

$$D_{\min}^2\left(C_1,C_2\right) = trace\ \left(C_2^TC_2+C_1^TC_1\right)-2\sup trace\left(C_2^TC_1R\right) \tag{9}$$

By using the singular value decomposition, the item $C_2^TC_1$ can be expressed as:

$$C_2^TC_1=V\Lambda U^T$$

where V and U belong to a special orthogonal group and Λ is the vector of eigenvalues of $C_2^TC_1$. Based on the properties of a special orthogonal group, if

$$R=UV^T \tag{10}$$

then the minimized sum of squares of residual error can be achieved and the result is as follows:

$$D_{\min}^2\left(C_1,C_2\right) = trace\ \left(C_2^TC_2+C_1^TC_1\right)-2trace\left(\Lambda\right) \tag{11}$$

Finally, the solution of the transformations is obtained that can lead to the best superposition based on the least square of the residual error between two configurations, and the translation and rotation transformations are represented by Eqs. (4) and (10), respectively.

2.2.2 Fixture Workspace Synthesis for Multi-Part Family

General Procrustes Analysis (GPA) is applied for fixture workspace synthesis involving more than two parts. Assume there are $N \geq 2$ fixture configurations represented by X_1, X_2, …, X_N, where each configuration is represented by a $k \times m$ matrix, k is the number of TEs, and m represents dimension. The GPA also uses similarity transformations to translate and rotate the configurations to minimize the total sum of squares of the distance of each pair of configurations $D(X_1, X_2, \ldots, X_N)$, which can be expressed as follows:

$$D\left(X_1,X_2,\ldots,X_N\right)= \frac{1}{N}\sum_{i=1}^{N}\sum_{j=i+1}^{N}\left\|\left(X_iR_i+L_kT_i\right)-\left(X_jR_j+L_kT_j\right)\right\|^2 \tag{12}$$

where R_i and T_i represent rotation and translation transformations and i represents the i-th fixture configurations. Equation (12) indicates that the approach is composed of all of the combinations of OPA for each pair of configurations. Based on this relationship, an efficient algorithm can be developed, the procedure for which is as follows:

(1) *Translation transformations for coordinate registration*

This is done using translation to center all of the configurations. After this step, for all X_i, $i = 1, \ldots, N$, the following holds,

$$X_i^TL_k=0$$

where L_k is a $k \times m$ matrix with all elements equal to one.

(2) *Rotation transformation*

For each configuration X_i, $i = 1, \ldots, N$, the average is computed for other $N-1$ configurations, which is represented by:

$$\bar{X}_{(i)}=\frac{1}{N-1}\sum_{j=1,j\neq 1}^{N}X_j$$

Consequently, for the two resulting configurations, X_i and $\bar{X}_{(i)}$, the OPA can be applied to obtain the necessary translation T_i and rotation R_i for X_i so as to achieve the best superposition with $\bar{X}_{(i)}$. Calculate $D(X_1, X_2, \ldots, X_N)$ using Eq. (12).

(3) *Repeat step (2)*

From Eq. (8), it can be seen that there are no more translations needed after the coordinate registration. Therefore, step (2) can be repeated for all $i = 1, \ldots, N$, and the resulting rotation transformations will make the N configurations iteratively closer and closer, so the value of $D(X_1, X_2, \ldots, X_N)$ will be decreased. The procedure will stop when $D(X_1, X_2, \ldots, X_N)$ cannot be reduced any further.

By using the GPA algorithm, the necessary translation [from Step (1)] and rotation [from Step (2)] for each configuration can be obtained, which leads to the best match for all configurations in terms of the least sum of squares of the coordinate differences between every pair of the configurations.

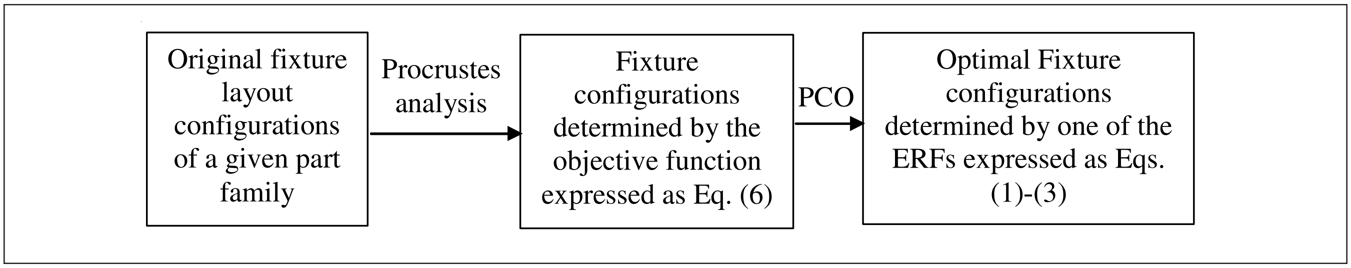

Figure 5
Fixture Configuration Transitions and Corresponding Analysis

2.3 Pairwise Configuration Optimization (PCO)

Section 2.2 presents a preliminary fixture workspace synthesis using analytical Procrustes analysis based on the objective function expressed as Eq. (6), to reduce search domain of the variables that need to be optimized. In this section, a pairwise configuration optimization method is developed to obtain the final fixture workspace layout required by the ERF functions expressed by Eqs. (1)–(3). The flow of configuration transitions and the corresponding analysis is described in *Figure 5*.

The challenge in conducting pairwise optimization for the final solution of fixture workspace synthesis is how to conduct it for a number of parts equal to any $N > 2$. If one part is simply fixed and the other $N - 1$ parts are rotated and translated, then the simultaneous optimizing of $3*(N–1)$ variables is required for 2-D cases. The complexity involved is excruciatingly high. To solve this problem, a Pairwise Configurations Optimization (PCO) method is used, which simultaneously deals with only three variables at any time. The basic procedure is illustrated in *Figure 6*. For N parts, assume that Y_i, $i = 1, \dots, N$, represents their fixture layout configurations obtained from the conducted Procrustes-based fixture workspace synthesis analysis (Section 2.2). For each configuration, Y_{ct} ($ct = 1, \dots, N$), the average configuration, $\overline{Y}_{(ct)}$, is computed based on all configurations except Y_{ct}.

$$\overline{Y}_{(ct)} = \frac{1}{N-1} \sum_{j=1, j \neq ct}^{N} Y_j$$

Then search optimization for the pair of configurations Y_{ct} and $\overline{Y}_{(ct)}$ is conducted based on one of the ERF functions represented by Eqs. (1)–(3). During the optimization, the configuration of $\overline{Y}_{(ct)}$ is fixed, and rotation and translation are made to Y_{ct}. For example, for 2-D cases in the XY plane, the search variables are T_x, T_y (translations in X and Y directions) and R (rotation in XY plane). The final result of the search optimization identifies rotation and translation transformations needed to transform Y_{ct} to satisfy one of the ERF functions. This search optimization process is repeated for every Y_{ct} ($ct = 1, \dots, N$) until the obtained value of the ERF function cannot be reduced any further. Then, the best match for the N configurations is determined, which meets the corresponding ERF function.

The PCO conducts all computations by using simultaneously only three variables. The currently existing method developed by Lee, Hu, and Ward (1999) uses simultaneously $3*(N–1)$ variables, where N is the number of parts. This results in the complexity of $O(K^{3*(N-1)})$, if the number of search steps

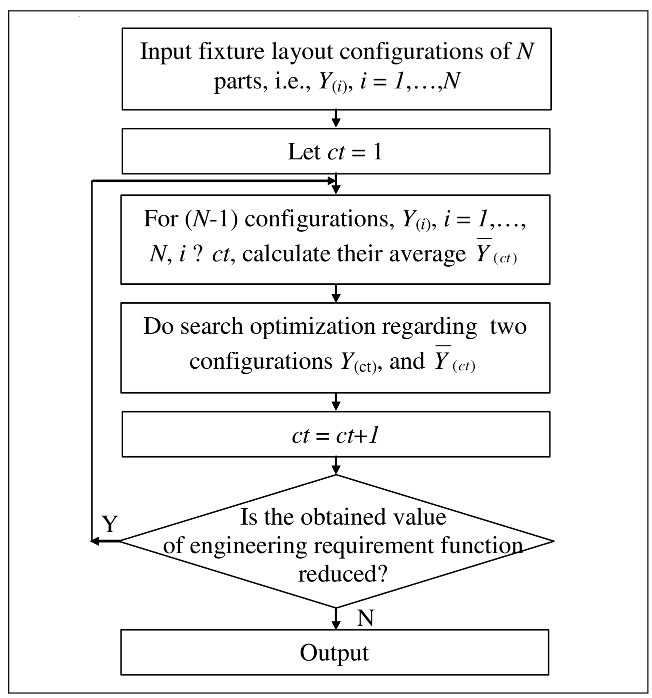

Figure 6
Procedure of the Pairwise Configuration Optimization (PCO)

for each variable is K. The method proposed in this paper deals simultaneously with only three variables, which results in the complexity of $O(K^3)$. Therefore, the complexity of optimization is drastically reduced by using the proposed PCO method.

The pairwise configuration optimization can be conducted with various ERF functions as required by specific industrial needs. As discussed in Section 2.1, this paper explores three ERF functions, described in Sections 2.3.1, 2.3.2, and 2.3.3, respectively.

2.3.1 Fixture Workspace Synthesis: Engineering Requirement Function I

The first ERF function for the fixture workspace synthesis is to minimize the FWE of the tooling element with the largest FWE within a given fixture. Let $(F_i)_{Min(Max)}$ ($i = 1, \ldots, k$) represent the diameters of the circles obtained based on ERF function I [Eq. (1)]. Assume that the obtained solution by applying Procrustes analysis for all F_i (see *Figure 2*) based on Eq. (6) is represented as:

$$(F_i)_{LS} \quad (i = 1,2,\ldots,k) \tag{13}$$

The diameter of the maximum circle obtained based on Eq. (6) is represented as $(F_{max})_{LS}$. From the definition of the least sum of squares, the following then must hold:

$$\left(\sum_{i=1}^{k}(F_i)_{LS}^2\right) < \left(\sum_{i=1}^{k}(F_i)_{Min(Max)}^2\right)$$

Assuming $(F_{max})_{Min(Max)}$ is the maximum amid $(F_i)_{Min(Max)}$, $i = 1, \ldots, k$, the following can be obtained:

$$\left(\sum_{i=1}^{k}(F_i)_{LS}^2\right) < k\left(F_{\max}\right)_{Min(Max)}^2 \tag{14}$$

It is known that Eq. (1) is the ERF function for the fixture workspace synthesis that minimizes the largest circle among the ones that enclose the corresponding locating point cluster of different configurations. Therefore, the following inequality holds:

$$\left(F_{\max}\right)_{Min(Max)} < \left(F_{\max}\right)_{LS} \tag{15}$$

Based on Eqs. (14) and (15), the following can be obtained:

$$\sqrt{\left(\sum_{i=1}^{k}(F_i)_{LS}^2\right)\Big/k} < \left(F_{ax}\right)_{Min(Max)} < \left(F_{\max}\right)_{LS} \tag{16}$$

Equation (16) actually provides the boundary condition for further pairwise configuration optimization, and it can be utilized to determine the range of variables that need to be optimized. Afterward, a simple exhaustive search optimization method can be applied to rapidly find the desired solution.

2.3.2 Fixture Workspace Synthesis: Engineering Requirement Function II

Here is minimized the total FWE for all tooling elements within a given fixture. The ERF function II is formulated as Eq. (2).

The notation in Eq. (2) is the same as in Eq. (1), and the expected configuration can still be described by *Figure 2*. Assume $(F_i)_{Sum}$, $i = 1, \ldots, k$, are the diameters of the circles obtained using Eq. (2). Then,

$$\sum_{i=1}^{k}(F_i)_{Sum} < \sum_{i=1}^{k}(F_i)_{LS}$$

where $(F_i)_{LS}$ is obtained using the Procrustes analysis. If the diameter of minimum circle among $(F_i)_{Sum}$, $i = 1, \ldots, k$, is notated as $(F_{min})_{Sum}$, the following inequality must hold:

$$\left(F_{\min}\right)_{Sum} < \sum_{i=1}^{k}(F_i)_{LS}\Big/k \tag{18}$$

Equation (18) can be utilized to narrow the range of the variables that need to be optimized during the further optimization based on ERF function II.

2.3.3 Fixture Workspace Synthesis: Engineering Requirement Function III

This ERF function is to (i) minimize the FWE of one selected tooling element and (ii) simultaneously to minimize the total FWE for all tooling elements. This scenario is much more specific than others because it has requirements for both a workspace of a selected tooling element and an overall fixture workspace. The ERF function for this case is expressed as Eq. (3). This is a two-step optimization. The first step is to minimize the workspace of a specified TE. Then, the second step will minimize the overall fixture workspace, meanwhile maintaining the specified TE workspace unchanged.

In Section 2.2, a point called a landmark is used to represent a locating area, and the landmark is a center of the minimum circle that encloses the corresponding locating area. The procedure for satisfying ERF function III is explained by using a simple example with the two fixture configurations shown in *Figure 7*. Assume that TE 6 was selected to be minimized. Thus, after the Procrustes analysis (*Figure 7a*), both TE 6's for parts A and B are coincided by using translation (*Figure 7b*). This can ensure that the size of workspace for TE 6 is minimized, and it is actually equal to the diameter of the larger circle among the two workspaces of TE 6 for parts A and B (as shown in *Figure 7b*; see *Figure 8* for details).

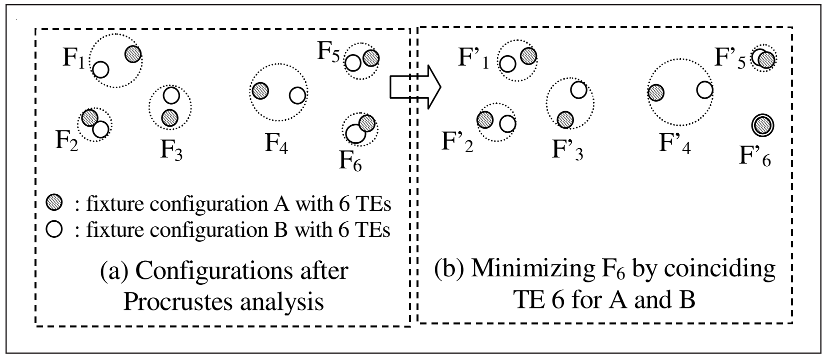

Figure 7
Processing for Engineering Requirement Function III

In *Figure 8*, which is, in fact, an enlarged view of the workspace of TE 6 in *Figure 7b*, it can be seen that the two landmarks belonging to the two configurations A and B (*Figure 8a*) become one point after they are coincided (*Figure 8b*), and consequently, the aforementioned larger workspace of the two TEs for fixture configurations A and B determines the required minimum workspace of TE 6.

Thus, after the specified workspace of TE 6 is minimized, the overall workspace also needs to be minimized. This can be done by rotating one of the configurations around the center of the coincided TE 6 to identify the overall minimum fixture workspace. In this way, the selected TE workspace (TE 6 in *Figure 7*) still remains minimized. Moreover, it can be seen that to remain the minimized size of the specified TE workspace it is not necessary to keep the two TE 6's for configuration A and B completely coincided. For example, as long as the distance "δ" (*Figure 8c*) between the two TE 6 landmarks for configurations A and B is small enough so that Circle 1 is always within Circle 2, the resulting workspace, F_6', is still minimized (F_6' is determined by the size of Circle 2). Therefore, this provides an extra capability (translations) to minimize the overall fixture workspace in addition to the aforementioned rotating operation.

In general, for k locator point clusters (in *Figure 7*, k = 6), the diameters of the corresponding k circles are represented as F_i', i = 1, …, k, based on the process described in *Figure 7b*. The selected TE workspace to be minimized is noted as F_p' and marked as D_{min}. It is assumed that F_i'', i = 1, …, k, represents the final solution, which meets both requirements of minimizing the FWE of one selected tooling element and simultaneously minimizing the total FWE for all tooling elements. During optimization of the overall fixture workspace, the selected workspace of TE 6 (Circle 2) that has been minimized will not change, that is,

$$F_p'' = F_p' = D_{\min} = D_{Circle2}$$

Based on the optimization requirement, it is known that:

$$\sum_{i=1}^{k} F_i'' < \sum_{i=1}^{k} F_i' \qquad (19)$$

It is assumed that the smallest circle among F_i'', excluding F_p'' is $F_{2nd\_\min}''$. Then, based on Eq. (19), the following relationship holds:

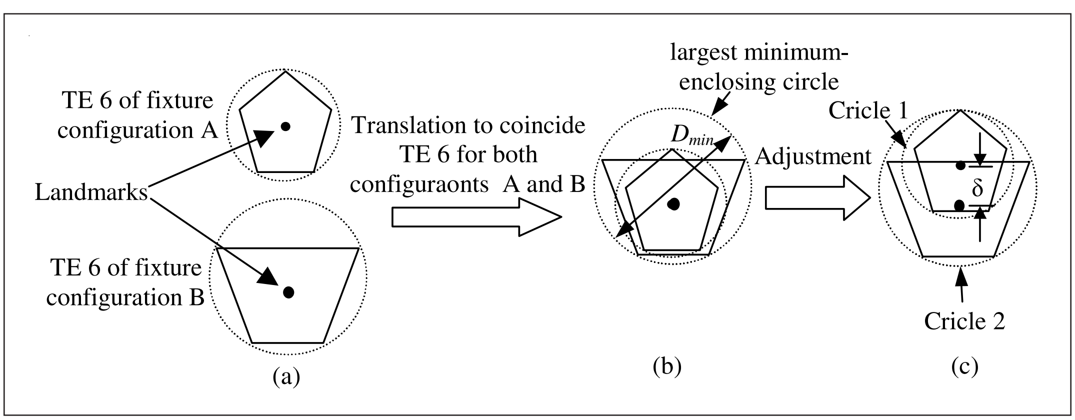

Figure 8
Enlarged View of Workspace TE 6 in *Figure 7*

$$F''_{2nd\_\min} < \left(\sum_{i=1}^{k} F'_i - D_{\min} \right) \Big/ k \qquad (20)$$

Equation (20) can be used to reduce the ranges of the variables that need to be optimized during the further optimization based on ERF function III.

The presented procedure can be summarized in the following way by using the example in *Figure 8*. After applying Procrustes analysis, the coinciding operation applied to the selected TE of configurations A and B illustrated in *Figure 8b* is conducted. Then, the minimized workspace will remain unchanged as long as Circle 1 is within Circle 2, as shown in *Figure 8c*. Under this constraint and based on Eq. (3), the pairwise optimization approach is applied and the final solution can be obtained, which satisfies both the requirements of (i) minimizing the FWE of one selected tooling element and (ii) simultaneously minimizing the total FWE for all tooling elements.

3. Case Studies

Two case studies are provided to demonstrate the advantages of the proposed method over the existing method. For case study 1 presented in Section 3.1, exactly the same data is used as given in Lee, Hu, and Ward (1999). Because Lee, Hu, and Ward do not specify the dimension unit of the original coordinate data, what is obtained in this paper is only the ratio of improvement of the method versus the method shown by Lee, Hu, and Ward. Case study 2 is presented in Section 3.2. One part (with dimension unit) taken from an automotive body assembly process is used to run fixture workspace synthesis by using both methods. The detailed results show the improved performance of the proposed method versus the method shown in Lee, Hu, and Ward.

3.1 Case Study 1: Comparison Study with Existing Method

In this case study, there is one part family with three parts, each of which has six locating areas whose coordinate data are listed in the Appendix in *Table A1*. Each locating area (TE workspace) is represented by a polygon.

Figure 9 shows the fixture configuration layout of the original coordinates without optimization. The corresponding locating areas are enclosed using a circle with the smallest diameter. There are six locating points for each part, so a total of six reconfigurable tools are needed, with each one holding three TEs (locators) corresponding to three parts. Among the six circles, the largest one has a diameter of 4.87.

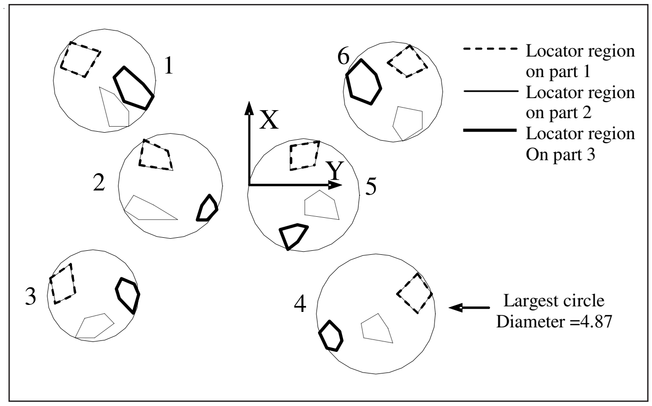

Figure 9
Original Fixture Configuration Layout of the Individual Part

Figures 10, *11*, and *12* describe the solutions by using the method proposed in this paper. *Figure 10* shows the solution based on ERF function I, and the diameter of the largest circle (being minimized) is 4.16 (TE 4). *Figure 11* shows the results based on ERF function II, and the optimized solution is that the sum of diameters of all circles (TEs) is 22.29. *Figure 12* depicts the solution based on ERF function III. The result shows that the diameter of Circle 6 (TE 6) is 1.84 and the sum of the diameters of all circles (TEs) is 24.02.

Table 1 presents a comparative summary of all results obtained using the method proposed in this paper and the method of Lee, Hu, and Ward (1999). It is noted that for ERF function I, the method described in this paper results in 3% improvement. For ERF function III, the method obtains improvement of 12%: the diameter of the smallest specified circle is reduced from 2.08 to 1.84 and the result of overall circles is also improved.

Based on the comparison of the results from the two methods, it can be seen that the proposed method is able to achieve equal or better solutions in dealing with various requirements. Moreover, the method is based on analytical Procrustes analysis and pairwise configuration optimization. Therefore, the algorithm is comparatively more efficient and the implementation is much easier than Lee, Hu, and Ward's method.

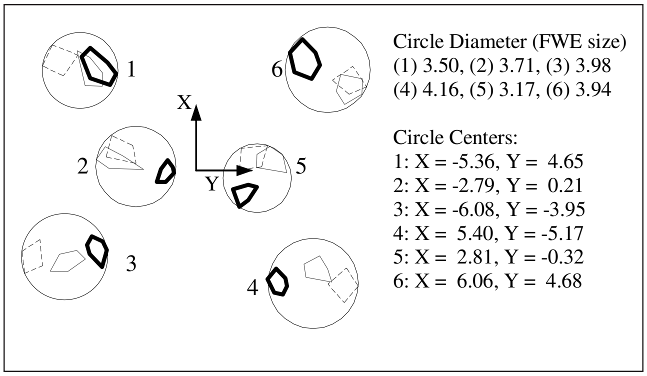

Figure 10
Fixture Workspace Analysis Based on ERF Function I

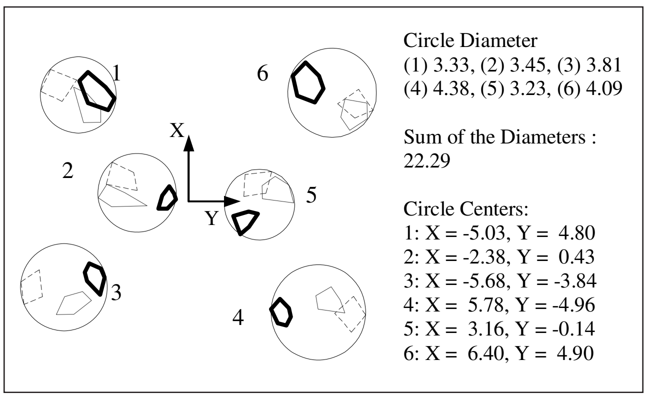

Figure 11
Fixture Workspace Analysis Based on ERF Function II

3.2 Case Study 2: Interpretion of Optimization Result with Industrial Reconfigurable Tools Selection

An underbody floor panel is selected for comparison, with approximate dimension of 2000 × 3000 (unit: mm). *Table A2* lists the TE locations for a part family with three parts, which have homothetic shapes with the parts shown in *Figure 9*. This case study compares both methods by conducting a reconfigurable tool selection procedure in the context of a fixture workspace synthesis problem. The underbody floor panel is around 250 times of the coordinate data in *Table A1*. *Figure 1b* shows commercial reconfigurable tools produced by Fanuc Robotics America Inc. This series of robots has two specifications, with cylindrical FWE of 500 mm (F-100) and 1024 mm (F-200) in diameter, respectively. Now assume that there is a case in which there is limited room for the workspace of TE 6 (*Figure 12*) due to the components, which might interfere with TE 6. This is a problem of fixture workspace synthesis with ERF function III. Therefore, the optimization approach presented in Section 2.3.3 is applied to obtain the optimized solution. The optimized solution results in a workspace diameter of TE 6 equal to 460 mm. By comparison, the solution output from the existing method in Lee, Hu, and Ward (1999) is 520 mm (shown in *Table 2*).

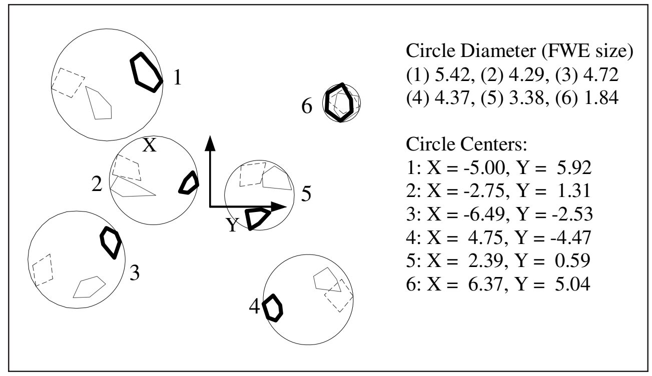

Figure 12
Fixture Workspace Analysis Based on ERF Function III

Table 2 summarizes optimization results by using both methods, which leads to a design decision in the selection of the best reconfigurable tools. For exactly the same coordinate data of part locating areas, the difference of the results from the method proposed in this paper (460 mm) and the existing method (520 mm) might result in a selection of different reconfigurable tools. Now, there are two types of reconfigurable tools available in the product catalog, namely, F-100 and F-200, with FWEs of 500 mm and 1024 mm, respectively. Because the workspace must be equal to or smaller than the available reconfigurable tools, based on the solution obtained from the proposed method, the reconfigurable tool named F-100 is applicable. However, if the result from the existing method is used, then the reconfigurable tool F-200 needs to be utilized. Because the cost and the size of F-200 are much higher and larger than F-100, unnecessary waste will occur. More importantly, the larger reconfigurable tool may not fit in the limited available space. Therefore, the tooling installation may become infeasible if the result from Lee, Hu, and Ward (1999) is adopted.

4. Summary

Today's rapidly changing market is creating a trend that adopts technologies of reconfigurable assembly systems capable of assembling multi-products on a single assembly line. In an attempt to address this trend, this paper presents an approach

Table 1
Comparison of Results Between Method in Lee, Hu, and Ward (1999) and Proposed Method in This Paper

	Engineering Requirement Functions	Results of Lee, Hu, and Ward (1999)*	Results of Proposed Method in This Paper	Improvement
Case 1	Minimization of largest workspace	Largest circle is # 6; Diameter = *4.29*	Largest circle is # 4; Diameter = *4.16* (*Figure 10*)	3%
Case 2	Minimization of total workspaces	Sum of circle diameter: *22.33*	Sum of circle diameter: *22.29* (*Figure 11*)	
Case 3	Minimization of a specified (TE 6) workspace and total workspaces	Diameter of circle #6: *2.08*; Sum of circle diameter: *24.13*	Diameter of circle #6: *1.84*; Sum of circle diameter: *24.02* (*Figure 12*)	12%

* Dimensional unit is not spedified in the data presented in Lee, Hu, and Ward (1999)

Table 2
Selection of Reconfigurable Tools Based on Optimization Results

Available Commercial Reconfigurable Tools		Minimum Workspace Obtained by Existing Method		Minimum Workspace Obtained by Proposed Method	
Type	FWE Size	Tool Selection	Workspace Diameter Size	Tool Selection	Workspace Diameter Size
F100	500 mm		*520* mm	X	*460* mm
F200	1024 mm	X			

of fixture workspace synthesis for a family of parts, which is critical for design of fixtures with reconfigurable tooling elements. The proposed approach applies analytical Procrustes analysis to narrow the search domain for variables and uses pairwise configuration optimization to identify the final solution based on various engineering requirement functions. The approach can rapidly and accurately find the solutions that minimize the fixture workspace for a family of parts with much less computational complexity than the existing method in the literature. The case studies compare the proposed approach with the existing method. The results demonstrate that, overall, the proposed method can obtain better solutions over the existing method by achieving greater improvement in both algorithm efficiency and solution accuracy.

Acknowledgment

The authors gratefully acknowledge the financial support of the NSF Engineering Research Center for Reconfigurable Manufacturing Systems (NSF Grant EEC95-92125) and National Science Foundation award NSF-DMII-0218208.

References

Apley, D.W. and Shi, J. (1998). “Diagnosis of multiple fixture faults in panel assembly.” *ASME Journal of Mfg. Science and Engg.* (v120, n4), pp793-801.

Asada, H. and By, A. (1985). “Kinematic analysis of workpart fixturing for flexible assembly with automatically reconfigurable fixtures.” *IEEE Journal of Robotics and Automation* (vRA-1, n2), pp86-94.

Cai, W.; Hu, S.J.; and Yuan, J.X. (1996). “Deformable sheet metal fixturing: principles, algorithms, and simulations.” *ASME Trans., Journal of Mfg. Science and Engg.* (v118, n3), pp318-324.

Ceglarek, D. (1998). “Multivariate analysis and evaluation of adaptive sheet metal assembly systems.” *Annals of the CIRP* (v47, n1), pp17-22.

Ceglarek, D. and Shi, J. (1995). “Dimensional variation reduction for automotive body assembly.” *Mfg. Review* (v8, n2), pp139-154.

Ceglarek, D. and Shi, J. (1996). “Fixture failure diagnosis for auto body assembly using pattern recognition.” *ASME Trans., Journal of Engg. for Industry* (v118, n1), pp55-65.

Chou, Y-C.; Chandru, V.; and Barash, M.M. (1989). “A mathematical approach to automatic configuration of machining Fixtures: analysis and synthesis.” *ASME Trans., Journal of Engg. for Industry* (v111, n4), pp299-306.

Crosilla, F. (1999). “Procrustes analysis and geodetic science.” Technical Report. Stuttgart, Germany: Dept. of Geodesy and Geoinformatics, Univ. of Stuttgart.

De Meter, E.C. (1995). “Min-max load model for optimizing machining fixture performance.” *ASME Trans., Journal of Engg. for Industry* (v117, n2), pp186-193.

Ding, Y.; Shi, J.; and Ceglarek, D. (2002). “Diagnosability analysis of multistage manufacturing processes.” *ASME Trans., Journal of Dynamic Systems, Measurement, and Control* (v124, n1), pp1-13.

Dryden, I. and Mardia, K. (1998). *Statistical Shape Analysis*. New York: John Wiley & Sons.

Gupta, S. and Krishnan, V. (1998). “Product family-based assembly sequence design methodology.” *IIE Trans.* (v30, n10), pp933-945.

Hu, S.J. (1997). “Stream-of-variation theory for automotive body assemblies.” *Annals of the CIRP* (v46, n1), pp1-6.

Hu, S. and Wu, S.M. (1992). “Identifying root causes of variation in automobile body assembly using principal component analysis.” *Transactions of NAMRI/SME* (v20), pp311-316.

Kong, Z. and Ceglarek, D. (2003). "Rapid deployment of reconfigurable assembly fixtures using workspace synthesis and visibility analysis." *Annals of the CIRP* (v52, n1), pp13-16.

Koren, Y.; Heisel, U.; Jovane, F.; Moriwaki, T.; Pritschow, G.; Ulsoy, A.G.; and Van Brussel, H. (1999). "Reconfigurable manufacturing systems." *Annals of the CIRP* (v48, n2).

Lee, J.; Hu, S.J.; and Ward, A.C. (1999). "Workspace synthesis for flexible fixturing of stampings." *ASME Trans., Journal of Mfg. Science and Engg.* (v121, n3), pp478-484.

Menassa, R.J. and DeVries, W.R. (1991). "Optimization methods applied to selecting support positions in fixture design." *ASME Trans., Journal of Engg. for Industry* (v113), pp412-418.

Moon, K.S. and Sutherland, J.W. (1992). "Improved image information via Procrustes analysis," *Quality Assurance* (v56), pp139-150.

Park, E. and Mills, J. (2002). "Three-dimensional localization of thin-walled sheet metal parts for robotic assembly." *Journal of Robotic Systems* (v19, n5), pp207-217.

Rekiek, B.; De Lit P.; and Delchambre, A. (2000). "Designing mixed-product assembly lines." *IEEE Trans. on Robotics and Automation* (v16, n3), pp268-280.

Rong, Y. and Bai, Y. (1996). "Machining accuracy analysis for computer aided fixture design verification." *ASME Trans., Journal of Mfg. Science and Engg.* (v118, n3), pp289-300.

Roy, U. and Sun, P.L. (1994). "Selection of preliminary locating and clamping positions on a workpiece for an automatic fixture design system." *Computer Integrated Mfg. Systems* (v7, n3), pp161-172.

Sakurai, H. (1992). "Automatic setup planning and fixture design for machining." *Journal of Manufacturing Systems* (v11, n1), pp30-37.

Stadzisz, P.C. and Henrioud, J.M. (1998). "An integrated approach for the design of multi-product assembly systems." *Computers in Industry* (v36, n1-2), pp21-29.

Travaini, E.; Pedrazzoli, P.; Rinaldi, R.; and Boër, C.R. (2002). "Methodological approach and reconfiguration tool for assembly systems." *Annals of the CIRP* (v51, n1), pp9-13.

Appendix

Table A1
Data for Candidate Locator Regions (Lee, Hu, and Ward 1999)

	Part One		Part Two		Part Three	
	X	Y	X	Y	X	Y
1	–7.8	4.6	–6.2	3.8	–5.6	4.2
	–6.8	4.2	–5.8	2.2	–5.2	3.2
	–6.2	5.2	–5.0	2.2	–4.3	2.9
	–7.4	5.6	–5.0	2.8	–4.0	3.4
			–5.6	3.5	–4.4	3.9
					–5.2	4.6
2	–4.6	0.6	–5.2	–1.2	–2.2	–1.6
	–3.2	0.4	–4.6	–1.6	–1.8	–1.6
	–3.4	1.2	–3.0	–1.6	–1.4	–1.2
	–4.4	1.6	–4.0	–1.0	–1.5	–0.9
			–4.8	–0.6	–1.7	–0.6
					–2.0	–-1.0
3	–8.2	–4.0	–7.2	–6.4	–5.5	–4.4
	–8.0	–5.0	–6.4	–6.4	–5.4	–4.8
	–7.2	–4.6	–5.6	–5.8	–4.8	–5.4
	–7.4	–3.4	–6.0	–5.4	–4.6	–4.6
			–6.8	–5.6	–4.8	–4.2
					–5.3	–4.0
4	6.0	–4.6	4.6	–6.4	2.8	–6.2
	6.8	–5.4	5.8	–6.6	3.1	–6.8
	7.4	–4.6	5.6	–6.0	3.5	–6.9
	6.8	–3.8	5.2	–5.4	3.7	–6.5
			4.5	–5.8	3.6	–6.1
					3.2	–5.7
5	2.6	0.6	2.2	–1.4	1.5	–2.8
	1.6	0.4	3.6	–1.6	1.9	–2.5
	1.6	1.4	3.4	–0.8	2.1	–2.2
	2.8	1.6	2.8	–0.4	2.3	–1.9
			2.2	–0.8	1.9	–1.8
					1.2	–2.0
6	5.6	4.8	5.8	2.2	4.1	3.4
	6.4	4.2	6.2	1.6	4.9	3.1
	7.2	4.6	7.0	2.1	5.3	3.7
	6.5	5.5	7.0	2.8	5.1	4.3
			6.0	3.0	4.5	4.9
					3.9	4.3

Table A2
Data for Candidate Locator Regions of the Part with Unit (Unit: mm)

	Part One		Part Two		Part Three	
	X	Y	X	Y	X	Y
1	–1950	1150	–1550	950	–1400	1050
	–1700	1050	–1450	550	–1300	800
	–1550	1300	–1250	550	–1075	725
	–1850	1400	–1250	700	–1000	850
			–1400	875	–1100	975
					–1300	1150
2	–1150	150	–1300	–300	–550	–400
	–800	100	–1150	–400	–450	–400
	–850	300	–750	–400	–350	–300
	–1100	400	–1000	–250	–375	–225
			–1200	–150	–425	–150
			0	0	–500	–250
3	–2050	–1000	–1800	–1600	–1375	–1100
	–2000	–1250	–1600	–1600	–1350	–1200
	–1800	–1150	–1400	–1450	–1200	–1350
	–1850	–850	–1500	–1350	–1150	–1150
			–1700	–1400	–1200	–1050
					–1325	–1000
4	1500	–1150	1150	–1600	700	–1550
	1700	–1350	1450	–1650	775	–1700
	1850	–1150	1400	–1500	875	–1725
	1700	–950	1300	–1350	925	–1625
			1125	–1450	900	–1525
					800	–1425
5	650	150	550	–350	375	–700
	400	100	900	–400	475	–625
	400	350	850	–200	525	–550
	700	400	700	–100	575	–475
			550	–200	475	–450
					300	–500
6	1400	1200	1450	550	1025	850
	1600	1050	1550	400	1225	775
	1800	1150	1750	525	1325	925
	1625	1375	1750	700	1275	1075
			1500	750	1125	1225
					975	1075

Authors' Biographies

Zhenyu Kong received his BS and MS degrees in mechanical engineering, both from Harbin Institute of Technology, China, in 1993 and 1995, respectively, and his PhD degree in industrial engineering at the University of Wisconsin-Madison in 2004. He is currently an assistant professor in the School of Industrial Engineering and Management at Oklahoma State University. His research interests include modeling and analysis of manufacturing processes for quality improvement, information technology for diagnosis of manufacturing processes, and reconfigurability and reusability of assembly systems. He is a member of SME, IIE, ASME, and INFORMS.

Dariusz Ceglarek received his PhD degree in mechanical engineering from the University of Michigan in 1994. He was with the research faculty at the University of Michigan from 1995 to 2000. In 2000, he became an assistant professor in the Det. of Industrial and Systems Engineering at the University of Wisconsin-Madison, where he rose to the rank of associate professor and professor in 2003 and 2005, respectively. His research interests include design and manufacturing with an emphasis on multistage production system convertibility, scalability, and diagnosability, ramp-up time, and variation reductions; integration of statistical methods and engineering models for root-cause identification of quality/variation faults; sensing systems/networks in manufacturing; and reconfigurable/reusable assembly systems. Dr. Ceglarek was elected as a fellow of CIRP and is a member of SME, NAMRI/SME, ASME, IIE, and INFORMS. He was chair of the Quality, Statistics and Reliability Section of INFORMS and is currently a program chair of the ASME Design-for-Manufacturing Life Cycle (DFM-LC) Conferences. He is an associate editor of the *ASME Transactions, Journal of Manufacturing Science and Engineering*. He also serves on the program review panel for the State of Louisiana Board of Regents R&D Program. He has received a number of awards for his work, including the 2003 CAREER Award from the National Science Foundation, 1998 Dell K. Allen Outstanding Young Manufacturing Engineer Award from the Society of Manufacturing Engineers, and Best Paper Awards by the ASME MED and DED divisions in 2000 and 2001, respectively.