Mode-based Decomposition of Part Form Error by Discrete-Cosine-Transform with Implementation to Assembly and Stamping System with Compliant Parts

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Abstract
A discrete-cosine-transformation (DCT) based decomposition method is proposed for modeling part form error, which decomposes the error field into a series of independent error modes. Compression, which ensures a compact model, is achieved by correlation reduction or mode truncation based on good energy compaction property of DCT. The part error related to assembly process (rigid body modes) and part distortion during manufacturing (deformation modes) can be separated and identified. DCT has been proven to be equivalent to least square regression with 2D cosine-base for modeling of part variation pattern. Estimation of these parameters in the model has also been developed. The proposed method was applied to model and evaluate assembly and stamping errors at one of the US stamping plant.

Key words:
Form error, Geometric tolerancing, Discrete cosine transformation

1 INTRODUCTION

Parametric statistical tolerance plays a prominent role in current industrial design practice, existing standards (ASME Y14.5.1M-1994), and in almost all previous tolerancing research. Geometric tolerances are also used in design to describe form and shape errors as well as the orientation and position relationships between features. Size, orientation and position tolerances of a polyhedral component can easily be parameterized and modeled by simply shifting the nominal geometry from a 3D solid model. The geometric and statistical tolerancing have also been commonly accepted. Therefore, it is important to be able to conduct geometric tolerancing statistically to avoid higher cost margin of overcompensated design through worst-case method.

When a more geometrically complex product is considered, the current parameterized tolerance techniques confront great difficulties. For example, in automotive industry the automotive body is comprised of hundreds of sheet metal parts. Reducing the geometric variation of an auto body is particularly important not only for functionality (proper functioning, avoidance of water leak, wind noise, aesthetics…) but also for manufacturability and suitability for assembly [1,2]. Dimensions due to geometric part complexity cannot simply define the specifications of most of these parts.

Current tolerance models are not yet satisfactory in simulating the effect of the manufacturing process statistically on part geometry. It is an enormous challenge to develop a model, which conforms to the tolerance zone suggested by standard (ASME Y14.5) and helps to generate instances statistically to simulate part manufacturing process statistics for tolerance analysis.

The objective of this paper is to develop a mode-based decomposition technique for form error representation. This technique provides a basis for establishing a surface-based variational model, which is suitable for implementation in statistical geometric tolerancing.

2 RELATED WORKS

In the last two decades a comprehensive research effort has been made to establish theoretic framework and application algorithms for geometric tolerancing.

In the work of Requicha [3], tolerance zone (TZ) and variational class concepts for representing geometric tolerances have been pioneered. Tolerance zone was defined as a region bounded by two of the same geometries, offset from the nominal feature surface (all actual features of the surface, which are within the zone belongs to the acceptable (in spec) variational class). Several techniques have been developed for computing offset surfaces in CAD models [4,5]. All these efforts focused on tolerance zone boundary generation, which would be utilized for representing tolerances of mechanical parts in CAD/CAM systems. Turner [6] proposed a higher order polynomial function to model form tolerance for the nominal planner part feature. Bezier's triangle fitting and triangle patches have also been proposed to represent planner surface by Gupta and Turner [7]. However, the Bezier surface does not interpolate all control points except for the first and the last points, thus the variation instances created can not be accurately controlled. Two independent surface variations have been developed for tolerance analysis from the simplification of surface mating as in a kinematic joint [8], wherein, only the boundaries of tolerance zone are used to develop variations, so it only permits worst case analysis. A sixteen-point bicubic surface interpolation method has been proposed by Roy and Li [9,10] which assures that all control points will be randomly selected in tolerance zone located on the interpolation surfaces. Combination of more than one surface patch is needed for representing variation class of a complex feature. One of the main problems with the above method is that too many control points are needed and the variations of these points are correlated (dependent), consequently statistical (Monte Carlo) tolerance methods are invalid.

There is a tremendous need for developing a tolerance model, which 1) supports current geometric tolerance standards; 2) helps to create variational class which can simulate production process statistically; and 3) is compact and based on independent variables.

3 PROBLEM STATEMENT

The objective of this paper is to develop a modeling methodology for part form error field representation, characterization and random simulation. Given the ensemble of real/actual surfaces as measured by CMM, and which represent the manufacturing process variation statistics, we will address the question on how to describe the statistical properties of the part form error field by developing unified math model with minimum number of independent variables.

Form error field in this paper is defined as the difference of actual and nominal surface feature(s). Two hypotheses are introduced to simplify the modeling process:

Smoothness assumption: form error field signal has sufficient smoothness such that the high spatial frequency components (short wavelength error such as surface roughness and waviness) are small and can be ignored. This assumption implies that form error is highly spatially correlated.

Height field assumption*:* the form error of a part feature (surface) can be represented as a height field function *f*(x,y) defined in 2D domain, which is a stationary random field process.

In general, part form error field can be sampled as discrete-space signals by CMM measurements. In order to minimize number of parameters in describing a form error field, the correlation reduction technique, such as orthogonal transforms commonly used in digital image processing, can be applied. The sampled error data set $f(n,m)=f(n\Delta x, m\Delta y)$, where n and m represent sample size in two axes, is transformed into spatial frequency domain. The used transformation has the property of selecting number of transform coefficients in the model to reduce correlation between error data points. For typical "smooth" error field a large amount of "signal energy" is expected to be concentrated in a small fraction of the transform coefficients. For two-dimensional square signal (sampled data) arrays, where the number of sample points is N^2, the forward and inverse transforms (models generation and reconstruction, respectively) are given as follows:

$$T(u,v)=\sum_{n=0}^{N-1}\sum_{m=0}^{N-1} f(n,m)g(n,m,u,v) \quad \text{- forward transform, and}$$

$$f(n,m)=\sum_{u=0}^{N-1}\sum_{v=0}^{N-1} T(u,v)h(n,m,u,v) \quad \text{- inverse transform} \qquad (1)$$

where T(u,v) are independent transformation parameters representing contribution of the error mode with space frequency of u and v in two axes x and y, respectively. The g(n,m,u,v) and h(n,m,u,v) are called the forward and inverse transformation kernels, respectively (see also Section 4). Transformation described by Eq. (1) can be interpreted as superposition of a series of rigid and deformation modes which represent the principle pattern of an error field *f*(x,y) (Figure 1). Each coefficient T(*u,v*) represent the different mode characterized by integer space frequency coefficients u and v in two axes, respectively.

Each mode can represent various manufacturing error patterns (Fig. 1). For example, the rigid body mode (Mode 1 or T(*0,0*)) can represent assembly fixturing positioning error; deformation mode (Mode 2 or 3: T(*1,0*) or T(*0,1*)) can represent part distortion (bending) during stamping operations (see case study 2 for further details).

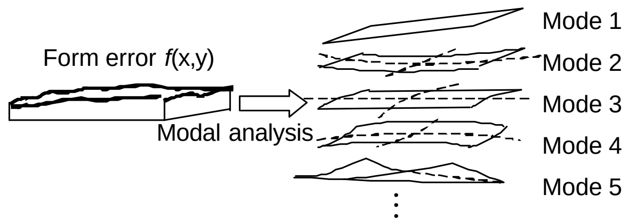


Figure 1. Error field mode-based decomposition

An important property of the presented transformations is that of energy compaction, i.e., the most energy (information) of the sampled data can be represented by a small number of the transformation coefficients T(*u,v*), that furthermore allows us to discard many coefficients or to truncate many trifling modes without seriously affecting the error field information. All transformations used presently in image processing are orthogonal, so the energy preservation holds between frequency domain and error field "pixel" domain (space domain represented by sampled measurement data). Thus, mode truncation criteria can be developed based upon this property. The significance of each mode can be identified as magnitude of the parameter T(u,v) and expressed as the level of signal energy represented by that mode. The ratio of the energy expressed by selected number of significant modes to the total energy of the signal (sampled data) can be used to characterize the energy compaction of the model.

4 2-DIMENSIONAL DCT

4.1 Single part mode decomposition via DCT transformation

Eq. (1) (coefficients T(*u,v*)) represents a class of generic orthogonal transformations. In this paper for the purpose of manufacturing form error modeling, we will investigate the discrete-cosine transformation (DCT) (with coefficients marked as C(u,v)) Let us assume that the realization function of the error field ensemble is represented by *f*(n,m) and can be obtained by CMM measurements of uniformly sampled parts. The two-dimensional forward DCT kernel is defined as:

$$g(n,m,0,0)=1/N$$

$$g(n,m,u,v)=\frac{2}{N}\cos[2\pi nu/(2N-1)]\cos[2\pi mv/(2N-1)]$$

for n,m=0,1,…N-1 and u,v=0,1,…N-1, where N represents sample size and number of modes. The inverse kernel h(n,m,u,v) is also of the same form. The two dimensional transformation pair is given for u,v=0,1,…,N-1 and n, m=0,1,…, N-1 as follows:

$$
\begin{cases}
C(0,0)= \dfrac{1}{N}\displaystyle\sum_{n=0}^{N-1}\sum_{m=0}^{N-1} f(n,m) \\
C(u,v)= \displaystyle\sum_{n=0}^{N-1}\sum_{m=0}^{N-1} f(n,m)g(n,m,u,v)
\end{cases}
\quad (3)
$$

$$
\begin{cases}
f(0,0)= \dfrac{1}{N}\displaystyle\sum_{u=0}^{N-1}\sum_{v=0}^{N-1} C(u,v) \\
f(n,m)= \displaystyle\sum_{u=0}^{N-1}\sum_{v=0}^{N-1} C(u,v)g(n,m,u,v)
\end{cases}
\quad (4)
$$

One of the advantages of the DCT transformation is that there is a fast algorithm available as in Discrete Fourier Transform (DFT), which allow high efficient computations. The DCT can be obtained from the DFT by mirroring the original N-point sequence to obtain a 2N-point sequence. Furthermore, DCT outperforms DFT in energy compaction, because DFT block or segment-based transforms will lead to discontinuities at block boundaries (this, so called Gibbs phenomenon, can be overcome by making DFT on the 2N-point mirrored sequence).

Coefficients of DCT and the associated modes can provide easy-to-interpret illustration of the part of subassembly error field. The superposition of the first three modes in Eq. (4), approximately represent the rigid modes of the form error field. For example, when u=v=0,

$f(n, m)|_{1st\ mode} = \dfrac{2}{N^2}\displaystyle\sum_{k=0}^{N-1}\sum_{l=0}^{N-1} f(k,l)$ which represents a horizontal plane (rigid body motion of the part/subassembly), when u=0 v=1 or v=0 u=1, we have inclined quasi-rigid modes as shown in Figure 2. Part form deformation contributes more to higher modes, as shown in Fig. 3. The error field mode decomposition concept will benefit the separation and identification of different error pattern sources, for example, in a subassembly or a final product rigid mode error represents datum or locator induced part position/orientation deviations, while the deformation modes reveal the error caused by parts manufacturing or assembly process characteristics such as die misalignment, springback, joining compliant parts and/or part interference.

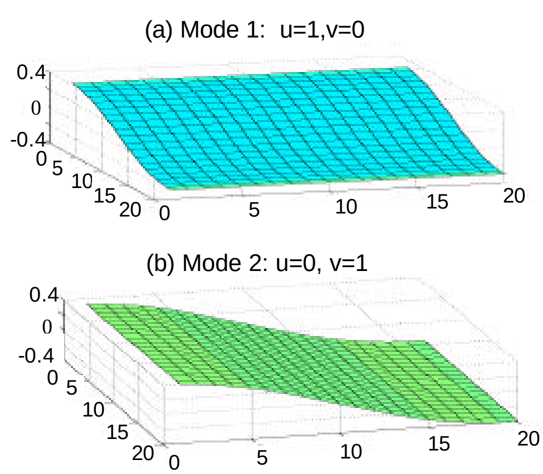


Figure 2 Rigid modes of part/subassembly form error field

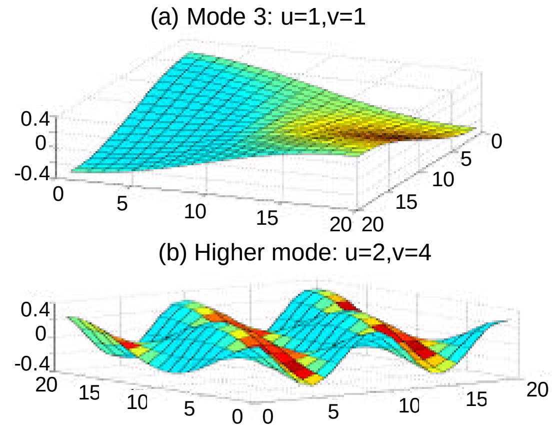


Figure 3 Higher modes of part form error field

4.2 DCT and cosine base regression for multiple part mode decomposition

The DCT transformation can be used to estimate part error field of a single part/subassembly based on the measurement data. However, in general, in manufacturing process it is important to estimate error field for a sample of measured parts/subassemblies not only for a single one. In this section we will provide the estimation of statistical properties of the C(u,v) coefficients by using regression analysis. We will prove that the DCT transform is equivalent to a 2D cosine base regression, i.e. the C(u,v) coefficients of the DCT transformation are equivalent to the regression coefficients $c_{u,v}$ (b) and their estimation **b** (Section 4.3). This will help to obtain the statistical property of coefficients C(u,v) in Eq. (4) by using estimation theory of regression model. Considering the continuous error field function in the form of *f*(x,y), we use the following regression model:

$$
f(x,y)= \sum_{u=0}^{\infty}\sum_{v=0}^{\infty} c_{u,v}\cos(uxp)\cos(vyp)
$$

The continuous function basis of {h(x,y,u,v)=cos(uxπ)cos(vyπ): u, v=0,1,…∞ } is orthogonal and represents deformation mode of order (u, v) where u, v represent spatial frequencies (samples per unit length). Similarly, it is easy to show that the discrete base vectors of:

$H^T_{u,v}$=[h(1,0,u,v), h(1,1,u,v), h(1,2,u,v)…h(1,N-1,u,v), h(2,0,u,v), h(2,1,u,v),…h(2,N-1,u,v)…, h(N-1,N-1,u,v)]^T

for u, v=0,1…, I-1, N≥I are also orthogonal, that is:

$$
(H^T_{u,v})(H_{s,t})=
\begin{cases}
N^2/4 & \text{for } u=s \text{ and } v=t \\
0 & \text{for } u\neq s \text{ or } v\neq t \text{ or both}
\end{cases}
\quad (5)
$$

and where N≥I implies that only a fraction of modes are considered in the model.

According to the smoothness assumption we can fit measurement data set of f(*n,m*) by using only a fraction of the basis:

$$
f(x,y)= c_0+\sum_{u=1}^{I-1}\sum_{v=1}^{I-1} c_{u,v}\cos(uxp)\cos(vyp) + \varepsilon(x,y)
$$
$$
=\mathbf{X}b+\varepsilon(x,y) \quad (6)
$$

where b={$c_0, c_{1,1}, c_{1,2}, c_{1,3}$ …, $c_{I-1,I-1}$}^{T}, the column of **X** is composed of a unit vector and **I** orthogonal base vectors ($H_{u,v}$) and ε(x,y) stands for random errors, so the least square estimation for measurements f(*n,m*) is as follows:

$$\hat{f} = X\mathbf{b} \quad \mathbf{b}=(X'X)^{-1}X'f \tag{7}$$

We will prove the equivalence of least squares estimation of as presented in Eq. (7) and DCT transformations presented in Eqs. (3) & (4). In fact, from Eq. (5), we have:

$$(X'X)^{-1} = 4/N^2\,[\mathbf{I}] \tag{8}$$

By inserting Eq. (8) into Eq. (7), and then comparing Eqs. (3) and (7), we see that $b_{u,v}$ are actually the coefficients $(2/N)*C(u,v)$ in Eq. (7).

4.3 Parameter estimation

The form error field can be considered as a random field when taking manufacturing process uncertainties into consideration. The parameters in the regression or DCT models are therefore, random variables, which are subject to a distribution induced by manufacturing process. Estimation methods in multiple regression theory can be used for parameter estimation. Assuming that $\varepsilon(n,m)\sim N(0,\sigma^2)$ and Gauss-Markov conditions hold, it can be concluded that **b** is unbiased estimation of b**:**

$E(\mathbf{b})=b$, and $Cov(\mathbf{b})= \sigma^2(X'X)^{-1}=4\sigma^2/N^2[\mathbf{I}]$

It implies the statistical independent property of model coefficients in **b.**

If we just keep small fraction of the most significant coefficients $\mathbf{b_1}$ in **b,** it is important to determine the goodness of the estimation. Considering the orthogonality of all column vectors of X, it can be proved that: $E(\mathbf{b_1})= b_1$. Thus, the estimation of $\mathbf{b_1}$ is still unbiased.

Test for statistical significance of any individual regression parameters, say β_j, can be conducted by using the hypothesis: H_0: $\beta_j=0$; H_1: $\beta_j\neq 0$. The test statistic for this hypothesis is:

$t_0 = \dfrac{b_j}{\sqrt{\hat{s}^2 a_{jj}}} = \dfrac{b_j N}{2\hat{s}}$, where $a_{jj}=4/N^2$ is the diagonal element of $(X'X)^{-1}$. If $|t_0| > t_{\alpha/1,\, N^2-(I-1)^2}$ the null hypothesis is rejected, it means the parameter β_j is statistically significant.

It is noteworthy that even if a parameter is statistically significant it still might be dropped off from the model because of its small energy contribution. In the mode truncation criteria all those modes, whose parameters are both energy and statistically significant, should remain.

Further we have the observation error estimation from the model:

$E[f(x,y)]=Xb$ and $\mathbf{C}ov(f(x,y))=E[(f-Xb)(f-Xb)']= \sigma^2\mathbf{I}$

Let $\mathbf{X}_0 = (\mathbf{X}_{00}, \mathbf{X}_{01}, \mathbf{X}_{02} \ldots \mathbf{X}_{0K})$, K<N, represent values of K bases at any 1~L points on form error. The predicted value of $f(x,y)$ is $\hat{f}_0 = X'_0\, \mathbf{b}$, and under the Gauss-Markov condition yields:

$E[\hat{f}_0]=X'_0\, b$ and $var(\hat{f}_0)=\mathbf{X}_0'\, cov(\mathbf{b})\, \mathbf{X}_0= 4\sigma^2/N^2\, [\mathbf{X}_0'\mathbf{X}_0]$.

Hence if $f_1, f_2, f_3, \ldots f_L$ are normally distributed:

$\hat{f}_0 - X'_0\, b \sim \boldsymbol{N}(0, 4\sigma^2/N^2\, [\mathbf{X}_0'\mathbf{X}_0])$

where $(1-\alpha)\times 100\%$ percent confidence interval (C.I.) and for the mean predicted value $X'_0\, b$ is given by:

$\hat{f}_0 \pm t_{r,\alpha/2}\, 2\hat{s}/N[\mathbf{X}_0'\mathbf{X}_0]^{1/2}$.

For future observation $\hat{f}_0$ we have the $(1-\alpha)\times 100\%$ percent C.I. or prediction interval as:

$$\hat{f}_0 \pm t_{r,\alpha/2}\, s[1+(4/N^2)\mathbf{X}_0'\mathbf{X}_0]^{1/2}$$

5 CASE STUDY

Case Study 1: A 4 points bilinear NURBS surface is used to simulate the form error surface. A grid with 400 nodes is used to represent CMM measured error data set (Figure 4 a). By using 99.9% energy compaction with only 3 independent variables (DCT coefficients), maximum error (Hausdorff distance) caused by the proposed method is 3%. Small amount of error can be seen at the corners of the error surface in Figure 4 b.

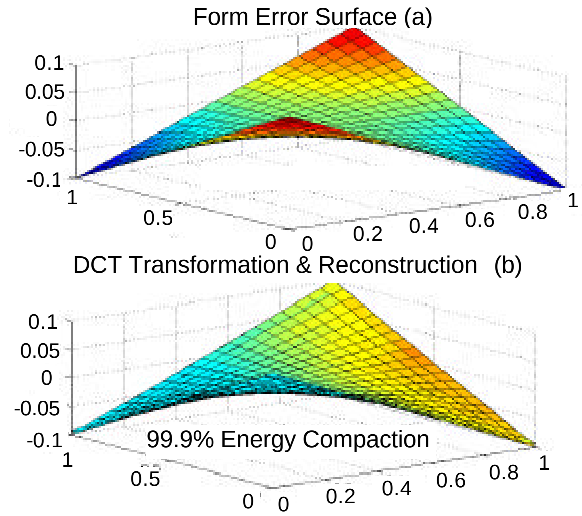


Figure 4 Comparison of original error and reconstructed error field

Case Study 2 This case study describes part error field estimation in one of the US stamping and assembly manufacturers. The analyzed stamped part with 24 measurement points grid is shown in Figure 5 (a). The parts were measured by CMM. The form error surface was obtained by calculating the differences between the measured and nominal data in z-direction (Figure 5 (b)). The DCT coefficients C(u,v) for 5 modes with 99% signal energy compaction are shown in Table 1. These coefficients provide the information of the significance of each individual mode's contribution to the total form error. Figure 5(c) presents reconstruction of the surface error field by using the DCT model. The relationship between truncation and energy compaction are shown in Figure 6. The mode shapes are shown in Figure 7. Maximum error (Hausdorff distance) caused by truncation is estimated at 6.3%. Table 2 presents the significance of each mode, their DCT coefficients and physical interpretation of the error field

Table 1 Reserved mode coefficients C(u,v) with 99% error signal energy compaction

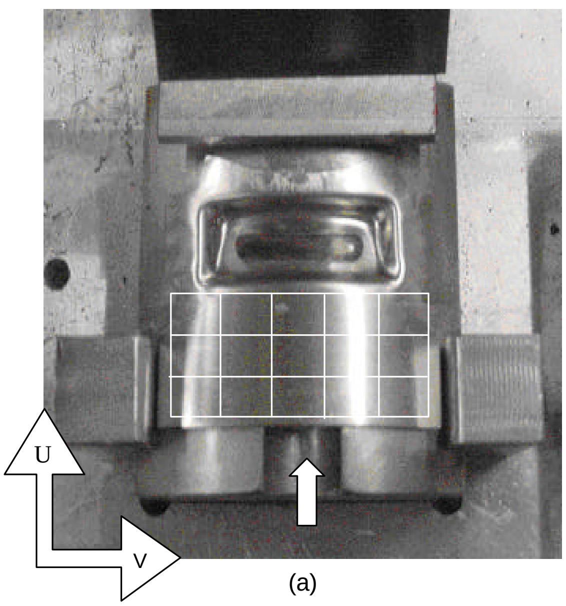
C(u,v)	1	2	3	4	5	6
1	0.04031	-0.03317	-0.02806	0	0	0
2	0.05151	-0.03432	0	0	0	0
3	0	0	0	0	0	0
4	0	0	0	0	0	0

described by each mode. The presented mode-based decomposition of the part/error field provides an important modeling tool for geometric tolerancing and manufacturing system diagnostics or process control.

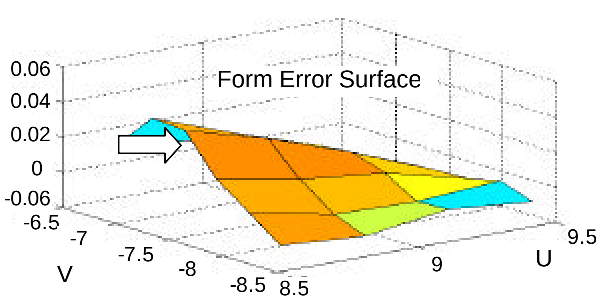
6 CONCLUSION

Based on discrete-cosine-transformation (DCT) method a mode-based decomposition scheme for form error has been proposed. The form error surface or field is represented as a combination of a series of mode components. The orthogonality of these modes ensures the independency of their coefficients, a fundamental prerequisite for statistic geometric tolerance analysis. Fewer significant modes are required to represent the error surface thereby, establishing more compact representation.

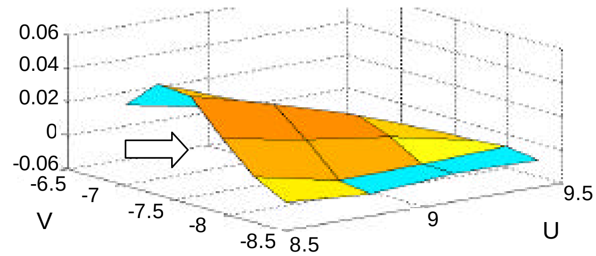
The proposed method also provides a base for tolerance-zone-constraint-driven variational representation since the extreme values of the truncated representation model is easy to determine when distribution of coefficients is known. The Tolerance Zone (TZ) constrained mode synthesis for form error simulation will be a focus in our future work.



(a)



(b)



(c)

Figure 5 A sheet metal part (a), its form error surface (b), and reconstructed error surface with 5 modes (c)

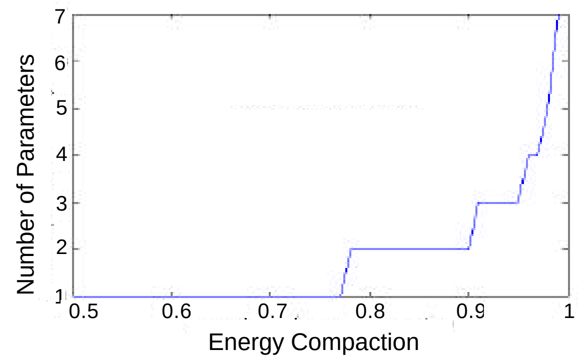
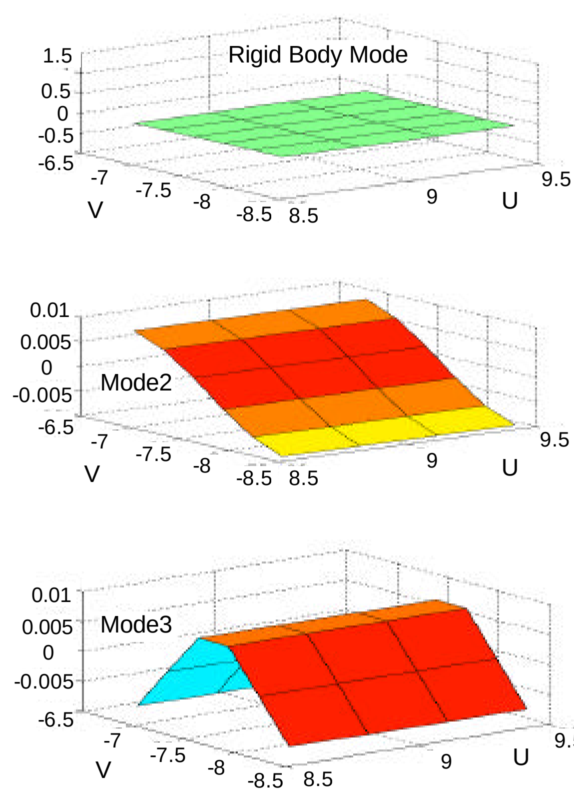
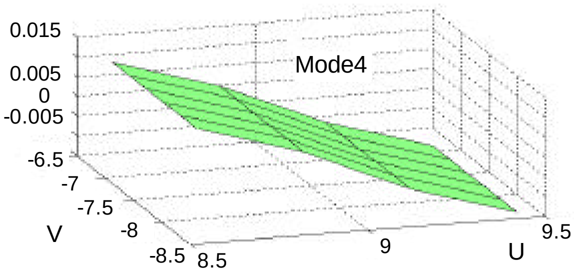


Figure 6 Energy compaction vs. mode truncation





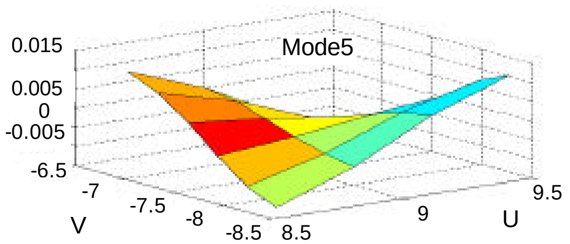


Figure 7 Decomposition of the form error: modes 1-5

Table 2 Case study 2: Modes significance and interpretation

Modes	Coefficients	Significance	Error Interpretation
Mode 1	C(0,0)	22.13%	Mean shift caused by fixture calibration
Mode 2	C(0,1)	14.98%	Springback error
Mode 3	C(0,2)	10.72%	Non-uniform material flow creating potential part wrinkles
Mode 4	C(1,0)	36.13%	Lower & upper die misalignment
Mode 5	C(1,1)	16.04%	Non-uniform strain distribution causing part twist in drawing operation

Another advantage of the method is the ability to separate and identify error pattern sources, since the most significant modes have an intuitive and easy to explain physical interpretation. The modes are related to the manufacturing process error sources such as part positioning error (lower order modes), part twisting, spring back, wrinkling and other etc., (higher modes). The mode shape, orientation and its significance (coefficient) provide more error pattern information than the raw data. The presented two case studies illustrate the presented methodology.

Although the proposed model is an attempt to bridge the gap between tolerance representation and the uncertainties from manufacturing processes, more challenges will be involved in collecting form error data and establishing the statistical distributions of the mode coefficients for specific manufacturing processes.

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