Multiobjective Optimization for Integrated Tolerance Allocation and Fixture Layout Design in Multistation Assembly

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Cost and dimensional variation of products are significant attributes in multistation assembly processes. These attributes depend on product/process tolerances and fixture layouts. Typically, tolerance allocation and fixture layout design are conducted separately without considering potential interrelations. In this work, we use multiobjective optimization for integrated tolerance allocation and fixture layout design to address interactions and to quantify tradeoffs among cost, product variation, and assembly process sensitivity. A nested optimization strategy is applied to a vehicle side frame assembly. Results demonstrate the presence and quantification of tradeoffs, based on which we introduce the concept of critical variation and critical budget requirements.
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1 Introduction

Rapid changes in the market place have led designers and manufacturers to continuously develop new products to satisfy the demands of more exigent customers. Those changes have significantly affected producers of complex products, such as automobiles and airplanes, because their products consist of a large number of components that must be designed, fabricated, and assembled. For example, automobiles have up to 10,000 components and complex manufacturing processes. To be competitive, designers and manufacturers must balance final product variation with cost effectiveness.

Assembly of complex products usually follows a sequential process with multiple stations, as depicted in Fig. 1 for an automobile body structure. Different components are put together at each station and then moved downstream to form the final product. Thus, variations in the components and manufacturing processes are propagated station by station to the final product.

Evaluation of multistation assembly systems depends on some critical features, known as key characteristics. Thornton [2] defined the key characteristics as quantifiable features of a product or its assemblies, parts, or processes whose expected variation from target have a significant impact on the cost, performance, or safety of the product. In our work, key dimensional features for products are referred to as key product characteristics (KPCs), also known as key output variables (KOPVs). They usually correspond to product features with a key role on product functionality and variation. The process variables that affect the KPCs are termed key control characteristics (KCCs), also known as key input variables (KIPVs). In assembly, the KCCs that control dimensional variation are the fixtures, i.e., sets of locators and clamps used to position and hold the parts. Dimensional variation of fixtures impacts the dimensional accuracy of KPCs. Excessive variation in the KPCs may cause assembly process difficulties at subsequent stations and reduce final product variation. For example, variations in an automotive body assembly process may cause wind noise, water leakage, and part-fitting problems. Reduction of dimensional variation in assembly will improve product quality but will also incur additional costs. Thus, process performance is measured by the capability of the process to deliver products of high quality at low cost. This capability is evaluated by relating variation to tolerance specifications. Stringent tolerances require more expensive machines and tools, and so it is important to incorporate tolerances in any variation-cost analysis.

Traditionally, tolerance allocation and fixture layout design have been studied separately. Tolerance allocation is used extensively to minimize cost and final product variations, especially for rigid part assembly processes [3–7]. For example, Ding et al. [7] proposed a process-oriented tolerance synthesis for rigid multistation assembly systems, where process tolerances were allocated by solving a nonlinear constrained optimization problem, using a tolerance-variation model that relates fixture tolerances to equivalent fixture variation.

Fixture layout design is used to decrease process sensitivity with respect to external variation. Much research has been done in fixture design for single parts considering sensitivity, accessibility, and stability [8–10]. The fixture layout problem for multiple parts assembled in multistation processes was first studied by Kim and Ding [11]. They formulated the problem as an optimization one, where the objective was to minimize the impact that fixture variation has on KPC variation. They considered a type of fixture used in sheet metal assembly known as the "3-2-1" fixture scheme (see Fig. 2). A 3-2-1 fixture scheme consists of three net contact (NC) blocks or supports, where two of them have locating pins (four-way and two-way pins). The three blocks position and also restrain the part in the direction normal to the plane (y direction), and the pins help to restrict the in-plane (x-z plane) motion of the part through their fitting into a hole and a slot (also known as locators) previously pierced on the part. The position of the hole and the slot and their interaction with the fixtures play an important role on the product quality, i.e., the position of the KPC points M_1 and M_2 in Fig. 2. In the work of Kim and Ding, the design variables were the position (x and z coordinates) of the hole and slot on each part.

The present work addresses integrated layout design and tolerance allocation for a 3-2-1 fixture scheme used in sheet metal assembly to reduce variation (i.e., improve quality) cost efficiently. To this end, we (i) use a model of the multistation assembly process that connects KCC variation with that of KPCs; (ii) derive a sensitivity index (SI) that relates fixture layout design with final product quality; (iii) determine relations that use tolerances to link cost with product and process variation; and (iv) formulate and solve a multiobjective design optimization problem that trades off cost, variation, and process sensitivity subject to geometric and process-related constraints. We formulate and solve

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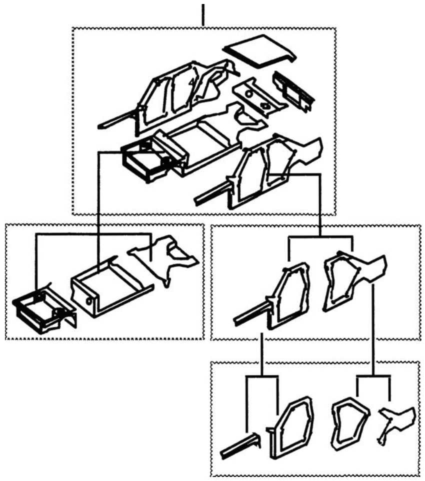

Fig. 1 Schematic of an automobile body structure assembly [1]

this problem using a nested optimization strategy. The remainder of the paper is organized as follows: In Sec. 2, we present the models used in the integrated formulation. In Sec. 3, we formulate this integrated design optimization problem to determine optimal fixture distribution, tolerance allocation, and dimensional variation concurrently. In Sec. 4, we demonstrate the proposed method on an automobile side frame example and introduce the new concept of critical variation and critical budget requirements. Finally, we draw conclusions in Sec. 5.

2 Required Models

In this section, we describe the required models for estimating variation propagation, process sensitivity, and cost.

2.1 Variation Propagation Model. In multistation assembly process, parts and processes variations are propagated station to station toward a subassembly or final product. This propagation process can be modeled to predict variation of final products. Several models have been proposed to study the variation propagation of dimensional features for rigid and compliant sheet metal parts [12–15].

Here, we use the rigid part state space model reported in Ref. [13]. This model determines the deviations of the parts $\mathbf{x}_k \in \mathbb{R}^n$ and the KPC deviations $\mathbf{y}_k \in \mathbb{R}^m$ at station k as

$$\mathbf{x}_k = \mathbf{A}_{k-1}\mathbf{x}_{k-1} + \mathbf{B}_k\mathbf{u}_k + \mathbf{w}_k \tag{1}$$

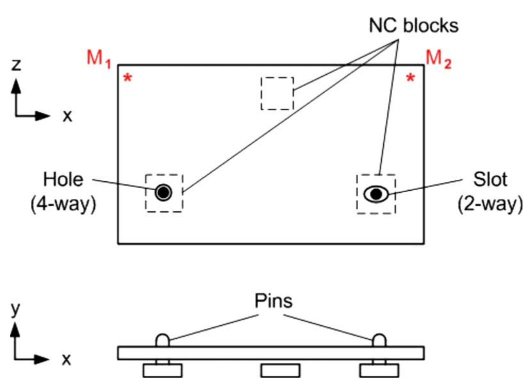

Fig. 2 Top and side views of the 3-2-1 fixture layout

$$\mathbf{y}_k = \mathbf{C}_k\mathbf{x}_k + \mathbf{v}_k, \quad k = \{1,2,\ldots,N\} \tag{2}$$

where $\mathbf{u}_k \in \mathbb{R}^p$ corresponds to the fixture deviations, $\mathbf{w}_k \in \mathbb{R}^n$ are the process disturbances and unmodeled factors, $\mathbf{v}_k \in \mathbb{R}^m$ is the measurement noise, and N is the number of stations. State matrices $\mathbf{A}_k \in \mathbb{R}^{n\times n}$, $\mathbf{B}_k \in \mathbb{R}^{n\times p}$, and $\mathbf{C}_k\mathbb{R}^{m\times n}$ depend on fixture layout, assembly sequence, locator scheme, and location of the measurement points in the multistation assembly system.

Due to the linear properties of the state space model, the deviations of the final product KPCs can be represented as a linear combination of the deviations of the fixtures in all the stations, the incoming part deviations $\mathbf{x}_0$, the external disturbances, and the measurement noise [16],

$$\mathbf{y}_N = \sum_{k=1}^{N} \mathbf{C}_N\Phi_{N,k}\mathbf{B}_k\mathbf{u}_k + \mathbf{C}_N\Psi_{N,0}\mathbf{x}_0 + \sum_{k=1}^{N} \mathbf{C}_N\Psi_{N,k}\mathbf{w}_k + \mathbf{v}_N \tag{3}$$

where $\Phi_{k,i} \equiv \mathbf{A}_{k-1}\mathbf{A}_{k-2},\ldots,\mathbf{A}_i$, $\Phi_{i,i} \equiv \mathbf{I}$, and $\Psi_{N,k} = \mathbf{C}_N\Phi_{N,i}$.

2.2 Sensitivity Index. Assuming that the main source of variation in the process are the fixtures (i.e., $\mathbf{w}_k \approx \mathbf{0}$, $\mathbf{v}_N \approx \mathbf{0}$, and $\mathbf{x}_0 \approx \mathbf{0}$), Eq. (3) can be simplified to

$$\hat{\mathbf{y}}_N \equiv \mathbf{D}\mathbf{u} = \sum_{k=1}^{N} \mathbf{C}_N\Phi_{N,k}\mathbf{B}_k\mathbf{u}_k \tag{4}$$

where $\mathbf{D} \equiv [\mathbf{C}_N\Phi_{N,1}\mathbf{B}_1, \mathbf{C}_N\Phi_{N,2}\mathbf{B}_2, \ldots, \mathbf{C}_N\mathbf{B}_N]$, $\mathbf{u} \equiv [\mathbf{u}_1^T, \ldots, \mathbf{u}_N^T]^T$, and $\hat{\mathbf{y}}_N$ is the fixture-induced product deviation.

Following this simplification, Kim and Ding [11] defined the SI for a multistation assembly system as

$$\mathrm{SI} \equiv \sup_{\mathbf{u}\neq 0} \frac{\mathbf{u}^T\mathbf{D}^T\mathbf{D}\mathbf{u}}{\mathbf{u}^T\mathbf{u}} = \lambda_{\max}(\mathbf{D}^T\mathbf{D}) \tag{5}$$

Using this index, it is possible to characterize each fixture or locator layout with the maximum possible variation that can be expected on the KPCs (worst case scenario). Therefore, an appropriate fixture layout should have small SI.

2.3 Variation and Cost Models. Without loss of generality, we assume that components, subassemblies, and processes have capabilities (C_p) [17] equal to 1. Therefore, product variation (measured in terms of the standard deviation σ) can be expressed in terms of final product tolerances $\mathbf{t}_N$,

$$\sigma = \frac{\|\mathbf{t}_N\|_\infty}{6} \tag{6}$$

The use of the infinity norm implies that the variation requirement is imposed on the KPCs with the largest variation. This representation of variation measure is one of several possible choices; other measures include the l_1-norm and the l_2-norm.

Much effort has been expended in determining the impact of tolerances on cost [3,18–20]. Manufacturing costs are both site and process dependent, and they are usually calculated based on empirical relations guided by an understanding of the process. Two models commonly used to relate cost and tolerances are based on reciprocal and exponential functions of the tolerances, and offer good data fit with a simple function. Here, the reciprocal function of tolerance (t) is chosen for the cost-tolerance relation. Then, the total cost (c) is computed as

$$c(\mathbf{t}) = \sum_{i=1}^{n} \frac{\alpha_i}{t_i} \tag{7}$$

where α_i corresponds to a fitted constant and the vector $\mathbf{t}$ contains the n tolerances of the system.

3 Problem Formulation

Figure 3 presents the relations among system inputs or design variables ($\mathbf{t}$, τ, and $\mathbf{p}$) and system outputs or attributes (c, σ, and

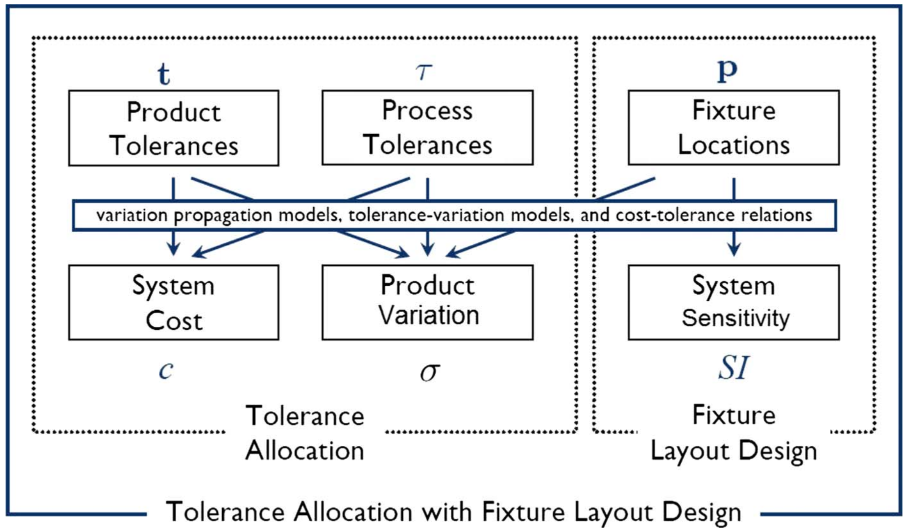

Fig. 3 Relations among system inputs and major system attributes

SI). The system cost c depends only on part and process tolerances $\mathbf{t}$ and τ. System sensitivity, measured by SI, depends only on the position of fixtures $\mathbf{p}$. Final product variation (measured in terms of final product tolerance) depends on fixture locations and process/part tolerances. Based on the aforementioned relations, we will analyze the cost-sensitivity and variation-sensitivity interactions.

3.1 Cost-Sensitivity Considerations. An appropriate fixture layout design reduces final product variability, leads to manufacturing cost reduction, and reduces the sensitivity of the system against process variation and disturbances. We study cost-sensitivity relations to investigate possible tradeoffs by formulating and solving the following two-objective optimization problem:

$$\min_{\mathbf{t},\tau,\mathbf{p}} \{c(\mathbf{t},\tau),\ \mathrm{SI}(\mathbf{p})\}$$
$$\text{subject to } \sigma(\mathbf{t},\tau,\mathbf{p}) \leq \sigma_s$$
$$\mathbf{g}(\mathbf{t},\tau) \leq \mathbf{0} \tag{8}$$
$$\mathbf{g}(\mathbf{p}) \leq \mathbf{0}$$

The design variables $\mathbf{t}$, τ, and $\mathbf{p}$ determine the total cost c and process sensitivity (SI), evaluated using Eqs. (7) and (5), respectively. The constraint $\sigma(\mathbf{t},\tau,\mathbf{p}) \leq \sigma_s$ ensures that the final product variation will satisfy the requirement σ_s. The constraints $\mathbf{g}(\mathbf{t},\tau) \leq \mathbf{0}$ represent lower and upper design bounds for $\mathbf{t}$ and τ. Finally, the constraints $\mathbf{g}(\mathbf{p}) \leq \mathbf{0}$ represent limitations on fixture locations imposed by part geometry: the locators must be positioned such that they are at least 35 mm away from any boundary (internal or external) of the part to avoid damage.

For this model we assume that changing the fixture layout is cost-free. We assume that the assembly system has not been built yet; if the assembly system were to be reconfigured, fixture layout costs would have to be taken into account. Therefore, the only costs considered in this research depend on product and process tolerances. The value of the tolerance-cost constants (α_i) are set all equal to 1.

The problem of Eq. (8) is difficult to solve due to the large number of design variables and the computational cost of the simulations. Thus, a nested optimization strategy is adopted to improve efficiency. Specifically, Eq. (8) is rearranged as

$$\min_{\mathbf{p}} \{\min_{\mathbf{t},\tau}\{c(\mathbf{t},\tau) | \sigma(\mathbf{t},\tau,\mathbf{p}) \leq \sigma_s,\ \mathbf{g}(\mathbf{t},\tau) \leq \mathbf{0}\},\ \mathrm{SI}(\mathbf{p})\}$$
$$\text{subject to } \mathbf{g}(\mathbf{p}) \leq \mathbf{0} \tag{9}$$

The resulting optimization process is shown in Fig. 4. The outer loop determines values for fixture layout design variables $\mathbf{p}$. Given $\mathbf{p}$, the sensitivity matrices are computed and used as parameters in variation propagation models for the inner loop optimization. Product and process tolerances $\mathbf{t}$ and τ are then allocated to achieve the minimum cost c while satisfying the variation requirement σ_s.

3.2 Variation-Sensitivity Considerations. We now study the relationship between final product variation and process sensitiv-

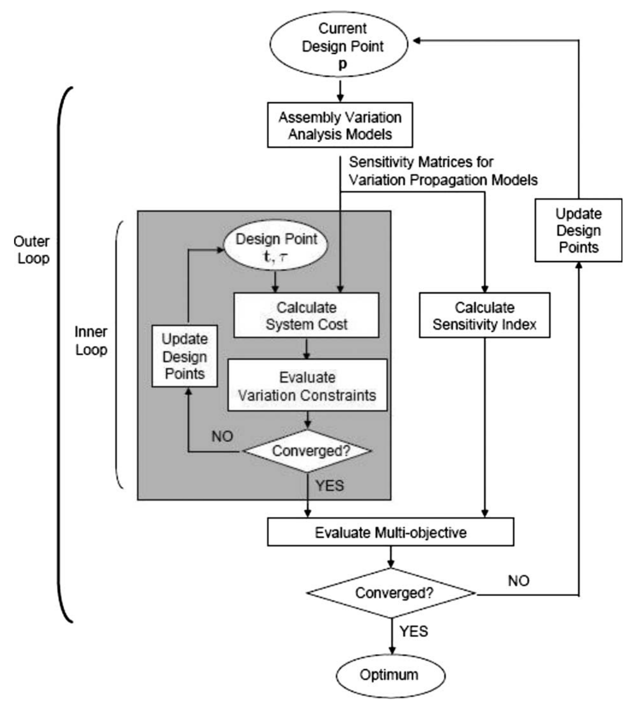

Fig. 4 Nested optimization strategy for the problem of Eq. (8)

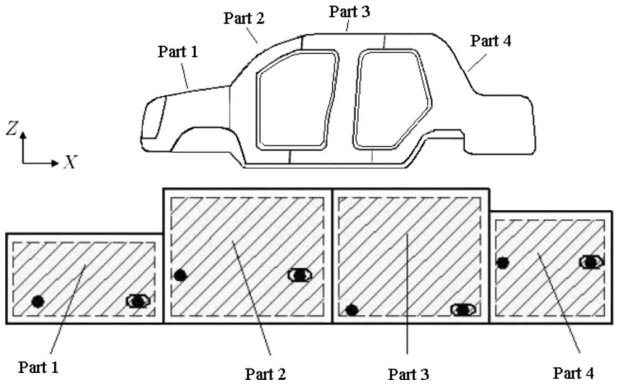

Fig. 5 Example: vehicle side frame model

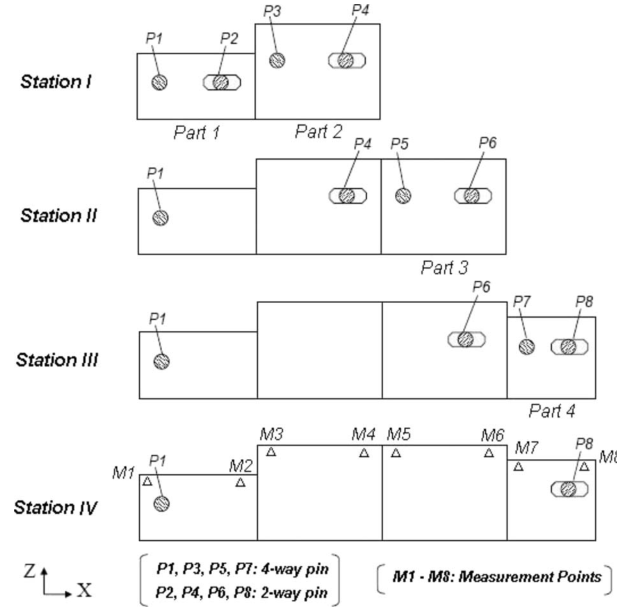

Fig. 6 Fixture layout in the rigid multistation assembly system

ity. To accomplish this, variation is considered as an objective and cost is treated as a constraint. A two-objective problem is reformulated as follows:

$$\min_{\mathbf{t},\tau,\mathbf{p}} \{\sigma(\mathbf{t},\tau,\mathbf{p}),\ \mathrm{SI}(\mathbf{p})\}$$

$$\text{subject to } c(\mathbf{t},\tau) \leqslant b \qquad (10)$$

$$\mathbf{g}(\mathbf{t},\tau) \leqslant \mathbf{0}$$

$$\mathbf{g}(\mathbf{p}) \leqslant \mathbf{0}$$

Applying the nested optimization strategy again, the problem of Eq. (10) is rewritten as

$$\min_{\mathbf{p}} \{\min_{\mathbf{t},\tau}\{\sigma(\mathbf{t},\tau,\mathbf{p})|c(\mathbf{t},\tau) \leqslant b, \mathbf{g}(\mathbf{t},\tau) \leqslant \mathbf{0}\},\ \mathrm{SI}(\mathbf{p})\} \qquad (11)$$

$$\text{subject to } \mathbf{g}(\mathbf{p}) \leqslant \mathbf{0}$$

The resulting optimization process is similar to that in Fig. 4.

4 Application to Vehicle Side Frame Assembly

We use a four-station assembly process of a sedan vehicle side frame to illustrate the concurrent tolerance allocation and fixture layout design. The two-dimensional rigid body panel assembly model is shown in Fig. 5. The panel consists of four parts: front wheel house (part 1), front passenger compartment (part 2), rear passenger compartment (part 3), and rear quarter panel (part 4). For simplicity, the parts are modeled as quadrilaterals, with dimensions 700×400 mm^2 for part 1, 750×600 mm^2 for part 2, 700×600 mm^2 for part 3, and 550×500 mm^2 for part 4, in the x and z directions.

The assembly sequence is presented in Fig. 6. Starting at level 4 (station I) parts 1 and 2 are assembled to form subassembly 1. Subassembly 1 and part 3 are assembled at station II to form subassembly 2. At station III, subassembly 2 and part 4 are joined together as the final assembly. Measurement points on the final assembly are inspected at station IV (measurement station).

In this study we focus on the possible variation of locating pins only; thus, $\{P_{4\text{way}}, P_{2\text{way}}\}$ is used as a simplified representation of a "3-2-1" fixture layout. Locating pins for parts, subassemblies, and final products for the case analyzed are shown in Fig. 6. The datum scheme (set of fixtures used to hold parts and subassemblies on each station) is

$$\{\{P_1,P_2\},\{P_3,P_4\}\}_{\mathrm{I}} \rightarrow \{\{P_1,P_4\},\{P_5,P_6\}\}_{\mathrm{II}} \rightarrow \{\{P_1,P_6\},\{P_7,P_8\}\}_{\mathrm{III}} \rightarrow \{\{P_1,P_8\}\}_{\mathrm{IV}}$$

Considering only the assembly stations (stations I, II, and II), there are 12 tolerance design variables for fixtures, and 16 position design variables for the fixtures (x-z position of each fixture).

4.1 Cost-Sensitivity Results. The problem of Eq. (9) is solved for $\sigma_s=0.33$ mm ($6\sigma_s=2$ mm) using the nested optimization strategy. A gradient-based optimization algorithm (the MATLAB implementation of sequential quadratic programming [21]) is used to solve the tolerance allocation problem in the inner loop. For the outer loop, the neighborhood cultivation genetic algorithm (NCGA) of the ISIGHT software package is used to generate the Pareto set [22]. The NCGA number of function evaluations was set to 10,000, the population size was set to 100, and the number of generations was set to 100; the one-point crossover method was used with a crossover rate of 1.0 and a mutation rate of 0.01.

As can be seen in Fig. 7 there exists a cost-sensitivity tradeoff. Specifically, a 12% decrease in sensitivity can result in an 18% increase in system cost. The tradeoff between system cost and sensitivity does not only exist for $\sigma_s=0.33$ mm; it also exists for $\sigma_s=0.3$ mm, 0.25 mm, and 0.22 mm, as presented in Fig. 8.

Fixture layout not only determines the SI, but also changes the parameters of the variation propagation models in the tolerance allocation problem and therefore affects cost. Thus, depending on the variation requirement constraint, there may exist a tradeoff between sensitivity and cost. In fact, the tradeoff between system cost and sensitivity does not exist for all variation requirements as presented in Fig. 9, where Eq. (9) was solved for several values of σ_s to further study the cost-sensitivity tradeoff.

The tradeoff becomes less significant as the variation requirement σ_s increases. At $\sigma_s=1.67$ mm, there is only a single solution, where all the 12 tolerances reach the upper bound of 2 mm. Therefore, the minimum cost for the system is $c=12\times 1/t=12\times 1/2(\mathrm{mm})=6$. The goal is then to find a fixture layout that provides the minimum SI. Thus, at $\sigma_s=1.67$ mm, there is only one solution that ensures both minimum system cost and minimum SI.

Based on this analysis, we introduce the *critical variation requirement* σ_c defined as the final product variation evaluated at the optimal fixture layout, with all tolerance variables at their upper design bounds. The problem in Eq. (8) or (9), solved at $\sigma_s=\sigma_c$, has only one solution. At this solution, all tolerances reach the upper design bounds. According to the cost-tolerance relations, the cost is the minimum for that assembly system. Additionally, the SI is the lowest for that assembly system.

In the example, the critical variation requirement σ_c is much larger than the expected product variation for a car-body structure, which should always be less than 0.33 mm ($6\sigma=2$ mm). There

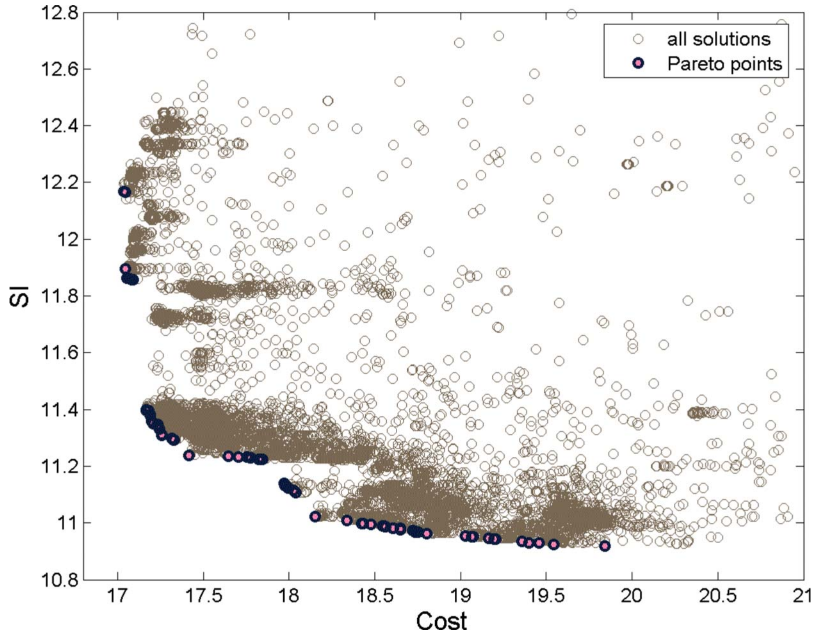

Fig. 7 Tradeoff between cost and sensitivity for σ_s=0.33 mm

are two ways to decrease σ_c for an assembly system: change station characteristics (assembly sequence or datum scheme) or decrease the upper design bounds for both product and process tolerance variables. For example, when the upper bound of the tolerance design is reduced from 2 mm to 1.5 mm, the critical variation requirement σ_c changes from 1.67 mm to approximately 1.17 mm.

4.2 Variation-Sensitivity Results. The problem of Eq. (11) is solved for budgets b=7, 10, 15, 20, 25, 30, 35, and 40. Once again, SQP is used as the inner loop optimizer and NCGA as the outer loop optimizer. The results are shown in Fig. 10. There exists a tradeoff between product variation and sensitivity. For example, at b=7, a 5% increase in sensitivity can result in a 13% decrease in final product variation.

In Fig. 10, we see also that the tradeoff between the final product variation and the SI diminishes as the budget increases. The critical budget requirement b_c is defined as the required cost of an optimal fixture layout when all tolerance variables are at their lower bound values. If the budget is less than the critical budget requirement, a choice has to be made between variation and SI along the Pareto curve. The design goal for the multistation assembly system then becomes one of decreasing the critical budget requirement. This can be realized by changing the station characteristics or increasing the lower design bounds for both product and process tolerance variables.

5 Conclusion

We introduced a multiobjective optimization formulation for integrated tolerance allocation and fixture layout design in multistation assembly processes. A nested optimization strategy was used to solve the formulated multiobjective problems that considered final product variation, assembly process sensitivity, and cost. Tradeoffs between cost and sensitivity and between variation and sensitivity were identified and quantified under the assump-

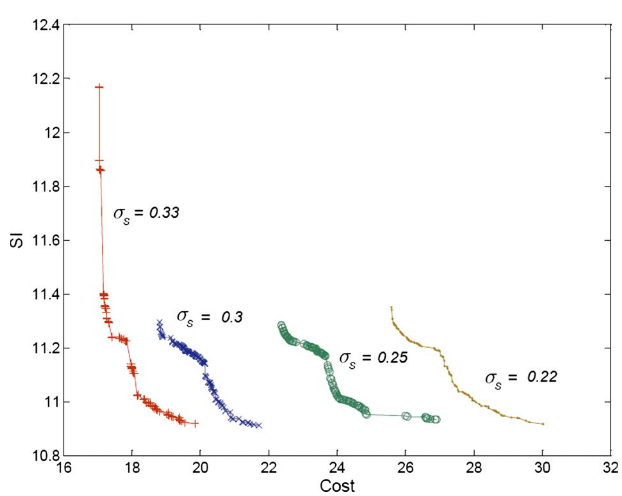

Fig. 8 Tradeoffs between cost and sensitivity for σ_s =0.22−0.33 mm

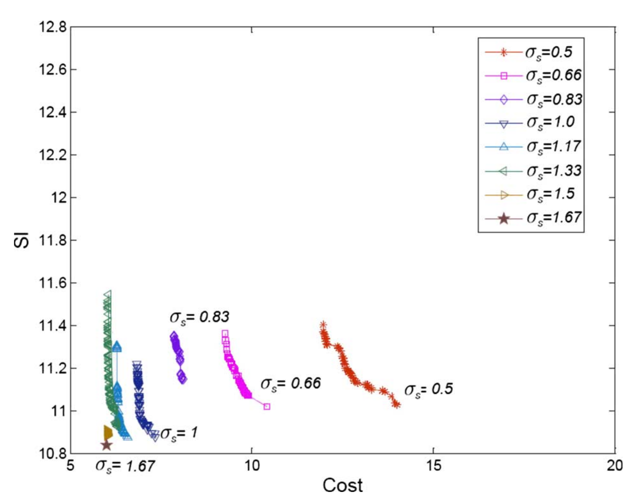

Fig. 9 Relation between cost and sensitivity for σ_s =0.5−1.67 mm

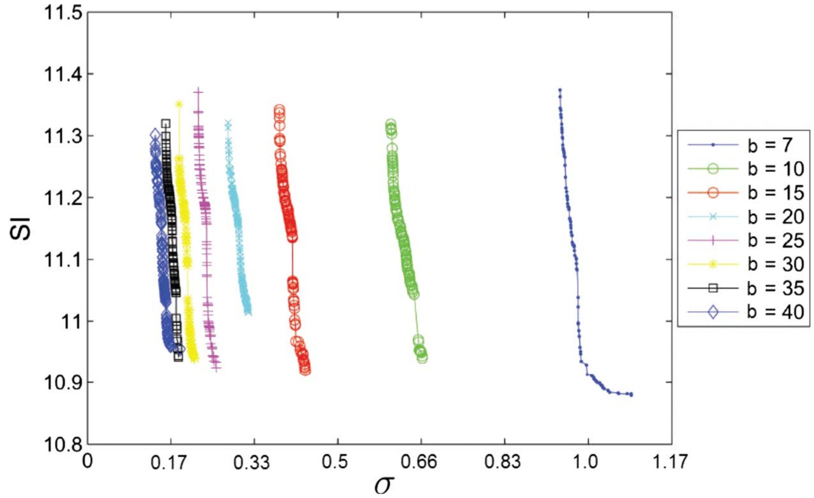

Fig. 10 Relation between variation and sensitivity for different budgets

tions that changing fixture layouts does not incur costs and that the considered system cost depends only on allocated product and process tolerances.

We found that the existence of cost-sensitivity and variation-sensitivity tradeoffs depend on the value of the active variation and budget constraints, respectively. Based on this finding, we introduced the new concept of critical variation and critical budget requirements as the values where the tradeoffs cease to exist. If current variation or budget requirements are far from their critical values, one has to choose a design from the Pareto set. Alternatively, one can change the design bound values for the product and process tolerance variables in order to change critical requirement values and therefore obtain a single design solution.

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