

# Rapid Deployment of Reconfigurable Assembly Fixtures using Workspace Synthesis and Visibility Analysis

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## Abstract

Due to rapid changes in recent market demands, shortened production ramp-up/launch and model changeover of new products with simultaneous manufacturing of family of products on a single production line is becoming inevitable. This requires systematic methods for rapid design and analysis of reconfigurable fixture workspace synthesis and visibility. This paper presents an integrated approach for rapid reconfigurable fixture deployment which is based on (1) the procrustes analysis integrated with a pairwise optimization for fixture workspace configurations synthesis; and (2) screen space transformation-based visibility analysis for rapid fixture calibration. A case study and simulations illustrate the proposed approach.

## Keywords:

Reconfigurable fixture, Workspace synthesis, Visibility

## 1 INTRODUCTION

### 1.1 Problem Description

In recent years due to significant diversification of market and overall increase in product quality and durability, shortening lead time of new production is inevitably becoming the prevailing trend.

In order to satisfy the ever-increasing demand in product variety, production systems need to be improved, i.e. to develop Reconfigurable Manufacturing and Assembly Systems (RMSs and RASs) [1][2]. Therefore **fixture workspace synthesis** for production of a family of parts on a single assembly line must be developed.

**Fixture calibration** is of paramount importance for product quality since a significant number of fixture-related failures are related to fixture installation and maintenance [3][4]. The application of RASs brings new challenges to fixture calibration since reconfigurable tooling elements need to be calibrated in multiple positions. However, currently there are no methodologies to determine the best positions of the measurement system such as laser tracker to fully calibrate a given fixture or minimize the number of setup positions of a measurement system.

The objective of this paper is to develop an integrated approach for rapid reconfigurable fixture deployment based on (1) Fixture Workspace Synthesis (FWS) which identifies the best superposition of locating layouts for a family of parts; and (2) Fixture Visibility Analysis (FVA) for tooling elements which minimizes number of setups of measurement equipment during fixture calibration.

### 1.2 Related Work

#### **Fixture Design and Analysis**

Many algorithmic and heuristic methods were developed to synthesize and analyze setup plans and fixture configurations for a single rigid part [5][6]. As for design and analysis of fixture for single compliant/flexible part, Menassa and DeVries [7] proposed optimization techniques to assist in the design and evaluation of 3-2-1 fixtures for prismatic parts. Cai *et al.* [8] proposed an "N-2-1" locating layout for sheet metal parts fixturing.

In the area of assembly systems for a family of parts, Lee *et al.* [9] presented a workspace synthesis analysis for

fixturing of family of stamped parts using the genetic algorithm.

#### **Fixture Visibility/Accessibility**

Spyridi, Spitz and Requicha [10][11] developed both analytical and discretized accessibility analysis methods. Chen and Woo [12] first developed the concept of visibility map and provided geometric algorithm.

The accessibility/visibility methods were applied to minimize the number of workpiece setup in CNC machining and CMM inspection [12] and compute the die-opening directions for removing fabricated workpieces [13].

In the case of tooling calibration addressed in this paper, the position and orientation of tooling elements are fixed, and thus, the problem is related to identifying measurement system position and orientation (setup) and this is different from the aforementioned research.

## 2 FIXTURE WORKSPACE SYNTHESIS (FWS) METHODOLOGY

The reconfigurable fixture uses reconfigurable tooling elements which can be freely reconfigured to any position within its functional work envelope (FWE). Therefore, the position of a locator is no longer at a single fixed point, but rather at any point within the FWE workspace.

Fixture Workspace Synthesis (FWS) for a family of parts basically entails seeking the best superposition of locators for all the parts. For a general case, assume a part family has  $N$  parts, each of which has  $k$  locating points, and define  $C_i$  ( $i=1,2,\dots,k$ ) as the minimum circle that contains  $N$  corresponding locating points (Fig. 1 depicts a case with  $N=2$  and  $k=6$ ). The problem of FWS can be expressed as minimizing the maximum diameter of  $C_i$  ( $i=1,2,\dots,k$ ) and thus the objective can be formulated as:

$$\text{Min}(\text{Max}(C_i)) \quad 1 \leq i \leq k \quad (1)$$

Let us define  $(C_m)_{\text{Min}(\text{Max})}$  as the largest circle among the  $k$  circles obtained based on Eq. (1). Then during fixture design, the diameter of the FWE ( $D_{\text{FWE}}$ ) for a given reconfigurable tooling element must be equal to or larger than  $(C_m)_{\text{Min}(\text{Max})}$  so as to ensure that the corresponding locating points can be contained within the FWE of the

reconfigurable tooling element. Thus, we define the maximum Reconfigurability Index (RI) as:

$$RI_{\max} = D_{FWE} / (C_m)_{\text{Min(Max)}} \quad (2)$$

Then for any fixture workspace configuration layout, its RI must be within the following range:

$$1 \leq RI \leq RI_{\max} \quad (3)$$

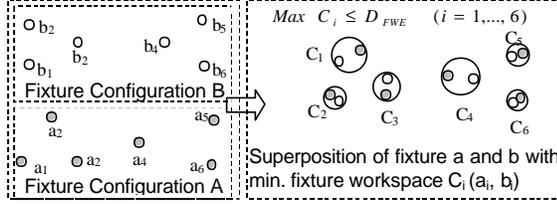


Figure 1: Expected configuration for two parts.

In order to overcome the challenges of simultaneously optimize the configurations of multiple parts, a fixture configuration synthesis method for a family of part is developed by applying procrustes-based pairwise optimization. (See [14] for more details)

## 2.1 Preliminary Configuration Layout Using Procrustes Analysis

In reconfigurable fixture design, each tooling element such as locator or clamp, is not just a point, rather it is a locating area. For simplicity, we use a circle to represent a potential tooling element position. Consequently, we can still utilize a point that is the center of the circle to represent a locating area, along with diameter of the circle.

For fixture synthesis, each part is represented by a set of locators or clamps, which are known as Tooling Elements (TEs). Two parts classified by the same TEs are recognized as identical in the sense of fixture configuration design. The procrustes analysis for fixture workspace configuration can be presented as follows:

(1) **Coordinate Registration.** A centered coordinate system is utilized for coordinate registration, firstly computing "centered" points whose coordinate is the arithmetic average of the TE coordinates of each configuration, and then translating each configuration to make the "centered" point coincident with the coordinate origin.

(2) **Similarity Transformations.** Assume  $C_1$  and  $C_2$  are two fixture configurations after coordinate registration and are represented by  $k \times m$  ( $k$  is the number of TEs and  $m$  represents dimensions) matrix. Let  $R$  and  $T$  be the rotation and translation transformations, respectively. Then if fitting  $C_1$  to  $C_2$  using  $R$  and  $T$  transformations, we can get:

$$C_2 = C_1 R + I_k T + e \quad (4)$$

where  $I_k$  is a  $k \times m$  matrix with all elements of one and  $e$  is residual error, which indicates the difference between two fixture configurations. By applying the least square method, the  $e$  can be minimized. The sum of square of  $e$  is:

$$D^2(C_1, C_2) = \|e^T e\| \quad (5)$$

The solution for minimizing the sum of square of the residual error can be obtained as:

$$T = 0 \quad (6)$$

$$R = UV^T \quad (7)$$

where  $U$  and  $V$  can be obtained by using the singular value decomposition  $C_2^T C_1 = V \Lambda U^T$  ( $U$  and  $V$  belong to special orthogonal group and  $\Lambda$  is the vector of eigen values of  $C_2^T C_1$ ).

By using this approach, the necessary translation and rotation transformations for each configuration can be obtained, which lead to the best match for the configurations.

## 2.2 Pairwise Configuration Optimization (PCO)

Although procrustes analysis is to minimize the distance between the corresponding TEs of the configurations, it is based on the sum of the square of the distances of TEs, and its objective function is expressed as:

$$\text{Min} \left( \sum_{i=1}^k C_i^2 \right) \quad (8)$$

The solution of  $C_i$  based on Eq. (8) is represented as:

$$(C_i)_{LS} \quad (i = 1, 2, \dots, k) \quad (9)$$

The diameter of maximum circle obtained based on Eq. (9) is represented as  $(C_m)_{LS}$ . Based on the requirement of reconfigurable fixture workspace analysis (Eq(1)), the following holds:

$$\sqrt{\left( \sum_{i=1}^k (C_i)_{LS}^2 \right) / k} < (C_m)_{\text{Min(Max)}} < (C_m)_{LS} \quad (10)$$

According to this relationship, the scope of search domains for the variables can be narrowed to a small range. Then a simple searching optimization method can be applied to find the desired solution rapidly.

For this optimization, the key point is how to efficiently accomplish it for multiple configurations. We used a method called Pairwise Configurations Optimization (PCO) that simultaneously deals with only two configurations at any time. For  $N$  parts, assume  $Y_i$ ,  $i=1, \dots, N$  are their configurations that have been processed by using the procrustes analysis. For each configuration  $Y_i$ , we compute the average of other  $N-1$  configurations  $\bar{Y}_{(i)}$ . We then conduct search optimization for the configuration pair of  $Y_i$  and  $\bar{Y}_{(i)}$  based on objective function represented in Eq. (1). Repeat this process until the obtained  $(C_m)_{\text{Min(Max)}}$  cannot be reduced any further. Then the best match for the  $N$  configurations (parts) is determined, which meets the requirement of objective function presented in Eq. (1).

## 3 FIXTURE VISIBILITY ANALYSIS (FVA) USING SCREEN SPACE TRANSFORMATION FOR MINIMIZATION OF NUMBER OF SETUPS

Figure 2 illustrates the visibility problem as applied to in-line fixture calibration along with the corresponding terminology. Observation Space (OS) is a predefined space in which the Measurement Equipment (ME) is allocated. Any position inside OS is called an Observation Position (OP). Measurement Targets (MTs) are those feature points of the tooling elements (locators/clamps) that need to be measured and calibrated by the ME. Any object that may block lines of sight between ME and MTs is called Measurement Obstacle (MO). Examples of MOs are: material handling device(s), robots and other tooling elements. A single ME setup is described as a single position of ME within OS. For in-line fixture calibration, each MT represents a single geometrical feature of a given tooling element (locator or clamps).

In order to solve the problem of minimum number of setups, the visibility should be checked for every MT and all the possible OPs within the OS to verify if the line of sight between ME and MT is blocked by some MOs or not. However, this method has extremely intensive computations of (1) heavy computation of intersections between straight lines and planes in 3D space, and (2) the number of surface polygons representing obstacles is quite large. In order to overcome these shortcomings, the Screen Space Transformation (SST) and a simplified modeling of measurement obstacles are presented in the ensuing sections.

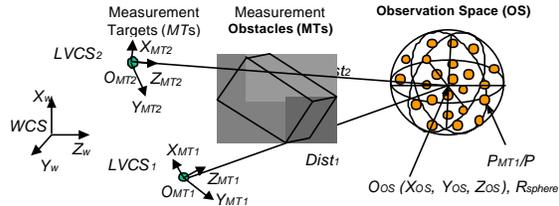


Figure 2: Setup of local view coordinate system (LVCS) for different MTs

Figure 2 also illustrates an example for which the proposed visibility algorithm is applied. All the objects are located in a World Coordinate System (WCS), represented by  $X_w$ ,  $Y_w$ , and  $Z_w$ . It is assumed that all measurement obstacles are convex polyhedrons.

First we setup the local view coordinate systems that take each measurement target as origin. Each observation space is represented as a sphere which can lead to a convenient setup for a local view coordinate system. In Fig. 2, two local view coordinate systems  $LVCS_1$  and  $LVCS_2$  take  $MT_1$  and  $MT_2$  as origin respectively. Due to the spherical shape of the OS, the orientation of the axis of each local view coordinate system can be conveniently determined.

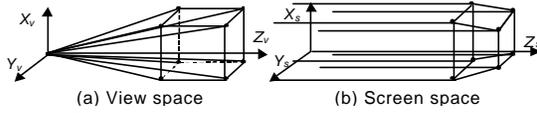


Figure 3: View space and screen space.

Next, SST is applied in each local view coordinate system. Figure 3(a) illustrates the view space which is the original 3D space before SST transformation. The Screen Space Transformation [15] can be utilized to transform the 3D view space to a 2D screen space where the original lines of sight become parallel to each other, and the perspective projection is converted to an orthogonal projection, as shown in Fig. 3(b). Thus, the computation can be simplified to a 2D domain that is perpendicular to the projection orientation. Therefore, a line of sight in 3D space is alleviated to a point in 2D space. Consequently, the computation of intersection between a line of sight and an obstacle polygon in 3D space can be converted to verifying if a point is within a polygon in 2D space. Thus, the computation can be simplified significantly.

Furthermore, after the SST, the measurement obstacle of 3D convex polyhedron becomes a 2D polygon by the orthogonal projection. Then the number of geometrical elements used to represent the MO can be significantly reduced by identifying the boundary edge of the 2D polygon. Therefore the computational complexity can be alleviated.

#### 4 INTEGRATION OF FIXTURE WORKSPACE SYNTHESIS AND FIXTURE VISIBILITY ANALYSIS

As discussed earlier, the applications of reconfigurable fixtures instead of dedicated fixtures, bring new challenges for fixture calibration and related visibility analysis. The reconfigurable tooling elements need to be calibrated in multiple positions (within the FWE of the reconfigurable tool) instead of a single/fixed position(s). The presented FWS method in section 2 minimizes necessary volume space (in 2D case – circles  $C_i$  as shown in Fig. 1) for each tooling element to assemble a family of parts in a given fixture. The resulting workspace synthesis of the reconfigurable fixture is measured as RI (Eq. (3)). It can be observed that for all  $RI \geq 1$ , the selected fixture is capable of assembling a given part family. However, the obtained minimized workspace may potentially increase fixture

calibration time and decrease accuracy due to tooling element visibility limitations. Thus the proposed integration of fixture workspace synthesis and visibility analysis is conducted in two steps: (1) minimize fixture workspace synthesis, as measured by RI; (2) maximize tooling element visibility by reducing RI, but still meet Eq. (3) (use full workspace capability of the fixture). This can be integrated as identifying tooling elements visibility within the range of  $1 \leq RI \leq RI_{max}$  as shown in Fig. 4.

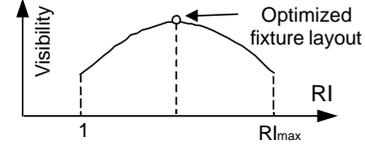


Figure 4: Relationship between the visibility and RI

As shown in Fig. 5, visibility is defined by whether the lines of sight between measurement equipment and measurement targets (tooling elements) are blocked by measurement obstacles or not. When one fixture element is being calibrated, all other tooling elements are considered as obstacles. Since all obstacles such as robots and material handling, are arranged in relation to fixture configuration, it is the fixture configuration that determines the results of visibility.

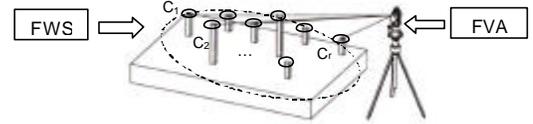


Figure 5: Relationship between FWS and FVA.

Figure 6 illustrates the relationship between FWA and FVS. First, as discussed in Section 2, the FWS determines the layout of all the tooling elements, and then it gives the fixture layout as an input to the FVA analysis. Based on the fixture layout, the FVA computes the corresponding visibility property which is presented in Section 3, subject to the constraint of  $1 \leq RI \leq RI_{max}$ .

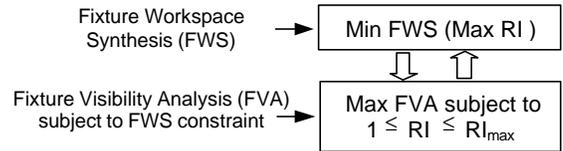


Figure 6: Integration between FWA and FVS

#### 5 CASE STUDY

The two parts in Fig. 7 are represented by their TEs  $a_i$  and  $b_i$  respectively, and they are arbitrarily placed. The centers of the TEs are the features that need to be calibrated. The reconfigurable fixture to be used for the reconfigurable fixture has a circular FWE with diameter of 3.5. First we perform the fixture workspace synthesis. The result that meets the requirement of Eq. (1) is obtained and shown in Fig. 8. The maximum circle of all the circles that wrap the corresponding locating point, i.e.  $(C_{ri})_{Min(Max)}$ , is obtained, which is equal to 3.0. Based on Eq. (2), then  $RI_{max} = 3.5/3 = 1.167$ . Assume that under some constraints, there are only three observation positions available (Fig. 8). It can be observed that although, Fig. 8 gives the best fixture workspace configuration, the visibility for the measurement equipment is not good since none of the three observation positions makes the measurement equipment visible to all the measurement targets (locators). In this case, two setups are needed.

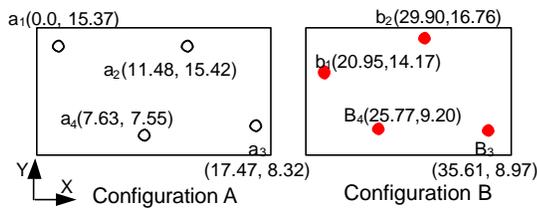


Figure 7: Two parts with arbitrary positions

For any other alternatively feasible fixture workspace configuration, their RI must meet:  $1 \leq RI \leq 1.167$  (Eq.(3)), based on which some adjustment can be made to the FWS module. Following the algorithm described in Fig. 4 and 6 we obtain the fixture workspace configuration shown in Fig. 9. The value of  $(C_m)_{Min(Max)}$  is 3.18, and correspondingly the RI is  $3.5/3.18 = 1.1$  which is smaller than the previous configuration. However, the resulting visibility is improved since all the measurement targets are visible to the measurement equipment from  $OP_2$  (only the TEs of the same part will block each other). That means only one setup is needed, so the best visibility is achieved.

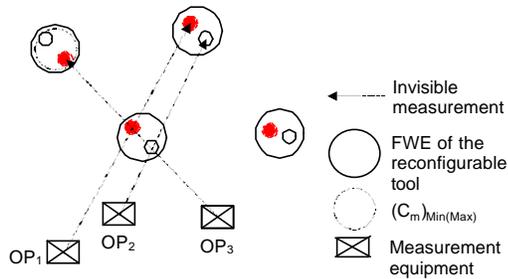


Figure 8: the fixture workspace configuration after the 1st FWS

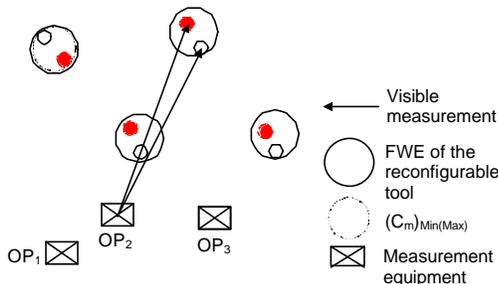


Figure 9: the final fixture workspace configuration

## 6 SUMMARY

This paper presents an integrated approach for rapid reconfigurable fixture deployment which is based on (1) fixture workspace synthesis which allocates the best superposition of locating layouts for a family of parts to be produced on a single reconfigurable fixture, and (2) fixture visibility analysis for tooling elements which allows to minimize number of setups of measurement equipment during fixture calibration. The integration of FWS and FVA provides an analytical tool for rapid fixture deployment in a new assembly system and allows for the optimization of reconfigurable fixture capability to produce a family of parts and fixture visibility to minimize number of setups. Provided case study also illustrates the proposed method.

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