MODE-BASED TOLERANCE ANALYSIS IN MULTI-STATION ASSEMBLY USING STREAM OF VARIATION MODEL

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ABSTRACT

Modeling of variation propagation is crucial in predicting product dimensional quality and general performance of manufacturing systems. This paper develops a variation propagation model for multi-station assembly processes which can be applied for analysis of various tolerance modes. The variation propagation model generalizes the current state space approach based variation propagation method from 2D to 3D cases. Additionally, various tolerance modes can be included into the model based on GD&T standards and industrial applications. Thus, by using the developed variation propagation model, the variations on the measurements are directly calculated, instead of using the standard Monte Carlo simulations with large sample size. The results of the industrial case study show that the proposed model can accurately predict variation propagation in multi-station assembly process.

INTRODUCTION

In large and complex multi-station assembly processes, there are many factors affecting product quality, among which, dimensional quality has a significant impact on the overall product quality and performance, as well as on the productivity and production cost. However, due to the extreme complexity of the multi-station assembly systems (Ceglarek and Shi, 1995), it is very challenging to ensure the required dimensional quality of assembled products.

Related Research

One critical research direction in the area of dimensional management is modeling and diagnosis of manufacturing processes. The research in this area integrates engineering CAD/CAM models with statistical methods and has grown rapidly since its inception in the early 1990s (Ding *et al*. 2000). This research started from statistical descriptions of variation patterns based on in-line measurements (Hu and Wu, 1992) and then included rule-based fault isolation approaches (Ceglarek *et al.*, 1994). Ceglarek and Shi (1996) developed a kinematic based engineering model for fault diagnosis of a single fault in a single station assembly process. In a similar vein, Rong *et al*. (2000) proposed a stiffness matrix based engineering model for dimensional fault diagnosis of compliant beam structures. Large amount of research work has been done regarding multiple fault modeling and diagnosis as well. For example, Barton and Conzalez-Barreto (1996) presented a process-oriented basis representation for diagnosis in multivariate processes; Apley and Shi (1996) constructed a fixture fault model using the geometric information of the panel and fixture; Chang and Gossard (1998) proposed a computational method for variation fault diagnosis in assembly processes. Carlson and Soderberg (2003) proposed a multi-fixture single station assembly diagnosis model for rigid part assembly. However, all of them are focused in the scenario of single stage/station. This limitation severely constrains the application of these methods.

A physical model that can characterize the multi-stage assembly process propagation of variation was proposed by Jin and Shi (1999). They applied the state space model to characterize the error propagation in multi-station assembly processes. This model was further enhanced by Ding *et al.* (2000). In machining area, Djurdjanovic and Ni (2001), Zhou et al. (2003), Huang and Shi (2004) also applied state space model to formulate the error propagation in multi-stage machining processes. Their research work led to some industrial applications. However, some significant limitations of the current models still exist, especially in assembly area. The major limitations include: (1) in assembly process, only "0-2-1" locating scheme instead of generic "3-2-1" is used (Ding *et al*. 2000); (2) the fixture locators deviations are assumed to be uncorrelated (Ding *et al*. 2002).

Objective and Organization

In order to overcome the limitations of existing methods, this paper is aimed at modeling both fixtures and parts variation in multi-station assembly processes, which provides the mathematical foundation for Stream of Variation Analysis (SOVA) (Ceglarek *et al.* 2004). An enhanced state space model is developed to incorporate both product and process information into product variability. Furthermore, based on various tolerance modes, accurate formulation of tolerance input is accomplished, which makes the model much more comprehensive and applicable for industrial applications. The rest of the paper is organized as four sections. First, a variation propagation model for multi-station assembly processes is presented using the state space model. And then the tolerance inputs of various tolerance modes based on GD&T standard and industrial experiences are formulated. Afterwards, a case study is provided to validate the proposed approach. Finally, the whole methodology is summarized.

VARIATION PROPAGATION MODEL FOR MULTI-STATION ASSEMBLY PROCESSES

Figure 1 shows a multi-station assembly process with *m* stations. Variable *i* is the station index. The product quality information (e.g., part dimensional deviations) at each station is represented by state vector x(*i*). The process faults (e.g., fixturing error and part fabrication error) are included as input u(*i*). The product quality measurement is denoted by y(*i*), which is not necessarily available at every station. Variables w(*i*) and v(*i*) are the process noise and measurement noise, respectively, and they are assumed to be mutually independent.

The part quality at current station x(*i*) is determined by the process error sources u(*i*), the incoming product quality x(*i-1*), and the process noise *v(i)*. The variation propagation can be integrated as a station-indexed state space model:

$$x(i)=A(i-1)x(i-1)+B(i)u(i)+v(i) \quad i \in [1,2,...,m] \qquad (1)$$

$$y(i)=C(i)x(i)+w(i) \quad i \in [1,2,...,m] \qquad (2)$$

where, A(*i*-1) is the state matrix, A(*i*-1)x(*i*-1) represents the effects of product quality from station *i*-1 to station *i*, B(*i*) is the input matrix, and B(*i*)u(*i*) represents how the product quality is affected by the process error sources at station *i*, C(*i*) is the observation matrix, determined by the distribution and number of measurement devices. w(i) represents measurement noise at each station.

Based on Eqs. (1) and (2), the relationship between process error sources u(i) and end of line measurement y can be expressed as follows,

$$\mathbf{y} = \mathbf{\Gamma u} + \boldsymbol{\varepsilon} \qquad (3)$$

where Γ is an $\mathbf{n} \times \mathbf{p}$ matrix, and represents collection of fault patterns related to fixture and/or part mating errors. There are a total of n measurements and p error sources represented by matrix Γ. ε is an $\mathbf{n} \times \mathbf{1}$ random vector, which is the combination of process noise and measurement noise in the assembly process. For detailed derivation, please refer to Ding *et al*. 2000.

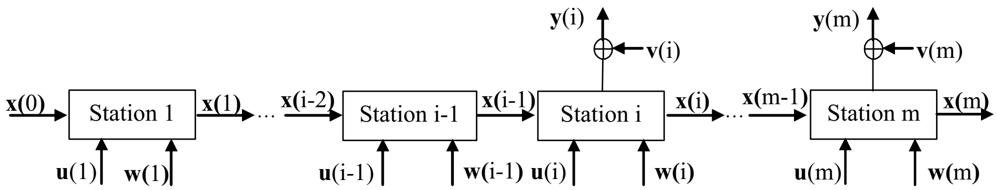

FIGURE 1. DIAGRAM OF A MULTI-STATION ASSEMBLY PROCESS

ε in Eq. (3) can be dropped by assuming that only the variations from fixture locators and part mating features significantly contribute to the variations on the measurements. Then we have the variation propagation analysis model,

$$\mathrm{cov}(\mathbf{y}) = \mathbf{\Gamma}\,\mathrm{cov}(\mathbf{u})\mathbf{\Gamma}' \quad (4)$$

where cov(.) represents the covariance matrix.

The method presented by Ding *et al*. (2000) simplifies the fixturing scheme as "0-2-1" (2D) instead of generic "3-2-1" (3D). It does not consider part mating feature errors, and includes only lap joints between assembled parts. Therefore, generic 3D variation propagation models are necessary that take into consideration of both fixture locator errors and part mating feature errors to more closely represent the real world assembly processes.

In Eq. (1), matrix A(i-1) reflects the impact of locating scheme changes between stations to the variation. The fundamental model which provides the relationship between tolerance input and output variation is given by matrix B(i). In the following analysis, the process of how to obtain matrix B(i) is introduced briefly.

The 3D variation propagation model considers 3-2-1 fixture layout and various types of planar mating features, such as lap joint, butt joint, and T joint, etc. The generic 3-2-1 fixture setup is shown in Fig. 2. These locators (P_1 to P_6) define a primary plane (by P_1, P_2 and P_3), secondary plane (by P_4 and P_5), and tertiary plane (by P_6). Point P_r is chosen as the reference point to describe the rigid body variation of the entire individual part. The relationship between the errors of the six locators and point P_r is derived using kinematic model represented by matrix F_s in Eq. (5). F_s is purely determined by the fixture locators layout, and it provides the necessary elements of matrix B(i) in Eq. (1). Fixture locator deviation is represented as Δf, which consists of the deviations of locators P_1-P_3 in Z direction (Δz_1, Δz_2, and Δz_3), locators P_4 and P_5 in X direction (Δx_4 and Δx_5,), and locator P_6 in Y direction (Δy_6). Thus, given the fixture locator error Δf, the deviation of rigid part ΔP_r, including translational deviations (Δx, Δy, and Δz) in X, Y and Z axes and rotational deviations ($\Delta \alpha$, $\Delta \beta$, and $\Delta \gamma$) around X, Y and Z axes, can be calculated by Eq. (5).

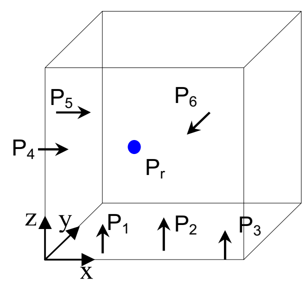

FIGURE 2. GENERIC 3-2-1 FIXTURE LAYOUT

In some cases of automotive sheet metal assembly, the three locators (P_1, P_2 and P_3 in Fig. 2), which define the primary plane, are provided by part mating features, instead of physical fixture locators. Eq. (5) provides a generic unified model for both part mating errors and fixture locator errors. It implies that the part to part mating errors can also be represented in the same framework by considering them as virtual fixture locators. Therefore, this 3D variation propagation model includes not only in plane 4way/2way induced deviation but also out-of-plane deviation due to the three fixture locator errors or part mating feature errors.

$$\mathbf{\Delta P_r} =_{\mathbf{r}} [\mathbf{\Delta x} \quad \mathbf{\Delta y} \quad \mathbf{\Delta z} \quad \mathbf{\Delta \alpha} \quad \mathbf{\Delta \beta} \quad \mathbf{\Delta \gamma}]'_r = \mathbf{F_s \Delta f} =$$

$$\begin{bmatrix} \frac{\mathbf{D_{y5r}}}{\mathbf{D_{y54}}} & \frac{\mathbf{D_{yr4}}}{\mathbf{D_{y54}}} & 0 & \frac{\mathbf{D_{zr4}D_{y32}}}{\mathbf{C}} & \frac{\mathbf{D_{zr4}D_{y13}}}{\mathbf{C}} & \frac{\mathbf{D_{zr4}D_{y21}}}{\mathbf{C}} \\ \frac{\mathbf{D_{xr6}}}{\mathbf{D_{y54}}} & \frac{\mathbf{D_{x6r}}}{\mathbf{D_{y54}}} & 1 & \frac{\mathbf{D_{z6r}D_{x32}}}{\mathbf{C}} & \frac{\mathbf{D_{z6r}D_{x13}}}{\mathbf{C}} & \frac{\mathbf{D_{z6r}D_{x21}}}{\mathbf{C}} \\ 0 & 0 & 0 & \mathbf{E} & \mathbf{F} & \mathbf{G} \\ 0 & 0 & 0 & \frac{\mathbf{D_{x32}}}{\mathbf{C}} & \frac{\mathbf{D_{x13}}}{\mathbf{C}} & \frac{\mathbf{D_{x21}}}{\mathbf{C}} \\ 0 & 0 & 0 & \frac{\mathbf{D_{y32}}}{\mathbf{C}} & \frac{\mathbf{D_{y13}}}{\mathbf{C}} & \frac{\mathbf{D_{y21}}}{\mathbf{C}} \\ \frac{1}{\mathbf{D_{y54}}} & \frac{-1}{\mathbf{D_{y54}}} & 0 & 0 & 0 & 0 \end{bmatrix} \bullet \begin{bmatrix} \mathbf{\Delta x_4} \\ \mathbf{\Delta x_5} \\ \mathbf{\Delta y_6} \\ \mathbf{\Delta z_1} \\ \mathbf{\Delta z_2} \\ \mathbf{\Delta z_3} \end{bmatrix} \quad (5)$$

$$\text{where, } \begin{cases} \mathbf{D_{xij} = (X_i - X_j)} \\ \mathbf{D_{yij} = (Y_i - Y_j)} \\ \mathbf{C = D_{x21}D_{y31} - D_{x31}D_{y21}} \\ \mathbf{E = 1 + (D_{x1r}D_{y32} + D_{y1r}D_{x23})/C} \\ \mathbf{F = (-D_{x1r}D_{y31} + D_{y1r}D_{x31})/C} \\ \mathbf{G = (D_{x1r}D_{y21} - D_{y1r}D_{x21})/C} \end{cases}$$

Figure 3 depicts a typical assembly application that includes part to part mating error. For example, among the six locators of part 3, P_{34} and P_{35}, which define the secondary plane, and P_{36}, which define the tertiary plane, are provided by physical fixture locators. However, P_{31}, P_{32} and P_{33}, which define the primary plane, are from the part to part mating feature between part 2 and part 3 (butt joint).

Compared with the variation propagation model presented by Ding *et al*. (2000), the proposed generic "3-2-1" fixture modeling is encapsulated into system matrices A, B and C in Eqs. (1) and (2), and input matrix u includes both fixture locator errors and part mating feature errors. Therefore, the variation propagation model applied in this paper is a significant enhancement of the model presented by Ding *et al*. (2000).

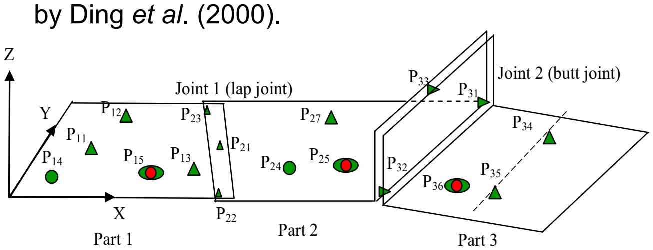

FIGURE 3. 3D ASSEMBLY WITH PART-TO-PART MATING ERROR

CONSIDERATION OF VARIOUS TOLERANCE MODES

Eq. (3) reveals that the variation of the measurements is a combination of both fault pattern matrix $\mathbf{\Gamma}$ and error sources u. $\mathbf{\Gamma}$ is purely determined by the geometrical structure of fixture and parts. Deviation of error sources u is decided by tolerance input from both fixtures and parts. Ding *et al*. (2002) assumes that the deviations of error sources are uncorrelated with each other. Although this is a typical case for tolerance analysis, in practice there are many cases in which the deviation of errors are not uncorrelated, but rather correlated in a certain type of mode.

3DCS^{TM} (2002) is one of most widely used simulation software in dimensional engineering. It has implemented different tolerance modes in its tolerance simulation software. The included tolerance modes are based on GD&T standards and industrial experiences. For a list of tolerances, there are three different types of modes, namely, independent, group and composite. The modes describe how the points (from fixture locators or part mating features) selected in the list will be toleranced with respect to each other. When independent mode is specified, each point in the tolerance list varies independently with one another. When group mode is specified, all the points in the tolerance list vary completely synchronized. When composite mode is specified, each point varies with certain correlation with other points in the tolerance list. In this section, aiming these three tolerance modes, corresponding covariance matrices of tolerance inputs are formulated. Consequently, the variation of measurements can be calculated by using Eq. (4) directly. In the following analysis, all the tolerances are assumed to be normally distributed. Therefore, by default setting the range of a tolerance is six times of the standard deviation (σ).

<u>Independent Tolerance Mode</u>

When independent tolerance mode is selected, the tolerances within the list are uncorrelated with each other. Therefore, the covariance matrix of the tolerance input has the simplest form. Usually, the tolerance value is input as the "range" associated with an assigned sigma number that specifies the ratio between range and standard deviation tolerances. The default sigma number is 3. Assume that the ranges of the tolerances are noted as r_1, r_2,… r_p, respectively, where p is number of tolerances in this list. Thus, the covariance matrix of the tolerances is follows,

$$\mathbf{Cov(u)} = \begin{vmatrix} \mathbf{(r_1/6)^2} & \mathbf{0} & \mathbf{...} & \mathbf{0} \\ \mathbf{0} & \mathbf{(r_2/6)^2} & \mathbf{...} & \mathbf{0} \\ \mathbf{...} & \mathbf{...} & \mathbf{...} & \mathbf{...} \\ \mathbf{0} & \mathbf{0} & \mathbf{...} & \mathbf{(r_p/6)^2} \end{vmatrix} \qquad (6)$$

Group Tolerance Mode

For group tolerance mode, the tolerances within the list are completely synchronized with each other. Therefore, the correlation between any pair of the tolerances is 1. Assume the ranges of the tolerance pair are r_1, and r_2, respectively. The covariance matrix of this tolerance pair is as follows,

$$\mathbf{Cov(r_1,r_2)} = \begin{vmatrix} \mathbf{(r_1/6)^2} & \mathbf{r_1r_2/36} \\ \mathbf{r_1r_2/36} & \mathbf{(r_2/6)^2} \end{vmatrix} \quad (7)$$

By applying Eq. (7) to every tolerance pair within the tolerance list on which the group tolerance mode is selected, then the corresponding covariance matrix for this tolerance list input can be formulated.

Composite Tolerance Mode

The composite mode tolerance is much more complex than the aforementioned two modes. In GD&T, the composite feature control frame contains a single entry of the geometric characteristics symbol followed by each tolerance and datum requirement, one above the other as shown in Fig. 4. Where composite control is used, the upper segment is referred to as the pattern locating control, which defines a Pattern-Locating Tolerance Zone Framework (PLTZF). The PLTZF is located from specified datums by basic dimensions. It specified the larger positional tolerance for the location of the pattern of features as a group (list). The lower segment is referred to as the feature relating control, which determines a Feature-Relating Tolerance Zone Framework (FRTZF). It governs the smaller positional tolerance for each feature within the pattern.

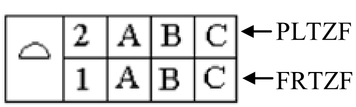

FIGURE 4. MULTIPLE FEATURE COMPOSITE CONTROL FRAME

For example, the composite tolerance control frame shown in Fig. 4 will control the deviation of all points within a 2mm zone relative to the part's datums. Furthermore, all points are controlled within a 1 mm zone relative to each other. In essence, the 1mm relative control zone will float within the larger 2mm control zone.

In fact, once a composite mode is applied to a set of tolerances, they are not independent with each other. The correlation between the multiple tolerance features controlled by the composite mode can be derived based on the values of PLTZF and FRTZF. Consequently, the covariance matrix of fixture deviation and part deviation can be obtained.

In Eq. (3), let u be a pair of tolerances, namely,

$$\mathbf{u} = |\mathbf{u_1} \quad \mathbf{u_2}|' \quad (8)$$

u_1 and u_2 are assumed to be correlated, which is a general case. Then, the covariance matrix of measurement y can be computed as,

$$\mathbf{cov(y)} = \mathbf{\Gamma \bullet cov(u) \bullet \Gamma'}$$

$$= \mathbf{\Gamma} \bullet \begin{vmatrix} \mathbf{var(u_1)} & \mathbf{cov(u_1,u_2)} \\ \mathbf{cov(u_1,u_2)} & \mathbf{var(u_2)} \end{vmatrix} \bullet \mathbf{\Gamma'} \quad (9)$$

Since u_1 and u_2 are correlated, $\mathbf{cov(u_1,u_2)} \neq \mathbf{0}$. The key issue becomes how to obtain $\mathbf{cov(u_1,u_2)}$.

Let u' represent the latent error sources hidden in u, which also has two components, but are uncorrelated, namely

$$\mathbf{u'} = |\mathbf{u_1'} \quad \mathbf{u_2'}|', \text{ with } \mathbf{cov(u_1',u_2')} = \mathbf{0} \quad (10)$$

The relationship between u and u' can be represented using an orthonormal matrix Q, namely,

$$\mathbf{u} = \mathbf{Qu'} \quad (11)$$

where $\mathbf{Q} = \begin{vmatrix} \mathbf{q_1} & \mathbf{-q_2} \\ \mathbf{q_2} & \mathbf{q_1} \end{vmatrix}$ with $\mathbf{0 < q_1 < 1}$, $\mathbf{0 < q_2 < 1}$, and $\mathbf{q_1^2 + q_2^2 = 1}$. Eq. (11) leads,

$$\mathbf{Cov(u)} = \mathbf{Q \bullet cov(u') \bullet Q'} \quad (12)$$

Because u_1' and u_2' in Eq. (10) are uncorrelated, Eq. (12) can be written as Eq. (13),

$$\begin{vmatrix} \mathbf{var(u_1)} & \mathbf{cov(u_1,u_2)} \\ \mathbf{cov(u_1,u_2)} & \mathbf{var(u_2)} \end{vmatrix} =$$

$$\begin{vmatrix} \mathbf{q_1^2var(u_1') + q_2^2var(u_2')} & \mathbf{q_1q_2var(u_1') - q_1q_2var(u_2')} \\ \mathbf{q_1q_2var(u_1') - q_1q_2var(u_2')} & \mathbf{q_1^2var(u_2') + q_2^2var(u_1')} \end{vmatrix} \quad (13)$$

where var(u_1) and var(u_2) are given already from the tolerance inputs, namely,

$$\begin{cases} \mathbf{var(u_1) = (range_{1.1}/6)^2} \\ \mathbf{var(u_2) = (range_{1.2}/6)^2} \end{cases} \quad (14)$$

In Eq. (14), $range_{1.1}$ and $range_{1.2}$ represent the range (6σ) of the tolerances of two error sources (PLTZF). Eq. (5) indicates,

$$\begin{cases} var(u_1) = q_1^2 var(u_1') + q_2^2 var(u_2') \\ var(u_2) = q_1^2 var(u_2') + q_2^2 var(u_1') \end{cases} \quad (15)$$

Since matrix Q in Eq. (11) is orthogonal, based on Eqs. (14) and (15), the following equality must hold,

$$\begin{aligned} & var(u_1) + var(u_2) = var(u_2') + var(u_1') \\ & = (range_{1.1}/6)^2 + (range_{1.2}/6)^2 \end{aligned} \quad (16)$$

Eq. (11) also leads,

$$u_1 - u_2 = (q_1 - q_2)u_1' - (q_1 + q_2)u_2' \quad (17)$$

Since u'$_1$ and u'$_2$ are uncorrelated, we can have the following expression,

$$\begin{aligned} & var(u_1 - u_2) \\ & = (q_1 - q_2)^2 var(u_1') + (q_1 + q_2)^2 var(u_2') \end{aligned} \quad (18)$$

From the definition of composite tolerance, the following result must hold,

$$var(u_1 - u_2) = 2 \bullet (range_2/6)^2 \quad (19)$$

in which range$_2$ is the value of FRTZF. Since Q is an orthonormal matrix, Eqs. (18) and (19) produce the following result,

$$\begin{aligned} & var(u_1') + var(u_2') - 2q_1 q_2 (var(u_1') - var(u_2')) \\ & = 2 \bullet (range_2/6)^2 \end{aligned} \quad (20)$$

Thus, based on Eqs. (5) and (20), the covariance between u$_1$ and u$_2$ can be obtained as follows,

$$\begin{aligned} & cov(u_1, u_2) = \\ & \frac{((range_{1.1}/6)^2 + (range_{1.2}/6)^2)}{2} - (range_2/6)^2 \end{aligned} \quad (21)$$

Finally, the correlation matrix of u is:

$$\begin{vmatrix} 1 & \rho \\ \rho & 1 \end{vmatrix} \quad (22)$$

where

$$\rho = \frac{1}{2}\left(\frac{range_{1.1}}{range_{1.2}} + \frac{range_{1.2}}{range_{1.1}}\right) - \frac{range_2{}^2}{range_{1.1} \bullet range_{1.2}} \quad (23)$$

For a valid correlation matrix, $|\rho| \le 1$, namely,

$$\frac{|range_{1.1} - range_{1.2}|}{\sqrt{2}} < range_2 < \frac{range_{1.1} + range_{1.2}}{\sqrt{2}} \quad (24)$$

Therefore, as long as the condition expressed by Eq. (24) holds, the latent errors represented by Eq. (11) can always be identified. In other words, if Eq. (24) holds, the orthogonal matrix Q must exist. Finally, by using Eq. (9), the covariance matrix of measurement y can be calculated.

For GD&T composite tolerance input, the pattern locating tolerance is the same for u$_1$ and u$_2$, namely,

$$range_{1.1} = range_{1.2} = range_1$$

Then, correlation matrix of u can be written as

$$\begin{vmatrix} 1 & 1 - (range_2 / range_1)^2 \\ 1 - (range_2 / range_1)^2 & 1 \end{vmatrix} \quad (25)$$

Let $k = \left(\frac{range_2}{range_1}\right)^2$, covariance matrix of tolerance input u is as follows,

$$\begin{aligned} & \begin{vmatrix} var(u_1) & cov(u_1, u_2) \\ cov(u_1, u_2) & var(u_2) \end{vmatrix} = \\ & (range_1/6)^2 \bullet \begin{vmatrix} 1 & 1-k \\ 1-k & 1 \end{vmatrix} \end{aligned} \quad (26)$$

Based on the definition of composite tolerance mode, range$_2$ cannot be larger than range$_1$ in Eq. (25). Therefore, for a valid composite tolerance, the range for *k* is,

$$0 \le k \le 1 \quad (27)$$

In other words, the correlation between u$_1$ and u$_2$ shall always be positive.

Finally, the covariance matrix of measurement y can be obtained using Eqs. (9) and (26). For a tolerance list which has more than two tolerances, Eq. (26) should be applied to every tolerance pair within the list. Consequently, the covariance matrix for the whole tolerance list can be formulated.

In Eq. (26), let *k*=1, then it becomes Eq. (6), which is the covariance matrix of tolerance inputs for independent mode. If *k*=0, then it is the same as Eq. (7), which is the covariance matrix of tolerance inputs for group mode. Therefore, essentially, both independent and group tolerance modes are special cases of the composite tolerance mode.

CASE STUDY

Figure 5 shows a multi-station 3DCS^{TM} analyst model of a simplified automotive floor pan, which includes three stations, through which total four (left floor pan, right floor pan, left bracket and right bracket) parts are assembled. The measurements are taken at station 3. One point on each individual part is measured. Each measurement point is measured in X, Y and Z directions. Therefore, there are a total of 12 measurements taken at station 3.

By using the state space model, matrix Γ in Eq. (3) is generated, which has dimension of $n \times p$, with n=12, the number of measurements and p = 33, the number of error sources in all 3 stations. Error sources T_{25}, T_{26}, T_{27}, which are fixture locators at station 3, are marked in Fig. 5

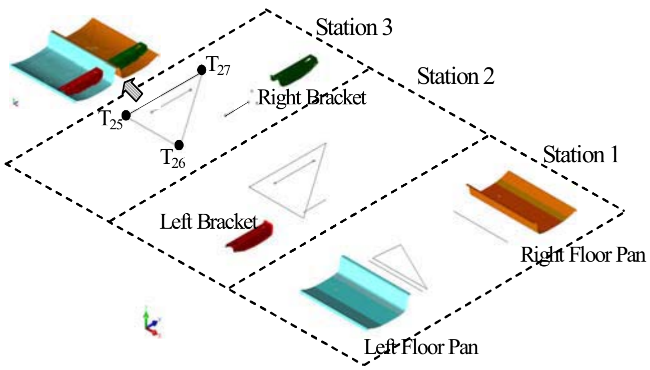

FIGURE 5. AN EXAMPLE OF MULTI-STATION ASSEMBLY PROCESS

In this case study, tolerance analysis for all the three types of tolerance modes is conducted. For independent, group, and composite tolerance modes, the corresponding matrix can be obtained using Eqs. (6), (7), and (25). Once the covariance matrix of the tolerance input is determined, then by using Eq. (4), the variances of measurements can be calculated. The results from proposed method and 3DCS^{TM} simulation are listed in Table 1. It can be noted that for all of these three modes, the results from the two methods matched very closely. Therefore, we conclude that the proposed model is able to predict variation propagation accurately.

SUMMARY

This paper presents a variation propagation model for multi-station assembly processes which can be applied for analysis of various tolerance modes. By using this model, the dimensional variation of the selected measurement points on the product can be directly calculated by using as inputs tolerances assigned to (i) fixture locators, and (ii) part fabrication. These direct computations do not require time consuming Monte Carlo simulations, which are currently widely used in industrial practice. The presented methodology expands the current SOVA model from 2D to 3D to include the "3-2-1" fixture locating scheme. Additionally, the developed methodology allows considering various tolerance modes in dimensional variation analysis. Moreover, the obtained analytical SOVA model can also be used for many other useful applications, such as tolerance allocation, system sensitivity analysis and fault diagnosis etc. One case study from the automotive assembly process validates the developed methodology.

TABLE 1 COMPARISON BETWEEN THE RESULTS (STD) FROM 3DCS AND PROPOSED METHOD

Measure-ments	Independent Mode			Group Mode			Composite Mode		
	3DCS	SOVA	Discrepancy (%)	3DCS	SOVA	Discrepancy (%)	3DCS	SOVA	Discrepancy (%)
M_1	0.1661	0.1651	0.6324	0.1198	0.1192	0.5141	0.1270	0.1277	0.4950
M_2	0.1306	0.1286	1.5658	0.1126	0.1127	0.1366	0.1143	0.1154	0.9696
M_3	0.0628	0.0625	0.5684	0.0834	0.0836	0.1903	0.0818	0.0805	1.5655
M_4	0.1632	0.1651	1.1356	0.1200	0.1193	0.6153	0.1283	0.1277	0.4283
M_5	0.1555	0.1532	1.5230	0.1399	0.1402	0.1520	0.1416	0.1423	0.5220
M_6	0.1084	0.1075	0.7860	0.0835	0.0836	0.0886	0.0885	0.0878	0.8244
M_7	0.1417	0.1416	0.1180	0.1388	0.1365	1.6735	0.1377	0.1374	0.2330
M_8	0.2226	0.2207	0.8755	0.1881	0.1865	0.8521	0.1907	0.1924	0.8811
M_9	0.1420	0.1400	1.4781	0.1454	0.1460	0.4165	0.1446	0.1450	0.2923
M_{10}	0.1155	0.1184	2.4313	0.1641	0.1662	1.2615	0.1585	0.1556	1.8791
M_{11}	0.1674	0.1666	0.4423	0.1661	0.1666	0.2809	0.1671	0.1666	0.2827
M_{12}	0.1607	0.1629	1.3483	0.1484	0.1462	1.4726	0.1490	0.1490	0.0027

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