All Eggs in One Basket: When Diversification Increases Portfolio Risk*

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November 7, 2025

Abstract

We show that when investment increases the probability of success of a project, diversifying a fixed budget across more uncorrelated projects increases overall portfolio risk. This result holds for portfolios comprised of projects with binary outcomes—success or failure—where the payoffs associated with each outcome are exogenous (and can be replaced by an entire exogenous payoff distribution). Based on this finding, we hypothesize that risk-averse managers in Venture Capital (VC) limit portfolio diversification to mitigate risk. We propose a testable prediction, based on the correlation between a VC portfolio's diversification and its subsequent returns, that distinguishes our hypothesis from the general premise that diversification reduces risk. Existing empirical evidence from the VC industry, showing a positive correlation between diversification and returns, is consistent with our hypothesis.

Keywords: Risk, Diversification, Venture Capital

*We are grateful for comments and discussions with Nittai Bergman, David Disatnik, Yaniv Grinstein, Ilan Kremer, Oren Levintal, Nadav Levy, Sharun Mukand, Ady Pauzner, Daniel Sgroi, and Marija Vukotic.

1 Introduction

Modern portfolio theory, pioneered by Markowitz (1952), establishes diversification as a fundamental principle for risk-averse investors, showing that combining assets with imperfect correlations can improve the risk-return profile. This principle holds unequivocally when asset return distributions are largely exogenous to an individual investor's allocation decisions. Indeed, conventional investment practice involves diversifying across numerous assets, where investors face exogenous return distributions that they can combine but not alter.

However, in several important contexts—such as internal corporate innovation, entrepreneurial finance, and venture capital—investment has a substantial impact on the outcome distributions. In particular, the probability of success for a project increases with the level of investment. In the venture capital (VC) context, evidence suggests that the investment package, which includes both funding and strategic guidance, increases the success probability of portfolio companies. For instance, Ewens et al. (2016) show that when VCs provide follow-on funding without new investors ("inside rounds")—effectively diversifying their support across existing portfolio companies—it leads to lower returns and higher failure rates. Similarly, Hellmann and Puri (2000) show that VC-backed firms bring products to market significantly faster than comparable non-backed firms, and Sørensen (2007) finds that companies funded by more experienced VCs are more likely to go public.[1]

We show that when investment primarily increases the probability of success (rather than the payoffs conditional on success), diversification—spreading a fixed budget across more ex-ante identical, independent projects—does not mitigate portfolio risk, but rather exacerbates it. Consequently, the anticipated trade-off between the potential for a higher probability of success from focused investment and the conventional benefit of risk reduction from diversification does not always hold. Our framework, therefore, offers a novel explanation for the observed limited diversification in certain investment contexts, one that does not rely on factors such as technological non-convexities, behavioral biases, market frictions, or agency problems.

The existing literature generally takes as given that diversification across uncorrelated assets (while holding risk, expected return, and other relevant characteristics constant) is risk-reducing. To the best of our knowledge, the only exception is provided by Ibragimov (2009) and Ibragimov et al. (2011), who demonstrate that diversification fails to reduce risk in the case of extremely heavy-tailed return distributions (relevant for insurance and systemic risk applications). Our contribution fundamentally differs: we show that when investment affects project success probabilities—a hallmark of strategic and operational investments, such as venture capital, new product development, and entrepreneurial ventures—diversification increases portfolio risk through a mean-preserving spread mechanism. Our framework could shed light on various instances of limited diversification and offers a potential resolution to a puzzling finding in the empirical literature concerning the effect of diversification on returns in venture capital funds. Our setting is not the first in which investment increases the probability of project success; this is a familiar modeling approach in the literature.

[1] As acknowledged in the literature, this finding may arise from both the positive effect of VC experience and from sorting, as experienced VCs tend to select better companies. (See also the comprehensive survey by Lerner and Nanda, 2020).

However, we are the first to show that, within this framework, diversification increases portfolio risk.

To illustrate our core mechanism, we first analyze a benchmark model in which a risk-averse investor allocates a fixed budget across numerous ex-ante identical and independent projects, each with a binary outcome (success or failure). Investment in each project increases its probability of success at a constant marginal rate, up to some maximum threshold. Given this setup, the portfolio's expected total return remains invariant to the allocation strategy. We show that any concentrated allocation second-order stochastically dominates (SOSD) any more diversified allocation. Consequently, in this setting, any risk-averse investor will optimally choose the least diversified allocation.

To obtain some intuition for this finding, consider a simple example. An investor has a fixed budget of 80 dollars to allocate between two ex-ante identical, independent projects. Each project has a binary outcome resulting in a return of zero for failure or 150 dollars for success. The probability of success is proportional to the investment in the project. Specifically, every dollar invested increases the success probability by one percent. Given these numbers, the portfolio's expected total return is constant at 50 percent regardless of the allocation strategy.

To see the effect of diversification, consider a shift away from the most concentrated allocation. If the investor allocates the entire 80 dollars to one project, the total payoff is 150 dollars with a probability of 80 percent, and zero dollars otherwise. Now, suppose the investor diversifies by allocating some of the budget to the second project. Any diversification introduces a non-zero probability of obtaining the maximum total payoff of 300 dollars (from both projects succeeding), and this probability increases further with more diversification. Crucially, this comes at the cost of a parallel increase in the probability of a total portfolio failure (a zero-dollar payoff, from both projects failing). This results in a mean-preserving spread of the portfolio return, making the diversified portfolio riskier and less desirable for any risk-averse investor.

We establish the robustness of our central finding through two key extensions that relax the simplifying assumptions of our baseline model. First, we generalize the project framework to encompass stochastic outcomes, moving beyond the binary success/failure case. Our results persist when the successful outcome is replaced by a lottery that first-order stochastically dominates (FOSD) the lottery associated with failure.

Second, we demonstrate that our qualitative result (diversification increases risk) is independent of the linearity assumption for the success probability function. By introducing a success probability function characterized by initially increasing marginal returns to investment, followed by diminishing marginal returns, we obtain a unique interior allocation that maximizes the portfolio's expected return—if the budget is above some minimum, the entire budget will not be allocated to one project. (A fixed cost can substitute for the initial range of increasing marginal returns). This structure explicitly enables a trade-off between maximizing the portfolio's expected return and minimizing its associated risk. We show that a risk-averse investor will rationally deviate from the return-maximizing allocation by reducing diversification (i.e., increasing concentration). This extension not only demonstrates the robustness of our core finding but also allows us to formally derive the empirical predictions discussed in the next section.

Our analysis, reinforced by these extensions, establishes that the nature of the investment

function is critical: when investment primarily increases the probability of success, a concentrated allocation reduces portfolio risk. Importantly, this finding is not limited to settings where investment is solely constrained to enhancing the probability of success. As we show in Katreniak et al. (2023), a risk-averse investor who can choose between these two investment margins would prefer to direct funds toward increasing the probability of success rather than increasing the conditional payoff, holding the expected return constant across the two margins (because the former, in comparison to the latter, reduces risk). This result supports the broader applicability of our framework, as its conclusions are pertinent even when enhancing the probability of success is simply a feasible investment option.

The distinction between these two investment margins can be illustrated by contrasting investment in production capacity with investment in product quality. The former affects the payoff conditional on success—the product finding demand, while the latter directly aims to increase the probability of success. While the boundary between these two investment types is often blurred, our proposition remains relevant so long as investors can distinguish between these two margins and allocate capital to optimize their portfolio decisions.

In the next section, we turn to our core applied contribution: the study of venture capital investment strategy. We begin by discussing existing explanations for focused investment within that industry and then describe our empirical prediction and the relevant findings in the literature.[2]

2 Venture Capital: An Empirical Prediction

Our core finding is particularly relevant to the venture capital (VC) industry, where concentration of investment is a documented strategy, and investment in each venture increases its probability of success.[3] While a large body of literature has addressed the trade-off between the benefits of diversification and those of concentration, these studies, when considering risk, take as given that diversification is risk-reducing—an assumption we demonstrate is not universally valid.

The existing literature has invoked several mechanisms to justify why concentrated investment might be preferred. Agency problems provide one main explanation (e.g., Jensen and Meckling, 1976; Holmström, 1982; Shleifer and Vishny, 1986), which often includes an emphasis on incomplete contracting (e.g., Aghion and Bolton, 1992; Hart and Moore, 1990; Kaplan and Strömberg, 2003). Several papers explicitly examine the trade-off between the advantages of diversification and a small portfolio and have identified several factors that can create an optimal portfolio diversification. These include the diminishing returns of a VC's effort in each investment (Kanniainen and Keuschnigg, 2003), and the ability to reallocate resources in the case of a startup failure (Fulghieri and Sevilir, 2009). Our theory adds a

[2] A few additional mechanisms are offered by the literature for explaining focused investment more broadly. These include herding behavior (Bikhchandani et al., 1992) and a variety of behavioral explanations such as overconfidence (Zacharakis and Shepherd, 2001); loss aversion (Benartzi and Thaler, 1995); reliance on heuristics (Jiang et al., 2024; Zion et al., 2010); and home bias (Heathcote and Perri, 2013).

[3] Beyond the evidence that VCs increase the likelihood of success of a venture (as noted above), VCs themselves believe their active involvement contributes to this success. This belief is critical for our theory's predictions. Gompers et al. (2020) document this self-perceived importance and provide evidence of significant active support, showing VCs dedicate 18 hours per week on average to portfolio companies.

very simple mechanism to these explanations: when investment increases the probability of success of each project, risk-averse agents reduce diversification to reduce risk.

VC managers may, on the margin, reduce a fund's risk at the expense of maximizing its expected returns, driven by concerns over their own employment risk. This well-documented agency problem, stemming from managers being more risk-averse than shareholders, is common in financial settings. For instance, studies show that managers of publicly traded firms have engaged in mergers to reduce undiversifiable employment risk (Amihud and Lev, 1981), diversification into new lines of business (May, 1995), or hedging against price fluctuations (Tufano, 1996), often to the detriment of shareholder returns.

If diversification reduces risk, as conventionally assumed, risk-averse VC managers would over-diversify relative to return-maximizing benchmarks, resulting in a negative correlation between diversification and observed returns. However, the empirical evidence reveals a different, puzzling pattern. Buchner et al. (2017) find that both industry and stage diversification are positively correlated with realized returns. They address this puzzle by proposing that diversification, because it reduces risk, induces fund managers to select riskier underlying investments with higher expected returns, leading to the observed positive correlation.[4]

Our theory offers an alternative explanation. We hypothesize that in the context of VC, where investment is directed at increasing the probability of success, diversification increases risk, and that risk-averse managers would rationally under-diversify their funds. This leads to the empirical prediction of a positive correlation between fund diversification and realized returns, which is directly contrary to the conventional prediction. Thus, our theory provides a simple, testable prediction that distinguishes our framework from the conventional view, without requiring the measurement of risk or the identification of causal effects. We formally derive this prediction in an extension of the model, which introduces a non-linear success probability function that generates an interior solution for the expected return-maximizing allocation.

As in Buchner et al. (2017), other studies report positive associations between certain forms of diversification and returns. Other studies, however, find the opposite. Lossen (2006) reports a positive association between industry diversification and returns, a negative association between stage diversification and returns, and no significant correlation with geographic diversification. Gompers et al. (2009) finds that specialized funds (those concentrating in particular industries or sectors) achieve higher rates of successful exits compared to highly diversified funds. Knill (2009) finds that stage diversification has complex, context-dependent effects on fund performance, influencing fund growth positively but time to exit negatively. Humphery-Jenner (2013) shows that both industry and geographic diversification are associated with higher fund IRRs and exit multiples, proposing that diversification improves knowledge sharing. They also suggest that diversification can reduce returns by spreading management effort too thinly. Finally, Cressy et al. (2014) finds that broader industry diversification reduced fund success rates, whereas diversification

[4]within a broader context of entrepreneurial investment, Moskowitz and Vissing-Jørgensen (2002) wonder about "A Private Equity Premium Puzzle?" They document that entrepreneurial investment (a form of private equity that includes early-stage VC) is extremely concentrated, yet they find the returns are surprisingly low (no higher than public equity).

across countries increased overall fund returns.

The relevance of our finding that diversification could increase risk is arguably more significant when VCs invest in early-stage startups, where the probability of success of each project is low and investment is, arguably, directed at increasing this probability. This is perhaps less so the case when VCs invest in later stages, where the probability of success is higher, and investment is directed at increasing the expected return conditional on success (e.g., scaling production capacity). This insight—that the effect of diversification on risk is conditional on the project stage—provides a new, testable hypothesis that can guide future empirical studies investigating the complex relationship between diversification and returns in venture capital.

It is worth noting that, consistent with the existing literature on VC focus, our theory relies on the practical constraints of limited syndication (the splitting of investment in a project among multiple investors). While syndication is a common practice that offers a mechanism for diversification and risk sharing (Lerner, 1994; Brander et al., 2002), its constraints are empirically evident. Despite the potential for broader investment, VCs typically maintain highly focused portfolios (less than 15 portfolio companies as documented by Kaplan and Strömberg, 2004). This suggests a trade-off where the benefits of control and non-pecuniary value-add (deep engagement) outweigh the potential benefits from extensive syndication, including diversification and risk reduction (see the survey by Da Rin et al., 2013).

3 Model

We analyze the optimal investment strategy of a risk-averse investor with a fixed budget who seeks to allocate resources across multiple projects. In our setting, an investor's additional investment in a project directly increases its probability of success. Our central result establishes that, under a wide range of conditions, the optimal portfolio is a corner solution that features a maximal feasible concentration of investment.

In the next subsection, we examine a benchmark case with binary project outcomes, where investment directly influences the success probabilities. We then generalize this framework to a setting where project outcomes are represented by lotteries with stochastic distributions. In both cases, we show that the optimal allocation concentrates resources maximally rather than diversifying. The extension to a non-linear probability function is presented in the following section.

3.1 Multiple Projects with Discrete Outcomes

Consider a risk-averse expected-utility-maximizer facing n identical investment opportunities. Each project $i \in \{1, \ldots, n\}$ yields a binary outcome: success generates a return $H > 0$,

[4]It is important to emphasize that our prediction is, of course, valid for an empirical test that controls for funds' size, as these papers do. Clearly, increasing a fund's size and proportionally increasing its diversification will reduce the portfolio's risk. It is also worth noting that, to the extent that employment contracts can offset the distortion of managers, the effect of marginal changes in diversification on returns would approach zero.

while failure generates a return $L = 0$.

The investor affects the success probabilities $(p_1, \ldots, p_n)$ through a costly investment. The cost function is linear: $c(p_1, \ldots, p_n) = \alpha \sum_{i=1}^{n} p_i$, which we normalize with $\alpha = 1$, and we assume that the success probabilities are bounded above by $\bar{p}$, reflecting technological constraints, although the results hold for $\bar{p} = 1$. Project returns are stochastically independent.

The investor's problem is to fully allocate the budget K by choosing $(p_1, \ldots, p_n)$ to maximize the expected utility. Let $R(p_1, \ldots, p_n)$ denote the random total return, which takes values $kH + (n-k)L$ for $k \in \{0, \ldots, n\}$. For any feasible allocation, the total expected return is $H \sum_{i=1}^{n} p_i = H \cdot K$, which is constant. Therefore, the agent maximizes expected utility:

$$\mathbb{E}u(R(p_1, \ldots, p_n)),$$

by minimizing the variance of the portfolio.

Proposition 1. *The agent's optimal choice of probabilities is to set $p_i = \overline{p}$ for the largest possible number of projects, n^*. For one project, j, the agent sets the probability to $p_j = K - n^*\overline{p}$, and for all remaining projects, the probability is set to $p_i = 0$.*

Proof. Suppose that $p_1, \ldots, p_n$ is an optimal choice of probabilities that is such that there exist two projects, i and j with probabilities $0 < p_j \leq p_i < \overline{p}$. Observe that shifting probability mass from project j to project i preserves the expectation of the total return from projects i and j, denoted $R(p_i, p_j)$ because:

$$\begin{aligned}\mathbb{E}[R(p_i, p_j)] &= p_i H + (1 - p_i)L + p_j H + (1 - p_j)L \\ &= (H - L)(p_i + p_j) + 2L.\end{aligned}$$

The cumulative distribution of the return from investment in projects i and j before the shift is given by:

$$F^{i,j}(x) = \begin{cases} 0 & x < 2L \\ (1 - p_i)(1 - p_j) & 2L \leq x < L + H \\ 1 - p_i p_j & L + H \leq x < 2H \\ 1 & 2H \leq x. \end{cases}$$

Observe that a shift of probability mass m from project j to project i decreases the height of the distribution $F^{i,j}$ at $2L$, because the derivative of $(1 - p_i - m)(1 - p_j + m)$ with respect to m is

$$p_j - p_i - 2m \leq 0.$$

Similarly, this shift of probability increases the height of the distribution $F^{i,j}$ at $2H$, because the derivative of $(p_i + m)(p_j - m)$ with respect to m is also equal to

$$p_j - p_i - 2m \leq 0.$$

It therefore follows that the distributions $F^{i,j}$ before and after the shift satisfy Property 6.D.2 in Mas-Colell et al. (1995, p. 198), or that the distribution $F^{i,j}$ after the shift Second-Order Stochastically Dominates the distribution $F^{i,j}$ before the shift. Therefore, by Proposition 6.D.2 in Mas-Colell et al. (1995, p. 199), every risk-averse expected utility maximizer weakly prefers to shift as much probability mass from p_j into p_i as possible. □

3.2 Multiple Projects with Stochastic Outcomes

We now extend the analysis to a setting where project outcomes are themselves random variables. Consider an investor allocating resources between two projects, where investment affects the probability of drawing from favorable versus unfavorable distributions.

Let H and L denote random variables representing high and low outcome distributions, with cumulative distribution functions F_H and F_L, respectively. We assume H first-order stochastically dominates L, meaning $F_H(x) \leq F_L(x)$ for all $x \in \mathbb{R}$ with strict inequality for some x. This ensures $E[H] > E[L]$ (Mas-Colell et al., 1995, pp. 195–196). We assume $\infty > E[H] > E[L]$, $E[H] > 0$, and $\inf \text{supp}(L) < 0$.

The investor faces two independent investment opportunities. For each project $i \in \{1, 2\}$, the payoff X_i is drawn from H with probability p_i and from L with probability $1 - p_i$. Investment costs are linear in probabilities, with total budget constraint $p_1 + p_2 = K < 2$, $p_i \leq \bar{p} \leq 1$.

The investor's total payoff equals $R = X_1 + X_2$, where X_1 and X_2 are independent. A risk-averse investor with von Neumann-Morgenstern utility u (where $u' > 0$ and $u'' < 0$) chooses (p_1, p_2) to maximize $E[u(R)]$ subject to the constraints $p_i \in [0, \bar{p}]$ and $p_1 + p_2 = K$.

3.2.1 Structure of Payoff Distributions

The distribution of total payoffs has a particularly tractable structure. Since H first-order stochastically dominates L, convolution preserves this ordering: $H+H \succeq_{FOSD} H+L \succeq_{FOSD} L + L$. Moreover, for any feasible allocation (p_1, p_2), expected total payoff equals

$$E[R(p_1, p_2)] = 2E[L] + K(E[H] - E[L]),$$

depending only on the budget K, not on its allocation across projects.

Proposition 2 (Mixture Representation)**.** *Assume $H_1, H_2 \overset{i.i.d.}{\sim} H$ and $L_1, L_2 \overset{i.i.d.}{\sim} L$, with independence across all draws. For allocation (p_1, p_2), the cumulative distribution of $R = X_1 + X_2$ is*

$$F_{(p_1,p_2)}(w) = \sum_{k=0}^{2} \pi_k(p_1, p_2) F_k(w)$$

where

$$\begin{aligned} \pi_0(p_1, p_2) &= (1 - p_1)(1 - p_2), & F_0(w) &= P(L_1 + L_2 \leq w), \\ \pi_1(p_1, p_2) &= p_1(1 - p_2) + (1 - p_1)p_2, & F_1(w) &= P(H_1 + L_2 \leq w), \\ \pi_2(p_1, p_2) &= p_1 p_2, & F_2(w) &= P(H_1 + H_2 \leq w). \end{aligned}$$

Proof. Define events $A_0 = \{X_1 = L, X_2 = L\}$, $A_1 = \{(X_1 = L, X_2 = H) \cup (X_1 = H, X_2 = L)\}$, and $A_2 = \{X_1 = H, X_2 = H\}$. By the law of total probability,

$$F_{(p_1,p_2)}(w) = \sum_{k=0}^{2} P(A_k) \cdot P(R \leq w | A_k).$$

The stated π_k and F_k follow directly from the independence and definitions. □

3.2.2 Optimality of Corner Solutions

The mixture representation reveals why concentrated allocations dominate diversified ones. Moving the probability mass from the less likely to the more likely project creates a mean-preserving contraction in the payoff distribution.

Theorem 1 (Dominance of Asymmetric Allocations)**.** *Let (p_1, p_2) be a feasible allocation with $p_1 > p_2 > 0$ and $p_1 < \bar{p}$. For sufficiently small $\epsilon > 0$, the allocation $(p_1 + \epsilon, p_2 - \epsilon)$ second-order stochastically dominates (p_1, p_2).*

Proof. Let allocation $A = (p_1, p_2)$ and $K = (p_1 + \epsilon, p_2 - \epsilon)$. From the mixture representation, the weights change by

$$\Delta\pi_0 = -\epsilon(p_1 - p_2) - \epsilon^2, \quad \Delta\pi_2 = -\epsilon(p_1 - p_2) - \epsilon^2, \quad \Delta\pi_1 = 2\epsilon(p_1 - p_2) + 2\epsilon^2.$$

This removes equal mass from $L + L$ and $H + H$ and adds it to $H + L$. The mean of R is preserved since $E[R_B] = E[R_A] = 2E[L] + K(E[H] - E[L])$. Since $E[L + L] < E[H + L] < E[H + H]$, the reallocation constitutes a mean-preserving contraction.

Define $\phi(w) = 2F_1(w) - F_0(w) - F_2(w)$. The CDF difference is

$$F_B(w) - F_A(w) = [\epsilon(p_1 - p_2) + \epsilon^2]\phi(w).$$

Integrating yields

$$\int_{-\infty}^{w} [F_A(t) - F_B(t)]dt = -[\epsilon(p_1 - p_2) + \epsilon^2] \int_{-\infty}^{w} \phi(t)dt \geq 0,$$

where the inequality follows because $\int_{-\infty}^{w} \phi(t)dt \leq 0$ for all w under a mean-preserving contraction by definition. By the Rothschild and Stiglitz (1978) characterization, $B \succeq_{SOSD} A$. □

Theorem 2 (Corner Solution Optimality)**.**

$$(p_1^*, p_2^*) \in \{(\bar{p}, K - \bar{p}), (K - \bar{p}, \bar{p})\}.$$

Proof. Suppose (p_1^*, p_2^*) is interior. If $p_1^* \neq p_2^*$, Theorem 1 gives a dominating allocation, violating optimality. If $p_1^* = p_2^* = K/2$, perturb to $(K/2 + \epsilon, K/2 - \epsilon)$; Theorem 1 implies SOSD and the strict concavity of u gives $E[u(R)]$ strictly higher. Thus no interior optimum exists and corners are optimal. □

3.3 Extension to n Projects with Stochastic Outcomes

We now extend our analysis from two projects to n projects, showing that the corner solution optimality persists in the general case. The key insight is that any improvement through pairwise transfers established in the two-project case can be iteratively applied to achieve maximal concentration.

3.3.1 Setup and Mixture Representation

Fix a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Let H and L be real-valued random variables with cumulative distribution functions F_H and F_L, respectively, such that:

- $H \succeq_{\text{FOSD}} L$ (i.e., $F_H(x) \leq F_L(x)$ for all x, with strict inequality for some x),
- $\mathbb{E}[|H|], \mathbb{E}[|L|] < \infty$ and $\mathbb{E}[H] > \mathbb{E}[L]$.

Let $(H_i)_{i=1}^n$ be i.i.d. copies of H and $(L_i)_{i=1}^n$ be i.i.d. copies of L, with all these variables mutually independent.

An *allocation* is a vector $\mathbf{p} = (p_1, \ldots, p_n) \in [0, \bar{p}]^n$ with $\sum_{i=1}^n p_i = K \in [0, n\bar{p}]$. For each project i, let $B_i \sim \text{Bernoulli}(p_i)$, independent across i and independent of $(H_j, L_j)_{j \leq n}$, and define

$$X_i = B_i H_i + (1 - B_i) L_i, \qquad R(\mathbf{p}) = \sum_{i=1}^n X_i.$$

Preferences are represented by an increasing, concave utility $u : \mathbb{R} \to \mathbb{R}$ with $\mathbb{E}[|u(R(\mathbf{p}))|] < \infty$ for all feasible $\mathbf{p}$.

Proposition 3 (Mixture Representation for n Projects)**.** *For any feasible* $\mathbf{p}$ *and any* $w \in \mathbb{R}$*,*

$$F_{\mathbf{p}}(w) = \mathbb{P}(R(\mathbf{p}) \leq w) = \sum_{k=0}^n \Pi_k(\mathbf{p}) F_k(w),$$

where

$$\Pi_k(\mathbf{p}) = \sum_{\substack{S \subseteq \{1,\ldots,n\} \\ |S|=k}} \prod_{i \in S} p_i \prod_{j \notin S} (1 - p_j) \quad \textit{and} \quad F_k(w) = \mathbb{P}\left(\sum_{i=1}^k H_i + \sum_{i=k+1}^n L_i \leq w \right).$$

Moreover, the expected total payoff is

$$\mathbb{E}[R(\mathbf{p})] = n\mathbb{E}[L] + K(\mathbb{E}[H] - \mathbb{E}[L]),$$

depending only on the budget sum $K = \sum_i p_i$*, not on its allocation.*

Proof. Let $K_n(\mathbf{p}) = \sum_{i=1}^n \mathbf{1}\{B_i = 1\}$ denote the number of H-draws. By the law of total probability,

$$F_{\mathbf{p}}(w) = \sum_{k=0}^n \mathbb{P}(K_n(\mathbf{p}) = k) \cdot \mathbb{P}(R(\mathbf{p}) \leq w \mid K_n(\mathbf{p}) = k) = \sum_{k=0}^n \Pi_k(\mathbf{p}) F_k(w),$$

using independence for both terms. The expectation follows from linearity: $\mathbb{E}[X_i] = p_i \mathbb{E}[H] + (1 - p_i)\mathbb{E}[L]$ and summation. □

3.3.2 Order Preservation Properties

Lemma 1 (Stochastic Orders Preserved Under Independent Sums)**.** *If $A \succeq_{SOSD} B$ and Z is independent of (A, B), then $A + Z \succeq_{SOSD} B + Z$. Similarly, if $A \succeq_{FOSD} B$, then $A + Z \succeq_{FOSD} B + Z$.*

Proof. For SOSD, let u be any increasing concave function with finite expectations and set $v(x) = \mathbb{E}[u(x+Z)]$. Then v is increasing and concave, so $\mathbb{E}[u(A+Z)] = \mathbb{E}[v(A)] \geq \mathbb{E}[v(B)] = \mathbb{E}[u(B+Z)]$. The FOSD case is analogous with u increasing. □

Lemma 2 (Ordering of Component Sums)**.** *For $k = 0, \ldots, n-1$, we have $F_{k+1} \succeq_{FOSD} F_k$.*

Proof. Write F_{k+1} as the law of $H_1 + S$ and F_k as the law of $L_1 + S$, where $S = \sum_{i=2}^{k+1} H_i + \sum_{i=k+2}^{n} L_i$ is independent of (H_1, L_1). Since $H \succeq_{\text{FOSD}} L$, Lemma 1 (FOSD part) gives the claim. □

3.3.3 Dominance Through Pairwise Transfers

For distinct indices $i \neq j$ and $\varepsilon \in \mathbb{R}$, define the transfer operator

$$T_{ij,\varepsilon}(\mathbf{p}) = \mathbf{p} + \varepsilon e_i - \varepsilon e_j,$$

where e_i is the i-th standard basis vector. The transfer is *feasible* if $0 \leq p_i + \varepsilon \leq \bar{p}$ and $0 \leq p_j - \varepsilon \leq \bar{p}$.

Theorem 3 (Pairwise Transfer Dominance)**.** *Let $\mathbf{p}$ be a feasible allocation with indices $i \neq j$ satisfying $0 < p_j \leq p_i < \bar{p}$. For any $\varepsilon \in (0, \min\{\bar{p} - p_i, p_j\}]$,*

$$R(T_{ij,\varepsilon}(\mathbf{p})) \succeq_{SOSD} R(\mathbf{p}) \quad \textit{with} \quad \mathbb{E}[R(T_{ij,\varepsilon}(\mathbf{p}))] = \mathbb{E}[R(\mathbf{p})].$$

If u is strictly concave and $F_H \neq F_L$, then $\mathbb{E}[u(R(T_{ij,\varepsilon}(\mathbf{p})))] > \mathbb{E}[u(R(\mathbf{p}))]$.

Proof. Decompose $R(\mathbf{p}) = Y + Z$, where $Y = X_i + X_j$ and $Z = \sum_{\ell \notin \{i,j\}} X_\ell$. Under $T_{ij,\varepsilon}(\mathbf{p})$, the component Z remains unchanged and independent of (X_i, X_j).

By Theorem 1 (the two-project result), the transfer $(p_i, p_j) \mapsto (p_i + \varepsilon, p_j - \varepsilon)$ yields $Y^\varepsilon \succeq_{\text{SOSD}} Y$ while preserving $\mathbb{E}[Y]$.

Since Z is independent of both Y and Y^ε, Lemma 1 (SOSD part) gives

$$R(T_{ij,\varepsilon}(\mathbf{p})) = Y^\varepsilon + Z \succeq_{\text{SOSD}} Y + Z = R(\mathbf{p}).$$

Expectation invariance follows from Proposition 3. Strict improvement in expected utility follows from strict concavity and non-degeneracy ($F_H \neq F_L$). □

3.3.4 Global Optimality of Corner Solutions

Lemma 3 (Finite Convergence to Corner)**.** *For any feasible allocation $\mathbf{p}$, there exists a finite sequence of feasible pairwise transfers that transforms $\mathbf{p}$ into a corner allocation (one where all but at most one coordinate equals either 0 or $\bar{p}$).*

Proof. Let $N(\mathbf{p}) = \#\{i : 0 < p_i < \bar{p}\}$ be the number of interior coordinates. If $N(\mathbf{p}) \leq 1$, we are done. If $N(\mathbf{p}) \geq 2$, pick indices i, j with $p_i, p_j \in (0, \bar{p})$ and, without loss of generality, $p_i \geq p_j$.

Apply transfer $T_{ij,\varepsilon}$ with $\varepsilon = \min\{\bar{p} - p_i, p_j\} > 0$. This sends either p_i to $\bar{p}$ or p_j to 0, so N decreases by at least one. Since $N(\mathbf{p})$ is finite and strictly decreases with each transfer, after at most $n - 1$ steps we reach $N \leq 1$, i.e., a corner allocation. □

Theorem 4 (Corner Solution Optimality for n Projects)**.** *For any risk-averse expected-utility-maximizer with increasing, concave utility u, and any feasible allocation $\mathbf{p}$, there exists a corner allocation $\mathbf{p}^*$ such that*

$$R(\mathbf{p}^*) \succeq_{SOSD} R(\mathbf{p}) \quad \textit{and} \quad \mathbb{E}[R(\mathbf{p}^*)] = \mathbb{E}[R(\mathbf{p})].$$

If u is strictly concave and $F_H \neq F_L$, then $\mathbb{E}[u(R(\mathbf{p}^*))] > \mathbb{E}[u(R(\mathbf{p}))]$ unless $\mathbf{p}$ is already a corner allocation.

Moreover, every optimal allocation takes the form (after appropriate relabeling):

$$p_i^* = \begin{cases} \bar{p} & \text{for } i = 1, \ldots, n^* \\ K - n^*\bar{p} & \text{for } i = n^* + 1 \\ 0 & \text{for } i > n^* + 1 \end{cases}$$

where $n^* = \lfloor K/\bar{p} \rfloor$.

Proof. Starting from $\mathbf{p}$, apply the finite sequence of transfers from Lemma 3. By Theorem 3, each transfer weakly improves the allocation in the SOSD sense while preserving the mean. The terminal corner allocation $\mathbf{p}^*$ therefore SOSD-dominates $\mathbf{p}$ with the same expectation.

By the Rothschild-Stiglitz characterization, SOSD dominance implies $\mathbb{E}[u(R(\mathbf{p}^*))] \geq \mathbb{E}[u(R(\mathbf{p}))]$ for any concave utility function, with strict inequality under strict concavity unless $\mathbf{p}$ was already a corner allocation.

The canonical form follows from the constraint $\sum_i p_i = K$ and the box constraints: we can have at most $n^* = \lfloor K/\bar{p} \rfloor$ projects at maximum investment $\bar{p}$, with any remainder $K - n^*\bar{p} \in [0, \bar{p})$ allocated to at most one additional project. □

[Connection to Majorization] Let $\succcurlyeq_{\text{maj}}$ denote the majorization order on $\mathbb{R}^n$. If $\mathbf{p}, \mathbf{q} \in [0, \bar{p}]^n$ satisfy $\sum_i p_i = \sum_i q_i$ and $\mathbf{p} \succcurlyeq_{\text{maj}} \mathbf{q}$, then $\mathbf{p}$ can be obtained from $\mathbf{q}$ by a finite sequence of pairwise transfers. By Theorem 3, $R(\mathbf{p}) \succeq_{\text{SOSD}} R(\mathbf{q})$ with equal means. Thus, among allocations with the same budget sum, greater concentration (in the majorization sense) yields lower risk in the SOSD sense.

4 Interior Solutions and Empirical Predictions

We now extend our analysis to non-linear cost functions, which generate interior solutions and enable us to derive testable empirical predictions about the relationship between diversification and returns.

Suppose the cost function $c(p)$ is continuously differentiable, concave up to some point $p_{\text{inf}} \in (0, 1)$, and convex thereafter. This S-shaped cost structure (U-shaped marginal cost

curve) is natural when low levels of investment have increasing returns (learning effects, fixed costs), while at higher levels, investment is subject to diminishing returns (resource constraints, management limits). As in our basic model, each project provides a reward of $H > 0$ with probability p and zero otherwise. Let q denote the inverse of the cost function, so $p = q(x)$ when investment x yields success probability p. To simplify the analysis, we treat the number of projects n as a continuous variable.

Let p_{RN} be the unique probability that minimizes the average cost $c(p)/p$. This occurs where the marginal cost equals the average cost: $c'(p_{RN}) = c(p_{RN})/p_{RN}$. Given our assumptions on c, such a point exists uniquely in $(p_{\text{inf}}, 1)$.

Proposition 4. *A risk-neutral agent with a budget K optimally chooses $n_{RN} = K/c(p_{RN})$ projects, investing $c(p_{RN})$ in each. This allocation maximizes expected total return.*

Proof. The expected return from n projects with an equal investment of K/n in each, is $\mathbb{E}[R(n)] = n \cdot q(K/n) \cdot H$. Taking the derivative and setting $p = q(K/n)$:

$$\frac{d\mathbb{E}[R]}{dn} = H\left[p - \frac{c(p)}{c'(p)}\right]$$

This equals zero when $c'(p) = c(p)/p$, uniquely identifying $p = p_{RN}$. The second derivative is negative, ensuring this is a maximum:

$$\frac{d^2\mathbb{E}[R]}{dn^2} = H\frac{d}{dp}\left(p - \frac{c(p)}{c'(p)}\right)\frac{dp}{dn} \tag{1}$$

The term $\frac{dp}{dn}$ is negative. The term $\frac{d}{dp}\left(p - \frac{c(p)}{c'(p)}\right)$ is positive at $p = p_{RN}$, as this point marks the minimum of the average cost function, which is U-shaped. Thus, the expected return is maximized at n_{RN}. □

Now consider the portfolio variance. For n projects with equal investment, each succeeding with probability $p = q(K/n)$:

$$V(n) = n \cdot p(1-p) \cdot H^2$$

The derivative with respect to n is:

$$\frac{dV}{dn} = H^2\left[p(1-p) + n\frac{dp}{dn}(1-2p)\right]$$

Since $p = q(K/n)$ and $q = c^{-1}$, we have $dp/dn = -c(p)/(nc'(p))$. Therefore:

$$\frac{dV}{dn} = H^2\left[p(1-p) - \frac{c(p)}{c'(p)}(1-2p)\right]$$

Proposition 5. *At the risk-neutral optimum n_{RN}, the variance is strictly increasing in n:*

$$\left.\frac{dV}{dn}\right|_{n=n_{RN}} = H^2 p_{RN}^2 > 0$$

Proof. At $n = n_{RN}$, we have $c(p_{RN})/c'(p_{RN}) = p_{RN}$. Substituting:

$$\left.\frac{dV}{dn}\right|_{n=n_{RN}} = H^2[p_{RN}(1-p_{RN}) - p_{RN}(1-2p_{RN})] = H^2 p_{RN}^2 > 0$$

□

This result has immediate implications for risk-averse agents. Since the expected return is maximized at n_{RN} (the derivative equals zero) while the variance is increasing, a risk-averse agent will choose fewer projects than n_{RN}. The agent accepts a lower expected return to reduce portfolio risk. This analysis is based on a simple envelope theorem argument: at the allocation that maximizes returns, the derivative of the return with respect to diversification is zero, whereas the derivative of the variance is strictly positive.

This section thus shows that our results are not limited to the case of a linear cost function. Moreover, it allows a formal derivation of our empirical prediction. We hypothesize that in venture capital funds, the investment package of funds and effort is viewed as a way to primarily increase the success probability of a project rather than the return given success, and therefore, risk-averse managers reduce diversification to reduce risk.

Assuming that not all managers are similarly risk-averse, it follows that the data will reveal a distribution in diversity and returns with a positive correlation between the two. A more risk-averse manager will choose to reduce diversification more than a less risk-averse manager, accepting a larger reduction in the expected return to achieve a lower variance. This is in contrast to the prediction of the conventional approach, which posits that investors over-diversify to reduce risk, leading to a negative correlation between diversification and returns. Thus, our model provides a testable hypothesis that can be evaluated using only data on returns and diversification, bypassing the need to measure risk directly or to identify any causal effects.

5 Conclusion

The existing literature on the trade-off between diversification and focused investment universally assumes that combining uncorrelated assets reduces portfolio risk. This assumption is valid when asset return distributions are exogenous to investor decisions. However, the literature often extends this assumption to settings where investment has a direct impact on a project's probability of success.

Our paper challenges this conventional view. We demonstrate that in such contexts, diversification does not necessarily reduce risk; in fact, when an investment's primary effect is to increase success probabilities, diversification increases risk. This finding is relevant for a wide range of strategic and operational investments, including venture capital, new product development, and entrepreneurial ventures.

While a large body of literature has sought to explain the limited diversity of investments in these fields through factors such as agency problems and informational asymmetries, we propose a simpler, alternative explanation. Our findings suggest that in the relevant cases, agents reduce diversification to reduce risk.

Our work provides a testable prediction that stands in direct opposition to the prediction of the conventional theory: in the equilibrium allocation of venture capital funds, the marginal effect of diversification on returns is positive. This prediction aligns with the empirical findings in the venture capital literature—findings that are puzzling under the conventional assumption that diversification reduces risk.

In conclusion, our paper presents a key counterintuitive finding that offers a new lens through which to view investment patterns in a variety of contexts. While we leave further extensions to future research, our core finding provides a fundamental re-evaluation of the risk-return relationship in a significant class of investments.

References

P. Aghion and P. Bolton. An incomplete contracts approach to financial contracting. *Review of Economic Studies*, 59(3):473–494, 1992.

Y. Amihud and B. Lev. Risk reduction as a managerial motive for conglomerate mergers. *Bell Journal of Economics*, 12(2):605–617, 1981.

S. Benartzi and R. H. Thaler. Myopic loss aversion and the equity premium puzzle. *Quarterly Journal of Economics*, 110(1):73–92, 1995.

S. Bikhchandani, D. Hirshleifer, and I. Welch. A theory of fads, fashion, custom, and cultural change as informational cascades. *Journal of Political Economy*, 100(5):992–1026, 1992.

J. A. Brander, R. Amit, and W. Antweiler. Venture-capital syndication: Improved venture selection vs. the value-added hypothesis. *Journal of Economics & Management Strategy*, 11(3):423–452, 2002.

A. Buchner, A. Mohamed, and A. Schwienbacher. Diversification, risk, and returns in venture capital. *Journal of Business Venturing*, 32(5):519–535, 2017.

R. Cressy, A. Malipiero, and F. Munari. European private equity: An empirical analysis of the performance of european private equity partnerships. *Journal of Private Equity*, 17 (2):64–79, 2014.

M. Da Rin, T. Hellmann, and M. Puri. A survey of venture capital research*. In *Handbook of the Economics of Finance*, volume 2, pages 573–648. Elsevier, 2013.

M. Ewens, M. Rhodes-Kropf, and I. A. Strebulaev. Insider financing and venture capital returns. *Working Paper*, 2016.

P. Fulghieri and M. Sevilir. Size and focus of a venture capitalist's portfolio. *Review of Financial Studies*, 22(11):4643–4680, 2009.

P. Gompers, A. Kovner, J. Lerner, and D. Scharfstein. Specialization and success: Evidence from venture capital. *Journal of Economics & Management Strategy*, 18(3):817–844, 2009.

P. A. Gompers, W. Gornall, S. N. Kaplan, and I. A. Strebulaev. How do venture capitalists make decisions? *Journal of Financial Economics*, 135(1):169–190, 2020.

O. Hart and J. Moore. Property rights and the nature of the firm. *Journal of Political Economy*, 98(6):1119–1158, 1990.

J. Heathcote and F. Perri. The international diversification puzzle is not as bad as you think. *Journal of Political Economy*, 121(6):1108–1159, 2013.

T. Hellmann and M. Puri. The interaction between proiamonuct market and financing strategy: The role of venture capital. *Review of Financial Studies*, 13(4):959–984, 2000.

B. Holmström. Moral hazard in teams. *Bell Journal of Economics*, 13(2):324–340, 1982.

M. Humphery-Jenner. Diversification in private equity funds: On knowledge sharing, risk aversion and limited attention. *Journal of Financial and Quantitative Analysis*, 48(5): 1545–1572, 2013.

R. Ibragimov. Portfolio diversification and value at risk under thick-tailedness. *Quantitative Finance*, 9(5):565–580, 2009.

R. Ibragimov, D. Jaffee, and J. Walden. Diversification disasters. *Journal of Financial Economics*, 99(2):333–348, 2011.

M. C. Jensen and W. H. Meckling. Theory of the firm: Managerial behavior, agency costs and ownership structure. *Journal of Financial Economics*, 3(4):305–360, 1976.

Z. Jiang, H. Liu, C. Peng, and H. Yan. Investor memory and biased beliefs: Evidence from the field. Technical report, National Bureau of Economic Research, 2024.

V. Kanniainen and C. Keuschnigg. The optimal portfolio of start-up firms in venture capital finance. *Journal of Corporate Finance*, 9(5):521–534, 2003.

S. N. Kaplan and P. Strömberg. Financial contracting theory meets the real world: An empirical analysis of venture capital contracts. *Review of Economic Studies*, 70(2):281–315, 2003.

S. N. Kaplan and P. Strömberg. Characteristics, contracts, and actions: Evidence from venture capitalist analyses. *Journal of Finance*, 59(5):2177–2210, 2004.

D. C. Katreniak, A. Khazanov, O. Moav, Z. Neeman, and H. Zoabi. Why not borrow, invest, and escape poverty? *arXiv preprint arXiv:2305.02546*, 2023.

A. Knill. Should venture capitalists put all their eggs in one basket? diversification versus pure-play strategies in venture capital. *Financial Management*, 38(3):441–486, 2009.

J. Lerner. The syndication of venture capital investments. *Financial Management*, 23(3): 16–27, 1994.

J. Lerner and R. Nanda. Venture capital's role in financing innovation: What we know and how much we still need to learn. *Journal of Economic Perspectives*, 34(3):237–261, 2020.

U. Lossen. The performance of private equity funds: Does diversification matter? Working Paper, University of Munich, 2006.

H. Markowitz. Portfolio selection. *Journal of Finance*, 7(1):77–91, 1952.

A. Mas-Colell, M. D. Whinston, and J. R. Green. *Microeconomic Theory*. Oxford University Press, New York, 1995.

D. O. May. Do managerial motives influence firm risk reduction strategies? *Journal of Finance*, 50(4):1291–1308, 1995.

T. J. Moskowitz and A. Vissing-Jørgensen. The returns to entrepreneurial investment: A private equity premium puzzle? *The American Economic Review*, 92(4):745–778, 2002.

M. Rothschild and J. E. Stiglitz. Increasing risk: I. a definition. In *Uncertainty in economics*, pages 99–121. Elsevier, 1978.

A. Shleifer and R. W. Vishny. Large shareholders and corporate control. *Journal of Political Economy*, 94(3):461–488, 1986.

M. Sørensen. How smart is smart money? a two-sided matching model of venture capital. *Journal of Finance*, 62(6):2725–2762, 2007.

P. Tufano. Who manages risk? an empirical examination of risk management practices in the gold mining industry. *Journal of Finance*, 51(4):1097–1137, 1996.

A. L. Zacharakis and D. A. Shepherd. The nature of information and overconfidence on venture capitalists' decision making. *Journal of Business Venturing*, 16(4):311–332, 2001.

U. B. Zion, I. Erev, E. Haruvy, and T. Shavit. Adaptive behavior leads to under-diversification. *Journal of Economic Psychology*, 31(6):985–995, 2010.