UNIVERSITY OF WARWICK

DEPARTMENT OF ECONOMICS

Diploma Exercise Sheet 3: Discrete and joint distributions

1. A contractor estimates the probabilities for the number of days required to complete a certain type of construction project as follows:

Table 1: Distribution of days of completion

Time (Days)	1	2	3	4	5
Probability	0.05	0.20	0.35	0.30	0.10

(a) What is the probability a randomly chosen project will take less than 3 days to complete?

(b) Find the expected time to complete.

(c) Find the variance of time required to complete a project.

(d) The contractor's cost is made up of two parts – a fixed cost of £10,000 plus £1,000 for each day taken to complete the project. Find the mean and standard deviation of total project cost.

(e) If 3 projects are undertaken, what is the probability that at least 2 of them will take at least 4 days to complete, assuming independence of individual project completion times.

(f) Consider the new distribution, which is the old distribution plus 10 days:

Time (Days)	11	12	13	14	15
Probability	0.05	0.20	0.35	0.30	0.10

What is the expected time and variance for time to complete this project and how do these relate to the answers to (a) and (b), respectively.

(g) Consider the new distribution, which is the old distribution times 5 days:

Time (Days)	5	10	15	20	25
Probability	0.05	0.20	0.35	0.30	0.10

What is the expected time and variance for time to complete this project and how do these relate to the answers to (a) and (b), respectively.

(h) Define $E(X)=\sum_{i=1}^{k} p_i x_i$ as the expected value of the random variable X and

$V(X)=E[X-E(X)]^2=E(X^2)-E(X)^2=\sum_{i=1}^{k} p_i (x_i - E(X))^2$ as the variance of that distribution. Show the in general, (i) $E(a+X)=a+E(X)$, (ii) $V(a+X)=V(X)$, (iii) $E(aX)=aE(X)$, (iv) $V(aX)=a^2V(X)$

2. From the university records for students who left university in 1993, we know the A-level points score of all students taking A-levels and their degree classification. Table 1 gives proportions of students in each of the classifications, by A-level category and degree class:

Table 1: A-level points score by degree class

	A-level points score				
Degree class	22	24	26	28	30
First	0.02	0.01	0.01	0.01	0.03
Upper second	0.20	0.06	0.06	0.05	0.06
Lower second	0.19	0.03	0.03	0.02	0.01
Third	0.13	0.02	0.02	0.02	0.02

(a) What is the mean number A-level points score? What is the standard deviation of the A-level points score?

(b) By allocating 5 points for a First class degree, 4 points for an Upper second class degree, 3 points for a Lower second class degree, 2 points for a Third class. What is the mean degree score and the standard deviation of the degree score?

(c) Calculate the covariance between A-level points score and degree score.

3. Consider the following bivariate distribution between Y and X

		x		
		-1	0	1
	1	0.2	0.1	0.1
y	2	0.1	0.0	0.2
	3	0.0	0.2	0.1

(a) Calculate $E(X)$, $E(Y)$

(b) Calculate $V(X), V(Y)$ and $\text{cov}(X,Y)$

(c) Write out the univariate distribution of $X+Y$. Using this univariate distribution calculate $E(X+Y), V(X+Y)$. How do these relate to your answers to parts (a) and (b)?

(d) Write out the univariate distribution of X-Y. Using this univariate distribution calculate $E(X-Y), V(X-Y)$. How do these relate to your answers to parts (a) and (b)?

(e) Define the joint probability density function for the random variables X, Y as $p(X,Y)$. Define the marginal density of X [Y] as $p(x)$ [$p(y)$], as $p(X) = \sum_y p(x,y)$ [$p(Y) = \sum_x p(x,y)$]. Now $E(X) = \sum_y xp(y)$, $E(Y) = \sum_y yp(y)$, $V(X) = \sum_x (x - E(X))^2 p(x)$, $V(Y) = \sum_y (y - E(Y))^2 p(y)$ and $\text{cov}(X,Y) = \sum_x \sum_y (x - E(X))(y - E(Y)) p(x,y)$. Show that: (i) $E(X+Y) = E(X) + E(Y)$, (ii) $V(X+Y) = V(X) + V(Y) + 2\,\text{cov}(X,Y)$