

## Exercise Sheet 4

$$1) p = 0.5$$

$$a) P_r(X \geq 4) = 0.5$$

$$b) P(X=7) = {}^7C_7 p^7 (1-p)^0 = 0.0078$$

$$\begin{aligned} c) P_r(X > 1) &= 1 - P_r(X \leq 1) = 1 - P_r(X=0) \\ &\quad - P_r(X=1) \\ &= 1 - 0.0078 - 0.05468 \\ &= 0.9375 \end{aligned}$$

$$2 \quad X \sim P(5)$$

$$a) \Pr(X \leq 2) - \Pr(X \leq 1) = 0.040$$

$$b) \Pr(X > 8) = 1 - \Pr(X \leq 8) \\ = 1 - 0.932 = 0.068$$

$$c) \Pr(2 \leq X \leq 5) = \Pr(X \leq 5) - \Pr(X \leq 1) \\ = 0.576$$

$$2) \Pr(Z > -1) = 0.841$$

$$\Pr(-1.70 < Z < 1.20) = 0.84$$

$$\Pr(Z < a) = 0.70 \quad a = 0.525$$

$$\Pr(Z < a) = 0.25 \quad a = -0.67$$

$$3/ \quad X \sim N(1200, 100^2)$$

$$\begin{aligned} a) \quad \Pr(X > 1000) &= \Pr\left(Z > \frac{1000 - 1200}{100}\right) \\ &= \Pr(Z > -2) = 0.977 \end{aligned}$$

$$\begin{aligned} b) \quad \Pr(1100 < X < 1300) \\ &= \Pr(-1 \leq Z \leq 1) = 0.682 \end{aligned}$$

$$\begin{aligned} c) \quad \Pr(X > b) &= 0.1 \\ \Pr\left(Z > \frac{b - 1200}{100}\right) &= 0.1 \end{aligned}$$

$$\frac{b - 1200}{100} = 1.28 \Rightarrow b = 1328$$

$$4) E \sim N(60, 18^2)$$

$$a) \Pr(E > b) = 0.9 \Rightarrow \Pr\left(Z > \frac{b-60}{18}\right) = 0.9$$

$$\frac{b-60}{18} = -1.28 \Rightarrow b = 36.96$$

$$b) \Pr(E < c) = 0.8 \Rightarrow \Pr\left(Z < \frac{c-60}{18}\right) = 0.8$$

$$\frac{c-60}{18} = 0.84 \Rightarrow c = 75.12$$

$$c) \Pr(E > 75) = 0.203 = p$$

$$\Pr(X=2) = 0.045$$

$$\Pr(X=1) = 0.324$$

$$\Pr(X \geq 1) = 0.365$$

$$6. \quad f(x) = \begin{cases} 1/20 & 10 < x < 30 \\ 0 & \text{otherwise} \end{cases}$$

$$Pr(X < 15) = \int_0^{15} \frac{1}{20} dx = 0.25$$

$$b) \quad Pr(X > 17) = \int_{17}^{30} \frac{1}{20} dx = 0.65$$

$$c) \quad E(\pi) = (17 - 15) \cdot Pr(\text{success}) = P, 300$$

$$d) \quad E(\pi) = (B - 15) [30 - B] / 20$$

$$\frac{\partial E(\pi)}{\partial B} = \frac{30 + 15 - 2B}{20} = 0$$

$$B = 22.5$$