“An economic model differs substantially from a purely mathematical model in that it is a combination of a mathematical model and its interpretation.” Ariel Rubinstein

Supply and Demand

Plan

- <u>Demand</u>
  - Deriving demand curve
  - Substitution and Income effects
  - Price elasticity of demand
  - Consumer welfare
- <u>Supply</u>
  - A law of supply?
  - Price elasticity of supply
  - Producer's welfare

Equilibrium:
Demand = Supply

Demand

Demand-Supply model

- Demand function for washing machines

$$Q_w^d = D(p_w)$$

  - Where: Q_c, quantity of washing machines demanded; p_c, price of washing machines.

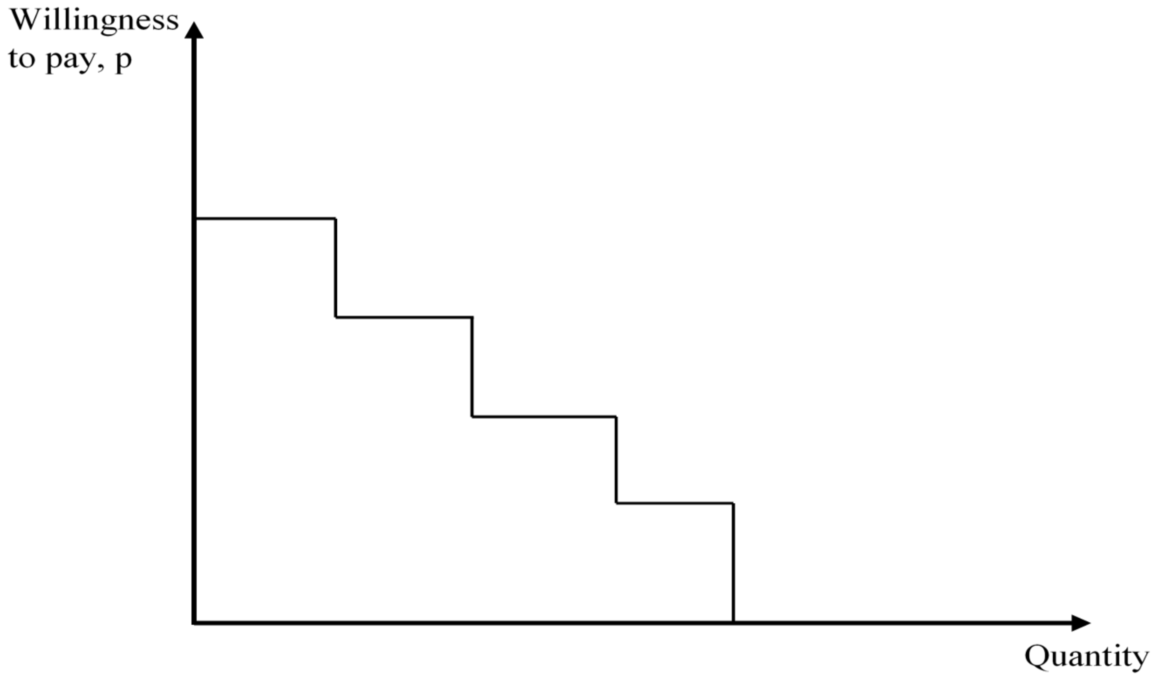
- Inverse demand
  - $p_w = D^{-1}(Q_w)$

- The demand function tells you how much will an individual purchase at any given price, given all other factors are held constant

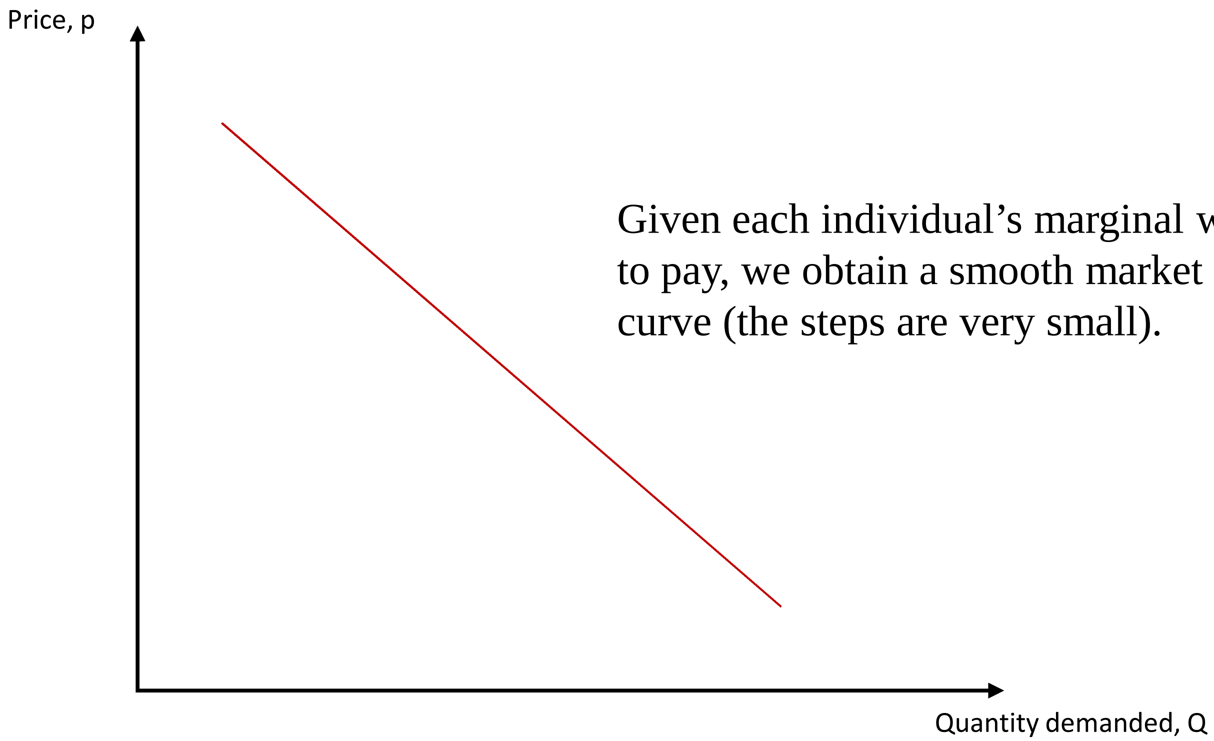
Deriving the demand curve - Marginal willingness to pay

- Ask the question, how much would an individual be willing to pay to purchase 1 baguette?
  - Then carry on asking… How much would the consumer be willing to pay to consume a second unit?
    - The change in the amount the consumer is willing to pay for 2 units, relative to 1 unit, is the marginal willingness to pay of the consumer.
- Given the price of substitutes/complements, income, preferences, tastes etc. we can derive a curve showing the quantity of a good demanded, at any given price.

Deriving the demand curve

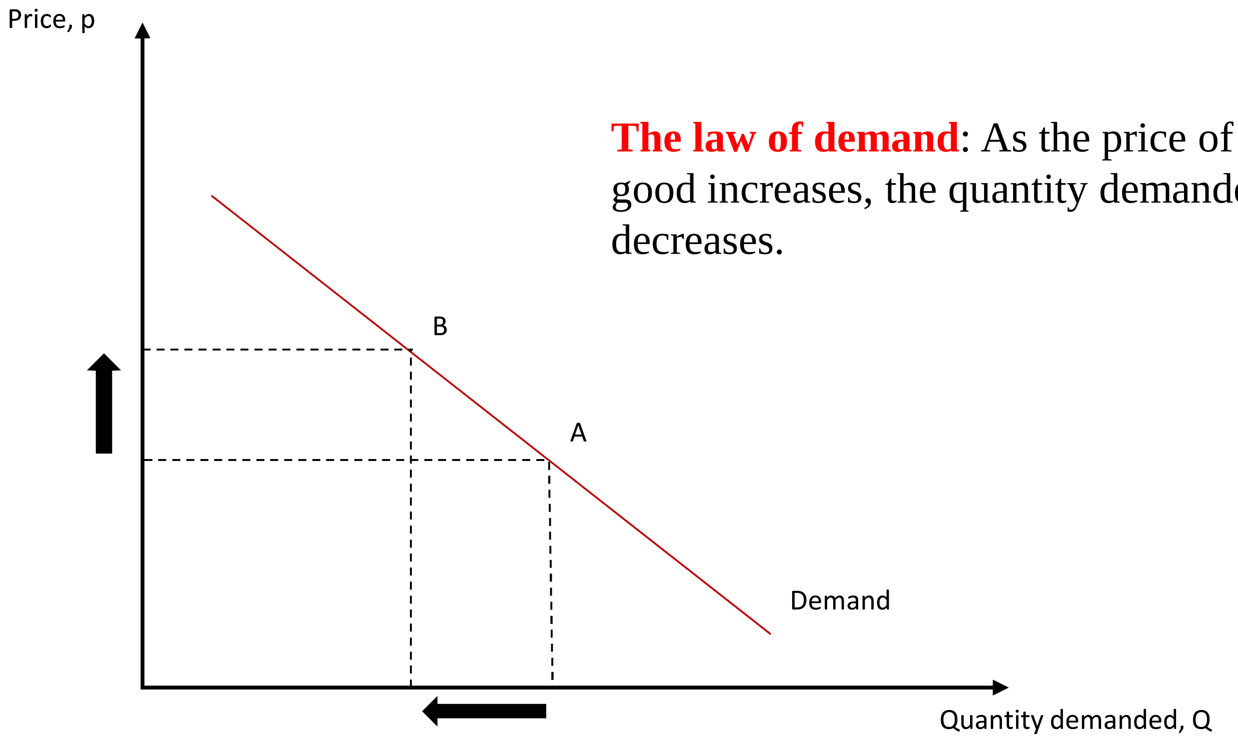


The demand curve



Given each individual's marginal willingness to pay, we obtain a smooth market demand curve (the steps are very small).

The demand curve and the law of demand



The law of demand: As the price of a good increases, the quantity demanded decreases.

An introduction to Income and substitution effects

- Holding everything else constant, the effect of a price change on the consumer's choice of consumption bundles can be divided into two components:
  - Income effect: Is the fraction of the total decrease in quantity demanded coming from the decrease in real income. The price of the good has increased, given a constant level of income, the consumer has less real income.
  - Substitution effect: Is the fraction of the total decrease in quantity demanded due to the substitution to consuming other goods instead, in order to keep the consumer as happy as before. It relates to the change in relative prices.
- The sum of these two effects is the total effect of a price increase. We will be looking at this in more details later (derivation and graphical interpretation).

Different types of goods

- Good can be said to be Normal or Inferior. To understand this, Let's look at the concept of income elasticity.
- It tell us how quantity demanded responds to changes in income.

$$\xi = \frac{\Delta Q^d / Q^d}{\Delta Y / Y} = \frac{\Delta Q}{\Delta Y}\frac{Y}{Q} = \frac{dQ}{dY}\frac{Y}{Q}$$

As income increases by 1%, quantity demanded increases by ξ%.

– A good is said to be normal, if $\xi > 0$

– A good is said to be inferior, if $\xi < 0$

Income/substitution effect for inferior goods

- When we discussed substitution/income effect earlier, we assumed the good to be normal. What if the good is inferior?
- Income effect
  - It will have the opposite sign! From the definition earlier, it is the decrease in quantity demanded from a decrease in real income. However, this isn't true for inferior good- As income decreases, quantity demanded will increase.
- Substitution effect
  - The effect will be of the same sign.

Factors affecting demand

- What factors affect quantity demanded?
  - The price
  - The price of substitutes/complements
  - Government regulation
  - Income
  - Tastes and preferences
- How would a change in any of those factors alter the demand curve?

Price elasticity of demand

- How much does quantity fall, when the price of a product increases?
  - Why do we not just look at the slope of the demand curve?
    - If we were to graph the same demand curve, but with different units on the horizontal axis, we would see a change in the responsiveness. However we are looking at the same product.
    - The slope is constant along the demand curve. Why does that matter?

Price elasticity of demand

- We therefore need a measure that does not depend on how price and quantity are measured.
  - Ratio of the *percentage* changes:

$$\varepsilon = \frac{\Delta Q^d / Q^d}{\Delta P / P} = \frac{\Delta Q}{\Delta P}\frac{P}{Q} = \frac{dQ}{dP}\frac{P}{Q}$$

Here units do not matter. If you say you have put on weight, by 15%, whether you are talking about kg or stones does not matter.

  - Provides a good way to compare the effect of a price change between commodities.

Price elasticity of demand

- For a linear demand curve we can derive:

$$Q = a - bp$$
$$\varepsilon = -b\frac{p}{Q}$$

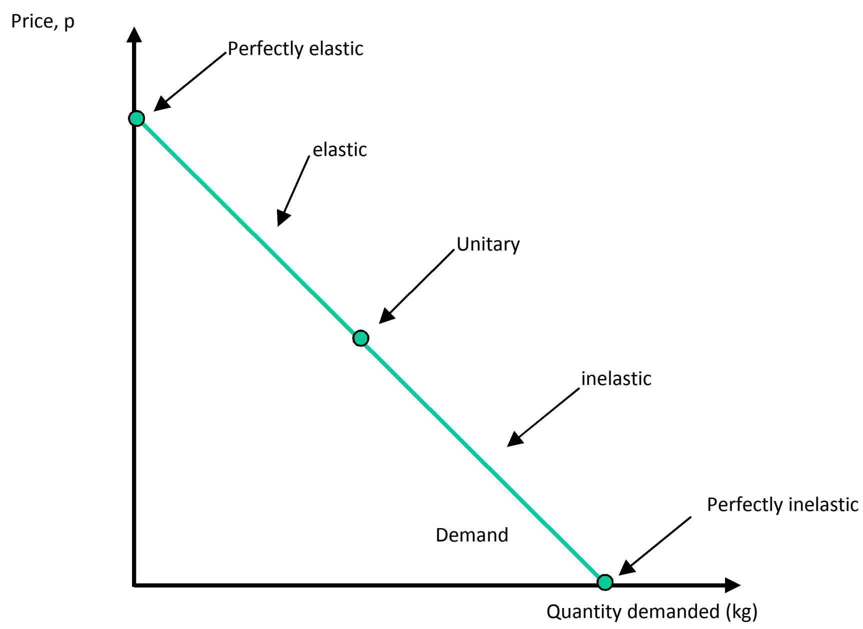
If $\varepsilon < -1$, price elasticity of demand is said to be elastic

If $\varepsilon = -1$, price elasticity of demand is said to be unitary

If $\varepsilon > -1$, price elasticity of demand is said to be inelastic

- The demand curve slopes downwards- Price elasticity will have a negative sign, which can seem counter-intuitive.

Price elasticity of demand- Linear demand curve

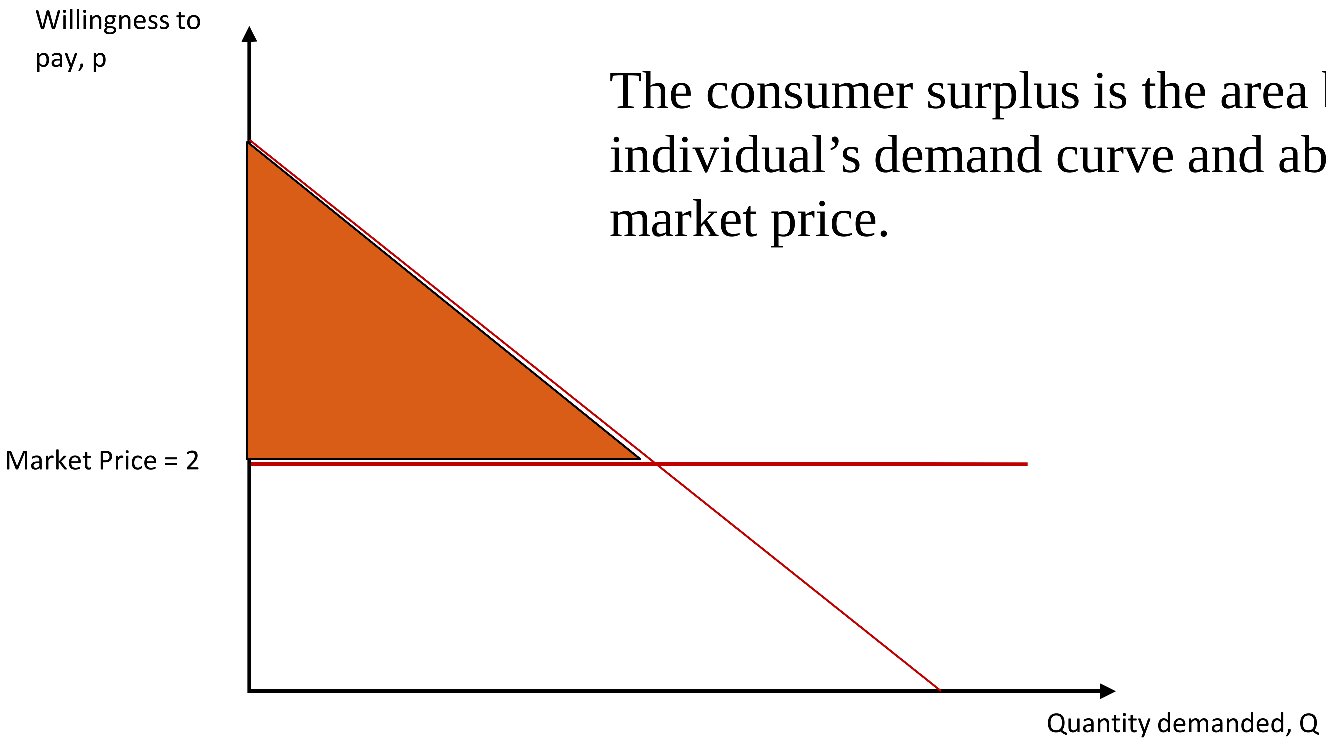


Consumer's welfare

- How can you measure how much *better off* a consumer is from consuming an extra unit of a good?
  - We need to decide how to measure welfare.

- Recall earlier the concept of the willingness to pay
  - Consider first the amount one is willing to pay for an amount x of a good.
  - Now consider how much the individual actually pays.
    - I am willing to pay quite a high amount for chocolate, say £5. A bar of chocolate doesn't cost £5. The difference between the price I am willing to pay, and the price I actually pay, is called the "consumer surplus", it is a measure of how better off I am.

Consumer's welfare



The consumer surplus is the area below the individual's demand curve and above the market price.

Supply

Demand-Supply model

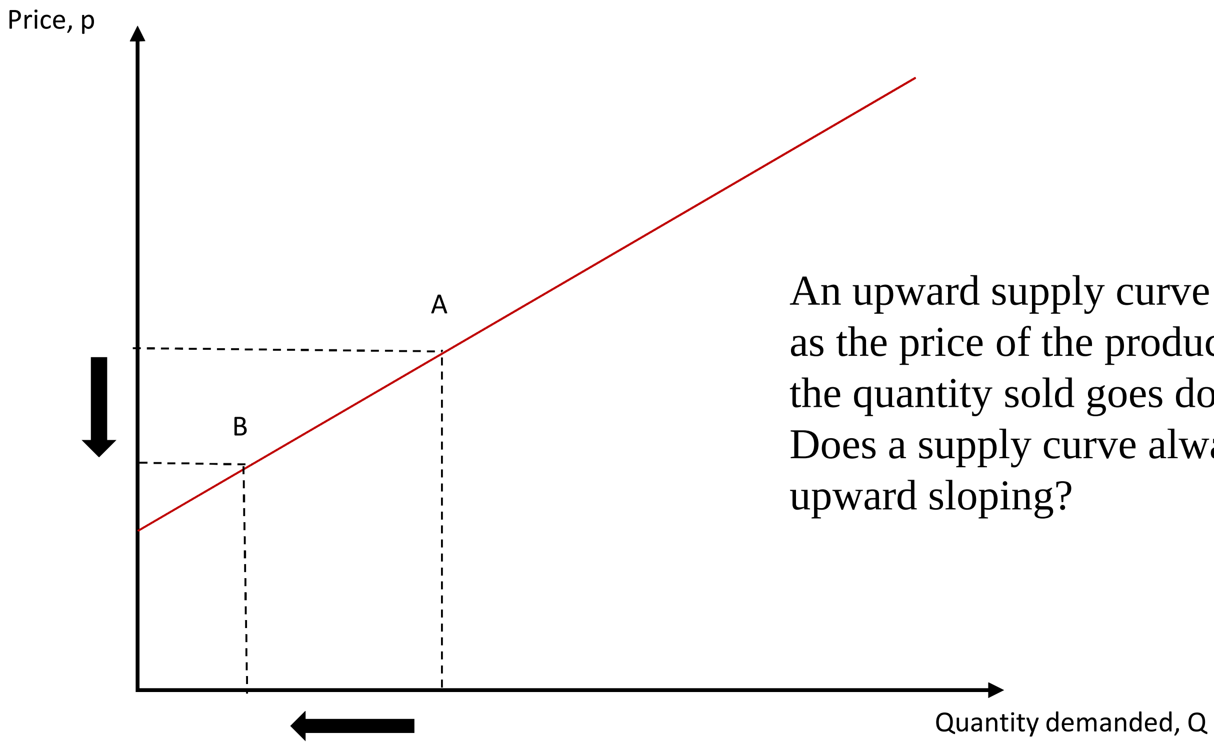
- Supply function for washing machines

$$Q_w^s = S(p_w)$$

  - Where: Q_w^s , quantity of washing machines supplied; p_w, price of washing machines.

- The supply function tells you how much a firm is willing to supply at any given price, ceteris paribus.
- We will be deriving the supply curve a bit later in the course.

The supply curve- Is there a law of supply?



An upward supply curve implies that as the price of the product goes down, the quantity sold goes down.
Does a supply curve always have to be upward sloping?

Factors affecting the supply curve

- What factors affect quantity supplied?
  - The price of inputs
  - Government regulations
  - Expectation of price changes
  - Weather
  - Profitability of goods in joint supply
  - Increase in supply of related goods

- How would a change in any of those factors alter the supply curve?

Price elasticity of supply

- As before, we look at a measure which does not relate to measures of price or quantity.
  - Ratio of the *percentage* changes:

$$\eta = \frac{\Delta Q^s / Q^s}{\Delta P / P} = \frac{\Delta Q^s}{\Delta P} \frac{P}{Q^s}$$

And again here, units do not matter.

  - If the price increases by 1%, quantity supplied will increase by η%

Price elasticity of supply

- For a linear demand curve we can derive:

$$Q = a + bp$$

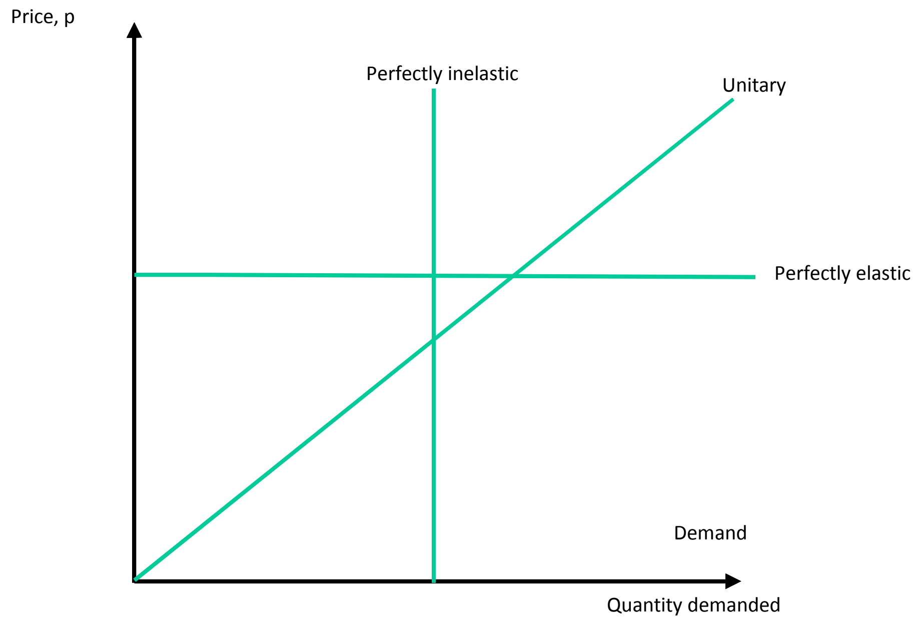
$$\eta = b\frac{p}{Q}$$

If $\varepsilon < 1$, price elasticity of supply is said to be <u>inelastic</u>

If $\varepsilon = 1$, price elasticity of supply is said to be <u>unitary</u>

If $\varepsilon > 1$, price elasticity of supply is said to be <u>elastic</u>

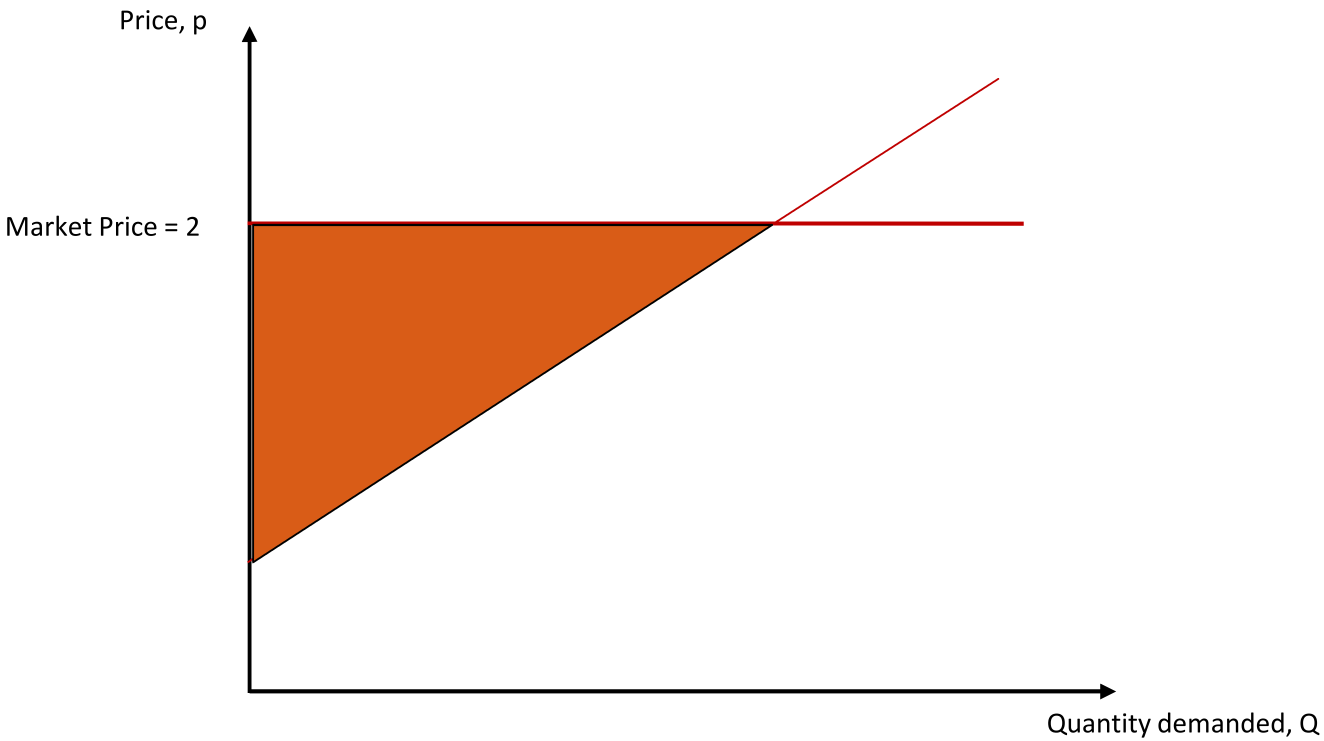
Price elasticity of supply



Producer's welfare

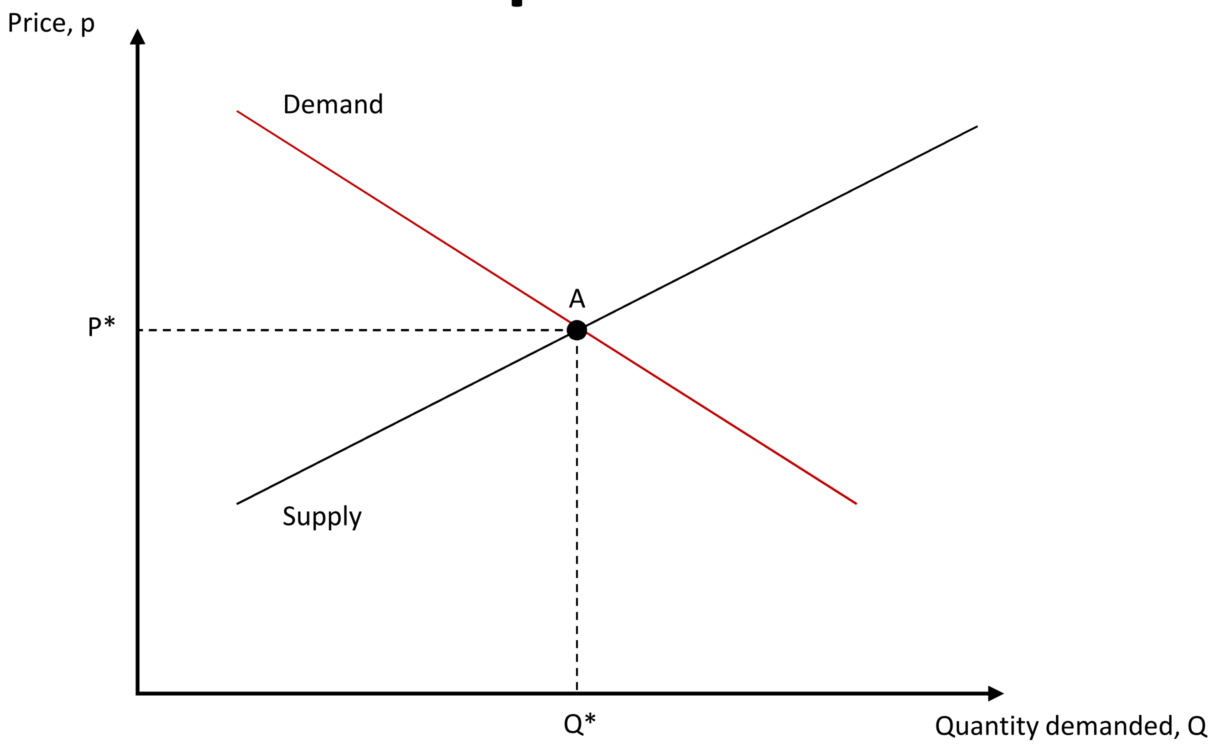
- How can you measure how much *better off* a producer is from producing an extra unit of a good?
  - Again, we are going to use a measure in terms of money.
- Here, we use the concept of marginal cost
  - When deciding how much to supply (in the short run), a firm looks at how much more it will cost to produce an extra unit of output. This change in cost from producing an extra unit is called the marginal cost.
  - The difference between the price at which the producer sells its product, and the cost of production, is the producer surplus.

Producer’s welfare



Equilibrium

Equilibrium



Equilibrium (continued)

- In equilibrium, there are no incentives to deviate. Both are demanding and supplying the amount they are willing to at that price.

$$Q^d = Q^s = Q^*$$
$$P = P^*$$

- What happens if the market price differs from P^*?

Equilibrium (continued)

- Assume demand and supply for wine are given by

$$Q^d = 20 - 2P$$
$$Q^s = 5 + 3P$$

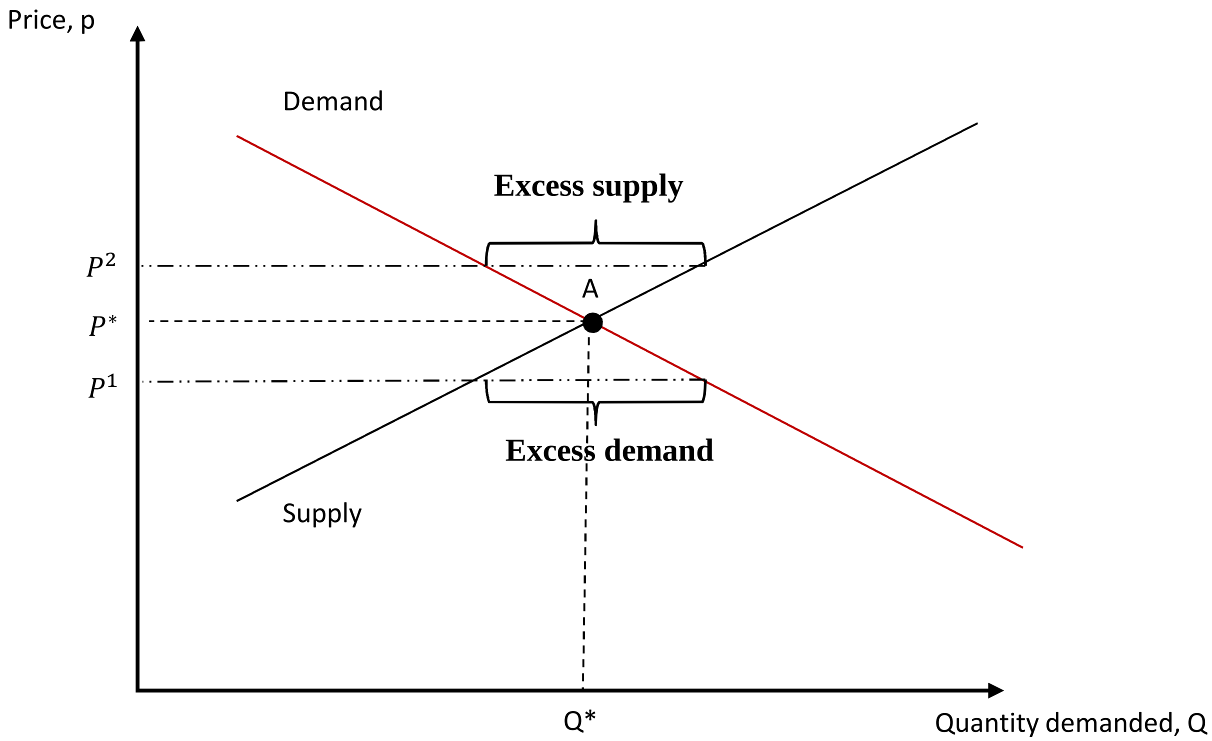
Setting $Q^s = Q^d$,

$$20 - 2P = 5 + 3P$$
$$25 = 5P$$
$$P^* = 5$$

To find the equilibrium quantity, plug P^* into Q^d (or Q^s):

$$Q^* = 20 - (2 * 5) = 10$$

Equilibrium



Shocks to demand/supply effects on equilibrium

- We have seen that the demand and supply curves can shift, in response to changes in different factors. If that happens, what happens to equilibrium?

  – Let's look at the market for baguettes. And how a change in the price of wheat affects the supply curve. How has the equilibrium changed?

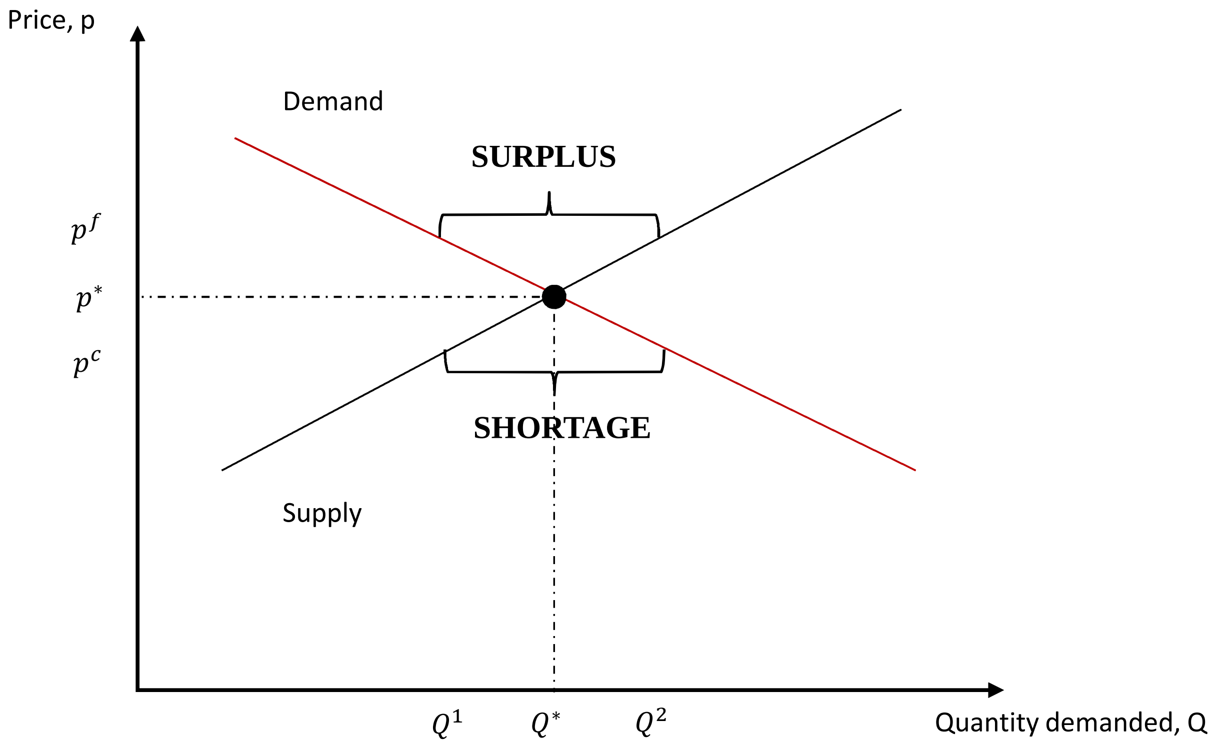
    - The cost of producing the baguettes have increased- at previous equilibrium price, the producer is not willing to supply as much. However at that price, demand hasn't changed. There is now **excess demand**!

Government intervention- Effects on equilibrium

- Surpluses/shortages can happen when the government puts a constraint on the pricing/quantity sold of some products.

- Suppose that the government puts a price floor on the price of wheat. What effect will that have on the quantity demanded and quantity supplied in equilibrium?

Government intervention- Effects on equilibrium



Microeconomics I – Term 1, Part 2

CONSUMER THEORY

Plan

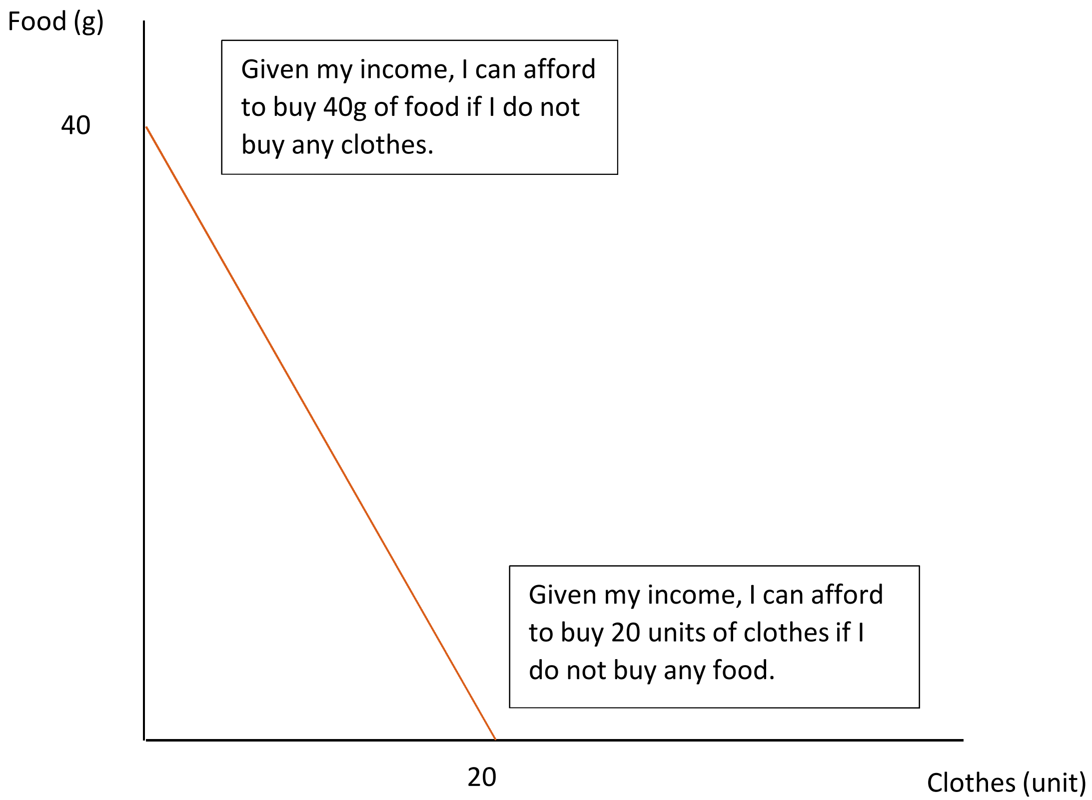
- Budget constraint
  - The feasible set
- Preferences
  - Properties
  - Indifference maps/curves and properties
  - Trade offs
- Optimal choice
- The utility function approach
  - Indifference curves
  - Marginal utility and optimization

Budget constraint I

- We saw earlier the factors affecting the quantity demanded by consumers.
- One of those was their income. I like to spend money on food and on clothes. Assume they cost £5/g/week and £10/unit/week respectively. Also assume that my weekly income is £200.

  - $I = F \times P^f$ if I decide to only consume food.
    - $200 = F \times 5 \rightarrow F = 40 = \frac{I}{Pf}$
  - $I = C \times P^c$ if I decide to only buy clothes.
    - $200 = C \times 10 \rightarrow C = 20 = \frac{I}{P^c}$

Budget constraint II



Budget Constraint III

- If instead, I want to balance my consumption between the two, it must be true that I do not spend more than my income:

$$I = F \times P^f + C \times P^c$$

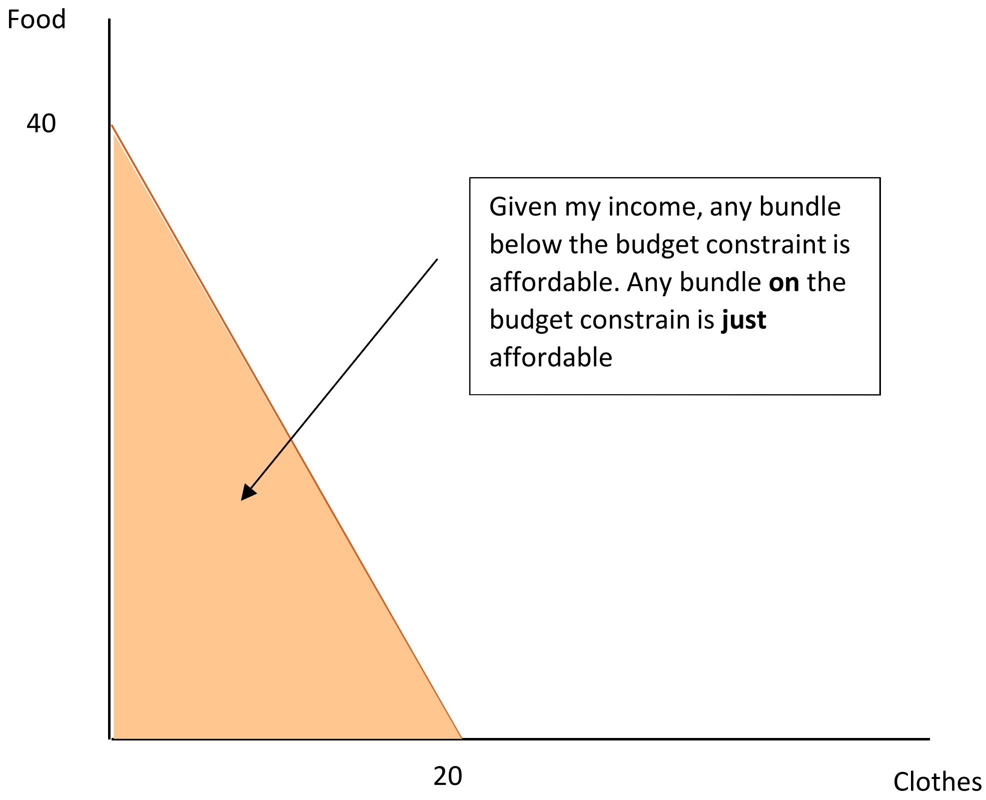
- We know the respective prices, so we can re-write the budget constraint such as:

$$I = 5F + 10C$$

- We have drawn the budget constraint with food on the vertical axis. Rewrite the budget constraint such as:

$$F = \frac{I}{5} - \frac{10}{5}C$$

The feasible set



Given my income, any bundle below the budget constraint is affordable. Any bundle **on** the budget constrain is **just** affordable

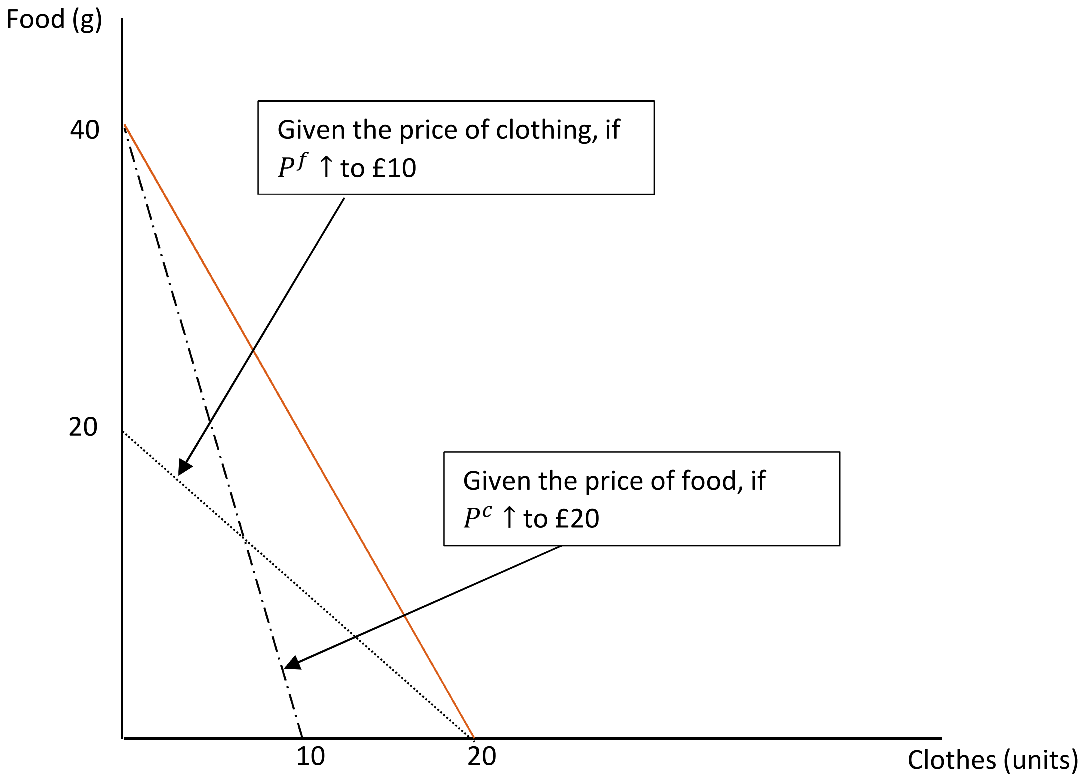
Budget constraint- shifts and movement

- We saw earlier that the slope of the budget constraint depends only on relative prices:

$$Slope = -\frac{P^c}{P^f}$$

- A change in the price of one of the good will lead to a rotation of the budget constraint
- A change in the income of the consumer will lead to a shift of the budget constraint
- What if both prices change by the same proportion?

The effect of a change in the price of one good



Preferences

- Preferences will tell us more about a consumer's choice.
- Preference ordering
  - For any choice between two bundles of goods, let's say X: (15,10) and Y: (12,12), the consumer can say whether she prefers X to Y, Y to X, or is indifferent between the two.
  - Not a quantitative statement!
- While *preferences* will differ between individuals, there are some properties we assume to be shared by all regarding *preference ordering*. These are useful to enable us to link those preferences to the constraint we have seen above.

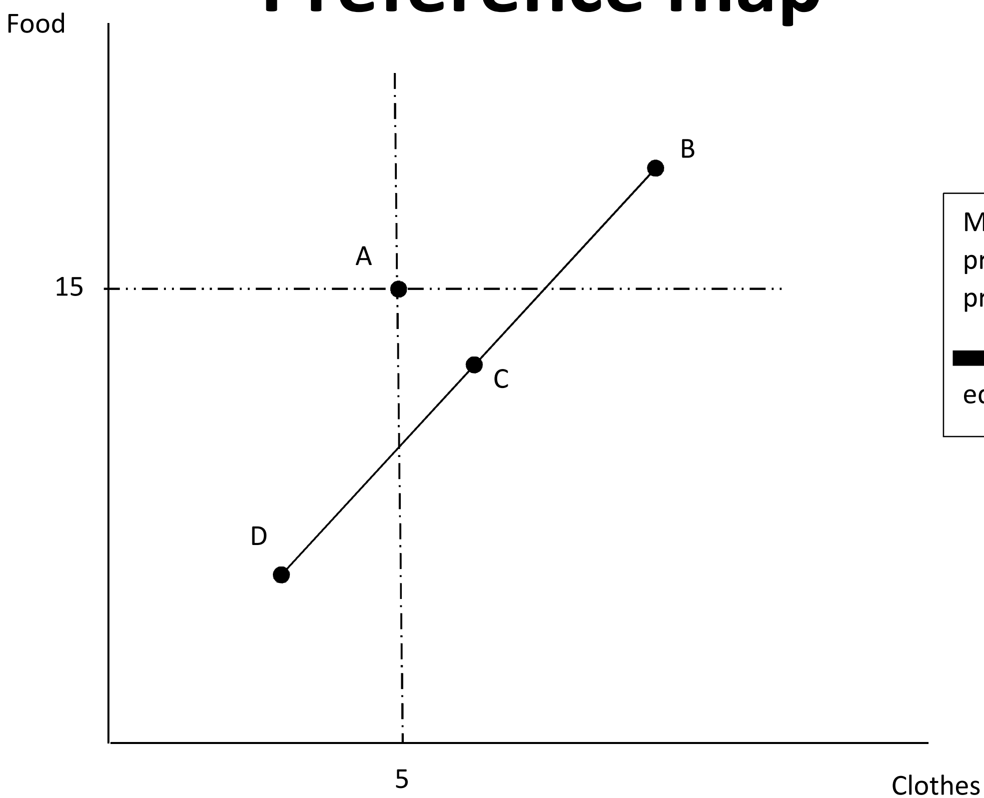
Properties of preference ordering

- Completeness
  - The consumer can always rank between a choice of bundles. Either the consumer prefers X to Y, Y to X, or is indifferent between the two.
- Transitivity
  - This assumption helps with the consistency of preference ordering. Assume there is a third bundle of goods Z, If X is preferred to Y, and Y is preferred to Z, it must be true that X is preferred to Z.
- More is better
  - Suppose two bundles, X and Y such that X: (10,15) and Y: (12,15). It must be true that B is preferred to A.

Mapping preferences

- We said earlier than we needed information about preferences to know which bundle the consumer would choose from his affordable set.
- Given the consumer’s income, the goods the consumer was choosing from and their relative prices, we obtained the budget constraint. Let’s map preferences in a similar way.
- For now, let’s ignore the constraint we talked about before, that is let’s ignore income and prices: We are going to map all possible bundles, assuming the properties above hold.

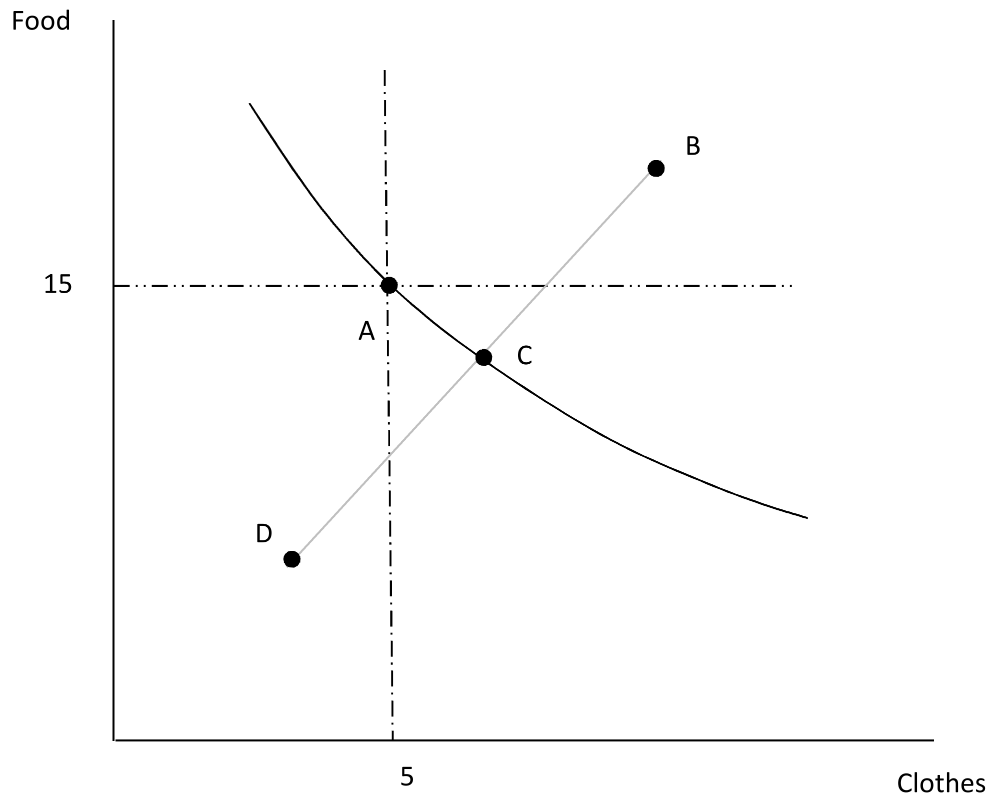
Preference map



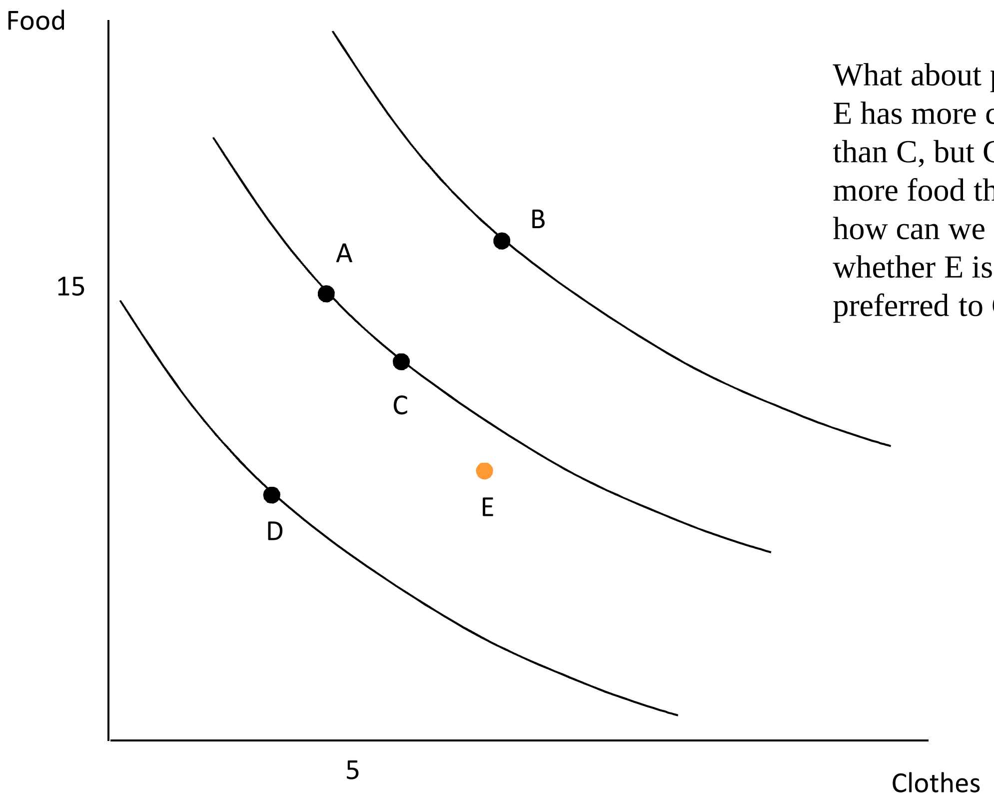
More is better tells us that B is preferred to A, and A is preferred to D.

➡ There must be a point C equally likeable to A on [DB]

Indifference curve



Indifference map



What about point E? E has more clothes than C, but C has more food than E, how can we tell whether E is preferred to C?

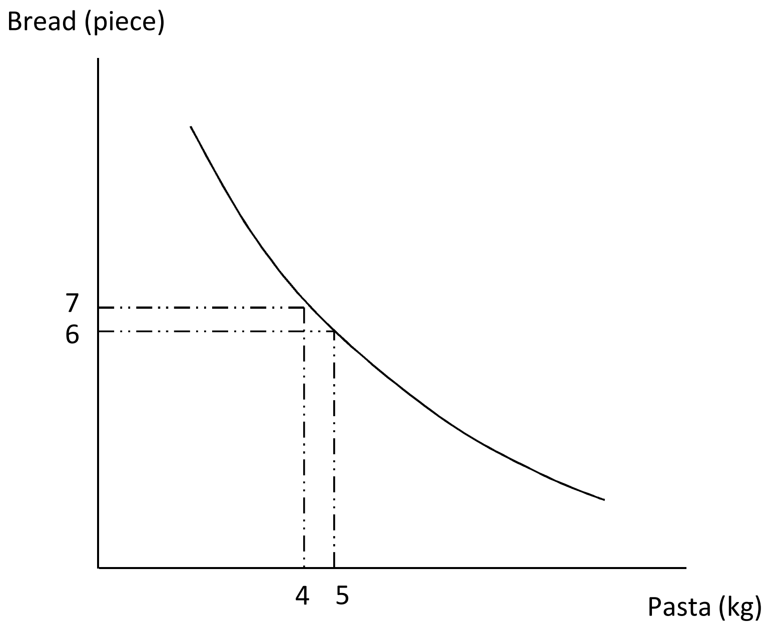
Properties of indifference curves

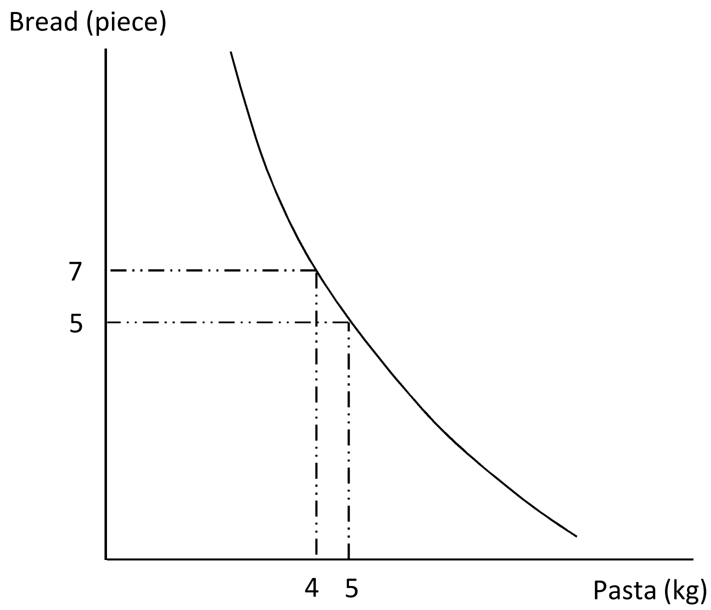
- Bundles on indifference curves further from the origin are preferred to ones on indifference curves closer to the origin
  - Follows from the more is better assumption and transitivity.
- Indifference curves are downward sloping
  - Follows from the more is better
- Indifference curves cannot cross
  - Follows from more is better and transitivity assumptions
- Indifference curves are continuous
  - Follows from the completeness assumption

Trade offs

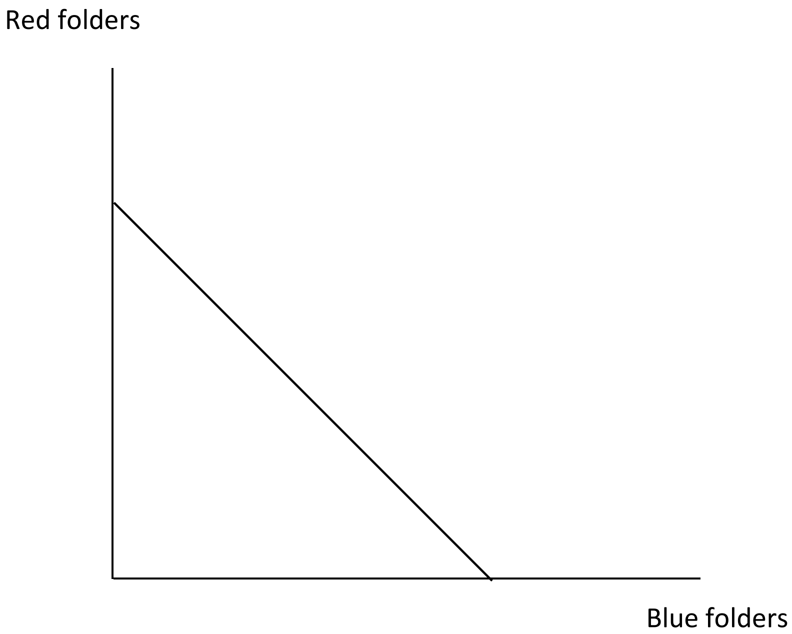
- We know that one is indifferent between two bundles on the same indifference curve. The rate at which a consumer is willing to substitute one good for another is called the **Marginal rate of substitution.**
  - MRS $= \left|\frac{\Delta F}{\Delta C}\right| =$ Absolute value of the slope of the indifference curve (for small changes).
  - We have seen that ICs are downward sloping- MRS is be a negative number and will be diminishing as one moves to the right of the indifference curve.
- Think about the shape of the indifference curve
  - People have different preferences

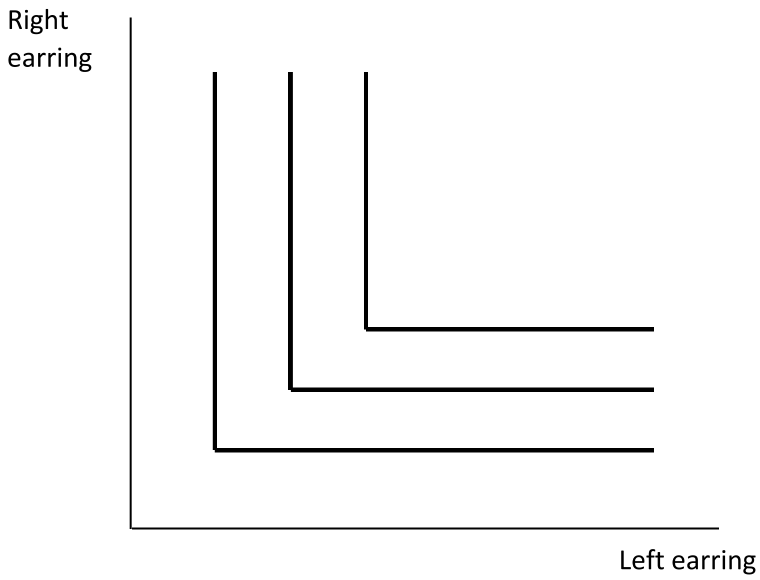
Different preferences





Perfect complements/Perfect substitutes





The most preferred <u>and</u> affordable bundle

- The budget constraint tells us what is affordable, indifference curve give us information about the utility from choosing a particular bundle.

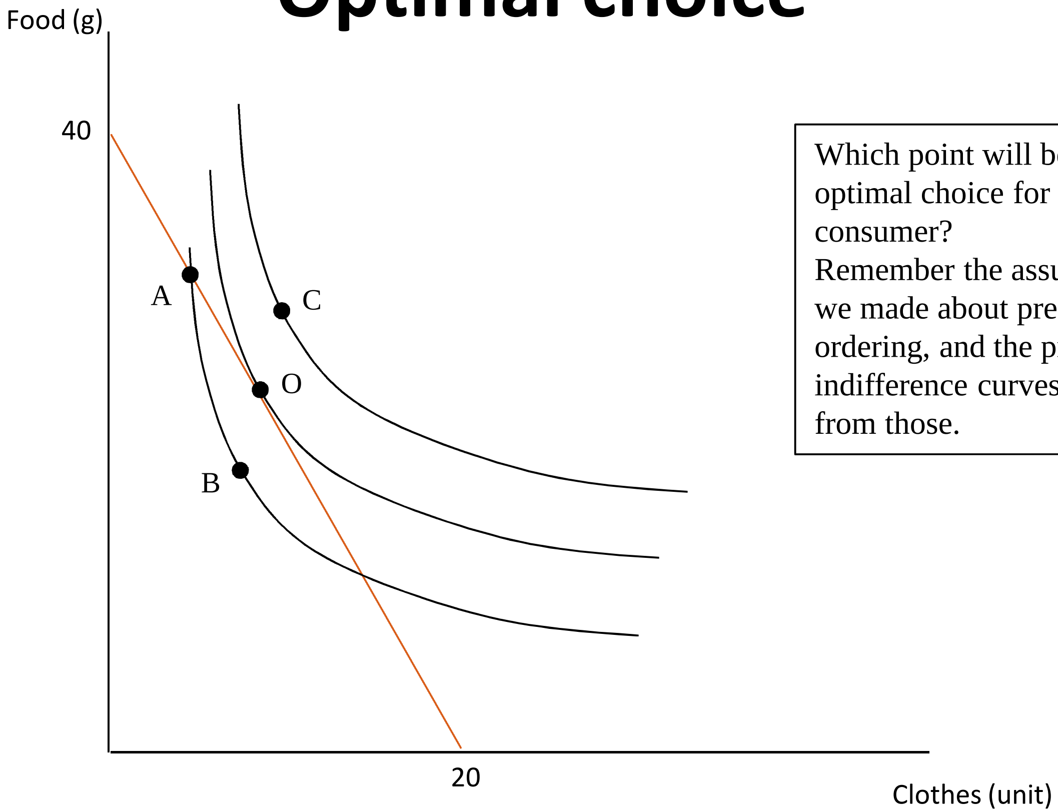
- For the choice to be optimal, it will have to give the consumer the highest possible utility, given the constraint he faces; his income.

Linking MRS to the slope of budget constraint

- Slope of the budget constraint: For a given income (expenditure), how much of one good does one have to give up to purchase an extra unit of the other good
- Slope of the indifference curve: How much of one good is one willing to give up to obtain an extra unit of the other good, to be as well as off as before.

- **The slope of the budget constraint give us the marginal cost of clothes in terms of food. The slope of the indifference curve give us the marginal benefit of clothes in terms of food.**

Optimal choice



Which point will be the optimal choice for the consumer?
Remember the assumptions we made about preference ordering, and the properties of indifference curves that follow from those.

Optimal choice

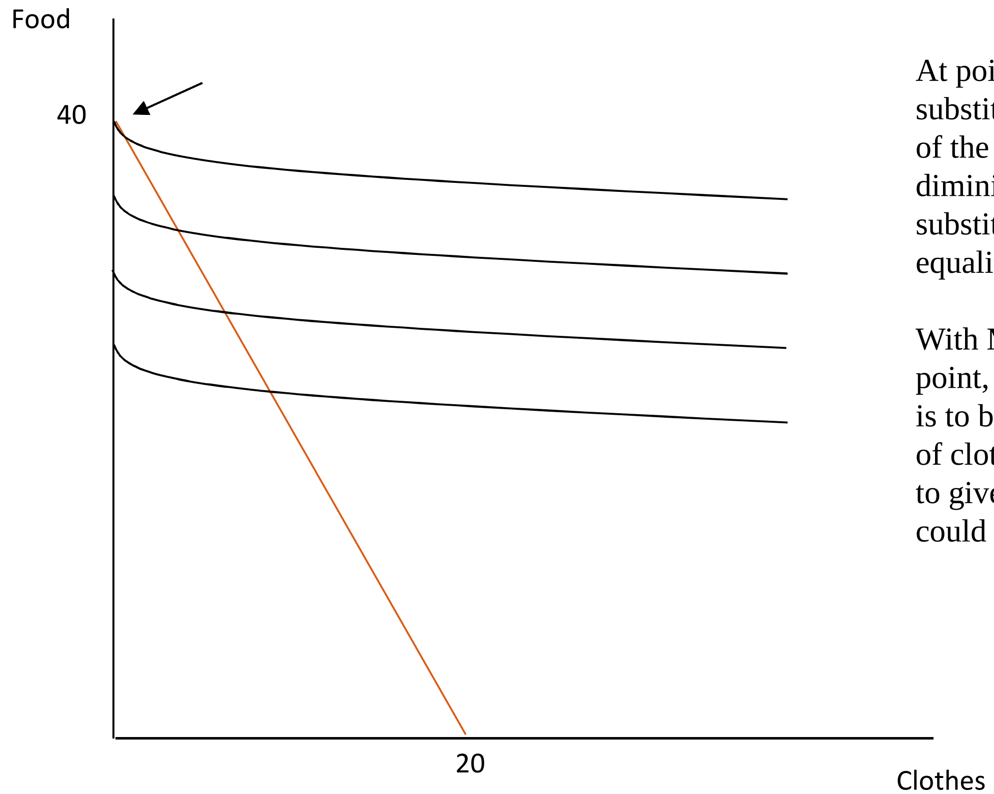
- The optimal choice for the consumer is the tangency point between the budget constraint and an indifference curve.
  - At this point, it will be true that the opportunity cost of clothes, in terms of food (the slope of the budget constraint) will equal the benefit of buying clothes rather than food (the slope of the Indifference curve).

$$MRS = \left| -\frac{P^c}{P^f} \right| = \frac{P^c}{P^f} = \left| \frac{\Delta F}{\Delta C} \right|$$

If $MRS < \frac{P^c}{P^f}$, the consumer gets less food (1) for giving up a unit of clothing than the amount of food the consumer could buy by not buying any clothes (2)

➡ The consumer will clearly be better off buying more food, and less clothes.

Corner solution



At point A the marginal rate of substitution is lower than the slope of the budget constraint, with diminishing marginal rate of substitution, the two never reach equality.

With MRS < slope of BC at every point, the best the consumer can do is to buy food only: For an extra unit of clothing, the consumer will have to give up more food than what he could buy, by not buying clothes.

The utility function approach

- The indifference map approach is not the only way to represent consumer preferences- We can also use a utility function.
- Using a utility function enables us to assign a number to the Utility derived from each bundle
  - With the indifference map, we knew which bundle was preferred if it was on a higher indifference curve, a relative ranking of bundles. A utility function approach enables us to assign a number to the utility from each bundle.
  - It is still true that all we know is the relative ranking of bundles.

The utility function approach (continued)

- In our Food/Clothes choice for the consumer, suppose that the utility function takes the following form:

$$U(F, C) = FC$$

  - F is g of food a week, and clothes the number of items.

- Using this utility function we can compare two bundles X and Y, where X has, say, 7g of food and 9 units of clothing, and Y has 8g of food and 8 units of clothing.

$$U(7,9) = 7 \times 9 = 63$$
$$U(8,8) = 8 \times 8 = 64$$

$\longrightarrow$ Y is preferred to X

Indifference curves II

- We have seen that indifference curves show all bundles equally preferable by the consumer.

- The utility function enables us to assign a number of utility units obtained from a consumption bundle. For the consumer to be indifferent between two different bundles, they have to provide him with the same utility
  - Above, bundle Y provided the consumer with 64 utility units; Any bundle providing the consumer with 64 utility units will be equally preferable to him

Indifference curve III

- Obtain an indifference curve by finding all bundles providing the consumer with a particular level of utility
  - $U(F,C) = F \times C$ If we want to obtain an indifference curve going through Y, we need all bundles of Food and clothing for which the consumer obtains 64 units of utility.

$$64 = F \times C$$
$$F = \frac{C}{64}$$

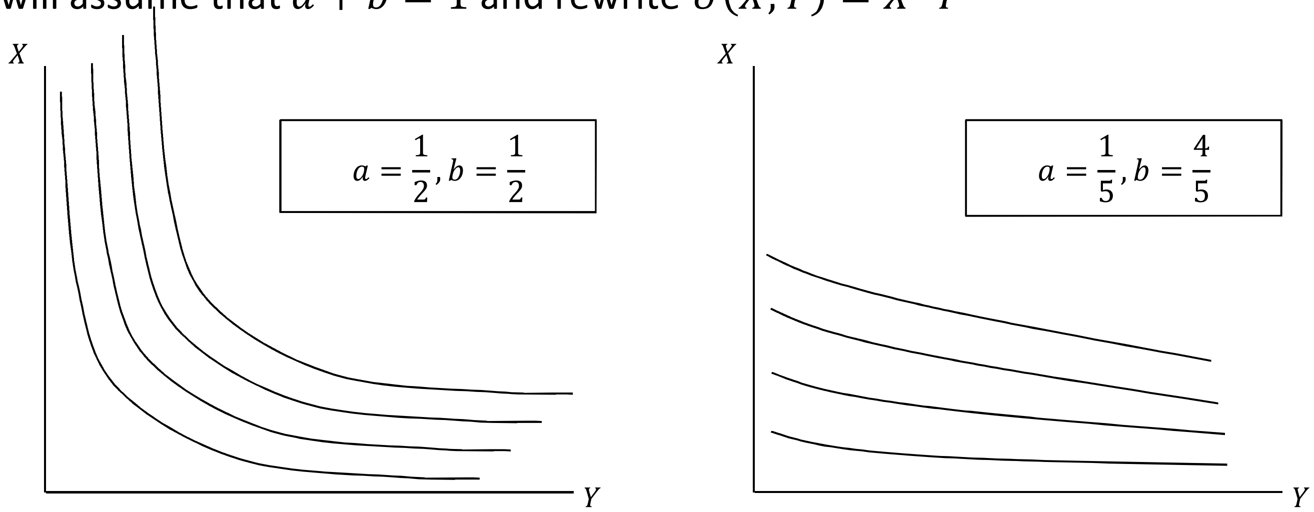
  - Some bundles satisfying this condition are (64,1), (1,64), (8,8), (32,2), (2,32)
- One can repeat this for different levels of utility and obtain a set of different indifference curves

Some utility functions examples

- Perfect substitutes
  - Remember the highlighter example above. All the consumer cares about is the overall number of folders, not how many red/blue folders there are in the mix. Considering the one-to-one relationship from above, the utility function would take the form $U(F^r, F^b) = F^r + F^b$
  - In general, the utility function will take the form $U(F^r, F^b) = aF^r + bF^b$
- Perfect complements
  - We looked above at the right earring/left earring where the relationship is one to one. The right earring will not provide any utility without a left earring. Considering this one-to-one relationship we obtain the following utility function $U(E^r, E^f) = \min\{E^r, E^f\}$.
  - In general (not a one to one relationship), we would have $U(E^r, E^f) = \min\{aE^r, bE^f\}$

Some utility functions examples

- Cobb-Douglas preferences
  - Very commonly used (also for production functions). Takes the general following form: $U(X,Y) = X^a Y^b$, where a and b represent the consumers preferences. We will assume that $a + b = 1$ and rewrite $U(X,Y) = X^a Y^{1-a}$



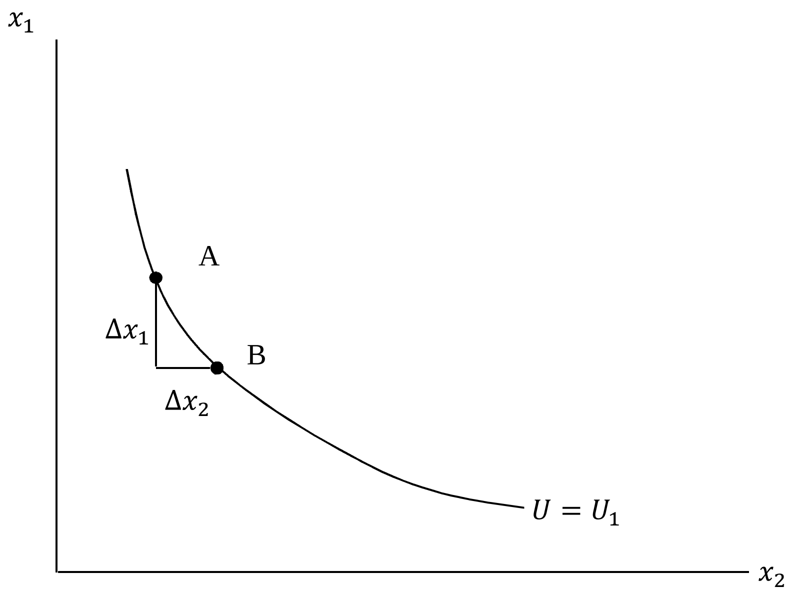
Marginal utility

- Remember the optimality condition before; The optimal choice for the consumer happened where the slope of the budget constraint was tangent to an indifference curve.

$$\frac{P^c}{P^f} = \left|\frac{\Delta F}{\Delta C}\right|$$

- Suppose a consumer consumes a bundle of goods $x_1$and x_2; (x_1, x_2)
  - We want to know how the consumer's utility change as you give the consumer a *little* more of x_1, or a *little* more of x_2, that is, the **marginal utility** with respect to good 1

Marginal utility



From point A to point B, the consumer loses Δx_1 and gains Δx_2. We also know that both A and B give the consumer the same utility U_1.

➡ Marginal utility lost from less x_1 must be offset by marginal utility gained from more x_2

Optimization and marginal utility

- $MU_1 = \frac{\Delta U}{\Delta x_1} = \frac{u(x_1+\Delta x_1, x_2)}{\Delta x_1} \longrightarrow \Delta U = MU_1 \times \Delta x_1$
- $MU_2 = \frac{\Delta U}{\Delta x_2} = \frac{u(x_1, x_2+\Delta x_2)}{\Delta x_2} \longrightarrow \Delta U = MU_2 \times \Delta x_2$

$$MU_1 \times \Delta x_1 = MU_2 \times \Delta x_2$$

This tells us that the marginal utility lost from consuming less x_1 must be offset by marginal utility gained from consuming more x_2.

Optimization and marginal utility

- $MU_1 \times \Delta x_1 = MU_2 \times \Delta x_2 \longrightarrow \frac{MU_1}{MU_2} = \frac{\Delta x_2}{\Delta x_1}$
  - We know that, for small enough Δx_1 and Δx_2, their ratio is the slope of the indifference curve at that bundle.
- Following the same logic as earlier; the optimal choice for the consumer is to choose a bundle at the tangency point between an indifference curve and the budget constraint we can rewrite the above as:

$$\frac{MU_1}{MU_2} = \frac{P_{x_1}}{P_{x_2}} \longrightarrow \frac{MU_1}{P_{x_1}} = \frac{MU_2}{P_{x_2}}$$

Microeconomics I – Term 1, Part 2

CONSUMER THEORY (II)

Laura Sochat

09/12/2015

Plan

- Solving for the optimal bundle
  - Langrangian method
  - An alternative method
- Using rational choice theory to derive a demand curve: A change in price
- Using rational choice theory to see the effect of a change in Income on consumption
- Income and substitution effects revisited

Maximising utility with algebra

- We want to maximise utility subject to the budget constraint

$$Maximise\ U(X,Y)\ subject\ to\ I = P_x X + P_Y Y$$

Lagrangian method

$$\max_{X,Y,\lambda} L = U(X,Y) - \lambda(P_x X + P_Y Y - \mathrm{I})$$

1. Obtain the F.O.C.s

$$\left.\begin{aligned} \frac{dL}{dX} &= 0 \\ \frac{dL}{dY} &= 0 \\ \frac{dL}{d\lambda} &= 0 \end{aligned}\right\}$$

Solve the system to find the values of X and Y, for which the Lagrangian is maximised, and the constraint binds.

Maximising utility with algebra

1. Obtain the F.O.C.s

$$\frac{dL}{dX} = 0$$
$$\frac{dL}{dY} = 0$$
$$\frac{dL}{d\lambda} = 0$$

$$\longrightarrow$$

$$\frac{dL}{dX} = \frac{dU}{dX} - \lambda P_x = 0 \qquad (1)$$
$$\frac{dL}{dY} = \frac{dU}{dY} - \lambda P_Y = 0 \qquad (2)$$
$$\frac{dL}{d\lambda} = I - P_x X - P_Y Y = 0 \qquad (3)$$

2. Solve for X, Y and λ from (1), (2) and (3)

$$\frac{(1)}{(2)} = \frac{\frac{dU}{dX} - \lambda P_x}{\frac{dU}{dY} - \lambda P_Y} = 0 \longrightarrow \frac{\frac{dU}{dX}}{\frac{dU}{dY}} = \frac{P_x}{P_Y}$$

Maximising utility with algebra- Some examples

- Suppose that $I = £120$, P_x=£6 and $P_Y = £3$, also assume that the consumer's preferences for goods X and Y, are defined by the following utility function:

  $a)\ \ U(X, Y) = XY$

  $b)\ \ U(X, Y) = X^{3/4}Y^{1/4}$

  Solve for the optimal bundle of X and Y

- An alternative method could be to substitute for X, or Y, within the utility function and solve for the first order condition:

$$\max_{X,Y} U(X, Y) subject\ to\ I = P_x X + P_Y Y$$

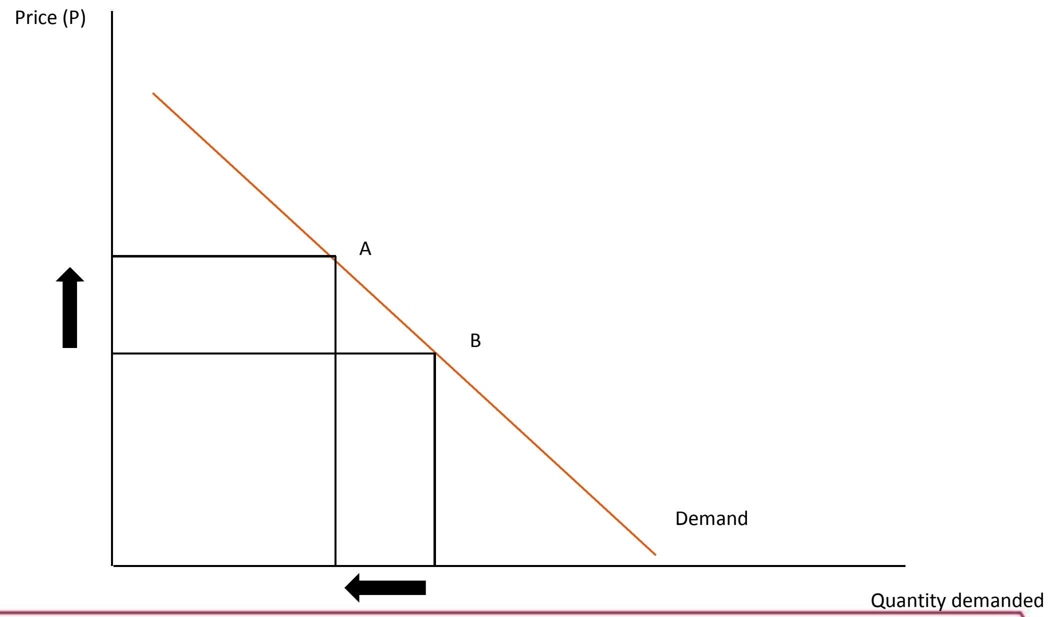
  $a)\ \ U(X, Y) = X^{1/3}Y^{2/3}$, also assume that $I = £24$, $P_x = £4$, and $P_Y = £2$

Given the values of Px, Py, and I, It is possible to solve for the optimal bundle.

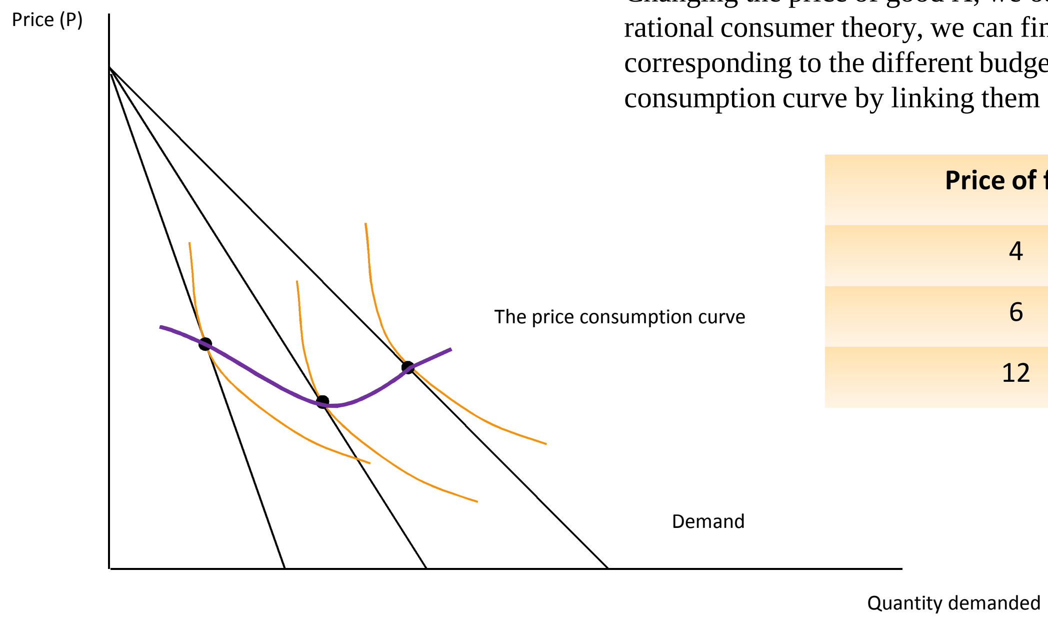
Note: With this alternative method, it may be useful to use monotonic transformation of the utility function

Using the rational choice model to derive individual demand: A change in the price of one good.

- Remember the demand curve we have seen before, giving us relationship between the price of a good and the quantity demanded of that good.



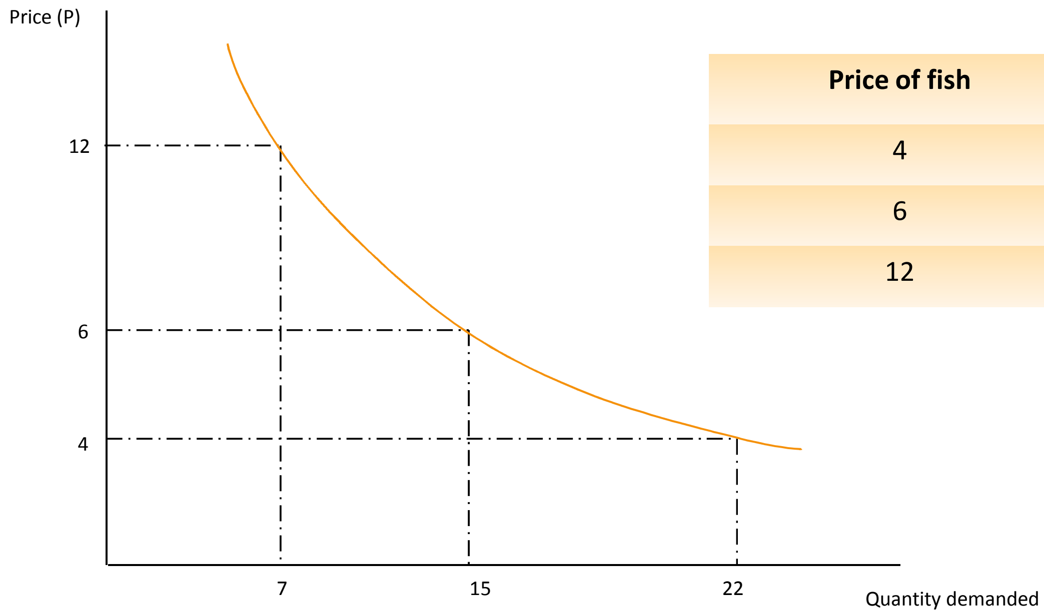
Using the rational choice model to derive individual demand: A change in the price of one good.



Changing the price of good X, we obtain different budget lines-Using rational consumer theory, we can find the optimal bundles corresponding to the different budget line and obtain the price-consumption curve by linking them

Price of fish	Quantity demanded
4	22
6	15
12	7

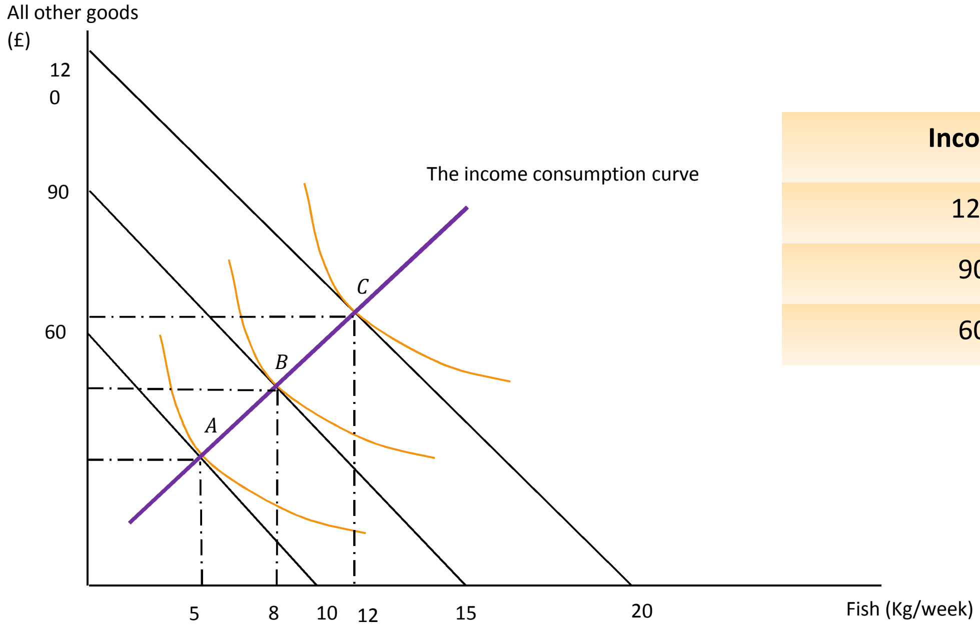
Using the rational choice model to derive individual demand: A change in the price of one good.



Price of fish	Quantity demanded
4	22
6	15
12	7

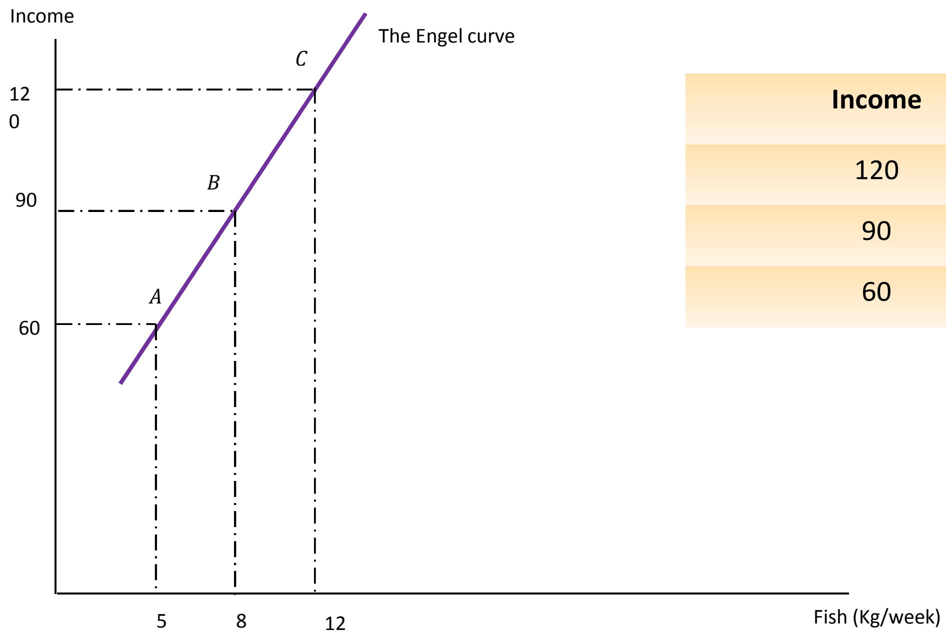
A change in Income: The Income-Consumption curve and the Engel curve

- Recall the effect of a change in income on the budget constraint: It leads to a shift in the budget constraint, and therefore to an increase in the feasible set.



Income	Quantity demanded
120	12
90	8
60	5

A change in Income: The Income-Consumption curve and the Engel curve



Income	Quantity demanded
120	12
90	8
60	5

Different types of goods

- Remember the concept of Income elasticity, telling us how quantity demanded responds to a change in income. It is given by:

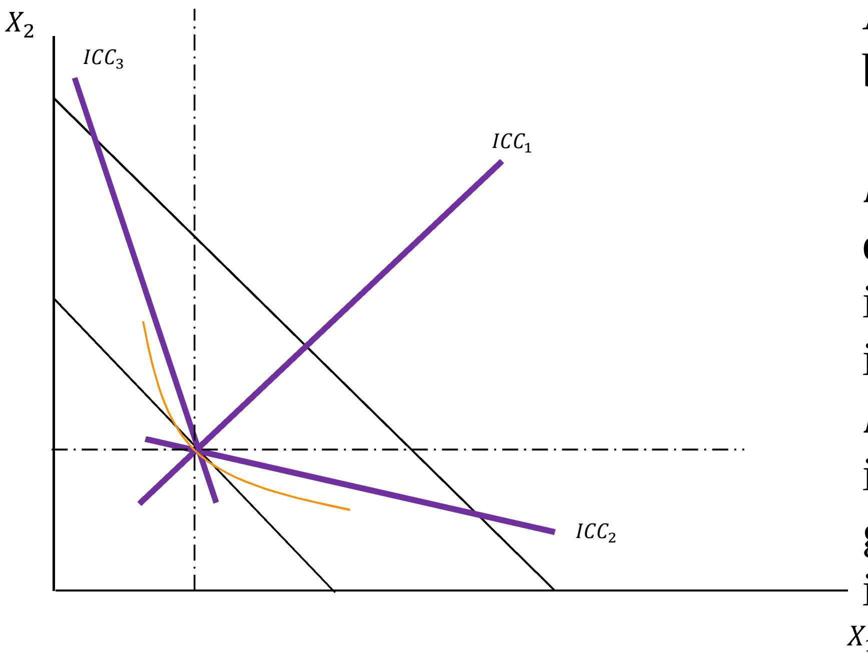
$$\xi = \frac{\Delta Q^d / Q^d}{\Delta Y / Y} = \frac{\Delta Q^d}{\Delta Y}\frac{\mathrm{Y}}{Q^d} = \frac{dY}{dQ^d}\frac{Q^d}{Y}$$

  – As income increases by 1%, quantity demanded increases by ξ%.

A good is said to be normal, if ξ>0, the quantity demanded of a normal good increases (decreases) as income increases decreases)

A good is said to be inferior, if ξ<0, the quantity demanded of an inferior good decreases (increases) as income increases (decreases)

Income elasticities and Income consumption curves



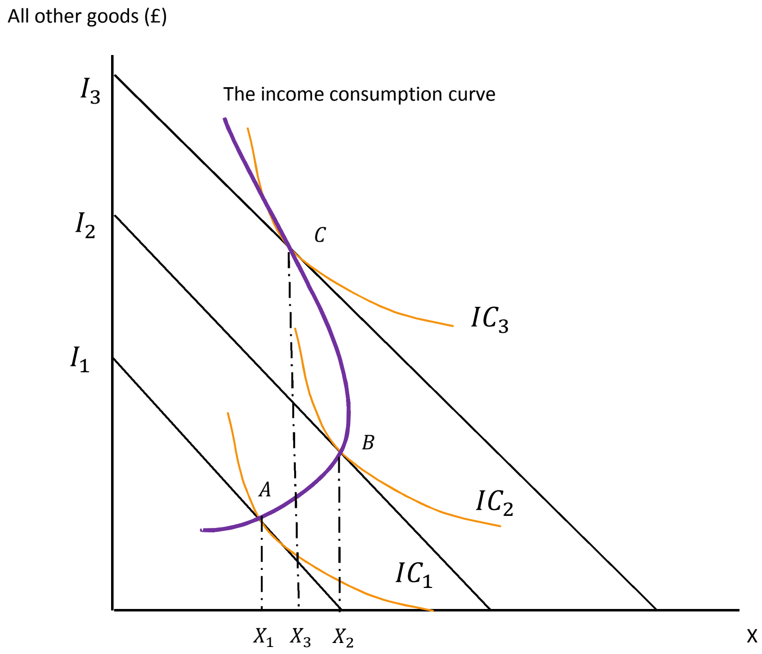
Assume income increases; The budget constraint shifts to the right.

ICC_1 : Both goods are normal, quantity demanded of both goods has increased following the increase in income

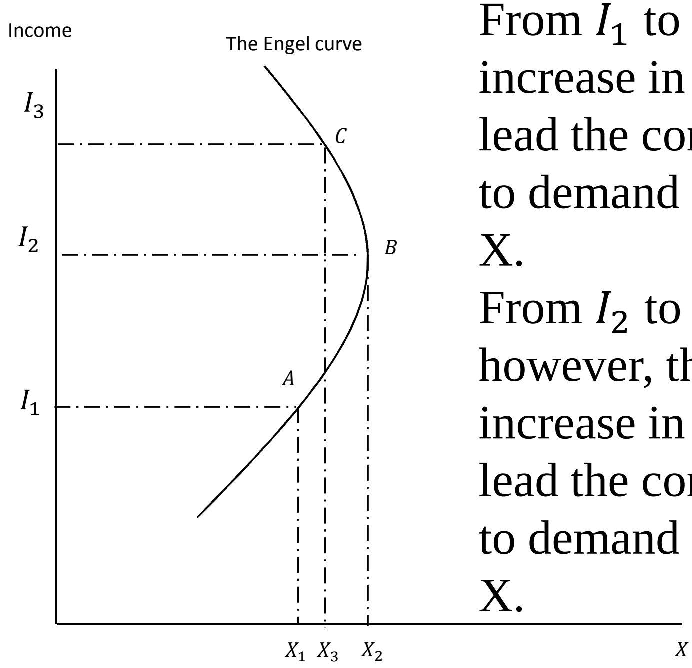
ICC_2 : X_1 is a normal good, while X_2 is inferior. Quantity demanded of good 2 has fallen following the increase in income

ICC_3 : Good 2 in normal, while good 1 is inferior.

The Engel curve when one of the good is both normal and inferior



The income consumption curve



The Engel curve

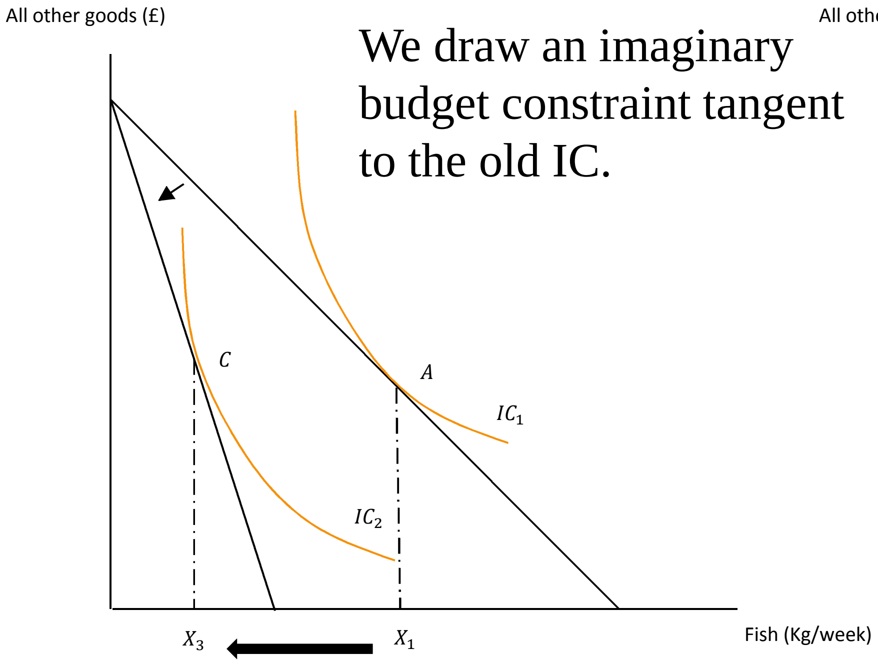
From I_1 to I_2, the increase in income lead the consumer to demand more of X.
From I_2 to I_3, however, the increase in income lead the consumer to demand less of X.

The income and substitution effects

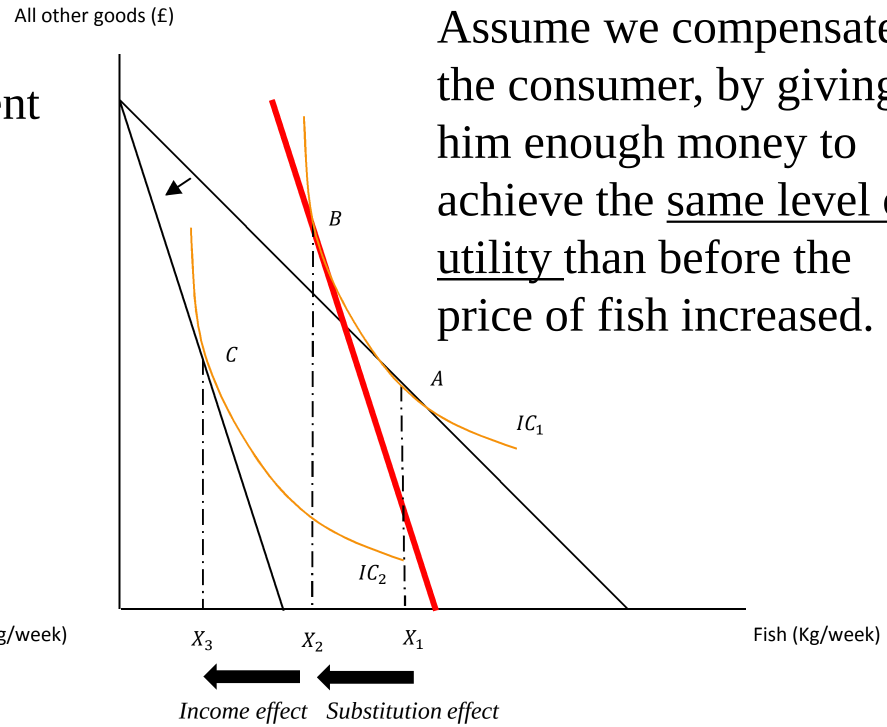
- From the law of demand, we know that an increase (decrease) in the price a good leads to an decrease (increase) in the quantity demanded of that good. We can divide the total effect of a price change into two effects:

- The *substitution effect* refers to the change in the relative price of the good. As the price of a good rises (falls), other goods become relatively cheaper (more expensive), making them more (less) attractive to the consumer.
- The *income effect* refers to the change in real income from a rise (fall) in the price of one good. The consumer is now poorer (richer), leading to a change in quantity demanded.

The income and substitution effects (Hicks) : A normal good



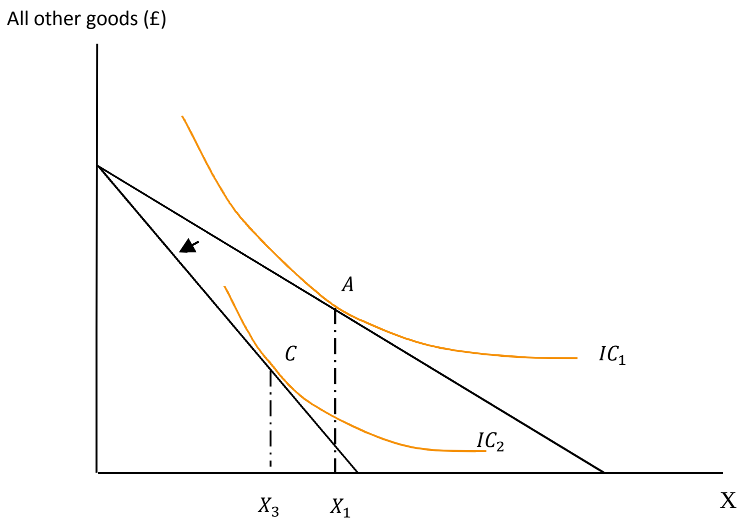
We draw an imaginary budget constraint tangent to the old IC.

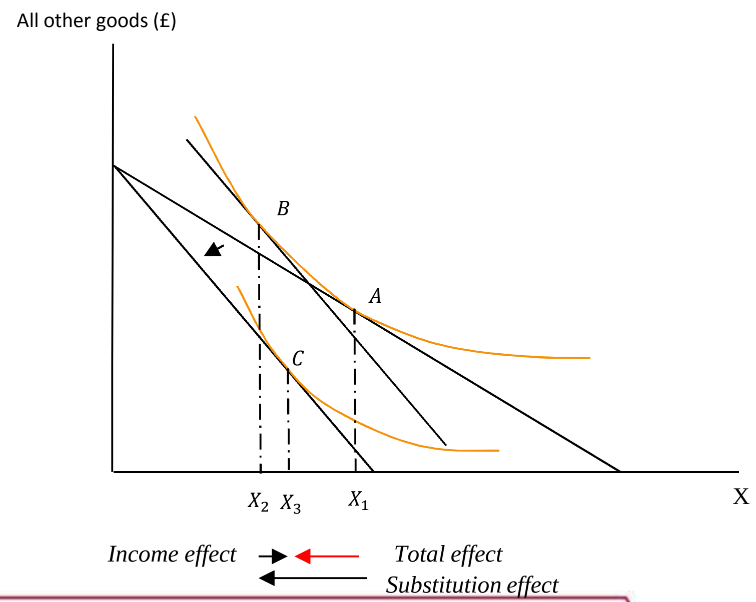


Assume we compensate the consumer, by giving him enough money to achieve the same level of utility than before the price of fish increased.

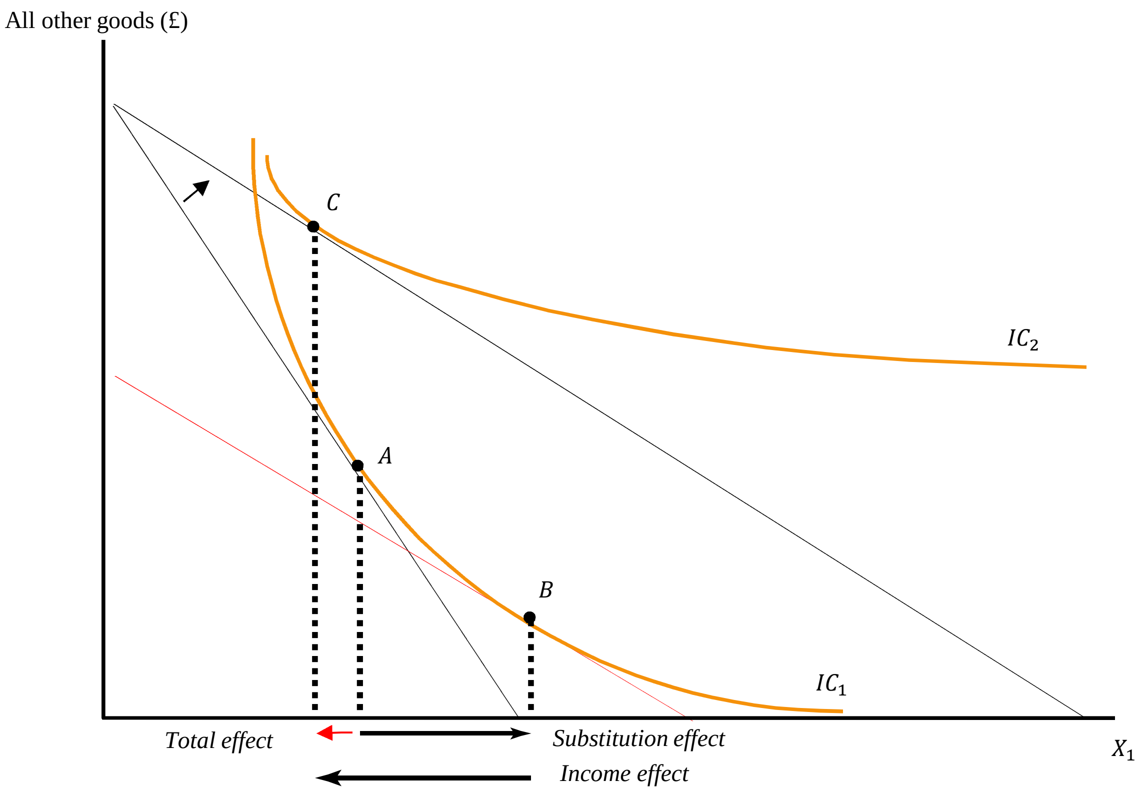
The income and substitution effects (Hicks) : An inferior good

- The income elasticity of an inferior good being negative, the income effect from a price increase will be positive, while the substitution effect is still negative.





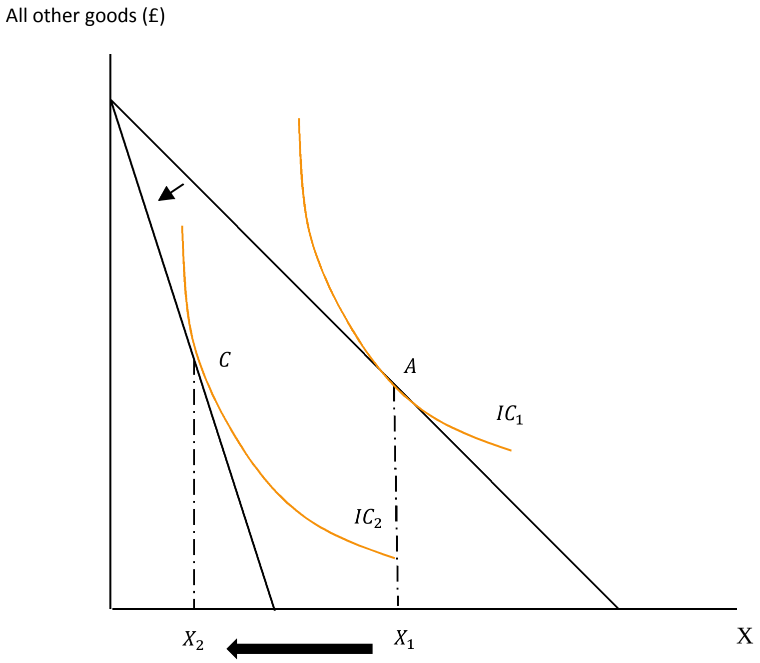
The income and substitution effect (Hicks) : A giffen good

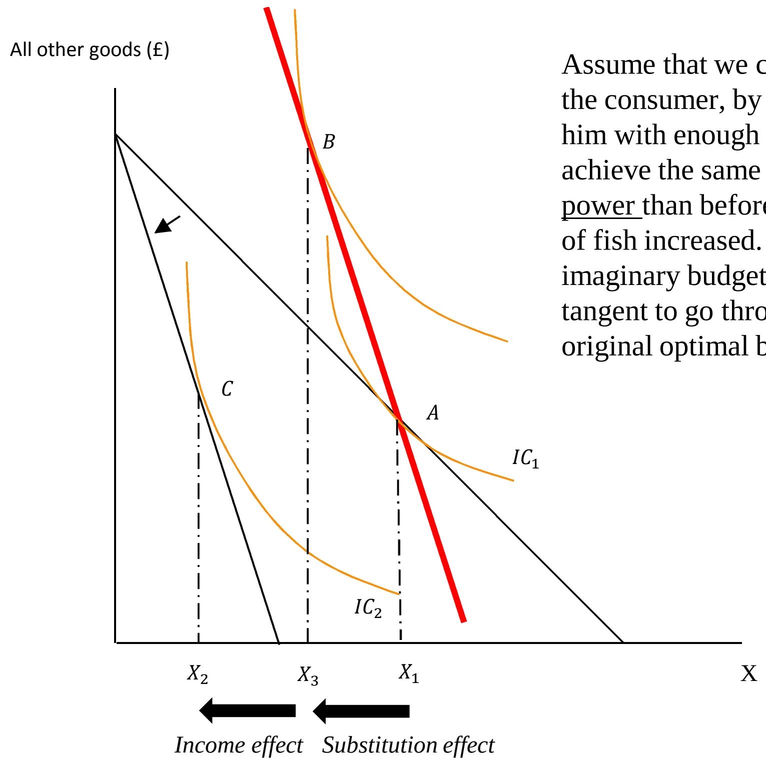


Suppose the price of X_1 falls, leading to a new (rotated) BL. X_1 is an inferior good: substitution effect leads to the consumer consuming more of good 1, while the income effect will lead the consumer to consume less of the good.

In this situation, the substitution effect is completely offset by the income effect.

The income and substitution effects (Slutsky) : A normal good





Assume that we compensate the consumer, by providing him with enough money to achieve the same purchasing power than before the price of fish increased. We draw an imaginary budget constraint tangent to go through the original optimal bundle.

An algebraic interpretation: The Slutsky equation

- We know that the observed effect is the sum of the income and substitution effect.

$$Observed\ response = subsitution\ effect + Income\ effect$$

Denote the change in quantity demanded ΔQ and the change in price as ΔP, we can denote the observed effect as $\frac{\Delta Q}{\Delta P}$

- The (Slutsky) substitution effect was derived earlier by compensating the consumer with enough income to keep the same purchasing power as before the change in price.

$$Substitution\ effect = \left(\frac{\Delta Q}{\Delta P}\right)_{comp}$$

An algebraic interpretation: The Slutsky equation

- The income effect will be the product of two terms:

  - The amount of money income (amount of pounds) lost from the increase in price:

Suppose that Marie consumes 3kg of sugar, and assume that the price of sugar goes up by 1£. She is therefore 3£ worse off, following the price increase. More generally, If the consumer consumes an amount Q_1 of a good, and its price increases by 1£, the consumer is - Q_1 worse off.

  - The change in quantity demanded of the good per unit of change in money income (per pound), denoted as $\frac{\Delta Q}{\Delta I}$

The income effect is the product of those two terms:

$$Income\ effect = -Q_1 \frac{\Delta Q}{\Delta I}$$

An algebraic interpretation: The Slutsky equation

- Putting the two together, we obtain the slutsky equation:

$$\frac{\Delta Q}{\Delta P} = \left(\frac{\Delta Q}{\Delta P}\right)_{comp} - Q_1 \frac{\Delta Q}{\Delta I}$$

- The substitution effect from a price increase will always be negative (as long as the diminishing marginal rate of substitution assumption holds).
- The sign of the income effect will depend on whether the good is normal or inferior.
- The magnitude of the income effect will depend on the size of Q_1

Application: Labor-Leisure choice

- Consider a consumer choosing how to spend his time. He has a choice between working, and consuming leisure (N).

$$H = 24 - N$$

- The consumer spends his total income on a variety of goods (a composite good) which costs £1 per unit.
  - How many goods the consumer buys depends on how much he earns; so does the cost the leisure. When the consumer isn't working, he is losing earnings.

- The consumer's utility depends on how many goods he buys, and how many hours he spends not working (consuming leisure)

$$U = U(Y, N)$$

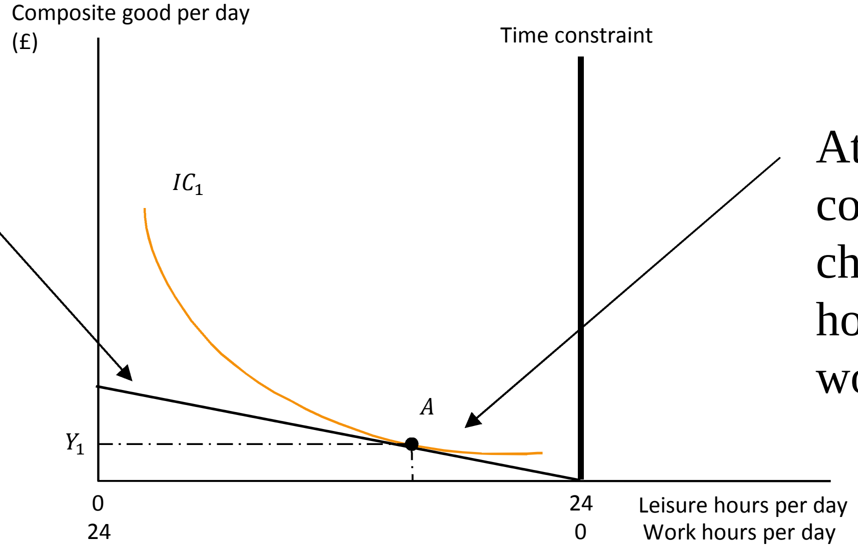
- The consumer's total income is given by :

$$Y = wH = w(24 - N)$$

Where w represents hourly wage

Application: Labor-Leisure choice

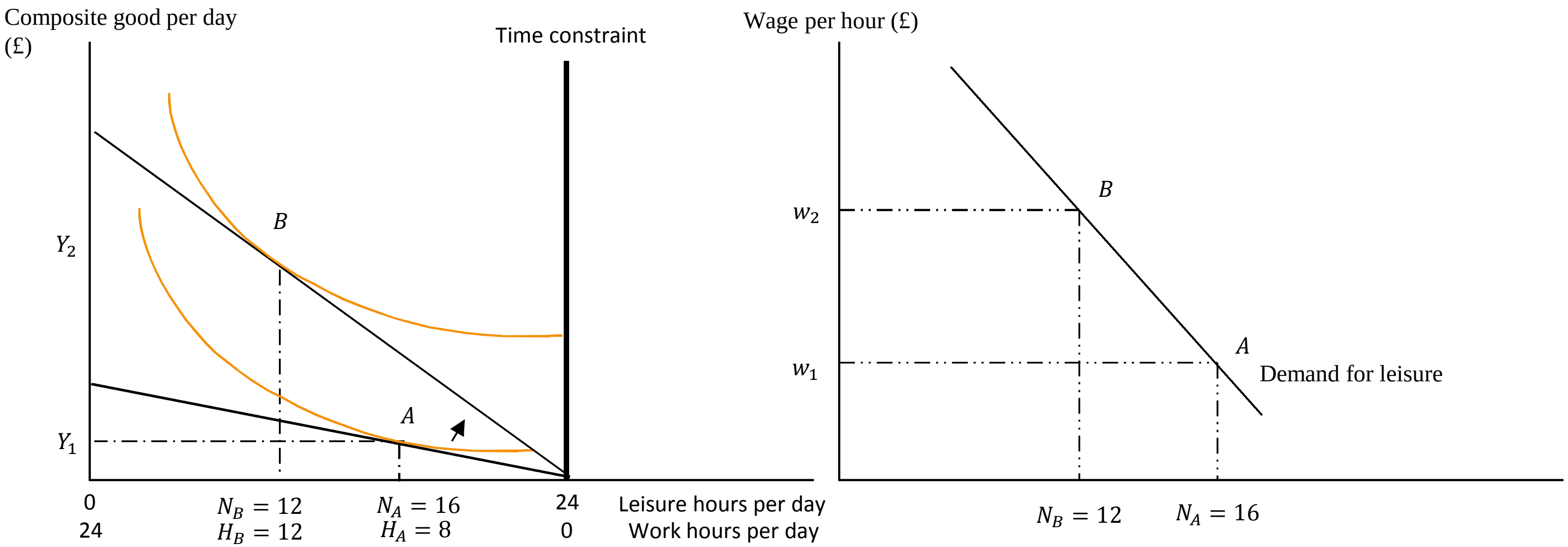
The slope of the budget constraint is given by $-w_1$, the price of one extra unit of leisure is an hour of foregone earnings working.



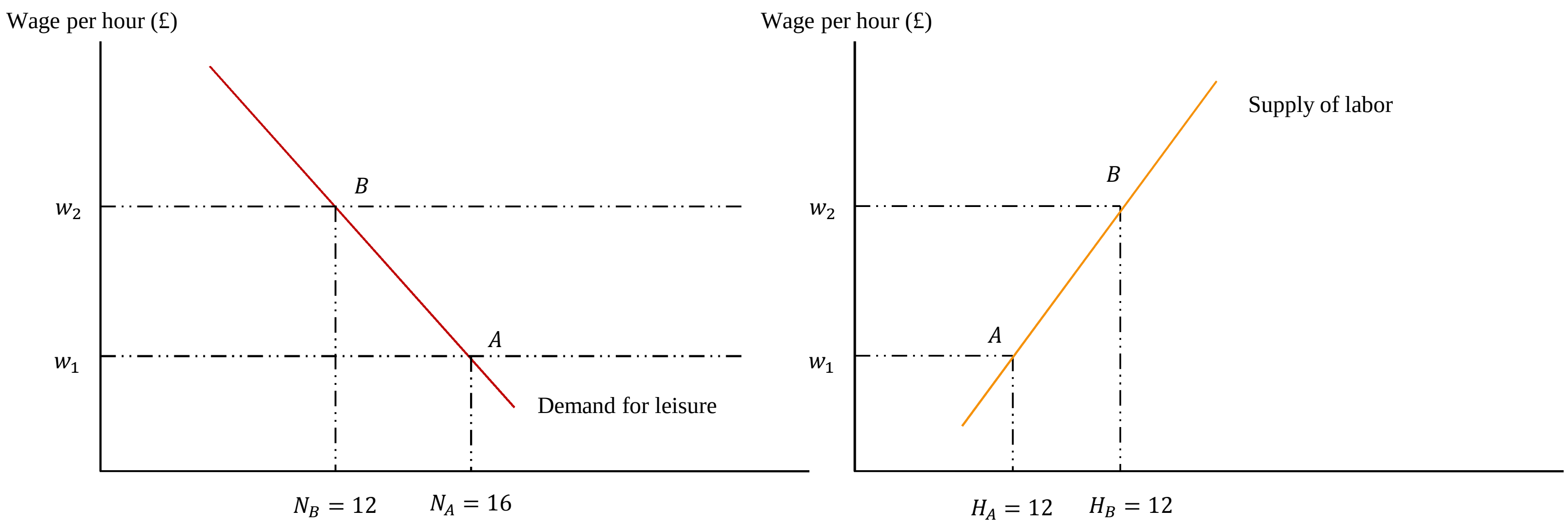
At point A, the consumer's optimal choice is to consume 16 hours of leisure, and work for 8 hours.

Application: Labor-Leisure choice

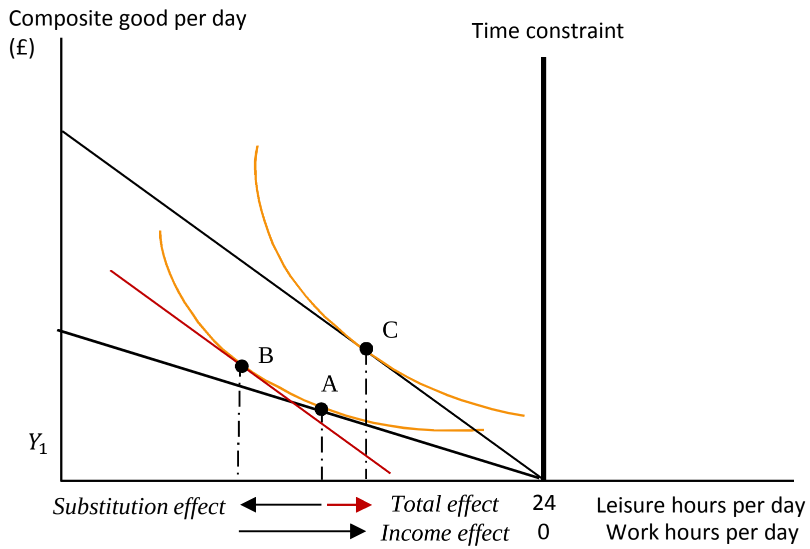
- We can now derive a demand curve for leisure. Increasing the wage from w_1 to w_2, we obtain a new rotated budget constraint and a new optimal bundle of work and leisure (12, 12).



Application: Labor-Leisure choice



Application: Labor-Leisure choice – Income and substitution effects (leisure is a normal good)



A to B is substitution effect: At the higher wage, leisure is now more expensive. The consumer substitutes leisure for work.
B to C is the income effect, with the now higher wage (and leisure being a normal good) the consumer consumes more leisure.

What happens if leisure becomes an inferior good after the wage increases above a certain threshold?

EC109 Microeconomics – Term 2, Part 1

FIRMS AND PRODUCTION

Laura Sochat

12/01/2015

Plan

- Production functions
  - One input
  - more than one input
  - Substitutability between inputs
- Returns to scale
- Technological progress

Production functions

- The labor (employees) and capital (factories, robots) a firm uses are called the factors of production, or inputs.
- The amount of goods and services produced are called the firm's output.

- Given a specific combination of inputs, the firm can produce a certain amount of outputs.
  - The production function tells us the maximum amount the firm can produce give a particular combination of inputs.
  - Assuming amount of capital used is denoted as K, amount of labor used as L, and quantity of output as Q, we can write the production function as:

$$Q=f(L,K)$$

Production functions

- Assume a production function for a single input.

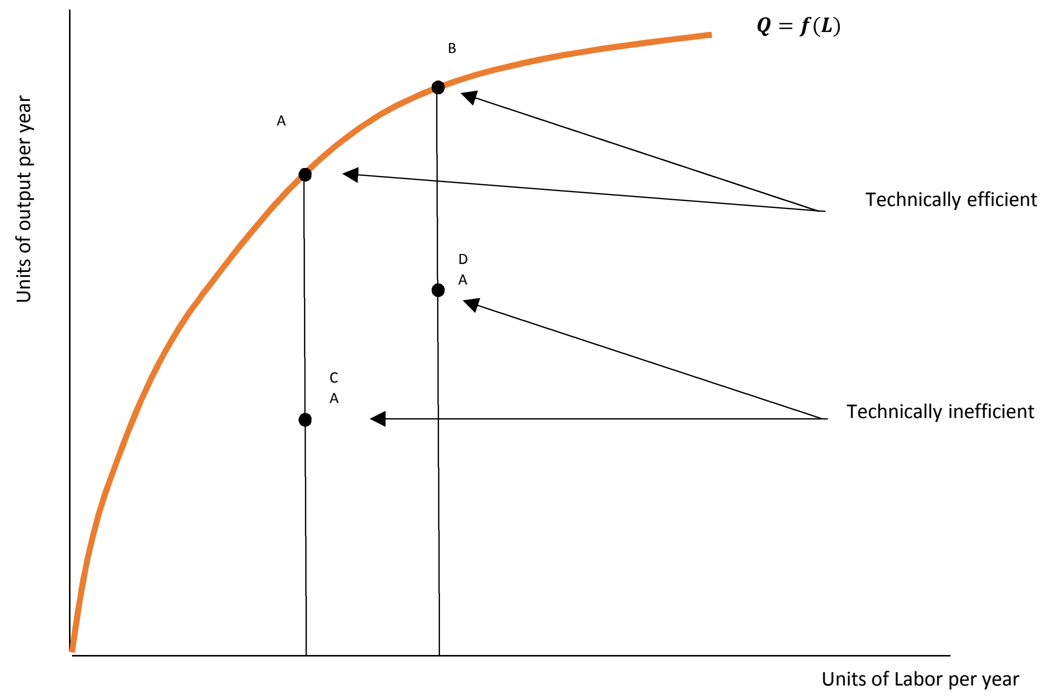
$$Q = f(L)$$

- From this function, we can also obtain the labor requirement function:

$$L = h(Q)$$

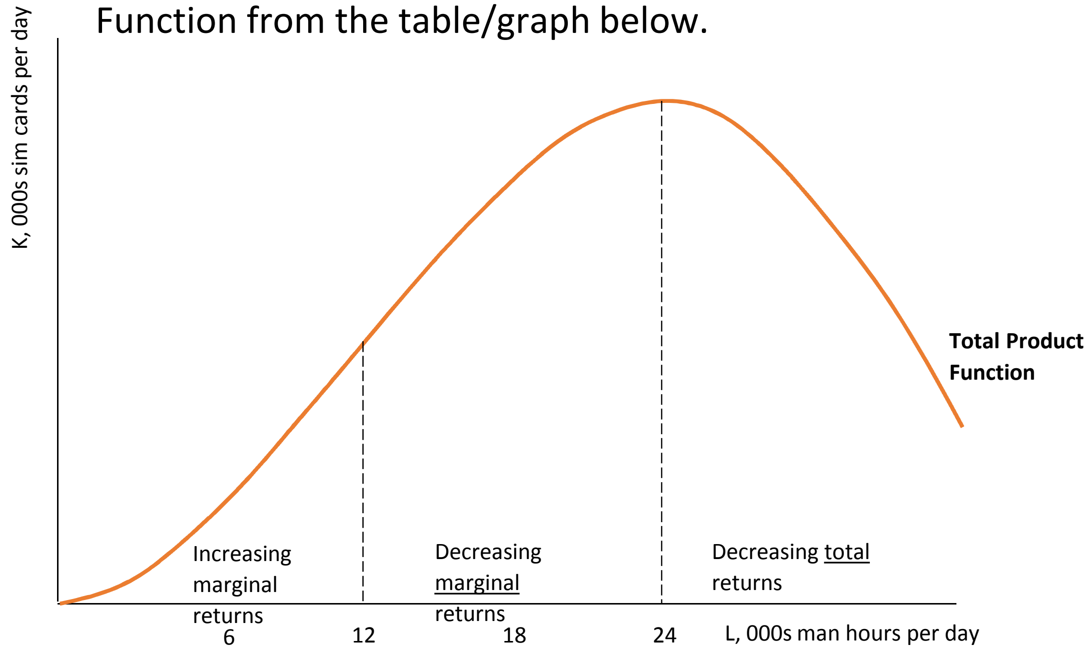
  - This function tells us the minimum amount of labor required to produce a certain amount of output.
  - For example, if the production function takes the form $Q = \sqrt{L}$, the labor requirement function then takes the following form: $L = Q^2$
  - What is the firm's labor requirement to produce an output of 10 units?

Technical efficiency with a single input



The total product function

- A single input production function depicting how output depends on the level of that input used in production, given a fixed amount of other inputs used. Consider the Total Product Function from the table/graph below.



L	Q
0	0
6	30
12	96
18	162
24	192
30	150

Marginal and average product

- Marginal product of Labor

$$MP_L = \frac{Change \text{ in total product}}{Change \text{ in quantity of labor}} = \frac{\Delta Q}{\Delta L}$$

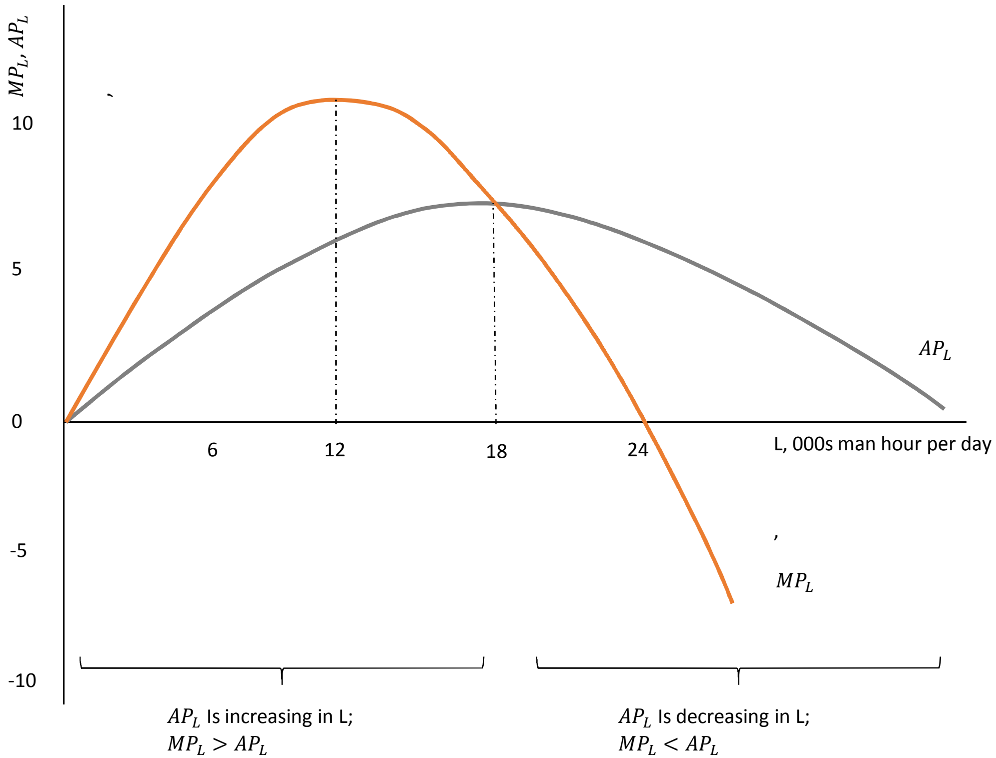
  - A notion of productivity of the firm, varies with the quantity of inputs used. It is the rate at which total output changes as the firm changes its quantity of labor. Analogous to the marginal utility concept seen earlier in the module.

- Average product of Labor:

$$AP_L = \frac{\text{Total product}}{\text{Quantity of labor}} = \frac{Q}{L}$$

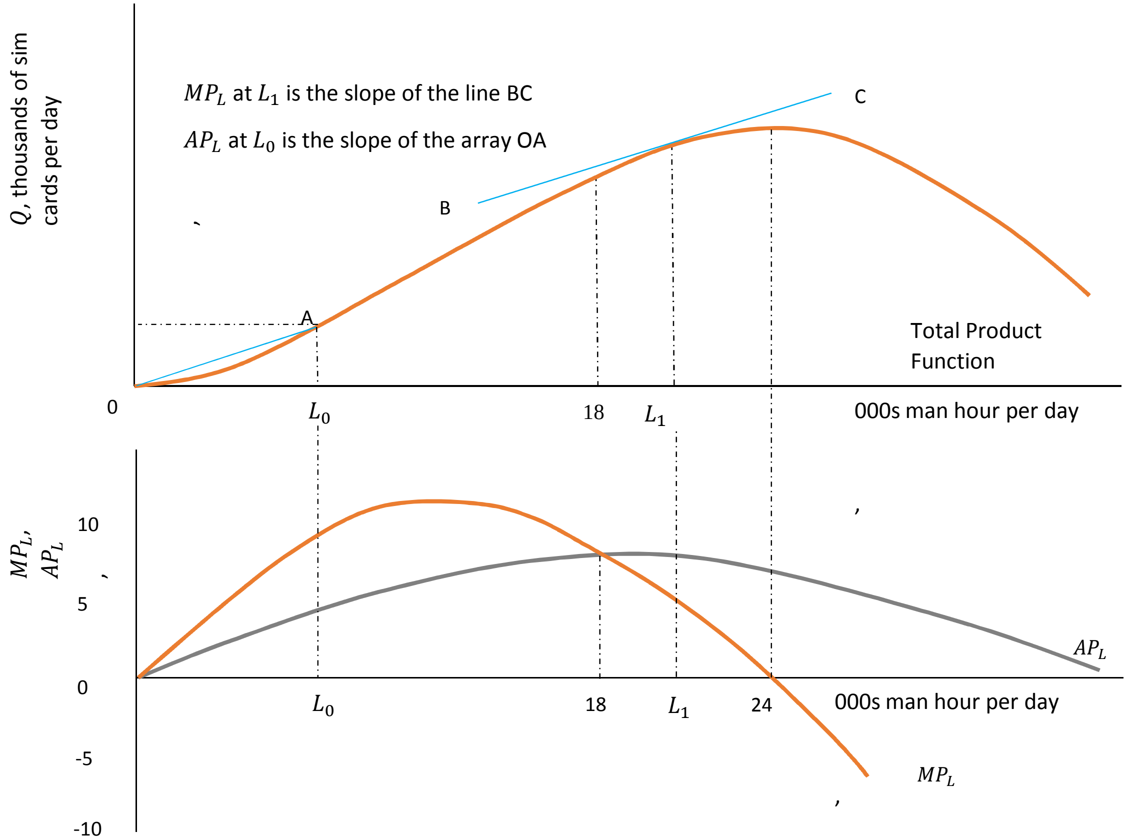
  - Another notion of productivity of the firm, it is the average amount of output produced by each unit of input used, here labor (usually the mean of comparison when you hear journalist comparing the productivity of workers in one country to another).

Relationship between Marginal and Average product



L	Q	$AP_L = \frac{Q}{L}$
6	30	5
12	96	8
18	162	9
24	192	8
30	150	5

Marginal and average product

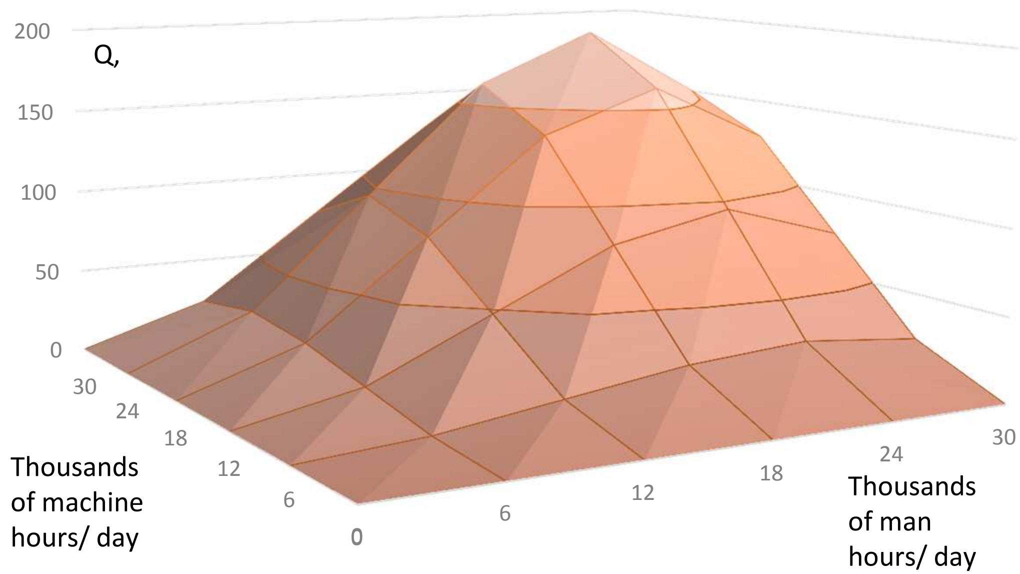


<u>Law of diminishing marginal returns</u>: Principle that as the usage of one input increases, the quantities of other inputs being held fixed, a point will be reached beyond which the marginal product of the variable input will decrease.

Two inputs- Robots and humans

- Assume now that production requires both Capital and Labor.

	K: 0	6	12	18	24	30
L: 0	0	0	0	0	0	0
6	0	5	15	25	30	23
12	0	15	48	81	96	75
18	0	25	81	137	162	127
24	0	30	96	162	192	150
30	0	23	75	127	150	117



Marginal products of Labor and Capital

- The marginal product of an input, just as we saw before, tells us how the quantity of output produced changes as the firm changes the quantity of one of its inputs. However, we now have more than one input:
- Marginal product of labor

$$MP_L = \frac{Change \text{ in quantity of output}}{Change \text{ in quantity of labor}}\bigg|_{\text{K held constant}} = \frac{\Delta Q}{\Delta L}\bigg|_{\text{K held constant}}$$

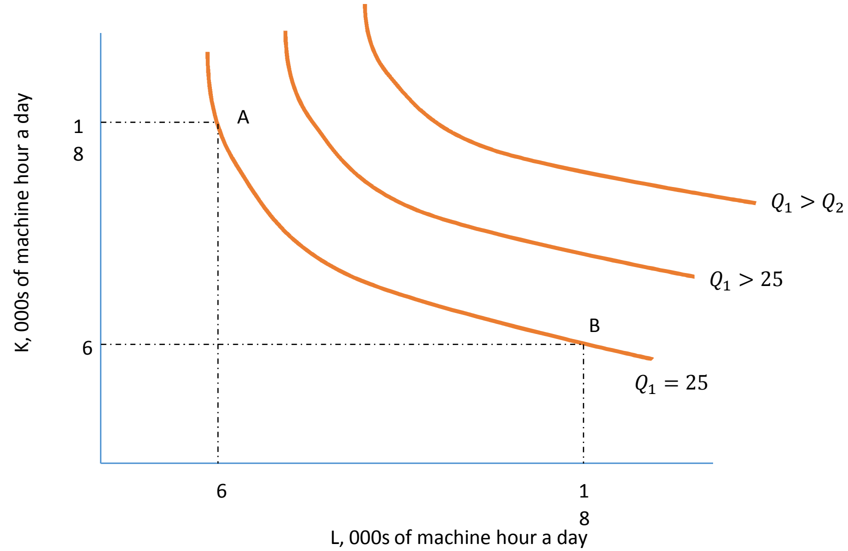
- Marginal product of capital

$$MP_L = \frac{Change \text{ in quantity of output}}{Change \text{ in quantity of capital}}\bigg|_{\text{L held constant}} = \frac{\Delta Q}{\Delta K}\bigg|_{\text{L held constant}}$$

ISOQUANTS

- Isoquants enable us to represent the production function. It is a curve that shows all combinations of labor and capital that can produce a given level of output.

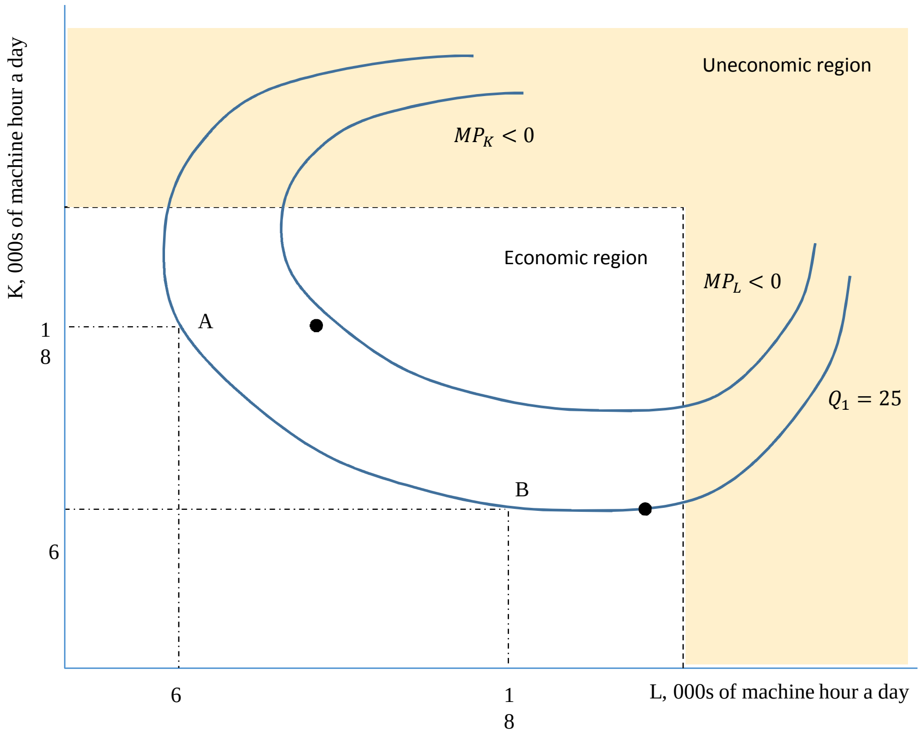
		K					
		0	6	12	18	24	30
L	0	0	0	0	0	0	0
	6	0	5	15	**25**	30	23
	12	0	15	48	81	96	75
	18	0	**25**	81	137	162	127
	24	0	30	96	162	192	150
	30	0	23	75	127	150	117



Deriving the equation of an isoquant

- Consider the production function whose equation is given by the formula $Q = \sqrt{KL}$.
  - What is the equation of the isoquant corresponding to Q=20.
  - For the same production function, what is the general equation of an isoquant, corresponding to any level of output Q?

Economic and uneconomic regions of production

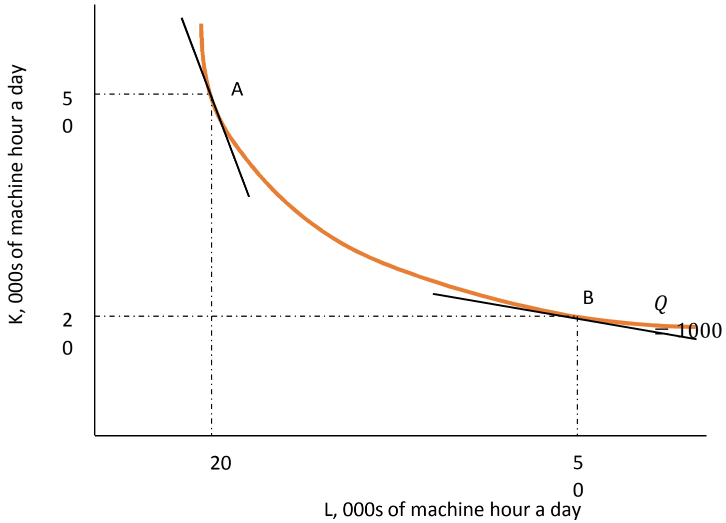


When an input's marginal return is negative, implying diminishing total return to that input: To stay on the same isoquant, the firm has to *increase* its use of the other input to compensate!

A firm wishing to minimize production costs, would never produce where this is the case : We refer to this area as the uneconomic region of production.

Marginal rate of Technical substitution

- The marginal rate of technical substitution (of labor for capital) is the rate at which capital can be reduced for every one unit increase in labor, and keeping output constant. It is defined as the absolute value of slope of the isoquant drawn with labor on the horizontal axis, and capital on the vertical axis.



Moving down the isoquant, the $MRTS_{L,K}$ diminishes in absolute value: **Diminishing marginal rate of technical substitution**. When a production function exhibits diminishing marginal rate of technical substitution, it is convex to the origin.

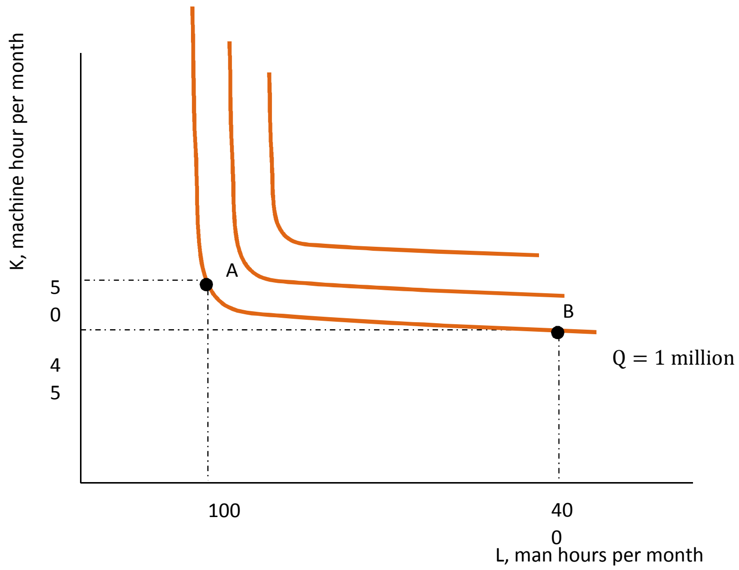
$MRTS_{L,K}$ and Marginal Products

- When we change the quantity of labor by ΔL units of labor and the quantity of capital by ΔK units of capital, the resulting change in output from this substitution can be written such as:

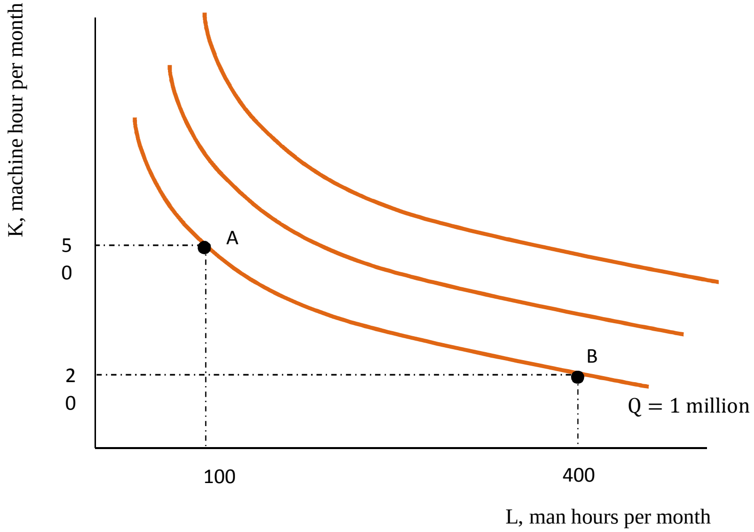
$$\Delta Q = (\Delta K)(MP_K) + (\Delta L)(MP_L)$$
$$= \Delta Q\ change\ in\ output\ from\ change\ in\ quantity\ of\ capital$$
$$+\ change\ in\ output\ from\ change\ in\ quantity\ of\ labor$$

$$=\succ -\frac{\Delta K}{\Delta L} = \frac{MP_L}{MP_K} =\succ \frac{MP_L}{MP_K} = MRTS_{L,K}$$

Describing a firm's input substitution opportunities graphically



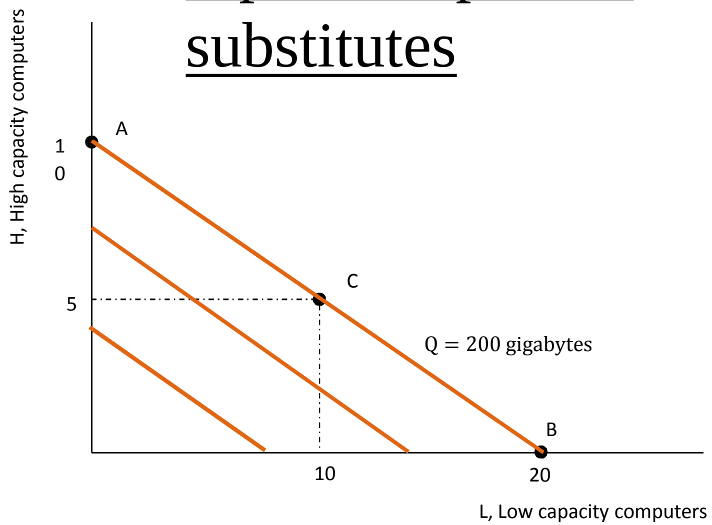
(a) Production function with **limited** input substitution opportunities



(b) Production function with **abundant** input substitution opportunities

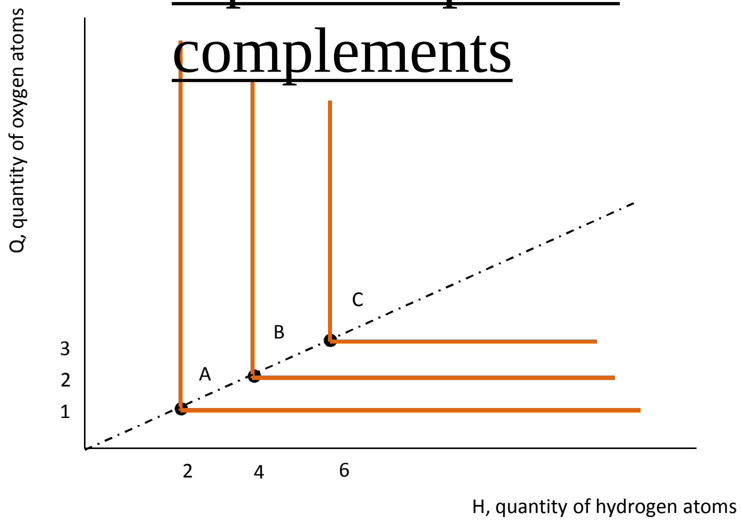
Special production functions: Linear and fixed proportions production functions

Inputs are perfect substitutes



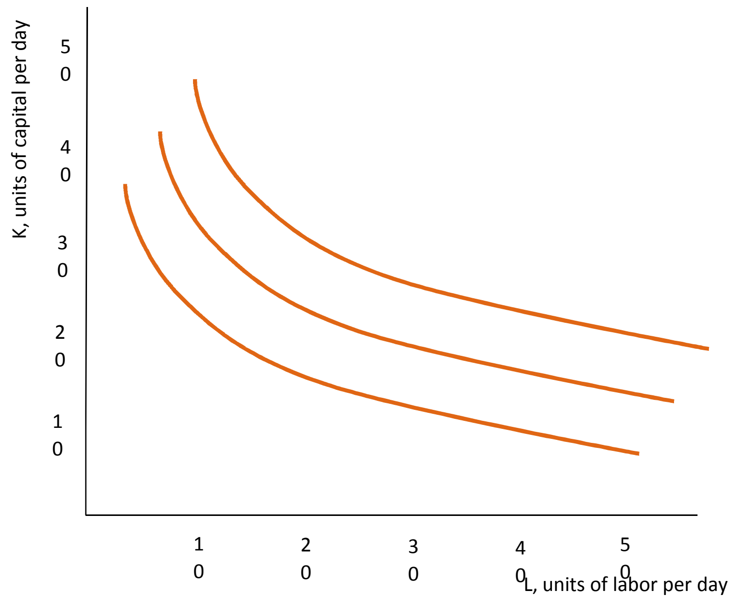
(a) Map of isoquants for data storing

Inputs are perfect complements



(b) Map of isoquants for molecules of water

Special production functions: Cobb Douglas production function



Cobb-Douglas production function:

$$Q = AL^{\alpha}K^{\beta}$$

- Unlike the fixed proportions function, labor and capital can be substituted for each other in variable proportions.
- Unlike a linear production function, the rate at which labor can be substituted for capital is not constant as you move along the isoquant

Returns to scale

- We know that with positive marginal products of labor and capital, a firm's total output must increase when the quantities of all inputs increase simultaneously. In other words, if we increase a firm's scale of operations, output will increase.
- We ask the question "By how much will output increase when all inputs are increased by a given amount simultaneously?"

$$Returns\ to\ scale = \frac{\%\Delta\ (quantity\ of\ output)}{\%\Delta\ (quantity\ of\ all\ inputs)}$$

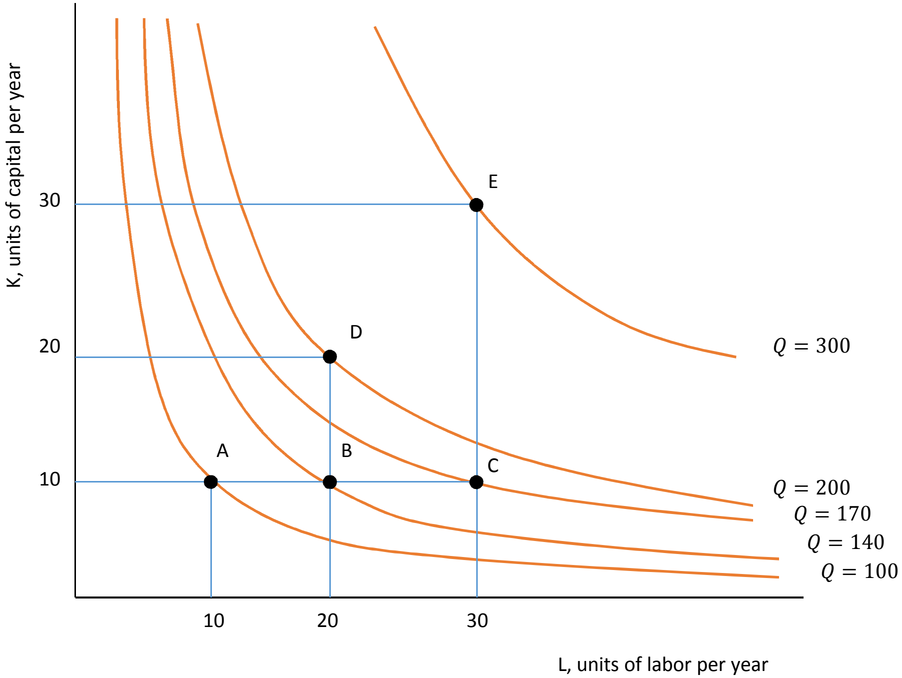
Returns to scale

- A firm uses two inputs, K and L, to produce output Q. Suppose that all inputs are 'scaled up' by λ ($\lambda > 1$). Denote by a the resulting increase in the quantity of output produced.

- If $a > \lambda$, there are increasing returns to scale. In this case, a proportionate increase in all input quantities results in a greater than proportionate increase in output.
- If $a = \lambda$, there are constant returns to scale. In this case, a proportionate increase in all input quantities results in the same proportionate increase in output.
- If $a < \lambda$, there are decreasing returns to scale. In this case, a proportionate increase in all input quantities results in a less than proportionate increase in output.

- Does a Cobb-Douglas production function, $Q = AL^{\alpha}K^{\beta}$, exhibit increasing, decreasing, or constant returns to scale?

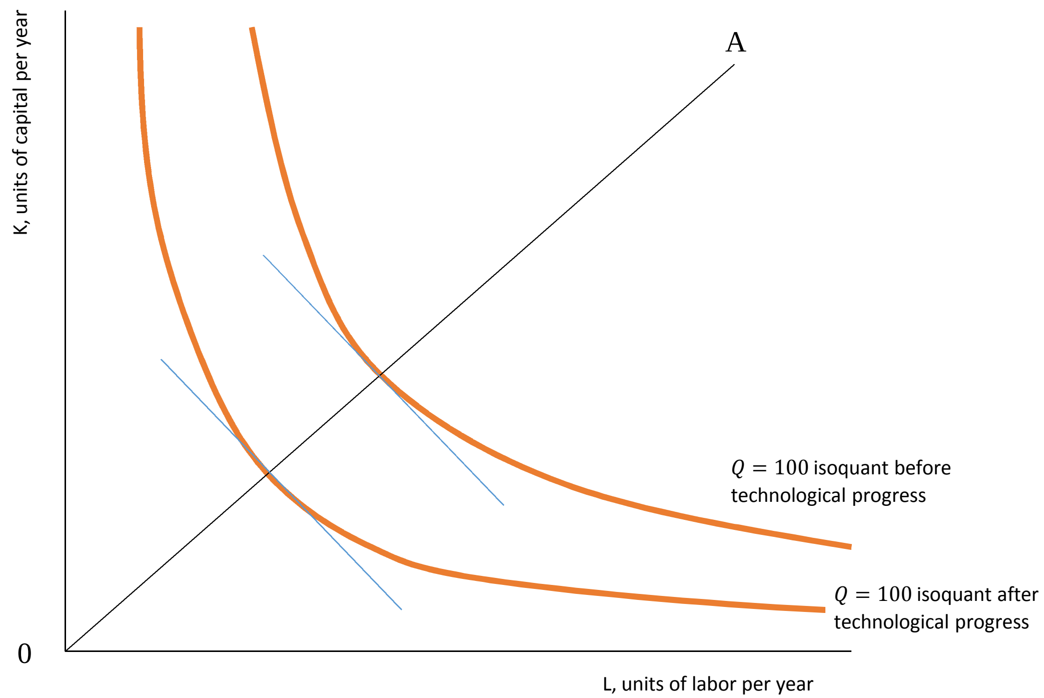
The distinction between returns to scale and diminishing marginal returns



- Returns to scale refer to the impact of an increase in all input quantities simultaneously
- Marginal returns refer to an increase in the quantity of a single input, such as labor holding quantities of all other inputs fixed.

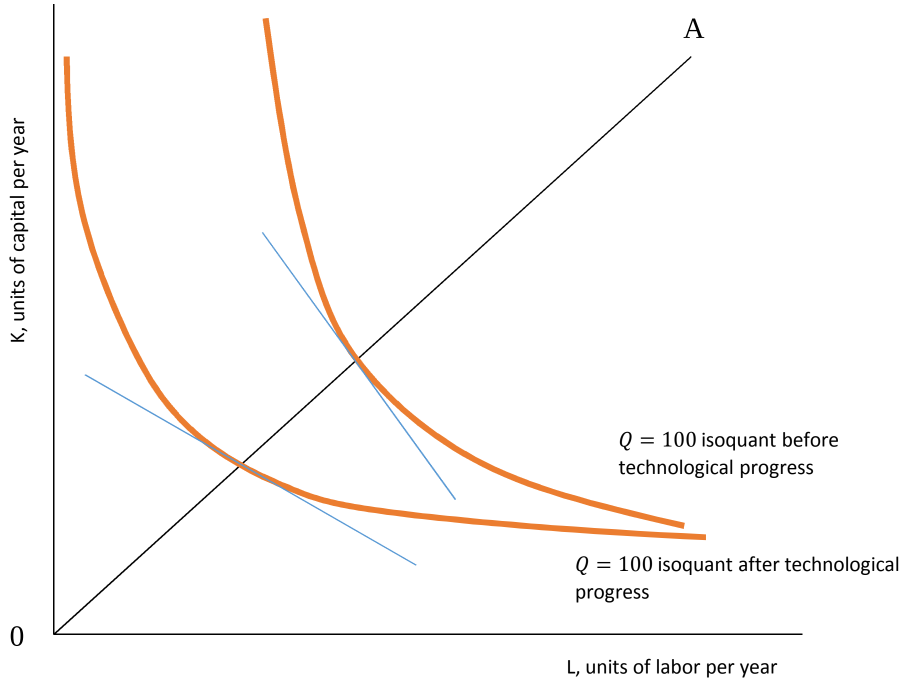
- The figure to the left illustrates a situation where there are diminishing marginal returns to labor, but constant returns to scale.

Neutral technological progress



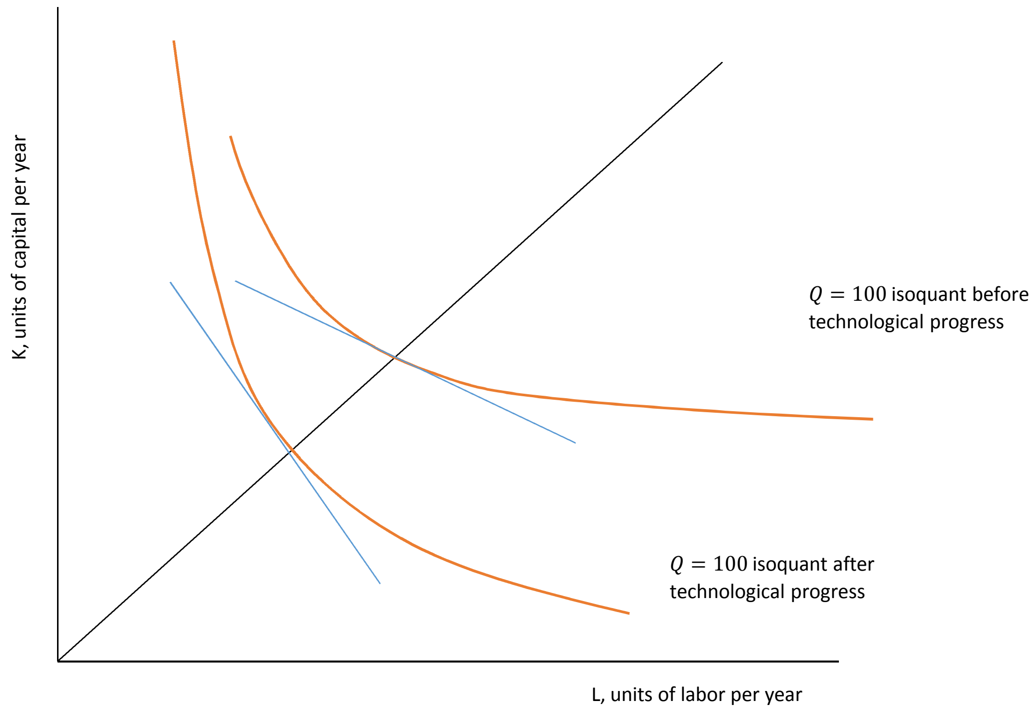
- The isoquant corresponding to a given level of output shifts inwards, but the $MRTS_{L,K}$ stays unchanged.

Labor-saving technological progress



- In this case too, the isoquant shifts inwards. However, as the figure shows, the $MRTS_{L,K}$ in this case decreases (in absolute value), reflecting an increase in the marginal product of capital relative to the marginal product of labor.

Capital saving technological progress



- The isoquant shifts inwards again. However, as the figure shows, the $MRTS_{L,K}$ in this case increases (in absolute value), reflecting an increase in the marginal product of labor relative to the marginal product of capital.

EC109 Microeconomics – Term 2, Part 1

Cost and cost minimization

Laura Sochat

19/01/2015

Plan

- Cost concepts for the firm’s decision making
  - Opportunity cost
  - Economic versus accounting costs
  - Sunk versus non sunk costs
- The cost minimization problem in the long run
- Comparative statics
  - Change in input prices
  - Change in output
- The cost minimization problem in the short run
  - Costs in the short run
  - Comparative statics

What do we mean by costs?

- What is the cost of an airline using the planes it owns for scheduled passenger services?
  - Crew salaries, fuel etc.
  - Foregone income from not renting the plane to someone else
- Costs don’t always refer to direct monetary transfers
  - Explicit costs refer to those costs needing a direct monetary outlay
  - Implicit costs refer to those costs not involving such a monetary outlay

Opportunity cost

- The opportunity cost is a notion referring to the value of sacrificed opportunities. It is the payoff associated with the best alternative to the choice made.
  - Includes both explicit and implicit costs
  - It is a forward looking concept
  - It depends on the decision made, and on current market prices
- Suppose you own your own business and are deciding whether to go out of business or to carry on your activities. The table below summarises your cost benefit analysis. What is your opportunity cost of remaining in business over the next year?

Remain in business	Go out of business
£100,000 on labor	£75,000 income
80 hours of your time	80 hours of your time
£80,000 on supplies	

Economic versus Accounting costs

- Economic costs are the sum of all explicit and implicit costs
- Accounting costs are the ones which would appear on an accounting statement- They are all explicit costs, incurred <u>in the past</u>

- Suppose a firm has an inventory of sheet steel which it bought for £1m. It could either use the sheet steel for manufacturing or resell it to other firms. Assume also that the price of the sheet steel has now gone up to £1.2m.
  - What cost would appear on an accounting statement?
  - What is the opportunity cost of using the sheet steel for manufacturing?

Sunk versus non sunk costs

- Another important concept for the firm’s decision making. It will enable the firm to assess the costs that <u>the decision</u> will actually affect.
- Sunk costs refer to the costs which the firm has already incurred. They therefore cannot be avoided by the firm.
- Non sunk costs refer to the costs which would only be incurred should a particular decision was taken. They can therefore be avoided by not taking such a decision.

- Suppose you go to the Opera. 30 minutes after the start of the movie, you realise that you do not like it. Should you stay?
  - What is the cost of staying?
  - What is the cost of leaving?

Cost minimization in the long run

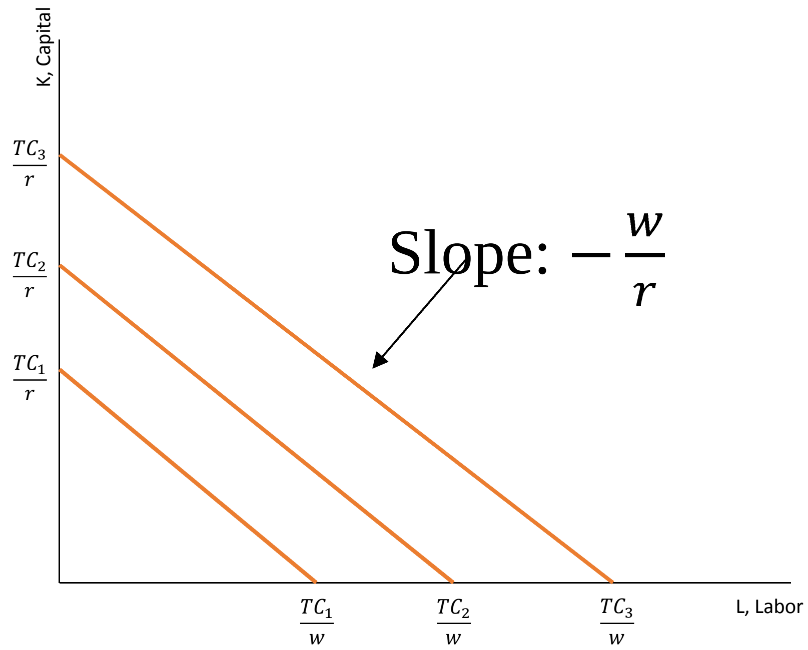
- Let’s go back to an important decision the firm needs to take- Out of all possible combinations of inputs possible to produce a given amount of output, which one will be the cost minimizing combination?
- The long run here refers to the situation where the firm is able to vary the quantities of all the inputs it uses. What can we say of the costs associated with a long run decision for the firm?
- The short run will refer to a situation where the firm faces some constraints as it will not be able to vary the quantities of all inputs.

- Suppose the firm uses both Labor and capital . The price of labor is the wage rate w and the price of capital is r. The firm has to produce amount Q of output over the next year. The firms costs of production are such that:
$$TC = wL + rK$$

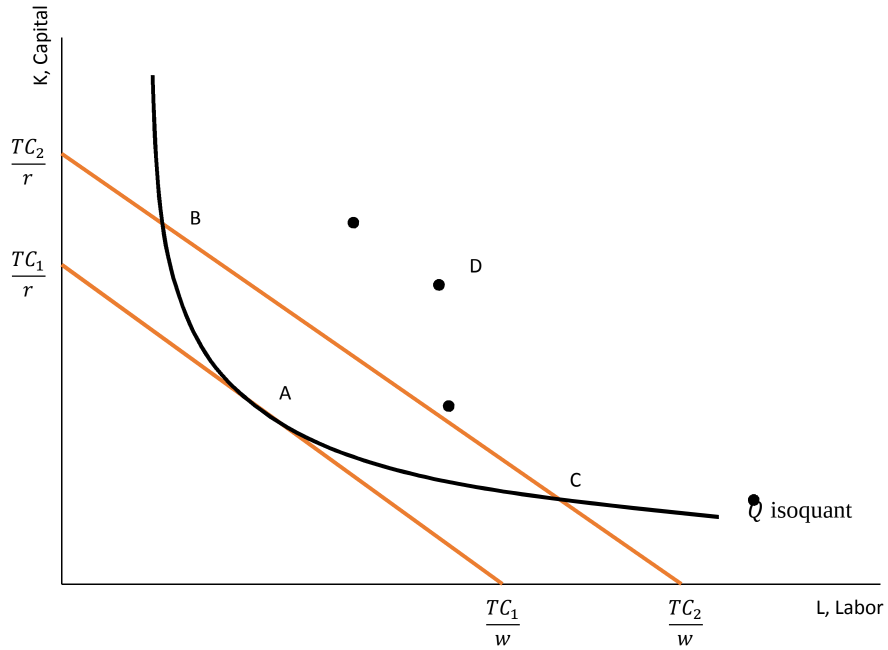
Isocost lines

- An isocost line represents a set of combination of labor and capital that have the same total cost for the firm.

For a cost level of £1m, and assuming that the price of labor, $w = £10$ and the price of capital is such that $r = £20$.
What is the equation describing the £1m isocost line?
More generally, what is the equation for an isocost line.



Graphical representation of the long run cost minimization problem



- Point D is off the isoquant
- Point B and C are technically efficient, but not cost minimizing
- Point A is technically efficient and cost minimizing

- At point A: Slopes are equal.

$$MRTS_{L,K} = \frac{w}{r}$$

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$

Cost minimization example

- Suppose the firm's production function is such that $Q = 50\sqrt{KL}$. Suppose also that the price of labor is $w = £5$/unit and the price of capital such that $\mathrm{r} =$ £20/unit.
  1. If the firm choose to produce 1000 units of output during the year, what would be the optimal input combination?

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$

$$L = 4K$$

$$L = \frac{400}{K}$$

$$\left.\begin{matrix} \\ \end{matrix}\right\} \quad K = 10, \quad L = 40$$

Cost minimization using Lagrangean method

- We can re write the cost minimization problem of the firm chosing Labor and Capital to minimize its cost such as:

$$\min_{L,K} wL + rK$$
$$subject\ to: f(L,K) = Q$$

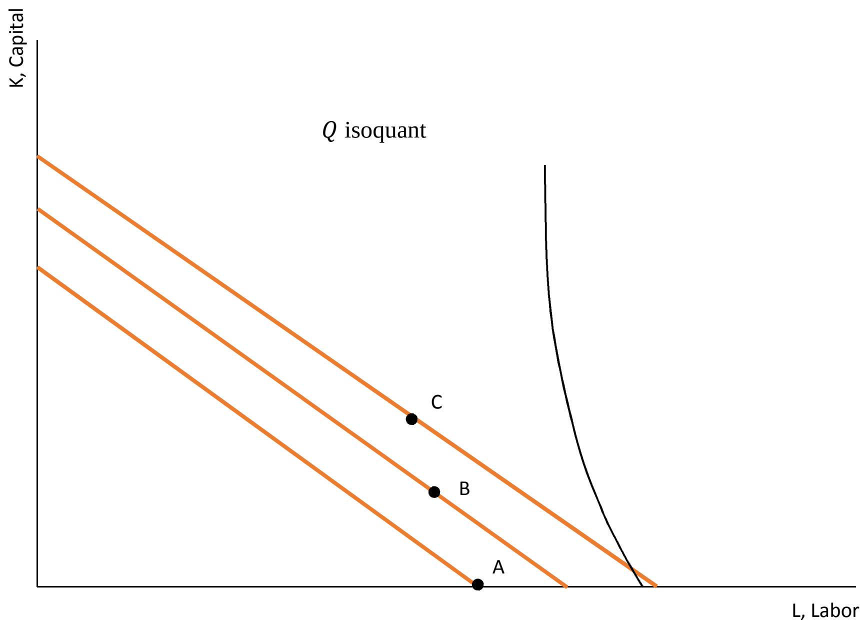
- Defining the Lagrangean function such as

$$\Lambda(L,K,\lambda) = wL + rK - \lambda(f(L,K) - Q)$$

$$\Rightarrow \quad \frac{MP_L}{MP_K} = \frac{w}{r} \quad ; \quad \mathrm{f(L,K) = Q}$$

Corner point solutions

- The cost minimizing input combination happens at a point where the firm uses no Capital:



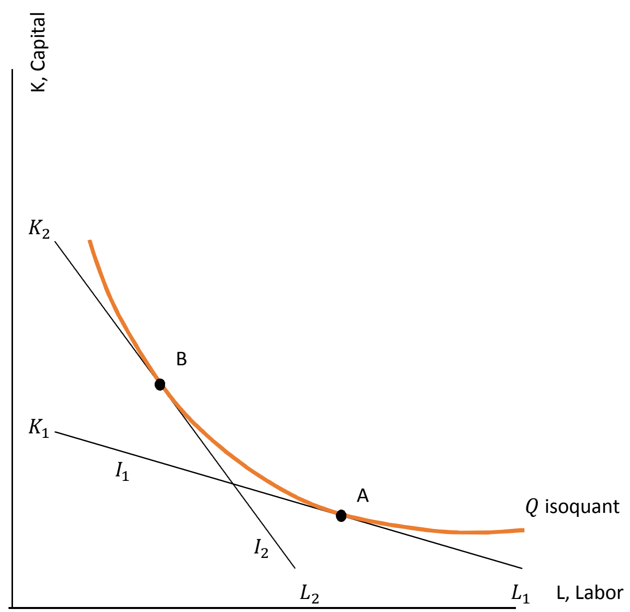
$\frac{MP_L}{MP_K} = \frac{w}{r}$ does not hold at any point along the isoquant: the isocost is flatter than the isoquant at every point.
The situation is such that:

$$\frac{MP_L}{MP_K} > \frac{w}{r}$$
$$\frac{MP_L}{w} > \frac{MP_K}{r}$$

- Every dollar spent on labour is more productive than every dollar spent on Capital.

Comparative statics: A change in input prices

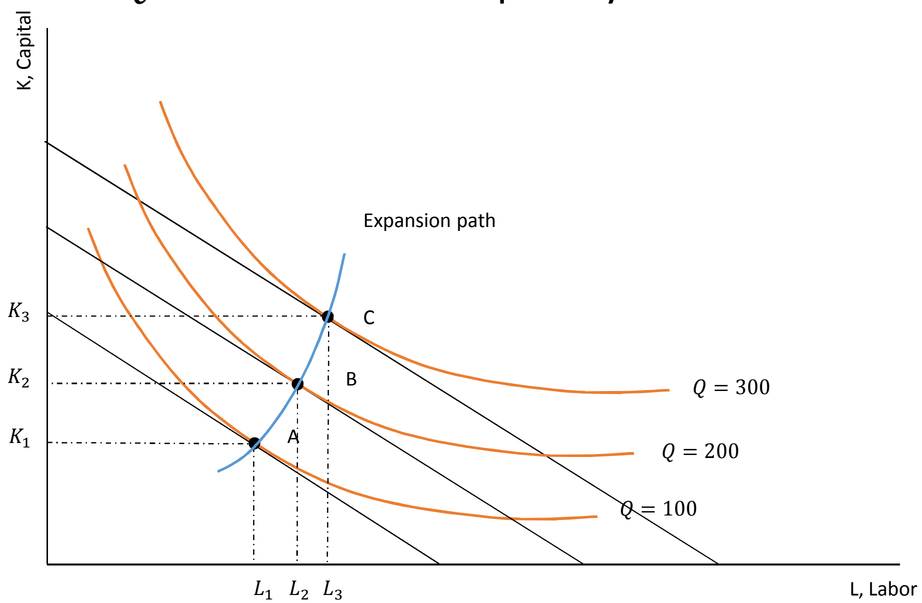
- Suppose that the price of capital, r , and the quantity of output, Q, are both held fixed. The figure below shows the effect of an increase in the price of labor, w.



- As the price of labour increases, the isocost line becomes steeper.
- With diminishing MRTS of labour for capital, the new optimal point is farther up the isoquant.
- The firm now uses more capital and less labour, as price of labour is relatively higher.
- Note two important assumptions
  - The firm was already using positive amounts of both inputs
  - The isoquants are smooth

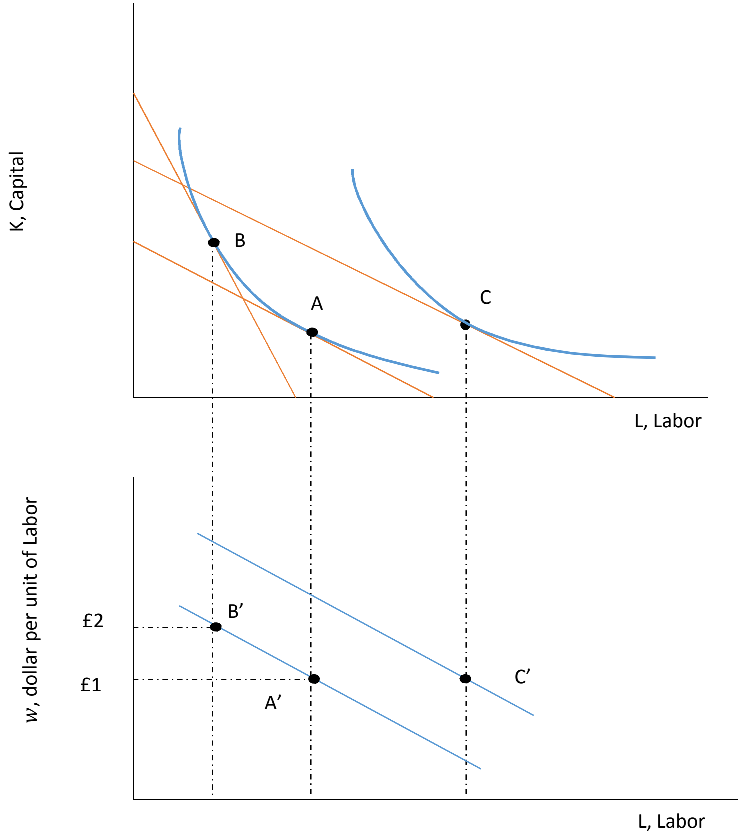
Comparative statics: A change in output

- Suppose that now the prce of both inputs stays unchanged but that the level of output the firm wants to produce first changes to $Q = 200$, from an initial level of $Q = 100$ and subsequently increase further, to $Q = 300$.



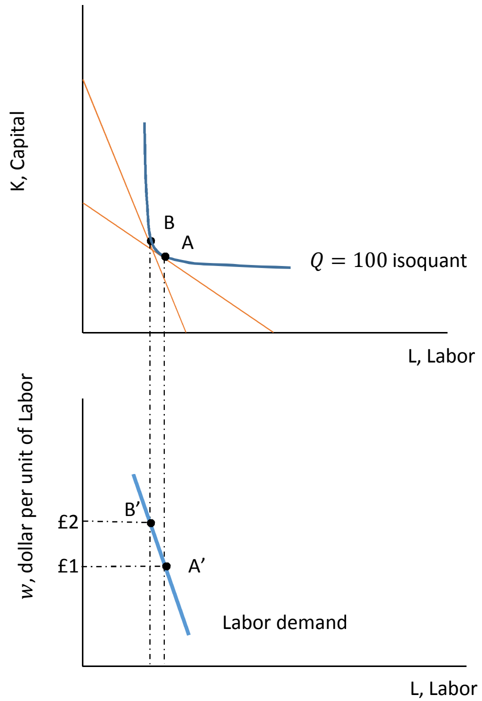
- The optimal combinations of inputs move north east as the quantity of output increases.
- The line linking the three different optimal bunbles is called the expansion path.
- Both capital and labor are normal goods.

Using comparative statics to derive input demand curves

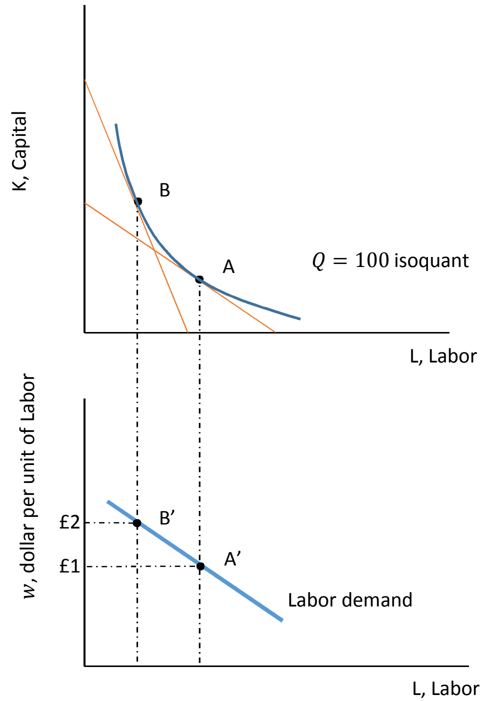


- The top graph shows both the effect of a change in the price of labor, and the effect of a change in output
- The bottom graph summarises the implications of the re optimisation of the firm: The labor demand curve
- As the quantity of output changes (and both the price of labor and capital stay unchanged), the labor demand curve shifts upwards.

The firm's subsitution opportunities and the elasticity of the input demand curve



(a) Limited substitution opportunities between inputs lead to an inelastic input demand curve



(b) Abundant substitution opportunities between inputs lead to an elastic input demand curve

Costs in the short run: Fixed and variable costs

- In the short run, the firm faces constraints in its ability to vary the quantity of some inputs. We will consider a case where the amount of capital the firm can use is fixed in the short run. We can rewrite the firm's total cost such that:

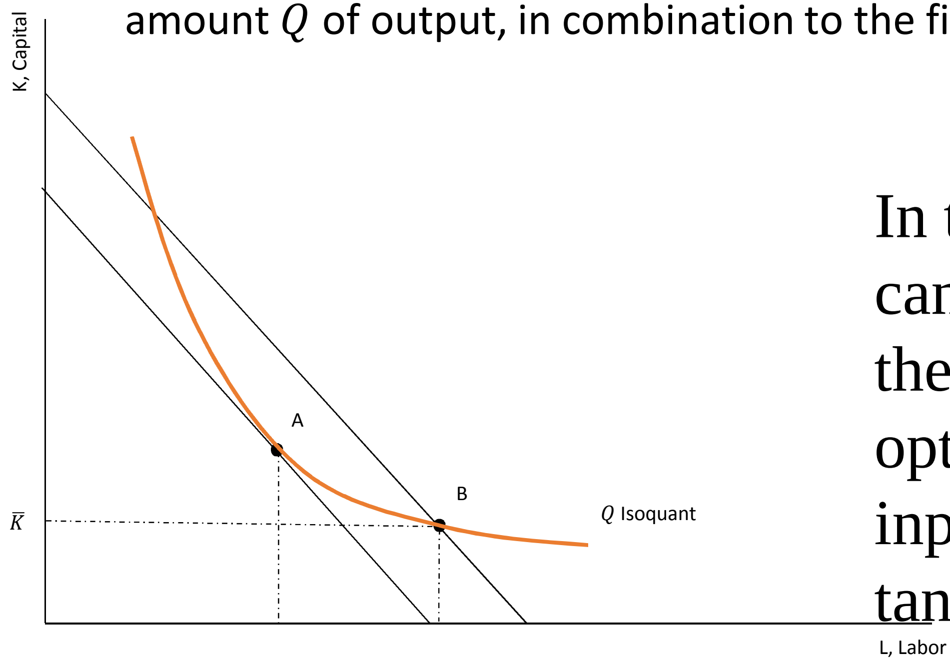
$$TC = wL + r\overline{K}$$

Where $\overline{K}$ represents the fixed amount of capital.

- The cost of labor constitues the firm's total variable cost- It will vary as the firm chooses to produce more or less output.
- The cost of capital constitutes the firm's total fixed cost- It will not vary as the firm produces more or less output.

Cost minimization in the short run

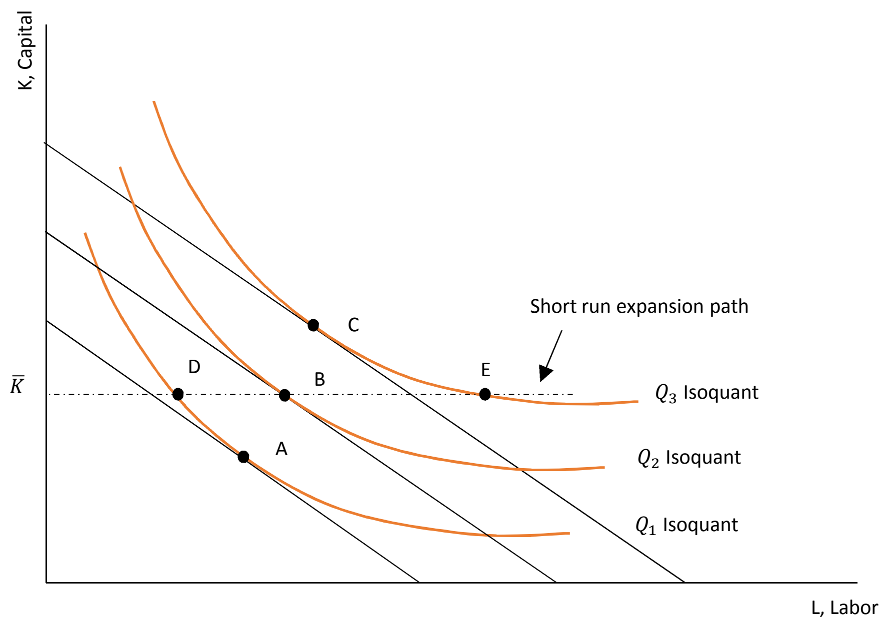
- The firm's only technically efficient combinaition of inputs occurs at point B, where it uses the minimum amount of labor which allows the firm to produce an amount Q of output, in combination to the fixed quantity of capital.



In the short run, the firm cannot substitute between the two inputs: The optimal combination of inputs does not involve a tangency condition.

Cost minimization in the short run

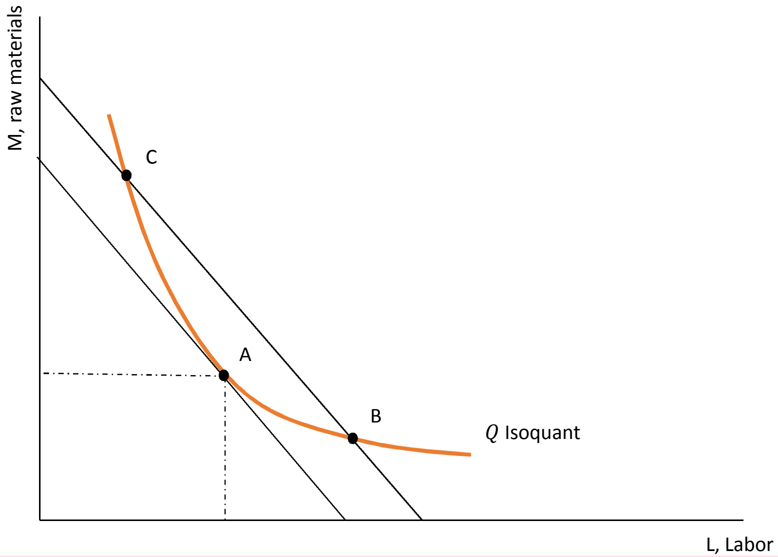
- The short run combination of inputs usually differs from the combination of inputs used in the long run: The firm typically operates at higher costs in the short run.



Consider point B.
In the long run, the firm will choose to produce at point B (it chooses freely between Labor and Capital).
In the short run, the firm can only use an amount $\bar{K}$ of capital to produce a level Q_2 of output:
In this case, the firm incurs the same cost in both time periods.

More than one variable input with one fixed input

- Let's assume now that the firm uses three inputs: Capital (K), Labor (L) and raw materials (M). Again assume that the quantity of Capital the firm can use is fixed at $\overline{K}$. The price of the three inputs are r, w and m respectively.



Production function is given by:
$Q = f(K, L, M)$

and total costs by: $wL + r\overline{K} + mM.$

Suppose the firm wants to produce an amount Q. The firm's optimal choice of inputs happens at the tangency where:

$$MRTS_{L,M} = \frac{MP_L}{MP_M} = \frac{w}{m}$$

EC109 Microeconomics – Term 2, Part 1

Cost Curves

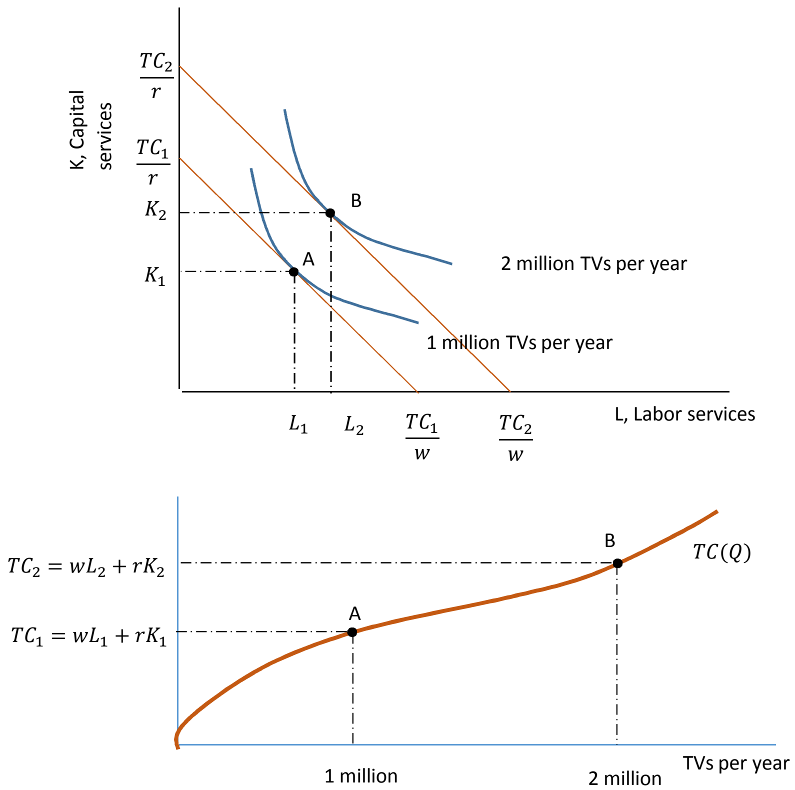
Laura Sochat

26/01/2016

Plan

- Long run total cost curves
  - Long run average and marginal cost curves
  - Economies and diseconomies of scale
- Short run cost curves
  - Relationship with long run total cost curves
  - Short run average and marginal cost curves
  - Relationship with long run average and marginal cost curves
- Economies of scope

Long run total cost curve



- As the level of output varies, holding input prices constant, the cost minimizing combination of input changes
- The long run total cost curve shows how minimized total cost varies with output, assuming constant input prices and that the firm chooses the input combination to minimize its costs.
- The long run total cost curve must be increasing in Q, and must be equal to 0, when $Q = 0$

Finding the total cost curve from a production function

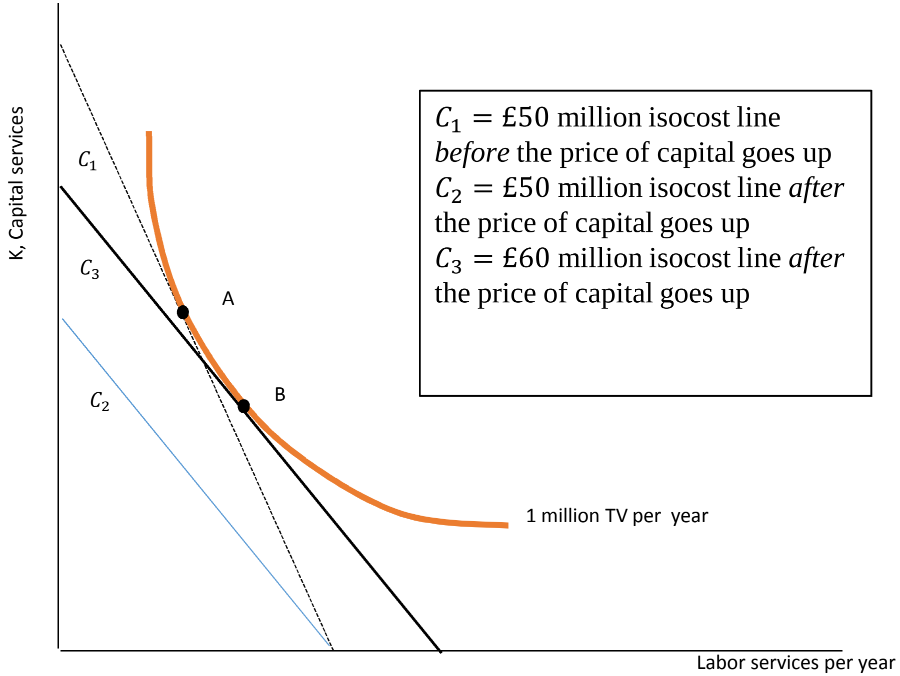
- Assume a production function of the form: $Q = 50\sqrt{KL}$

a) How does minimized total cost depend on the output level, and the input prices, for this production function?
b) What is the graph of the long run total cost curve when $w = 25$ and $r = 100$?

Remember that we earlier derived:

$$L = \frac{Q}{50}\sqrt{\frac{r}{w}}, \text{ and } K = \frac{Q}{50}\sqrt{\frac{w}{r}}$$

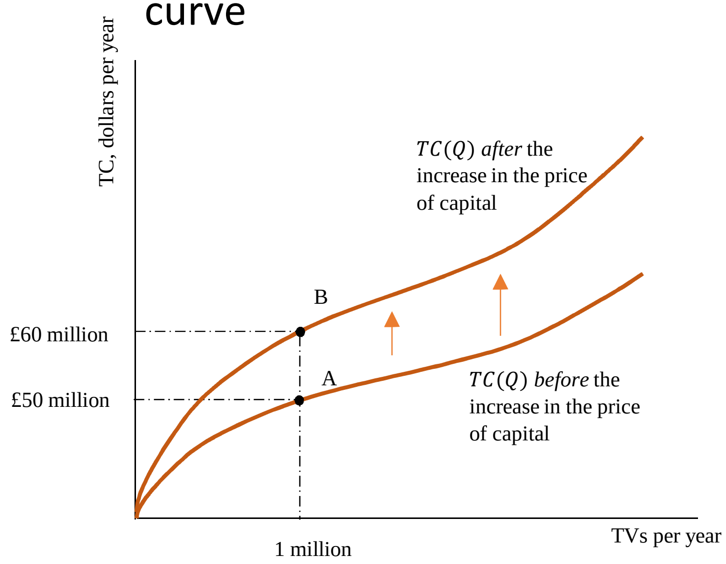
How does the long run cost curve shift when input prices change?



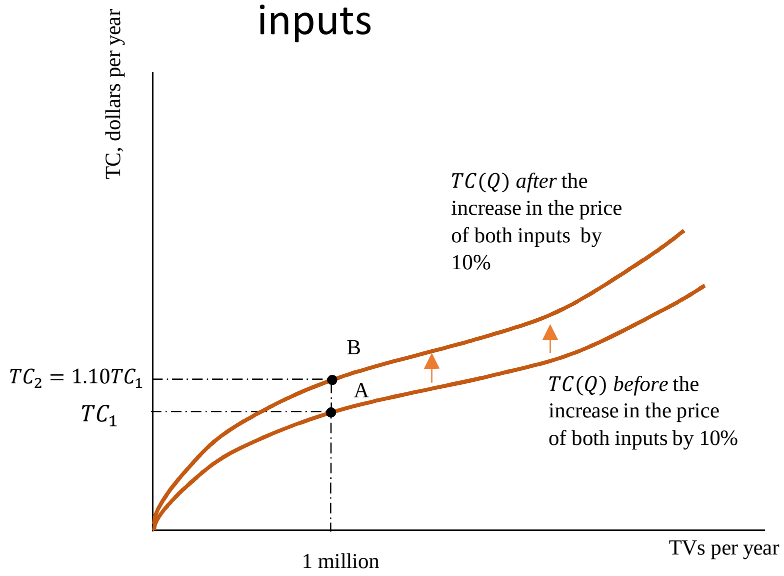
- Starting from point A, where the firm produces 1 million televisions, on isocost line C_1 where it cost the firm £50 million.
- After the price increase, the cost minimising input combination occurs at point B, where total cost is greater than it was at point A, say £60 million.

Long run total cost curve: Change in the price of inputs

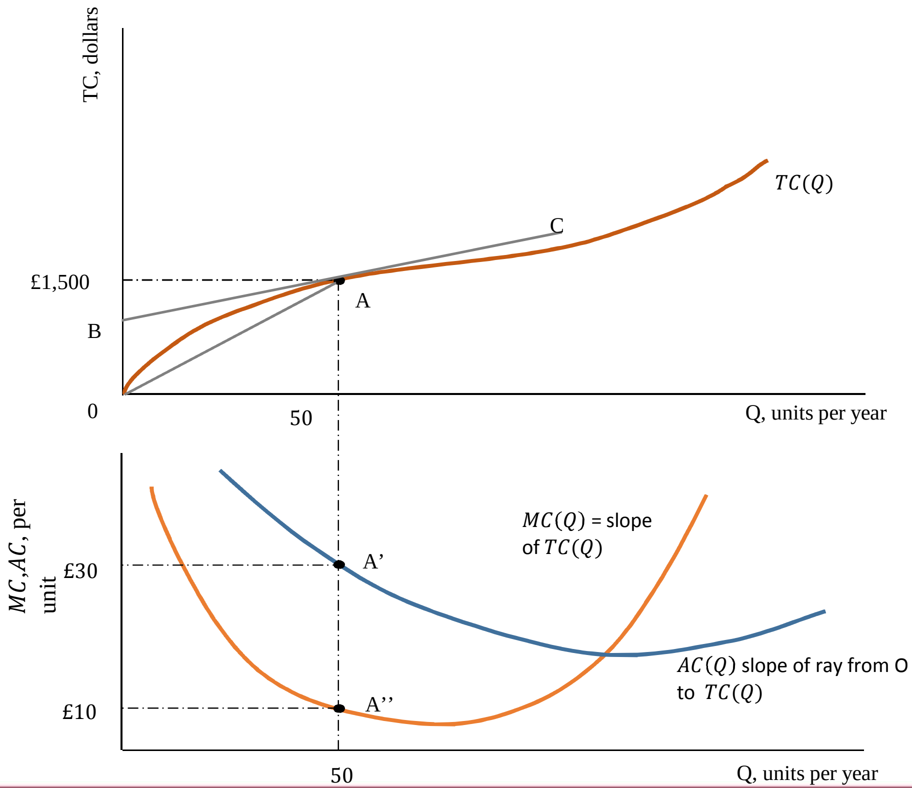
- The effect of an increase in the price of capital on the TC(Q) curve



- The effect a proportional increase in the price of both inputs



Long run average and marginal cost curves



- Long run average cost:

$$AC(Q) = \frac{TC(Q)}{Q}$$

- Long run marginal cost:

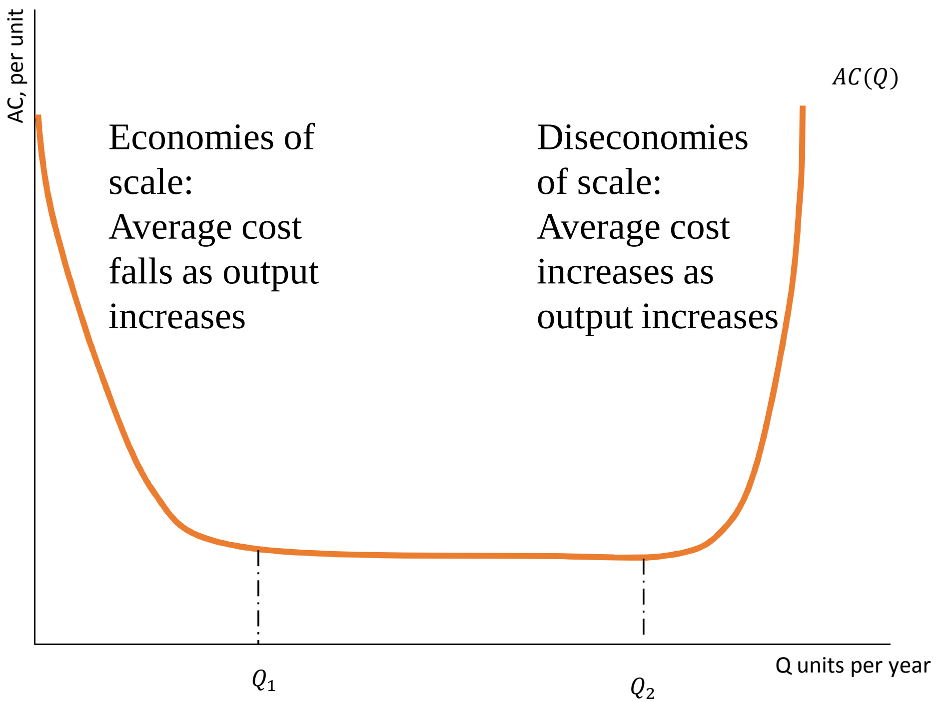
$$MC(Q) = \frac{\Delta TC(Q)}{\Delta Q}$$

- The relationship between the two is such that:
  - When AC is decreasing in Q, $AC(Q) > MC(Q)$
  - When AC is increasing in Q, $AC(Q) < MC(Q)$
  - When AC is at a minimum, $AC(Q) = MC(Q)$

Economies and diseconomies of scale

- We saw before that when a firm exhibits increasing returns to scale, output increases more than proportionally to an proportional increase in both inputs: The firm's average cost falls as output increases.
  - If a firm's average cost decreases as output increases, the firm is said to enjoy Economies of Scale.
- when a firm exhibits decreasing returns to scale, output increases less than proportionally to an proportional increase in both inputs: The firm's average cost increase as output increases.
  - If a firm's average cost increases as output increases, the firm is said to enjoy Diseconomies of Scale.
- when a firm exhibits constant returns to scale, output increases proportionally to an proportional increase in both inputs: The firm's average cost stays unchanged as output increases.
  - If a firm's average cost neither increases or decreases as output increases, the firm does not enjoy economies, or diseconomies of scale.

Economies and diseconomies of scale



The smallest quantity at which LRAC attains its minimum efficient scale (MES).

The size of the MES relative to the size of the market indicates the significance of economies of scale in particular industries

Large MES-market size ratio shows significant economies of scale.

Some examples of production functions

	Production functions		
	$\mathbf{Q = L^2}$	$\mathbf{Q = \sqrt{L}}$	$\mathbf{Q = L}$
L(Q)	$L = \sqrt{Q}$	$L = Q^2$	$L = Q$
TC(Q)	$TC(Q) = w\sqrt{Q}$	$TC(Q) = wQ^2$	$TC(Q) = wQ$
AC(Q)	$AC(Q) = w/\sqrt{Q}$	$AC(Q) = wQ$	AC(Q)=w
How does long run average cost vary with output	Decreasing	Increasing	Constant
Economies/ diseconomies of scale?	Economies of scale	Diseconomies of scale	Neither
Returns to scale?	Increasing	Decreasing	Constant

The output elasticity of total cost as a measure to the extent of Economies of scale

- Output elasticity of total cost is the percentage change in total cost per 1 percent change in output.

$$\epsilon_{TC,Q} = \frac{\frac{\Delta TC}{TC}}{\frac{\Delta Q}{Q}} = \frac{\Delta TC}{\Delta Q}\frac{Q}{TC}$$

- Recall that $AC(Q) = \frac{TC(Q)}{Q}$; $MC(Q) = \frac{\Delta TC(Q)}{\Delta Q}$
- We can therefore rewrite the output elasticity of total cost such as:

$$\epsilon_{TC,Q} = \frac{MC}{AC}$$

The output elasticity of total cost as a measure to the extent of Economies of scale

- Taking account of the relationship between long run average and marginal cost corresponds with the way average cost varies with output. We can tell the extent of economies of scale, using the output elasticity of total cost.

Value of $\epsilon_{TC,Q}$	MC versus AC	How AC varies as Q increases	Economies/ diseconomies of scale
$\epsilon_{TC,Q} < 1$	$MC < AC$	Decreases	Economies of scale
$\epsilon_{TC,Q} > 1$	$MC > AC$	Increases	Diseconomies of scale
$\epsilon_{TC,Q} = 1$	$MC = AC$	Constant	Neither

Short run total cost curve

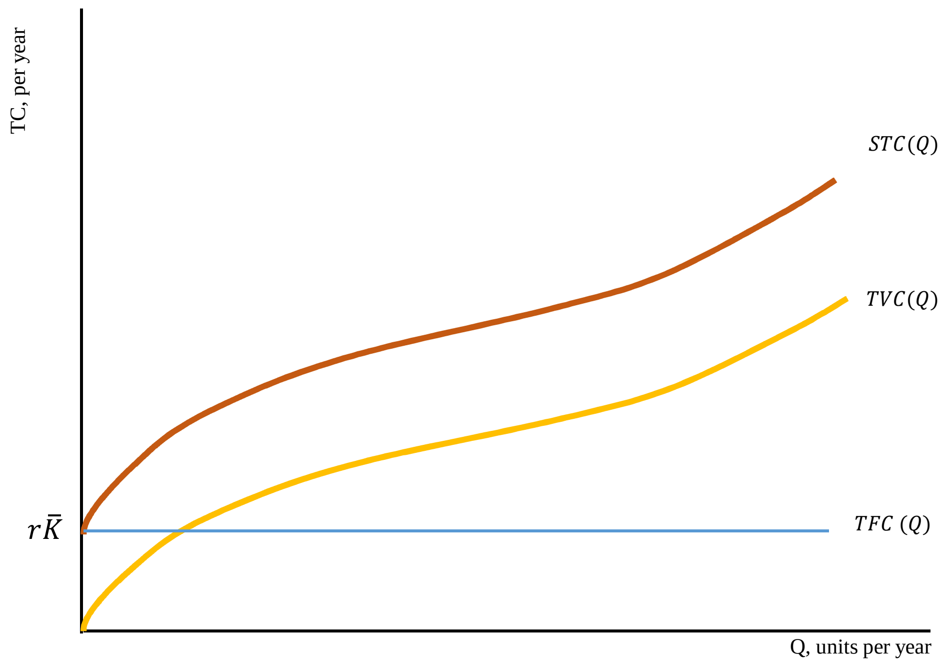
- We have seen when looking at the firm's cost minimization problem, that in the short run the firm faces both fixed and variable costs. The firm's short run total cost will be the sum of those two components.

$$STC(Q) = TVC(Q) + TFC(Q)$$

- Assuming the firm is constrained by the amount of capital it can use, $\overline{K}$, and that the price of capital is r, we can rewrite the expression for the short run total cost of the firm as:

$$STC(Q) = TVC(Q) + r\overline{K}$$

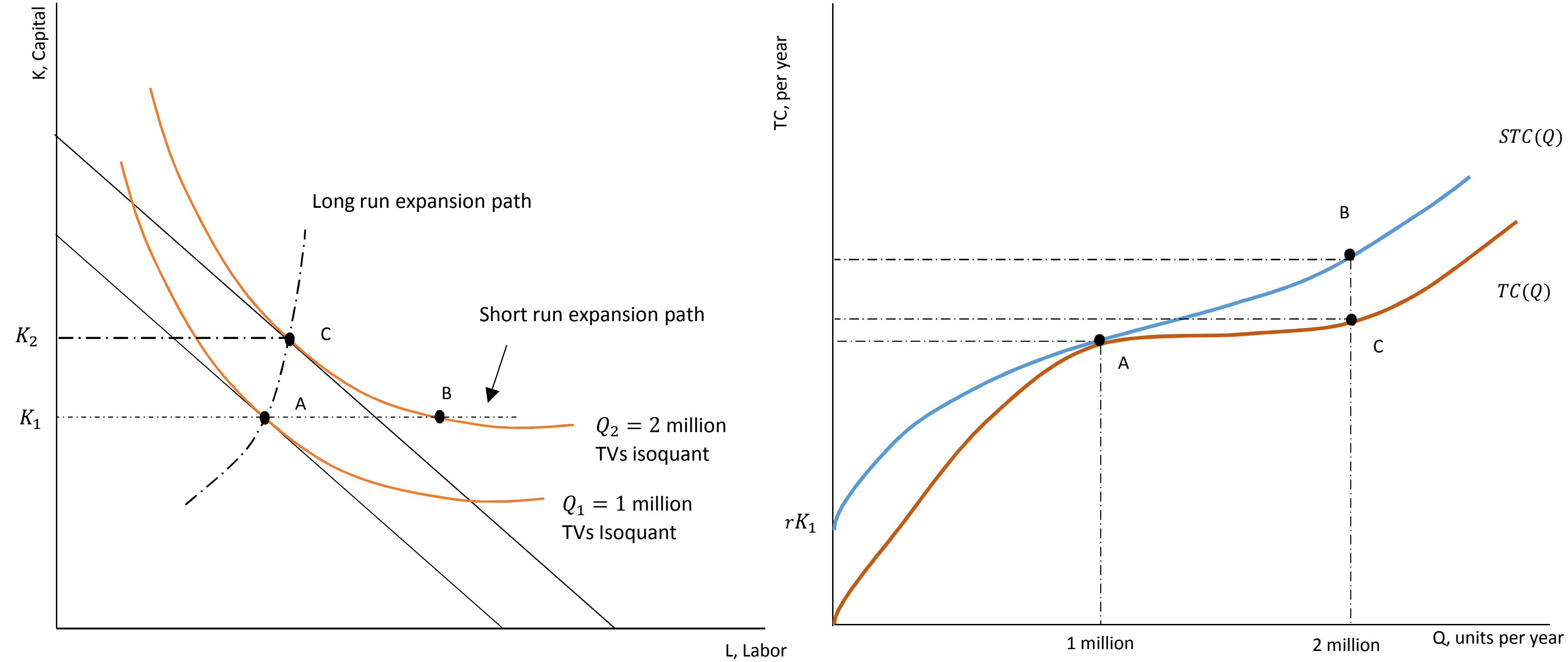
Short run total cost curve



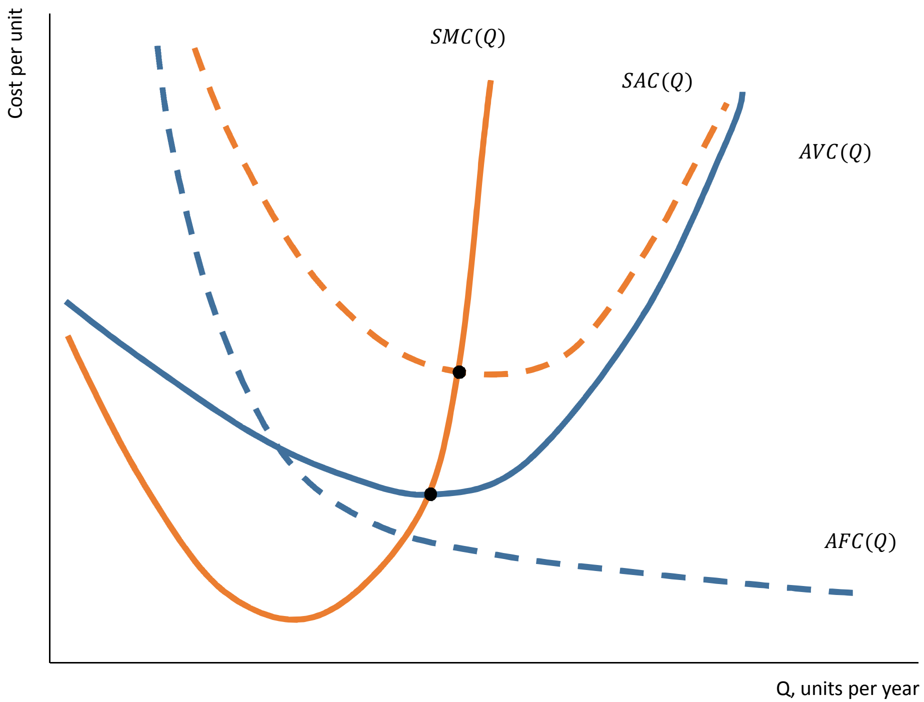
- Let's go back to the production function we have been using:
  $$Q = 50\sqrt{KL}$$
- Assume again that $w = 25$ and that $r = 100$. If capital is fixed at a level $\overline{K}$
- What is the short run total cost curve? What are the total variable and total fixed cost curves?

$$STC(Q) = \frac{Q^2}{100\overline{K}} + 100\overline{K}$$

Relationship between the long run and short run total cost curves



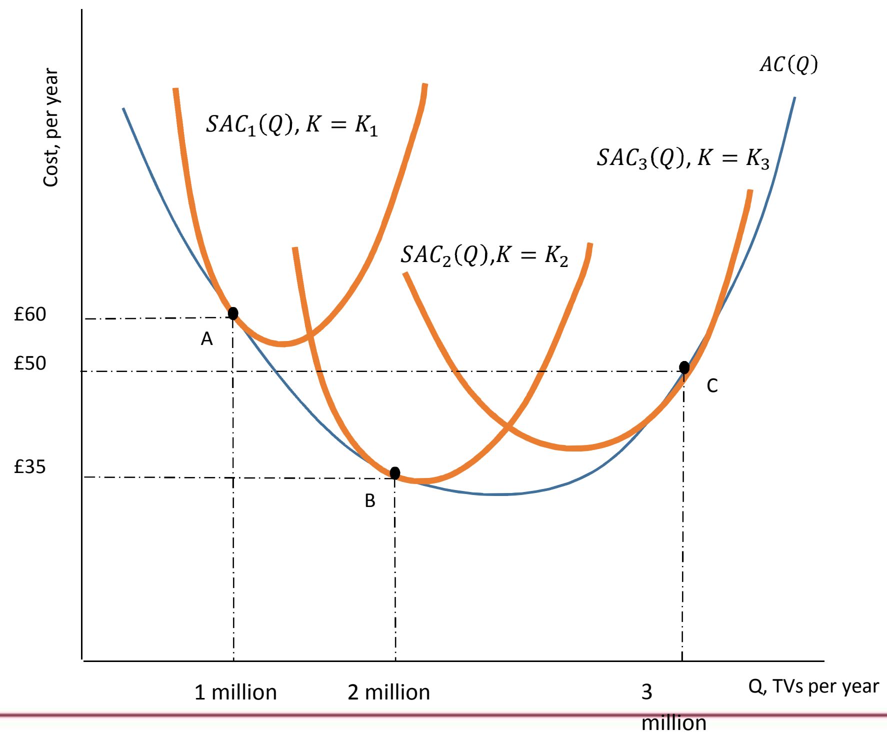
Short run average and marginal cost curves



- Short run average cost:
$$SAC(Q) = \frac{STC(Q)}{Q}$$
- Short run marginal cost:
$$SMC(Q) = \frac{\Delta STC(Q)}{\Delta Q}$$
- Average fixed and variable cost:
$$AFC = \frac{TFC}{Q} \; ; \; AVC = \frac{TAC}{Q}$$

Where we can write that:
$$SAC(Q) = AFC + AVC$$

The long run average cost curve as an envelope curve



- The long run average cost curve forms a boundary around the set of shot run average cost curves corresponding to different levels of output and fixed input.
- Each short run average curve corresponds to a different level of fixed capital.
- Point A is optimal for the firm to produce 1 million TVs per year, with fixed level of capital K_1.

Economies of scope

- We have so far been looking at firms producing a single product. Consider now a firm which produces two different products. The firm's total cost will depend on the quantity of product 1 it manufactures (Q_1), and on the quantity of product 2 (Q_2).

- When it is less costly for a single firm to produce both products, relative to two separate firms manufacturing the product separately, that is, when we have that:

$$TC(Q_1, Q_2) < TC(Q_1, 0) + TC(Q_2, 0)$$

- Efficiencies have arisen, which are called economies of scope
- The additional cost of producing Q_2 units of the second product, when the firm is already producing Q_1 units of the first product is lower that the additional cost of producing Q_2 when the firms does not manufacture product 1.

$$TC(Q_1, Q_2) - TC(Q_1, 0) < TC(0, Q_2) + TC(0,0)$$

EC109 Microeconomics – Term 2, Part 1

Perfectly competitive markets

Laura Sochat

02/02/2016

Plan

- Economic Profits
- The profit maximising output level
- Perfect competition introduced
- The short run perfectly competitive equilibrium
  - Price determination, the firm's short run supply curve, the market short run supply curve
- The long run perfectly competitive equilibrium
  - Price determination, the firm's long run supply curve, the market long run supply curve
  - Constant, increasing and decreasing cost industries

Economic Profit

- Profit can be thought of as being the difference between the amount of money the firm takes in, and the amount of money it pays out.

$$Economic\ Profit = Total\ revenue - Total\ Economic\ Costs$$

- Remember the difference between Economic and Accounting costs. We will always be referring to Economic costs when talking about Economic Profit.
- When a firm wants to maximise profits, it looks at all decisions it is taking: Choice of output levels, input levels, advertising, and take those decisions at the same time.

$$Economic\ Profit = Total\ revenue - Total\ Economic\ Costs$$

- Total revenue curve

A firm's total revenue (assuming the firm sells all its output at the same price), is defined as the quantity of output it sells, times the price at which it sells its output.

$$Total\ revenue = Quantity\ of\ output\ sold \times Price$$

A firm can derive its total revenue curve from its firm's specific demand function (NOT the same as the market demand we have seen before):

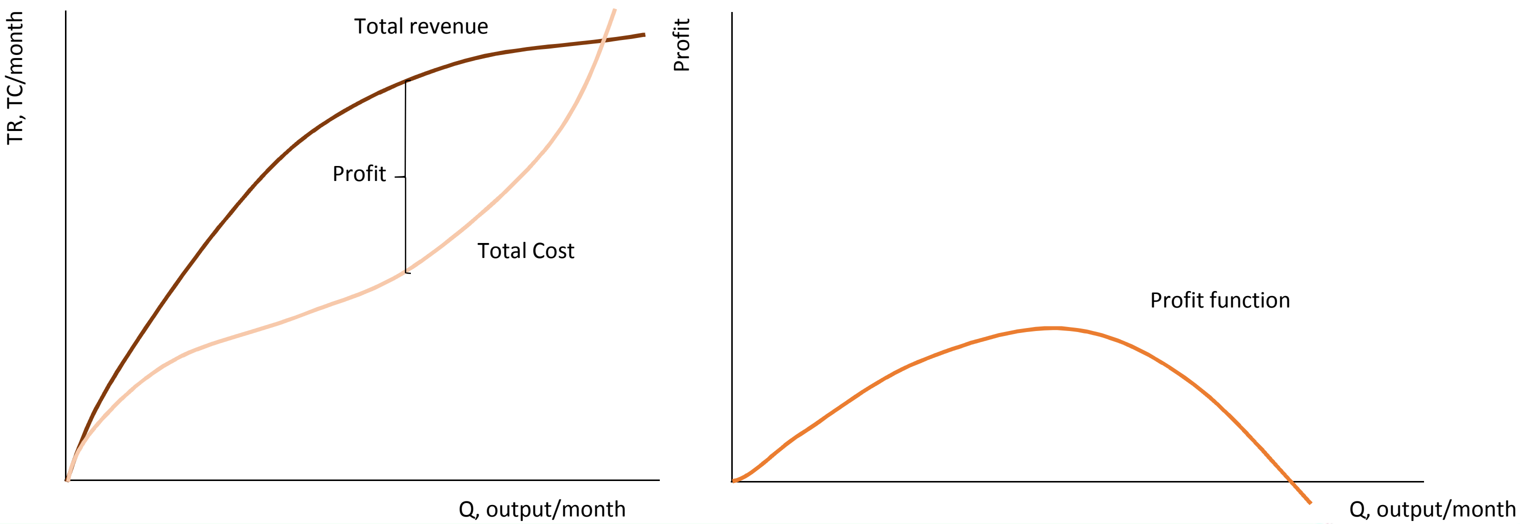
A schedule showing the quantity of a single firm's output demanded for any price charged by that particular firm.

- Total cost curve

Schedule that relates the firm's total cost to the amount of output produced

Maximizing Profit

- When choosing how much output to produce, the firm must therefore look for the point at which the total revenue curve is at the greatest distance above the total economic cost curve:



Maximizing Profit- The optimal level of output

- For a firm to maximise its profits, it cannot be the case that changing the level of output it produces increases its profits.
- Remember the concept of marginal cost seen earlier in the course; a similar concept, marginal revenue, exists for the change in total revenue given a change in output produced by the firm.

$$Change\ in\ profit = marginal\ revenue - marginal\ cost$$

- The firm should produce more output whenever the amount of money the additional sale of that output brings to the firm exceeds the additional cost incurred by the production of those additional unit
- Similarly, the firm should produce less output whenever marginal revenue is lower than its marginal cost, as profits would fall.
  - The profit maximising level of output occurs where marginal revenue = Marginal cost

Perfectly competitive market: What is perfect competition?

- Perfectly competitive markets have four characteristics:
  - The industry is fragmented
  - Firms produce undifferentiated products
  - Consumers have perfect information about prices all sellers in the market set
  - There is equal access to resources (e.g. technology)

- Those characteristics imply the following working of perfectly competitive markets
  - Sellers and buyers act as price takers: This comes from the fact that markets are fragmented
  - There is a law of one price: Firms produce undifferentiated products and consumers have perfect information regarding prices
  - There is free entry into the industry: All firms have access to the same technology and inputs.

Profit maximisation by a price taking firm

- As mentioned above, a firm's marginal revenue refers to the change in the firm's total revenue as the firm change the amount of output it produces.
- In the case of a price taking firm, its specific demand function will be a horizontal line at the market price: The firm can sell as much output as it wishes, at that price.

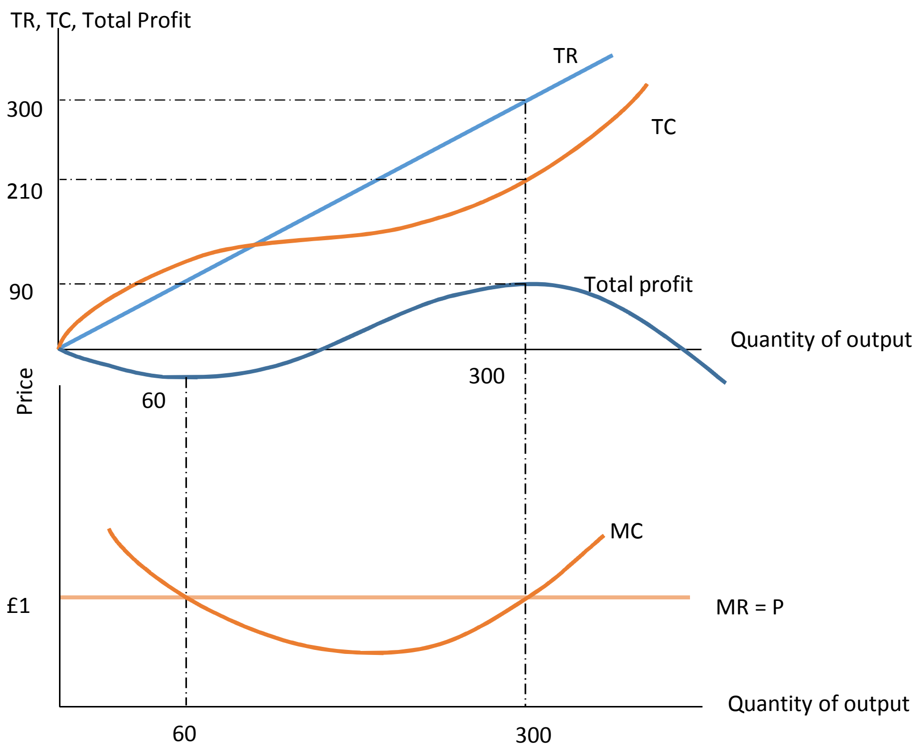
- For a price taking firm, the revenue earned from the sale of an additional unit of output, is the revenue from the marginal unit itself (the revenue earned from the other units is unaffected)
- Therefore; a price taking firm's marginal revenue is always equal to the price that it takes as given.
  - The profit maximising level of output occurs where the price equals to marginal cost

Total revenue, total cost and profits of a price taking firm

- Assuming the price of roses is expected to be £1, the table below summarises the information available to the price taking firm, trying to maximise its profits.

Q	TR(Q)	TC(Q)	π
0	0	0	0
60	60	95	-35
120	120	140	-20
180	180	155	25
240	240	170	70
300	300	210	90
360	360	300	60
420	420	460	-40

Profit maximisation by a price taking firm- Optimal output level



- The total revenue curve has a slope of 1 = Marginal revenue.
- Total profit is maximised at $Q = 300$
- For any $60 < Q < 300$, the firm increases profits by increasing the quantity of output it produces. In other words, marginal revenue is higher than marginal cost; $P > MC$.
- Consider $Q = 60$, at this point, we also have $P = MC$!

Price determination in the short run

- Consider a situation where:
  - The number of firms is fixed
  - At least one of the input used in production is fixed
- Remember the cost structure of a firm in the short run, when it produces a quantity Q of output:

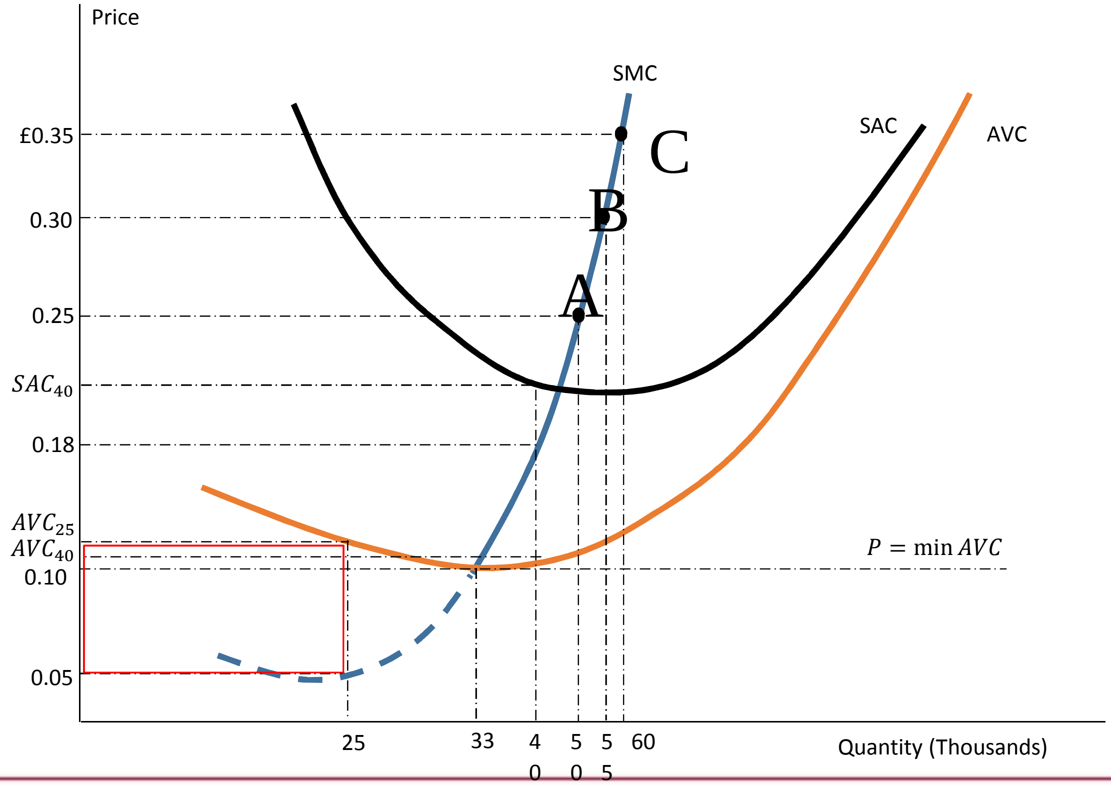
$$STC(Q) = \begin{cases} SunkFC + NonSunkFC + TotalVC(Q), \text{ when } Q > 0 \\ SunkFC, \text{ when } Q = 0 \end{cases}$$

- In what follows, we derive the supply curve for a price taking firm in different scenarios:
  - All the firm’s fixed costs are sunk
  - Some fixed costs are sunk, some others are non sunk

The short run supply curve when all fixed costs are sunk

- What information do we want to obtain through the short run supply curve? It will tell us how the firm's profit maximising output decision changes as the market price changes.
- In the situation where all fixed costs are sunk, we have that $NonsunkFC = 0$, and therefore, $TotalFC(Q) = SunkFC$
- Consider three possible market prices:
  - $P = £0.25$
  - $P = £0.30$
  - $P = £0.35$
- Applying our profit maximisation rule, $P = MC$, we can obtain the firm's profit maximising level of output at each possible price level.

The short run supply curve when all fixed costs are sunk

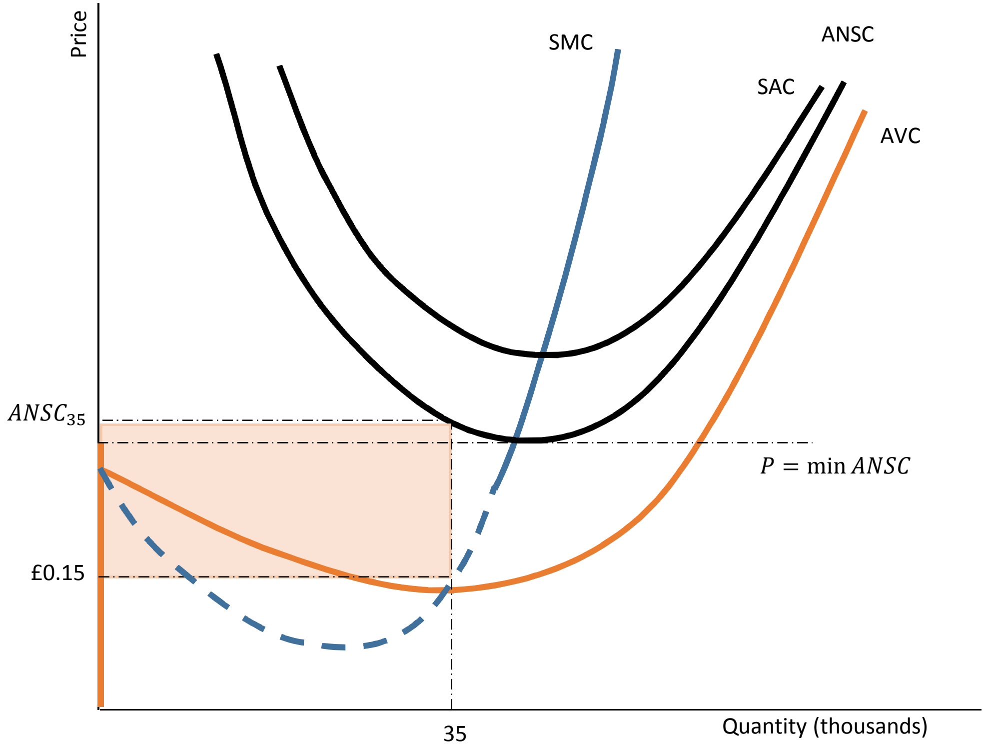


- The firm's supply curve corresponds to its marginal cost curve. But is this true at all possible price levels?
- On the graph above, consider the firm's decision at price £0.05: At this price, the firm faces a loss: It faces sunk fixed costs, and the difference between the price at which it sells the roses and the average variable costs it faces.

The short run supply curve when all fixed costs are sunk

- From the figure above, the firm will never produce when the price is lower than it average variable cost, as it would cut its loses (to its sunk fixed costs only), by shutting down.
- What does that tell us about a firm's short run supply curve?
  - At any price below the firm's short run (minimum) average cost, the firm produces no output (at any $P < 0.10$ on the figure above). This price is called the shut down price, and the firm's supply curve is a line along the vertical axis, at $Q = 0$.
  - For a market price above the firm's minimum average variable cost, it will produce a positive amount of output, and the firm's short run supply curve will coincide with its marginal cost curve.
- What can we say about the firm's decision at a price of £0.18, for example? Wouldn't the firm be experiences negative economic profits at this price?

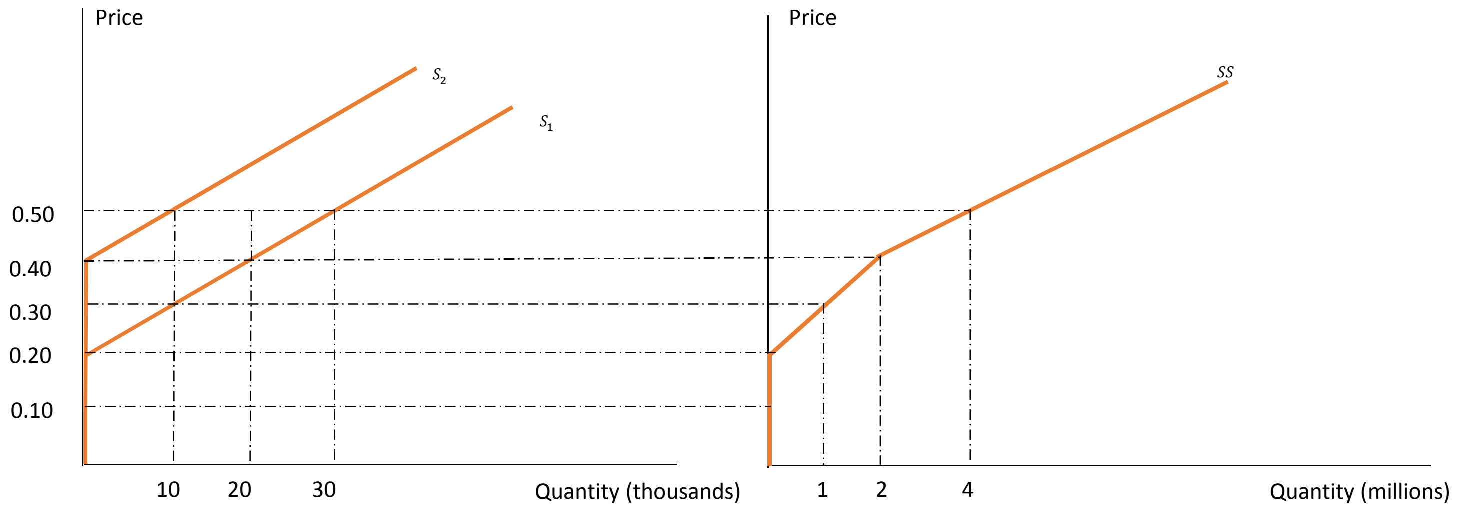
The short run supply curve when some fixed costs are sunk, and some others non sunk



- In this scenario, we have that:
  $$TFC = SFC + NSFC$$
- The shutdown rule differs in this case. We need a new curve, the firm's average non sunk cost curve, defined as the sum of the AVC, and ANSFC $\left({}^{SNFC}/_{Q}\right)$
- Suppose $P = £0.15$. At this price, the firm incurs a loss:
  $$Loss = SFC + 35{,}000(ANSC - £0.15)$$
- The firm's supply curve is vertical for prices below its minimum's ANSC, and coincides with the SMC curve for prices above that level.

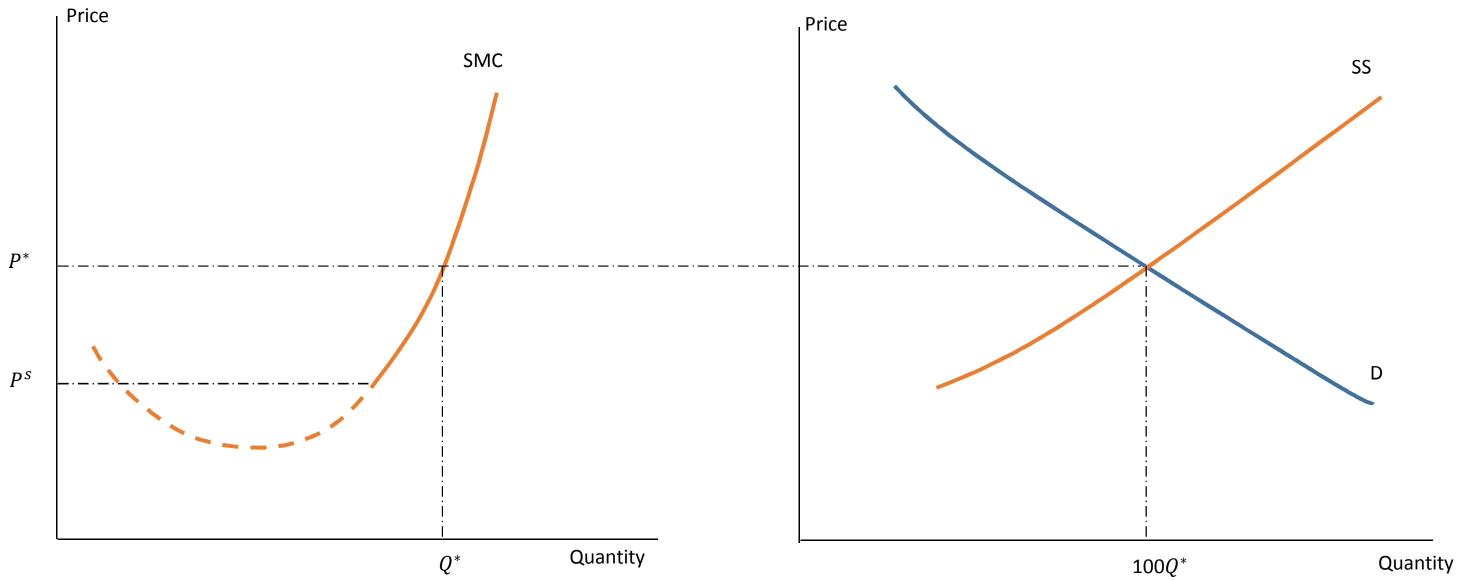
The short run market supply curve

- Number of firms in the market is fixed in the short run: market supply an any price will equal the sum of the quantities that each established firm supplies at that price.
- Assume there are two types of firms such as: there are 100 firms of type 1, each with a supply curve given by S_1 with a shut down price of £0.20/rose, and 100 firms of type 2, each with a supply curve given by S_2, with a shut down price of £0.40/rose.

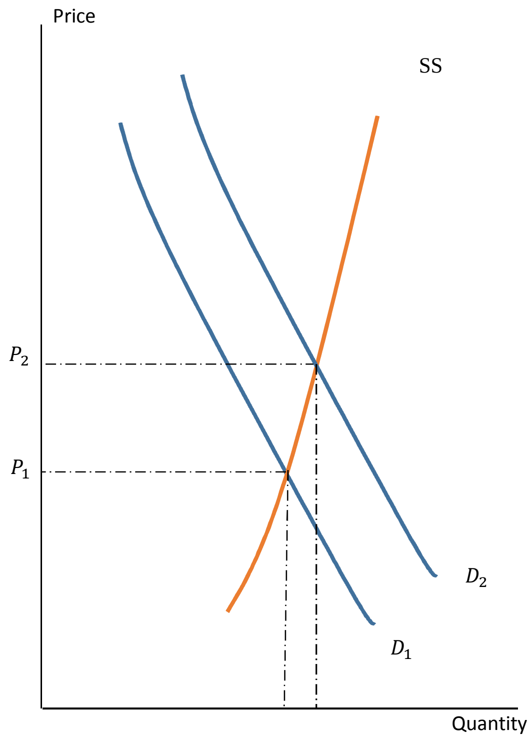


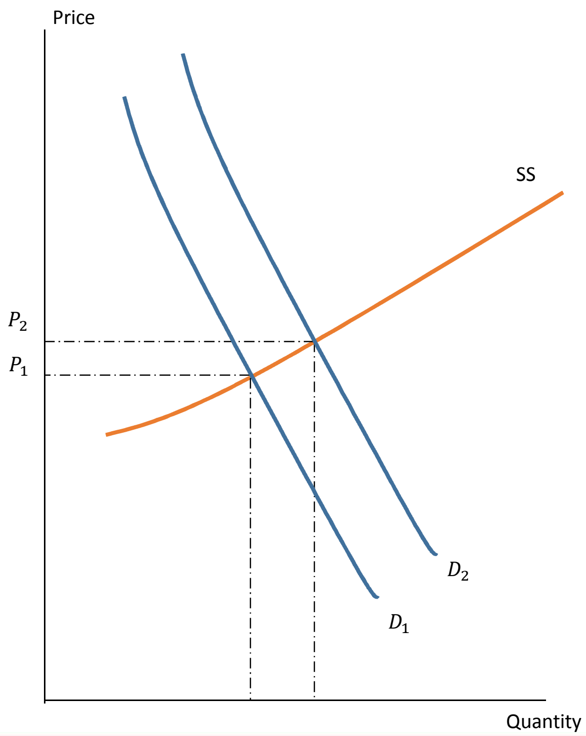
Short run perfectly competitive equilibrium

- The short run perfectly competitive equilibrium occurs when quantity demanded by consumers equals quantity supplied by all firms in the market. Below shows a situation where there are 100 identical producers.



Comparative statics and the Price elasticity of supply

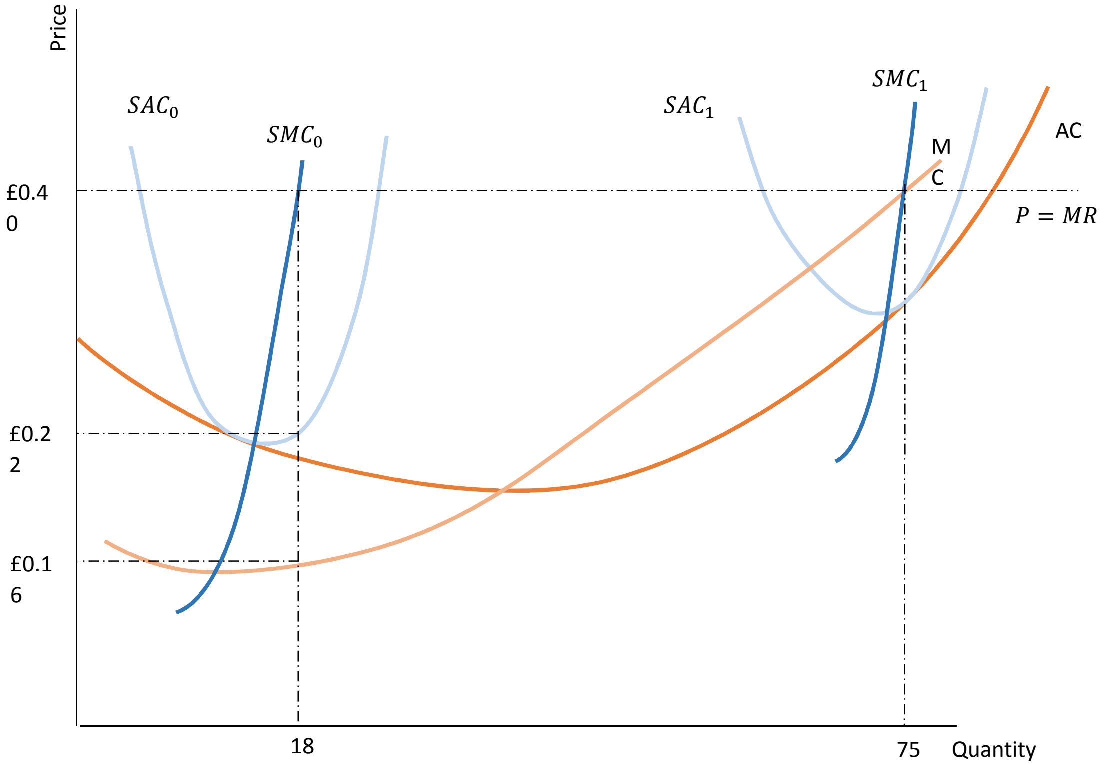




A given shift in demand in a market with relatively inelastic supply will have a more dramatic impact on the market price than the same shift in demand in a market with relatively elastic supply.

What would happen to equilibrium price and quantity if the number of firms in the market increases?

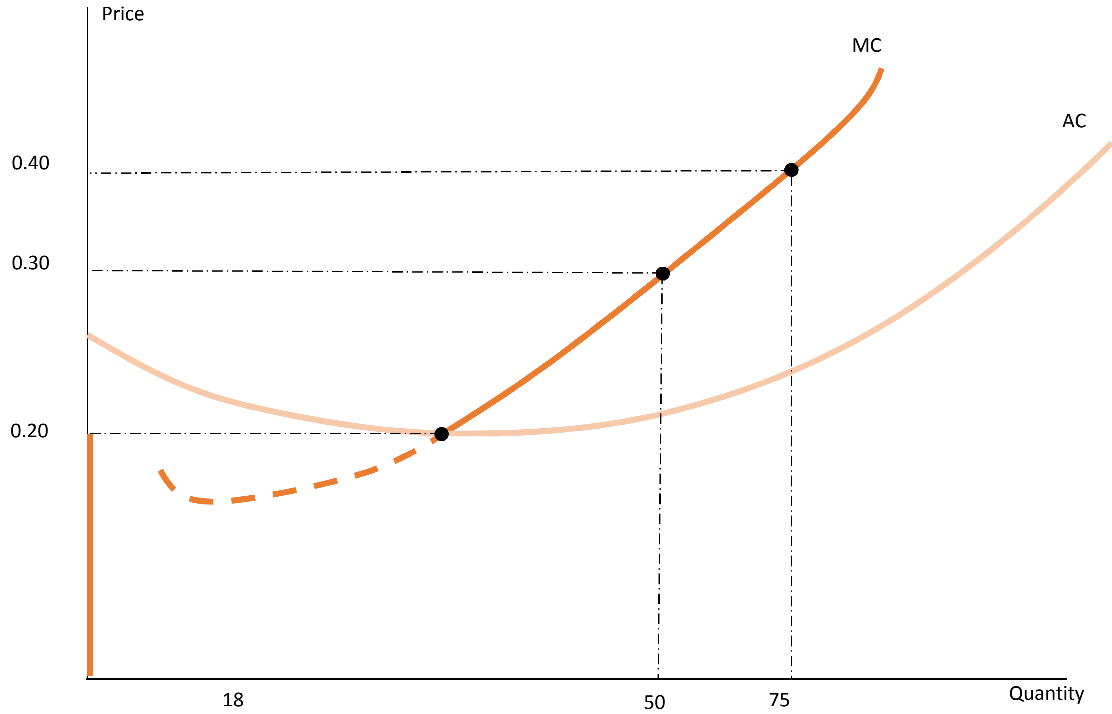
Price determination in the long run



Suppose $P = £0.40$;

- In the short run, $P = SMC$ implies a short run profit maximising quantity of output of 18,000.
- In the long run, the firm can increase its profits (by increasing its plant size, for example) and produce at the long run profit maximising output level of 75,000, where $P = MC$

The long run supply curve of a price taking firm

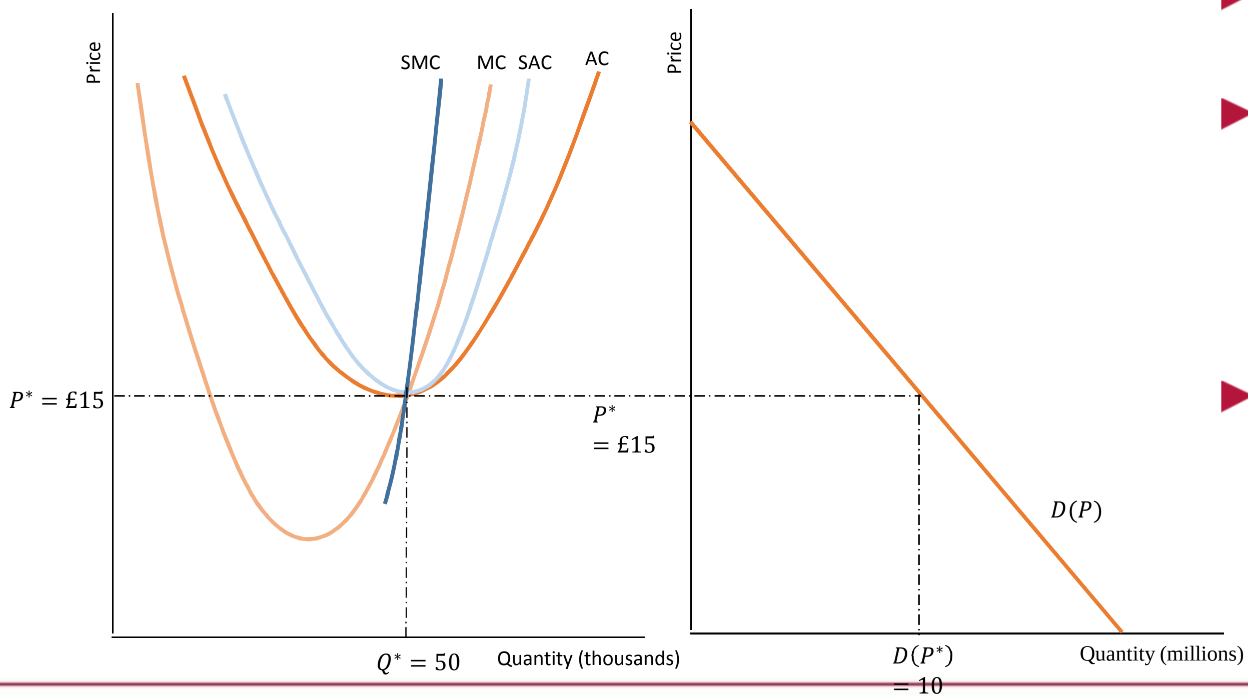


- For prices above the minimum level of long run average cost, the firm's long run supply curve coincides with its long run marginal cost curve.
- For prices below, it is a vertical line, along the vertical axis.

Perfectly competitive equilibrium in the long run: Free entry

- In the long run, the number of firms is not fixed anymore: Firms can enter and leave the industry. More particularly, if there are potential for any firm to enter the market and make positive economic profit, it will do so.
- The long run perfectly competitive equilibrium will therefore happen at a price such that supply equals demand, and firms have no incentive to enter, or to exit the industry.
- One can characterise a long run equilibrium regarding the equilibrium price, quantity and number of firms with respect to three conditions:
  - Firms choose how much to produce to maximise their profits, and inputs to minimise their costs : $P^* = MC(Q^*)$
  - Each firm in the industry will earn zero profits : $P^* = AC(Q^*)$
  - Demand equal supply : $D(P^*) = n^*Q^* \rightarrow n^* = {}^{D(P^*)}/_{Q^*}$

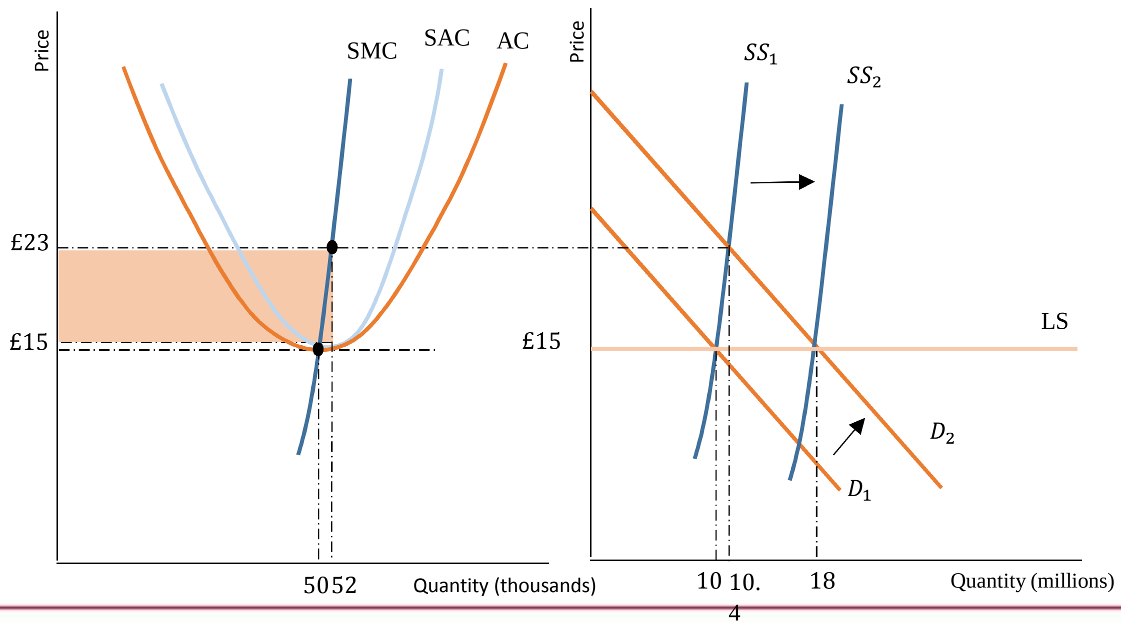
Perfectly competitive equilibrium in the long run: Free entry



- Graph shows the three above conditions.
- Because we have that P=MC=AC, we have a situation where each firm produces at its minimum efficient scale.
- This leads to the condition that the number of firms is determined by market demand over the minimum efficient scale.

Long run market supply curve, constant cost industries

- The long run market supply curve will tell us the total quantity of output that will be supplied in the market at various price, after all long run adjustments have been made.



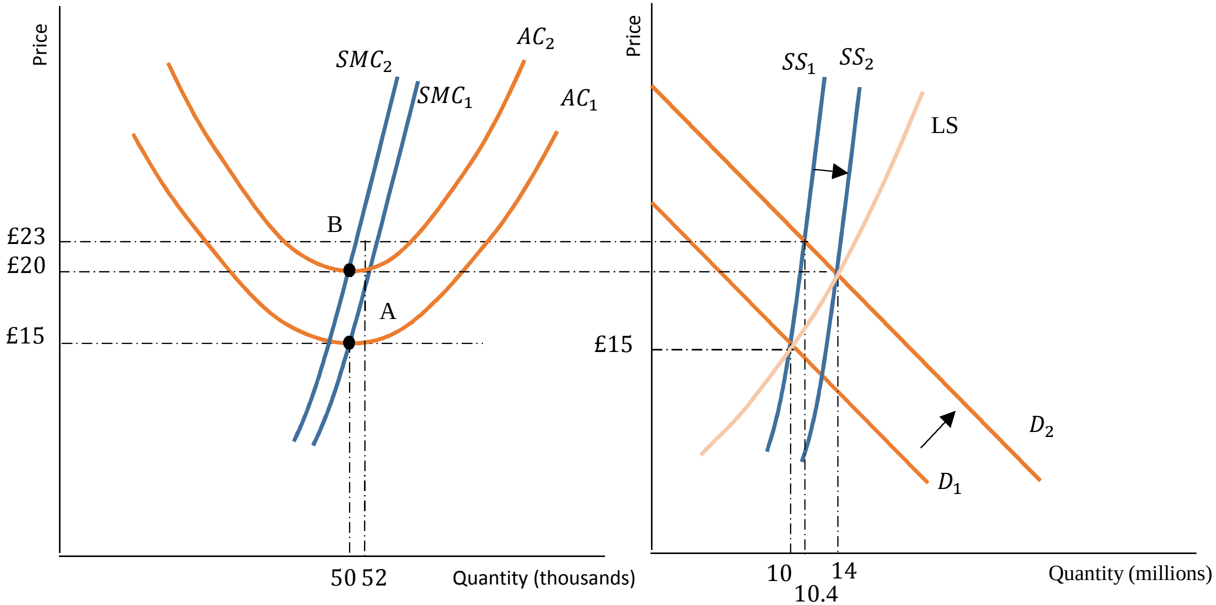
In a perfectly competitive market that is initially at long run equilibrium, additional market demand will lead to new firms entering.

The price may increase in the short run, in the long run new entry will drive equilibrium price back to its original level.

The long run supply curve is a horizontal line corresponding to equilibrium price

Long run market supply curve, Increasing cost industries

- This situation refers to a case where the expansion of the industry output leads to an increase in the price of an input (specific input).



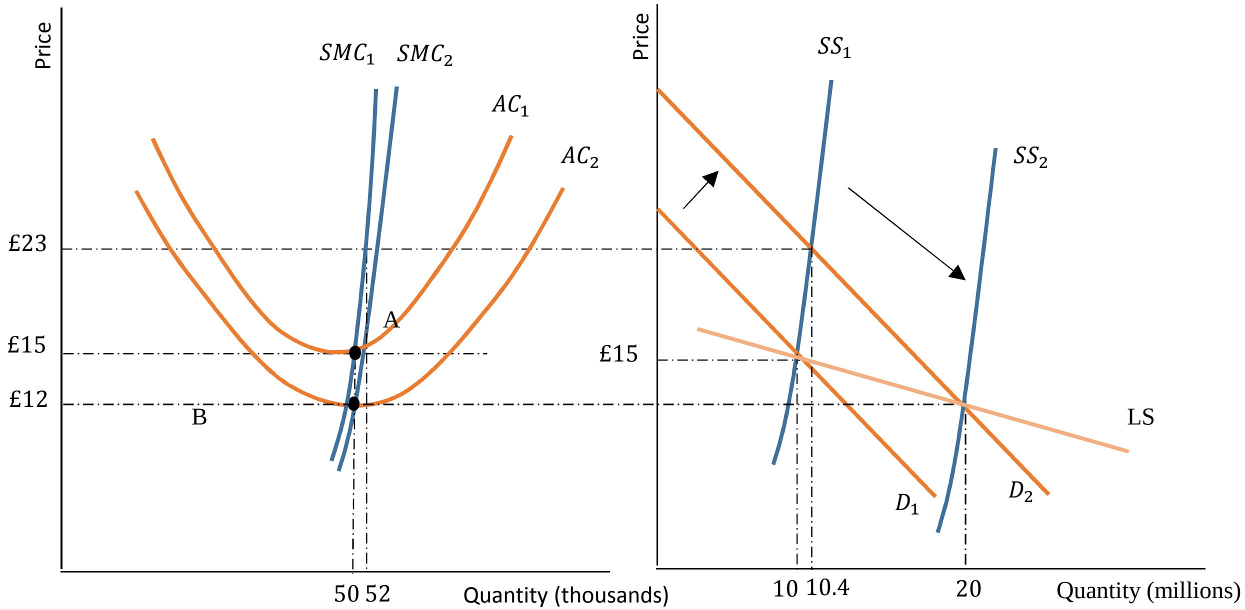
As industry output increases, prices of inputs increase; shifting up the long and short run cost functions.

After long run adjustments have taken place, the price becomes is now higher (£20).

In an increasing cost industry, the long run supply curve is upward sloping, compensating for the higher minimum AC.

Long run market supply curve, Decreasing cost industries

- This situation refers to a case where the expansion of the industry output leads to an decrease in the price of an input.



As industry output increases, prices of inputs decreases; shifting down the long and short run cost functions.

After long run adjustments (entry of new firms) have taken place, the price becomes is now lower (£12).

In an decreasing cost industry, the long run supply curve is downward sloping, reflecting the lower minimum AC.

EC109 Microeconomics – Term 2, Part 1

Perfectly competitive markets: Applications

Laura Sochat

09/02/2016

Plan

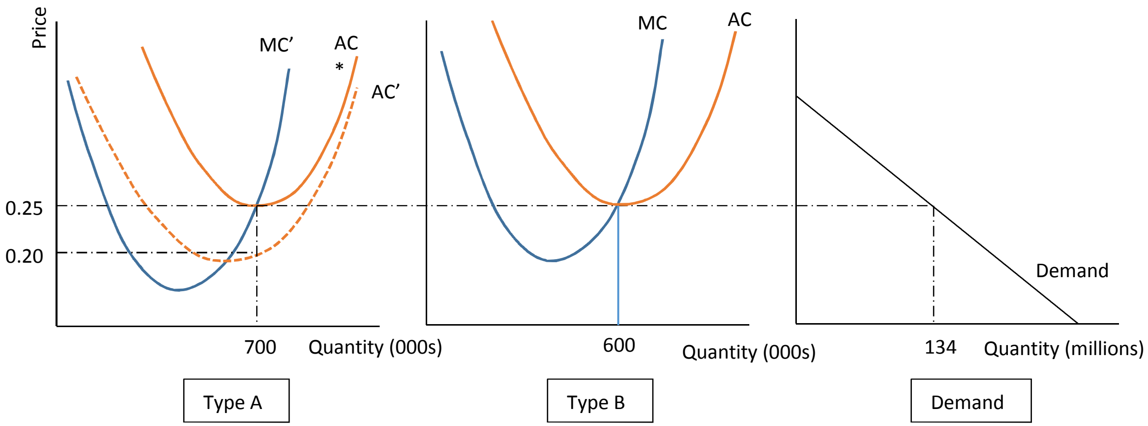
- Economic rent
- Producer and consumer surplus recap
- Welfare effect on the perfectly competitive equilibrium of:
  - An excise tax
  - A price ceiling
  - An import quota

Economic Rent

- Economic rent is defined as the difference between the maximum amount a firm is willing to pay for the services of an input and the input's reservation price (the return the input owner could get by using the input in its best alternative, outside the industry)
- Suppose, for the rose example seen above, that there are two types of growers: Type A and type B. Suppose that type A growers are much more productive than type B growers
- Assume also that both types of growers get paid the same salary of £70,000 by being employed in any firms producing roses, and that their reservation price is also equal to £70,000.
- We also assume that there are 20 type A growers only available for hiring and that the market price is equal to $P = £0.25$
- What can we say about the average and marginal cost curves of firms who hire type A growers? What would be the Economic rent in this situation?

Economic Rent

- Firms employing type A growers have lower marginal and average cost curves (MC' and AC'), and produce 700,000 roses at an average cost of £0.20.
- Firms employing type B growers have marginal and average cost such as AC and MC, and produce 600,000 roses at an average cost of £0.25.



- Type A firms save £0.05 in costs per rose due to the higher productivity of type A growers.
- To find the economic rent, we need to find the highest amount type A firms are willing to pay to hire type A growers.

Producer surplus and consumer surplus

- Producer surplus is defined as the difference between the price at which output is sold, and the marginal cost of producing the output.
  - It is the area above the supply curve and under the market price

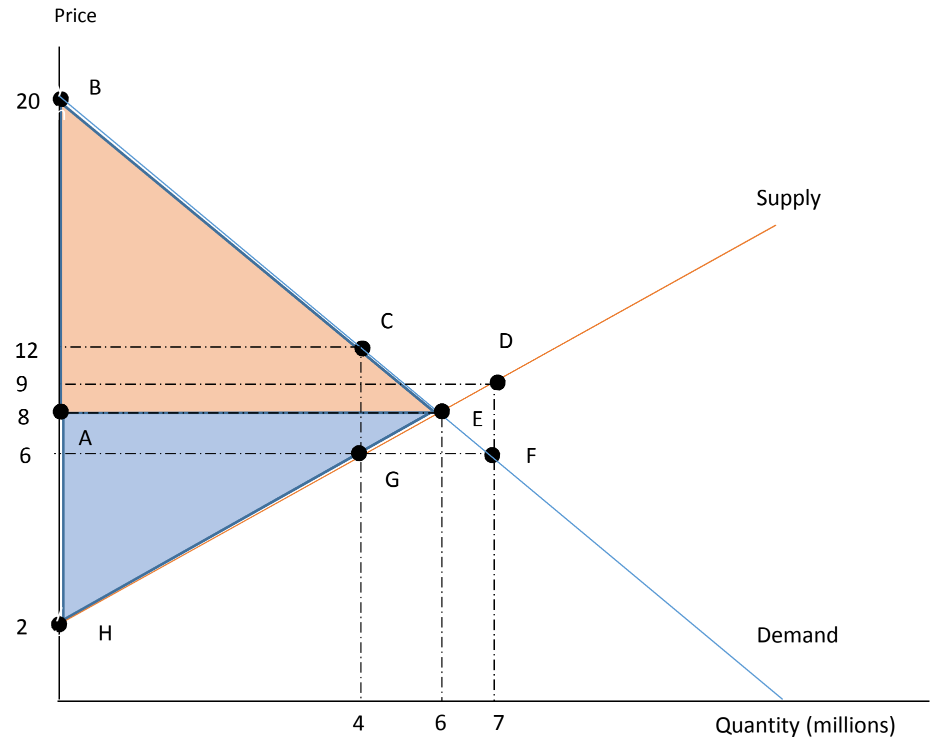
- Consumer surplus is defined as the difference between the price consumers are willing to pay for a good, and the price at which it sells.
  - It is the area below the demand curve and above the market price

- Deadweight loss is defined as the reduction in net economic benefits resulting from an inefficient allocation of resources

Introduction

- The analysis that follows will focus on a single market, and is called partial equilibrium analysis
  - We will be looking at the effect of a government intervention, for example, on a single market's equilibrium.
- Looking at the effect government intervention in one market might have on all markets simultaneously is called General Equilibrium analysis, and you will be examining this later in the course.
  - General equilibrium analysis is more complex, and the results might differ from the ones we will be looking at.
- All assumptions made before still hold (see previous lecture)
- There are no externalities

Efficient allocation of resources in perfectly competitive markets

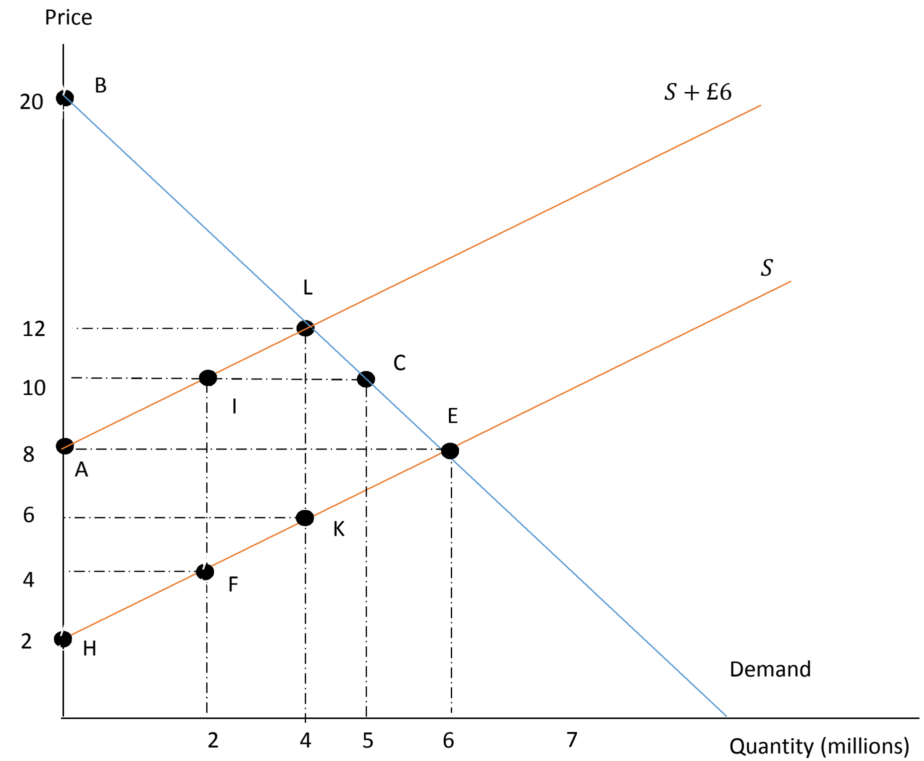


- Equilibrium happens at point E, where $P^* = 8$, and $Q^* = 6$.
- Total surplus (sum of producer and consumer surplus) $=$ £54 million
- Total surplus is maximised at the equilibrium price and quantity.
- Would it be efficient for the market to produce 4 million units? 7 million units?

An excise tax

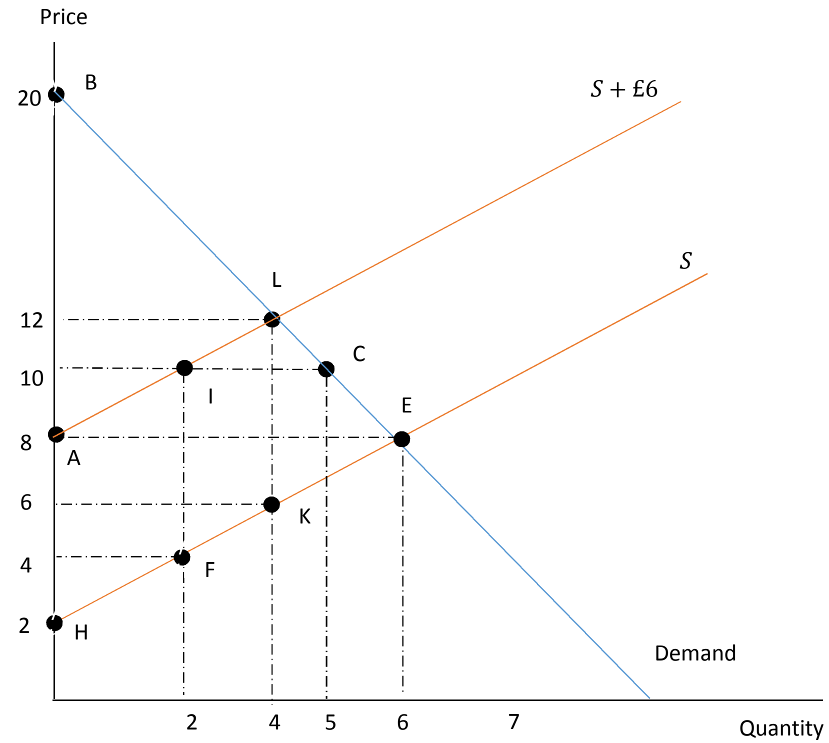
- A tax on a specific commodity, such as alcohol or tobacco is called an excise tax. We will be looking at the effect of such a tax on the price paid be consumers and producers, holding the price in all other markets constant (partial equilibrium analysis).
- In the analysis below, we will consider a market such as:
  - We are initially in equilibrium
  - The government decides to introduce an excise tax of £6 on each unit sold
- We will look at the effect of the tax on producer and consumer surplus, as well as on government budget (tax receipts), and on deadweight loss, defined as the reduction in net economic benefits due to an inefficient allocation of resources.
- In what follows, P^d refers to the price paid by consumers, P^s, the price received by producers.

Partial equilibrium analysis of an excise tax



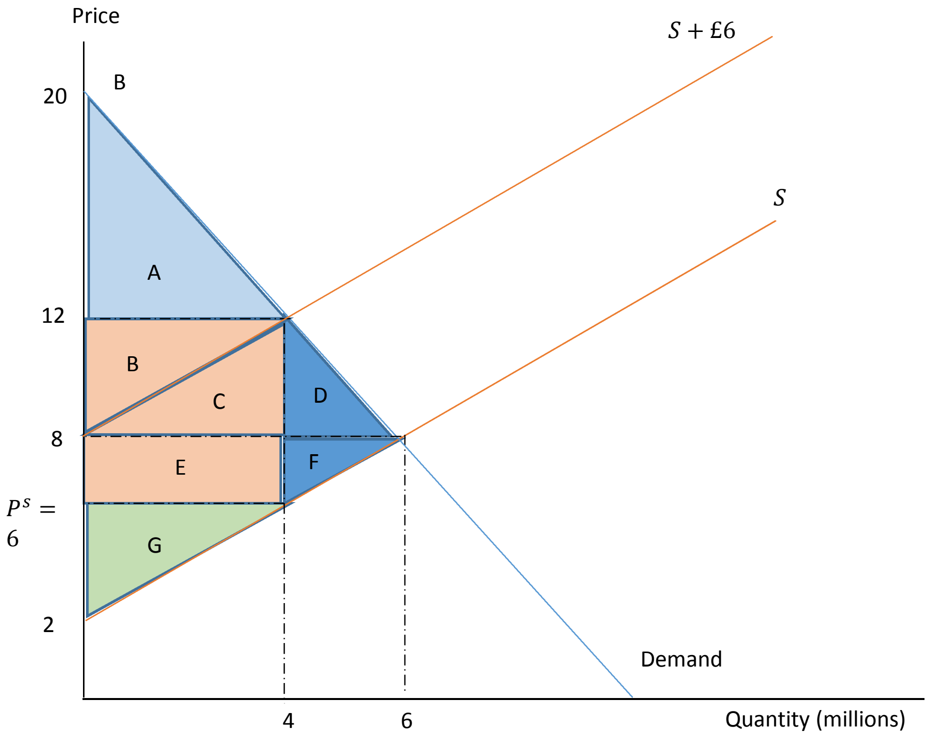
- Assuming the seller has the responsibility of collecting the tax, this creates a wedge between P^s and P^d. The curve labelled $S + £6$, represents an hypothetical supply curve, representing the wedge created by the tax.
- Suppose that in this market, the price is £10. With the tax, producers only receive $10 - 6 = £4$.
- At $P^d = £10$, consumers demand 5 millions units
- At $P^s = £4$, producers are willing to supply 2 million.
- There is excess demand

Partial equilibrium analysis of an excise tax



- Where does equilibrium happen? At the intersection of the demand and $S + £6$ curves, where $P^* = 12$ and $Q^* = 4$ million.
- In this case, we have that:
  - $P^d = £12$,
  - $P^s = £6$, and
  - The government collects a tax of £6 on each unit of output sold.

Partial equilibrium analysis of an excise tax



- Consumer surplus:
  - before the tax: $A + B + C + D = £36$ million)
  - After the tax: $A = £16$ million
- Producer surplus:
  - Before the tax: $\mathrm{E} + F + G = £18$ million
  - After the tax: $\mathrm{G} = £8$ million
- Tax receipts:
  - Before the tax: 0
  - After the tax: $B + C + E = £24$ million
- The tax has decreased consumer surplus by £20 million, producer surplus by £10 million, and has increased tax receipts by £24 million.
- Deadweight loss is therefore equal to $£30 - £24 = £6$ million (area D + F)

The effect of a price ceiling on the perfectly competitive equilibrium

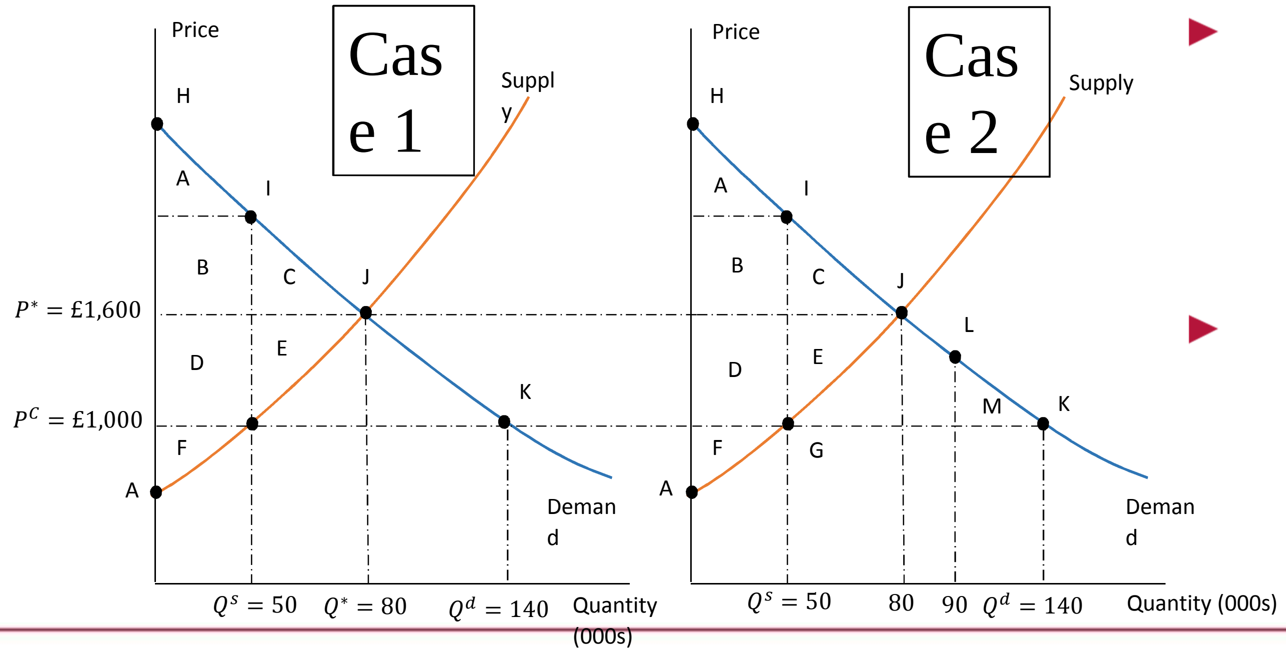
- A price ceiling refers to a situation where the government sets a maximum allowable price.

- The following effects will be observable:
  - Market clearing will not happen, due to excess demand
  - The market will be under producing, relative to the equilibrium level
  - Producer surplus will be lower than without the price ceiling
  - Some of the lost producer surplus will be transferred onto consumers
  - Consumer surplus may increase or decrease as a result of the price ceiling
  - There will be deadweight loss

- The following example looks at rent controls.

The effect of a price ceiling on the perfectly competitive equilibrium

- Point J represents the efficient equilibrium, where $P^* = £1600$ and $Q^* = 80{,}000$. Assume now that a price ceiling is set by the government at $P^C = £1000$



- In the first case, only consumers with the highest willingness to pay rent the available houses (H to I on the demand curve)
- In the second case, consumers with lowest willingness to pay rent the available housing (L to K on the demand curve)

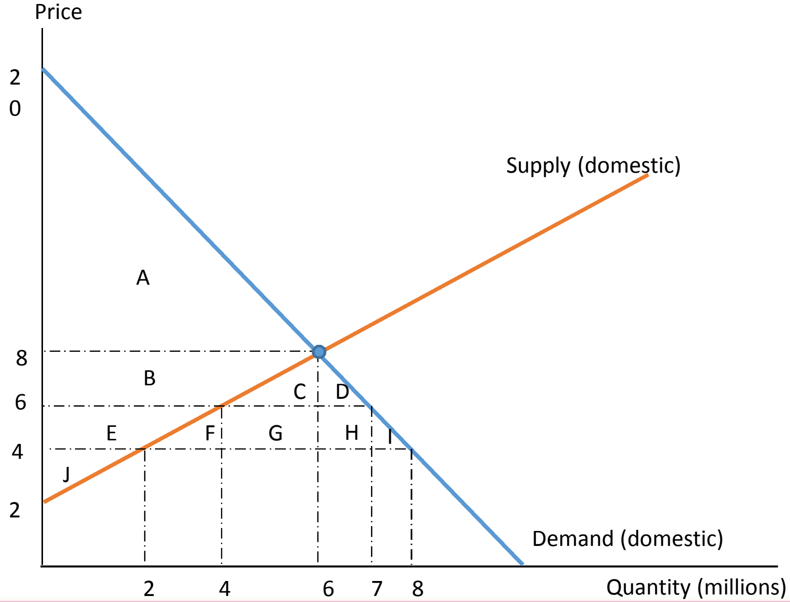
The effect of a price ceiling on the perfectly competitive equilibrium

- Let's compare the situation before, and after the government sets the price ceiling in both cases:
- In the first case, only consumers with the highest willingness to pay rent the available houses (Y to U on the demand curve)
- In the second case, consumers with lowest willingness to pay rent the available housing (T to X on the demand curve)

	No price ceiling	With rent control		Impact	
		Case 1	Case 2	Case 1	Case 2
Consumer surplus	$A+B+C$	$A+B+D$	M	$D-C$	$-A-B-C+M$
Producer surplus	$D+E+F$	F	F	$-D-E$	$-D-E$
Total surplus	$A+B+C+D+E+F$	$A+B+D+F$	$M+F$	$-C-E$	$-A-B-C-D-E+M$
Deadweight loss	$none$	$C+E$	$A+B+C+D+E-M$	$C+E$	$A+B+C+D+E-M$

The effect of an import quota on the perfectly competitive equilibrium

- Government sometimes restrict the amount of a good that can be imported into a country (they restrict free trade): We will compare the case of free trade with the extreme case of complete prohibition and to a quota restricting the amount that can be imported to a positive amount.



- With complete prohibition, equilibrium happens where demand equals supply and $P^* = 8$, and $Q^* = 6$.
- With free trade, assuming the world price is £4, domestic producers are willing to supply 2 million units of the good, while consumers demand 8 million units: With free trade, imports equal 6 million units.
- Now suppose the government wants to support a price of £6, what would be the amount of the import quota needed?

Effect of import quota on perfectly competitive equilibrium

		Effect of a quota		Impact of a quota	
	Free trade (no quota)	Total prohibition (quota = 0)	Quota = 3 million	Impact of prohibition	Impact of quota = 3 million
Domestic consumer surplus	$A + B + C + D + E + F + G + H + I$	A	$A + B + C + D$	$-B - C - D - E - F - G - H - I$	$E + F + G + H + I$
Domestic producer surplus	J	$B + E + J$	$E + J$	$B + E$	E
Total surplus	$A + B + C + D + E + F + G + H + I + J$	$A + B + E + J$	$A + B + C + D + E + J$	$-C - D - F - G - H - I$	$-F - G - H - I$
Deadweight loss	*none*	$C + D + F + G + H + I$	$F + G + H + I$	$C + D + F + G + H + I$	$F + G + H + I$

Abhinay Muthoo
Department of Economics
University of Warwick
Spring Term 2014

Lecture Notes for EC109, Microeconomics

Topic 1: Game Theory
Topic 2: Monopoly and Oligopoly

1 An Introduction to Game Theory

1.1 Preliminaries

A game is a situation in which the fate (or, payoff) of a player in the game depends *not only* on the actions of that player, but *also* on the actions of the other players involved in the game. This would suggest that all human interaction (economic or otherwise) is a game of some sort. Hence, it is important to study game situations.

Two extremely simple, but rather interesting, games are described below. In fact, both of these games capture the essence of two different types of economic interactions.

The **Battle of Sexes** game. In this game there are two players, Romeo and Juliet. It is about 6pm and they are unable to communicate with each other. However, they must decide where to spend the evening. Each must decide whether to go to the *theater* to see the ballet or to the *colosseum* to see the big fight. If they end up at different performances, then they become rather upset and hence each receives zero utility. However, if they meet at the same venue, then each receives two utils. In addition, Romeo gets another util if they meet at the *colosseum*, while Juliet gets an additional util if they meet at the *theater*. All this information is compactly summarised in the table below.

		Juliet	
		theater	*colosseum*
Romeo	*theater*	2,3	0,0
	colosseum	0,0	3,2

The **Battle of Sexes** game.

Notice that the payoff a player gets depends on the choices made by *both* players. This game, interpreted more generally, illustrates those economic situations in which agents are involved in a game of *coordination*.

The **Prisoners' Dilemma** game. This game involves two players. Each player has two feasible actions, denoted by C and D. The action C can be interpreted as *cooperate*, while the action D can be interpreted as *don't cooperate*. The payoffs associated with the possible outcomes are described below; the first number in each cell denotes player 1's payoff.

		player 2	
		C	D
player 1	C	3,3	0,4
	D	4,0	1,1

The **Prisoners' Dilemma** game.

This game illustrates the gist of many economic and social phenomena. As we shall see, it demonstrates rather strikingly the conflict between *individual* rationality and *collective* rationality, a theme that runs through much of economics, and indeed through much of human interaction in general.

1.2 Simultaneous-move games: the basic framework

Simultaneous-move games are games in which the players have to choose their actions (or, *strategies*) "simultaneously". There are three basic ingredients in such a game. First, we need to list all the players involved in the game. Second, we must specify the set of feasible actions, or strategies, available to each player. And third, we must specify the payoffs to the players for each possible outcome. Let us introduce some notation to make all this a bit more precise.

Definition of a simultaneous-move game.

(i) List of *Players*: There are N players (where $N \geq 2$), who may be denoted by player 1, player 2, player 3,..., player $N-1$, player N.

(ii) *Strategy sets*: Let A_i denote the feasible set of strategies available to player i (where $i = 1, 2, ..., N-1, N$).

(iii) *Payoffs*: Let $P_i(a_1, a_2, a_3, ..., a_{N-1}, a_N)$ denote the payoff to player i when player 1 chooses strategy $a_1 \in A_1$, player 2 chooses strategy $a_2 \in A_2$,...., and player N chooses strategy $a_N \in A_N$.

[**NB.** The payoff to player 1 depends not only on her strategy a_1, but also on the strategies $(a_2, a_3, ..., a_N)$ of the *other* players. And similarly, for each of the other players.]

A basic feature of simultaneous-move games is that the players choose their respective strategies "simultaneously". This does *not* necessarily mean at the same instant in time. What this means is that each player chooses her strategy in *ignorance* of the choices made by the other players.

The objective of each player is to choose a strategy that would maximise her payoff. However, this is not a standard optimisation problem. The payoff of each player is a function not only of her choice variable, but also of the choice variables of the other players. How could a player therefore attempt to maximise her payoff if she has only *partial* control over it? This is the basic problem that a *theory* of games has to solve.

We shall assume (unless stated to the contrary) that all the players have *complete information* on all aspects of the game. In particular, each player (i) knows who the players of the game are, (ii) knows the feasible set of strategies available to each player, and (iii) knows the payoffs to all players for each possible outcome.

Before moving on, attempt to solve the *Battle of Sexes* and the *Prisoners' Dilemma* games. If you were in Romeo's (or, Juliet's) shoes, where would you go? And, in the *PD* game, would you *cooperate* or *not cooperate*?

1.3 Nash equilibrium

For expositional convenience we shall restrict attention to simultaneous-move games with *two* players. What follows can be generalized straightforwardly to the $N > 2$ player case.

Let $G = (A_1, A_2, P_1, P_2)$ denote a two player simultaneous-move game, where A_1 and A_2 are the players' strategy sets, and P_1 and P_2 their payoff functions. Our objective is to predict the outcome of this game. That is, we want to predict the strategy choices that will be made by the players assuming that they are *rational*.

It has been argued that if $a_1^* \in A_1$ and $a_2^* \in A_2$ is the outcome (or, *solution*) of the game G, then the strategy pair (a_1^*, a_2^*) must constitute a *Nash Equilibrium* of the game G. Hence, after describing a simultaneous-move game G, it is useful to find the Nash equilibria of the game.

Definition of a Nash Equilibrium (NE, for short).

A strategy pair (a_1^*, a_2^*), where $a_1^* \in A_1$ and $a_2^* \in A_2$, constitutes a **Nash Equilibrium** of the game G if and only if

(i) $P_1(a_1^*, a_2^*) \geq P_1(a_1, a_2^*)$ *for any* $a_1 \in A_1$, *and*

(ii) $P_2(a_1^*, a_2^*) \geq P_2(a_1^*, a_2)$ *for any* $a_2 \in A_2$.

That is, in words, (i) player 1 cannot make himself better off by choosing a strategy a_1 different from a_1^*, *given* that player 2 is choosing strategy a_2^*, and (ii) player 2 cannot make herself better off by choosing a strategy a_2 different from a_2^*, *given* that player 1 is choosing the strategy a_1^*.

The following argument is one way to motivate, or justify, the Nash equilibrium concept. Suppose the players are *rational* and have calculated their respective *rational* strategies: denote them by a_1^* and a_2^*. And suppose that this pair of strategies does *not* constitute a NE. By the definition of a NE, it follows that, then, at least one of the players (say player 1) could *unilaterally* deviate to a different strategy, a_1' say, and make himself better off (i.e., $P_1(a_1', a_2^*) > P_1(a_1^*, a_2^*)$). But, this should contradict the *rationality* of the strategy a_1^*. Hence, the strategy pair (a_1^*, a_2^*) *must* be in a NE.

1.4 Methods of finding Nash equilibria

Two different methods will be suggested. The first method is especially appropriate in games in which each player has only a few strategies, but also could be useful when the players' payoff functions are *not* differentiable. The second method is quite powerful when the payoff functions are differentiable.

Method of Elimination

Basically, this method involves conducting the "Nash test" for each and every feasible strategy pair (a_1, a_2). Consider the strategy pair (a_1, a_2), and investigate whether either player can make a *unilateral* deviation (to an alternative strategy) and become better off. If neither player can benefit from such *unilateral* deviations, then strategy pair (a_1, a_2) passes the "Nash test", and therefore constitutes a NE. Otherwise, the strategy pair (a_1, a_2) does *not* constitute a NE.

Apply this method to the *Battle of Sexes* and the *Prisoners' Dilemma* games.

Reaction Function Approach

Consider the following argument, or mind experiment, conducted by player 1.

Suppose player 2 were to choose some *arbitrary* strategy $a_2 \in A_2$. Which strategy of mine would then maximise my payoff? Clearly, player 1 needs to solve the following *standard* optimisation problem:

$$\max_{a_1 \in A_1} P_1(a_1, a_2)$$

given that a_2 is *fixed* (i.e., held constant). Differentiating P_1 with respect to a_1, keeping a_2 constant, the first order condition is:

$$\frac{\partial P_1(a_1, a_2)}{\partial a_1} = 0 \tag{1}$$

Then solving for a_1 in terms of a_2, we obtain that a_1 is some function of a_2:

$$a_1 = R_1(a_2) \tag{2}$$

This function describes player 1's *best response*, or optimal reaction, to each feasible strategy of player 2.

Similarly, one obtains player 2's *best response*, or optimal reaction, to each feasible strategy of player 1:

$$a_2 = R_2(a_1) \tag{3}$$

Having derived the players' *reaction functions*, finding the set of all Nash equilibria is trivial, since any pair of strategies (a_1, a_2) constitutes a NE *if and only if* (a_1, a_2) is a solution to equations (2) and (3).

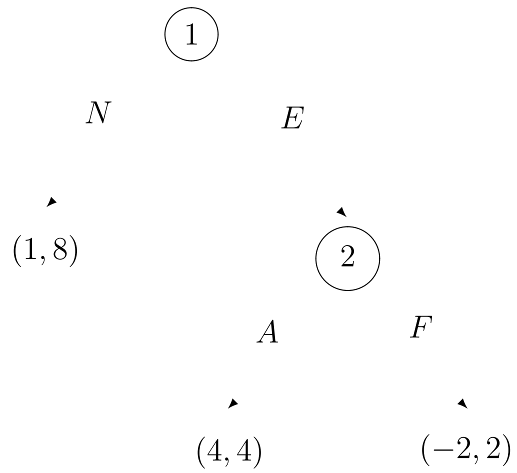
Finally, note that a simultaneous-move game may either have no NE, or exactly one NE, or many NEa. Finding all the NEa of a game can sometimes be quite easy, but sometimes quite difficult.

1.5 Sequential games: an example

So far we have studied *static* games, or games in which the players choose their respective actions *simultaneously*. This section begins our study of *dynamic* games; that is, games in which players choose their actions *sequentially*. Chess is an example of a two player sequential game; this is a rather complex dynamic game in which the players make moves (or, choose actions) alternately. Many real life (economic and non-economic) situations are also rather complex dynamic games, although perhaps not as complex as Chess. However, in order to understand some of the basic ideas and concepts of dynamic games, we can fruitfully restrict attention to an extremely simple class of sequential games. In particular, this understanding should allow you to study some economic and social phenomena that involve dynamic game interactions.

Let us begin with a simple example, which can be interpreted as a highly stylized model of entry-deterrence. Currently in a particular market (say, a market for computer software) there exists one producer-seller (or, firm), namely *IBM*. Being the *single* seller, IBM is making huge profits, say £8 million. *Richard Branson* is considering whether or not to enter this market. If he decides not to enter, then he can devote his resources to some other adventure/enterprise and make a profit of £1 million. However, if he does enter the market and starts competing with IBM, then both his profit and IBM's profit will depend on IBM's choice of action. If IBM *accepts* this entry (i.e., acquiesces) then the two share the market, and each makes a profit of £4 million. On the other hand, IBM may choose to *fight* this entry (i.e., be aggressive). In that case IBM makes a profit of £2 million and Richard Branson gets so damaged that he ends up making a *loss* of £2 million.

This economic situation is a two-player sequential game, in which Richard Branson (say, player 1) has to **first** of all decide between entering (E) or not entering (N) the market. If N is chosen, then the game ends. But if E is chosen, **then** IBM (player 2) has to decide between acquiescing (A) of fighting (F) to this entry. Following the choice made by IBM the game then ends. The game is illustrated below, with the first number in each cell the payoff to player 1.



The **Entry-Deterrence** game.

1.6 Simple two-player dynamic games: the basic framework

We restrict attention to two player sequential games with each player having just one move. Let the two players be denoted by 1 and 2. **First** player 1 moves: he chooses an action (or, strategy) a_1 from the set A_1 of feasible actions. Player 2 then observes the particular action *chosen* by player 1. **And then**, player 2 moves: she chooses an action (or, strategy) a_2 from the set $A_2(a_1)$ of feasible actions, where $A_2(a_1)$ emphasizes that the set of available actions to player 2 may be influenced by the action a_1 chosen by player 1. Let $P_1(a_1, a_2)$ denote the payoff to player i (where $i = 1, 2$) if player 1 chooses action a_1 and player 2 chooses action a_2. This completes the ingredients of a simple two-player sequential game. One crucial difference from a two-player simultaneous-move game is that player 2 **gets to see** player 1's choice **before** she makes her own choice. Another difference is that the *set* of feasible actions available to player 2 can be influenced by the action *chosen* by player 1.

The *Entry-Deterrence* game fits into this general framework, with $A_1 = \{N, E\}, A_2(N) = \emptyset$ (where $\emptyset$ means that the set $A_2(N)$ is empty; no actions are available) and $A_2(E) = \{F, A\}$.

As in simultaneous-move games, the players have complete information on all aspects of the sequential game, and want to maximise their respective payoff. Each player's payoff depends on *both* players' choice variables. Once again, the objective is to predict the strategy choices made by the players, given that they both are *rational*.

1.7 Analysing the entry-deterrence game

Suppose player 2 (IBM) chooses to fight (F) if player 1 were to enter (E) the market. In that case, player 1's best response is to stay out (choose N). Indeed, the strategy pair (N, F) constitutes a NE of this sequential game. However, this NE is *not* sensible for the following reason: that player 2's choice (F) of fighting is **not credible**. Player 1 argues that *if* I were to enter the market (choose E, instead of N), then player 2 would *not* carry out his *ex-ante* plan of fighting, because *ex-post* it is in player 2's interest to accept my entry. Hence, player 1 would not choose N, since he expects a *rational* player 2 to choose A (and *not* F). The sensible prediction is that player 1 will choose E and player 2 will choose A, which constitutes another NE of this game.

To summarise our analysis, we note that the *Entry-Deterrence* game has *two* NE*a*. One NE involves an **incredible** "threat", while the other NE can be said to involve a **credible** "threat". Since players should not be influenced by incredible threats, the former NE is not sensible.

1.8 Subgame perfect equilibrium

We now describe a general method of solving a sequential game of the type described in section 1.6. The objective is to describe a *procedure* to identify the NE*a* involving credible threats. A NE that does *not* involve an incredible threat is known as a **subgame perfect equilibrium (SPE, for short).** The SPEa are computed by the following *backwards induction* method.

Suppose player 1 has chosen some action $a_1 \in A_1$, and now player 2 has to choose an action $a_2 \in A_2(a_1)$. Clearly player 2 will choose the action that maximises her payoff **given** that player 1's action has already been made. That is, player 2 solves the following standard optimisation problem.

$$\max_{a_2 \in A_2(a_1)} P_2(a_1, a_2)$$

given that a_1 is *fixed* (has already been chosen and is *known* to player 2).

The solution to this problem will, in fact, be player 2's *best response* function:

$$a_2 = R_2(a_1)$$

Now proceeding *backwards* to derive player 1's optimal action, we note that player 1 will be aware of this fact, that if he chooses $a_1 \in A_1$, then player 2 will react by choosing $a_2 = R_2(a_1)$. Hence, player 1 will choose that action which maximises his payoff subject to the constraint that $a_2 = R_2(a_1)$. That is, player 1 will maximise $P_1(a_1, a_2)$ by choosing $a_1 \in A_1$ **s.t.** $a_2 = R_2(a_1)$. Let a_1^* denote the solution to this optimisation problem. Then the SPE outcome is: $[a_1^*, a_2^* = R_2(a_1^*)]$.

Finally, notice that the SPE concept is a *refinement* of the NE concept, designed to exclude NEa that involve incredible threats, or incredible plans. In a simultaneous-move game, every NE is a SPE; however, in a sequential game this need not be the case.

2 Imperfectly Competitive Markets: Monopoly and Oligopoly

2.1 Preliminaries

A fundamental characteristic of market economies is that the value (or, the price) of a commodity is determined by the interaction of the sellers and the buyers of the commodity in question. Understanding the forces that determine the price of a commodity (e.g., the wage rate of some job) is central to the study of microeconomics. One important determinant of the market price of a commodity is the *number* of firms that produce and sell the commodity.

This chapter explores the relationship between the market outcome (i.e., the *price(s)* at which the commodity is sold and the total *quantity* sold) and the number of firms producing and selling the commodity. The role of some other parameters (e.g., the degree of quality uncertainty) on the market outcome will be studied later on.

2.2 The basic environment

We consider a market for a homogenous commodity produced and sold by N firms, or sellers, where $N \geq 1$. The firms have an identical and constant marginal cost of production, namely $c \geq 0$. Thus, firm i's cost of producing a quantity q_i is cq_i (where $i = 1, 2, 3, ..., N$).

We assume that the buyers, or consumers, are "weak", in that they can neither set the price at which trade is to occur nor set the total quantity to be traded. However, they will certainly not buy any quantity at any price. Let $D(p)$ denote the total quantity demanded if the price is p. This market demand function, which is downward sloping (i.e., $D^{'}(p) < 0$), constrains the firms' voracious appetites (i.e., to sell a large quantity at a high price).

2.3 Monopoly

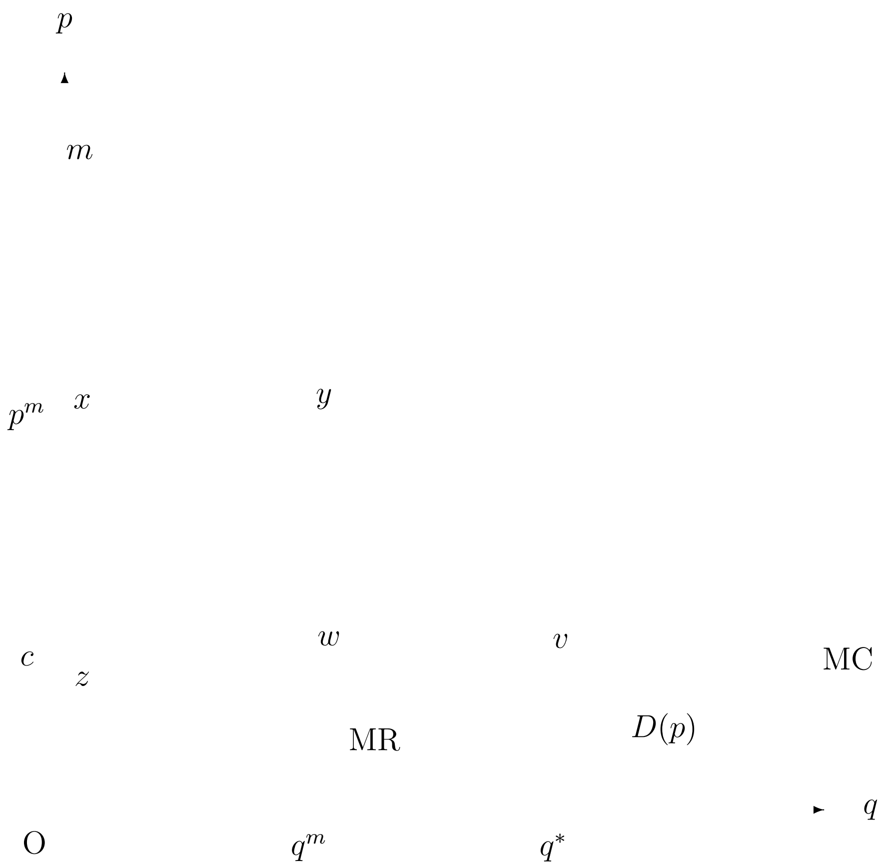
First, let us consider markets with just *one* firm (i.e., $N = 1$). The monopolist could, for example, set the quantity q to be produced and sold. In that case the price p will have to be such that this quantity is actually bought by the buyers, i.e., $D(p) = q$. Letting $P(q)$ denote the inverse of the market demand function (i.e., $P(q) = D^{-1}(q)$), the monopolist's profit Π^m by choosing $q \geq 0$ is as follows:

$$\Pi^m = qP(q) - cq.$$

The monopolist will choose q to maximise Π^m. Therefore, the first-order condition is:

$$qP'(q) + P(q) = c \tag{4}$$

The Left-Hand-Side (LHS) of equation (4) denotes the marginal revenue, while the RHS the marginal cost. The diagram below illustrates the market outcome: (p^m, q^m).



The monopoly price $p^m > c$, the marginal cost of production. This is evident also from equation (4), since $P'(q) < 0$. Indeed, the area of the rectangle $xywz$ is the monopoly profit and the area of the triangle mxy the consumer surplus.

The area of the triangle ywv is known as the *deadweight-loss*, which measures the degree of *inefficiency* of the monopoly outcome. This is because of the following argument. *Suppose* the buyers could offer the monopolist the following deal: set price equal to c, produce a quantity q^* (see diagram above) and we will pay you a sum equal to the area of the rectangle $xywz$ plus a penny. Clearly, the monopolist is better off (since she gets a penny extra). And, consumer surplus has *increased* by an amount equal to the previous deadweight-loss minus a penny. Unfortunately, such an offer is difficult to make for several reasons; for example, because it requires all the many consumers to get together and agree to make such an offer. Nevertheless, the point of the above argument remains: that the monopoly outcome (p^m, q^m) is Pareto-inefficient, since there exists a different outcome $(p = c, q = q^*)$ which would be prefered by both the monopolist and the consumers.

Verify that if the monopolist sets the *price* at which she will sell (rather than quantity, as above), with quantity sold being equal to $D(p)$, then the monopoly outcome is *unaffected*: it is as described above.

2.4 Durable Goods Monopoly: Coase's Conjecture

To be discussed in lecture: read: R.Coase, 1972, "Durability and Monopoly", *Journal of Law and Economics*, pp. 143-149.

2.5 Price Competition: Bertrand

We now add another firm/seller to the market discussed above. We first assume that the two firms, 1 and 2, will simultaneously set (or, announce) the *prices* p_1 and p_2 at which they are, respectively, willing to sell. All the buyers then observe both prices and buy only from the cheaper seller. Hence, if $p_1 < p_2$, then the quantity sold by firm 2 is zero, while the quantity q sold by firm 1 equals $D(p_1)$. Symmetrically if $p_2 < p_1$. And if $p_1 = p_2$, assume that each firm gets half the market demand. Therefore, the profit to each duopolistic firm i is:

$$\Pi_i^B = \begin{cases} (p_i - c)D(p_i) & \text{if } p_i < p_j \\ 0 & \text{if } p_i > p_j \\ \frac{(p_i-c)D(p_i)}{2} & \text{if } p_i = p_j, \end{cases}$$

where $j \neq i$ and $j = 1, 2$.

Clearly the profit (or, payoff) to each firm i depends on the price p_j set by her opponent. The *unique* Nash equilibrium (NE, for short) of this two-player simultaneous-move game is: $p_1^B = p_2^B = c$. That is, both firms set price equal to the marginal cost of production. This can be shown by the method of elimination. Here is the gist of the argument. Consider a pair of prices (p_1, p_2) such that $p_1 > p_2 \geq c$. This cannot be a NE, since firm 2 could increase her profit by unilaterally raising her price by a bit. And if

$p_1 = p_2 > c$, then either firm can increase profit by unilaterally decreasing its price by a bit. The final possibility $p_1 = p_2 = c$ is immune to profitable unilateral deviations.

This result (known as the Bertrand Paradox) is rather striking, since with just another seller the profit to each seller becomes zero. Note that this outcome, however, is Pareto-efficient.

Three important assumptions that are responsible for this paradox are: (i) homogeneous products, (ii) constant marginal costs of production and (iii) one-shot interaction. Relaxing either one of these assumptions would allow the two firms to earn positive profits.

2.5.1 Tacit Collusion: Infinitely Repeated Betrand Model

To be discussed in lecture.

2.6 Quantity competition: Cournot

Now we assume that the two sellers will simultaneously choose the *quantities* q_1 and q_2 that they, respectively, will produce and sell. The price p at which the commodity is sold is therefore $P(q_1 + q_2)$. Hence the profit to each duopolist i $(i = 1, 2)$ in this quantity-setting, Cournot, model is:

$$\Pi_i^C = q_i P(q_1 + q_2) - cq_i.$$

Let us assume that $P(q_1 + q_2) = k - (q_1 + q_2)$, where $k > c$.

In order to derive the Nash equilibria we first compute the firms' reaction functions. Differentiating Π_i^C with respect to q_i, keeping q_j constant, we obtain:

$$(k - c - q_j) - 2q_i = 0,$$

that is:

$$q_i = \frac{k - c - q_j}{2},$$

where $j \neq i$. This is player i's best response (reaction) function. Thus,

$$2q_1 = k - c - q_2$$

and

$$2q_2 = k - c - q_1.$$

Solving these two equations, we obatin the unique Nash equilibrium:

$$q_1^C = q_2^C = \frac{k-c}{3}.$$

Hence, the sum $q_1^C + q_2^C = \frac{2(k-c)}{3}$, and the market price $p^C = \frac{k+2c}{3}$, which is *strictly* greater than c. Indeed, therefore, each firm earns *positive* profits, unlike in the *price-setting*, Bertrand, model.

The monopoly outcome and profit are independent of whether the monopolist chooses quantity or price. This is not so with two firms. Duopolists would prefer competing in (or, setting) quantities rather then prices. Because in this Cournot model duopolists do earn some profit, it would appear to be a better model of duopolistic markets than Bertrand's model.

2.6.1 Tacit Collusion Revisted: Infinitely Repeated Cournot Model

To be discussed in lecture.

2.7 Cournot with N sellers

Let us compute the Nash equilibrium of the Cournot model with an arbitrary number N of firms/sellers. The profit to each firm i (where $i = 1, 2, 3, ..., N$) is:

$$\Pi_i^C = \big[k - (q_1 + q_2 + q_3 + ... + q_N)\big]q_i - cq_i.$$

Differentiating with respect to q_i, keeping all the other quantities $(q_1, q_2, ..., q_{i-1}, q_{i+1}, ..q_N)$ fixed, we obtain seller i's reaction function:

$$\Big[k - c - \sum_{j=1}^{N} q_j\Big] - q_i = 0.$$

This gives N equations in N unknowns. Impose symmetry: since the sellers are identical we expect $q_i = q^C$, for all $i = 1, 2, 3, ..., N$. Hence solve one equation in one unknown:

$$\big[k - c - Nq^C\big] - q^C = 0.$$

Therefore:

$$q^C = \frac{k-c}{N+1}.$$

Total quantity sold $Nq^C = \frac{N(k-c)}{N+1}$, and the market price $p^C = \frac{k+Nc}{N+1}$. Notice that, as N increases, p^C decreases and, moreover, as $N \to \infty$, $p^C \to c$. Thus, as the number of sellers in this Cournot model becomes arbitrarily large, the equilibrium market price coverges to the marginal cost of production. Similarly, verify that as N increases the equilibrium profit per firm decreases and, moroever, that as $N \to \infty$ the equilibrium profit of each firm converges to zero.

2.8 Leadership: Stackelberg

Return to the two seller market. And assume that seller 2 chooses her quantity *after observing* the quantity produced by seller 1. Assume that market price $P = k - (q_1 + q_2)$, where $k > c$. Let us derive the subgame perfect equilibrium of this *sequential* game.

First, derive seller 2's best response, or reaction, function. This is:

$$q_2 = \frac{k - c - q_1}{2}.$$

Now derive seller 1's optimal choice of q_1. Seller 1's profit is $\Pi_1^S = \big[k - (q_1 + q_2)\big] - cq_1$. Seller 1 will solve the following optimisation problem: Choose $q_1 \geq 0$ to maximise Π_1^S subject to the constraint that $q_2 = \frac{k-c-q_1}{2}$. Substituting for q_2, we obtain that

$$\Pi_1^S = \left[k - c - q_1 - \left[\frac{k - c - q_2}{2}\right]\right] q_1$$

That is,

$$\Pi_1^S = \left[\frac{k - c - q_1}{2}\right] q_1.$$

Therefore,

$$\frac{d\Pi_1^S}{dq_1} = \left[\frac{k - c - q_1}{2}\right] - \frac{q_1}{2} = 0.$$

Hence,

$$q_1^S = \frac{k - c}{2}.$$

Consequently, the subgame perfect equilibrium outcome is:

$$q_1^S = \frac{k-c}{2} \text{ and } q_2^S = \frac{k-c}{4}.$$

Notice that $q_1^S > q_2^S$. The leader, Firm 1, sells more than the follower, Firm 2. The total quantity sold is $\frac{3(k-c)}{4}$, which exceeds the total quantity sold in Cournot's, simultaneous-move, setting. Market price $p^S = \frac{k+3c}{4}$ is therefore lower than the Cournot price. In fact, the profit of the leader $\pi^L > \pi^C > \pi^F$, where π^C denotes the profit to each firm in Cournot's model, and π^F is the profit of the follower. The moral seems to be that: *The first-mover has an advantage.*

Elizabeth Jones – Topic 1

EC109

General Equilibrium And Welfare

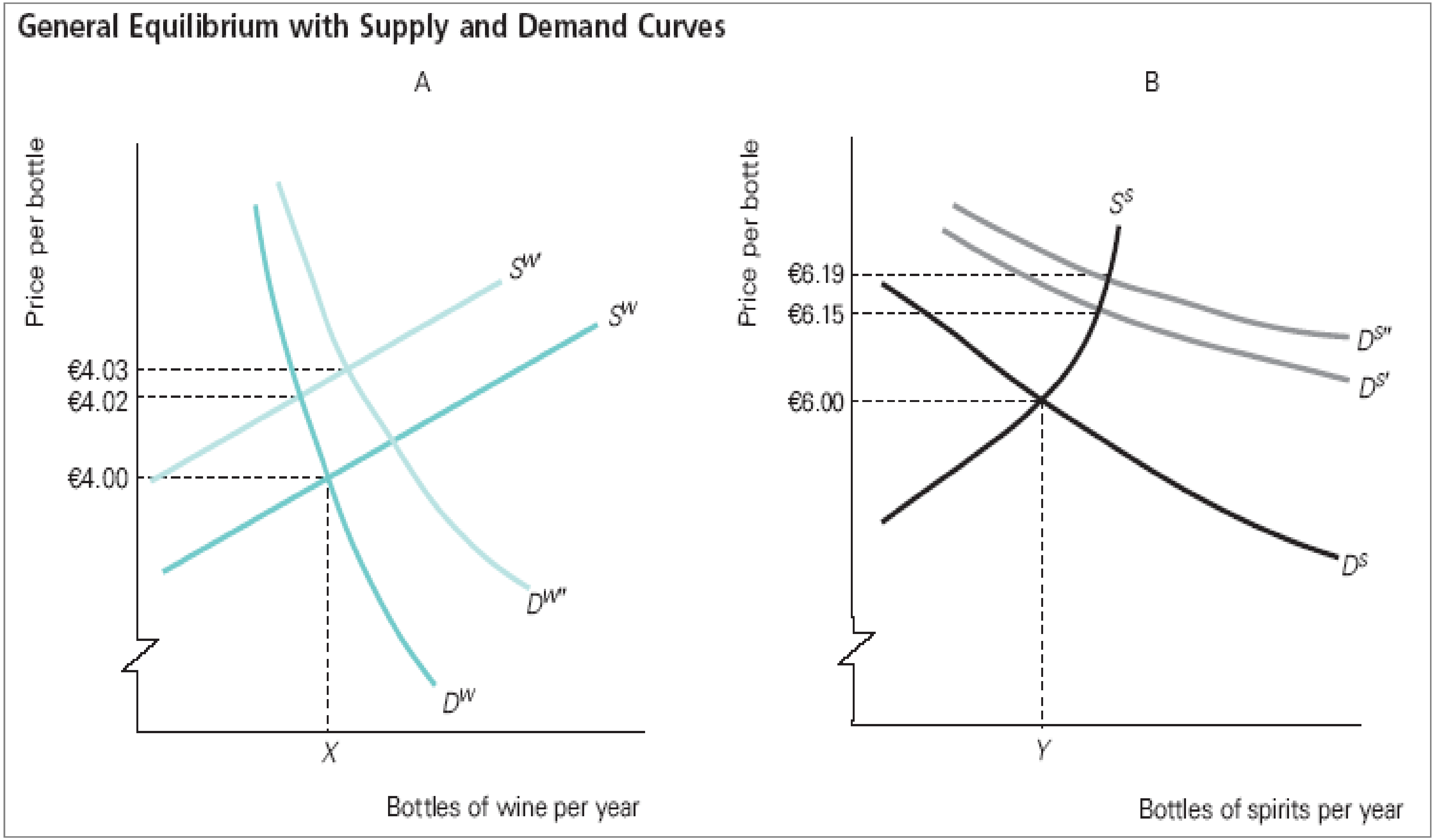
Topics

- General Equilibrium
- Edgeworth Boxes
- Pareto optimality
- The Welfare Theorems

Partial Equilibrium

- Most of your theory so far has focused on partial equilibrium analysis
  - It ignores the idea that different markets are inter-related through changes in input prices, substitution effects and welfare effects
- 2004: 4p tax levied on a bottle of wine
- Before tax all markets were in equilibrium
  - What's the difference between partial and general equilibrium analysis?

Increase in taxation on wine



General Equilibrium

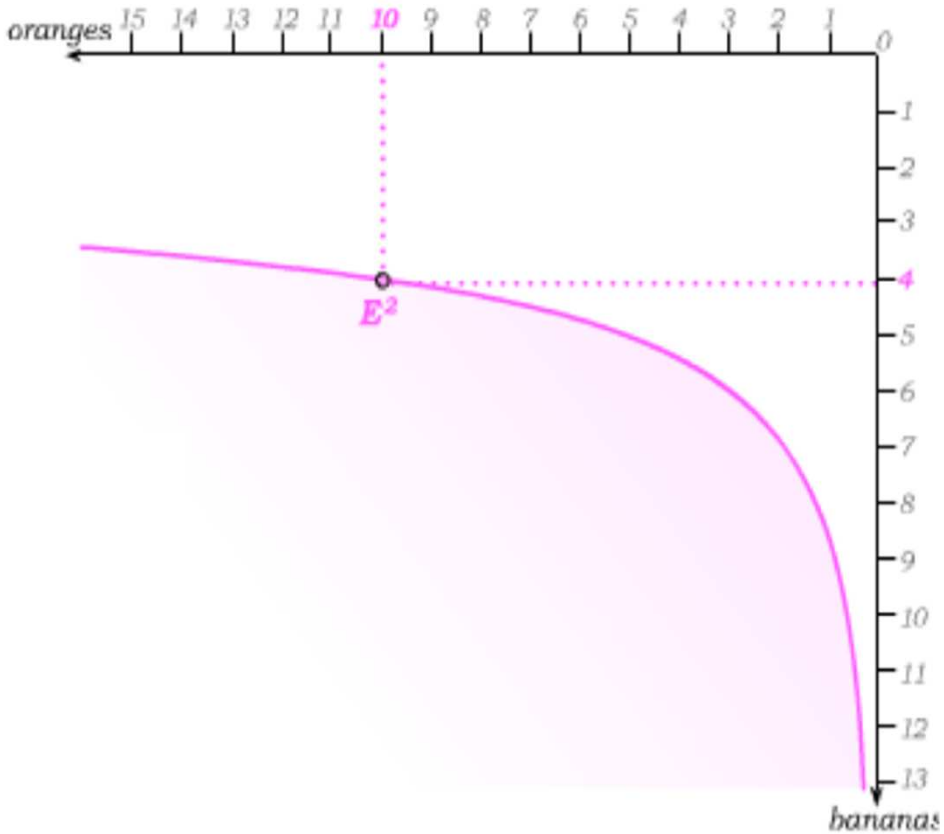
- We now view the economy as a system of related markets
  - It endogenizes prices and incomes
  - Analysis of the interactions of demand and supply in several markets to determine the market prices of several goods
- GE models can be developed showing production, exchange and consumption
- The importance for policy analysis

Pure Exchange

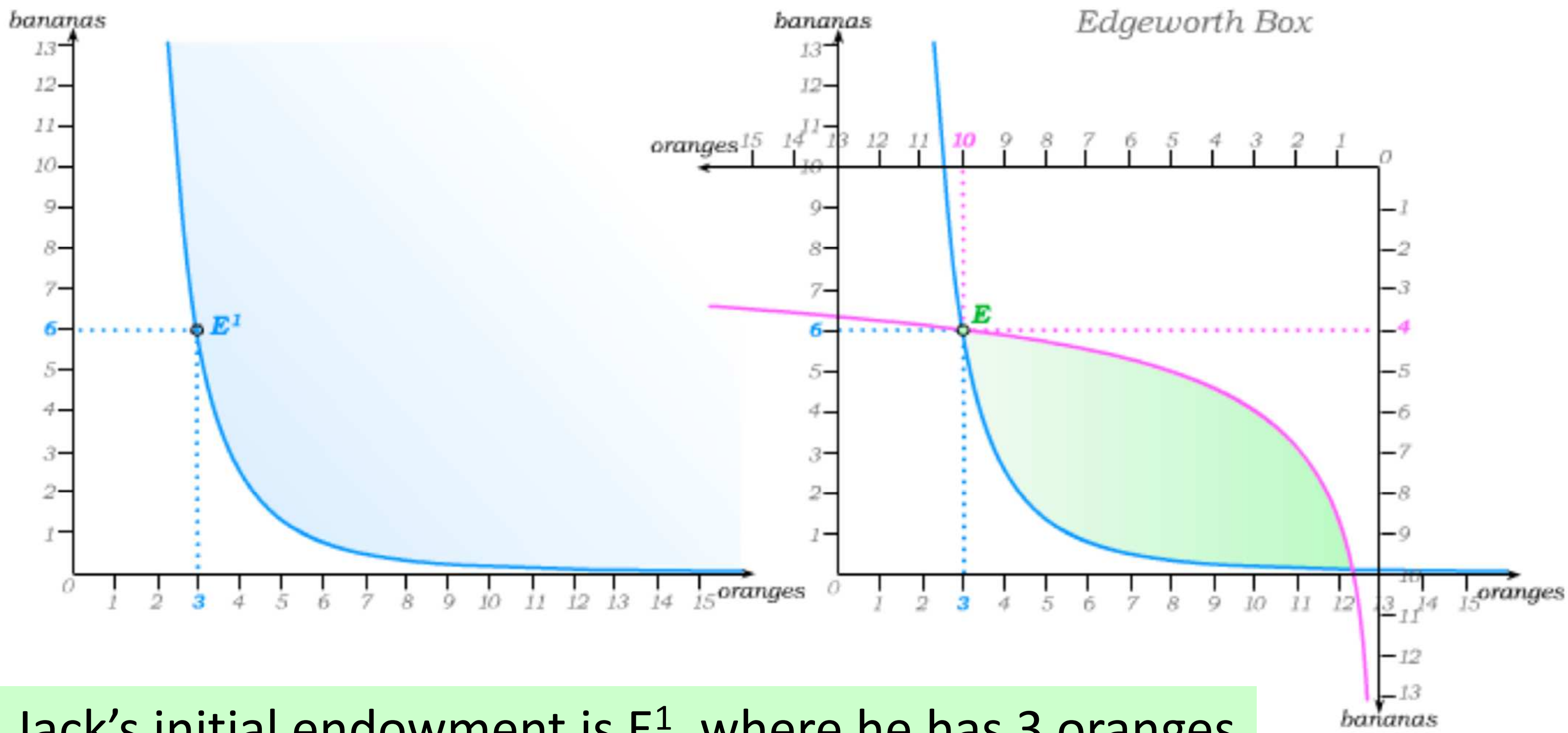
- '2 x 2' Economy
  - The simplest possible world
  - 2 consumers and 2 goods
  - No production, only exchange
  - Consumers have an initial endowment of goods (a bundle of the 2 goods) and try to maximise utility
  - We represent preferences using a utility function and thus indifference curves
  - For each good in the economy, there is a competitive market

‘2 x 2’ Pure Exchange Economy

Jill’s preferences give us her indifference curve.
Her initial endowment is given by E^2 with 10 oranges and 4 bananas.

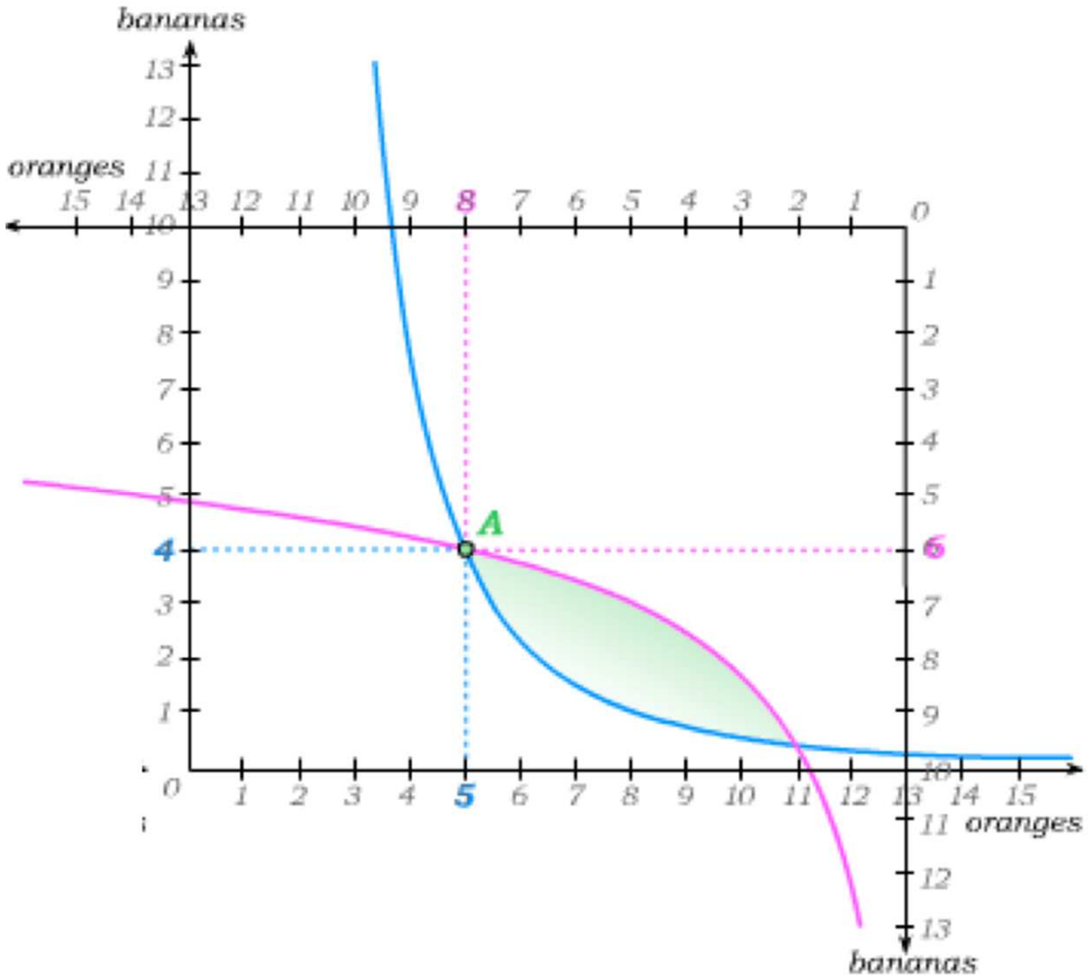


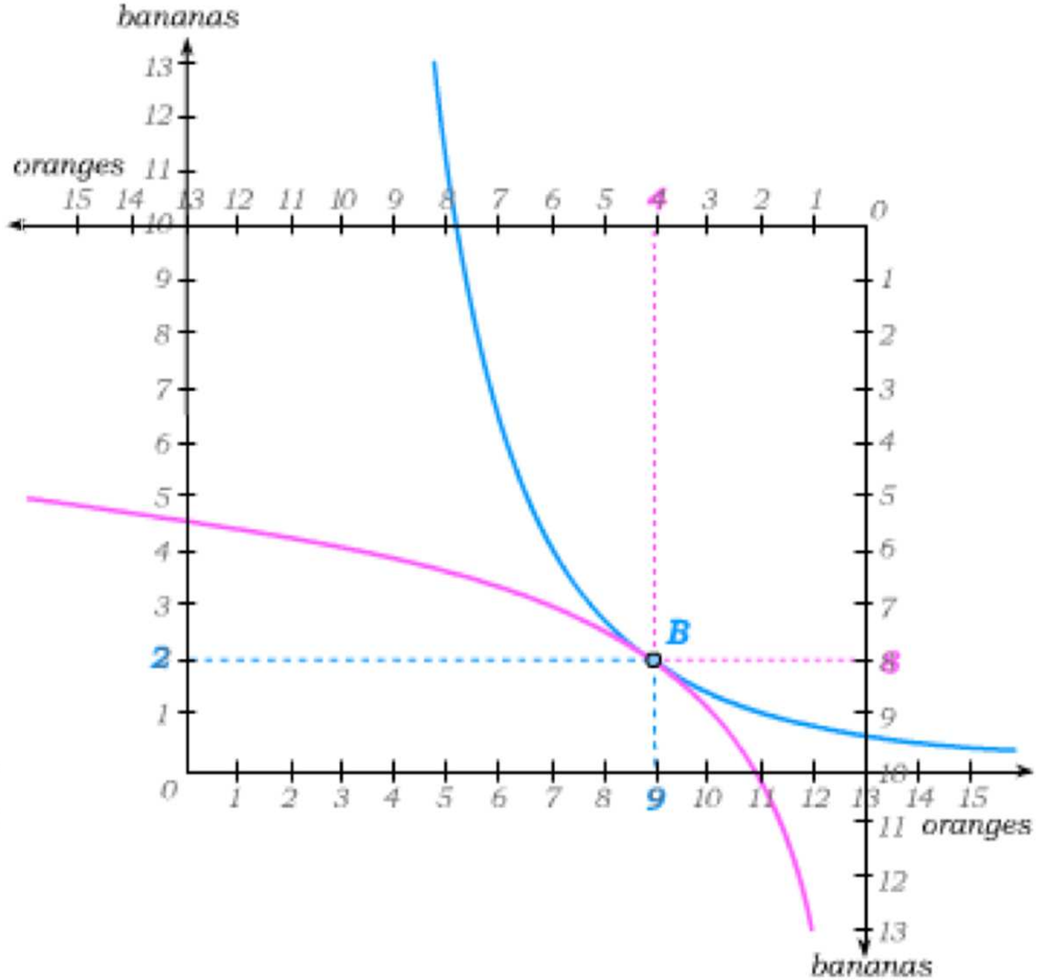
‘2 x 2’ Pure Exchange Economy



Jack’s initial endowment is E^1, where he has 3 oranges and 6 bananas.
Combining Jack and Jill’s indifference curves on one diagram gives us the Edgeworth Box.

The benefits of trade



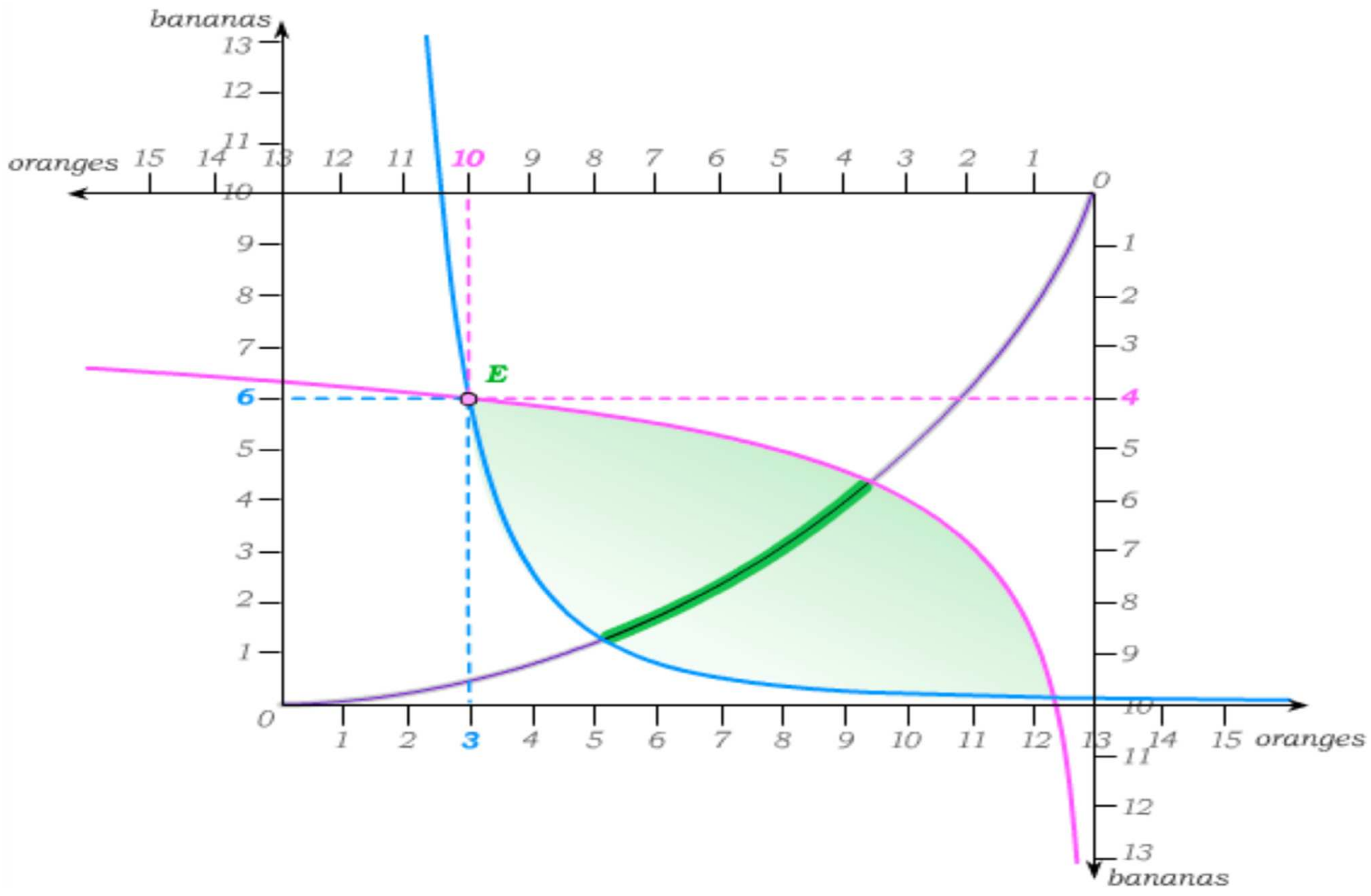


Jack and Jill will keep trading from point E to A etc. until no further mutually beneficial trades can be made. This occurs at point B, which is an efficient division of the economy's endowments.

Pareto Efficiency and the contract curve

- Pareto efficiency occurred at point B
- Are Pareto efficient points unique?
- The contract curve
- We want to be on the contract curve, but is every point achievable?
  - Those allocations that we can achieve through voluntary trade, given the endowments ‘E’ are part of the ‘Core’

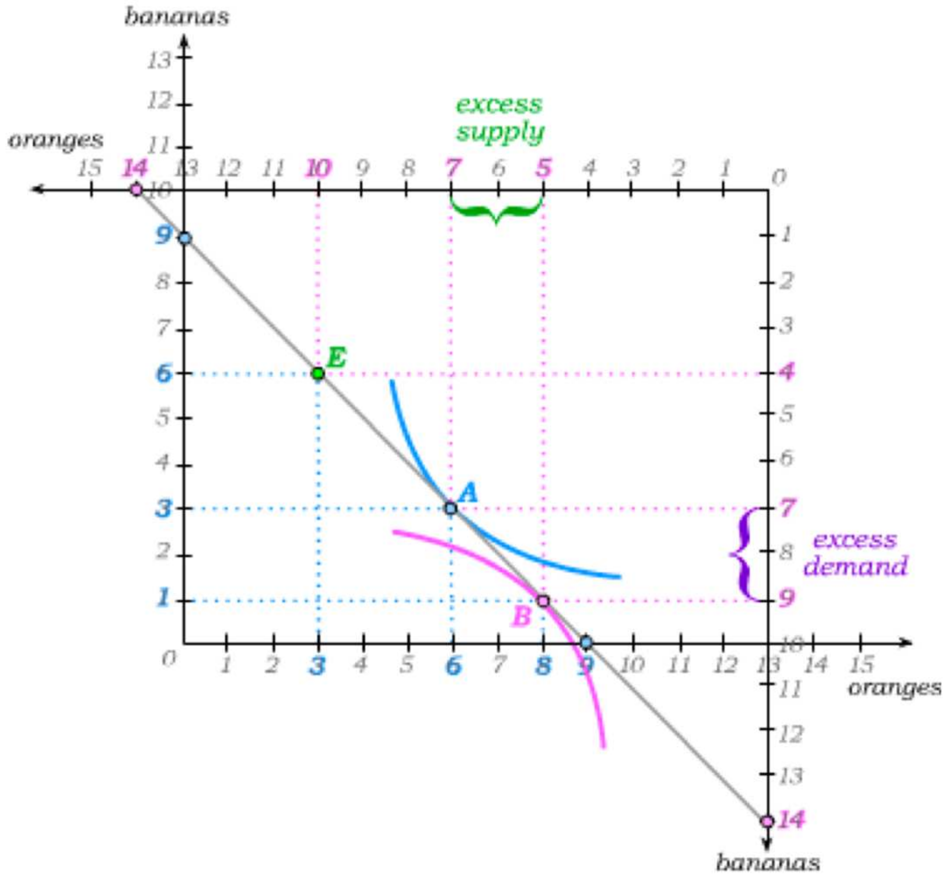
The Contract Curve and the Core

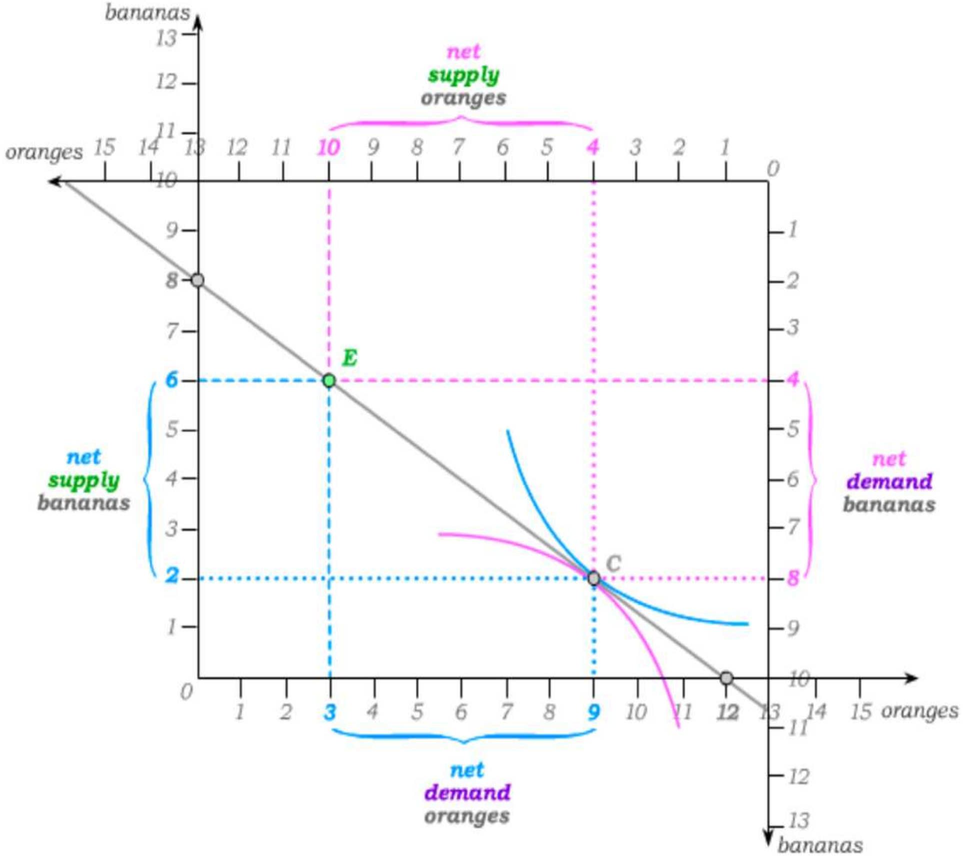


Competitive Equilibrium

- A trading process that mimics competition to find equilibrium prices
  - If we are in a disequilibrium with excess supply/demand, what will happen to prices?
- By adjusting prices, we can find a vector of prices which allows the exchange economy to reach a competitive equilibrium (CE)

Competitive Equilibrium





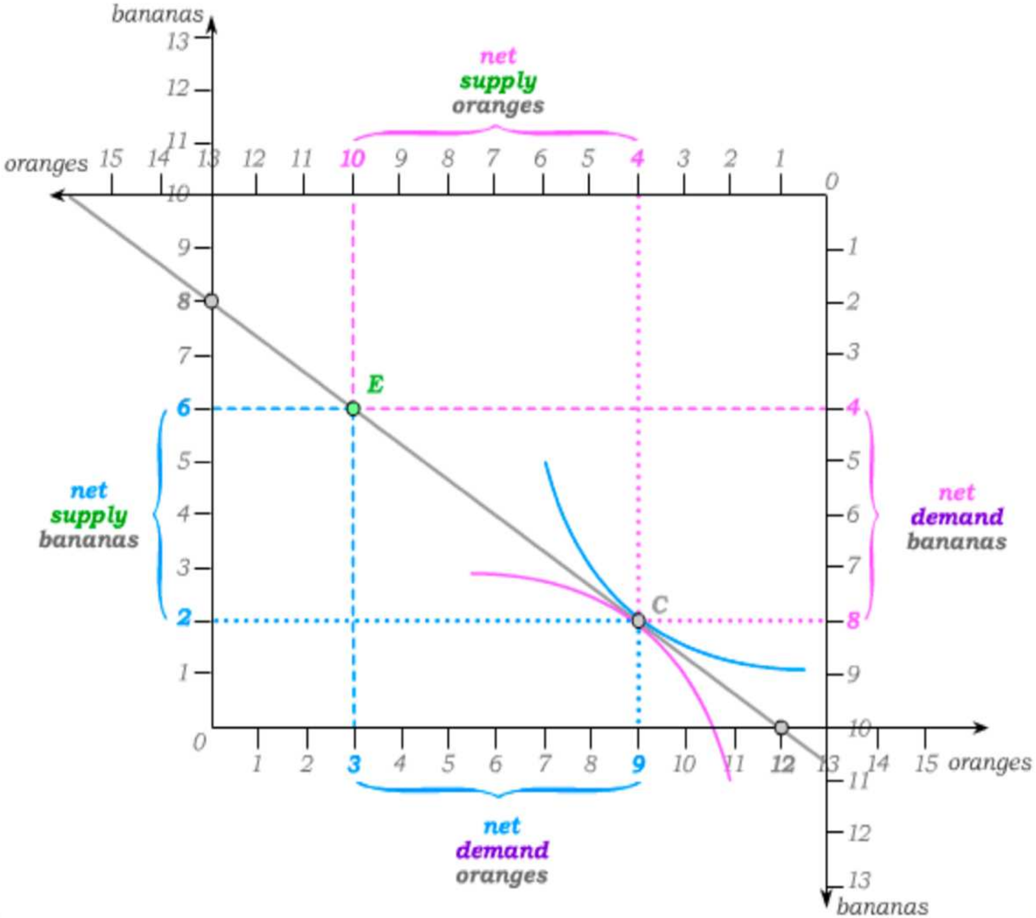
By changing the prices, Jack and Jill will optimise at the same point (C). Demand equals supply in both markets.

The Welfare Theorems

- We try to combine the market solution of competitive equilibrium …
- With the social ideology of Pareto efficiency
- This leads us to 2 key fundamental theorems of welfare economics
  - The First Welfare Theorem
  - The Second Welfare Theorem

The First Welfare Theorem

- If the competitive equilibria in an economy are Pareto efficient (under certain assumptions), the First Welfare Theorem will hold

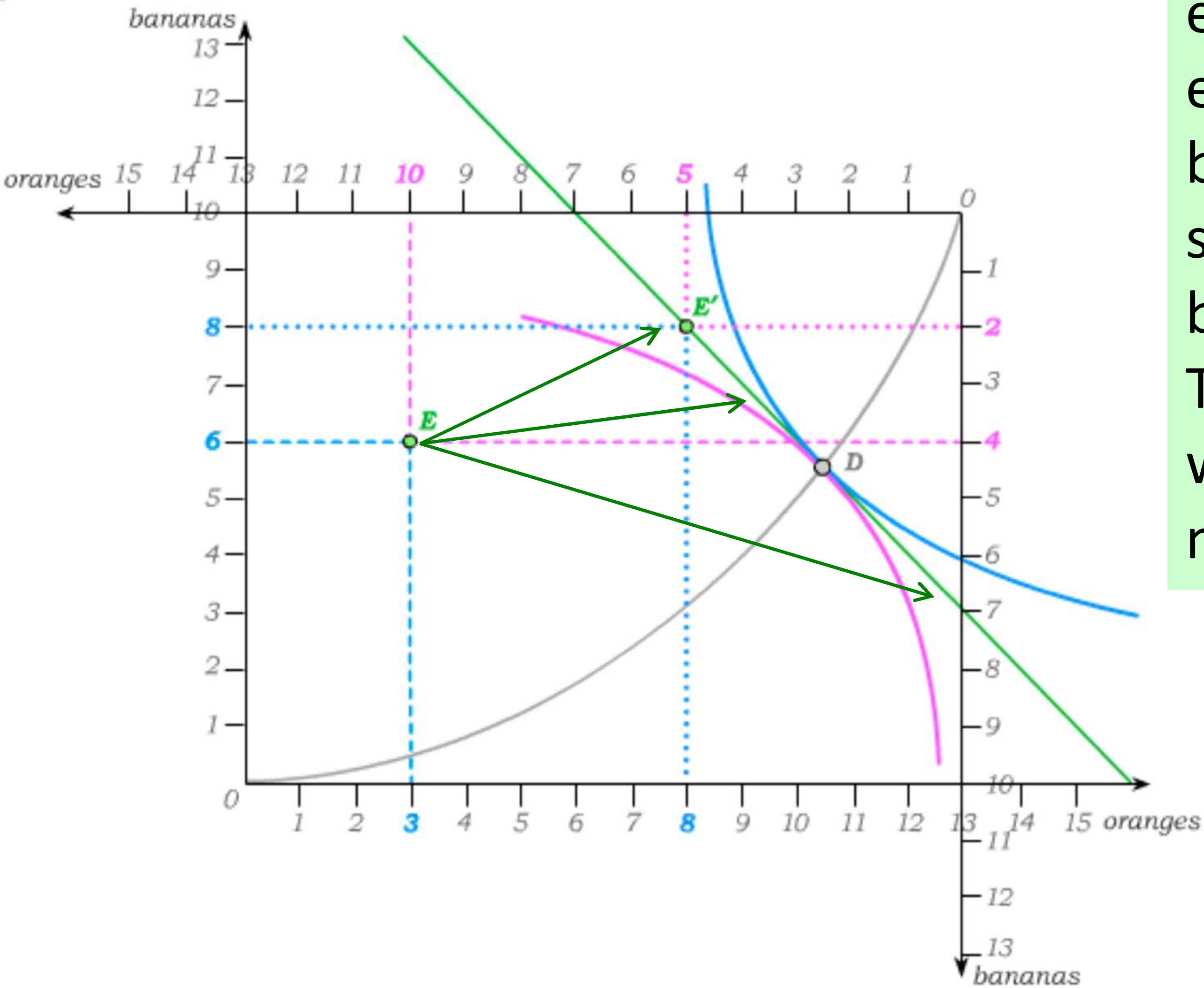


Does this theorem tell us anything about equity?

The Second Welfare Theorem

- This can be seen as the inverse of the first
- Given a Pareto efficient allocation of goods, can it also be a market equilibrium?
  - The First Theorem says that competitive equilibria are efficient
  - The Second Theorem says any efficient allocation can be a competitive equilibrium allocation, so long as the government can redistribute endowments without shrinking the economy in the process

The Second Welfare Theorem



For D to be an equilibrium, the endowment point must be redistributed to somewhere on the budget line, such as E′. The market mechanism will then lead to the new equilibrium at D.

Government's role

- Pick a 'societally preferred' Pareto optimal outcome and redistribute initial resources
  - Tax endowments and redistribute - The First Theorem guarantees Pareto optimality!
  - The Second Theorem says that taxing an agent's endowment is non-distortionary
  - The role of prices
- The trade-off between efficiency and equity
  - The problem of deadweight losses
  - Evidence of this trade-off

Conclusion

- GE is important for policy analysis
  - It ensures that policy makers consider the wider effects of any changes
- The Edgeworth Box is a useful tool with which to analyse decisions by consumers
  - It helps to distinguish between efficiency and equity considerations
- There is likely to be a trade-off between efficiency and equity and there is evidence of such a trade-off in many markets

Self-study questions

- What is the difference between partial equilibrium analysis and general equilibrium analysis? Use an example to explain the importance of GE.
- If two consumers have DVDs and books to consume, illustrate their indifference curves in an Edgeworth box.
- Show a situation in this Edgeworth box of an excess demand/supply for the goods.
- How can a competitive equilibrium emerge?
- At which point is consumption Pareto optimal?
- What does the contract curve show?
- Explain why Pareto optimal allocations are not unique.

Self-study questions

- What is the role of the budget constraint within the Edgeworth Box?
- Explain the First Fundamental Welfare Theorem.
- Using the Edgeworth Box explain the principle behind the Second Fundamental Welfare Theorem.
- Why do the consumers' initial endowments have to be on the budget constraint?
- What does the Second Welfare Theorem suggest about the role of government?
- Do the Welfare Theorems imply that there is a trade-off between efficiency and equity? Why or why not?
- In reality, is there an efficiency-equity trade-off?

Elizabeth Jones – Topic 2

EC109

Market Failures and Solutions

Topics

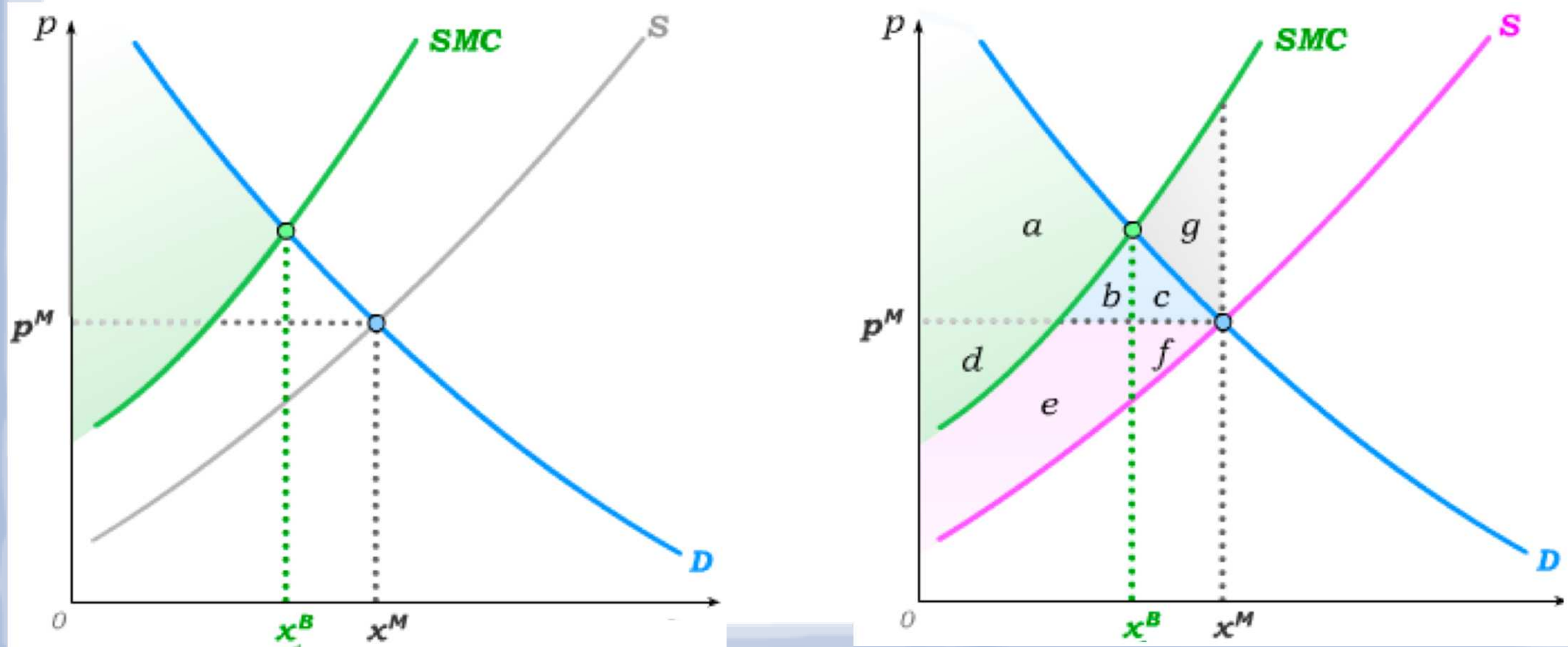
- Externalities
- Public goods
- Crowding
- Solutions

Externalities

- Even when markets are perfectly competitive, deadweight losses can still emerge
- This is the problem of externalities - directly imposed on non-market participants
  - Why do they matter?

Deadweight Loss from a Negative Externality

Competitive and social market outcomes. Consumers and producers receive the same surpluses with and without an externality, but a deadweight loss now emerges (= area g), because of the total social cost of pollution (b + c + e + f + g).



Pigouvian Tax

- If externalities exist, government policy can be implemented to enhance efficiency and thus reduce or eliminate a deadweight welfare loss
- A negative externality in production leads to the market over-producing
- The role of a Pigouvian tax is to eliminate the DWL
  - Find a ‘t’ that equates the firm’s private optimum with the social optimum
  - But, this requires an incredible amount of information

Calculating Pigouvian Taxes

- $X_d = \frac{A - p}{\alpha}$ and $X_s = \frac{B + p}{\beta}$
- We can find the competitive equilibrium price and quantity
- Suppose each unit of x produces δ units of $C0_2$ and this pollution increases quadratically with additional pollution dumped into the air. The externality cost is: $C_E(x) = (\delta x)^2$
- We can find the socially optimum quantity and the prices needed to give this
- We can calculate the optimal Pigouvian tax

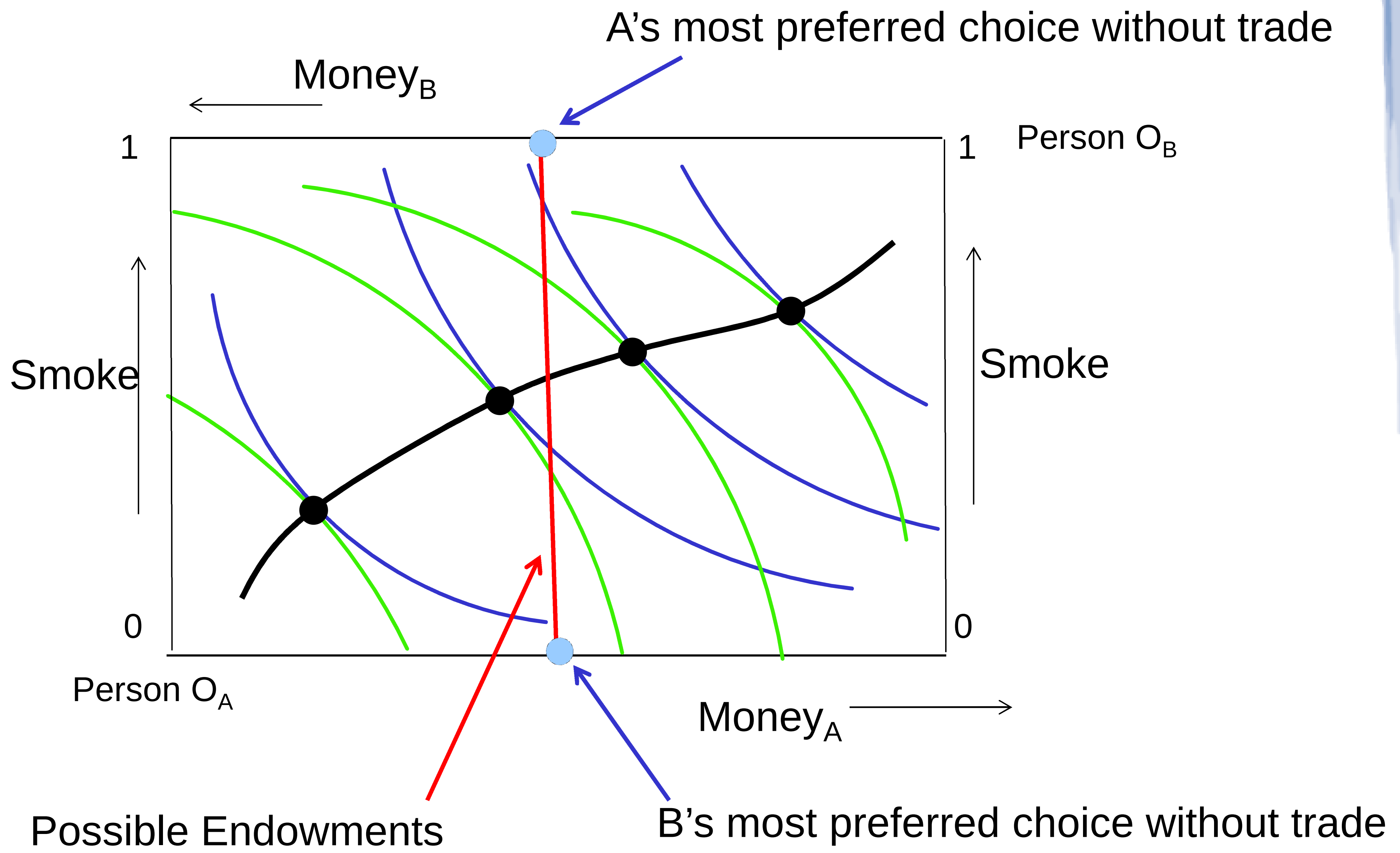
Solutions to externalities

- Or can we engineer another market solution?
- Create a new market for the externality
  - Pollution permits
  - This may require an additional agent (e.g. the government) to coordinate the market
- What's the benefit of a pollution permit system?

Consumption Externalities

- If externalities exist, a competitive equilibrium need not be Pareto efficient
  - Say, A cares about B’s consumption of cigars
  - Each person buys the best affordable bundle, but can we still make them both better off?
- We assign property rights to one agent
- Assume two roommates A and B
  - Both like money
  - A likes smoke, B likes clean air
  - Both have an endowment of £100

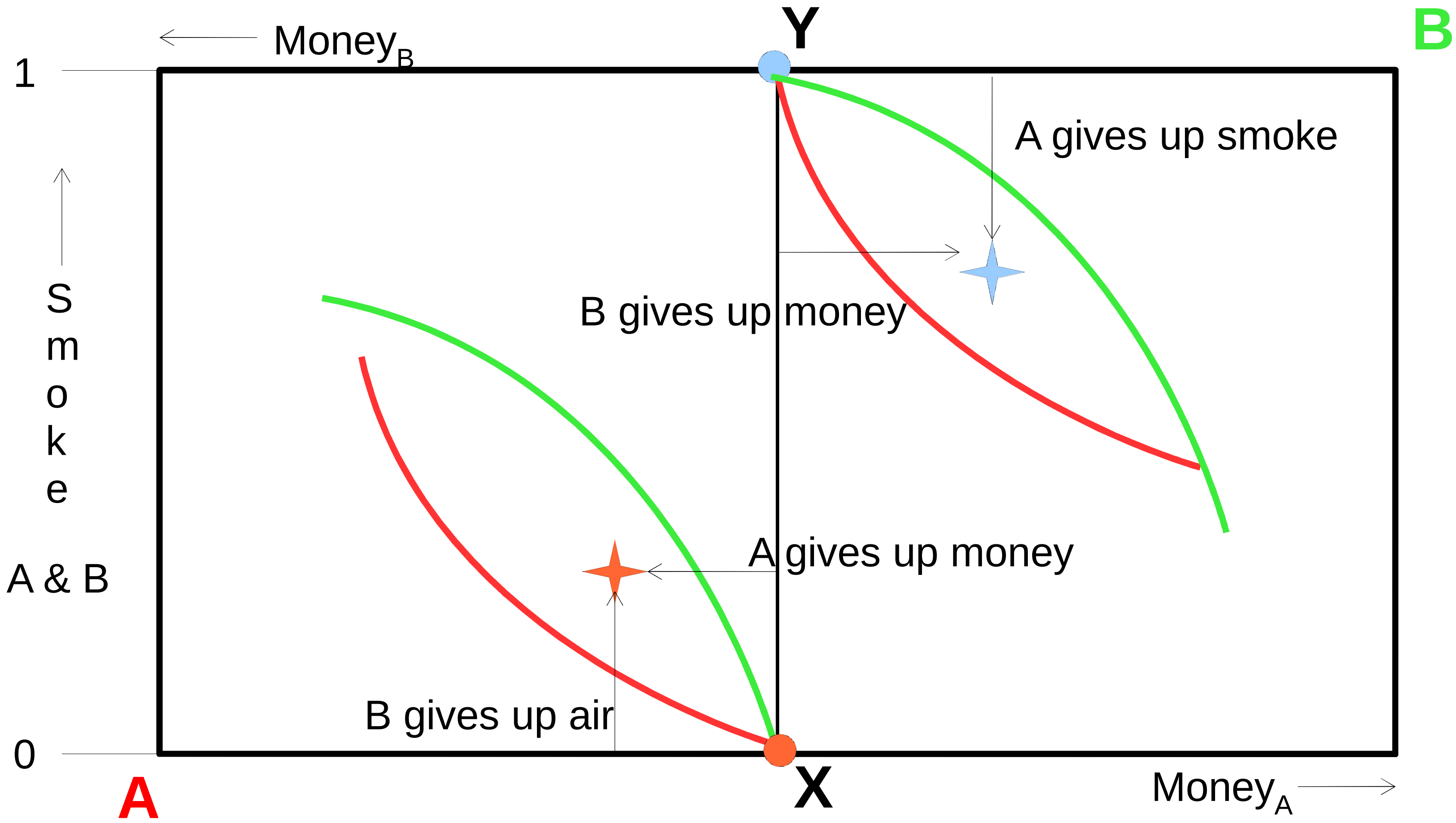
Consumption Externalities



Consumption Externalities

- Both A and B's most preferred choice, given their endowment of £100 is inefficient if they cannot trade money for smoke/clean air
  - Either there is too much or too little smoke
- What happens if the air in the room is assigned to either agent A or B?
  - There is now the ability to sell 'rights to smoke' or 'rights to clean air'
- We can show how the extension of property rights will lead to a Pareto optimal outcome

- **X:** B has the ‘property rights’
- **Y:** A has the ‘property rights’
- ● and ● are not necessarily Pareto Optimal >> trade



Property Rights

- So by clearly allocating property rights for the ‘missing market’ we can solve the problem and negotiation can occur between the agents
  - As long as a price mechanism exists to allow trade and negotiation is costless
  - Ronald Coase noticed this and won the Nobel Prize in 1991 for his work in this area
- The practical problems with externalities often arise because of poorly defined property rights
  - In reality, it is very hard to define property rights, especially for non-tangible goods

Public Goods

- They can be consumed by more than one individual at a time
- No-one can be prevented from consuming them
- Most goods lie in the continuum between a public good and a private good
  - Fireworks
  - National defence
  - Local policing

Non-rivalry and Crowding

- Some goods are non-rival to a point, but as N gets larger, enjoyment falls and costs may increase
- Consider a fireworks display… or a local police force… or a cinema
- To what extent are these ‘goods’ non-rival?
- How can we find the optimal level of public goods?

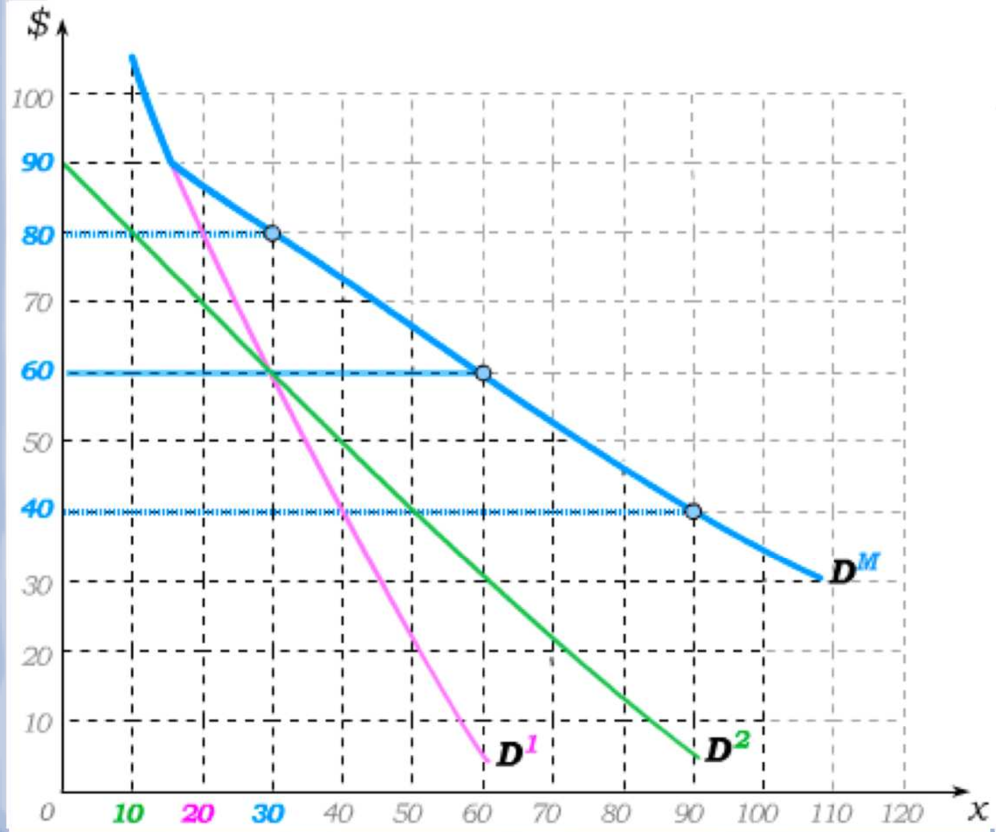
Demand, or *MWTP*, for Public Goods

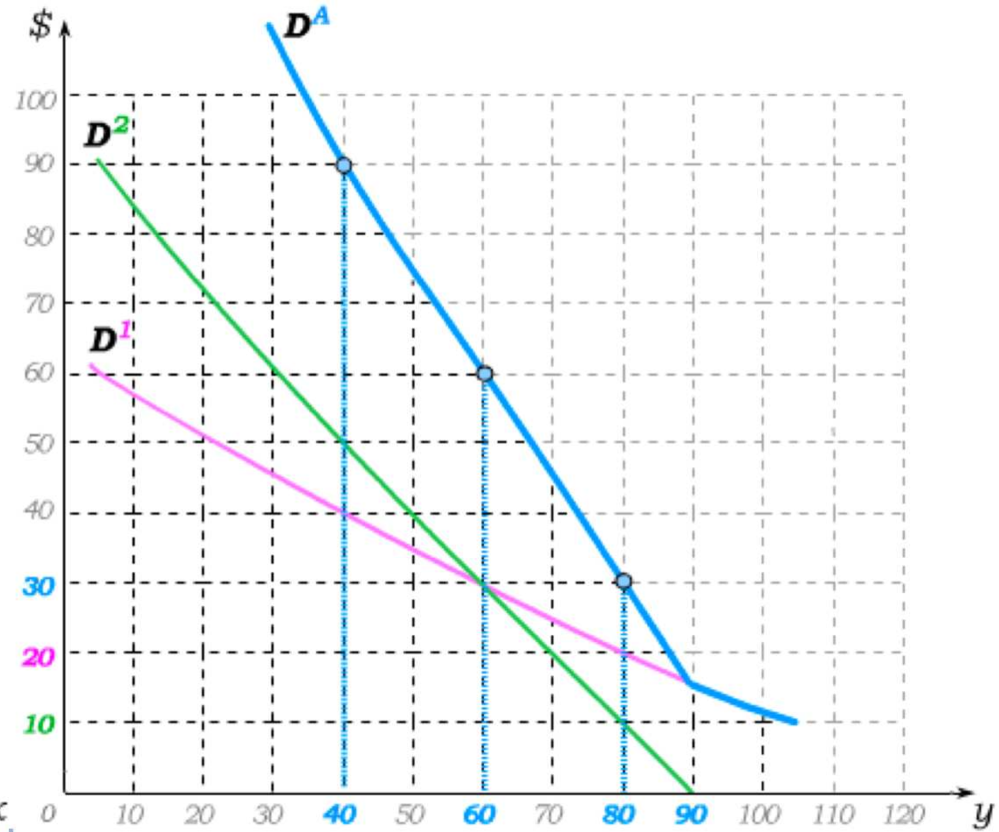
In deriving *market demand* for ***pure private goods***, we began with demands for 2 individuals.

At each price level, we then ***add the demand curves*** *"horizontally"* to get *total demand.*

To find *total demand* for *pure public goods*, we similarly begin with two initial demand curves.

But now each quantity can be consumed by *both individuals* – and so we ***add demand curves "vertically"*** to determine the *total willingness to pay* for that quantity.





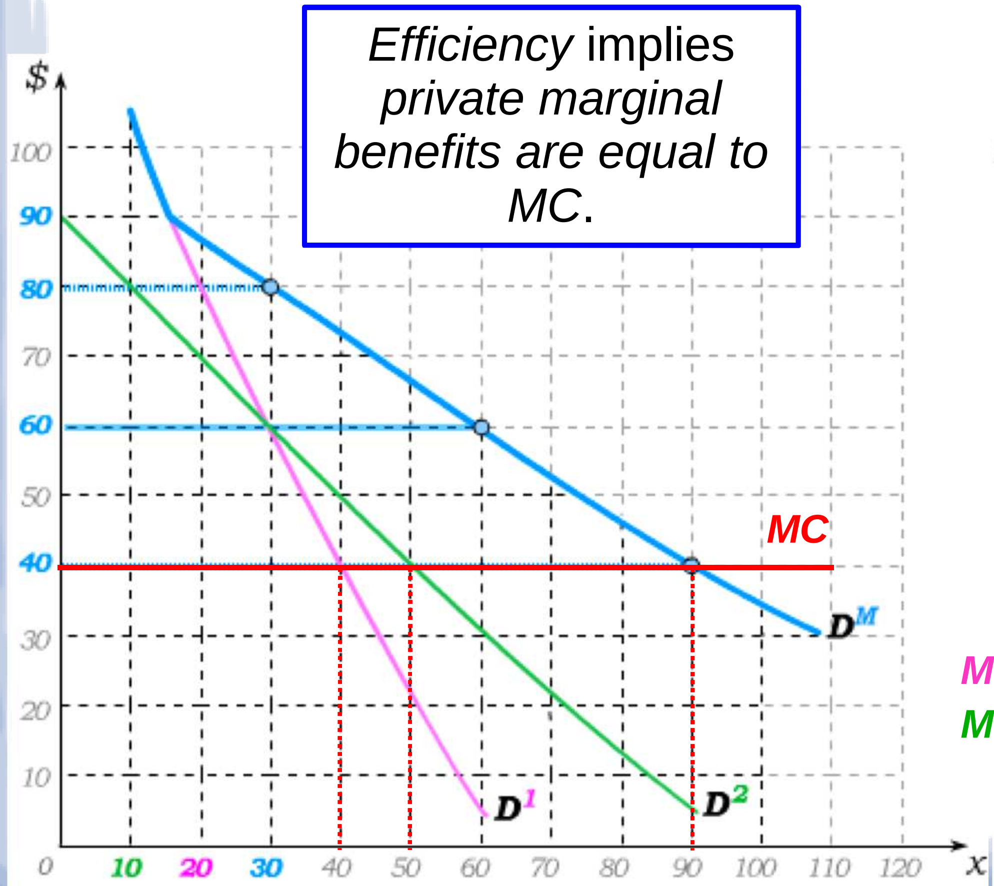
Efficient Output Level: Private vs. Public

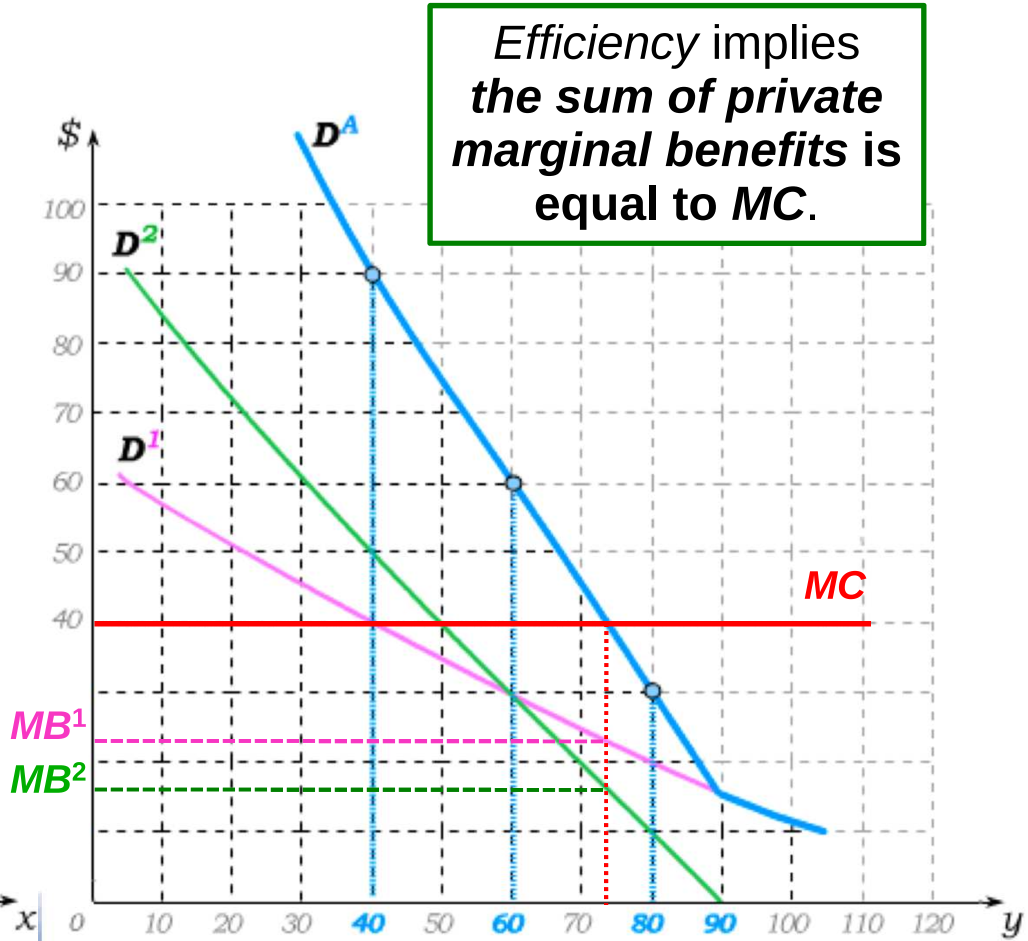
For a given *marginal cost* of production, the ***efficient private good output level*** occurs where $MC=D^M$.

Consumer 1 consumes where $MC=D^1$ and consumer 2 consumes where $MC=D^2$.

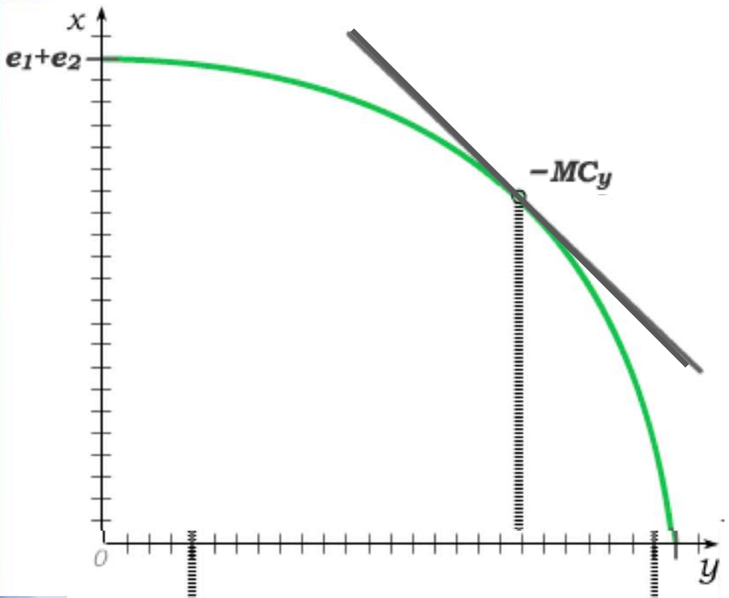
For a given *marginal cost* of production, the ***efficient public good output level*** occurs where $MC=D^A$.

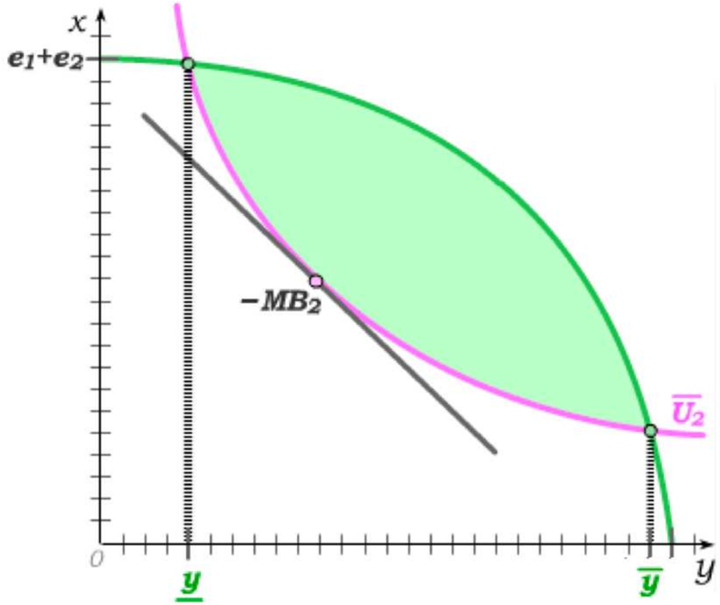
At this efficient quantity, $D^1 + D^2 = MC$.





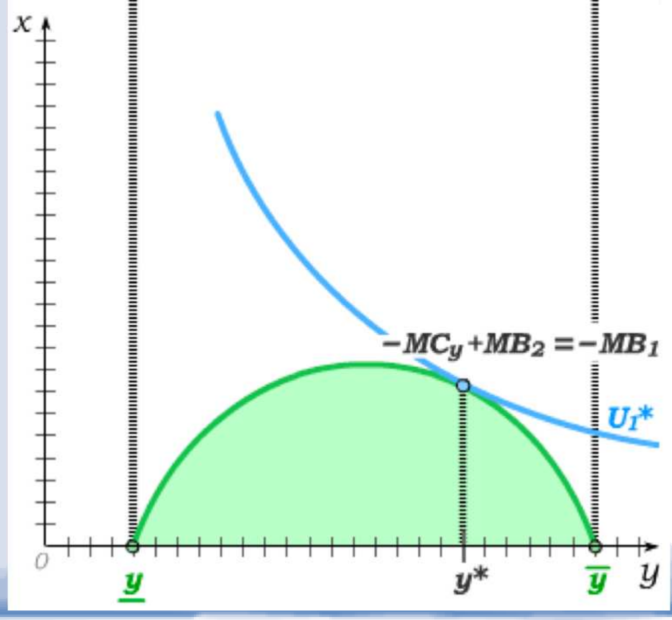
Suppose 2 individuals are *endowed* with ***private*** *good* levels e_1 and e_2 and public goods can be produced with a decreasing returns to scale production technology.





Then the ***production possibilities frontier*** results in a *convex* set of possible pairs of private and public goods.

Now suppose we guarantee *individual 2* utility $\overline{U}_2$



What can we say about points $\underline{y}$ *and* $\overline{y}$?

How can we use this analysis to find the efficient public good level, y*?

At the ***efficient public good level*** y^*,

$$MB_1 + MB_2 = MC_y.$$

Public Goods

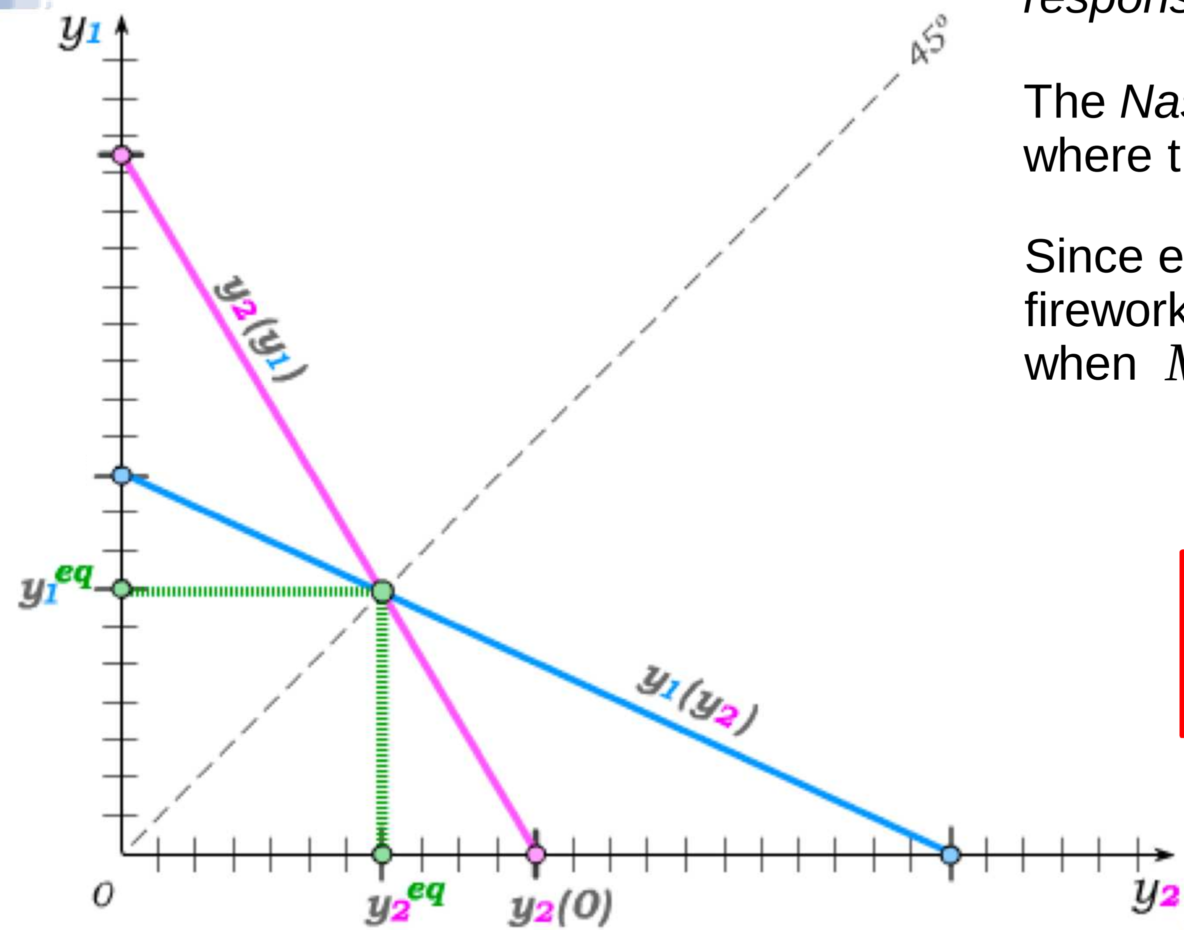
- Return to the smoker vs non-smoker scenario
  - What's the problem if we introduce a 3rd roommate who is a non-smoker and allocate property rights to the non-smokers?
- The externality of the public good suggests a potential failure of the First Welfare Theorem
  - When I buy the public good, it affects not only me, but everyone else too
- But Pareto efficiency is a social condition
  - Individuals only consider the benefit to themselves. Consider the case of fireworks…

Fireworks Provision and free-riding

Suppose two neighbours are *individually* going to put on a fireworks display. How many fireworks will be launched by each person?

If *1* thinks *2* will not launch any fireworks, what is *1's* best response?

What happens to *1's* best response as *2's* quantity of fireworks becomes > 0?



The symmetric is true for *individual 2*'s *best response* to *individual 1*'s contribution.

The *Nash equilibrium contributions* then occur where the *best response functions* cross.

Since each benefits from the other's fireworks, both will stop buying fireworks when $MB_1 + MB_2 > MC$.

Too little (relative to the efficient level) of the public good is therefore provided….

Government Intervention

- Society has evolved ways of inducing cooperation to enable the creation of public goods and projects
- In many cases, governments provide public goods and tax individuals to finance them
- Alternatively, governments subsidise private firms to produce them
- The Warm Glow Effect

Summary

- The Welfare Theorems fall down when externalities exist
  - Various market and non-market solutions exist
- All goods have varying degrees of rivalry and excludability and lie in the continuum between pure public and private goods
- Government intervention through compulsory taxation or provision may be needed in missing markets
- By exploiting interdependencies of welfare, we can overcome the free-rider problem

Self-study questions

- What is an externality? Illustrate both positive and negative ones on consumption and production.
- How can government intervention tackle externalities?
- Why is market structure an important consideration when designing policy to tackle externalities?
- Why are property rights so important?
- What are the characteristics of public goods?
- How is crowding relevant in the case of non-rivalry?
- Why is a public good a special case of an externality?
- Explain how we arrive at the Nash equilibrium of free-riding for public goods.

Self-study questions

- Consider the case of a cinema. To what extent is crowding likely to be an issue?
- What about the problem of crowding when it comes to the TV in your lounge?
- Is loud music at 3am in Halls a public good?
- Could property rights be imposed in the case above?
- How do we find the efficient level of a public good?
- What is the Warm Glow effect?
- If we all care about the distribution of income, we will agree to a progressive tax system? Will it be sufficiently progressive?

Elizabeth Jones – Topic 3

EC109

Uncertainty, Insurance and Asymmetric Information

Topics

- Uncertainty and risk
- Expected Value and Expected Utility
- Insurance
- Moral hazard and adverse selection
- Solutions

Uncertainty$_{(V12)}$

- Most economics you have studied has considered a world of certainty
  - But people always face risks
- Because people face risks, there is a potential market for products that reduce risk
  - Insurance, warranties, financial planning
- This topic looks at risk and uncertainty and how this affects the decision making of economic agents

André-Francois Raffray Trenton Times, 1995

- 1965: He would pay a 90-year-old woman $500 a month until she died, then move into her grand apartment.
  - On 25/12/95, Raffray died, having paid her $184,000 for an apartment he never lived in
  - That night, Jeanne Calment dined on foie gras, duck thighs, cheese and chocolate cake at a nearby nursing home. She was the world's oldest person at 120.
  - Life is full of uncertainty

Uncertainty

- For consumers (and producers) uncertainty throws a spanner in the works of choice
  - Say you want to choose between staying in and watching a film, or going out
  - You (think you) know how much utility you would derive from the film
  - However, there are 2 possible 'going out' scenarios that could occur

Uncertainty

- Scenario 1:
  - All your friends go out too
  - You go to a place where you like the music
  - You catch the last bus home
  - You would prefer this to watching the film
- Scenario 2:
  - None of your friends go out
  - You end up in Coventry
  - You miss the bus and sleep in a field
  - You would prefer to watch the film instead

Uncertainty

- Your decision in the previous case will be influenced by how likely it is that scenario 1 or scenario 2 occurs
  - If scenario 1 is very likely, perhaps you go out
  - If scenario 2 is very likely you might not
  - Thus, the probability distribution matters
- Risk – where we know the probability distribution
- Uncertainty – where we don't know the probability distribution

Expected Value

- A numerical example
  - I roll a fair die: you win the value of the die in £ if it's an even number and lose the value in £ if it's an odd number
  - I charge you 50p to play the game
  - Would you play? Let’s estimate the outcome
- On average, you will lose £1 1/6th of the time
  - But you will win £2 1/6th of the time
  - You will lose £3 1/6th of the time etc...

Expected Value

- **Expected** winnings in any given game are:

EV = (-£1 x 1/6) + (£2 x 1/6) + (-£3 x 1/6) + (£4 x 1/6) + (-£5 x 1/6) + (£6 x 1/6) = 50p

- EV captures the benefit of the game to you
  - You can't win 50p in 1 game – it is like a *long term average* of the game
  - As playing costs 50p: net winnings: Zero
- Any game like this is actuarially fair
  - It favours neither player

Relation of 'Utility' to 'Consumption'

- Husband specialises in earning income
- Wife specialises in running the household
- Probability that husband dies, leaving wife's Standard of Living much lower = β
  - Probability that husband lives = (1- β)
  - Assume that wife's utility depends only on husband's earned income
  - Assume also that wife can measure her utility from different consumption bundles

Relation of 'Utility' to 'Consumption'

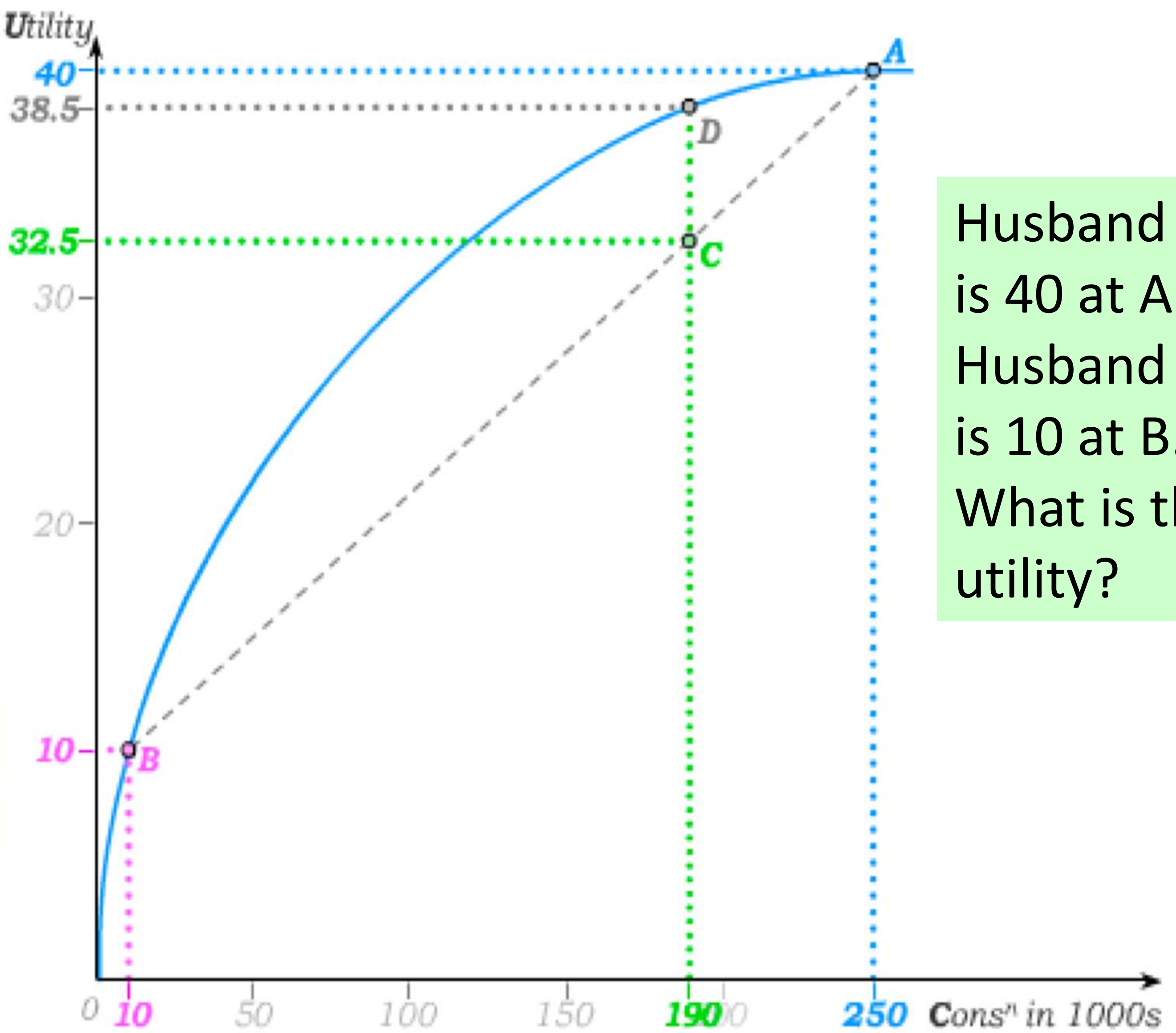
- Husband earns £250,000, but if he dies, wife will be left with £10,000.
- Husband is not popular at work and there is a 25% chance that someone will have him killed
  - $\beta = 0.25$

EV = (0.75 x 250,000) + (0.25 x 10,000) = 190,000

- Expected values are defined as:

$u(x_i,\pi_i) = \Sigma\, \pi_i x_i$ $\quad$ *π_i is prob.; x_i is value of event i*

Relation of 'Utility' to 'Consumption'



Husband lives >> Wife's utility is 40 at A.
Husband dies >> Wife's utility is 10 at B.
What is the wife's expected utility?

Attitudes to Risk

- Averse – won't play an actuarially fair game
  - $U(p_1W_1 + p_2W_2) > p_1U(W_1) + p_2U(w_2)$
  - The level of utility associated with the expected wealth > expected utility of wealth
- Seeking – will play an actuarially fair game
  - $U(p_1W_1 + p_2W_2) < p_1U(W_1) + p_2U(w_2)$
  - The level of utility associated with the expected wealth < expected utility of wealth
- Neutral – indifferent between playing and not
  - $U(p_1W_1 + p_2W_2) = p_1U(W_1) + p_2U(w_2)$
  - The level of utility associated with the expected wealth = expected utility of wealth

Risk and Insurance

- Empirically, risk-aversion is the most common attitude to risk.
  - Agents tend to prefer certainty
- Are they willing to pay for this?
  - Can certainty be 'supplied' in a market?
  - Is there a profit to be made?
- Insurance can change a gamble
  - As people are typically risk-averse, they will pay high premiums to pass on the risk
- Let's return to the husband and wife case

Actuarially fair Insurance

- Suppose wife purchases an insurance policy: Premium = £20,000; Insurance pay-out = £80,000; Probability of husband dying = β.
  - Wealth if husband lives

    $= Y - P$
  - Wealth if husband dies

    $= Y – P – (Loss – I)$
- Wife's Expected Wealth

$$(1 - \beta)[Y - P] + \beta[Y - P - (Loss - I)]$$

$$Y - P - \beta(Loss - I)$$

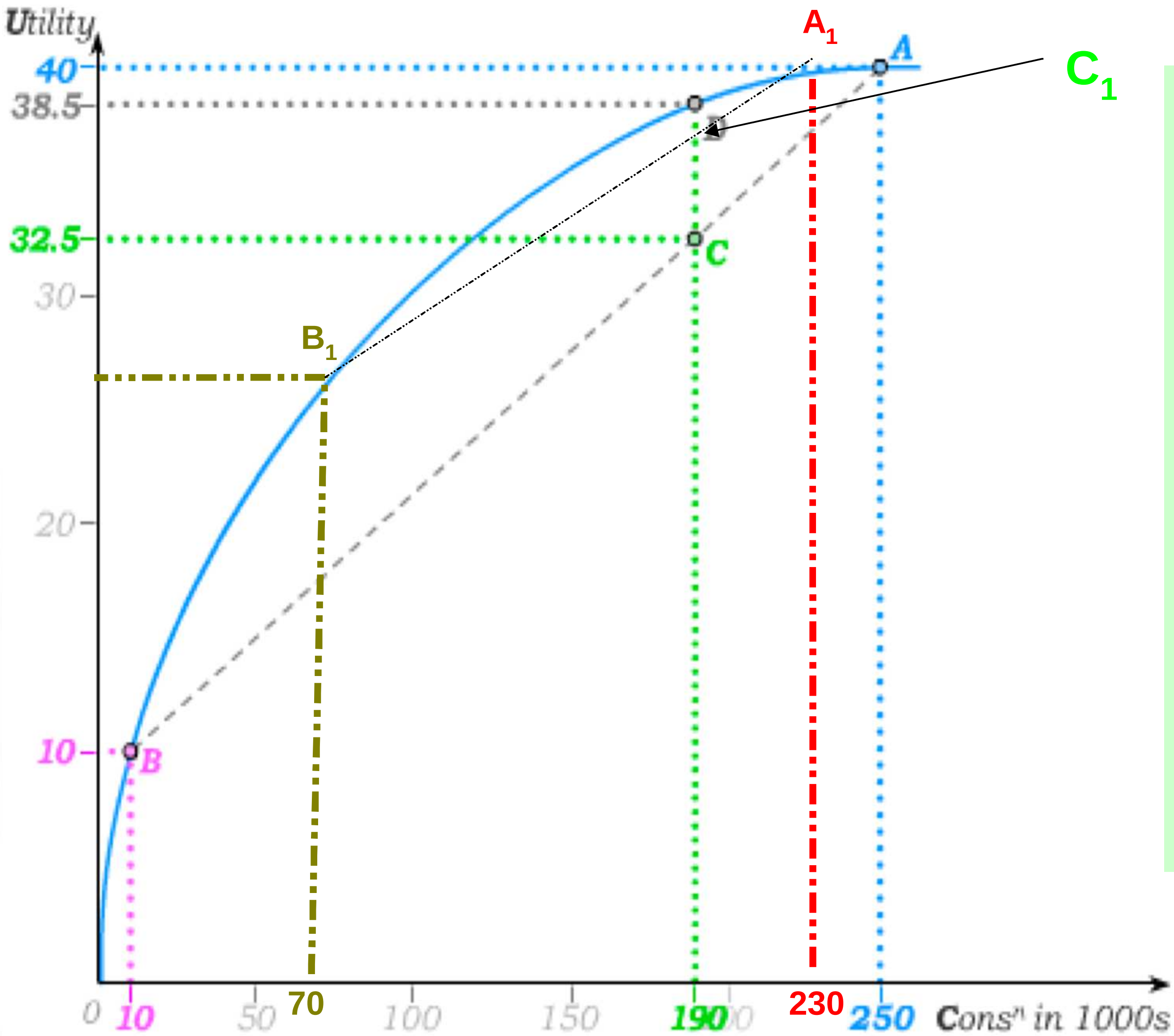
Actuarially fair Insurance

- Will the wife take out insurance?
  - If husband lives: 0.75 probability of £230,000
  - If husband dies: 0.25 probability of £70,000

Her expected consumption is still £190,000

- An insurance contract is actuarially fair iff the expected value of the gamble remains unchanged

Actuarially fair Insurance



Points A and B are good and bad outcomes with no insurance.
EU was at C = 32.5
A1 and B1 are good and bad outcomes with insurance.
EU at C1 = 36.5
Wife should take out insurance policy.

Car Insurance

- Assume car value = W.
- Risk of sustaining a loss of D, at a probability = γ
- Insurance, A, can be purchased at a cost of p per unit decrease in D.
- How can we determine a household's optimal level of car insurance, A*?
- Wealth with no loss
  - Wealth = W – pA
- Wealth with a loss
  - Wealth = W – pA – (D - A)

Car Insurance

- Household's expected wealth is:
  - $(1 - \gamma)(W - pA) + \gamma[W - pA - (D - A)]$
    - $W - pA - \gamma(D - A)$
- Assume the following expected utility function

$$U = (1 - \gamma)\ln(W - pA) + \gamma\ln[W - pA - (D - A)]$$

- The efficient level of insurance, A*, is determined by maximising EU and taking FOC

$$\max_{A} U = (1 - \gamma)\ln(W - pA) + \gamma\ln[W - pA - (D - A)]$$

$$\partial U/\partial A = -p(1 - \gamma)(W - pA)^{-1} + \gamma(1 - p)[W - pA - (D - A)]^{-1} = 0$$

Car Insurance

- Assume price of insurance is actuarially fair.
  - $pA = \gamma A$ >> $p = \gamma$
- The F.O.C is now:

$$-\gamma(1 - \gamma)(W - \gamma A)^{-1} + \gamma(1 - \gamma)[W - \gamma A - (D - A)]^{-1} = 0$$
$$>> (W - \gamma A)^{-1} = [W - \gamma A - (D - A)]^{-1}$$

- If A* = D, the equality will hold
  - Household's wealth = expected wealth
  - A risk-averse individual will fully insure against all losses

Actuarially Unfavourable Insurance

- Asymmetric information implies premium paid by household > expected loss
  - $pA > \gamma D$
- This means that household's wealth with insurance will be less than its expected wealth
- Households won't fully insure, but accept some risk
  - The more unfavourable the insurance, the less will be purchased and the more likely that risk averse consumers will be uninsured

Actuarially Unfavourable Insurance

- But...
- As long as the difference in premium and expected loss ($pA - \gamma D$) is below the maximum amount a household is willing to pay for avoiding risky outcomes it will purchase the insurance
  - $pA - \gamma D < W - \gamma D - C$
  - $pA < W - C$ where C = certainty equivalence of wealth

Asymmetric information

- This involves *hidden information that impacts others adversely* because the information can be used to 'take advantage' of the person on the other side of the market
- Two key problems result from this:
  - Adverse Selection
  - Moral Hazard
- Both problems arise because one agent has more information than the other

A different Market for Lemons

- Grade Insurance
  - Students purchase insurance to guarantee grade x at a given price
  - If insured student's grade < x, I grudgingly raise it to x and receive a payment.
- Assume a perfectly competitive market
- Distribution of grades is:
  - A = 10%; B = 25%; C = 30%, D = 25%, F = 10%
  - I require £c for every 1 letter grade rise

Grade Insurance

- Assumptions
  - Only ‘A’-insurance is available
  - Everyone must buy it
  - Students don’t change their behaviour
- We can now determine the premium p_A.

$$P_A = 0.1(4c) + 0.25(3c) + 0.3(2c) + 0.25c = 2c$$

- What happens if we don't enforce purchase of insurance?
  - Premium must rise, if we assume that only weaker students purchase insurance

Grade Insurance

- Assume all students will pay 2c for a 1 grade rise and 0.5c for each further 1 grade increase
- If there are 100 students, how many will purchase insurance at a premium of 2c?
  - 90%
- What’s the cost to the insurance company?

  (4c x 10) + (3c x 25) + (2c x 30) + (c x 25) = 200c
  - Average cost per student = 2.2c
- To make 0 profit, premium must be 2.2c
- But, at p = 2.2c, the B students won't buy!

Grade Insurance and Adverse Selection

- B students are willing to pay 2c, but premium is 2.2c >> they don't buy
- If only F, D and C buy >> premium must rise
  - (4c x 10) + (3c x 25) + (2c x 30) = 175
  - Average cost per student = 2.69c
- But at p = 2.69c, C students won't buy!
  - Premium must rise to 3.29c
- But at p = 3.29c, D students won't buy!
  - Premium must rise to 4c
- Now, even the F students won't buy!

Adverse Selection

- Students have more information about their ability and hence about the cost of insurance
  - High 'cost' students will buy more insurance than low 'cost' students
  - The competitive equilibrium means no insurance is sold >> inefficient.
- Students who buy insurance impose a negative externality in the market by raising the AC of insurance (and the premium)
  - Therefore, a market equilibrium doesn't exist

Grade Insurance and Moral Hazard

- Will students work as hard with insurance?
- Assume all students react in the same way and achieve 1 grade less than before
  - We can calculate the additional cost to the insurance company: premium rises to 2.9c
  - Anticipating moral hazard >> premiums rise
- Students now differ in their change in behaviour
  - Insurance company has less information and doesn't know who will exhibit more moral hazard
  - The insurance market again unravels

Grade Insurance Summary

- Adverse Selection alone imposes an externality, raises the price of insurance and can end the market
- Moral hazard by itself can be dealt with by pricing premiums. But if level of moral hazard can't be determined, adverse selection results.
  - Market may again disappear
- To resolve this, I could impose conditions on insurance
  - Compulsory attendance, taking exams etc.

Solutions

- Gather more information
- Laws
- Screening
- Incentive Schemes
  - Lower Premiums for those less likely to claim
- Excesses
- No Claims Bonus
- Signals
  - Make it clear that you are a good risk

Signalling

- You can't simply say 'I will get an A'
  - Those who will get an F also say 'I'll get an A'
- In the 'used car market' those selling good quality cars could charge a higher price
  - But sellers of bad quality cars just match it
- A strong signal must have a lower cost for sellers of high quality goods
  - Full warranties
    - Can they cause moral hazard and adverse selection?

An Application - Health insurance

- It gives you an expected wealth with certainty, rather than risky outcomes
  - Certainty gives utility
- If insurer can't distinguish between healthy and unhealthy people, an average premium will be paid
  - Healthy people won't pay it
  - So, insurer covers only unhealthy people!
    - Cutler and Reber; 1998: Harvard Insurance

Conclusion

- As uncertainty exists, insurance exists
  - People are typically risk-averse and so take out insurance, because certainty gives utility
  - The utility of expected wealth > expected utility of wealth
- Moral hazard and adverse selection develop naturally from an insurance market
  - Solutions do exist, but they may not be complete

Self-study questions

- What is the difference between risk and uncertainty?
- How is expected value calculated?
- How do expected value and expected utility differ?
- Explain the different attitudes towards risk. Make sure you can explain the shape of the utility functions for each attitude.
- In the husband/wife case: what happens when (a) the premium rises to £40,000 and the pay-out to £160,000 and (b) the premium rises to £60,000 and the pay-out to £240,000?
- Why will a risk-averse individual fully insure, as long as premiums are actuarially fair?

Self-study questions

- What is meant by actuarially unfavourable insurance?
- How is the expected value calculated when an individual takes out insurance?
- What is moral hazard? Use an example to explain why this is a problem.
- What is adverse selection? Use an example to explain why it occurs and the likely outcomes.
- Explain the market for lemons. You can use any example!
- Why does adverse selection arise?

Self-study questions

- What solutions exist to combat adverse selection and moral hazard?
- Why does a signal have to have different costs for high and low cost individuals?
- Why do people take out fully comprehensive car insurance?
- Why is 3rd party insurance compulsory, but fully comprehensive insurance is voluntary?
- Is education just a signal to an employer that you are a good risk? Is there a problem of moral hazard and adverse selection in the market for employment?