Introduction to Macroeconomics

Diploma Induction

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Overview

Key macroeconomic variables and some data
- Aggregate output
- Aggregate price level
- Growth rates
- A note on long term growth rates
- Unemployment
- Trends and cycles
- Components of expenditure

IS-LM model
- Goods market
- Money market
- IS-LM equilibrium

Intertemporal choice
- Consumption in a 2-period Fisher model
- Interest rate changes, income and substitution effect

Macroeconomics

Focus

- The performance, structure, behaviour of an economy as a whole
    - rather than of individuals, firms and individual markets

Topics

- aggregate output, aggregate unemployment, aggregate price level, aggregate consumption, aggregate investment, aggregate saving, international trade, interest rates, exchange rates
- growth and business cycles
- policy (fiscal, monetary, trade, ...)

Measures

- GDP, unemployment rate, CPI and GDP deflator, inflation rate, etc

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GDP: Gross Domestic Product

Definitions:

- Total market value of all final goods and services produced in an economy over a given period of time
- The total income earned by factors of production in an economy over a given period of time
- The total expenditure on final goods and services in the economy during a given period of time

GDP: Gross Domestic Product

Considerations:

- units of measurement
- length of periods
- total vs per capita
- nominal vs real
- growth rates, components, etc

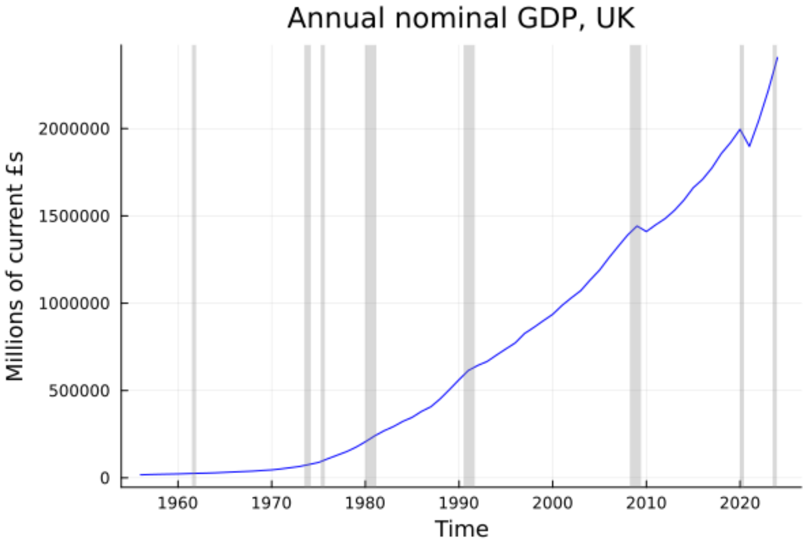
Nominal GDP

Measure of output at current prices:

- Let $\{q_{i,t}\}_{i=1}^{N}$ be the quantities of each of the N goods produced during period t
- Let $\{P_{i,t}\}_{i=1}^{N}$ be the prices of each of the N goods during period t

Then

$$\text{Nominal GDP}_t = \sum_{i=1}^{N} q_{i,t} \times P_{i,t}$$



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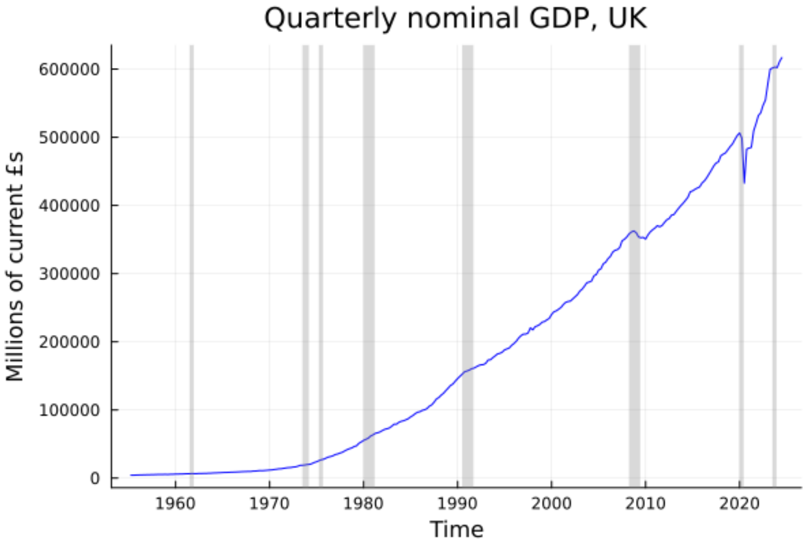

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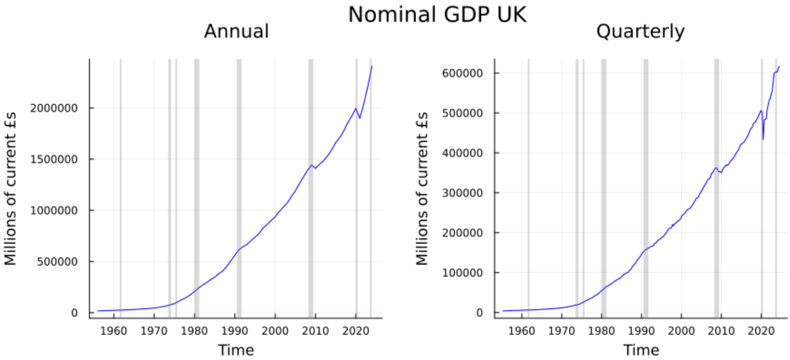

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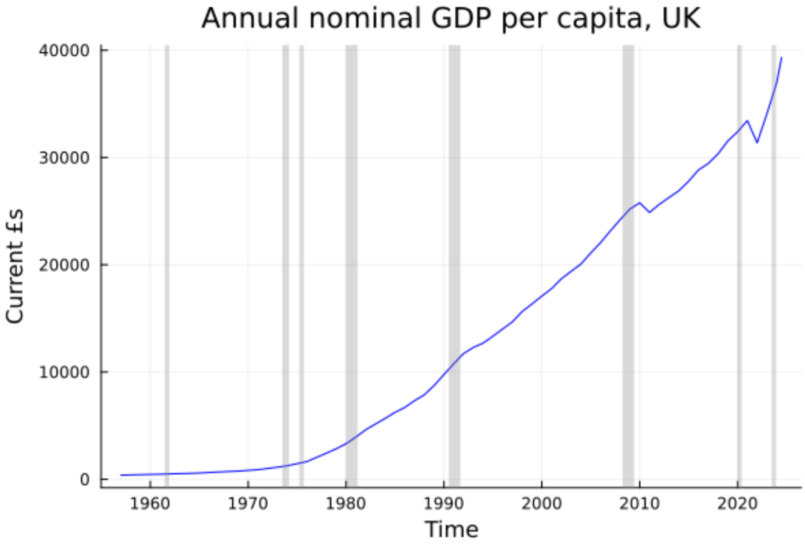
Per capita GDP

Measure of output produced per person or equivalently income per person

Then

$$\text{GDP per capita}_t = \frac{\text{GDP}_t}{\text{Population}_t}$$



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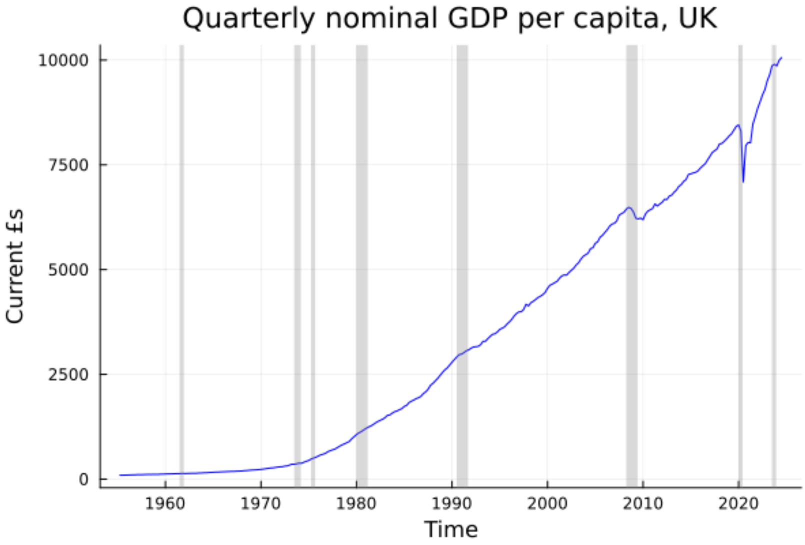

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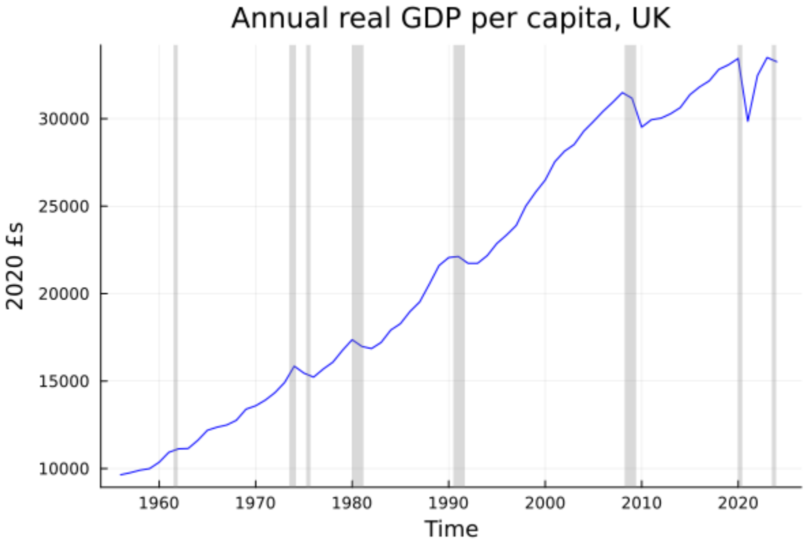
Real GDP

Measure of output at constant prices:

- Let $\{q_{i,t}\}_{i=1}^{N}$ be the quantities of each of the N goods produced during period t
- Let $\{P_{i,tBase}\}_{i=1}^{N}$ be the prices of each of the N goods during a specific period $tBase$

Then

$$\text{Nominal GDP}_t = \sum_{i=1}^{N} q_{i,t} \times P_{i,tBase}$$



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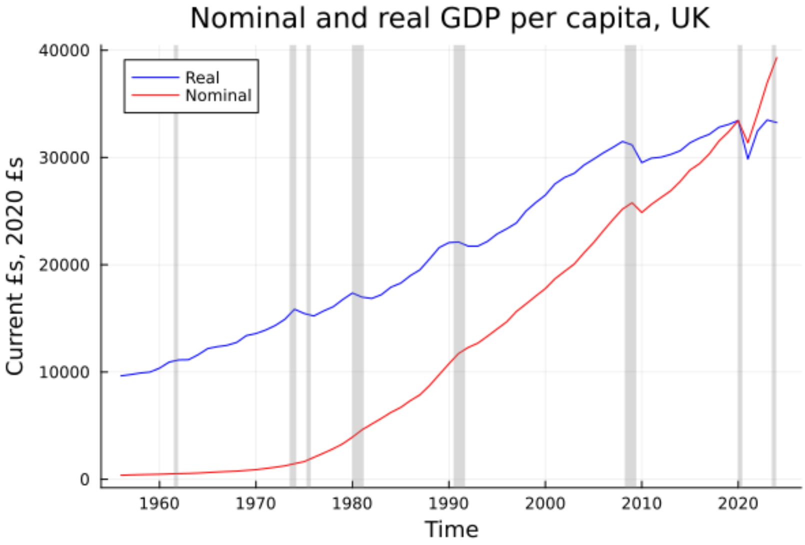

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Aggregate price level measures

GDP deflator:

Measure of price level, P_t, such that

$$\text{Nominal GDP}_t = \text{Real GDP}_t \times P_t$$

$$P_t = \frac{\text{Nominal GDP}_t}{\text{Real GDP}_t}$$

CPI:

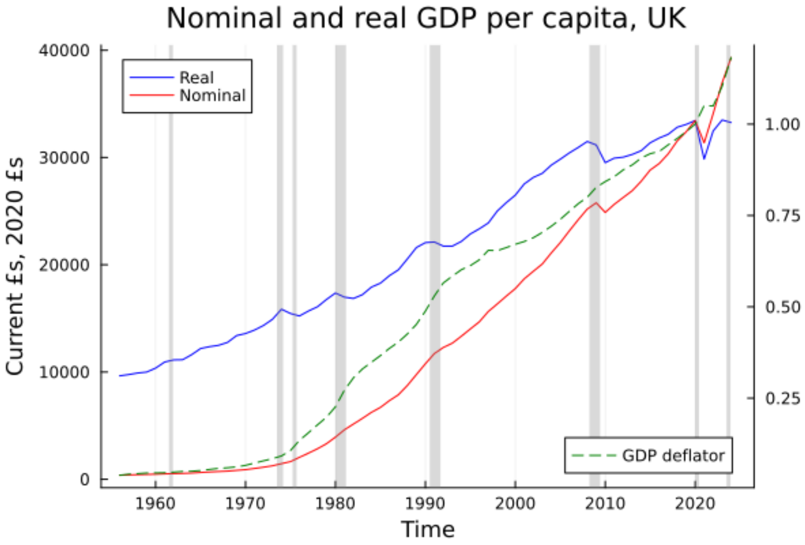
Measure of price level based on the current price of a fixed basket.

- Let $\{c_i\}_{i=1}^N$ be the quantities of N goods in a representative consumer basket. Then the price of the basket at time t is

$$P_t = \sum_{i=1}^N c_i \times P_{i,t}$$

Time	Nominal GDP pc	Real GDP pc	GDP deflator
1955-12-31	376.00	9642.00	0.04
⋮	⋮	⋮	⋮
2017-12-31	31534.00	32820.00	0.96
2018-12-31	32397.00	33082.00	0.98
2019-12-31	33443.00	33443.00	1.00
2020-12-31	31369.00	29852.00	1.05
2021-12-31	34078.00	32468.00	1.05
2022-12-31	36966.00	33497.00	1.10
2023-12-31	39315.00	33257.00	1.18

Source: Office for National Statistics



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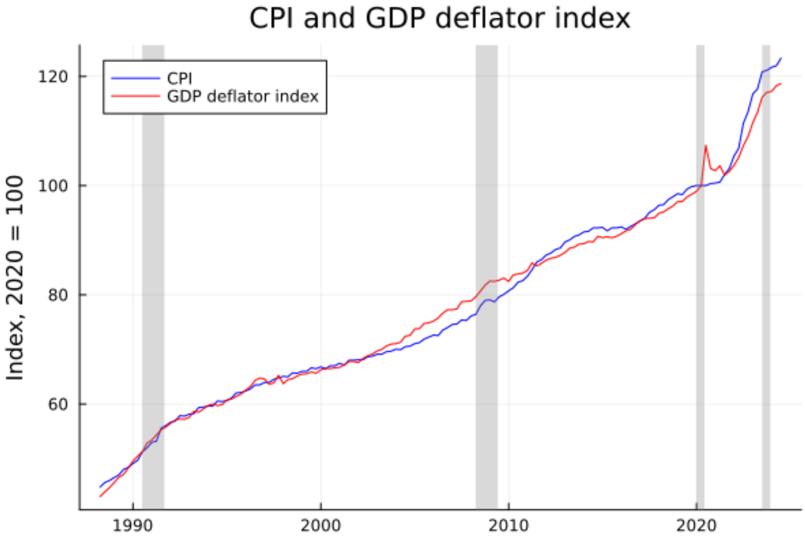

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Growth rates

Given a time changing quantity x_t with t denoting a period of time the growth rate of x from $t-1$ to t is the percentage change for the period.

$$g_t^x = \frac{x_t - x_{t-1}}{x_{t-1}}$$

- if y_t is GDP per capita in year t then the annual GDP per capita growth rate is
$$g_t^y = \frac{y_t - y_{t-1}}{y_{t-1}}$$
- if y_t is GDP per capita in quarter t then the quarterly GDP per capita growth rate is also
$$g_t^y = \frac{y_t - y_{t-1}}{y_{t-1}}$$
but now t denotes quarters.

Growth rates

Given a time changing quantity x_t with t denoting a period of time the growth rate of x from $t-1$ to t is the percentage change for the period.

$$g_t^x = \frac{x_t - x_{t-1}}{x_{t-1}}$$

- if p_t is the aggregate price in year t, under whichever measure, then the annual growth rate of the price level is

$$g_t^p = \frac{p_t - p_{t-1}}{p_{t-1}}$$

- the growth of the annual price level is called price inflation and the growth rate of prices, the inflation rate, is commonly denoted by π_t.

Time	Real GDP pc	Real GDP pc growth rate
2017-12-31	32820.000	...
2018-12-31	33082.000	0.008
2019-12-31	33443.000	0.011
2020-12-31	29852.000	-0.107
2021-12-31	32468.000	0.088
2022-12-31	33497.000	0.032
2023-12-31	33257.000	-0.007

Source: Office for National Statistics

Time	Nominal GDP pc	Real GDP pc	GDP deflator
2022-12-31	9503.00	8359.00	1.14
2023-03-31	9644.00	8342.00	1.16
2023-06-30	9860.00	8326.00	1.18
2023-09-30	9900.00	8295.00	1.19
2023-12-31	9859.00	8249.00	1.20
2024-03-31	9994.00	8288.00	1.21
2024-06-30	10059.00	8315.00	1.21

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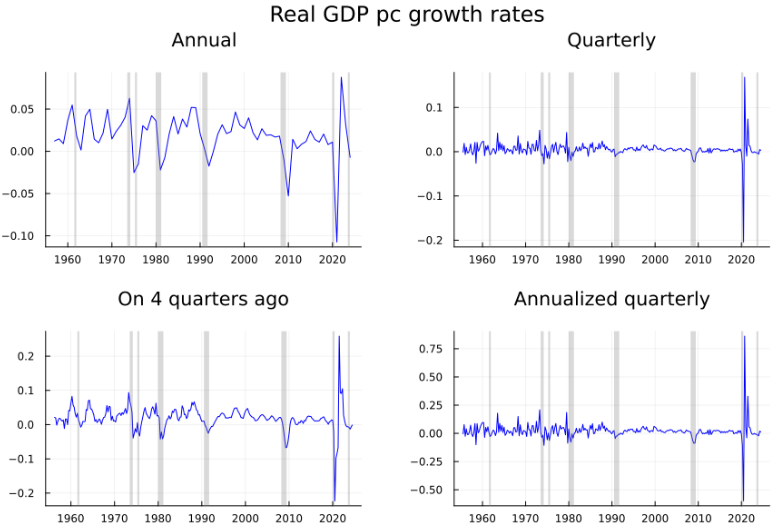

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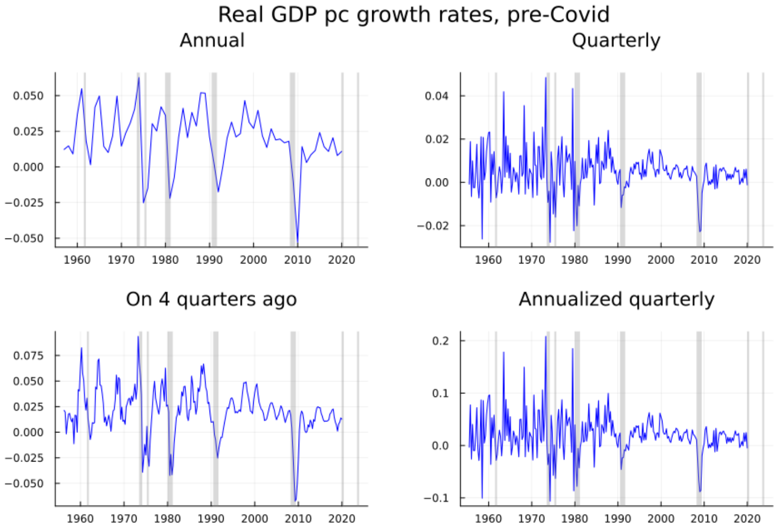

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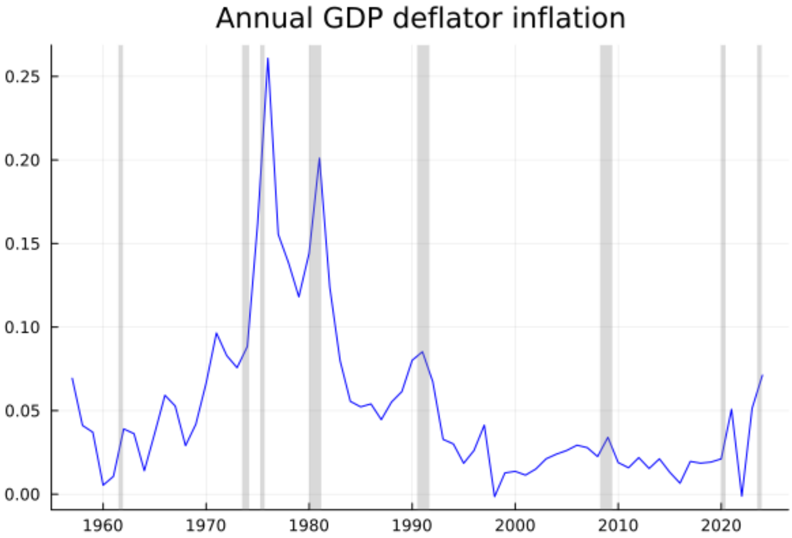

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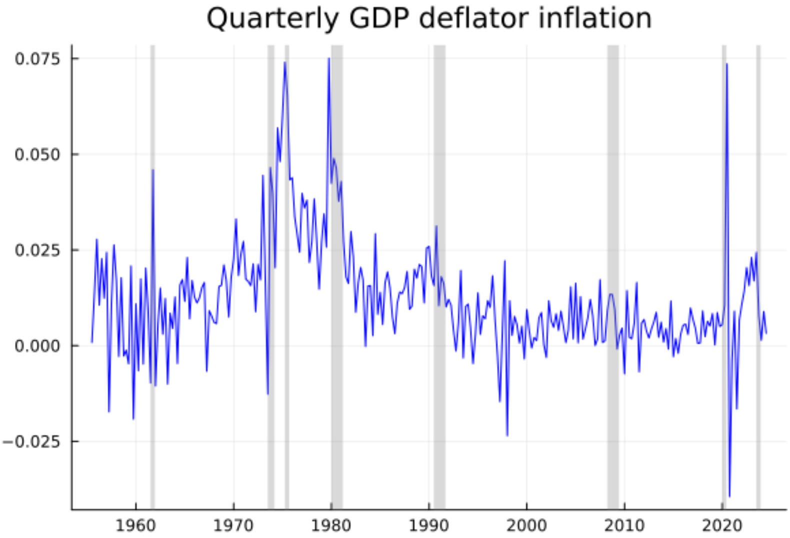

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Long-term growth rates

By inspection it previous figures make apparent a certain slowdown of GDP growth since the Great Recession.

We can evaluate this by finding the average annual growth rates over longer periods of time.

If x_t grows on average by rate of g per period, then for T periods

$$x_T = x_0(1+g)^T$$

Conversely, given two observations of x_t T periods apart, the average per-period growth rate is

$$g = \left(\frac{x_T}{x_0}\right)^{1/T} - 1$$

Average annual growth rate from 1955 to 2006 is

$$30937 = 9642(1+g)^{2006-1955} \Rightarrow g = \left(\frac{30937}{9642}\right)^{1/51} - 1 = 2.3\%$$

Average annual growth rate from 2009 to 2023 is

$$39315 = 24876(1+g)^{2023-2009} \Rightarrow g = \left(\frac{39315}{24876}\right)^{1/14} - 1 = 0.8\%$$

Time	Nominal GDP pc	Real GDP pc	GDP deflator
1955-12-31	376.00	9642.00	0.04
⋮	⋮	⋮	⋮
2006-12-31	24200.00	30937.00	0.78
⋮	⋮	⋮	⋮
2009-12-31	24876.00	29518.00	0.84
⋮	⋮	⋮	⋮
2023-12-31	39315.00	33257.00	1.18

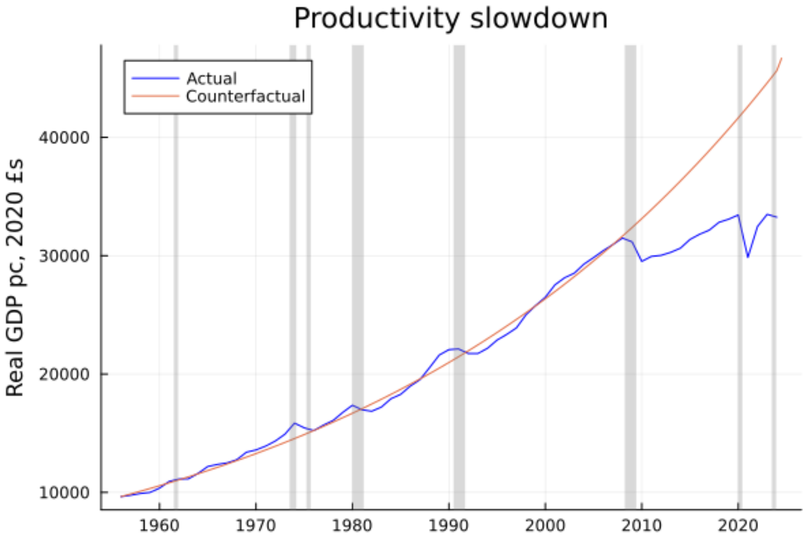
Growth slowdown

Similarly we can construct average annual growth rates per decades and confirm that this slowdown is specific to the most recent decades

Decade	Average annual growth rate per decade
1955-1965	0.025
1965-1975	0.021
1975-1985	0.022
1985-1995	0.021
1995-2005	0.027
2005-2015	0.004

As another way to visualize the slowdown, consider a counterfactual economy which grows constantly at the same average growth rate as UK in 1955-2006 for the whole time period. The real GDP per capita in this economy at time t is

$$y_t = 9642 \times 1.023^{t-1955}$$



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A note on use of log transformations

Often time series for GDP (and other growing quantities) are represented graphically on a log scale.

The reason is that the slope of a graph on a log scale equals (approximately) the growth rate. If

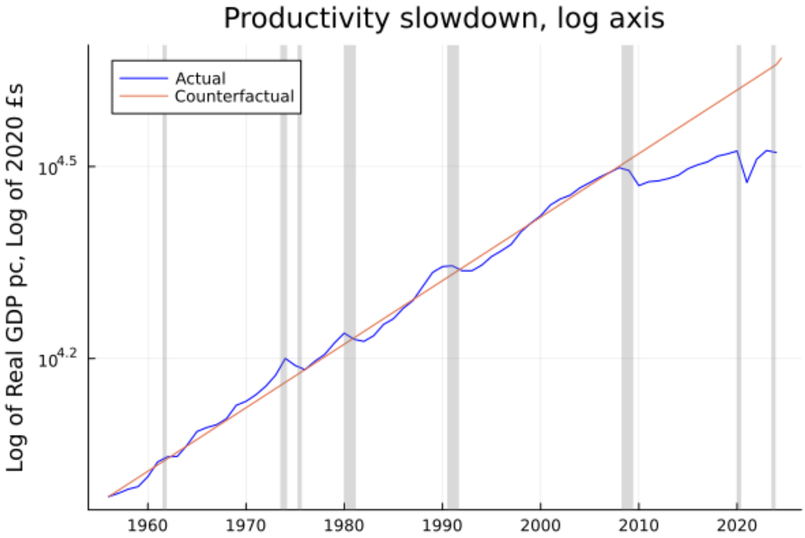
$$x_t = x_0(1+g)^t$$

then

$$\ln(x_t) = \ln(x_0) + t \times \ln(1+g) \approx \ln(x_0) + t \times g$$

Another way to think of the same: if $x(t)$ is a continuous function of time then

$$\frac{d \ln x(t)}{dt} = \frac{dx/dt}{x}$$



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Unemployment

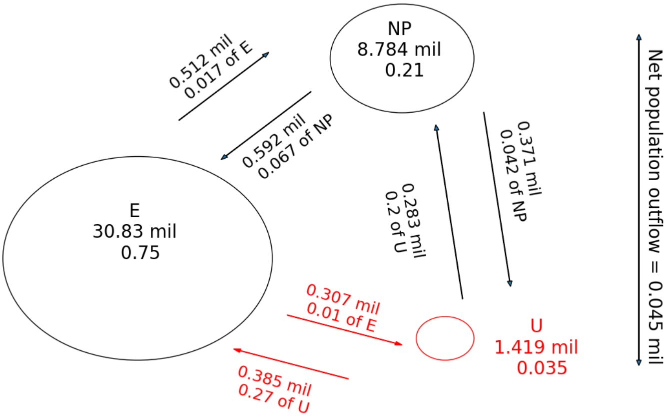
Individuals of working population are classified into the following groups by labour-market status.

- (E)mployed: have a job
- (U)nemployed: don't have a job but actively searching
- (N)on-participants: don't have a job and not searching

An important macroeconomic indicator, the unemployment rate, is defined as

$$u_t = \frac{U_t}{U_t + E_t}$$

UK labour market in Q1, 2017



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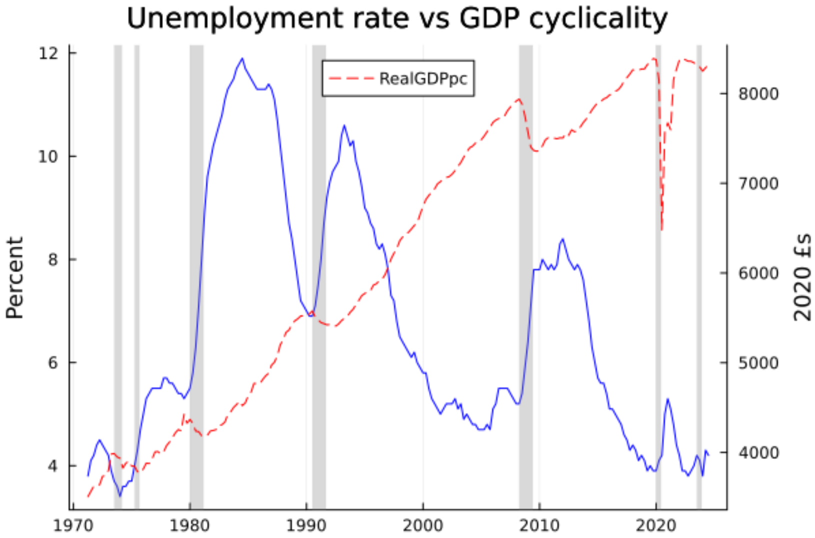

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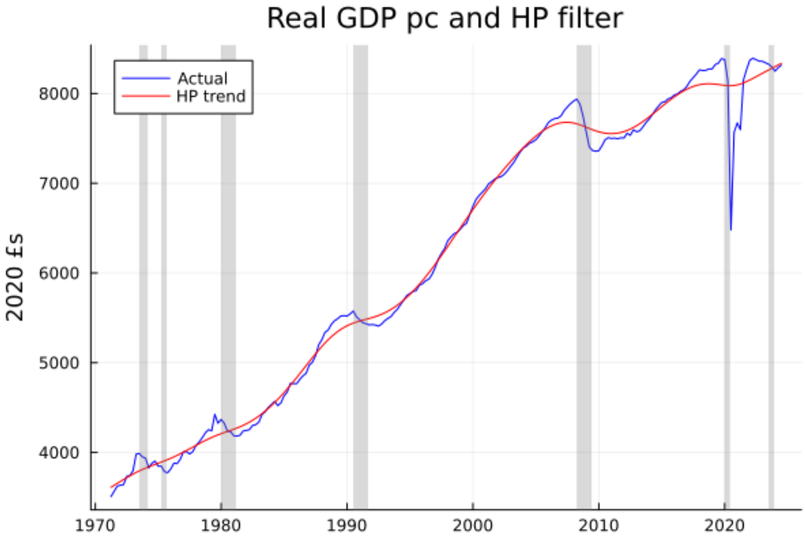

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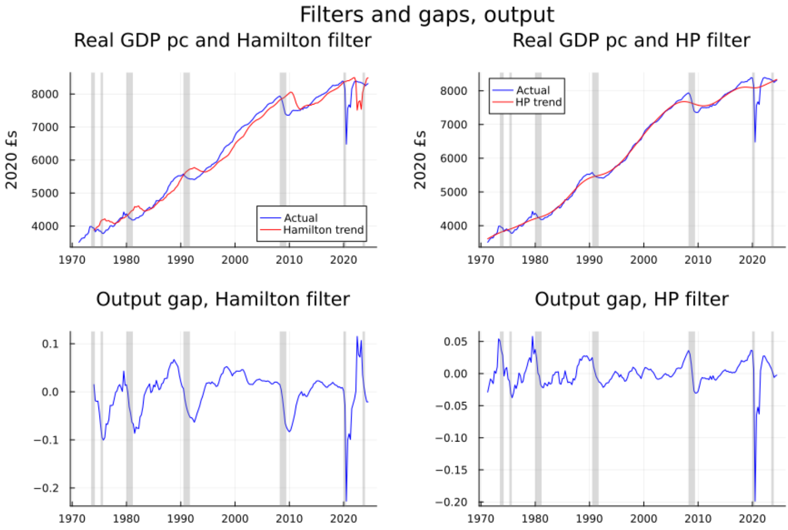

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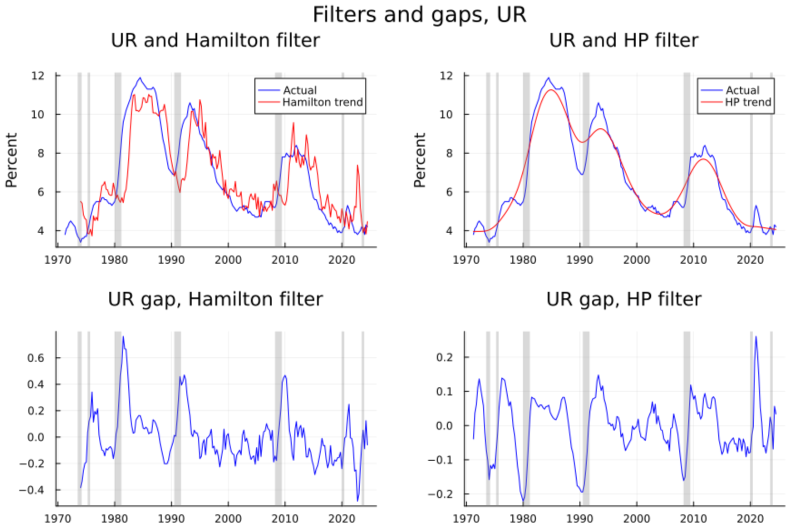

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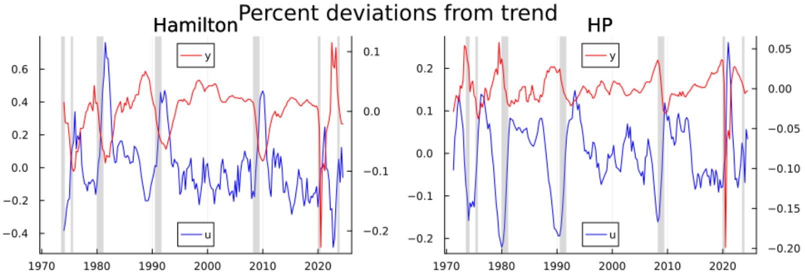

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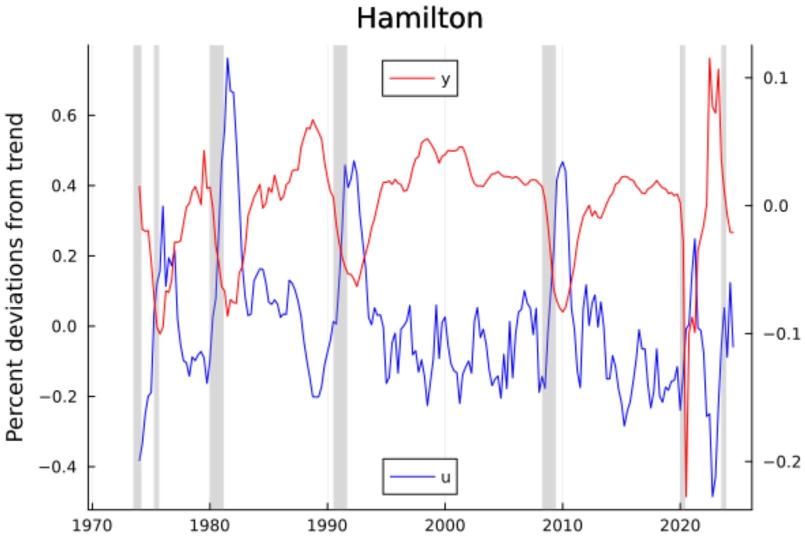

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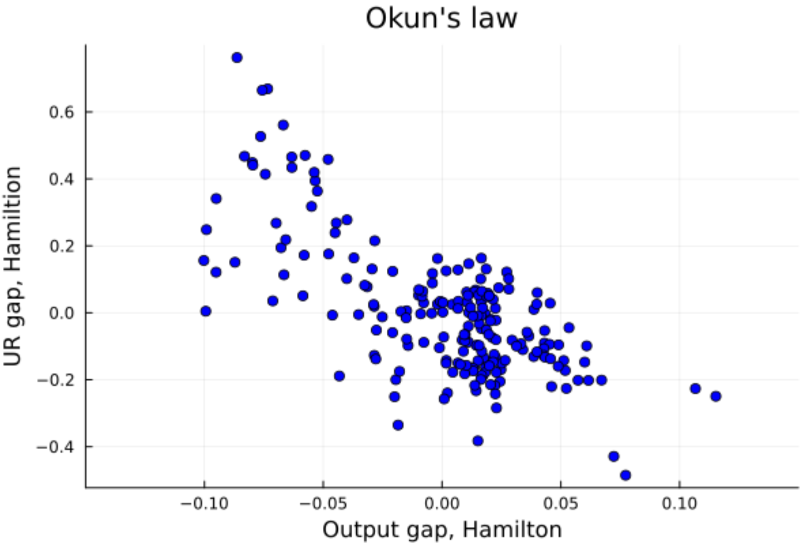

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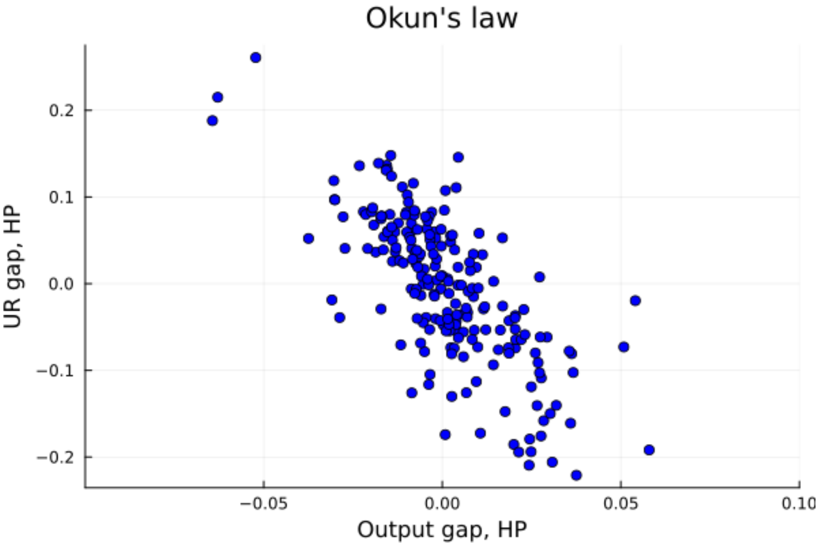

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GDP and components of expenditure

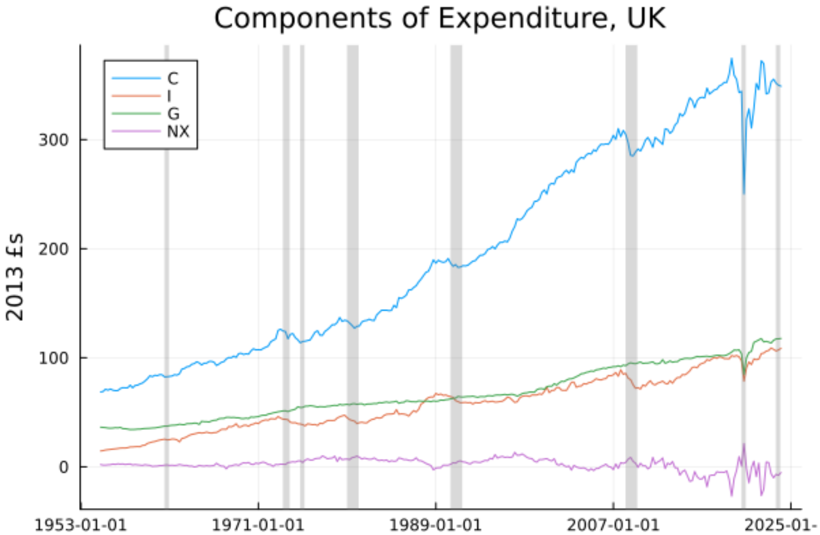
Each transaction contributing to GDP, Y, is classified into one of the components of expenditure

- C, consumption
- I, investment
- G, government spending
- NX, net export

As GDP is the sum of all expenditures

$$Y = C + I + G + NX$$

The four components have different levels and cyclical behaiour.



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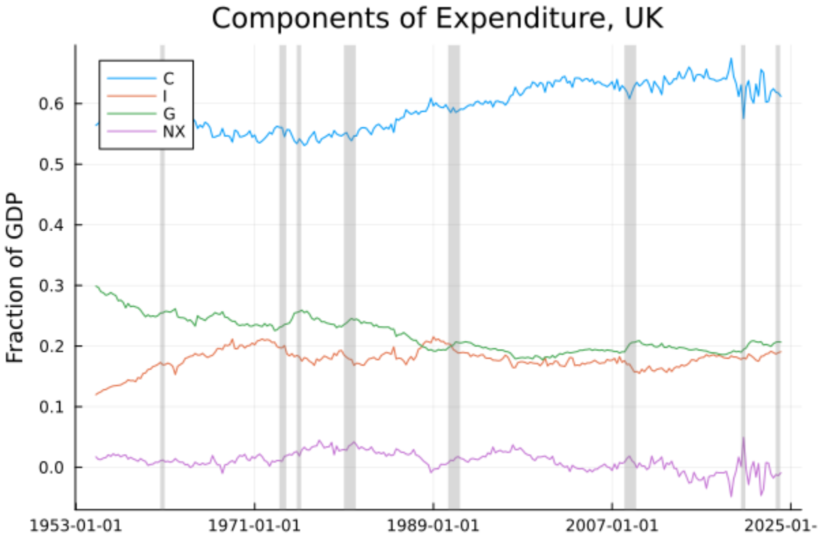

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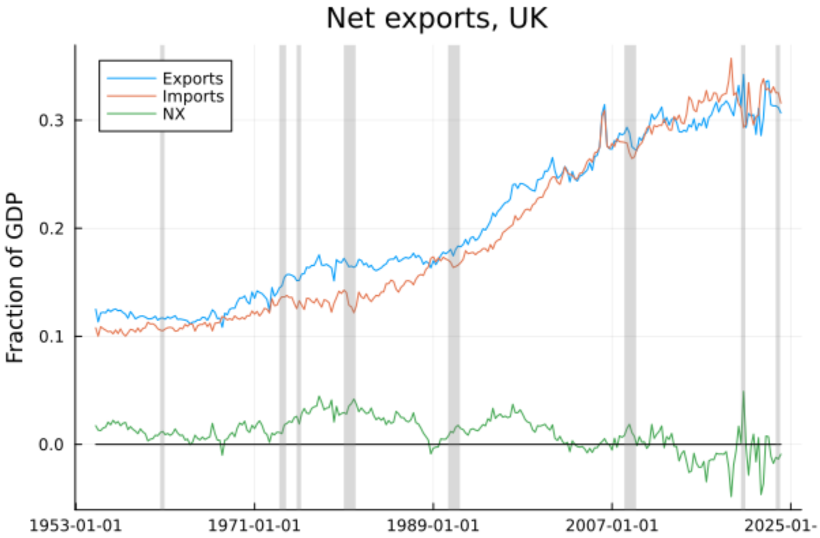

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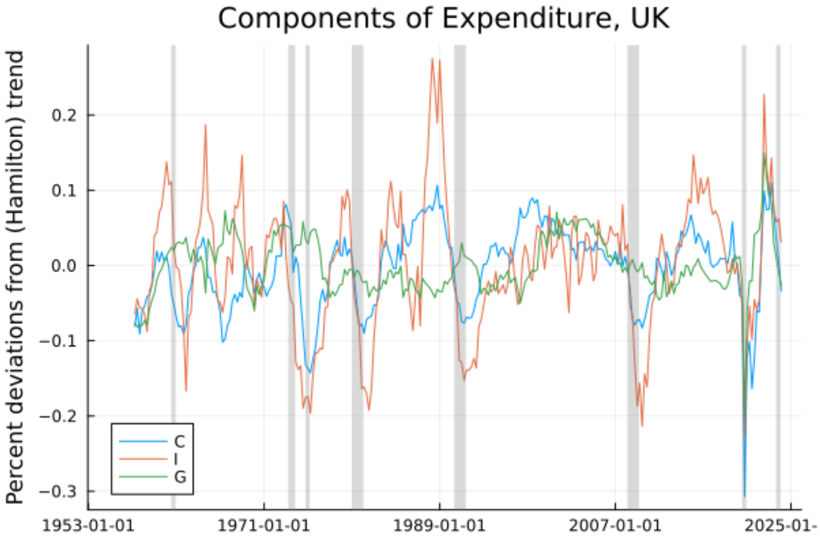

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IS-LM model

Developed by John Hicks in 1937 is the canonical mathematical model of the old Keynesian school

- dominant framework of macroeconomic analysis until the 1970s
- no longer
- still useful framework for thinking about key macro questions as first intuition
- Term 1 of EC108 is all about IS-LM logic

IS-LM model: building blocks

- Equilibrium in the goods market
  - Theory of consumption: Keynesian consumption function
  - Theory of investment: Keynesian investment function
  - Equilibrium notion: income equals expenditure
- Equilibrium in the money market
  - Theory of liquidity preference
  - Equilibrium notion: money supply equals money demand
- Joint equilibrium: theory of joint determination of output and interest rate

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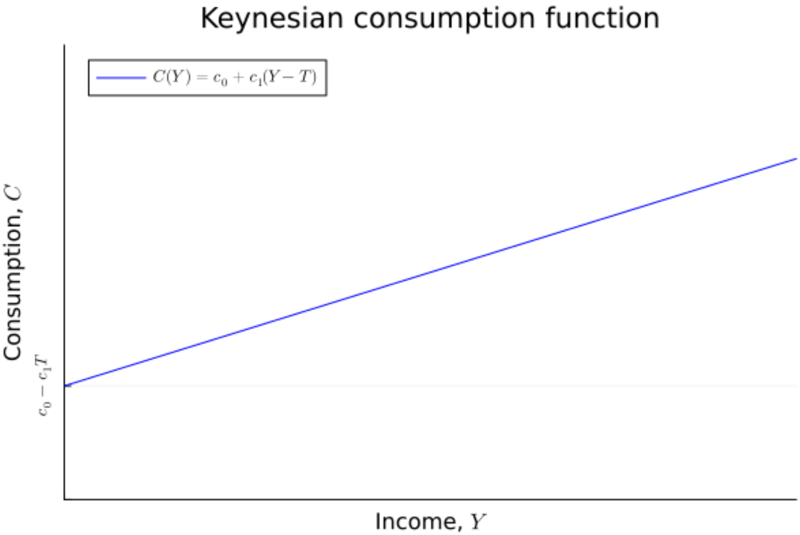
Consumption

Assumption: Keynesian consumption function

$$C(Y) = c_0 + c_1(Y - T)$$

Crucially, consumption depends on income only contemporaneously.

- $c_0 > 0$, autonomous consumption
- $0 < c_1 < 1$, marginal propensity to consume
- $Y - T$, disposable income


Keynesian consumption function
$C(Y) = c_0 + c_1(Y - T)$
Consumption, C
$c_0 - c_1 T$
Income, Y

Investment

Assumption: Investment depends negatively on (real) interest rate

$$I(r) = i_0 - i_1 r$$

Rationale: Time value of money

- If I can borrow and lend at the same interest rate r, then I am indifferent between
- £1 today, £1$(1+r)$ in a year, £1$(1+r)^2$ in two years, ..., £1$(1+r)^n$ in n years,...
- Conversely, £1 in n years has the **present value** of

$$\pounds\frac{1}{(1+r)^n}$$

today (discounting).

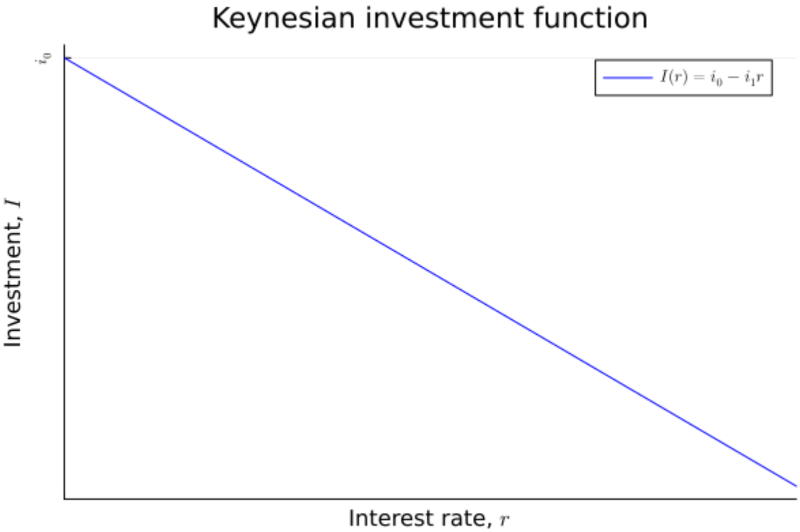
Investment, cont'd

Assumption: Investment depends negatively on (real) interest rate

$$I(r) = i_0 - i_1 r$$

Rationale: Investment projects typically involve

- (often large) cost incurred early on
- with benefits accruing in future
- Higher r reduces the present value of future payoffs hence the net present value of investment projects, leading to a decline in investment.


Keynesian investment function
i_0
$I(r) = i_0 - i_1 r$
Investment, I
Interest rate, r

Goods market equilibrium

Total expenditure in economy is

$$E = C + I + G + NX$$

Assume closed economy

$$E = C + I + G$$

and exogenous G, above assumptions imply expenditure depends on income and interest rate according to

$$E(Y, r) = C(Y) + I(r) + G = c_0 + c_1(Y - T) + i_0 - i_1 r + G$$

But by definition, expenditure is also equal to income in a macroeconomy

$$E = Y$$

Goods market equilibrium

In equilibrium output (and expenditure) are determined by

$$E = E(Y, r) = C(Y) + I(r) + G = \\ c_0 + c_1(Y - T) + i_0 - i_1 r + G \quad (1)$$

$$E = Y \quad (2)$$

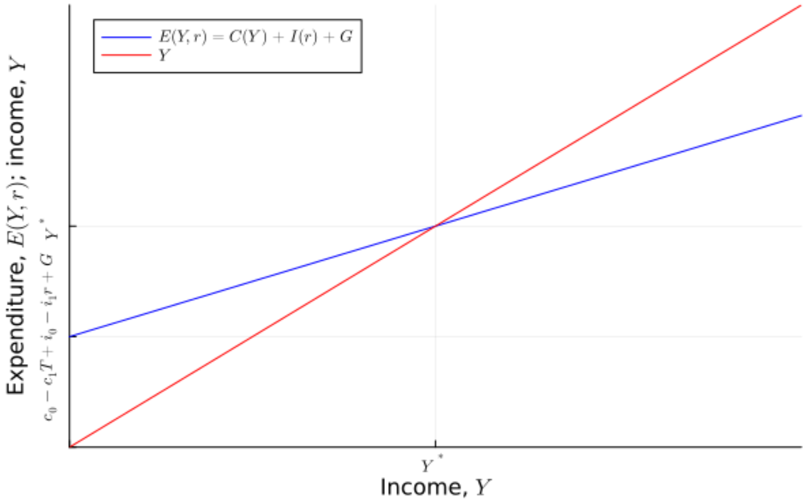
Hence equilibrium output, Y^* solves

$$Y^* = c_0 + c_1(Y^* - T) + i_0 - i_1 r + G$$

or equivalently

$$Y^* = \frac{c_0 - c_1 T + i_0 - i_1 r + G}{1 - c_1}$$

Keynesian cross



Keynesian cross, comparative statics

Given

$$Y^* = \frac{c_0 - c_1 T + i_0 - i_1 r + G}{1 - c_1}$$

some comparative statics results are

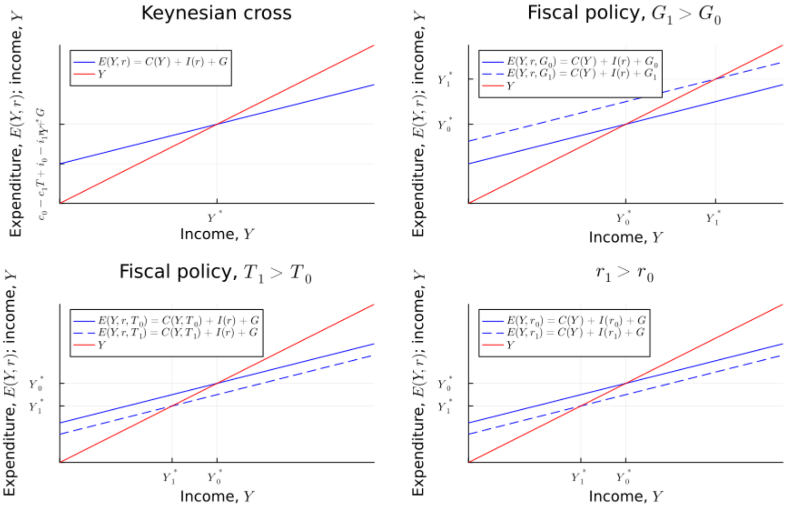
$$\frac{\partial Y^*}{\partial G} = \frac{1}{1 - c_1} > 1 \qquad \text{(GSM)}$$

$$\frac{\partial Y^*}{\partial T} = -\frac{c_1}{1 - c_1} < 0 \qquad (3)$$

$$\frac{\partial Y^*}{\partial r} = -\frac{i_1}{1 - c_1} < 0 \qquad (4)$$

(GSM) is known as the *Keynesian government spending multiplier*.

Comparative statics



Keynesian cross and IS curve

In the full IS-LM model both Y and r are endogenous. (4) shows that the relationship between Y and r across all possible goods market equilibria is negative.

Seen as a relationship between Y and r the equilibrium condition

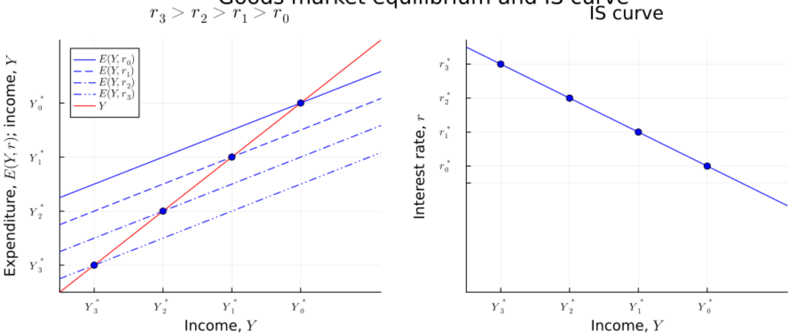
$$Y = \frac{c_0 - c_1 T + i_0 + G}{1 - c_1} - \frac{i_1}{1 - c_1} r \tag{IS1}$$

or equivalently

$$r = \frac{c_0 - c_1 T + i_0 + G}{i_1} - \frac{1 - c_1}{i_1} Y \tag{IS2}$$

is known as the IS curve (habitually plotted in (Y, r)-plane)

Goods market equilibrium and IS curve



Outline

Key macroeconomic variables and some data

- Aggregate output
- Aggregate price level
- Growth rates
- A note on long term growth rates
- Unemployment
- Trends and cycles
- Components of expenditure

IS-LM model

- Goods market
- Money market
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Intertemporal choice

- Consumption in a 2-period Fisher model
- Interest rate changes, income and substitution effect

Money demand

The rationale behind the modelling of the money market in the IS-LM model is the **theory of liquidity preference**.

Main idea: I have fixed wealth, which I decide how to allocate between *liquid* assets (money) and *illiquid* assets (say bonds).

- I can make transactions with money, but not with bonds. If I want to make more transactions, I will allocate more of my weath to money.
  - How much transactions I want to make likely depends on how much stuff I want to buy, which depends positively on my income. Hence, money demand will depend positively on Y
- Money does not provide returns but illiquid assets do (e.g., interest in case of bonds).
  - When r is high, I would like to allocate more wealth to bonds rather than money. Hence, money demand will depend negatively on interest rates.

Money market equilibrium

Hence, the demand for *real money balances*, $\left(\frac{M}{P}\right)^d$, will depend positively on Y and negatively on r.

$$\left(\frac{M}{P}\right)^d = \left(\frac{M}{P}\right)^d (Y, r) = m_y Y - m_r r$$

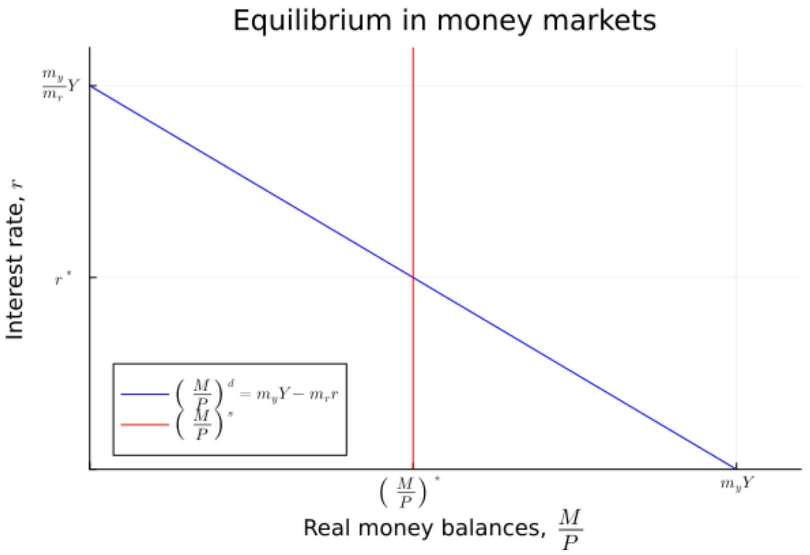
or equivalently

$$r\left(Y, \left(\frac{M}{P}\right)^d\right) = \frac{m_y}{m_r} Y - \frac{(M/P)^d}{m_r}$$

Suppose that the supply of real money balances, $\left(\frac{M}{P}\right)^s$, is exogenous (e.g., determined by the central bank).

In money market equilibrium

$$\left(\frac{M}{P}\right)^d = \left(\frac{M}{P}\right)^s$$


Equilibrium in money markets
$\frac{m_y}{m_r}Y$
r^*
Interest rate, r
$\left(\frac{M}{P}\right)^d = m_y Y - m_r r$
$\left(\frac{M}{P}\right)^s$
$\left(\frac{M}{P}\right)^*$
$m_y Y$
Real money balances, $\frac{M}{P}$

Money market equilibrium

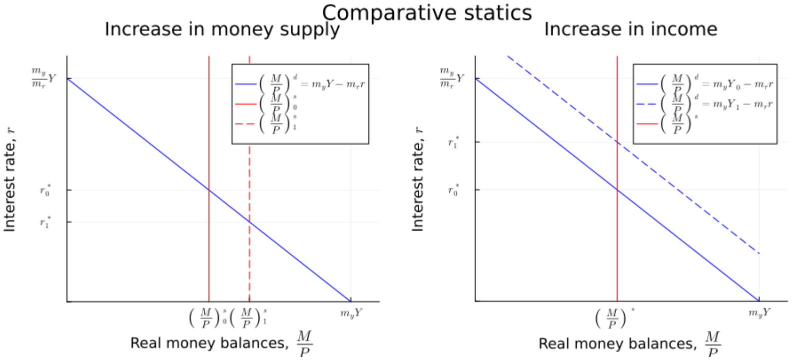
Hence the equilibrium interest rate is

$$r^* = \frac{m_y}{m_r} Y - \frac{(M/P)^s}{m_r}$$

Some comparative statics results

$$\frac{\partial r^*}{\partial (M/P)^s} = -\frac{1}{m_r} < 0 \quad (5)$$

$$\frac{\partial r^*}{\partial Y} = \frac{m_y}{m_r} > 0 \quad (6)$$


Comparative statics
Increase in money supply
Increase in income
Interest rate, r
Real money balances, M/P
m_y Y − m_r r
m_y Y_0 − m_r r
m_y Y_1 − m_r r
m_y Y

Money market and IS curve

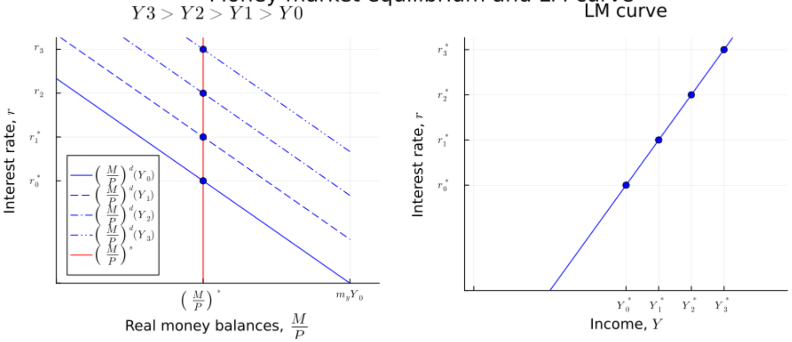
In the full IS-LM model both Y and r are endogenous. (6) shows that the relationship between Y and r across all possible money market equilibria is positive.

Seen as a relationship between Y and r the equilibrium condition

$$r = \frac{m_y}{m_r} Y - \frac{(M/P)^s}{m_r} \tag{LM}$$

is known as the LM curve (habitually plotted in (Y, r)-plane)

Money market equilibrium and LM curve



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IS-LM equilibrium

When both goods market and money market are in equilibrium

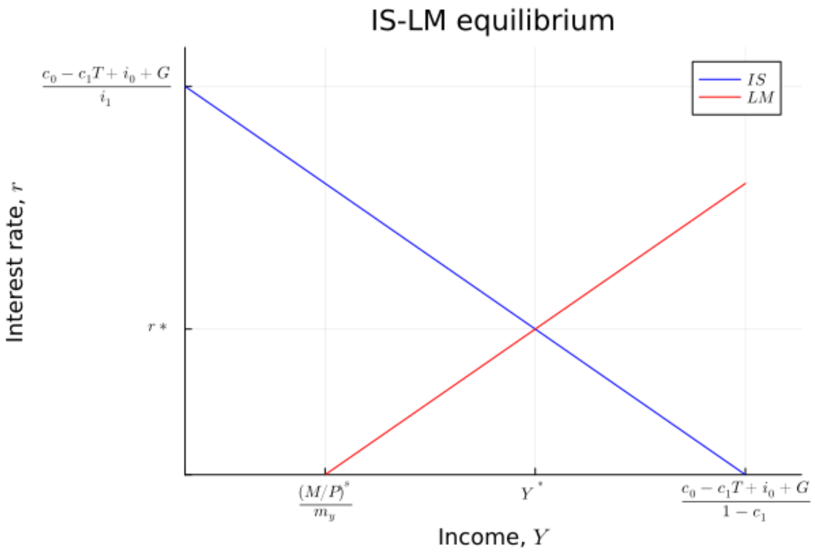
$$r = \frac{c_0 - c_1 T + i_0 + G}{i_1} - \frac{1 - c_1}{i_1} Y \tag{IS}$$

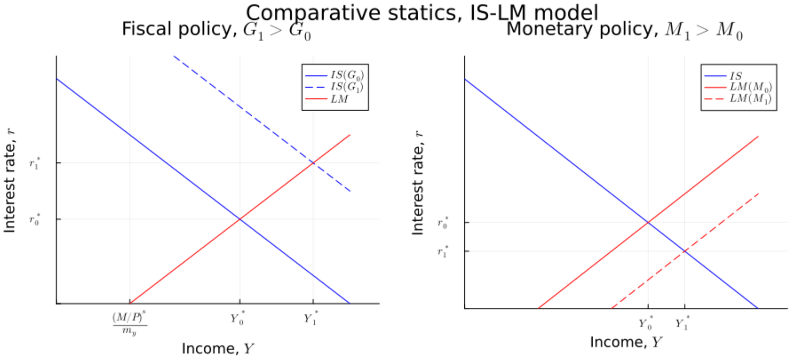
$$r = \frac{m_y}{m_r} Y - \frac{(M/P)^s}{m_r} \tag{LM}$$

Hence the equilibrium Y and r are

$$Y^* = \frac{\frac{c_0 - c_1 T + i_0 + G}{i_1} + \frac{(M/P)^s}{m_r}}{\frac{m_y}{m_r} + \frac{1 - c_1}{i_1}}$$

$$r^* = \frac{\frac{c_0 - c_1 T + i_0 + G}{1 - c_1} - \frac{(M/P)^s}{m_y}}{\frac{m_r}{m_y} + \frac{i_1}{1 - c_1}}$$


IS-LM equilibrium
IS
LM
Interest rate, r
$\frac{c_0 - c_1 T + i_0 + G}{i_1}$
$r*$
$\frac{(M/P)^s}{m_y}$
Y^*
$\frac{c_0 - c_1 T + i_0 + G}{1 - c_1}$
Income, Y


Comparative statics, IS-LM model
Fiscal policy, $G_1 > G_0$
$IS(G_0)$
$IS(G_1)$
LM
Interest rate, r
r_1^*
r_0^*
$\frac{(M/P)^s}{m_y}$
Y_0^*
Y_1^*
Income, Y
Monetary policy, $M_1 > M_0$
IS
$LM(M_0)$
$LM(M_1)$
Interest rate, r
r_0^*
r_1^*
Y_0^*
Y_1^*
Income, Y

Readings

You can find a primer on the IS-LM model in most introductory macroeconomics textbooks. A good example is Blanchard (2020).

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Intertemporal consumption-saving choice

- In the IS-LM model, expenditure is modelled on the basis of empirical, behavioural, aggregate relationships.
- An important feature of modern macro models is that they have microeconomic foundations instead.
- In particular, consumptions comes out of microeconomic modelling of optimal behaviour in intertemporal context.
- Another feature of IS-LM model is that it is static. Modern models tend to be dynamic.
- In what follows we consider a simple microfounded 2-period intertemporal choice model of consumption and saving which illustrates key intuitions about modern modelling in a simplified context.

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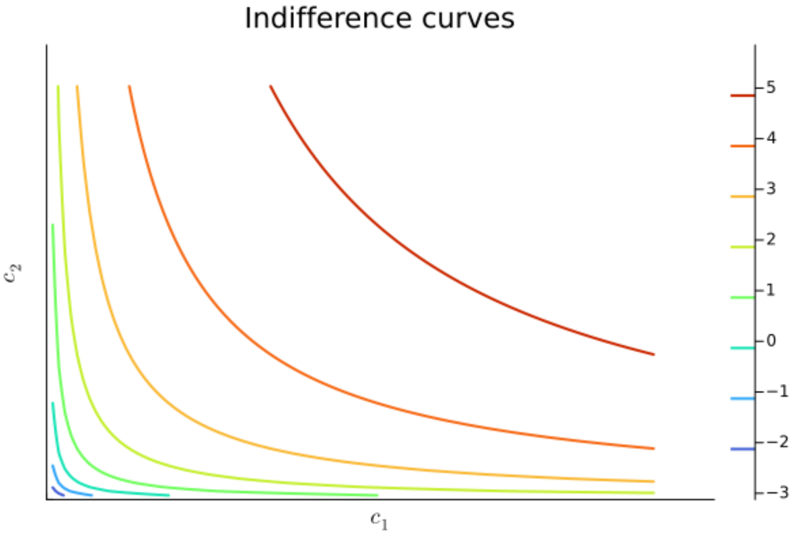
2-period model of consumption, environment, 1

- Individual lives for two periods, 1 and 2.
- They receive exogenous income Y_1 in period 1, and Y_2 in period 2.
- They have preferences over how much to consume in period 1, C_1, and period 2, C_2, defined by a utility function

$$u(C_1, C_2) = v(C_1) + \frac{1}{1+\rho} v(C_2)$$

  where $v(.)$ is some univariate function, and ρ is a utility discount rate.
  - You can think that ρ measures relative impatience and is positive.
- In what follows, for illustration, assume $v(.) = \ln(.)$

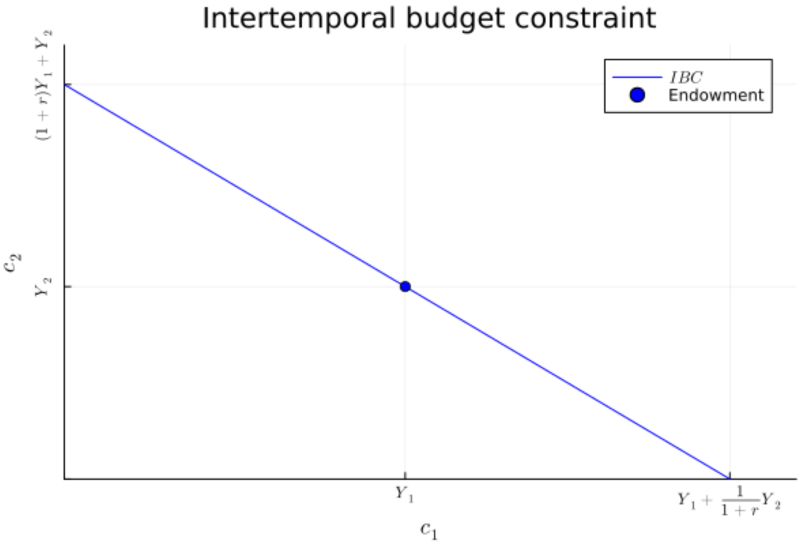

Indifference curves
c_2
c_1
5
4
3
2
1
0
−1
−2
−3

2-period model of consumption, environment, 2

- Can borrow/lend at exogenous interest rate r.
- Given income Y_1 in period 1, and Y_2 in period 2, all just affordable choices of consumption are defined by

$$C_1 + \frac{1}{1+r} C_2 = Y_1 + \frac{1}{1+r} Y_2$$

This is known as the **intertemporal budget constraint**.


Intertemporal budget constraint
IBC
Endowment
c_2
$(1+r)Y_1 + Y_2$
Y_2
c_1
Y_1
$Y_1 + \frac{1}{1+r}Y_2$

Optimal choice

Individuals' optimal choice of consumption (in period 1 and 2) is the solution of a **constrained optimization problem**:
Generally

$$\max_{C_1, C_2} u(C_1, C_2)$$
$$\text{s.t. } IBC$$

Under our assumptions

$$\max_{C_1, C_2} \ln(C_1) + \frac{1}{1+\rho} \ln(C_2) \tag{7}$$
$$\text{s.t. } C_1 + \frac{1}{1+r} C_2 = Y_1 + \frac{1}{1+r} Y_2$$

You should know 3 methods of solving constrained optimization problems.

Solving (7), Direct substitution method

Given

$$\max_{C_1, C_2} \ln(C_1) + \frac{1}{1+\rho}\ln(C_2)$$
$$\text{s.t. } C_1 + \frac{1}{1+r}C_2 = Y_1 + \frac{1}{1+r}Y_2$$

using BC substitute for one of the arguments in terms of the other into the objective function, maximize (the now unconstrained and univariate objective function), substitute back into the BC:

$$(7) \Leftrightarrow \max_{C_1} \ln C_1 + \frac{1}{1+\rho}\ln\left[(1+r)\left(Y_1 + \frac{Y_2}{1+r} - C_1\right)\right]$$

$$\text{FOC: } \frac{1}{C_1} - \frac{1}{(1+\rho)(Y_1 + \frac{Y_2}{1+r} - C_1)} = 0$$

$$C_1^* = \frac{1+\rho}{2+\rho}\left(Y_1 + \frac{Y_2}{1+r}\right), C_2^* = \frac{1+r}{2+\rho}\left(Y_1 + \frac{Y_2}{1+r}\right)$$

Solving (7), "Tangency" method

$$\max_{C_1, C_2} \ln(C_1) + \frac{1}{1+\rho}\ln(C_2)$$
$$\text{s.t. } C_1 + \frac{1}{1+r}C_2 = Y_1 + \frac{1}{1+r}Y_2$$

Intuitively the optimal choice lies at a point of tangency between an indifference curve and the budget constraint, i.e., their slopes are the same. Analytically

$$\left.\frac{dC_2}{dC_1}\right|_{u(C_1,C_2)=u*} = \left.\frac{dC_2}{dC_1}\right|_{IBC} \Leftrightarrow -\frac{\partial u/\partial C_1}{\partial u/\partial C_2} = -\frac{1}{1/(1+r)} \tag{8}$$
$$\frac{1/C_1}{1/((1+\rho)C_2)} = (1+r) \Rightarrow C_2 = \frac{1+r}{1+\rho}C_1 \tag{9}$$

Equations (8) (in general) and (9) (in particular) are known as the Euler equations of the problem.

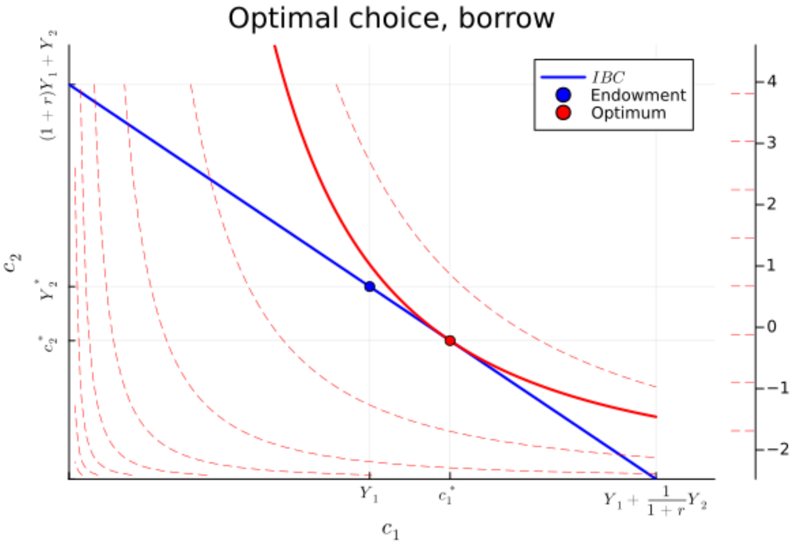
Solving (7), "Tangency" method, 2

Now solve the system of equations defined by the IBC and the EE

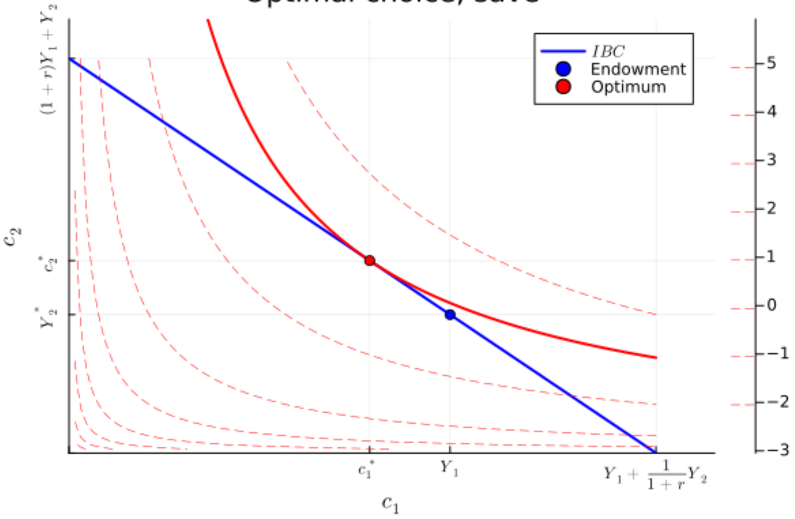
$$C_1 + \frac{1}{1+r} C_2 = Y_1 + \frac{1}{1+r} Y_2$$
$$C_2 = \frac{1+r}{1+\rho} C_1$$

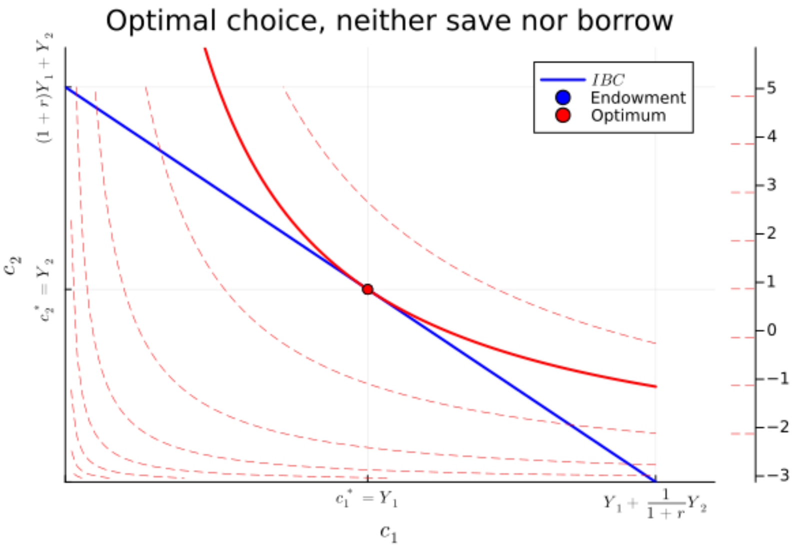
to get (as before)

$$C_1^* = \frac{1+\rho}{2+\rho} \left(Y_1 + \frac{Y_2}{1+r} \right)$$
$$C_2^* = \frac{1+r}{2+\rho} \left(Y_1 + \frac{Y_2}{1+r} \right)$$


Optimal choice, borrow
IBC
Endowment
Optimum
c_2
$(1+r)Y_1 + Y_2$
Y_2^*
c_2^*
c_1
Y_1
c_1^*
$Y_1 + \frac{1}{1+r}Y_2$
4
3
2
1
0
−1
−2

Optimal choice, save





Optimal choice, neither save nor borrow
IBC
Endowment
Optimum
$(1+r)Y_1 + Y_2$
c_2
$c_2^* = Y_2$
$c_1^* = Y_1$
$Y_1 + \frac{1}{1+r}Y_2$
c_1
5
4
3
2
1
0
−1
−2
−3

Solving (7), Lagrange multipliers method

Given

$$\max_{C_1, C_2} \ln(C_1) + \frac{1}{1+\rho} \ln(C_2)$$
$$\text{s.t. } C_1 + \frac{1}{1+r} C_2 = Y_1 + \frac{1}{1+r} Y_2$$

the Lagrangian of the problem is the function

$$\mathcal{L}(C_1, C_2, \lambda) = \ln(C_1) + \frac{1}{1+\rho} \ln(C_2) + \lambda(Y_1 + \frac{1}{1+r} Y_2 - C_1 - \frac{1}{1+r} C_2)$$

and the FOCs of the problem are

$$\mathcal{L}_{C_1} = 0 \Rightarrow \quad \frac{1}{C_1} = \lambda \tag{10}$$

$$\mathcal{L}_{C_2} = 0 \Rightarrow \quad \frac{1}{(1+\rho) C_2} = \frac{\lambda}{1+r} \tag{11}$$

$$\mathcal{L}_{\lambda} = 0 \Rightarrow \quad C_1 + \frac{1}{1+r} C_2 = Y_1 + \frac{1}{1+r} Y_2 \tag{12}$$

Solving (7), Lagrange multipliers method, 2

Note that by eliminating λ using (10) and (11) we obtain the Euler equation (9) again. Further, as (12) is the IBC, the FOCs collapse again to

$$\begin{aligned} C_1 + \frac{1}{1+r} C_2 =& Y_1 + \frac{1}{1+r} Y_2 \\ C_2 =& \frac{1+r}{1+\rho} C_1 \end{aligned}$$

implying (as before)

$$\begin{aligned} C_1^* &= \frac{1+\rho}{2+\rho} \left(Y_1 + \frac{Y_2}{1+r} \right) \\ C_2^* &= \frac{1+r}{2+\rho} \left(Y_1 + \frac{Y_2}{1+r} \right) \end{aligned}$$

Properties of consumption functions

$$C_1^* = \frac{1+\rho}{2+\rho}\left(Y_1 + \frac{Y_2}{1+r}\right)$$
$$C_2^* = \frac{1+r}{2+\rho}\left(Y_1 + \frac{Y_2}{1+r}\right)$$

Note: In sharp contrast to the Keynesian consumption function, here consumption depends on income not only contemporaneously, but consumption in each period depends on income in all periods.

Consumption is now more tightly linked not to current income, but to the present value of all incomes (as seen in expression above) and to **permanent income** (as seen on next slide).

Consumption further depends on interest rates, impatience, and other properties of utility function.

Permanent income

Permanent income is the counterfactual constant level of income, Y^p, yielding the same present value as the actual (possibly time-varying) stream of income. E.g., in our model

$$Y^p + \frac{Y^p}{1+r} = Y_1 + \frac{Y_2}{1+r} \Rightarrow Y^p = \frac{1+r}{2+r}\left(Y_1 + \frac{Y_2}{1+r}\right)$$

Hence

$$C_1^* = \frac{1+\rho}{2+\rho}\left(Y_1 + \frac{Y_2}{1+r}\right) = \left(\frac{1+\rho}{2+\rho}\right)\left(\frac{2+r}{1+r}\right) Y^p$$
$$C_2^* = \frac{1+r}{2+\rho}\left(Y_1 + \frac{Y_2}{1+r}\right) = \left(\frac{2+r}{2+\rho}\right) Y^p$$

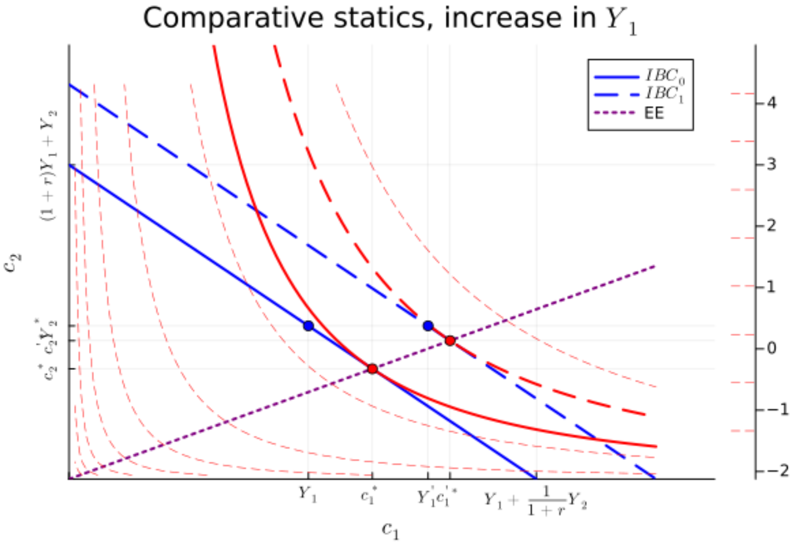
and if $r = \rho$ optimal choice involves always consuming permanent income.

Comparative statics

$$C_1^* = \frac{1+\rho}{2+\rho}\left(Y_1 + \frac{Y_2}{1+r}\right)$$
$$C_2^* = \frac{1+r}{2+\rho}\left(Y_1 + \frac{Y_2}{1+r}\right)$$

Some comparative statics results

$$\frac{\partial C_1}{\partial Y_1} = \frac{1+\rho}{2+\rho}, \quad \frac{\partial C_1}{\partial Y_2} = \frac{1+\rho}{(2+\rho)(1+r)}, \quad \frac{\partial C_1}{\partial r} = -\frac{(1+\rho)Y_2}{(2+\rho)(1+r)^2}$$
$$\frac{\partial C_2}{\partial Y_1} = \frac{1+r}{2+\rho}, \quad \frac{\partial C_2}{\partial Y_2} = \frac{1}{2+\rho}, \quad \frac{\partial C_2}{\partial r} = \frac{Y_1}{2+\rho}$$


Comparative statics, increase in Y_1
IBC_0
IBC_1
EE
c_2
$(1+r)Y_1+Y_2$
c_2^*
$c_2^{'*}$
Y_2
c_1
Y_1
c_1^*
$Y_1^{'}$
$c_1^{'*}$
$Y_1+\frac{1}{1+r}Y_2$
4
3
2
1
0
−1
−2

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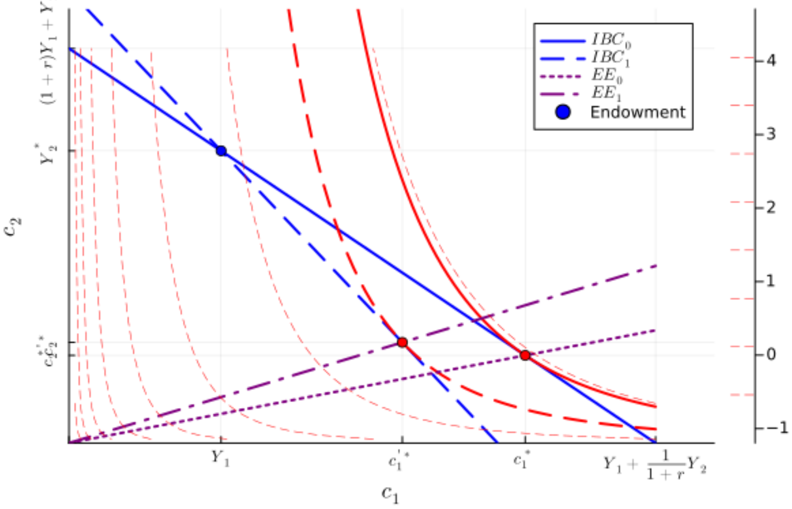
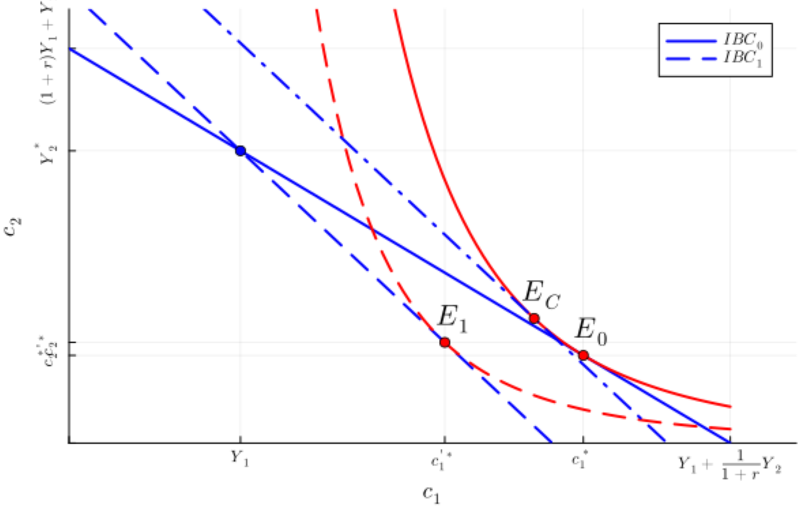
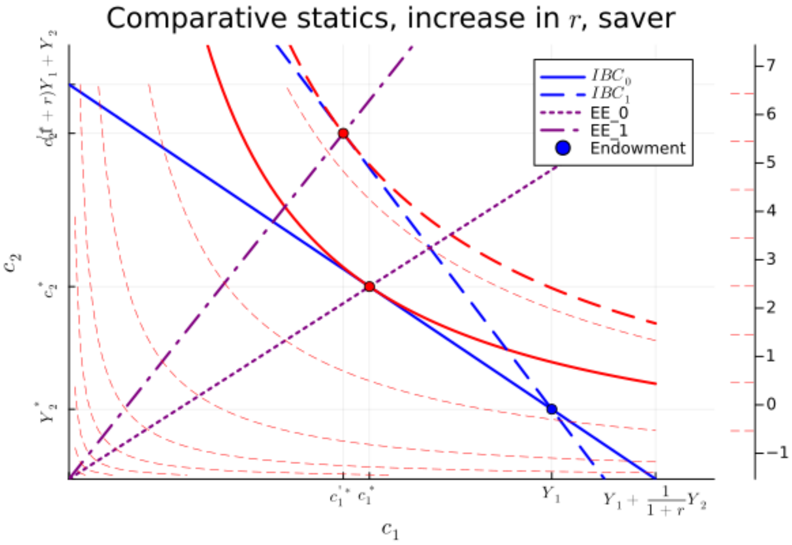
Comparative statics, increase in r, borrower

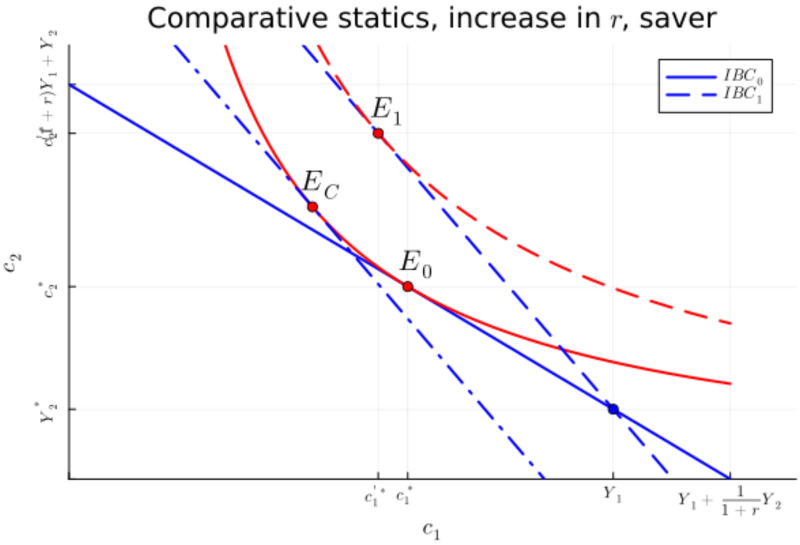


Comparative statics, increase in r, borrower





Comparative statics, increase in r, saver
IBC_0
IBC_1
EE_0
EE_1
Endowment
c_2
c_1
$(1+r)Y_1 + Y_2$
c_2^*
Y_2^*
$c_1^{'*}$
c_1^*
Y_1
$Y_1 + \frac{1}{1+r}Y_2$
7
6
5
4
3
2
1
0
−1


Comparative statics, increase in r, saver
IBC_0
IBC_1
E_1
E_C
E_0
c_2
c_2^*
Y_2^*
$(1+r)Y_1 + Y_2$
c_1
$c_1^{'*}$
c_1^*
Y_1
$Y_1 + \frac{1}{1+r}Y_2$

Income and substitution effects

- In graphs above, following an interest rate increase, optimal choice moves from point E_0 to point E_1
- The total effect on consumption can be seen as combination of two separate effects:
  - Point E_c on the graphs denotes a counterfactual optimal bundle which would prevail if consumer faces the new interest rate, but is exactly compensated (e.g., through a positive or negative income transfer) to be just as well off as before the change.
  - The change of consumption from E_0 to E_c is called the **substitution effect (SE)**.
  - The change of consumption from E_0 to E_c is called the **income effect (IE)**.

Income and substitution effects, 2

- The SE from an increase in r always lowers C_1 and increases C_2 - consumer substitutes away from current to future consumption, as latter becomes relatively "cheaper".
- The IE from an increase in r, decreases consumption in both periods for a borrower
  - as borrower gets "poorer" with increase in r
- ... but increases consumption in both periods for a saver
  - as saver gets "richer" with increase in r

Readings

You can find a primer on intertemporal choice in most introductory and intermediate microeconomics textbooks. A good example is Varian (2010).

References I

Blanchard, O. (2020). *Macroeconomics Global Edition*. Pearson.

Varian, H. R. (2010). *Intermediate Microeconomics: A Modern Approach*. W.W. Norton and Co., New York, eighth edition.