EC109: Microeconomics I
2019 – 2020

University of Warwick

Department of Economics

Terms 1 and 2

Professor Elizabeth Jones

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Welcome!!

- This is the first part of a 2-year long microeconomics module: EC109 and EC202
- EC109 Module Leader: Professor Elizabeth Jones
  - Social Sciences S0.79
  - Elizabeth.H.Jones@warwick.ac.uk
  - Advice and Feedback Hours: See my personal website
- EC109 Lecturer: Dr. Andrew Harkins
  - Social Sciences S1.113b
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  - Advice and Feedback Hours: See personal website

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Organisation

- Lecture times (2 x 1 hour lectures) term 1:
  - Tuesday 2 – 3pm in R0.21
  - Thursday 12 – 1pm in OC1.05
- Lecture times (2 x 1 hour lectures) term 2:
  - Tuesday 2 – 3pm in R0.21
  - Thursday 12 – 1pm in OC1.05
- 8 x 1 hour Workshops (fortnightly meetings weeks 3 – 10; 17 - 24)
- 8 x 1 hour classes (fortnightly meetings weeks 3 – 10; 17 - 24)
  - Sign up to a class time/group: stick to it
  - Only the UG office can give you permission to switch to another group (not your tutor)
- Class/Workshop materials will be on the module webpage

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Assessment

- 2 x Tests (20% in total)
  - One covering term 1 topics, worth 10%
  - One covering term 2 topics, worth 10%
  - Further details of time/location will be made available
  - New format this year – all MCQs
- Exam (80%)
  - Past papers available online

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Resources

- The Syllabus
- Lectures and the lecturers
- Advice and Feedback hours: lecturers and tutors
- Support and Feedback Classes and Class Tutors
- Revision Sessions
- Textbooks
- Online resources
- Forum
- Tabula

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Microeconomics Textbooks

- ‘Intermediate Microeconomics’ by Varian (WW Norton)
- ‘Microeconomics' by Perloff (Pearson) - easier
  - A suite of online resources is available if you purchase a new textbook with an access code
- ‘Game Theory: An Introduction’ by Tadelis (Princeton)
- There are many other intermediate microeconomics textbooks that will cover the material – find one that suits you

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Module Aims

- Across the two years, you will study a range of topics and learn to
  - Develop analysis which combines mathematical, graphical and intuitive skills
  - Apply theoretical concepts and analytical tools
  - Understand how theoretical concepts can be applied to various economic situations
  - Develop critical analysis skills and an ability to question rational economic thought
  - Understand the policy implications of microeconomic theory

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Module Structure

- View EC109 as part 1 of your micro modules
- Topics are split between years 1 and 2
- Aim: by the end of year 2, you can approach:
  - Mas-Colell's 'Microeconomic Theory' or
  - Varian's 'Microeconomic Analysis'
- We start by showing you the final goal: where your studies of microeconomics can take you
- 1 lecture by a researcher in an area of applied microeconomics (8/10/19): Robbie Akerlof
  - Content is non-examinable, but link to and application of theory can be examined

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The 2 year road map

- The plan for your 2 years in microeconomics
  - The core topics; The core skills; The tools of the trade

EC109

- Consumer Theory
- Producer Theory
- Market structure and firm behaviour
- Intro to Game Theory
- Partial equilibrium

EC202

- Choice under Uncertainty
- Game Theory
- General equilibrium
- Market failure

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EC109

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CONSUMER THEORY

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The Topics

- Budget Constraints and the feasible set
- Preferences
- Indifference curves and utility functions
- Revealed Preferences
- Optimisation
- Comparative statics
- Changes in welfare
- Applications

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An example

- If you and I go shopping in our respective local towns, why is it unlikely we will each come out of the shop with the same amount of each good in our baskets?
- If I buy more milk than you, what are the possible explanations?
- Differences in behaviour can emerge from
  - Different tastes
  - Different circumstances
- We optimise subject to our constraints

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BUDGET CONSTRAINTS

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Budget constraint I

- Income and prices affect the quantity consumers demand
  - Income can be determined exogenously as an amount, M
  - Or determined endogenously from resources
- Assume my weekly income is £200 and I spend money on food at £5/g and clothes at £10/unit.
  - M= $F \times P_f$ if I decide to only consume food.
    - $200 = F \times 5 \rightarrow F = 40 = \frac{M}{P_f}$
  - M= $C \times P_c$ if I decide to only buy clothes.
    - $200 = C \times 10 \rightarrow C = 20 = \frac{M}{P_c}$

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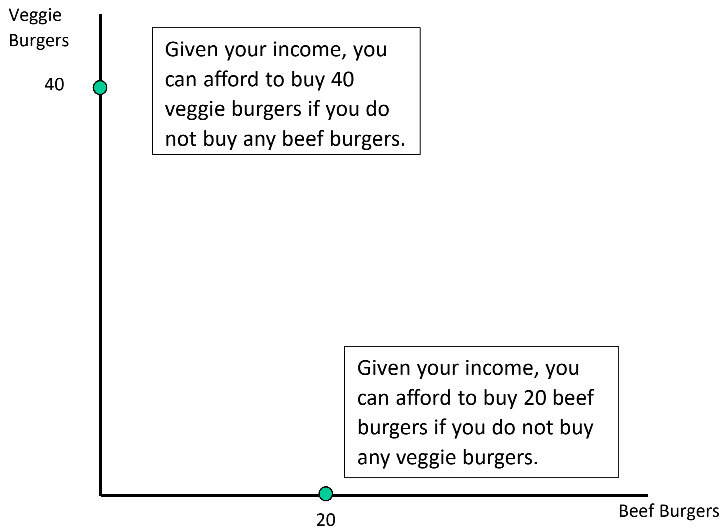
Budget constraint II



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Budget Constraint III

- If you invite meat and veggie lovers to the BBQ, you may want more balanced consumption, but it must be true that you do not spend more than your income:
$$M \geq V \times P_V + B \times P_B$$
- With veggie burgers on the vertical axis, constraint is:
$$V = \frac{M}{P_V} - \frac{P_B}{P_V} B$$
- We can now determine the vertical intercept and the slope of the budget constraint

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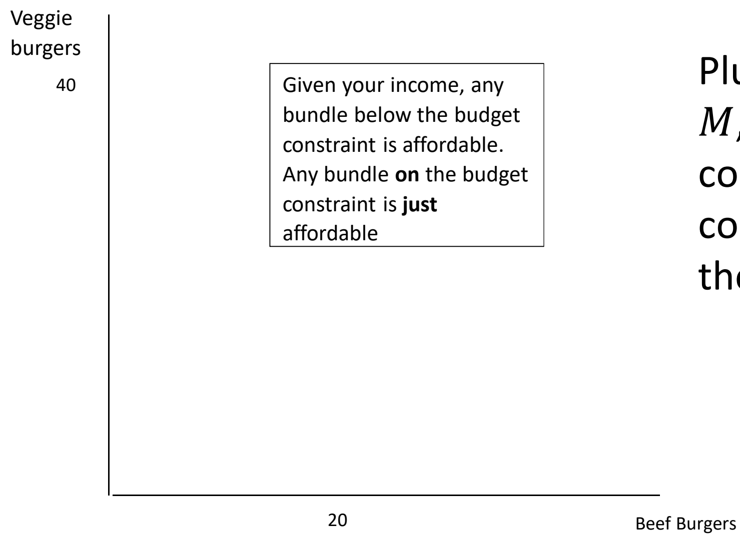
The feasible set



Plugging in the values for M, P_V and P_B allows us to construct the budget constraint and determine the feasible set:

$$200 = 5V + 10B$$

$$V = \frac{200}{5} - \frac{10}{5}B$$

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Budget constraint IV

- The slope measures the rate the market ‘substitutes’ good 1 for good 2: it’s the opportunity cost of consuming good 1
- If we consume more good 1, Δx_1, by how much must good 2 change to continue to satisfy the budget constraint?

$$p_1 x_1 + p_2 x_2 = m \qquad (1)$$

$$p_1(x_1 + \Delta x_1) + p_2(x_2 + \Delta x_2) = m \qquad (2)$$

- Subtract (1) from (2) to find: $p_1 \Delta x_1 + p_2 \Delta x_2 = 0$
- Thus: $\frac{\Delta x_2}{\Delta x_1} = -\frac{p_1}{p_2}$

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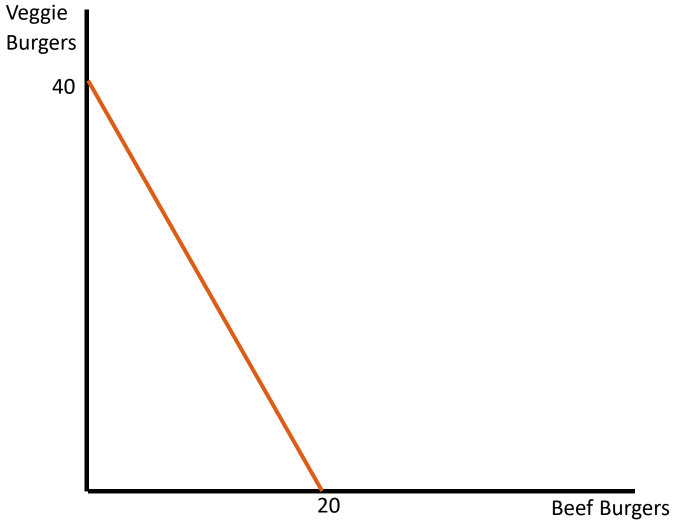
Changing prices and income



Returning in each case to $M = £200;\ P_V = £5;\ P_B = £10$

- Income falls to £100
- Price of beef burgers falls from £10 to £8
- Both prices double
- Prices of veggie and beef burgers rise and income falls

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PREFERENCES

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Preferences

- We assume consumers choose what they want the most
- Specifying preferences tells us something about a consumer's choice.
- Consider the bundle (x_1, x_2) and compare it with (y_1, y_2) to determine the preference ordering:
  - Strict preference $\succ$
  - Weak preference $\succcurlyeq$
  - Indifference $\backsim$
- We only care about ordinal relations

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Properties of Preferences

- Completeness
  - The consumer can always compare/rank bundles. Either $\mathrm{X} \succ Y, or\ Y \succ X, or\ X \backsim Y$
- Transitivity
  - If $X \succcurlyeq Y$ and $Y \succcurlyeq Z$ then $X \succcurlyeq Z$
- Continuous
  - If X is preferred to Y, and there is a third bundle Z which lies within a small radius of Y, then X will be preferred to Z.
  - Tiny changes in bundles will not change preference ordering

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Well-behaved preferences

- Monotonicity (non-satiation)
  - We are talking about *goods* and not *bads* >> More is better!
  - Consider two bundles X and Y. If Y has at least as much of both goods, and more of one, then $(y_1, y_2) \succ (x_1, x_2)$
- Convexity
  - Averages are better than extremes (or at least not worse)
  - An average of two bundles on the same indifference curve will be (at least weakly) preferred, for any $0 < t < 1$

$$z = (tx_1 + (1-t)y_1, tx_2 + (1-t)y_2) \succcurlyeq (x_1, x_2)$$

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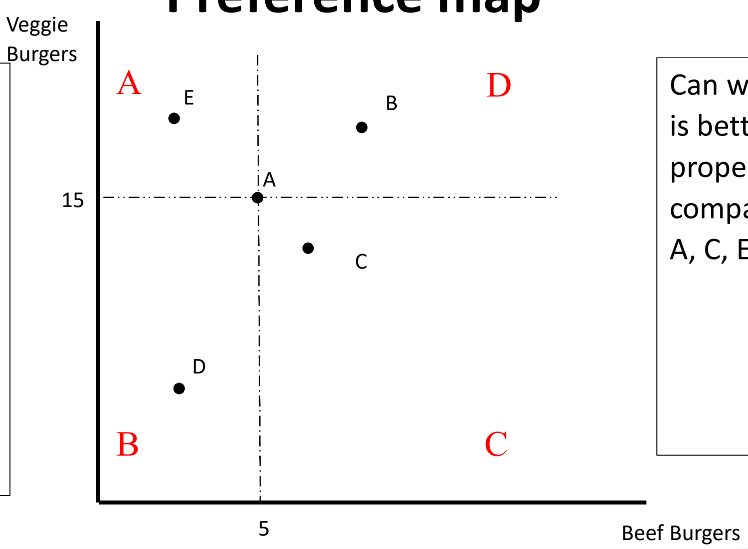
Preference map

More is better tells us …



Can we use more is better property to compare bundles A, C, E?

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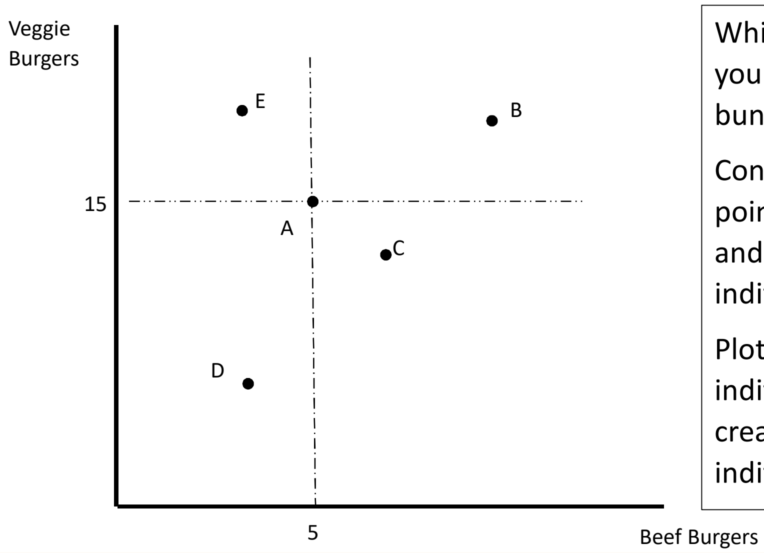
Indifference curves I



Which bundles do you like equally to bundle A?

Connecting these points (bundles A, C and E) creates an indifference curve.

Plotting other indifference curves creates an indifference map.

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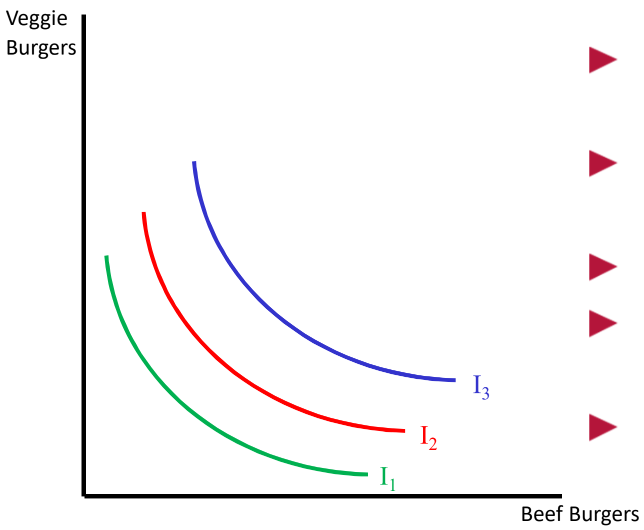
Indifference Curves II



- Bundles on I_3 are preferred to bundles on I_2 etc.
- Indifference curves are continuous
- Indifference curves cannot cross
- Most people's indifference curves are convex to the origin
- Indifference curves are downward sloping

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Indifference curves III

- Perfect Substitutes – goods with a constant rate of substitution
- Perfect Complements – goods that are always consumed together
- 'Bads' – a good that you dislike
- Neutral goods – a good that you don't care about
- Satiation – an overall best bundle: too much AND too little is worse

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Utility I

- Utility Functions describe preferences, assigning higher numbers to more-preferred bundles
  - All combinations of two goods that give an individual the same level of utility lie on the same indifference curve
- The further from the origin, the higher the utility
- Bundle $(x_1, x_2) \succ (y_1, y_2)$ iff $u(x_1, x_2) \succ u(y_1, y_2)$
- We are interested in ordinal and not cardinal utility
  - We care about which bundle is preferred and not by how much

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Utility II

What will the utility functions look like for …

- Perfect Substitutes:
- Perfect Complements:
- Cobb-Douglas
  - Between the two extremes: Imperfect Substitutes
  - The simplest example of well-behaved preferences
  - $U(x_1, x_2) = x_1 x_2$ (or $U(x_1, x_2) = {x_1}^2 {x_2}^2$)
  - $U(x_1, x_2) = x_1^{\frac{3}{4}} x_2^{\frac{1}{4}}$
  - More generally: $U(x_1, x_2) = x_1^{\alpha} {x_2}^{1-\alpha}$

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Monotonic transformations

- Applying monotonic transformations to a utility function creates a new function but with the same preferences
  - Transforms a set of numbers into another set; preserving the order
  - We can't work back from optimal demands for exact utility function
- Consider a utility function $U(x_1, x_2, \dots, x_n)$. Some examples of monotonic transformations:
  - $\log(U(x_1, x_2, \dots, x_n))$
  - $U(x_1, x_2, \dots, x_n) + k$
  - $U(x_1, x_2, \dots, x_n)^{\mathrm{n}}$
  - $\exp(U(x_1, x_2, \dots, x_n))$
  - $\gamma(U(x_1, x_2, \dots, x_n))$
  - $\sqrt{U(x_1, x_2, \dots, x_n)}$

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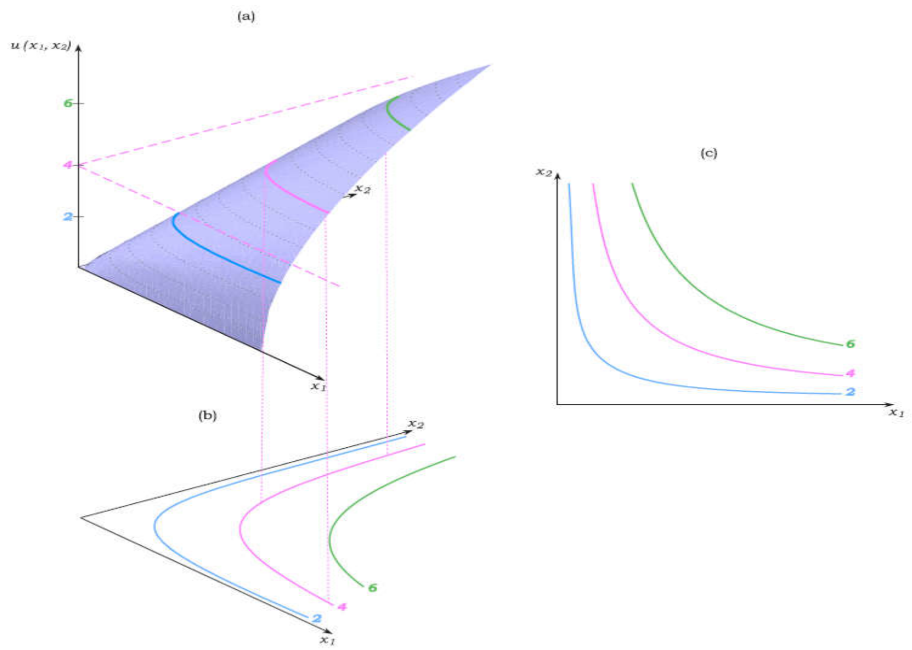

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Behavioural insights I

Which factors affect utility?

- Psychological attitudes
- Peer group pressures
- Personal experiences
- The general cultural environment

Ceteris paribus

- Only consider choices among quantifiable options
- Hold constant other things that affect behaviour

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Behavioural insights II

Do the axioms always hold? Are consumers truly rational?

- Too many choices/much information; how choices are framed: Prospect Theory
- Loss Aversion: the disutility of giving up an object is greater than the utility associated with acquiring it (cognitive bias)
- What's the default option? (e.g. buying something online)

► Bounded rationality

- Behaviour is influenced by our environment and the information we have
- Poor feedback restricts information

Can behaviour be influenced to become more rational?

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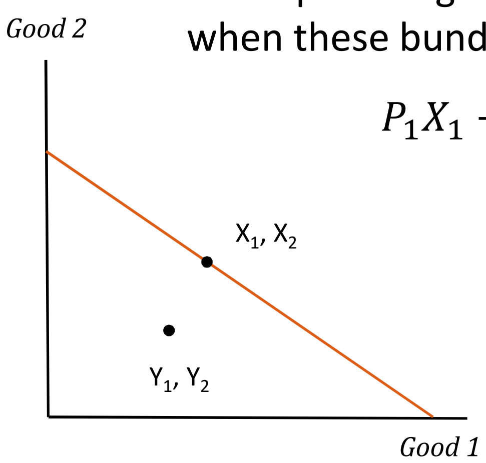
Revealed Preferences



If an optimising consumer chooses (X_1, X_2) over (Y_1, Y_2), when these bundles are different, it must be that:

$$P_1 X_1 + P_2 X_2 = M \quad \text{and} \quad P_1 Y_1 + P_2 Y_2 \leq M$$

$$P_1 X_1 + P_2 X_2 \geq P_1 Y_1 + P_2 Y_2$$

If this inequality is satisfied, then (X_1, X_2) is directly revealed preferred:

$$(X_1, X_2) \succ (Y_1, Y_2)$$

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WARP and SARP

- If $(X_1, X_2) \succ (Y_1, Y_2)$, then it can't be that $(Y_1, Y_2) \succ (X_1, X_2)$
  - WARP refers to directly revealed preferences
  - SARP refers to directly and indirectly revealed preferences
- Say that at prices (P_1, P_2), bundle (X_1, X_2) is bought when (Y_1, Y_2) is affordable
- This means that if (Y_1, Y_2) is purchased at prices (Q_1, Q_2), then (X_1, X_2) must be unaffordable

$$P_1X_1 + P_2X_2 \geq P_1Y_1 + P_2Y_2$$

AND NOT $$Q_1Y_1 + Q_2Y_2 \geq Q_1X_1 + Q_2X_2$$

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The Marginal Rate of Substitution I

- Joe consumes a given bundle, (x_1, x_2)
  - If we reduce his consumption of good 1, by Δx_1… how much good 2, Δx_2, is needed to put Joe back on the same indifference curve?
- The rate at which the consumer is just willing to substitute one good for the other: $\frac{\Delta x_2}{\Delta x_1}$
- With Δx_1 being a small change, we talk about the *marginal* rate of substitution (MRS)
  - This is the slope of an indifference curve at a particular point
  - It will be negative (which we also know from monotonicity)

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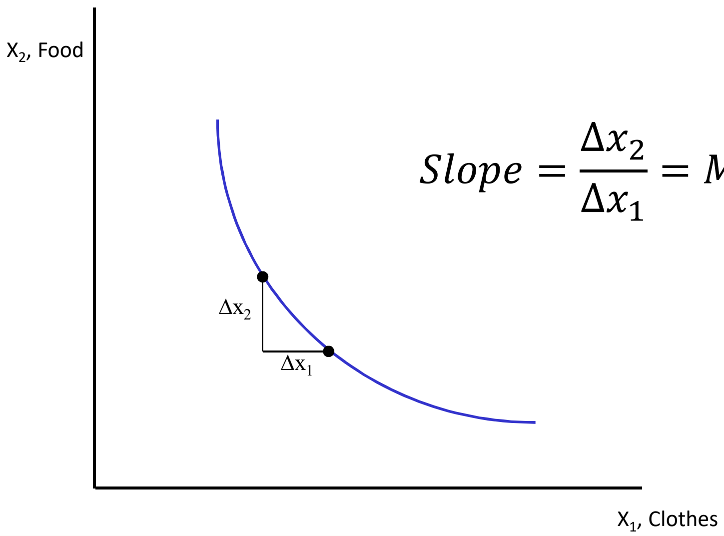
The Marginal Rate of Substitution II



$$Slope = \frac{\Delta x_2}{\Delta x_1} = MRS$$

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The Marginal Rate of Substitution III

$$MU_1 = \frac{\Delta U}{\Delta x_1} = \frac{u(x_1+\Delta x_1, x_2)-u(x_1, x_2)}{\Delta x_1}$$

$$\Delta U = MU_1 \Delta x_1$$

- To keep utility constant (ΔU = 0), if x_1 ↑, x_2 ↓

$$\Delta U = MU_1 \Delta x_1 + MU_2 \Delta x_2 = 0$$

- We know $MRS = \frac{\Delta x_2}{\Delta x_1}$ and so $MRS = \frac{MU_1}{MU_2}$

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The Marginal Rate of Substitution IV

- We refer to small changes in $x_1$and x_2 and so

$$du = \frac{\partial u(x_1, x_2)}{\partial x_1} dx_1 + \frac{\partial u(x_1, x_2)}{\partial x_2} dx_2 = 0$$

$$\frac{dx_2}{dx_1} = \frac{\frac{\partial u(x_1, x_2)}{\partial x_1}}{\frac{\partial u(x_1, x_2)}{\partial x_2}} = \text{MRS}$$

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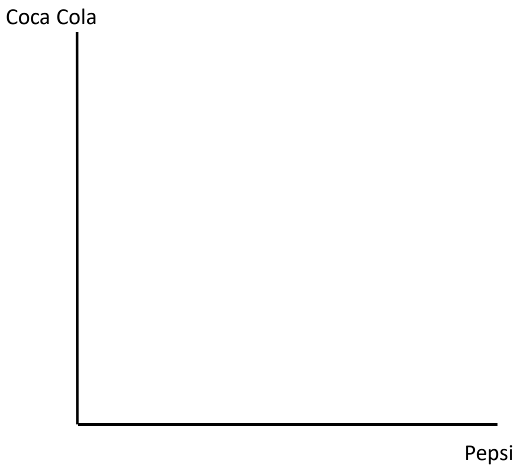
The MRS and Perfect Substitutes



- $U(C, P) = aC + bP$
  - $MRS =$
- What happens to the MRS as we move down the indifference curve?

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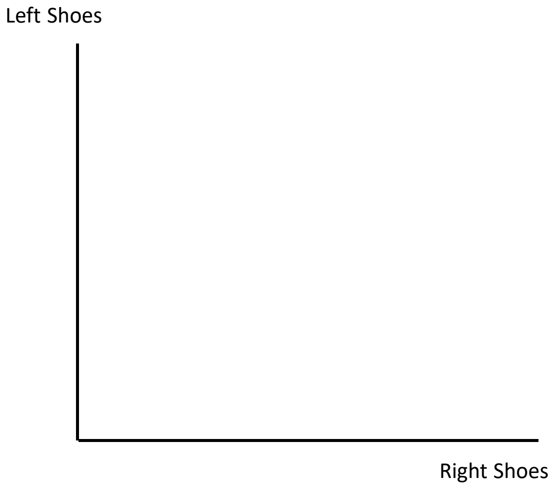
The MRS and Perfect Complements



- $U(S_R, S_L) = min\{aS_R, bS_L\}$
- MRS is:

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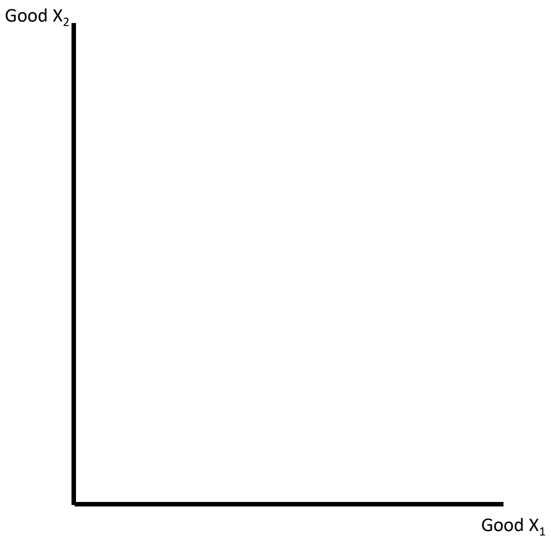
The MRS and Cobb-Douglas preferences



- $U(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}$
- MRS is:
  - $\frac{MUx_1}{MUx_2} =$
- A monotonic transformation
  - $\mathrm{U}(x_1, x_2) = tx_1^{\alpha} tx_2^{1-\alpha}$
  - $MRS =$
  - MRS does not depend on the utility representation

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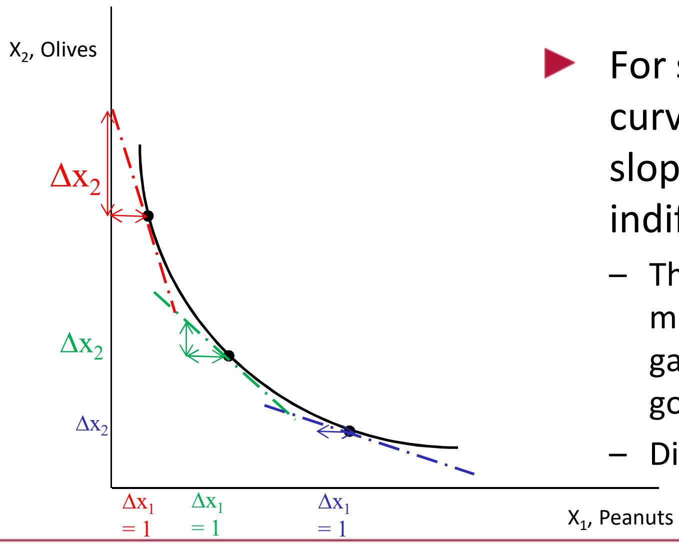
Diminishing Marginal Rate of Substitution



- For strictly convex indifference curves, what happens to the slope as we move down the indifference curve?
  - The more of a good you have, the more willing you are to sacrifice it to gain an additional unit of another good
  - Diminishing MRS (absolute value)

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Homothetic tastes I

- A situation where the consumer’s preferences depend solely on the ratio of good 1 to good 2
- Homothetic tastes give rise to indifference maps where the MRS is constant along any ray from the origin.

$$(x_1, x_2) \succ (y_1, y_2)$$
$$(tx_1, tx_2) \succ (ty_1, ty_2)$$

- When income rises, demand rises by the same proportion

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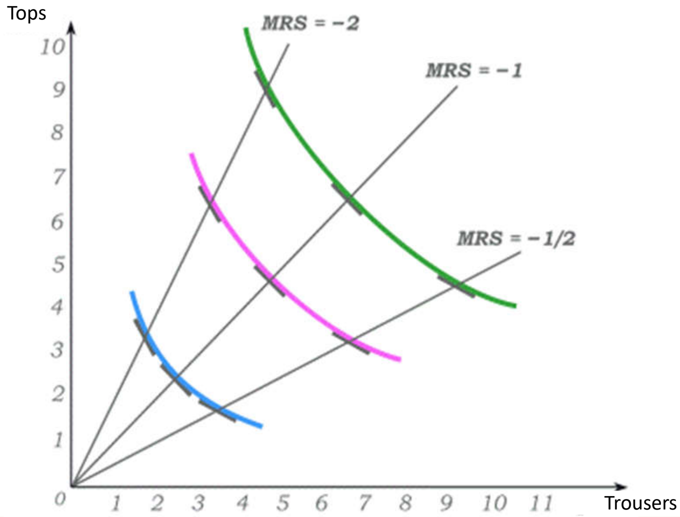
Homothetic tastes II



- The relative quantity of each good remains constant along any ray from the origin
- When income is scaled up or down by $t > 0$, the demanded bundle scales up or down by the same amount.
  - The ratio of trousers to tops remains constant as income changes

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Quasilinear tastes

- Tastes are linear in one good, but may not be in the other good:

$$u(x_1, x_2) = v(x_1) + x_2$$

- With quasilinear tastes, indifference curves are vertical translates of one another.
- In this case, the MRS is constant along any vertical line from the x-axis.

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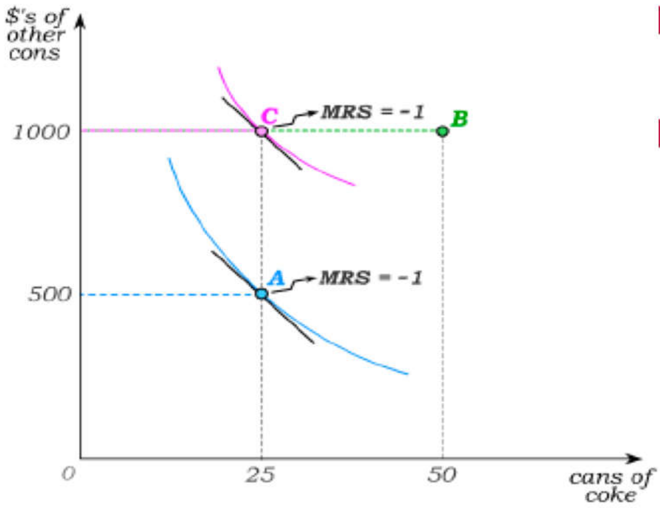
Coke and all other consumption



- At A, MRS = -1: we will trade $1 for 1 can of coke.
- At B, will we value the 50th can of coke the same as the 25th?
  - More likely that the 25th can is valued the same regardless of how much in other consumption we undertake

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Elasticity of Substitution I

- How does the consumer substitute between x_1 and x_2?
- The degree of substitutability measures how responsive the bundle of goods along an IC is to changes in the MRS
- The elasticity of substitution is defined as:

$$\sigma = \left| \frac{\%\Delta(x_2/x_1)}{\%\Delta MRS} \right|.$$

  - Perfect substitutes: perfect substitutability: σ = ∞
  - Perfect complements: no substitutability: σ = 0
  - Cobb-Douglas: σ = 1

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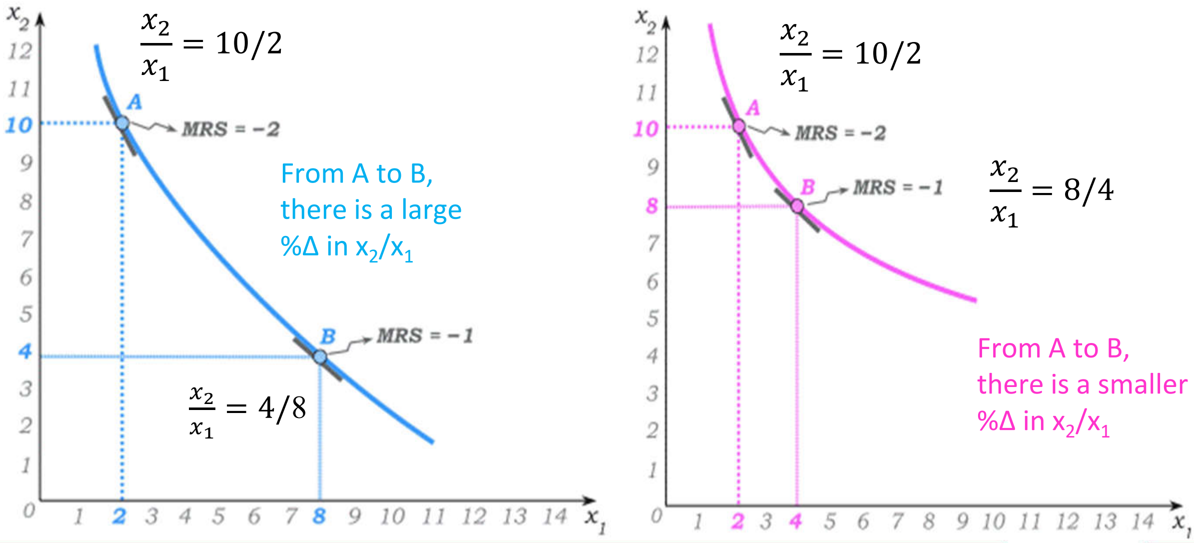
Elasticity of Substitution II



Elasticity of Substitution III

$$\sigma = \left| \frac{\%\Delta(x_2/x_1)}{\%\Delta MRS} \right| .$$

- Consider a 1% change in the MRS
- The less curved the IC, the more x_2 has to fall and the more x_1 has to increase for the MRS to have changed by 1%
- Thus, the less curvature in the IC, the greater is the % $\Delta x_2/x_1$ required for the MRS to change by 1%, which implies a higher elasticity of substitution σ
  - i.e. Perfect Substitutes

OPTIMISATION

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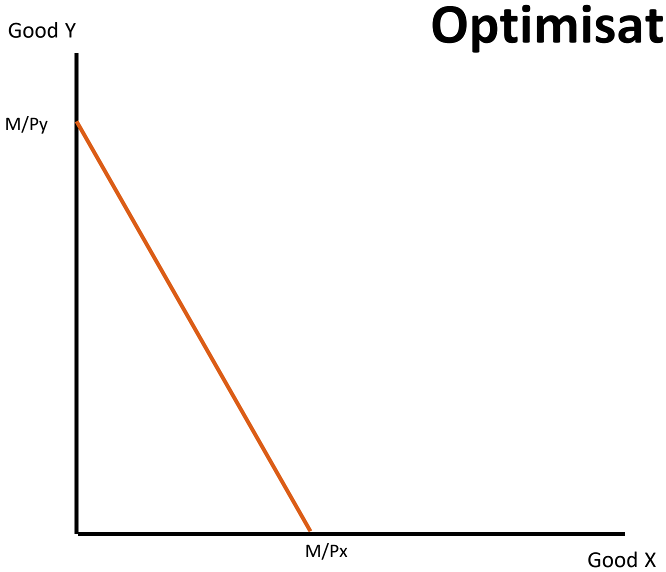
Optimisation I



Consumers choose the best bundle within their feasible set

Remember

Slope of budget constraint: $\frac{Px}{Py}$

=

Slope of indifference curve:

$\text{MRS} = \frac{MUx}{MUy}$

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Optimisation II

- Consumers maximise utility subject to a budget constraint
- This occurs at a tangency, where

$$MRS = \frac{\partial u(x,y)/\partial x}{\partial u(x,y)/\partial y} = \frac{MUx}{MUy} = \frac{Px}{Py}$$

- Why must this hold for an interior solution?
  - Assume two goods X and Y
  - What would happen if: $\frac{MU_X}{MU_Y} \neq \frac{P_X}{P_Y}$?

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Optimisation III

Say: $MRS = \frac{\Delta Y}{\Delta X} = \frac{MU_X}{MU_Y} = \frac{1}{1} > \frac{P_X}{P_Y} = \frac{2}{3}$

- To consume 1 more unit of X, Joe is willing to give up 1Y (MRS)
- To consume 1 more unit of X, the market only requires Joe to give up $\frac{2}{3}$ Y (slope of budget constraint)
- If Joe is willing to give up 1 unit of Y, he'll certainly give up only $\frac{2}{3}$ Y
- Consumption of X will rise and Y will fall until: $\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$

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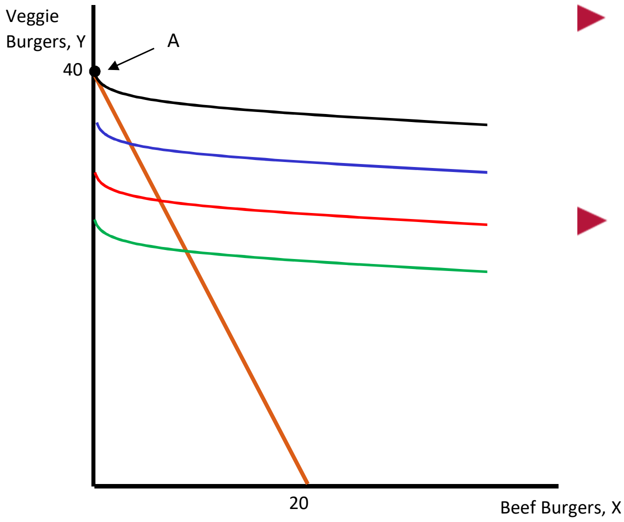
Optimisation IV: Corner Solutions



- If the optimal choice involves consuming both goods, then the tangency condition must hold
  - A necessary condition
- What happens at A?

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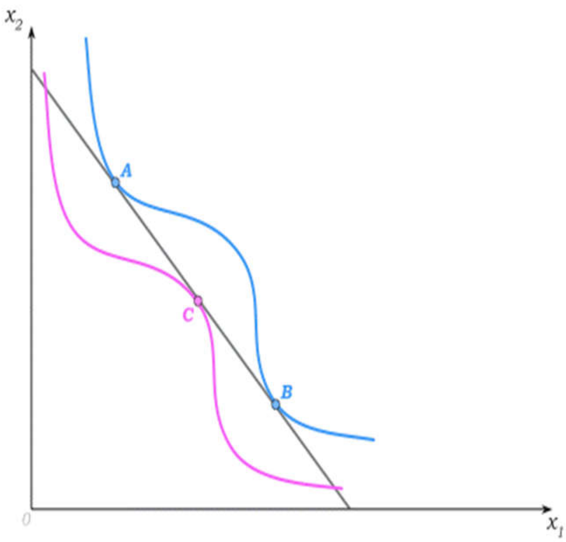
Optimisation V: non-convexity



- The tangency condition is necessary for optimality, but it is not sufficient unless preferences are convex
- If we have a tangency, we don’t always have an optimal choice
  - Points A, B and C are all interior tangency conditions

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Equi-marginal Principle I

- We can find the optimum bundle by setting the MRS equal to the price ratio and using the budget constraint.
- The solutions give the Marshallian demands – demand is dependent on prices and income: $x^*(p_1, p_2, M)$
- An example: $u = x_1^{\alpha} x_2^{1-\alpha}$, with P_1, P_2, M
- Find the MRS and set equal to the price ratio: $\frac{x_2}{x_1} = \frac{P_1}{P_2}$
- Rearrange to give (say): $P_1 x_1 = P_2 x_2$
- Plug into the budget constraint

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Equi-marginal Principle II

- $M = P_1 x_1 + P_2 x_2$ But $P_1 x_1 = P_2 x_2$
- $M = P_1 x_1 + P_1 x_1$ >> $M = 2P_1 x_1$
- So, we can find: $x_1 = \frac{M}{2P_1}$
- Plug the demand for x_1 into the budget constraint:
- $M = P_1 \frac{M}{2P_1} + P_2 x_2$ >> $2M = M + 2P_2 x_2$
- $x_2 = \frac{2M - M}{2P_2} = \frac{M}{2P_2}$
- Demand is dependent on prices and income

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Marshallian demands I

- What if we have a utility function where there isn't a tangency?
- Perfect Substitutes
- Perfect Complements
  - In each case, think about it logically…
  - In the diagram, what do we know about how consumers will allocate their income?
  - What do we know about the point at which consumers maximise their utility?

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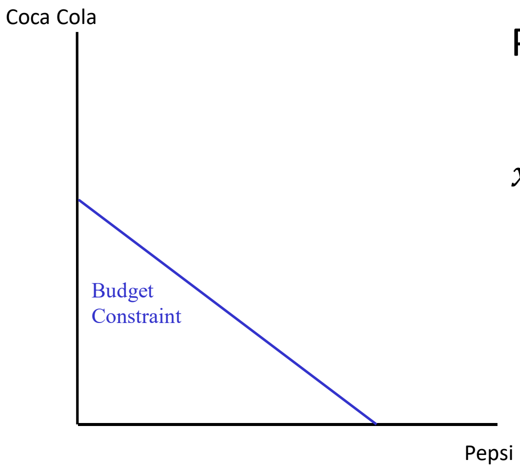
Marshallian demands II



Perfect Substitutes:

$$x_1^* = \begin{cases} M/P_1 & if\ P_1 < P_2 \\ 0 < x_1^* < M/P_1 & if\ P_1 = P_2 \\ 0 & if\ P_1 > P_2 \end{cases}$$

- Maximise utility on highest IC subject to BC:
- Compare slopes of BC and IC to determine which good is consumed

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Marshallian demands III

- Perfect Substitutes example:
- $U(x_1, x_2) = 5x_1 + 3x_2$; $Px_1 = 4, Px_2 = 3, M = 200$
- What is the MRS?
- What is the price ratio?
- In this case:
- IC is than BC so only good is consumed

$x_1 =$; $x_2 =$

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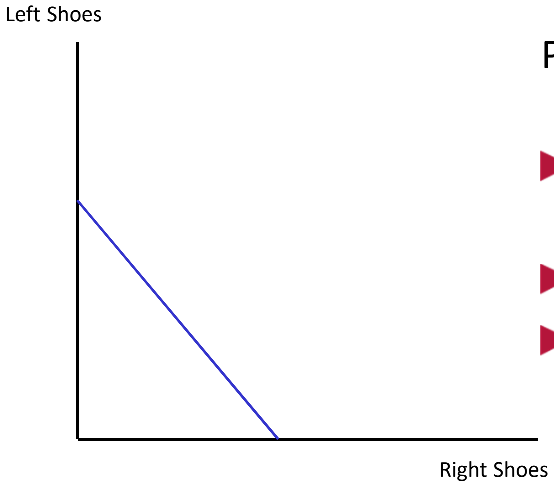
Marshallian demands IV



Perfect complements:

- $x^* = M/(P_1 + P_2)$

- Consumer will optimise at the kink, but…
- MRS is undefined at kink
- Find expression for Indifference curve at the kink and combine this with the Budget constraint to solve

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Marshallian demands V

- Perfect Complements example:
- $U(x_1, x_2) = \min\{4x_1, 3x_2\}$; $Px_1 = 4, Px_2 = 3, M = 200$
- How many units of x_1 and x_2 do we have at the kink? (1)
- Find an expression for the BC. (2)
- Substitute (1) into (2):
- Solve: $x_1 =$; $x_2 =$

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Marshallian demands VI

- Cobb-Douglas: $u = x_1^c x_2^d$

$$x_1^* = \frac{c}{c+d}\frac{M}{P_1} \quad \text{and} \quad x_2^* = \frac{d}{c+d}\frac{M}{P_2}$$

- It is convenient to write Cobb-Douglas utility functions with exponents that sum to 1
  - Raise utility to power $1/(c+d)$ ≫ $v = x_1^{\alpha} x_2^{1-\alpha}$
- A fixed proportion of income is spent on each good
  - The size is determined by the exponents, e.g. good 1 = α
  - Marshallian demand: $x_1^* = \frac{\alpha M}{P_1}$ and $x_2^* = \frac{(1-\alpha)M}{P_2}$

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COMPARATIVE STATICS

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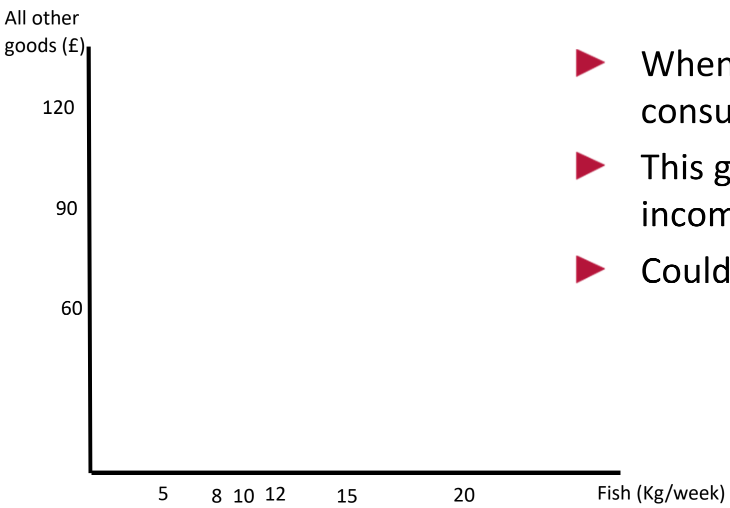
Changing optimal consumption I



- When income rises, optimal consumption moves: A – B – C.
- This gives the income-offer or income-consumption curve
- Could this curve's shape vary?

Income	Quantity demanded
120	12
90	8
60	5

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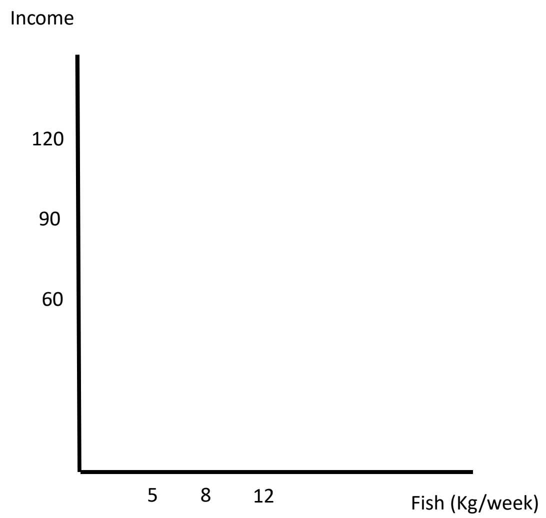
The Engel curve



- We derive the Engel curve from the income-offer curve
  - Holding prices constant, how does quantity demanded vary with income?
- For a normal good, as income rises, quantity demanded rises
- For an inferior good, as income rises, quantity demanded falls
  - The Engel curve may not slope upwards

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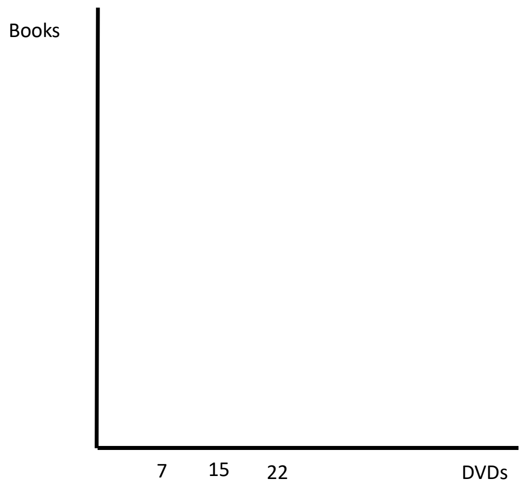
Changing optimal consumption II



- When price falls, optimal consumption moves: A – B – C.
- This gives the price-offer or price-consumption curve

Price of DVDs	Quantity demanded
12	7
6	15
4	22

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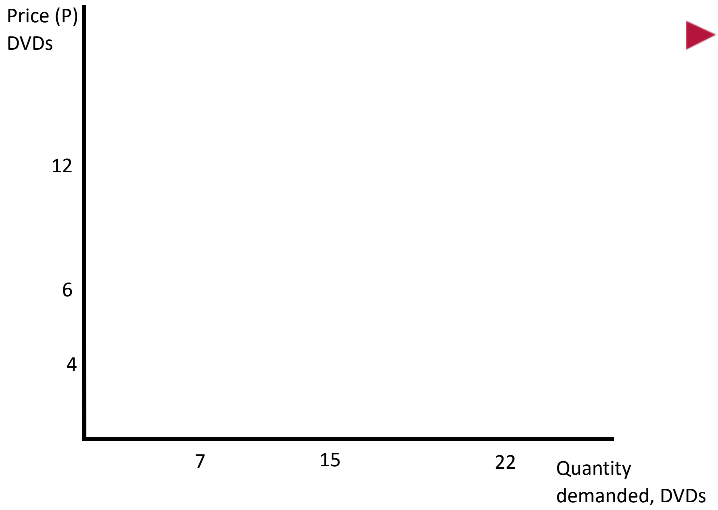
The Demand Curve I



- We derive the demand curve for DVDs from the price-offer curve
  - Holding income, other prices and preferences constant, how does quantity demanded change following a price change?
  - What happens to demand if income, other prices or preferences change?

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The Demand Curve II

- The law of demand
- The *substitution effect:* As $P_x \downarrow$, good Y becomes relatively more expensive and less attractive to the consumer.
- Even remaining on the same IC, the price ratio changes
  - Optimal demands will change to where: MRS = NEW price ratio
- The *income effect:* As $P_x \downarrow$, real income rises. The consumer now feels richer, so quantity demanded changes.
  - As real income changes, the consumer must move to a new IC

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Marshallian demand elasticities I

- When price falls, quantity demanded rises
  - By how much?
- The price elasticity of demand ε_{x,p_x} measures the percentage change in quantity demanded in response to a percentage change in a good's own price.

$$\varepsilon_{x,p_x} = \frac{\Delta x/x}{\Delta p_x/p_x} = \frac{\Delta x}{\Delta p_x}\frac{p_x}{x} = \frac{\partial x}{\partial p_x}\frac{p_x}{x}$$

- What is the sign of price elasticity of demand?

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Marshallian demand elasticities II

- Income elasticity of demand $\xi_{x,M}$ measures the percentage change in quantity demanded in response to a percentage change in income

$$\xi_{x,M} = \frac{\Delta x/x}{\Delta M/M} = \frac{\Delta x}{\Delta M}\frac{\mathrm{M}}{x} = \frac{\partial x}{\partial M}\frac{M}{x}$$

- Cross price elasticity of demand, e_{x,p_y} measures the percentage change in quantity demanded in response to a percentage change in the price of another good

$$e_{x,p_y} = \frac{\Delta x/x}{\Delta p_y/p_y} = \frac{\Delta x}{\Delta p_y}\frac{p_y}{x} = \frac{\partial x}{\partial p_y}\frac{p_y}{x}$$

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CONSUMER THEORY IN PRACTICE

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Income and substitution effects

- Two goods: apples and bananas and price of apples falls
- Apples are relatively cheaper (substitution effect), so the consumer can afford to buy more (income effect)
  - How many more are bought just because they are cheaper?
  - What's the change in demand ONLY due to the substitution effect?
- Graphical analysis: what we do
  - When price falls (BC pivots), real income rises … but ignore that by
  - … taking some income from the consumer (shift new BC) so that …

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Income and substitution effects

- Either (1)
  - Utility is the same after the price change as it was before the price change (that way the consumer is no better off despite the lower price)
- Or (2)
  - Purchasing power is the same after the price change as it was before the price change (the consumer has just enough income to buy the original bundle)
- (1): Hicksian
- (2): Slutsky
- Both only consider the substitution effect (the change in demand purely because the good is cheaper)

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What's the point?

- Government is about to introduce an energy tax and is concerned about the impact on pensioners
  - How much less energy will pensioners consume at this higher price?
- Government doesn't want pensioners worse off - key votes!
  - By how much should their pension rise so they are no worse off?
- As a government economist, you will be asked to find:
  - The change in demand due to the substitution effect
  - Original welfare of pensioners, to calculate required 'compensation'
  - Then it's interesting to see how energy consumption has changed with the tax and the increased pension payments

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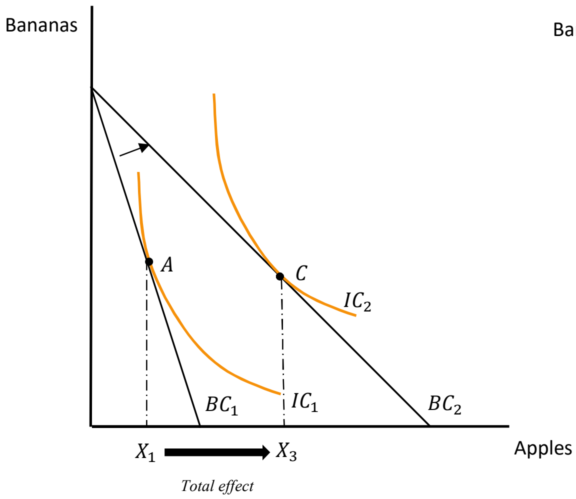
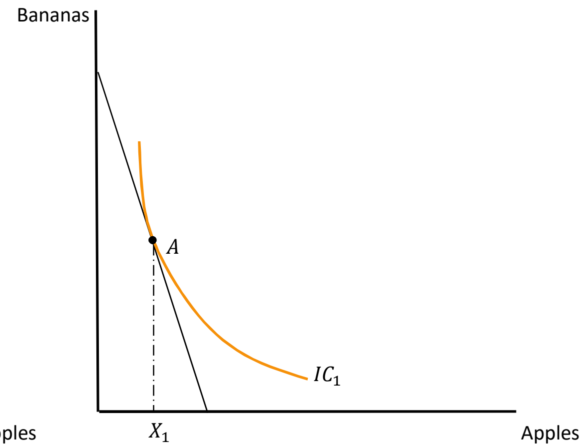
The Hicks income and substitution effects: A normal good





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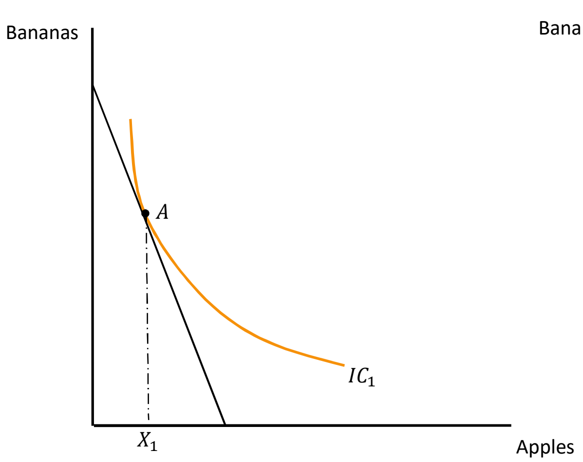
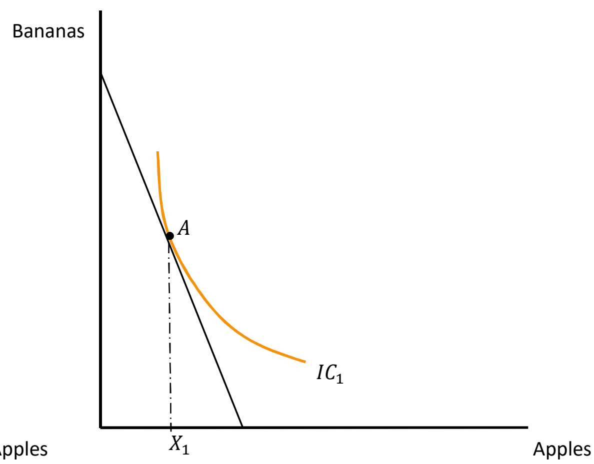
The Hicks income and substitution effects:





Inferior Good

Giffen Good

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Slutsky Substitution

- The Hicks substitution compensates the consumer so that he can remain on the same indifference curve
  - It keeps utility constant (following the price change)
- The Slutsky substitution compensates the consumer so that he can still consume his original bundle
  - It keeps purchasing power constant (following the price change)
- The process is the same as the Hicks substitution effect, but now the compensated budget line won't be tangential to the original indifference curve…
- It will pass through the original consumption bundle

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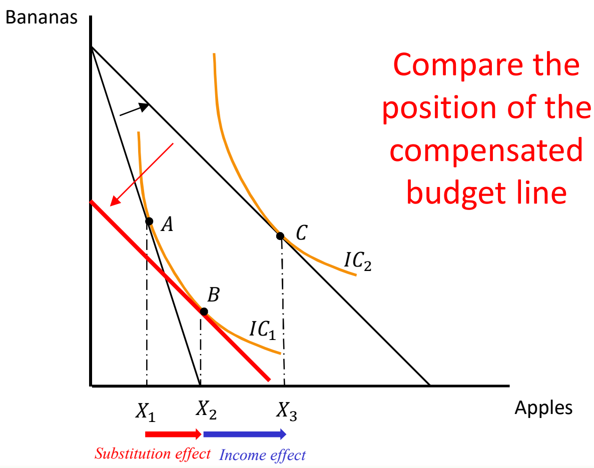
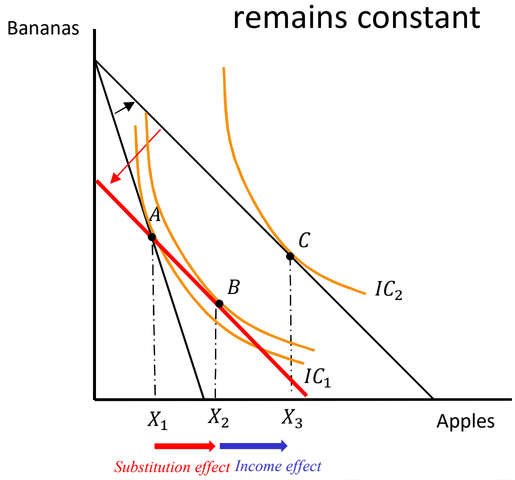
Slutsky versus Hicks: Normal good

Utility remains constant

Purchasing power remains constant



Compare the position of the compensated budget line



Hicksian

Slutsky

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Hicksian: The Dual Problem I

- Find the initial bundle (at original price ratio)
  - Utility maximisation: Consume on highest IC subject to BC
- Find the change in demand due to the substitution effect
  - Expenditure minimisation: what is the least costly way of achieving the original level of utility at the new price ratio?
- Find the new bundle (at new price ratio)
  - Utility maximisation (now also takes into account the income effect)

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Slutsky substitution

- Find the change in income needed to make the original bundle affordable at new price, P_1'

Original Income: $M = P_1 x_1 + P_2 x_2$

New Income: $M' = P_1{}' x_1 + P_2 x_2$

$$\Delta M = M' - M = P_1' x_1 - P_1 x_1 = x_1 \Delta P_1$$

- The change in income tells us the new amount of income the consumer needs such that they can just buy the original bundle at the new set of prices

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Deriving Demand Curves

- We have already derived the Marshallian Demand Curve (taking into account substitution and income effects)
- We can also derive a Hicksian demand curve, which just considers the substitution effect and keeps utility constant
- And a Slutsky demand curve, which also just considers the substitution effect, but this time keeps purchasing power constant
- Consider how the shapes will change if we have (i) a normal good, (ii) an inferior non-Giffen good (iii) a Giffen good

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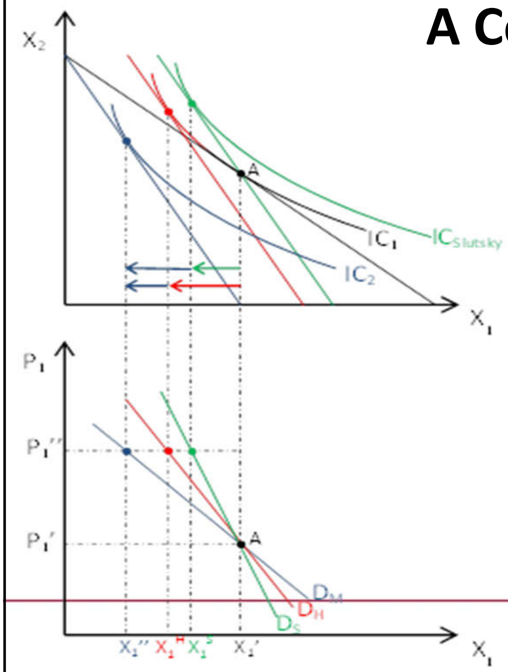
A Comparison: A Normal Good



D_M = Marshallian or own price demand curve

D_H = Hicksian Demand Curve (keeps utility constant)

D_S = Slutsky Demand Curve (keep purchasing power constant)

OPTIMISING MATHEMATICALLY

Optimising mathematically: The Primal

- If we know income, prices and the utility function:

$$\max_{x_1,x_2,\lambda} L = U(x_1, x_2) + \lambda(M - P_1x_1 - P_2x_2)$$

and solve for the Marshallian demands: $x^*(p_1, p_2, M)$

  - Demand is homogenous of degree zero
    - $x^*(p_1, p_2, M) = x^*(kp_1, kp_2, kM) \quad for\ k > 0$
  - If utility is monotonic then the budget binds
- Tangency is necessary, not sufficient. Only sufficient if preferences are convex
  - If preferences are not convex, check SOC to ensure a maximum

The Lagrange multiplier

- Tangency implies:
- $\lambda^* = \frac{MU_1}{P_1} = \frac{MU_2}{P_2} = \cdots = \frac{MU_i}{P_i} = \cdots = \frac{MU_n}{P_n}$
- λ^* is the marginal utility of an extra £ of expenditure
  - The marginal utility of income
  - £1 of extra income will increase utility by λ
- Price is the consumer's evaluation of the utility of the last unit consumed
- $P_i = \frac{MU_i}{\lambda}$ for every i

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A reminder: The Dual Problem I

- Find the initial bundle (at original price ratio)
  - Utility maximisation: Consume on highest IC subject to BC
- Find the change in demand due to the substitution effect
  - Expenditure minimisation: what is the least costly way of achieving the original level of utility at the new price ratio?
- Find the new bundle (at new price ratio)
  - Utility maximisation (now also takes into account the income effect)

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An example

$u = \left(x_1^{0.5}, x_2^{0.5}\right); P_1 = 20;\ P_2 = 10; M = 200$

$L = x_1^{0.5} x_2^{0.5} + \lambda(M - P_1x_1 - P_2x_2)$

FOCs

$\frac{\partial L}{\partial x_1} = 0.5x_1^{-0.5}x_2^{0.5} - \lambda P_1 = 0$

$\frac{\partial L}{\partial x_2} = 0.5x_1^{0.5}x_2^{-0.5} - \lambda P_2 = 0$

$\frac{x_2}{x_1} = \frac{P_1}{P_2}$ >> $P_1x_1 = P_2x_2$

$\frac{\partial L}{\partial \lambda} = M - P_1x_1 - P_2x_2 = 0$ >> $M = 2P_1x_1$ $\gg$ $x_1^* = \frac{M}{2P_1}$

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The Dual Problem II

$u(x_1, x_2) = x_1^{0.5}x_2^{0.5}$ $P_1 = 20; P_2 = 10; M = 200.$ $P_1 \downarrow to\ 10$

Step 1

$\max u(x_1, x_2) = x_1^{0.5}x_2^{0.5}\ subject\ to\ P_1x_1 + P_2x_2 \leq M$

$x_1^* = \frac{M}{2P_1} = \frac{200}{(2\times 20)} = 5$ and $x_2^* = \frac{M}{2P_2} = \frac{200}{(2\times 10)} = 10$

Step 2

- Find the level of utility at the optimal bundle:
- $u(x_1, x_2) = x_1^{0.5}x_2^{0.5} = 5^{0.5}10^{0.5} = 7.071 \ldots$

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Hicksian: Expenditure minimisation

- For the substitution effect, we want you (the consumer) to receive a given level of utility, say v, at lowest cost
  - Minimise your expenditure subject to a given level of utility
- Choose $x_1, x_2, \dots, x_n$ to solve the following problem:

$$min\, p_1x_1 + p_2x_2 + \cdots + p_nx_n \qquad s.t. \quad U(x_1, x_2, \dots, x_n) \geq v$$

- The solution to the problem gives the Hicksian demands of the form: $H_1^*(p_{1,}p_2, v)$
  - These are also known as the compensated demands

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The Dual Problem III

Step 3: find the compensated budget

$$\min \exp = 10x_1 + 10x_2 \ \textit{subject to}\ x_1^{0.5}x_2^{0.5} = 7.071$$

$$L = 10x_1 + 10x_2 + \lambda(7.071 - x_1^{0.5}x_2^{0.5})$$

$$\frac{\partial L}{\partial x_1} = 10 - 0.5\lambda x_1^{-0.5}x_2^{0.5} = 0$$

$$\frac{\partial L}{\partial x_2} = 10 - 0.5\lambda x_1^{0.5}x_2^{-0.5} = 0$$

$$\frac{x_2}{x_1} = 1 \quad \gg \quad x_2 = x_1$$

$$\frac{\partial L}{\partial \lambda} = 7.071 - x_1^{0.5}x_2^{0.5} = 0$$

$$x_1 = x_2 = 7.071$$

Substitution effect: x_1 rises from 5 to 7.071

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The Dual Problem IV

Step 4

$$\max u(x_1, x_2) = x_1^{0.5} x_2^{0.5} \ \ subject\ to\ P_1 x_1 + P_2 x_2 \leq M$$

$$\frac{\partial L}{\partial x_1} = 0.5 x_1^{-0.5} x_2^{0.5} - \lambda P_1 = 0$$

$$\frac{\partial L}{\partial x_2} = 0.5 x_1^{0.5} x_2^{-0.5} - \lambda P_2 = 0$$

$$\frac{x_2}{x_1} = \frac{P_1}{P_2} \quad \gg \quad \frac{x_2}{x_1} = 1$$

$$x_1 = x_2$$

$$P_1 x_1 + P_2 x_1 = M \quad \gg \quad x_1^* = \frac{M}{(P_1 + P_2)} = \frac{200}{(10 + 10)} = 10$$

Income effect: x_1 rises from 7.071 to 10

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Connecting the results I

- In both cases, Ps and M are given and we choose optimal x_i
- The solutions to the primal and dual must be consistent
- Utility maximisation yields the Marshallian demands
  - $x_1^* = x_1^*(p_1, p_2, M)$
  - $x_2^* = x_2^*(p_1, p_2, M)$
- These ordinary demands are plugged into the utility function and allow us to find the actual level of utility:

$$v(p_1, p_2, M) = U(x_1^*(p_1, p_2, M), x_2^*(p_1, p_2, M))$$

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Indirect Utility Function

$$v(p_1, p_2, M) = U(x_1^*(p_1, p_2, M), x_2^*(p_1, p_2, M))$$

- This is called the indirect utility function, where optimal level of utility depends indirectly on prices and income
- It has the following properties:
  - It is non-increasing in every price, decreasing in at least one price
  - Increasing in Income
  - Homogeneous of degree zero in price and income

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Connecting the results II

- Utility maximisation gives Marshallian demands and Indirect Utility Function
- Expenditure minimisation yields the Hicksian demands
  - $x_1^* = h_1^* \ (p_1, p_2, v)$
  - $x_2^* = h_2^* \ (p_1, p_2, v)$
- We can use these Hicksian demands to find the minimum expenditure needed to achieve a given level of utility, v

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Expenditure Function

$$M = E(p_1, p_2, v) = p_1 H_1^*(p_1, p_2, v) + p_2 H_2^*(p_1, p_2, v)$$

- This is called the Expenditure function, which maps prices and utility to minimal expenditure
- It has the following properties:
  - It is non-decreasing in every price, increasing in at least one price
  - Increasing in utility
  - Homogeneous of degree 1 in all prices p

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Connecting the results III

- Look at the symmetry

$$L = u(x_1 x_2) + \lambda\left(M - \sum_{i=1}^{n} p_i x_i\right)$$

$$L = \sum_{i=1}^{n} p_i x_i + \lambda(v - u(x_1 x_2))$$

- Constraint in primal becomes objective in dual

$$v = V(p_i, M) = V(p_i, E(p_i, v))$$

$$M = E(p_i, v) = E(p_i, V(p_i, M))$$

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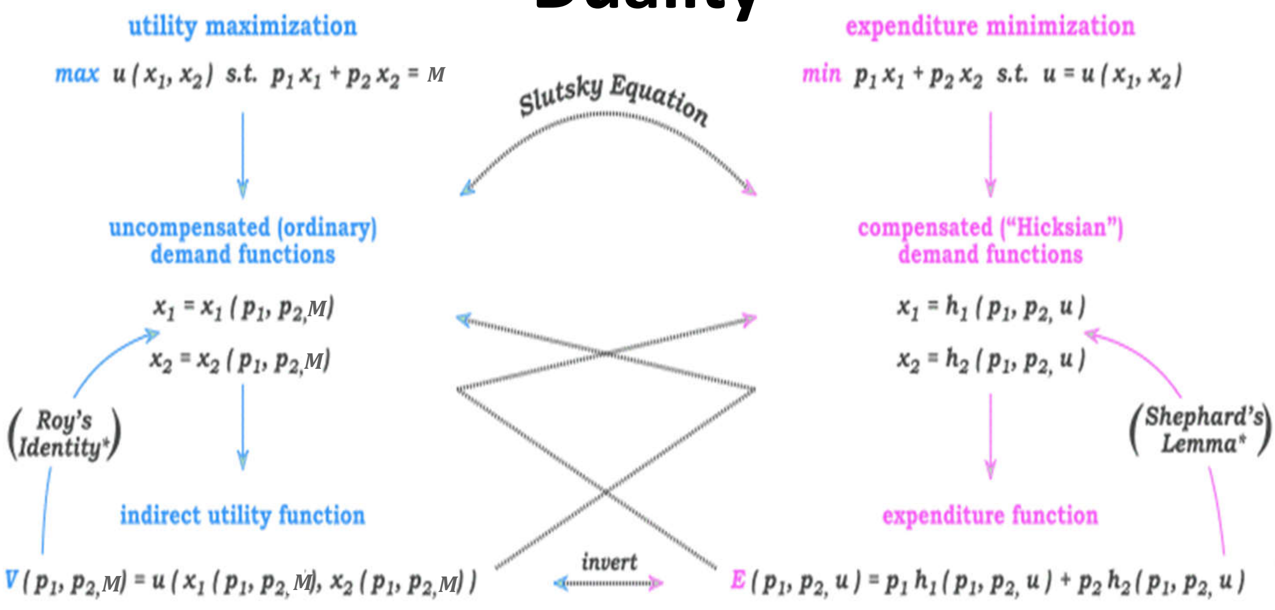
Duality



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A familiar example: but now Slutsky

$u(x_1, x_2) = x_1^{0.5} x_2^{0.5}$ $\quad P_1 = 20; P_2 = 10; M = 200.$ $\quad P_1 \downarrow to\ 10$

- We already know initial bundle is: $x_1^* = 5$ and $x_2^* = 10$
- Hicksian demands were: $x_1^H = x_2^H = 7.071$
- New bundle: $x_1^M = x_2^M = 10$
- But, Hicksian demands compensated consumer such that his utility was unaffected by price change
- Slutsky compensates the consumer such that at the new price ($P_1 = 10$), he can still consume: $x_1^* = 5$ and $x_2^* = 10$
  - To consume this bundle, by how much must income change?

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Slutsky equation I

- Find the change in income needed to make the original bundle (5, 10) affordable at new price, P_1'

Original Income: $M = P_1 x_1 + P_2 x_2$

New Income: $M' = P_1' x_1 + P_2 x_2$

$$\Delta M = M' - M = P_1' x_1 - P_1 x_1 = x_1 \Delta P_1 = 5(10 - 20) = -50$$

- Income must fall by 50, from 200 to 150
- This gives consumer just enough income to purchase the original bundle at the new price ratio

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Slutsky equation II

$$L = x_1^{0.5} x_2^{0.5} + (150 - P_1 x_1 - P_2 x_2)$$

$$x_1 = \frac{M}{(P_1 + P_2)} = \frac{150}{(10 + 10)} = 7.5 = x_2$$

- Under Hicks substitution, x_1 increases from 5 to 7.071
  - Income effect then causes x_1 to rise from 7.071 to 10
- Under Slutsky substitution, x_1 increases from 5 to 7.5
  - Income effect then causes x_1 to rise from 7.5 to 10

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Slutsky equation III

Substitution effect: $\Delta x_1^s = x_1(P_1', M') - x_1(P_1, M)$

Income effect: $\Delta x_1^n = x_1(P_1', M) - x_1(P_1', M')$

Total effect: $\Delta x_1 = x_1(P_1', M) - x_1(P_1, M) = \Delta x_1^s + \Delta x_1^n$

- Express as rates of change by defining Δx_1^m as $-\Delta x_1^n$
- $\Delta x_1 = \Delta x_1^s - \Delta x_1^m$
- Divide each side by ΔP_1 $\quad \dfrac{\Delta x_1}{\Delta P_1} = \dfrac{\Delta x_1^s}{\Delta P_1} - \dfrac{\Delta x_1^m}{\Delta P_1}$

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Slutsky equation IV

$\dfrac{\Delta x_1}{\Delta P_1} = \dfrac{\Delta x_1^s}{\Delta P_1} - \dfrac{\Delta x_1^m}{\Delta P_1}$ and recall: $\Delta M = x_1 \Delta P_1$ >> $\Delta P_1 = \dfrac{\Delta M}{x_1}$

Replace denominator in term 3 $\quad \dfrac{\Delta x_1}{\Delta P_1} = \dfrac{\Delta x_1^s}{\Delta P_1} - \dfrac{\Delta x_1^m}{\Delta M} x_1$

1. Rate of change of x_1 following a change in P_1, holding M fixed (total effect)
2. Rate of change of x_1 as P_1 changes, adjusting M to keep old bundle affordable (substitution effect)
3. Rate of change of x_1, holding prices fixed and adjusting M (income effect)

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WELFARE

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A change in welfare I

- In policy debates it is important to be able to ***quantify*** how consumer "welfare" is affected by changing prices.
  - How is welfare affected if fuel tax rises, or a carbon tax is introduced?
  - A key part of economists' role in government, regulators, consulting
- We can't look at utility directly (ordinal utility), so we use a proxy – income or money
- There are 3 ways that changes in welfare can be measured
  - Consumer Surplus
  - Compensating variation
  - Equivalent variation

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Consumer Surplus I

- We use compensated demand (MWTP) to measure it
- MWTP tells us how much each unit of a good is valued *given we are consuming at some bundle **A**.*
- TWTP for all q^A *units* is: $\sum_{i=1}^{A} MWTP$
  - The area under the MWTP curve
- The difference between the TWTP and the actual amount paid gives the consumer surplus
- When price changes, we measure the change in welfare by measuring the change in consumer surplus

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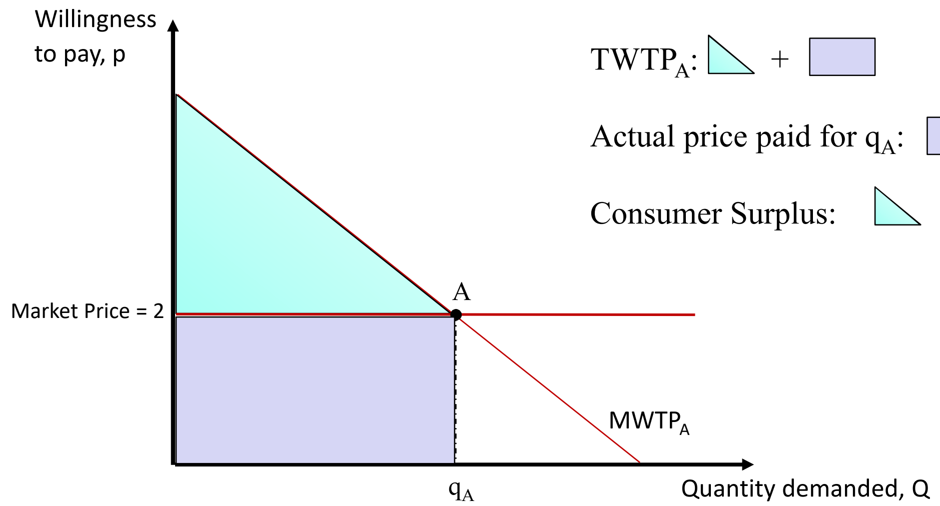
Consumer surplus II



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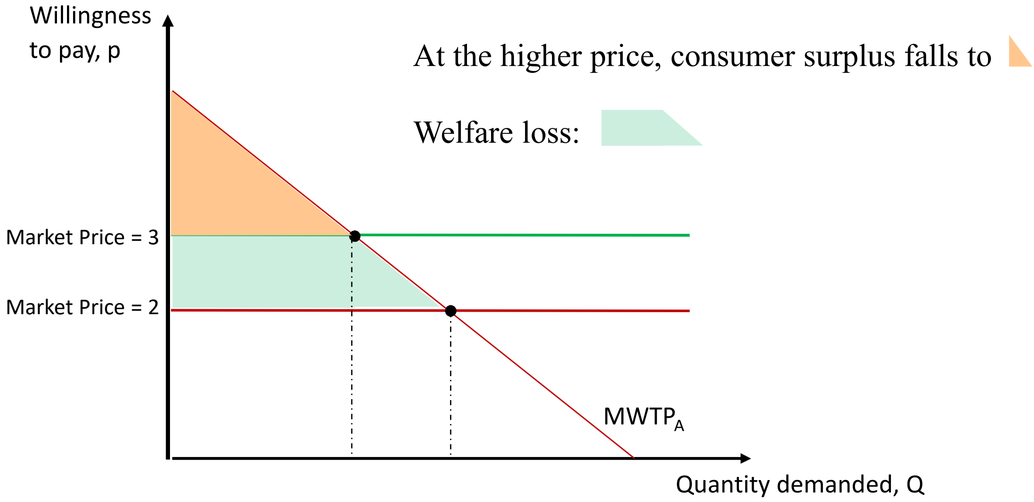
Consumer surplus III



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Compensating Variation I

- Say the price of a good rises …
- How much money would the government have to give the consumer **after** the price change to make him just as well off as he was **before** the price change?
  - You should recognise this idea!
- How far should we shift the **new** budget line so that it is just tangential to the **original** indifference curve?

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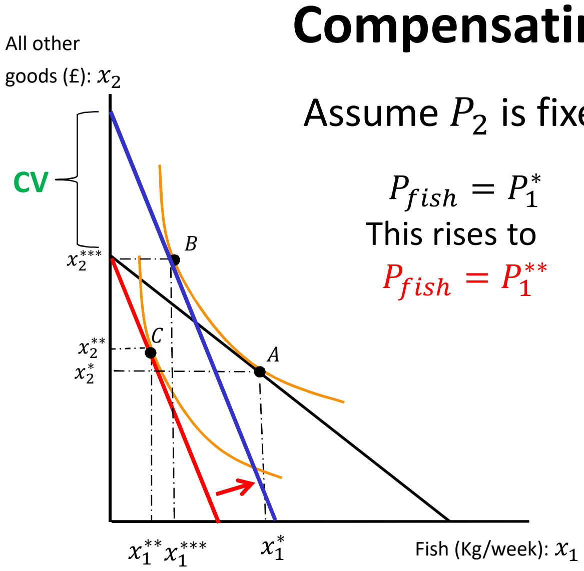
Compensating Variation II



Assume P_2 is fixed

$P_{fish} = P_1^*$
This rises to
$P_{fish} = P_1^{**}$

$$E_1(P_1^*, P_2, U_0) = P_1^* x_1^* + P_2 x_2^*$$

$$= P_1^{**} x_1^{**} + P_2 x_2^{**}$$

$$E_2(P_1^{**}, P_2, U_0) = P_1^{**} x_1^{***} + P_2 x_2^{***}$$

$$CV = E_2 - E_1$$

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Compensating Variation III

- Shephard's Lemma: derivative of expenditure function with respect to P_x is the compensated demand function

$$H(P_1, P_2, v) = \frac{\partial E(P_1, P_2, v)}{\partial P_1}$$

$$CV = E_2(P_1^{**}, P_2, U_0) - E_1(P_1^*, P_2, U_0)$$

- CV is the integral of the Hicksian demand
- This integral is the area to the left of the Hicksian demand curve between $P_1^{**} > P_1^*$

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Equivalent Variation I

- Say the price of a good rises …
- How much money would have to be taken away from the consumer **before** the price change to make him just as well off as he would be **after** the price change?
  - What is the maximum amount you are willing to pay to avoid the price change?
- How far must we shift the **original** budget line so that it is just tangential to the **new** indifference curve?

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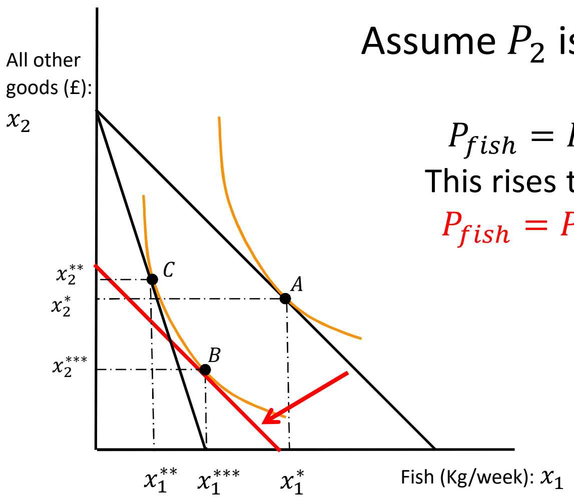
Equivalent Variation II



Assume P_2 is fixed

$P_{fish} = P_1^*$
This rises to
$P_{fish} = P_1^{**}$

$$M_1 = P_1^* x_1^* + P_2 x_2^*$$

$$= P_1^{**} x_1^{**} + P_2 x_2^{**}$$

$$M_2 = P_1^* x_1^{***} + P_2 x_2^{***}$$

$$\text{EV} = M_1 - M_2$$

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A change in welfare II

- Consumer surplus, compensating variation and equivalent variation can give different values
  - The same change in price can lead to different changes in welfare, depending on how we measure it
- £1 is worth differing amounts at different prices
- CS, CV and EV will only be equal to each other if tastes are quasilinear
  - As here, there is no income effect

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APPLICATIONS

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Endowments of goods I

- An endowment: A bundle of goods owned by a consumer and tradable for other goods:(e_1, e_2)
- Say you have e_1 of good 1, but would choose $x_1 > e_1$
  - This means you are a net demander/buyer of good 1
  - If you would choose $x_1 < e_1$, you are a net supplier/seller of good 1
- Income is determined by your endowments and prices
- Consumer's choice set depends on endowments and prices

$$C(P_1, P_2, e_1, e_2) = \{(x_1, x_2) | P_1 x_1 + P_2 x_2 \leq P_1 e_1 + P_2 e_2\}$$

- The budget constraint will always pass through (e_1, e_2)

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Endowments of goods II

- Say you bought 5 pairs of jeans (x_1) at £20 each and 10 jumpers (x_2) at £10 each from John Lewis for your partner
  - But they say that you bought the wrong ones and wrong quantities!
- You return to John Lewis, but don't have the receipt
  - You get a Gift Certificate for your goods at the current prices
  - $M = 5P_1 + 10P_2$
- If prices are unchanged, budget constraint is unchanged
- If prices have changed, budget constraint must still pass through (e_1, e_2) but will now have a different slope and intercepts

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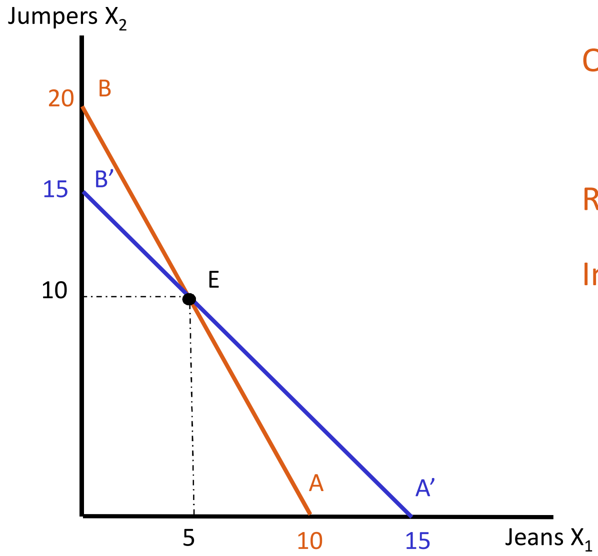
Endowments of goods III



Original budget constraint (AB):

$$P_1x_1 + P_2x_2 = 5P_1 + 10P_2$$

Rearranging gives: $x_2 = 5\frac{P_1}{P_2} + 10 - \frac{P_1}{P_2}x_1$

Intercept = 20 and Slope = -2

If jeans are on sale at 50% off, then Jeans in E are worth £10 not £20

Budget constraint pivots around E (A'B')

Intercept = 15 and slope = -1

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Endogenous versus exogenous income

- Utility maximisation with exogenous income implies:
  - $x_1^*(p_1, p_2, M) = \frac{M}{2P_1}$ $\quad$ x_1 is independent of P_2
  - When P_1 changed, we held money income constant
- Utility maximisation with endogenous income implies:
  - $x_1^*(p_1, p_2, e_1, e_2) = \frac{\alpha M}{2P_1}$ where $M = p_1e_1 + p_2e_2$
  - x_1 now depends on P_1 and P_2
  - Now when P_1 changes, your money income changes too
- What happens to demand for x_1 when its price changes?

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Slutsky: the endowment income effect I

$$\frac{\Delta x_1}{\Delta P_1} = \frac{x_1^s}{\Delta P_1} - \frac{\Delta x_1^m}{\Delta M} x_1$$

- Substitution effect remains the same
- But there are now two income effects to consider
  - Previously: say P_1 falls. Real income rises and so x_1 is affected (ordinary income effect: money income remains fixed)
  - But now when P_1 falls, your endowment and thus your money income is affected (endowment income effect)

$$\frac{\Delta x_1}{\Delta P_1} = \frac{x_1^s}{\Delta P_1} - \frac{\Delta x_1^m}{\Delta M} x_1 + \text{endowment income effect}$$

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Slutsky: the endowment income effect II

- When P_1 changes, the price of the endowment changes and so money income changes and this changes demand

Endowment income effect = (i) Change in income when price changes x (i) Change in demand when income changes

- Consider term (i): We know: $P_1 e_1 + P_2 e_2 = M$
  - Change in income when price changes = $\frac{\Delta M}{\Delta P_1} = e_1$
- We already have an expression for term (ii): $\frac{\Delta x_1^m}{\Delta M}$

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Slutsky: the endowment income effect III

- Endowment income effect: $\frac{\Delta x_1^m}{\Delta M}\frac{\Delta M}{\Delta P_1} = \frac{\Delta x_1^m}{\Delta M} e_1$
- Revised Slutsky equation: $\frac{\Delta x_1}{\Delta P_1} = \frac{x_1^s}{\Delta P_1} + (e_1 - x_1)\frac{\Delta x_1^m}{\Delta M}$
- Substitution effect is always negative: $P_1 \uparrow \rightarrow \downarrow x_1$
- With a normal good: ordinary income effect > 0
- Size of total effect depends on sign of $(e_1 - x_1)$:
  - Net demander: Price of normal good rises, demand must fall
  - Net supplier: It depends on the magnitude of the positive combined income effect, versus negative substitution effect

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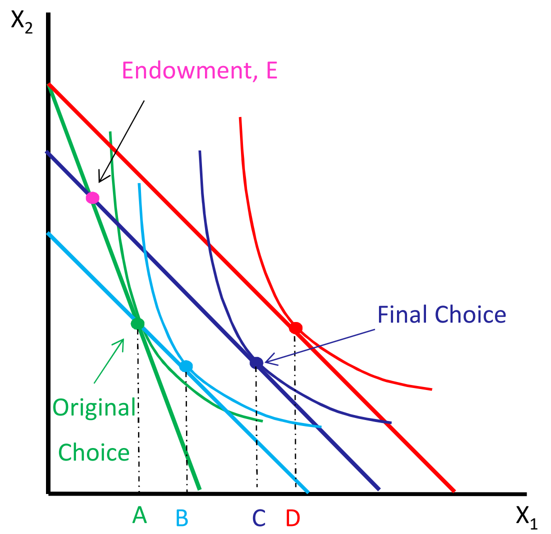
Endowments of goods IV



- Original budget constraint (A)
- Price of x_1 falls
- Substitution effect: A-B
- Ordinary income effect (holding money income fixed): B-D
- Endowment income effect (changes value of endowment and hence income): D-C
  - Must go through Endowment, E
- Final budget constraint (C)

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Intertemporal Choice I

- Should you go to university?
  - No: You'll have a low income as a student and will incur huge debts
  - Yes: You'll have a higher income as a graduate and pensioner
- We can apply the consumer choice model to consider:
  - How much debt should you accumulate as a student?
  - How will the amount of debt depend on the interest rate?
  - How much should you save for retirement?
- This model is crucial to understanding saving decisions
- We treat two time periods (t = 1, 2) exactly like two goods

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Intertemporal Choice II

- I can earn £10,000 (m_1) this summer and then travel next summer, earning £0 (m_2). Assume r = 10%
  - If c_1 = 0, I could consume $[m_1(1+0.1)+m_2]$ next summer.
  - For every £1 consumed today, next year's consumption falls by (1+r)
  - The most I have for consumption next summer is what I would have had if c_1 = 0 minus (1+r) times my actual consumption this summer

$$c_2 \leq m_1(1+r)+m_2-(1+r)c_1$$

Rearranging gives:

$$(1+r)c_1+c_2 \leq m_1(1+r)+m_2 \qquad \text{Future value}$$

$$c_1+c_2(1+r)^{-1} \leq m_1+m_2(1+r)^{-1} \qquad \text{Present value}$$

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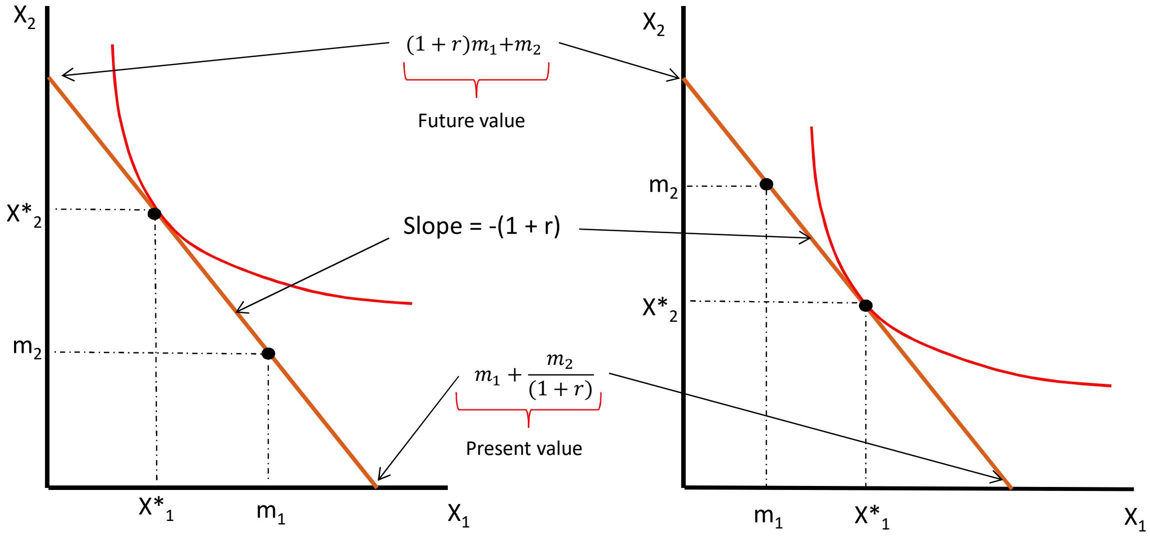
Lenders and Borrowers



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Intertemporal Choice III

- Opportunity cost of consuming £1 today is (1+r) tomorrow
- In the N period model, opportunity cost is $(1+r)^n$
- £10,000 invested today at interest rate r yields:
  - After 1 year: £10,000 (1 + r)
  - After 2 years: £10,000(1 + r)(1 + r) = 10,000$(1+r)^2$
  - After n years: £10,000$(1+r)^n$
- Budget Constraint becomes:

$$(1+r)^n c_1 + c_n \leq m_1(1+r)^n + m_n$$

  - This considers income and consumption in period 1 and n
  - What about the years in between?

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Intertemporal Choice IV

- Assume I have an idea about my earning in all periods 1 – n
  - This gives n different endowments across n years $(m_1, m_2, \dots, m_n)$
  - Assume constant r across all years
- If I consume nothing until the last year, I have m_n plus:
  - Penultimate year's endowment and 1 year's interest: $m_n + (1+r)m_{n-1}$
  - Plus second to last year's endowment and 2 year's interest, plus…
- So the maximum I could consume in last period is:
  $C_n = m_n + (1+r)m_{n-1} + (1+r)^2 m_{n-2} + \cdots + (1+r)^{n-1} m_{n-(n-1)}$

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Intertemporal Choice V

- The actual amount I can consume depends on how much I consumed in the previous periods

$$C_n = m_n + (1+r)m_{n-1} + (1+r)^2 m_{n-2} + \cdots + (1+r)^{n-1} m_1$$

$$-(1+r)c_{n-1} - (1+r)^2 c_{n-2} - \cdots - (1+r)^{n-1} c_1$$

Or

$$C_n + (1+r)c_{n-1} + (1+r)^2 c_{n-2} + \cdots + (1+r)^{n-1} c_1$$

$$= m_n + (1+r)m_{n-1} + (1+r)^2 m_{n-2} + \cdots + (1+r)^{n-1} m_1$$

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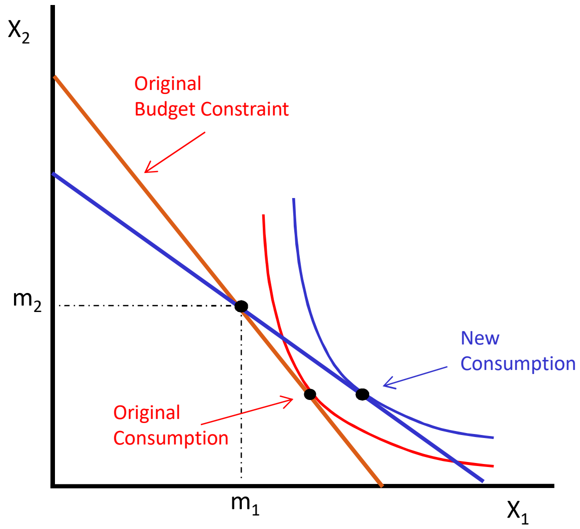
Comparative Statics: Borrowers



- What happens to the borrowing decision as ‘r’ changes?
- A decrease in ‘r’ pivots the budget constraint
- A borrower remains a borrower
- By Revealed preference, a borrower will be unambiguously better off
- If ‘r’ increases, a borrower may remain a borrower or may switch to become a lender

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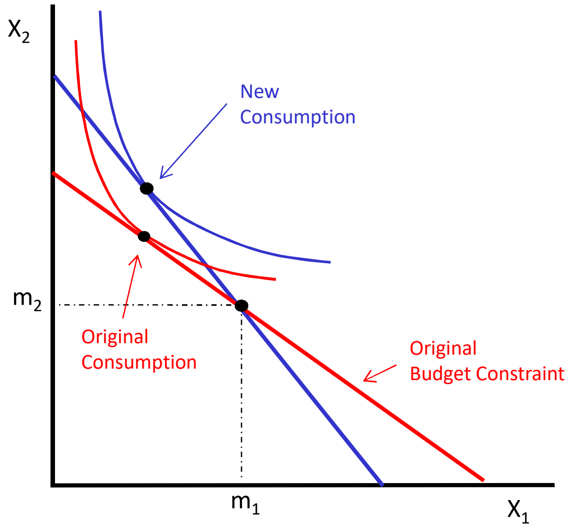
Comparative Statics: Lenders



- What happens to the borrowing decision as ‘r’ changes?
- An increase in ‘r’ pivots the budget constraint
- A lender remains a lender
- By Revealed preference, a lender will be unambiguously better off
- If ‘r’ decreases, a lender may remain a lender or may switch to become a borrower

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Applications

- We can apply the consumer choice model to many areas and it can give important insights to policy-makers
  - Should taxes be increased on certain goods?
  - If interest rates change, how will this affect the behaviour of savers and borrowers?
  - If income tax rises, what happens to the supply of labour?
  - If in-work or out-of-work benefits change, how will this affect people's incentive to work?
- You will look at a further application in seminars

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EC109

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Production Theory

Elizabeth Jones

The Topics

- Production functions
- Isoquants and MRTS
- Returns to scale
- Cost functions and cost curves
- Cost minimization
- Expansion paths
- Comparative statics
- Short versus long run
- Profit functions
- Supply functions

The firm's problem

Firms make choices:

- Which inputs should be used, e.g. capital and labour?

Firms face constraints:

- Technological constraints, e.g. how easy it is to convert inputs into outputs?
- Which combinations of inputs will produce a given level of output?
- Economic constraints that derive from the prices of inputs and outputs

Maximise profit: difference between revenue and costs

- Given input prices, what's the cheapest way to produce a given quantity?
- Given output prices, how much should the firm produce?

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Production Functions

Production functions

- Firms convert inputs into outputs: the inputs used are the firm’s factors of production
- The amount of goods and services produced is the firm's output (Q)
- Certain combination of inputs will produce given amounts of output

- The production function tells us the maximum amount of a good the firm can produce using various combinations of inputs.
- We often assume just 2 inputs (N = 2) with capital (K) and labour (L), giving a production function of the form:

$$Q = f(K, L)$$

- We also typically assume that Q is concave and monotonic

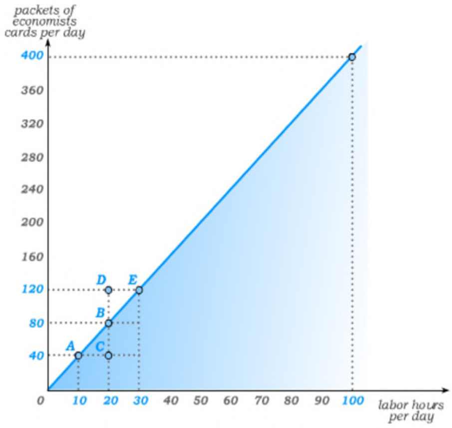
A short run one input/output model

- Consider a case where a producer converts one input into one output!
  - SR: one factor of production is fixed, so only one input is varied to increase output
- A production plan: shows the number of labour hours (L) needed to produce a given level of output (Q)

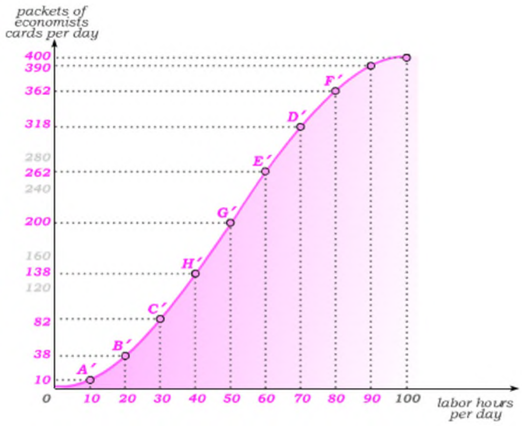
$$Q = f(L) \qquad \gg \qquad L = h(Q)$$

- The producer’s choice set is the set of production plans that are technologically feasible (inputs are sufficient to produce the output)
- The production function: the set of production plans, with no input waste
- As more workers are employed, output rises: by how much?
  - As more workers are added to a fixed factory space, the additional output produced by each last worker may rise to begin with, but is then likely to fall

Production functions



- B uses 20 labour hours to produce 80 units
- What can we say about C and D?



Why does the slope on this production function initially get steeper and then get shallower?

Marginal and Average Product

- Marginal product of Labour: The additional output produced by one more worker (or labour hour)

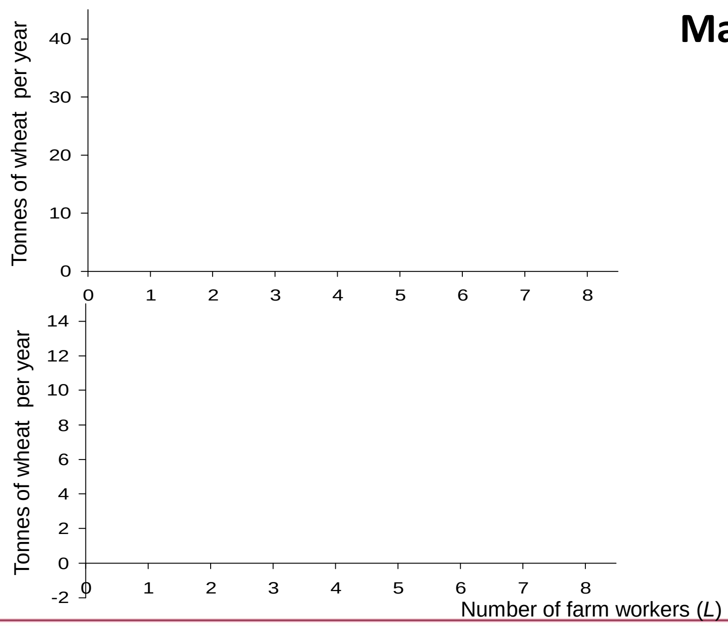
$$MP_L = \frac{\textit{Change} \text{ in total product}}{\textit{Change} \text{ in quantity of labour}} = \frac{\Delta Q}{\Delta L} = \frac{\partial Q}{\partial L} = f_L$$

- Linear Production function: Constant MP_L
- We normally assume diminishing marginal productivity (flatter function)

$$\frac{\partial MP_L}{\partial L} = \frac{\partial^2 f}{\partial L^2} < 0$$

- Average product of Labour:

$$AP_L = \frac{\text{Total product}}{\text{Quantity of labour}} = \frac{Q}{L} = \frac{f(K,L)}{L}$$



Marginal and Average Product

From the total product function, we can derive the marginal and average product of labour curves.

Point D: **<u>Law of diminishing marginal returns</u>**:

As variable input increases, with others held fixed, a point will be reached beyond which the marginal product of the variable input will decrease.

Point E: maximum output ($MP_L = 0$)

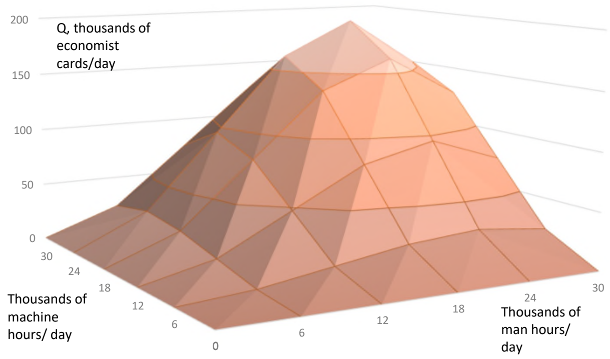
MP_L at $5L$ is the slope of AB

AP_L at $2L$ is the slope of OC

Two inputs - Robots and humans

Assume now that production requires both Capital and Labour.

L \ K	0	6	12	18	24	30
0	0	0	0	0	0	0
6	0	5	15	25	30	23
12	0	15	48	81	96	75
18	0	25	81	137	162	127
24	0	30	96	162	192	150
30	0	23	75	127	150	117



We can now find marginal products of capital and labour

$$MP_L = \frac{Change \text{ in total product}}{Change \text{ in quantity of } labour} = \frac{\Delta Q}{\Delta L} = \frac{\partial Q}{\partial L} = f_L$$

$$MP_K = \frac{Change \text{ in total product}}{Change \text{ in quantity of } capital} = \frac{\Delta Q}{\Delta K} = \frac{\partial Q}{\partial K} = f_K$$

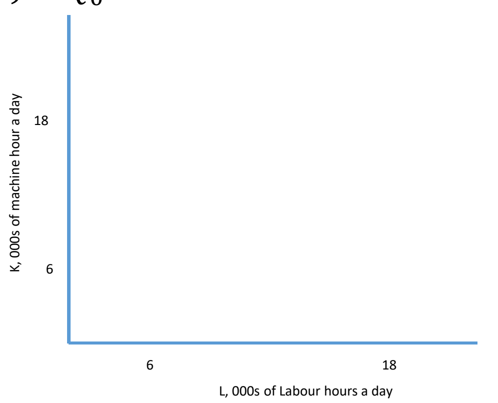
Isoquants

Isoquants

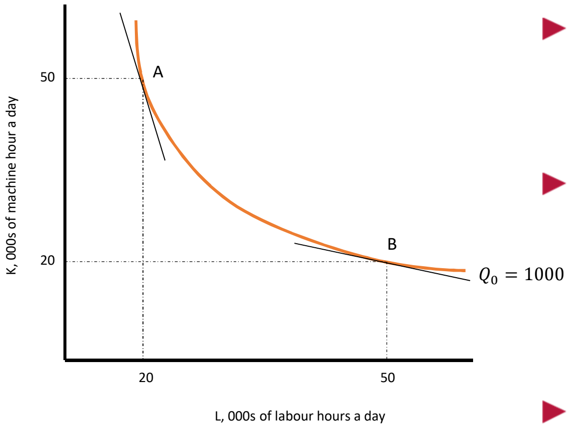
- Isoquants show all combinations of labour and capital that produce a given level of output (Q_0)

$$f(K, L) = Q_0$$

L \ K	0	6	12	18	24	30
0	0	0	0	0	0	0
6	0	5	15	**25**	30	23
12	0	15	48	81	96	75
18	0	**25**	81	137	162	127
24	0	30	96	162	192	150
30	0	23	75	127	150	117



Marginal rate of Technical Substitution



- Isoquants slope down: Keeping output constant, more labour means less capital
  - Isoquants are monotonic, thin and do not cross
- The $MRTS_{L,K}$ (of labour for capital) is the slope of the isoquant
  - How many K must be given up to use 1 more L, while keeping output constant
  - The $MRTS_{L,K}$ diminishes in absolute value as we move down the isoquant
- Isoquants are convex to the origin.

$MRTS_{L,K}$ and Marginal Products

- Analysis here is similar to consumer theory
- Take the total differential of the production function:

$$dq = \frac{df}{dL} \cdot dL + \frac{df}{dK} \cdot dK = MP_L \cdot dL + MP_K \cdot dK$$

- Along an isoquant $dq = 0$:

$$MP_L \cdot dL = -MP_K \cdot dK$$

$$MRTS_{L,K}(L\ for\ K) = \left.\frac{dK}{dL}\right|_{q=q_0} = \frac{MP_L}{MP_K}$$

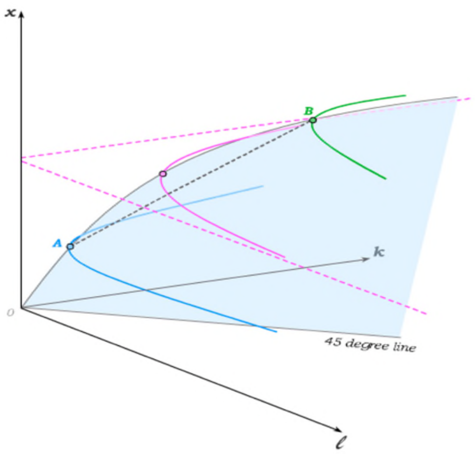
- How many units of capital the firm can substitute for one unit of labour
- The shape of the isoquant determines the rate of technical substitution

Isoquants versus indifference curves

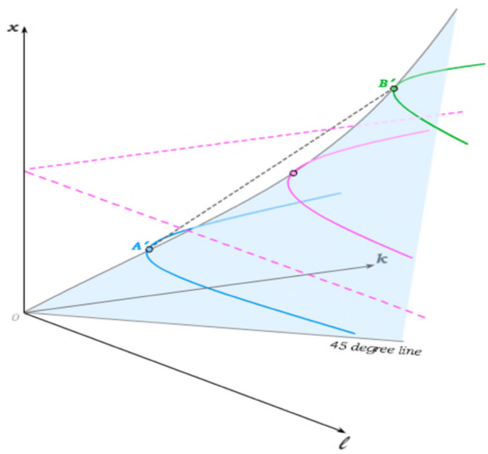
- Indifference curves represents tastes, while isoquants arise from production functions, which are the technological constraints faced by producers
- Utility is not measureable, meaning there is no objective interpretation as to the numbers accompanying indifference curves, beyond the ordering.
- Isoquants reflect output which is measurable
  - Doubling all values associated with an indifference map leaves us with the same tastes as before
  - Doubling all values associated with isoquants alters the production technology, with the new technology producing twice as much output from any bundle of inputs (Returns to scale)

Convexity under Producer Theory

Vertical and horizontal slices are convex: concave production function



Only horizontal slices are convex: quasi-concave production function



Isoquants and Returns to scale I

- With $MP_L, MP_K > 0$, if all inputs increase simultaneously, total output must increase: by how much?

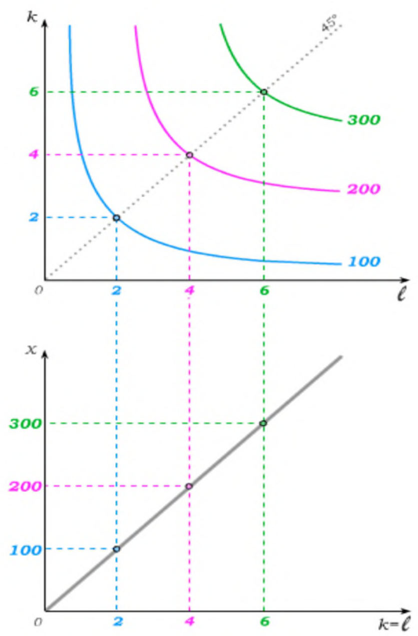
$$Returns\ to\ scale = \frac{\%\Delta\ (quantity\ of\ output)}{\%\Delta\ (quantity\ of\ all\ inputs)}$$

- Consider a homogeneous production function, such that:

$$f(tK, tL) = t^k f(K, L) = tQ$$

  - $k > 1$ implies IRS
  - $k = 1$ implies CRS
  - $k < 1$ implies DRS

- For a Cobb-Douglas production function $f(K, L) = AL^{\alpha}K^{\beta}$
  - $\alpha + \beta > 1$ implies IRS
  - $\alpha + \beta = 1$ implies CRS
  - $\alpha + \beta < 1$ implies DRS



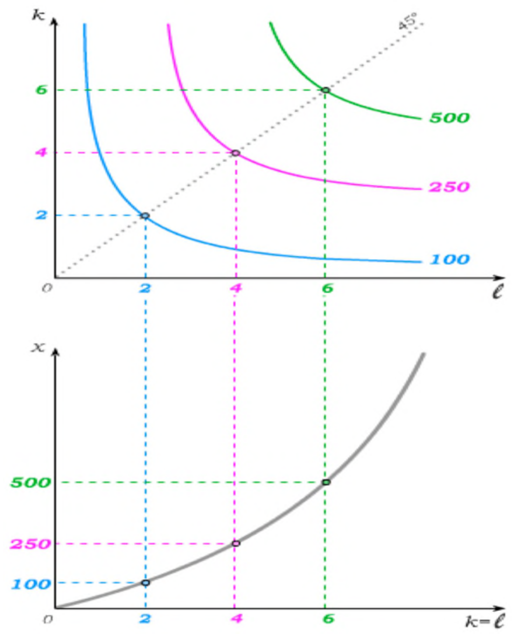
Isoquants and Returns to scale II

The vertical slice lying on this ray has output Q on vertical axis and both inputs on horizontal axis

The linear shape of the vertical slice indicates CRS

All isoquants are radial expansions of one another: a CRS production function is **homothetic** (isoquant labels increase proportionately with the inputs)

MRTS depends only on ratio of K to L and not on scale of production



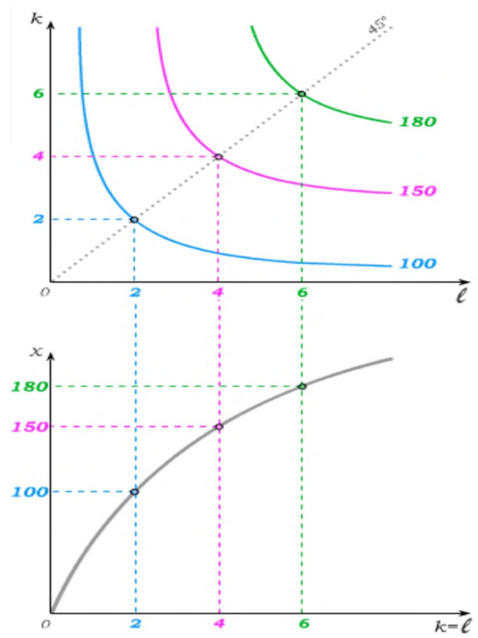
Isoquants and Returns to scale III

Same map, but the output associated with each isoquant has changed

This leads to an increasing slope on the vertical slice and thus IRS

As inputs double, output increases by more than double (isoquant labels increase proportionately more than inputs)

An IRS production function that is homogeneous is homogeneous of degree greater than 1: exponents sum to > 1



Isoquants and Returns to scale IV

Another map, with changed output for each isoquant

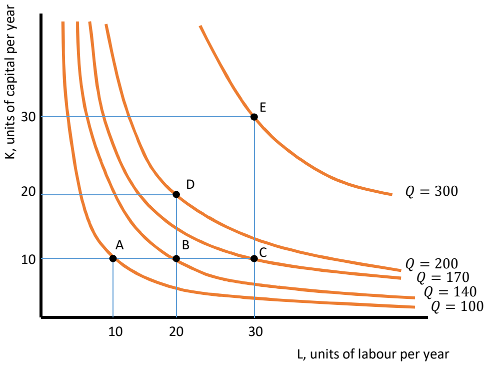
This isoquant map gives rise to an decreasing slope on the vertical slice and thus DRS

As inputs double, output increases by less than double (isoquant labels increase proportionately less than inputs)

A DRS production function that is homogeneous is homogeneous of degree less than 1: exponents sum to < 1

Returns to scale and diminishing marginal product I

Here we have diminishing marginal returns to labour, but constant returns to scale.



Consider: $f(L,K) = L^{\alpha}K^{\beta}$

$$MP_L = \alpha L^{\alpha-1}K^{\beta}$$

$$MP_K = \beta L^{\alpha}K^{\beta-1}$$

The function has diminishing MP iff:

$$\frac{\partial MP_L}{\partial L} = \alpha(\alpha-1)L^{\alpha-2}K^{\beta} < 0$$

$$\frac{\partial MP_K}{\partial K} = \beta(\beta-1)L^{\alpha}K^{\beta-2} < 0$$

If exponents $\alpha, \beta > 0$, the MP_L and MP_K will be diminishing iff each exponent < 1 (only then is derivative of MP_L and MP_K negative)

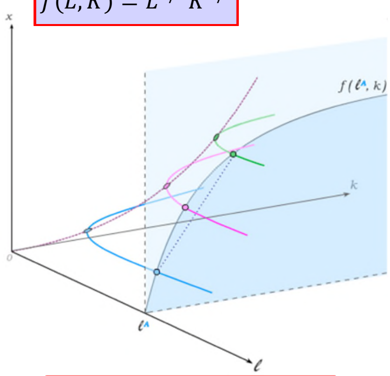
Returns to scale and diminishing marginal product II

$\boxed{f(L,K) = AL^{\alpha}K^{\beta}}$

$\boxed{MP_K = \beta AL^{\alpha}K^{\beta-1}}$

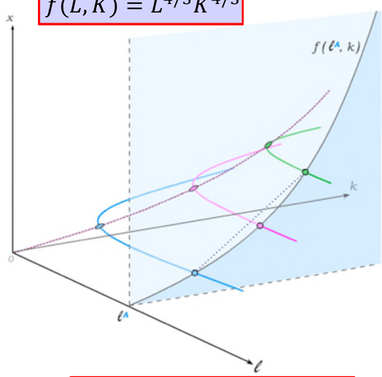
$\boxed{\frac{\partial MP_K}{\partial K} = \beta A(\beta-1)L^{\alpha}K^{\beta-2}}$

$\boxed{f(L,K) = L^{2/3}K^{2/3}}$



$\alpha+\beta > 1$ implies
Increasing Returns to Scale

$\boxed{f(L,K) = L^{4/3}K^{4/3}}$

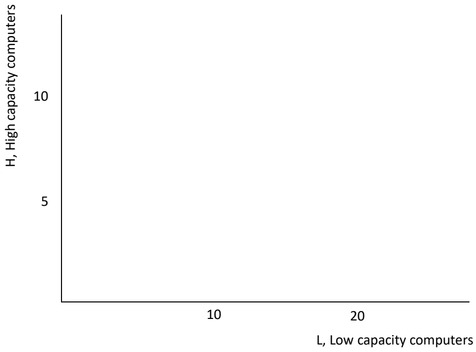


$\beta < 1 \; implies \; \frac{\partial MP_K}{\partial K} < 0$
Diminishing MP

$\beta > 1 \; implies \; \frac{\partial MP_K}{\partial K} > 0$
Increasing MP

Linear production functions

Elasticity of substitution: σ = ∞



Map of isoquants for data storing

$$Q = f(L, H) = \alpha L + \beta H$$

Low capacity and high capacity computers are perfect substitutes (ratio of 2:1)

MRTS = $-\frac{\alpha}{\beta}$ and is constant along linear isoquants

Constant returns to scale

$$Q = f(tL, tH) = \alpha tL + \beta tH = t(\alpha L + \beta H) = tf(L, H)$$

Fixed proportions production functions

$$Q = f(L, H) = \min\{\alpha H + \beta \mathrm{O}\}\ \alpha, \beta > 0\}$$

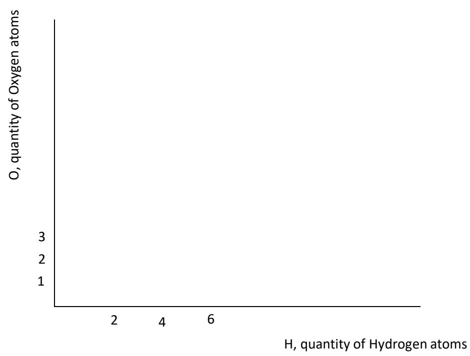
Oxygen and Hydrogen atoms are perfect complements, used in a fixed ratio

The Oxygen-Hydrogen ratio is fixed at $\frac{\beta}{\alpha}$ (slope of straight line) and firm operates along the ray where this is constant

If $\alpha H < \beta O, then\ Q = \alpha H$ (Hydrogen is the binding constraint) and vice versa

If $\alpha H = \beta O$ both inputs are fully utilised

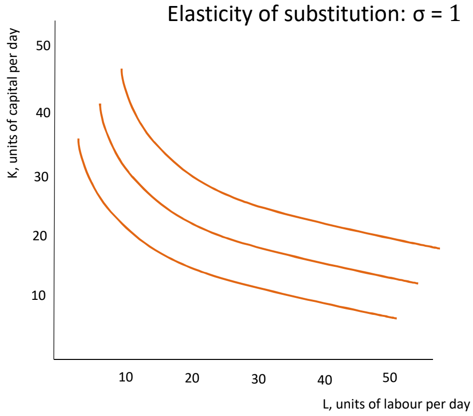
Elasticity of substitution: σ = 0



Map of isoquants for molecules of water

Cobb Douglas production function

Elasticity of substitution: σ = 1



$$Q = AL^{\alpha}K^{\beta} \quad \alpha, \beta, A > 0$$

Inputs substitutable in variable proportions along an isoquant

It can exhibit any returns to scale, depending on if $(\alpha + \beta) >, <, = 1$

$$MRTS = \frac{MP_L}{MP_K}$$

The CD function is linear in logarithms:

$$lnQ = lnA + \alpha lnL + \beta lnK$$

α is elasticity of output with respect to L
β is elasticity of output with respect to K

WARWICK

Costs

Economic Costs I

- What is the cost of an airline using the planes it owns for scheduled passenger services?
  - Crew salaries, fuel etc.
  - Foregone income from not renting the plane to someone else; time
- Costs don't always refer to direct monetary transfers
  - Explicit costs refer to those costs needing a direct monetary outlay
  - Implicit costs refer to those costs not involving such a monetary outlay
- The <u>economic cost</u> of an input is its opportunity cost
  - The remuneration the input would receive in its best alternative employment
  - It includes both explicit and implicit costs
  - A forward looking concept; depends on decision made and current market prices
- Accounting costs: all explicit, incurred <u>in the past</u> - on accounting statements

Economic Costs II

- Sunk costs: costs that are already incurred and so cannot be avoided
  - They do not (or should not) affect production decisions going forwards
  - Behavioural insights?
- Say I run a factory which, last year, emitted illegal pollution. I became aware of this at the start of the year and quietly fixed it. Then I receive a £10,000 fine for the pollution and am required to fix the problem. I've already fixed it, so now I just have to pay the fine.
  - This is a current cost for my business (according to the accountant)
  - But, does the size of the fine depend on my current production decisions?
  - No: regardless of whether or how much I produce now and in the future, the fine is based on something that happened in the past. It does not affect economic choices I currently face
  - It is not an economic cost of production

The firm's problem

- Firms aim to maximise profits (difference between TR and TC)
- Total Revenue: $TR = PQ = Pf(K, L)$
- Total Costs: $TC = wL + rK$ where w; r = cost of labour and capital respectively
- Profits:

$$\pi = PQ - (wL + rK) = Pf(K, L) - (wL + rK)$$

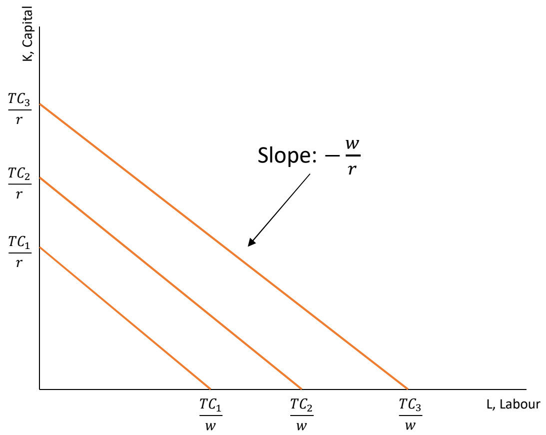
- Two options to solve the firm's problem:
  a) One Step solution: Choose (Q, K, L) to maximise π
  b) Two-step solution:
     - Minimise costs for a given output level, Q_0
     - Choose output to maximise π

Isocost curves

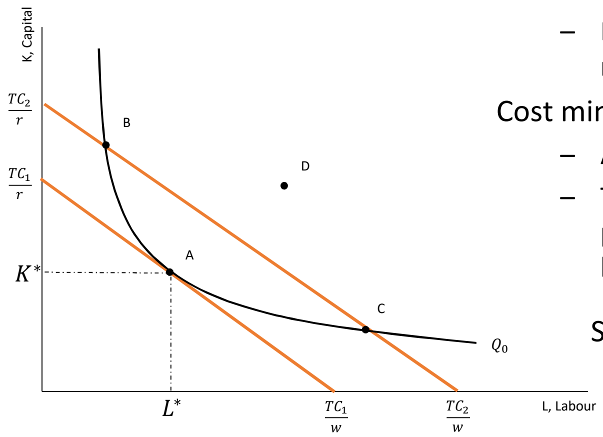
Isocosts: All the combinations of labour and capital that a producer could afford to purchase at a given set of input prices (w, r) and a total allowable cost level TC.

Assume $w = £10$ and $r = £20$ and a 'budget' of £1m.

We apply the same principles here as with the budget constraint in consumer theory



Cost Minimisation I



$$\min_{L,K} wL + rK \quad subject\ to: \quad f(L,K) = Q_0$$

- Shift isocost until it is tangential to isoquant
- B and C are technically efficient, but not cost minimising

Cost minimisation occurs at A:

- A is technically efficient and cost minimising
- The rate at which K can be traded for L in the production process = rate at which they can be traded in the marketplace

Slope of isoquant = Slope of isocost

$$MRTS_{L,K} = \frac{MP_L}{MP_K} = \frac{w}{r}$$

Cost Minimisation II

- The firm's cost minimisation problem is similar to EMP

$$\min_{L,K} wL + rK \quad subject\ to: \quad f(L,K) = Q_0$$

- Set up the Lagrangian:

$$\nabla = wL + rK + \lambda(Q_0 - f(L,K))$$

- FOCs:

$$\frac{\partial \nabla}{\partial L} = w - \lambda {}^{\partial f}/_{\partial L} = 0$$

$$\frac{\partial \nabla}{\partial K} = r - \lambda {}^{\partial f}/_{\partial K} = 0$$

$$\frac{w}{r} = \frac{{}^{\partial f}/_{\partial L}}{{}^{\partial f}/_{\partial K}} = MRTS_{L,K}$$

$$\frac{\partial \nabla}{\partial \lambda} = Q_0 - f(L,K) = 0$$

Cost Minimisation III

$$\frac{w}{r} = \frac{\partial f/_{\partial L}}{\partial f/_{\partial K}} = \frac{MP_L}{MP_K} = MRTS_{L,K}$$

- Rearranging:

$$\frac{f_K}{r} = \frac{f_L}{w} = \lambda \qquad or \qquad \frac{w}{f_L} = \frac{r}{f_K} = \lambda$$

- The Lagrange multiplier, λ, shows how much the optimal value of the objective function will change following a change in the constraint
  - By how much will costs increase when output constraint is increased marginally
- Solving the minimisation problems yields the optimal factor demands:

$$L^* = L^*(r, w, Q); \qquad K^* = K^*(r, w, Q)$$

  - These are derived or conditional factor demands, as input demand depends on Q
  - Cost function: $TC(r, w, Q) = wL^*(r, w, Q) + rK^*(r, w, Q)$

Cost minimisation IV: $Q = L^\alpha K^\beta$; $P_L = w$; $P_K = r$

$$\nabla = wL + rK + \lambda(Q_0 - L^\alpha K^\beta)$$

$$\frac{\partial \nabla}{\partial L} = w - \lambda \alpha L^{\alpha-1} K^\beta = 0$$

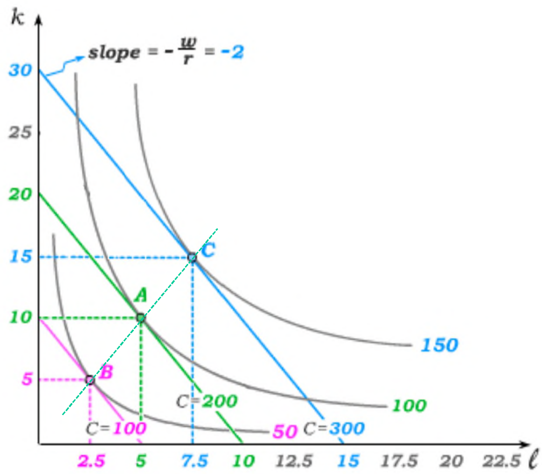
$$\frac{\partial \nabla}{\partial K} = r - \lambda \beta L^\alpha K^{\beta-1} = 0$$

$$\frac{w}{r} = \frac{\alpha}{\beta}\frac{K}{L} \gg K = L\frac{w}{r}\frac{\beta}{\alpha}$$

$$\frac{\partial \nabla}{\partial \lambda} = Q_0 - L^\alpha K^\beta = 0$$

- Solve for: $L^* = Q_0^{\frac{1}{\alpha+\beta}} \left(\frac{\alpha r}{\beta w}\right)^{\frac{\beta}{\alpha+\beta}}$
- Plug expression for L* into $K = L\frac{w}{r}\frac{\beta}{\alpha}$ and solve for: $K^* = Q_0^{\frac{1}{\alpha+\beta}} \left(\frac{\beta w}{\alpha r}\right)^{\frac{\alpha}{\alpha+\beta}}$
- Cost function: TC $= wQ_0^{\frac{1}{\alpha+\beta}} \left(\frac{\alpha r}{\beta w}\right)^{\frac{\beta}{\alpha+\beta}} + rQ_0^{\frac{1}{\alpha+\beta}} \left(\frac{\beta w}{\alpha r}\right)^{\frac{\alpha}{\alpha+\beta}}$

Expansion Path



The set of optimal combinations of L and K (tangency of isoquants and isocosts)

– How inputs increase with increases in output

The expansion path does not have to be a straight line or start at the origin

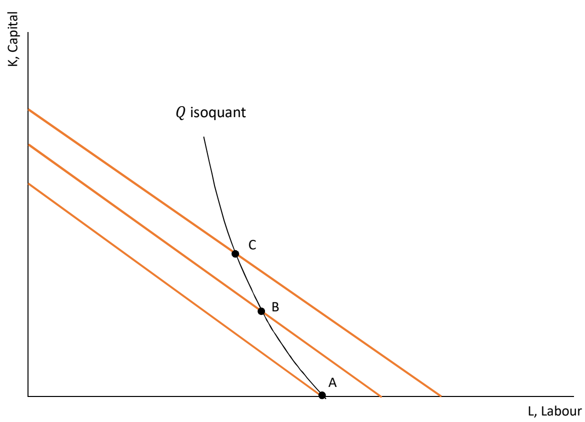
– It depends on the shape of the isoquants
– The use of some inputs may increase faster than others as output expands

In previous case: $Q = AL^{\alpha}K^{\beta}$

– MRTS depends on ratio of two inputs: $\frac{w}{r} = \frac{\alpha}{\beta}\frac{K}{L}$
– Production function is homothetic and expansion path is a straight line

Corner solutions

The cost minimizing input combination occurs where the firm uses no Capital:



Tangency condition doesn't hold at any point, as the isocost is flatter than the isoquant at all points:

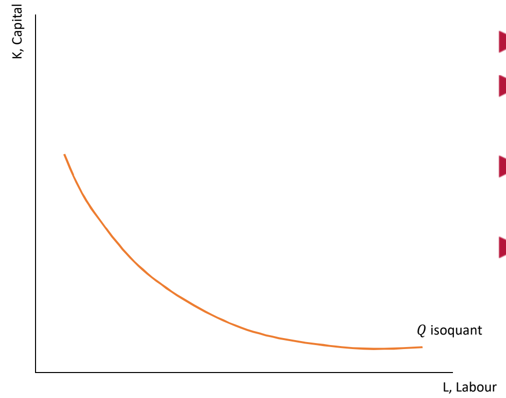
The situation is such that:

$$\frac{MP_L}{MP_K} > \frac{w}{r} \quad \gg \quad \frac{MP_L}{w} > \frac{MP_K}{r}$$

Every dollar spent on labour is more productive than every dollar spent on Capital.

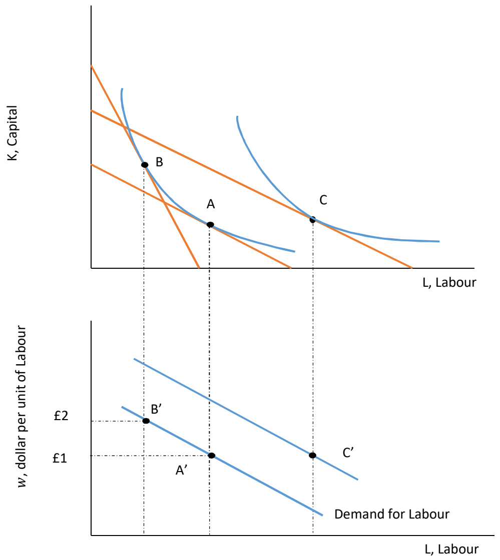
Comparative statics: A change in input prices

Suppose the price of capital, r , and the quantity of output, Q, are both held fixed. What happens if there is an increase in the price of labour, w?



- The isocost curve becomes:
- With diminishing $MRTS_{L,K}$, where is the new optimal point?
- What does this mean for the quantities of capital and labour employed?
- Note two important assumptions needed for those results:
  - $K, L > 0$
  - Convex isoquants

Using comparative statics to derive input demand curves



- The top diagram shows the effect of an increase in the price of labour and a change in output
  - As w rises (r falls) optimisation moves from A to B
  - As quantity of output changes (keeping w and r constant) the isoquant (and isocost) shift out
- The bottom diagram summarises the implications for the firm for its demand for labour
  - Following change in input prices, firm moves up its demand for labour curve A' to B'
  - Following increase in quantity of output, the labour demand curve shifts

Cost Functions I

- We have already seen that the firm's TC is a function of output and input prices
- Total Cost function: $\mathrm{C}(r, w, Q) = wL^*(r, w, Q) + rK^*(r, w, Q)$
  - With fixed prices, we often note $\mathrm{C}(Q)$
- Average Cost (AC) reflects costs per unit of output

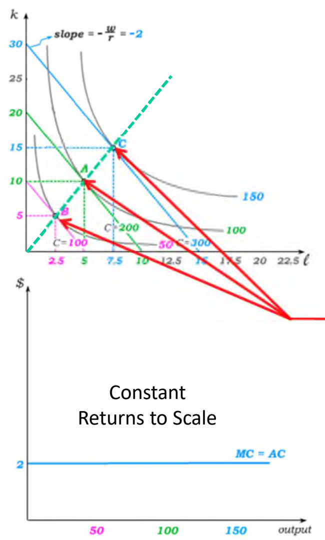
$$\mathrm{AC}(r, w, Q) = \frac{C(r, w, Q)}{Q}$$

- Marginal Cost (MC) reflects the change in total costs following a change in output

$$\mathrm{MC}(r, w, Q) = \frac{\partial C(r, w, Q)}{\partial Q}$$

- Consider the case of a CRS production technology

Cost Functions II: CRS



For instance, all input bundles on this isoquant will produce **100** units of *x "without wasting inputs"* – all represent ***technologically efficient*** ways of producing **100** units .

When w =20 and r =10, a budget – or ***isocost*** – of \$300 is sufficient to reach production plans on this isoquant.

But a production plan is not ***economically efficient*** unless its inputs are the ***least-cost way of reaching the output level***.

For every output quantity, there is usually one such *economically efficient* input bundle where

$$\boxed{TRS = -\frac{w}{r}}$$

And when technology is *homothetic*, the economically efficient input bundles lie on the same ray from the origin.

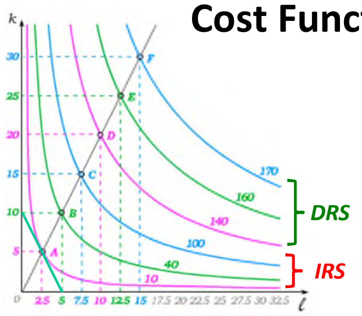
For each output level, we can then read off the ***cost*** of production *assuming the firm **cost-minimizes***.

From the *(total)* ***cost*** curve, we can derive the *marginal* and *average* cost curves

Cost Functions III

- A concave production function yields a convex cost function
- An IRS production function at all Q has a continuously falling AC curve
  - MC lies below AC at all levels of output
- A production function that is concave for positive quantities but requires a fixed cost
  - Total cost starts at a positive value when q = 0
  - AC is u-shaped and starts at an infinite level; reaches a minimum and then rises, pulling the MC up
- A non-concave production function (e.g. cubic) yields a decreasing then increasing total cost function (as we'll see next)
  - Total costs rise more and more rapidly once diminishing returns set in
  - A possible explanation is a 3rd factor of production that is fixed as inputs expand

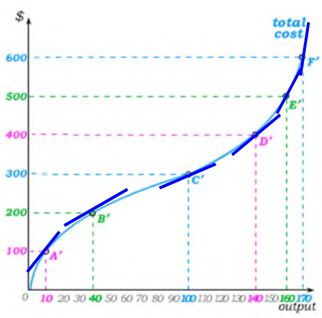
Cost Functions IV: IRS then DRS

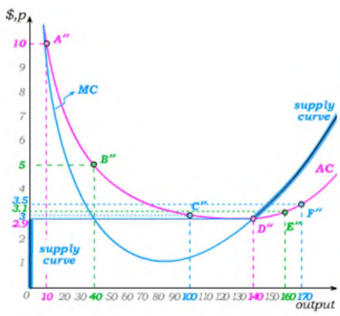


Suppose ***A*** is the cost-minimizing input bundle to produce **10** units of output.

With homothetic technologies, this implies ***all*** cost-minimizing input bundles lie on the same ray.

From ***A***, we can calculate the ***cost (A')*** and ***average cost (A'')*** of producing 10 units of output (given input prices $w = 20$ and $r = 10$).





And repeating this for all other output quantities, we get the *(total)* ***cost*** and ***average cost curves***.

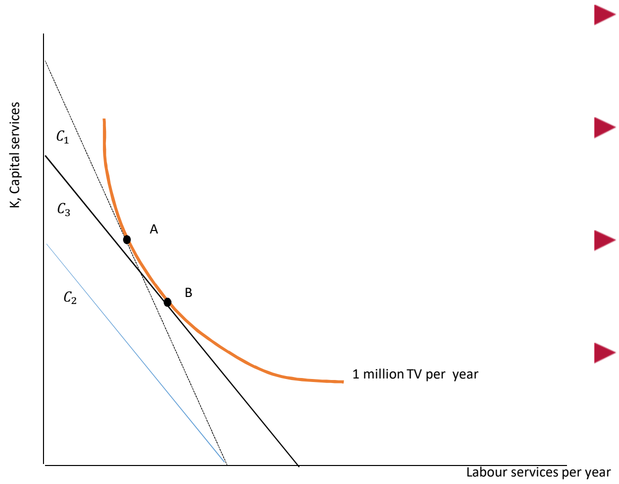
The slope of the (total) *cost* curve becomes the ***marginal cost curve***.

The MC curve that lies above AC becomes the firm's **supply curve**

Properties of cost functions

- Cost functions are homogenous of degree 1 in input prices
  - Doubling all input prices won't change level of inputs used; inflation shifts cost curves up
- Cost functions are non-decreasing in Q and in input prices
- If $\mathrm{Q} = \mathrm{f}(L, K)$ is convex, it exhibits IRS and $\mathrm{C}(\mathrm{w}, \mathrm{r}, Q)$ is concave in Q:
  - MC(Q) and AC(Q) fall as Q rises: The firm benefits from **economies of scale**
- If $\mathrm{Q} = \mathrm{f}(L, K)$ is concave, it exhibits DRS and $\mathrm{C}(\mathrm{w}, \mathrm{r}, Q)$ is convex in Q:
  - MC(Q) and AC(Q) rise as Q rises: The firm suffers from **diseconomies of scale**
- With CRS, output increases proportionally to an increase in all inputs: AC(Q) remains constant: no economies or diseconomies of scale
  - When costs are minimised, the firm is productively efficient: **minimum efficient scale**
- AC(Q) is increasing when MC(Q) ≥ AC(Q) and decreasing when MC(Q) ≤ AC(Q)

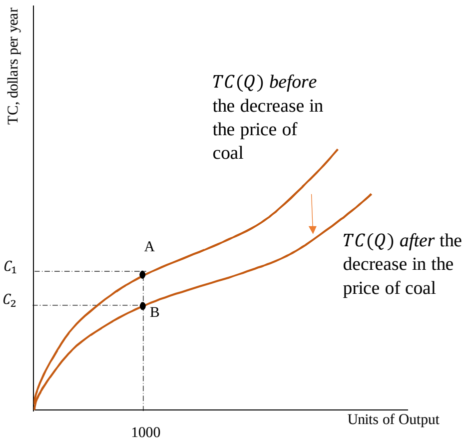
Shifting cost curves I



- Any change in technology or input prices will shift the isocost curve and hence the cost function
- Starting from point A, where the firm produces 1 million televisions, on isocost line C_1.
- After the price of capital increases, the cost minimising input combination to produce 1 million units occurs at point B
- But, total cost is now greater than it was at point A

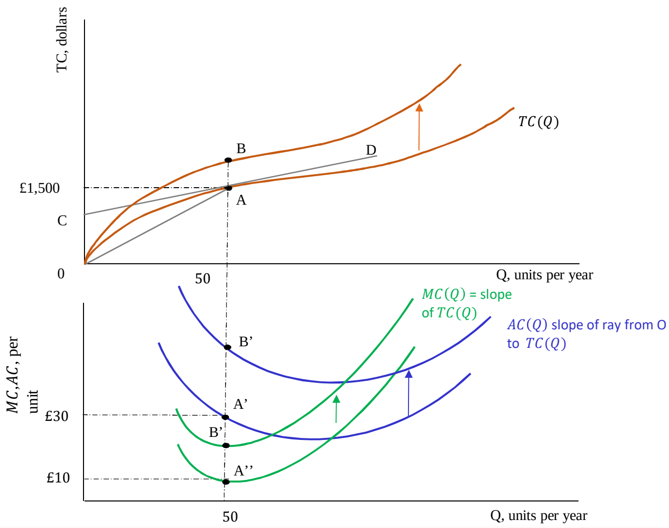
$$C_1 = C_2 < C_3$$

Shifting cost curves II



- Coal fields in Pennsylvania and West Virginia were opened in 19th century, cutting the price of coal
  - Iron producers substituted coal for wood
  - Fuel and cost curves for iron output shifted downwards
- Cyberspace: acquiring information is now relatively cheaper with 'IT'
- This tilts LRTC down and cuts the cost for producing a given level of output (A to B)
  - Or level of output increases for the same costs
- This may partly explain the breaking up of big companies from mid-1970s as the cost of information for smaller firms fell

Shifting cost curves III



- Slope of CD =Marginal Cost
- Slope of 0A = Average Cost
- An increase in costs
  - Tilts TC(Q) upwards
  - AC shifts upwards
  - MC shifts upwards
- The relationship between MC and AC is such that:
  - When AC is decreasing in Q, $AC(Q) > MC(Q)$
  - When AC is increasing in Q, $AC(Q) < MC(Q)$
  - When AC is at a minimum, $AC(Q) = MC(Q)$

Costs in the short run: Fixed and variable costs

- In the short run, the firm faces constraints in its ability to vary the quantity of some inputs. We will consider a case where the amount of capital the firm can use is fixed in the short run. We can rewrite the firm's total cost such that:

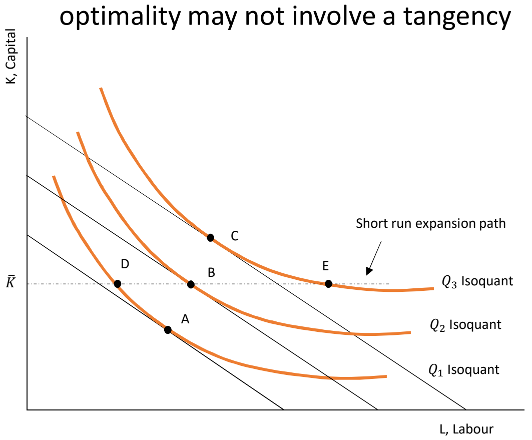
$$TC = wL + r\overline{K}$$

Where $\overline{K}$ represents the fixed amount of capital

- The cost of labour constitutes the firm's total variable cost
  - It will vary as the firm chooses to produce more or less output
- The cost of capital constitutes the firm's total fixed cost
  - It will not vary as the firm produces more or less output

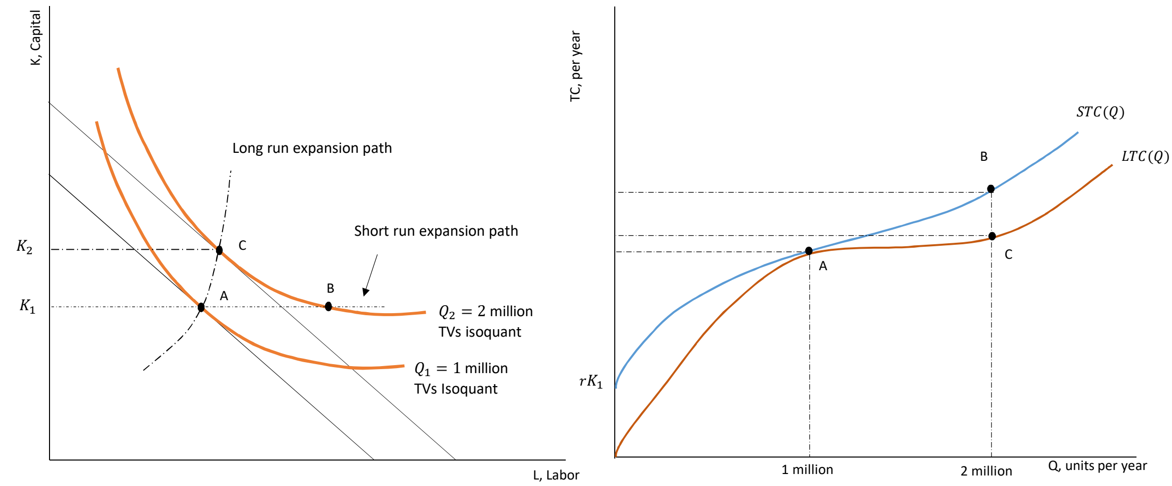
Cost minimization in the short run I

SR: firms can't substitute between inputs: optimality may not involve a tangency

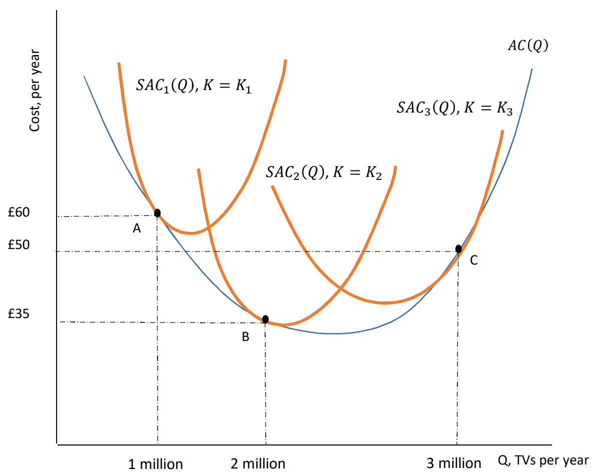


- LR: optimise at A, B, C (min. cost)
- SR: $\overline{K}$ units of capital must be used to produce any output
  - D (E): only technically efficient combination of inputs to produce Q_1 (Q_3): combining minimum quantity of L with fixed $\overline{K}$
  - To produce Q_2, the firm incurs the same costs in long run and in short run
- Combination of inputs used in SR and LR tend to be different
- Cost are typically higher in the SR than in the LR

Relationship between the long run and short run total cost curves



The long run average cost curve as an envelope curve



- The long run average cost curve forms a boundary around the set of shot run average cost curves corresponding to different levels of output and fixed input.
- Each short run average curve corresponds to a different level of fixed capital.
- Point A is optimal for the firm to produce 1 million TVs per year, with fixed level of capital K_1.

Profit Maximisation

Profit Maximisation: 2 step problem

- Minimise costs for a given output level (already done)
- Choose output to maximise π (i.e. revenue minus costs)

- Step 2 involves: $max_q \pi = pq - C(w,r,q)$
- This is an unconstrained maximisation problem

$$FOC: \quad \frac{\partial \pi}{\partial q} = p - \frac{\partial C(w,r,q)}{\partial q} = 0 \qquad \gg \qquad p = \frac{\partial C(w,r,q)}{\partial q} = MC(q)$$

- If revenue from last unit exceeds cost from last unit, then produce more
- As $\frac{\partial MC(q)}{\partial q} \geq 0$ MC must slope upwards at the optimum
- Solving yields optimal output: $q^*(w,r,p)$
- And Profit: $\pi(p,w,r) = pq^* - C(w,r,q^*)$

Two Step problem: $f(L,K) = L^{1/3}K^{1/3}$

- We already know the cost function for $f(L,K) = L^{\alpha}K^{\beta}$

$$\text{TC} = wQ^{\frac{1}{\alpha+\beta}}\left(\frac{\alpha r}{\beta w}\right)^{\frac{\beta}{\alpha+\beta}} + rQ^{\frac{1}{\alpha+\beta}}\left(\frac{\beta w}{\alpha r}\right)^{\frac{\alpha}{\alpha+\beta}} \qquad \text{Now set} \quad \alpha = \beta = {}^{1}\!/_{3}$$

$$\text{TC} = wQ^{\frac{3}{2}}\left(\frac{\alpha r}{\beta w}\right)^{\frac{1}{2}} + rQ^{\frac{3}{2}}\left(\frac{\beta w}{\alpha r}\right)^{\frac{1}{2}} = 2Q^{\frac{3}{2}}(rw)^{\frac{1}{2}}$$

- We want to maximise profits: $\pi = pq - C(w,r,q) = pq - 2Q^{\frac{3}{2}}(rw)^{\frac{1}{2}}$

$$FOC: \qquad p = MC \qquad \gg \qquad p = 3Q^{\frac{1}{2}}(rw)^{\frac{1}{2}}$$

$$Q^{*}(p,r,w) = \frac{p^2}{9rw} \qquad and \qquad \pi^{*}(p,r,w) = \frac{p^3}{27rw}$$

Remember from earlier: $Q = L^{\alpha}K^{\beta}; P_L = w; P_K = r$

$$\nabla = wL + rK + \lambda(Q_0 - L^{\alpha}K^{\beta})$$

$$\frac{\partial\nabla}{\partial L} = w - \lambda\alpha L^{\alpha-1}K^{\beta} = 0$$

$$\frac{\partial\nabla}{\partial L} = r - \lambda\beta L^{\alpha}K^{\beta-1} = 0$$

$$\frac{w}{r} = \frac{\alpha}{\beta}\frac{K}{L} \gg K = L\frac{w}{r}\frac{\beta}{\alpha}$$

$$\frac{\partial\nabla}{\partial\lambda} = Q_0 - L^{\alpha}K^{\beta} = 0$$

- Solve for: $\quad L^{*} = Q_0^{\frac{1}{\alpha+\beta}}\left(\frac{\alpha r}{\beta w}\right)^{\frac{\beta}{\alpha+\beta}}$

- Plug expression for L* into $K = L\frac{w}{r}\frac{\beta}{\alpha}$ and solve for: $\quad K^{*} = Q_0^{\frac{1}{\alpha+\beta}}\left(\frac{\beta w}{\alpha r}\right)^{\frac{\alpha}{\alpha+\beta}}$

- **Cost function: $\text{TC} = wQ_0^{\frac{1}{\alpha+\beta}}\left(\frac{\alpha r}{\beta w}\right)^{\frac{\beta}{\alpha+\beta}} + rQ_0^{\frac{1}{\alpha+\beta}}\left(\frac{\beta w}{\alpha r}\right)^{\frac{\alpha}{\alpha+\beta}}$**

Profit Maximisation: One Step problem

- L and K used to produce Q = f(L, K) (priced at p), with $P_L = w$; $P_K = r$
  - Assume firm is a price taker in both input and output markets
- Firm's profit maximisation problem involves choosing Q, L, K to:

$$\max \pi = pq - wL - rK = pf(L, K) - wL - rK$$

- This is another unconstrained maximisation problem

$$FOC: \quad p\frac{\partial f(L,K)}{\partial L} - w = 0 \qquad and \qquad p\frac{\partial f(L,K)}{\partial K} - r = 0$$

- These yield the optimal input demands: $L^*(p, w, r) \quad and \quad K^*(p, w, r)$
- We can then derive the supply function: $Q^*(p, w, r) = f(L^*K^*)$
- And the Profit function: $\pi^* = pq^* - wL^* - rK^*$

Same problem: One-Step: $f(L, K) = L^{1/3}K^{1/3}$

$$\pi = pq - wL - rK = pL^{\frac{1}{3}}K^{\frac{1}{3}} - wL - rK$$

FOCS:

$$\frac{\partial \pi}{\partial L} = \frac{1}{3}PL^{-\frac{2}{3}}K^{\frac{1}{3}} = w \quad (1) \qquad and \qquad \frac{\partial \pi}{\partial K} = \frac{1}{3}PL^{\frac{1}{3}}K^{-\frac{2}{3}} = r \quad (2)$$

Use (1) to find expression for K: $K = 27\left(\frac{w}{p}\right)^3 L^2$ (3)

Plug (3) into (2) to find optimal L^*:

$$\frac{1}{3}PL^{\frac{1}{3}}K^{-\frac{2}{3}} = r \quad \gg \quad \frac{1}{3}PL^{\frac{1}{3}}[27\left(\frac{w}{p}\right)^3 L^2]^{-\frac{2}{3}} = r \quad \gg$$

$$L^* = \frac{p^3}{27w^2r} \qquad (4)$$

One step problem: $f(L,K) = L^{1/3}K^{1/3}$

- To find optimal K^*, plug equation (4) into equation (3)

$$K = 27\left(\frac{w}{p}\right)^3 L^2 = 27\left(\frac{w}{p}\right)^3 \left[\frac{p^3}{27w^2r}\right]^2$$

$$K^* = \frac{p^3}{27wr^2} \qquad (5)$$

- To find optimal Q^*: plug optimal inputs (4), (5) into production function

$$Q = L^{1/3}K^{1/3} = \left(\frac{p^3}{27w^2r}\right)^{\frac{1}{3}}\left(\frac{p^3}{27wr^2}\right)^{\frac{1}{3}} \gg \quad Q^* = \frac{p^2}{9wr} \qquad (6)$$

- This is the supply function
  - Compare it with optimal Q from two step problem
  - They are identical

One step problem: $f(L,K) = L^{1/3}K^{1/3}$

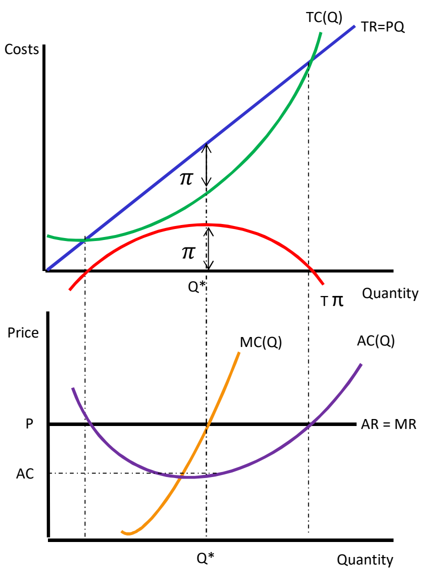
- Use (4), (5) and (6) to find Profit function

$$\pi^* = pQ^* - wL^* - rK^* = p\frac{p^2}{9wr} - w\frac{p^3}{27w^2r} - r\frac{p^3}{27wr^2}$$

$$\pi^*(p,w,r) = \frac{p^3}{27wr}$$

- Compare this with the profit function derived from two-step solutions
  - Again, it is identical
- Once we know the price of labour and capital, we can then solve to find how much will be supplied at each price and how much profit the firm will make

Profit and Supply Functions



Profit function

- Profit is maximised at Q* where:
  $MC = MR$
- Profits are given by:
  $\pi = pQ^* - c(Q^*)$
- Total profit on top diagram is:
  $\pi = TR - TC$
- Total profit on bottom diagram is:
  $\pi = [p - AC(Q^*)]Q^*$
- $\pi(p, r, w)$ increases in p and decreases in w, r
- $\pi(p, r, w)$ is homogeneous of degree 1 in p, r, w

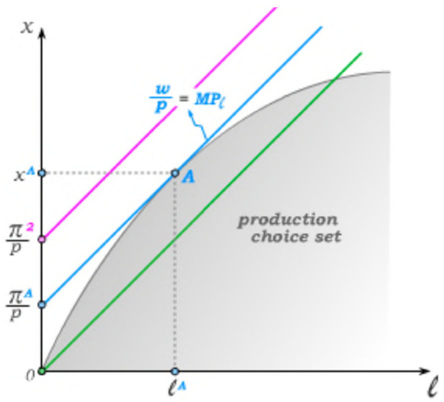
Isoprofit curves I

- When more of one input is used, ΔL, output rises by $\Delta Q = MP_L \Delta L$
- The value of this extra output is $\mathrm{p}MP_L\Delta L$ (where p = price of output)
- The cost of this additional output is: $\mathrm{w}\Delta L$ (where w = price of labour)
- If the value of using one extra L is greater than its cost, profits rise if one more L is used
- When profits are at a maximum, any change in how much L is used will cause profits to fall.
- At a profit maximising choice of inputs, the value of the marginal product of labour should equal the price of labour

$$\mathrm{p}MP_L(L^*, \overline{K}) = w$$

Isoprofit curves II

Assume K is held fixed and x denotes firm's output



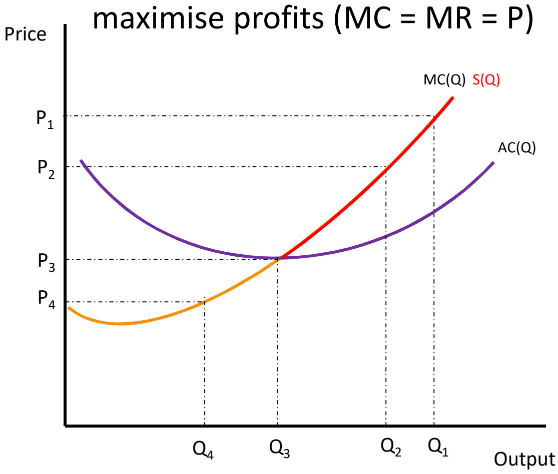
- Profits are: $\pi = px - wL - r\overline{K}$
- Solving yields: $x = \frac{\pi}{p} + \frac{r}{p}\overline{K} + \frac{w}{p}L$

  which describes the isoprofit lines
  - All combinations of the input goods and output good that give the same level of profit
  - A bit like a firm's indifference curves
  - Slope = $\frac{w}{p}$ Vertical intercept = $\frac{\pi}{p} + \frac{r}{p}\overline{K}$

  Which measures profits plus fixed costs
  - As fixed costs are fixed, the only thing that changes as we move between isoprofit curves is the level of profit (higher profit, higher isoprofit curve)
- Profit maximisation: point on production function with highest isoprofit curve

Supply I

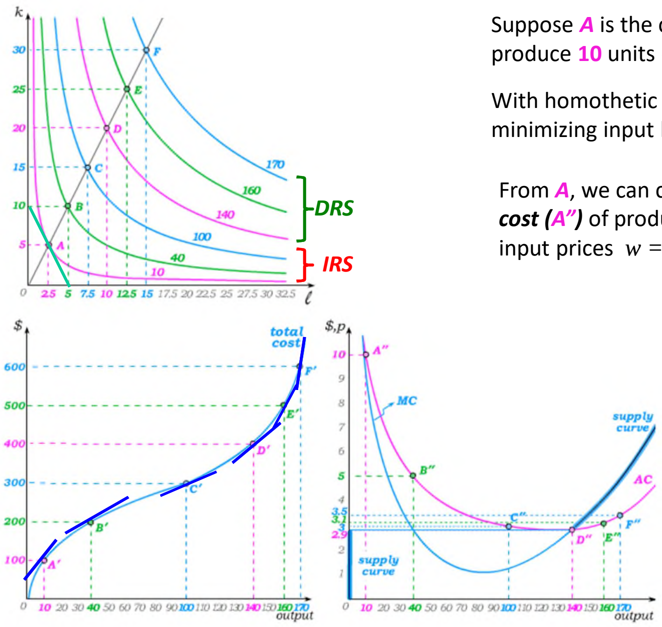
We assume firms aim to maximise profits (MC = MR = P)



- P_1: $\mathrm{P} > AC \;\gg Q = Q_1 \;\gg \pi > 0$
- P_2: $\mathrm{P} > AC \;\gg Q = Q_2 \;\gg \pi > 0$
- P_3: $\mathrm{P} = AC \;\gg Q = Q_3 \;\gg \pi = 0$
- P_4: $\mathrm{P} < AC \;\gg$
  - Firm doesn't respond by producing Q_4
  - When $\pi < 0$ the firm produces $\mathrm{Q} = 0$
- MC curve shows how much will be produced at any given price
- But if:

  $TR < TC \text{ or } P < AC; Q = 0$

Recall the slide from earlier… (or if not, here it is!!!)



Suppose ***A*** is the cost-minimizing input bundle to produce **10** units of output.

With homothetic technologies, this implies ***all*** cost-minimizing input bundles lie on the same ray.

From ***A***, we can calculate the ***cost (A')*** and ***average cost (A'')*** of producing 10 units of output (given input prices w =20 and r =10).

And repeating this for all other output quantities, we get the *(total)* ***cost*** and ***average cost curves***.

The slope of the (total) *cost* curve becomes the ***marginal cost curve***.

The MC curve that lies above AC becomes the firm's supply curve

Supply II

- Long run Supply (MC above AC)
- Short run Supply (MC above AVC) - only needs to cover variable costs
- Shape of supply depends on: shape of MC and whether there are FC
- The supply function is homogeneous of degree 0 (in p, r, w)
  - If prices double, profit equation scales up, so optimal output is unaffected
- Supply will slope upwards
- Long run supply will be flatter than short run supply, due to fixed input, capital

Conclusion: Combining topics 1 and 2

- In topic 1, we looked at the consumption decision
  - We derived consumer demand
- In topic 2, we have considered the production decision
  - We have derived producer supply
- We have seen that both the demand and supply functions depend on the market price of the good
- Combining these theories allows us to understand the fundamental workings of a competitive market
  - The interaction between consumers and producers
  - How equilibrium is determined in a market via the price mechanism which sends signals between the two sides