

# **EC130 FOUNDATIONS OF ECONOMIC ANALYSIS**

2004-2005

DEPARTMENT OF ECONOMICS

UNIVERSITY OF WARWICK

## **Topic 4**

### **Supply Analysis: Revenues and Costs**

Elasticity Revisited

Average, Marginal and Total Revenues

Average, Total and Marginal Costs

Economies and Diseconomies of Scale

Short-run and Long-run Decisions

Profit-Maximisation

***Learning objectives*** - what you should know and understand after this part of the lecture course:

- (i) The relationship between demand and the firm's revenue curves
- (ii) How to derive cost curves from production functions
- (iii) The relationship between short-run marginal cost and returns to labour
- (iv) The difference between long-run and short-run costs
- (v) The concept of profit maximisation

OHP47

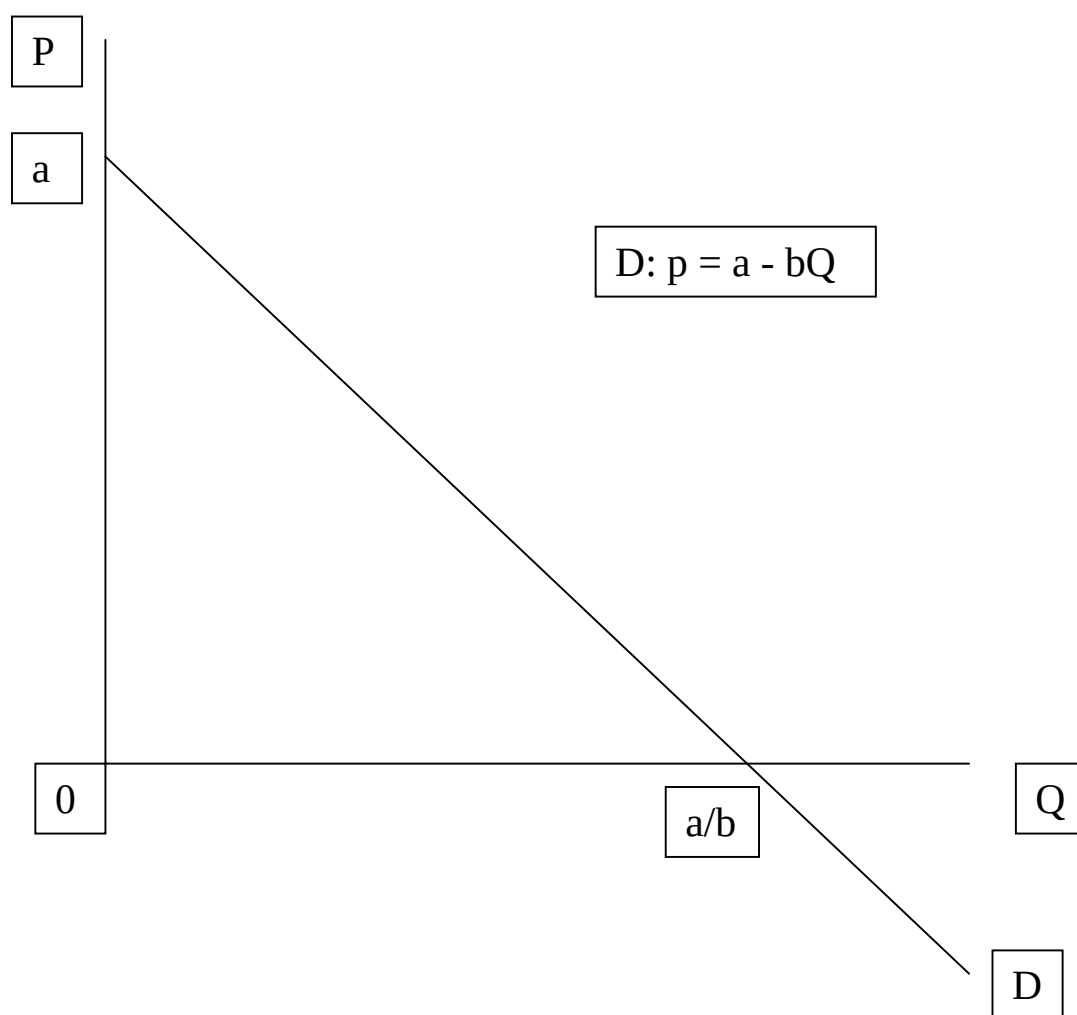
TOPIC 4

**1. Revenues**

Assume the firm faces a downward-sloping demand curve.

We can ask, “At what point will the firm choose to locate itself along this demand curve? I.e., what will be its chosen price-quantity combination?”

Figure 4.1



### OHP48

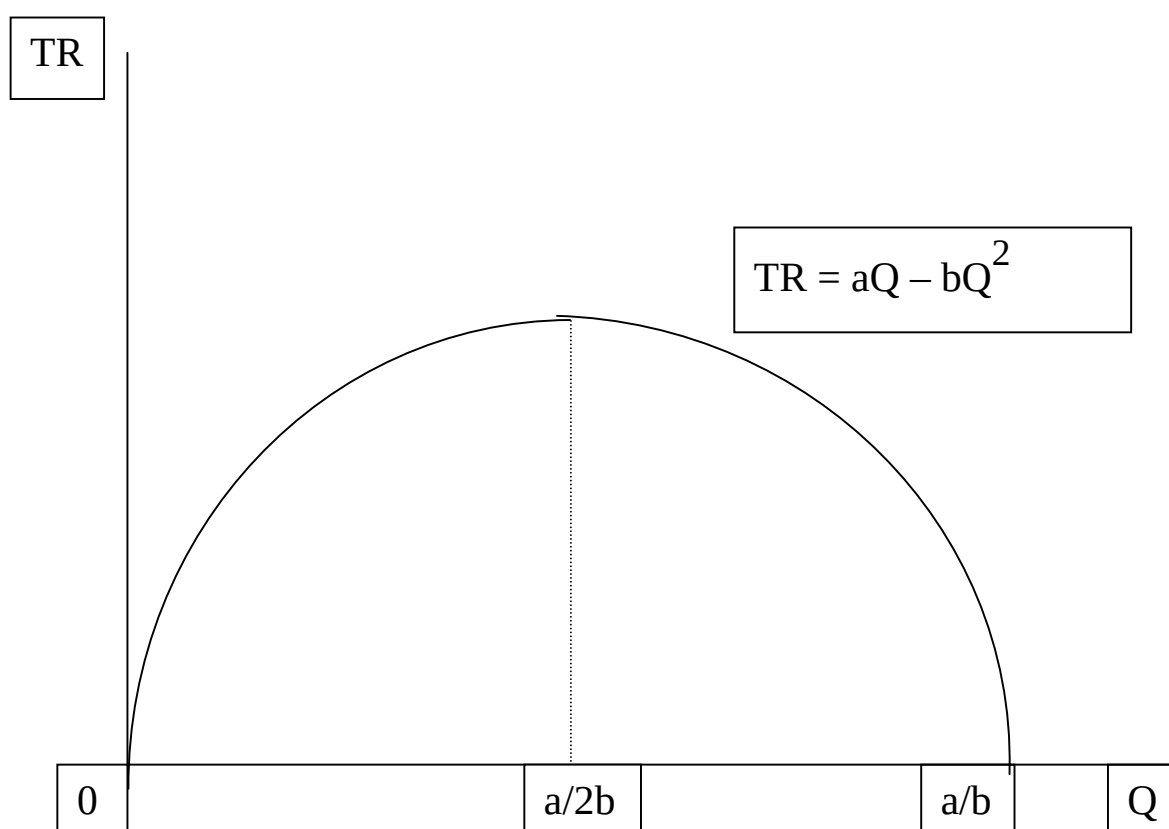
Algebraically,

$$p = a - bQ \quad (\text{the 'inverse' demand curve})$$

$$TR = \text{price} \times \text{quantity} = pQ = (a - bQ)Q$$

$$\Rightarrow TR = aQ - bQ^2$$

Figure 4.2



(SEE PAGE 4 FOR EXTRA DISCUSSION)

Note that the slope of the total revenue curve measures the ‘marginal revenue’. It tells us by how much total revenue rises for a one-unit increase in quantity.

When total revenue is maximised, marginal revenue (i.e. the slope of the total revenue curve) is zero.

Suppose the firm wants to choose output to maximise total revenue. When output is close to zero, an increase in output (i.e. sales) will cause TR to rise (i.e. the marginal revenue is positive) and hence the firm will want to increase output. This will continue whilst ever the MR is positive. Once MR is zero, the firm will not want to raise output further as to do so causes MR to become zero: i.e. TR falls as output expands further.

The MR can be written as:  
 $MR = a - 2bQ$ .

And it can be drawn in  
Figure 1.

Can you work out what it  
looks like?

*Optional mathematical  
interpretation:*

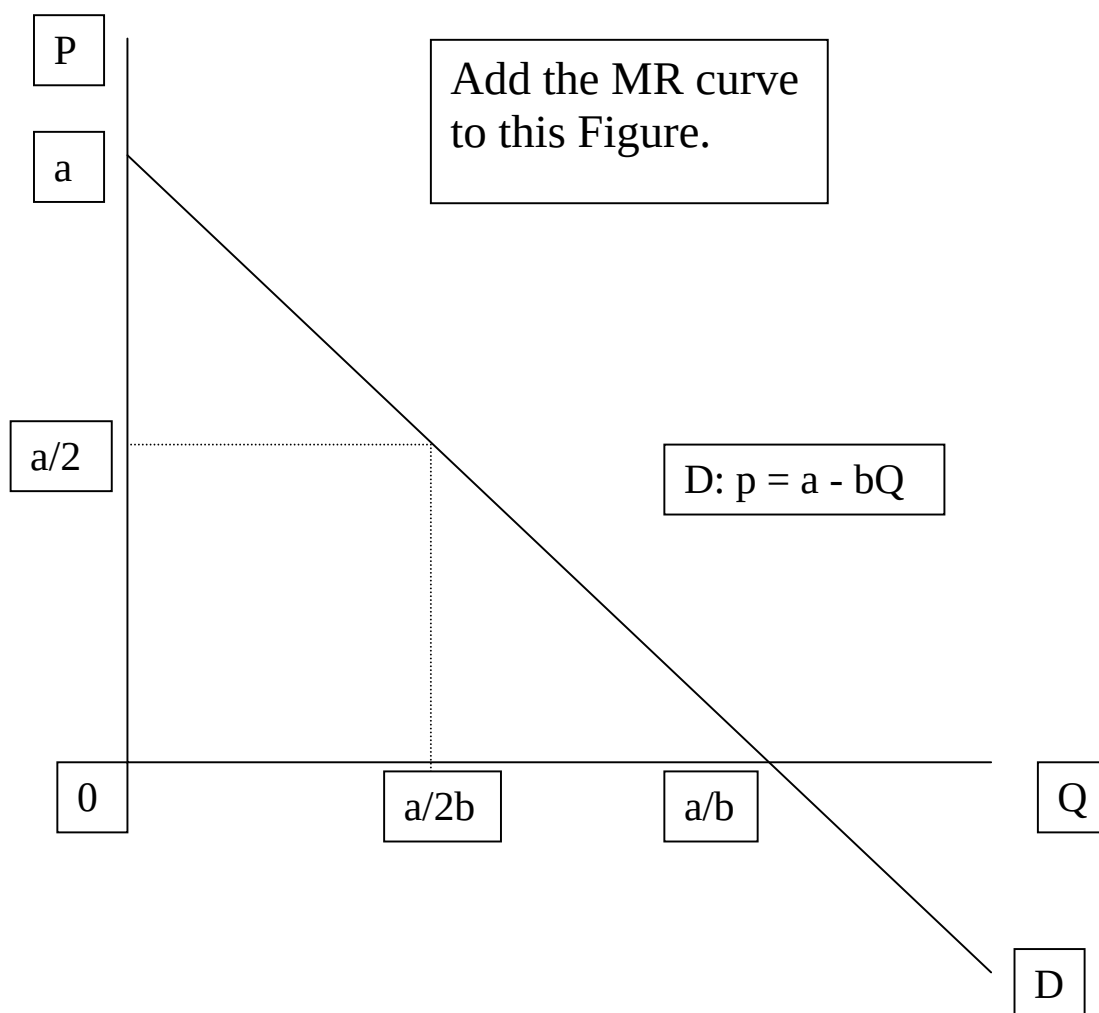
$$MR = dTR/dQ = a - 2bQ$$

$$MR = 0 \Rightarrow Q = a/2b$$

So total revenue is maximised when  $Q = a/2b$ , i.e. half-way between the origin and where the demand curve cuts the Q-axis. Hence,  $p = a/2$  when total revenue is maximised. This is shown in Figure 4.3.

## OHP49

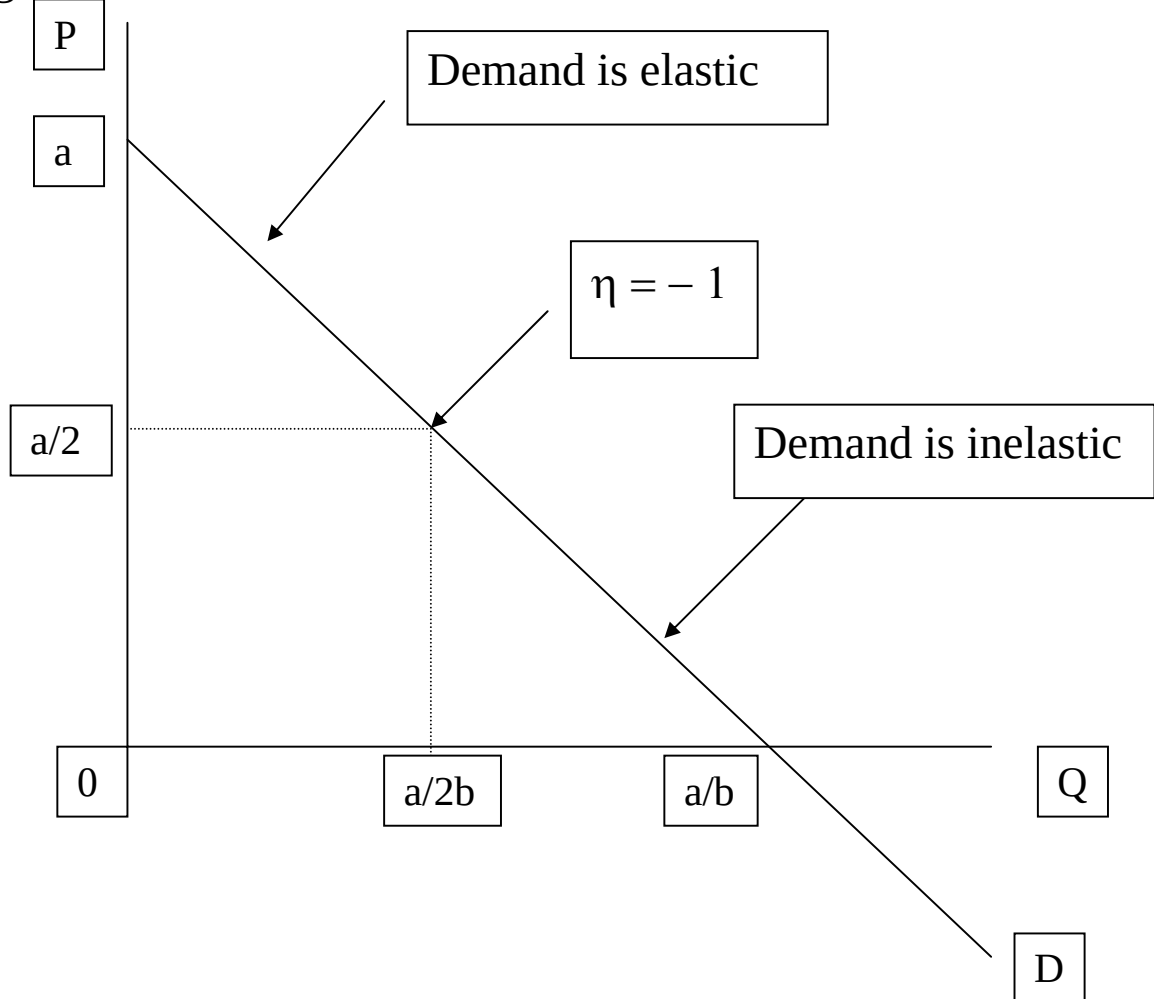
Figure 4.3



We shall now show that at the point at which TR is maximised (ie where  $MR=0$ ), the price elasticity of demand is unity ( $\eta = -1$ ). First see Figure 4.4.

## OHP50

Figure 4.4



Suppose price is high (e.g. above the level  $p = a/2$ ). Then demand is elastic. Hence, a reduction in price will lead to a disproportionately large increase in demand. The firm's total revenue will increase. Conversely, when demand is inelastic, a fall in price will lead to a disproportionately small increase in demand and total revenue will fall. Total revenue is thus maximised when  $\eta = -1$ .

To see the details of all this, notice that a cut in price has two effects on total revenue.

On the one hand, each unit is now sold at a lower price and this will tend to reduce the firm's TR.

On the other hand, the lower price means that firms can sell more units and this will tend to raise revenue.

When demand is elastic the former effect dominates, and when demand is inelastic, the latter effect is dominant.

When  $\eta = -1$ , the two effects just cancel out. MR is zero and TR is maximised.

### ***OPTIONAL EXTRA***

*This can be demonstrated more formally:*

$$\text{TR} = \text{price} \times \text{quantity} = p \cdot Q$$

Remember also that price itself is a function of quantity (the price you can charge depends upon how much you are trying to sell). Thus,

$$\text{TR} = p(Q) \cdot Q$$

$$\Rightarrow \text{MR} = d(\text{TR})/dQ = p \cdot dQ/dQ + Q \cdot dp/dQ$$

$$\Rightarrow \text{MR} = p + Q \cdot dp/dQ.$$

Think now about the ECONOMICS of this.

We now show more formally that  $\eta = -1$  when  $MR = 0$ .

We then show the way in which the elasticity changes as we move down along the straight line demand curve.

### OPTIONAL EXTRA

*Formal proof that  $\eta = -1$  when  $MR = 0$ .*

Recall that,

$$MR = p + Q.(dp/dQ) \quad (1)$$

and that,  $\eta = (dQ/dp).(p/Q)$

or,  $1/\eta = (dp/dQ).(Q/p)$

Now, multiply both sides by  $p$ ,

$$p/\eta = (dp/dQ).Q \quad (2)$$

Substituting (2) in (1),

$$MR = p + p/\eta$$

or

$$MR = p (1 + 1/\eta)$$

So, when  $\eta = -1$ ,  $MR = 0$ . QED

## OHP51

Why does the elasticity change as we move down along the (straight line) demand curve?

Recall that  $\eta = (dQ/dp).(p/Q)$

$dQ/dp$  does not change as we move down the demand curve.

Why?

Let  $p = a - bQ$ .

Then  $dp/dQ = -b$  is just the slope of the demand curve, and of course this is constant along a straight line by definition. Thus  $dQ/dp$  does not change either.

$p/Q$  (the term in the second pair of brackets in the definition of the elasticity) does change as we move down the demand curve, however. How?  $p$  falls and  $Q$  rises. Thus,  $p/Q$  falls.

Hence, the absolute size of  $\eta$  falls too. In other words, the demand curve becomes less elastic as we move down it. It must therefore be inelastic above the value  $p = a/2$  and inelastic below this value.

## OHP52

Let us now introduce COSTS into the analysis.

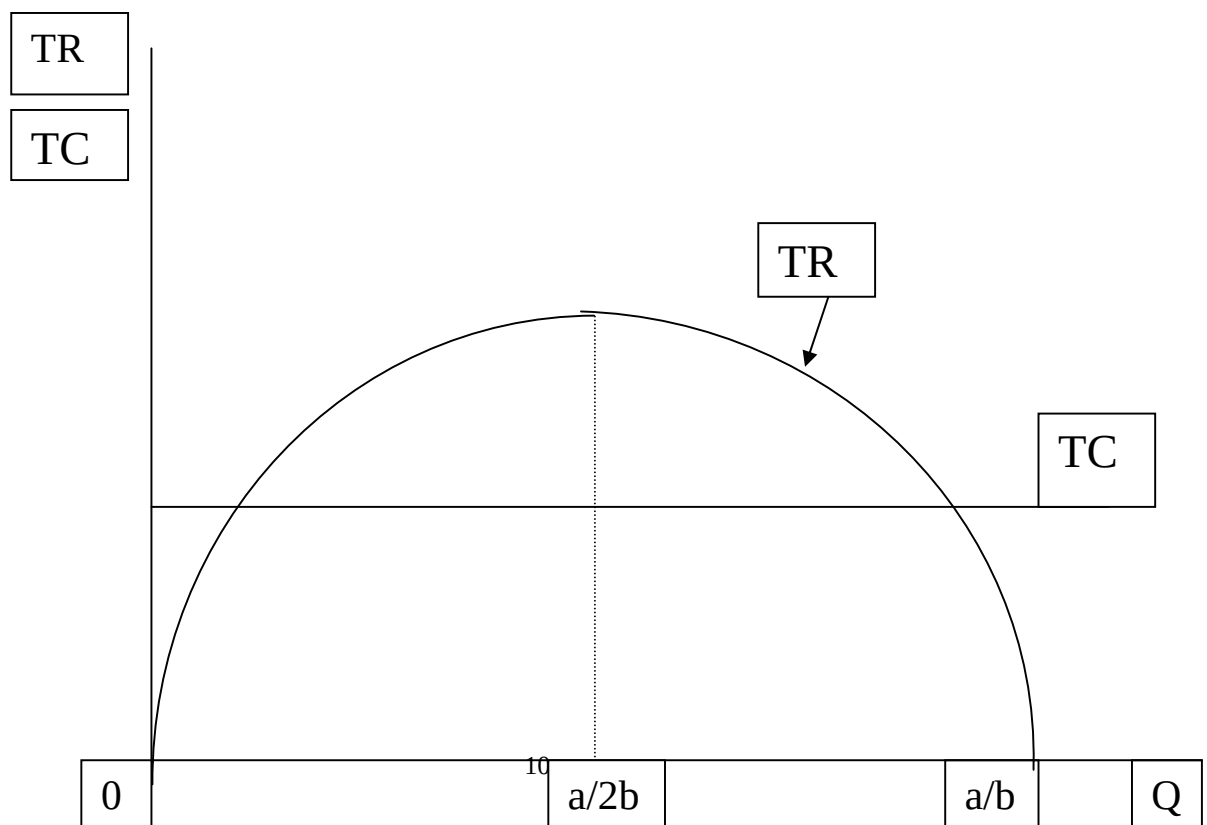
So far we have assumed that the firm is just interested in maximising TR, total revenue. But in reality, it is likely that firms want to maximise profits: which are given by total revenue *minus* total costs.

I.e.,  $\pi = TR - TC$

We know all we need to know about TR. But what about TC?

The simplest assumption would be that Total Costs are just some fixed amount: e.g., you are putting on a concert in the Students Union and your only costs are one thousand pounds for the band.

Figure 4.5



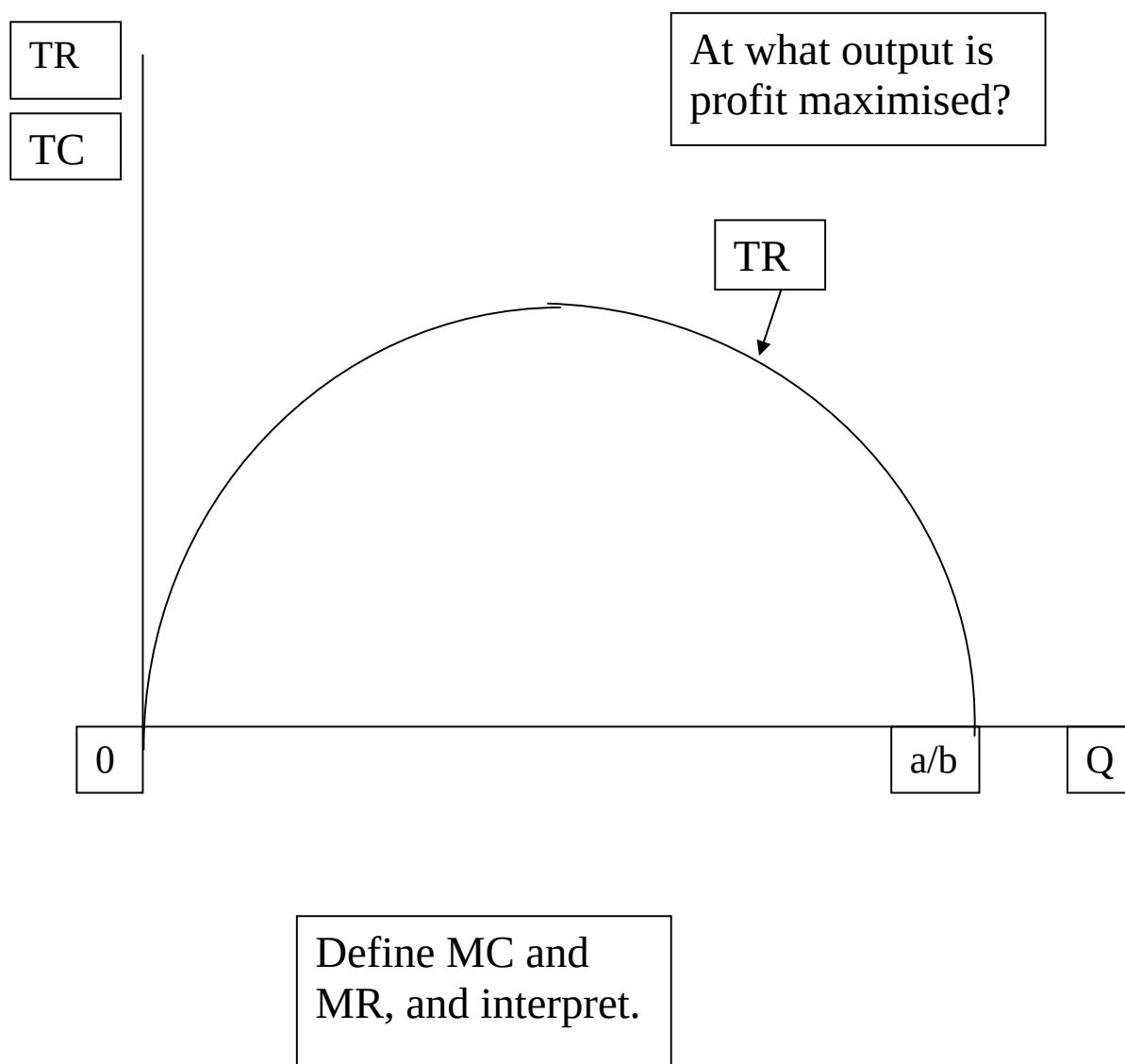
If Total Costs are constant (independent of the level of output) – as shown in Figure 4.5 – then what is the profit-maximising firm's choice of output? How does this differ from the level chosen by the TR-maximising firm we analysed earlier?

Suppose now that Total Costs are increasing in the level of output: as seems more realistic for most goods. E.g., if you want to produce more cars you probably have to hire more workers and buy in more components and other inputs. Suppose that there are no 'fixed' costs but only 'variable' costs and that each unit is produced at the same constant cost: e.g., ten thousand pounds per unit. How would we draw the Total Cost curve? Add it to Figure 4.6. And say where the new profit-maximising output level is.

## OHP53

Figure 4.6

Add the TC curve if costs are increasing.



The profit-maximising output level occurs when the distance between the TR and the TC curves is maximised (because  $\pi = TR - TC$ ).

In the diagram this is at the point on the TR curve at which the line parallel to the TC curve just touches (is a tangent to) the TR curve. (Be sure you can explain why this is.)

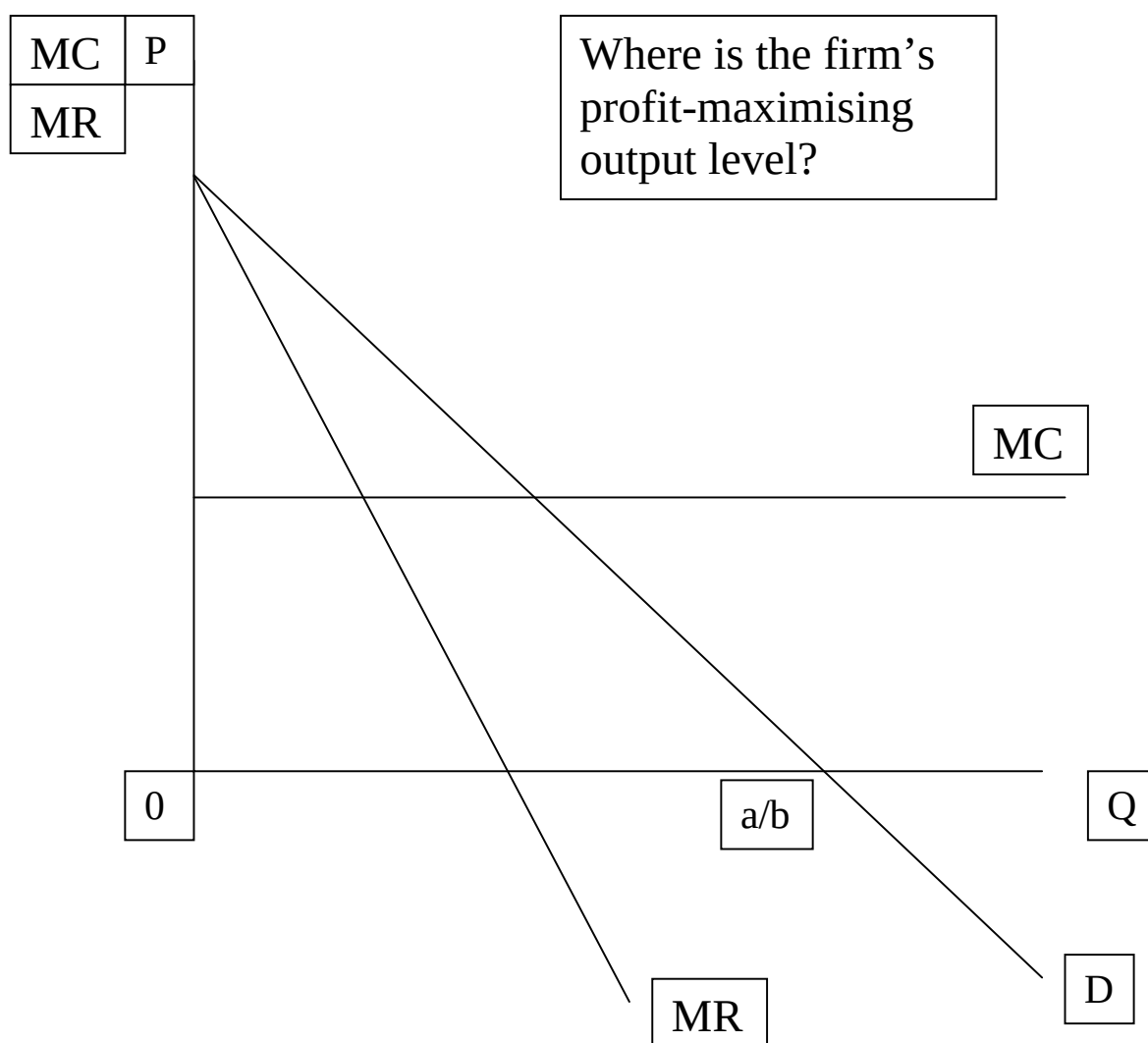
At this tangency point, the TC and TR curves have the same slope. The slope of the TR curve is given by the MR (we have already seen this). Similarly, the slope of the TC curve is given by the firm's 'Marginal Cost'. Be sure you can define this.

In Figure 4.6, as you should have developed it, the TC curve is a straight line through the origin. Its slope is constant because we have assumed that costs per unit are constant. In other words, the MC is constant: each extra unit costs the same to produce as each of the previous units produced.

We can show the profit maximising outcome in Figure 4.6 with TC and TR on the vertical axis. But it will be more convenient to use a diagram with MR and MC on the vertical axis. This is shown in Figure 4.7. (Note. In Figure 4.3 you should have already shown the MR curve in a diagram.)

# OHP54

Figure 4.7

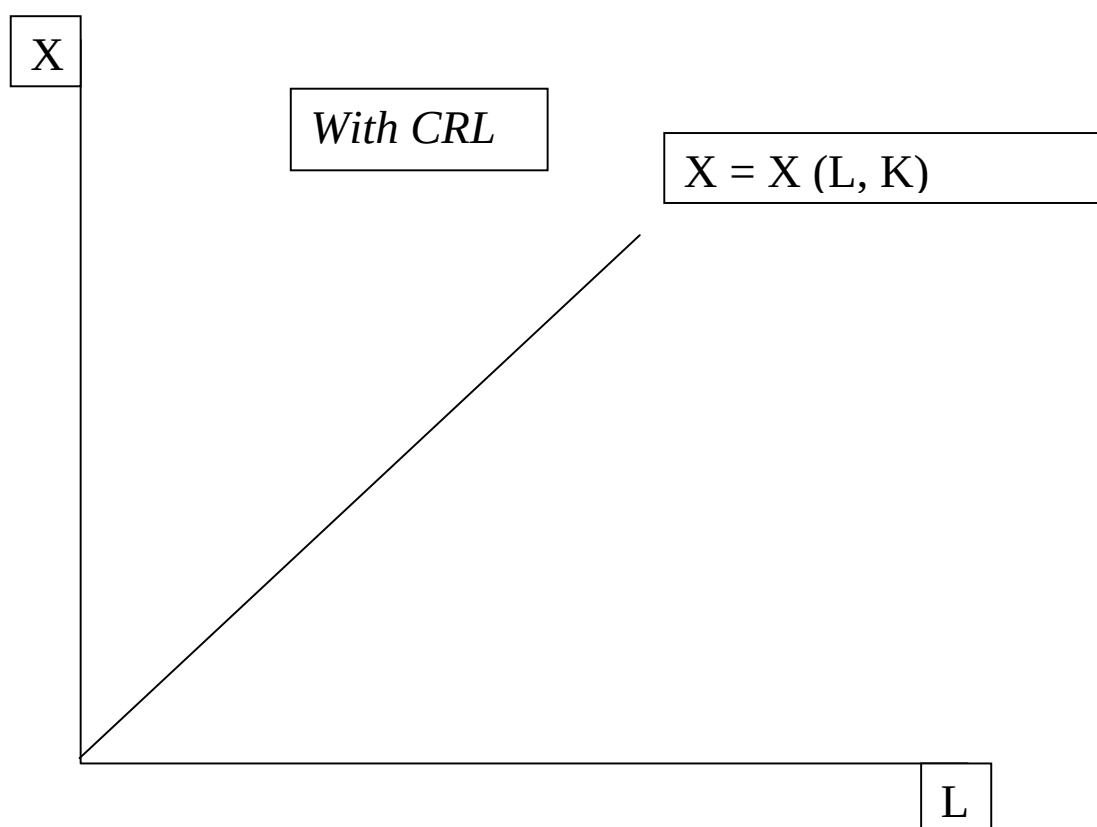


Explain why profits are maximised at the point (that in the lecture) we indicate in Figure 4.7.

## OHP55

So far we have assumed that the MC curve is constant. We now turn to examine what factors will determine the shape of the firm's cost curves.

Figure 4.8



$MPL = dX/dL =$  the change in output when one more unit of labour (e.g. one more worker) is employed.

If the MPL is constant, we say that there are constant returns to labour (CRL). We can show MPL in a different diagram:

### ***Production functions, production decisions and costs.***

We denote the production function by

$$X = X(L, K)$$

In the short-run, capital input  $K$  is fixed. Only labour input  $L$  is variable. A possible shape for the production function is shown in Figure 4b.1.

If  $L = 0$ , then  $X = 0$ . Why?

*The slope of the production function shows how much more output can be produced when one extra unit of labour input is employed. I.e., it shows the marginal product of labour:*

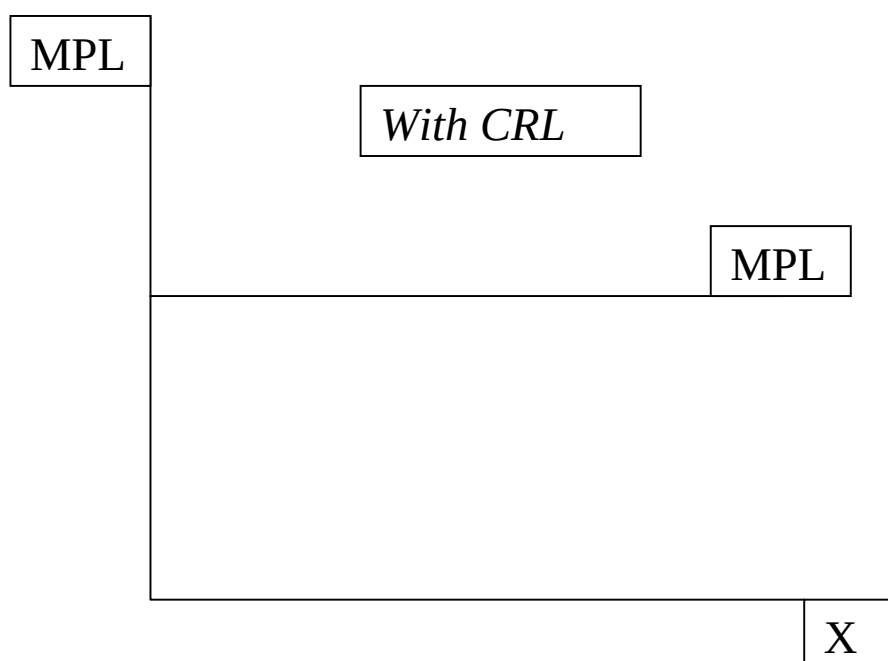
$MPL = dX/dL =$  the change in output when one more unit of labour (e.g. one more worker) is employed.

*We can easily show this in Figure 4.8. Similarly, it is easy to show in the figure that if the production function is linear, then this means that the marginal product of labour is constant (i.e., it doesn't change according to the level of labour employed). Check you can see this from the figure.*

If the MPL is constant, we say that there are constant returns to labour (CRL). We can show MPL in a different diagram:

## OHP56

Figure 4.9



In Figures 4.8 and 4.9 we assumed CRL.

But there are two obvious alternative assumptions about the MPL.

First, under Decreasing Returns to Labour (DRL) the MPL of labour falls as output increases (see Figures 4.10 and 4.11). [Notice how the production function gets flatter as  $X$  increases in Figure 4.10]

Second, under Increasing Returns to Labour (IRL) the MPL of labour rises as output increases (see Figures 4.12 and 4.13). [Notice how the production function gets steeper as  $X$  increases in Figure 4.14]

OHP57

Figure 4.10

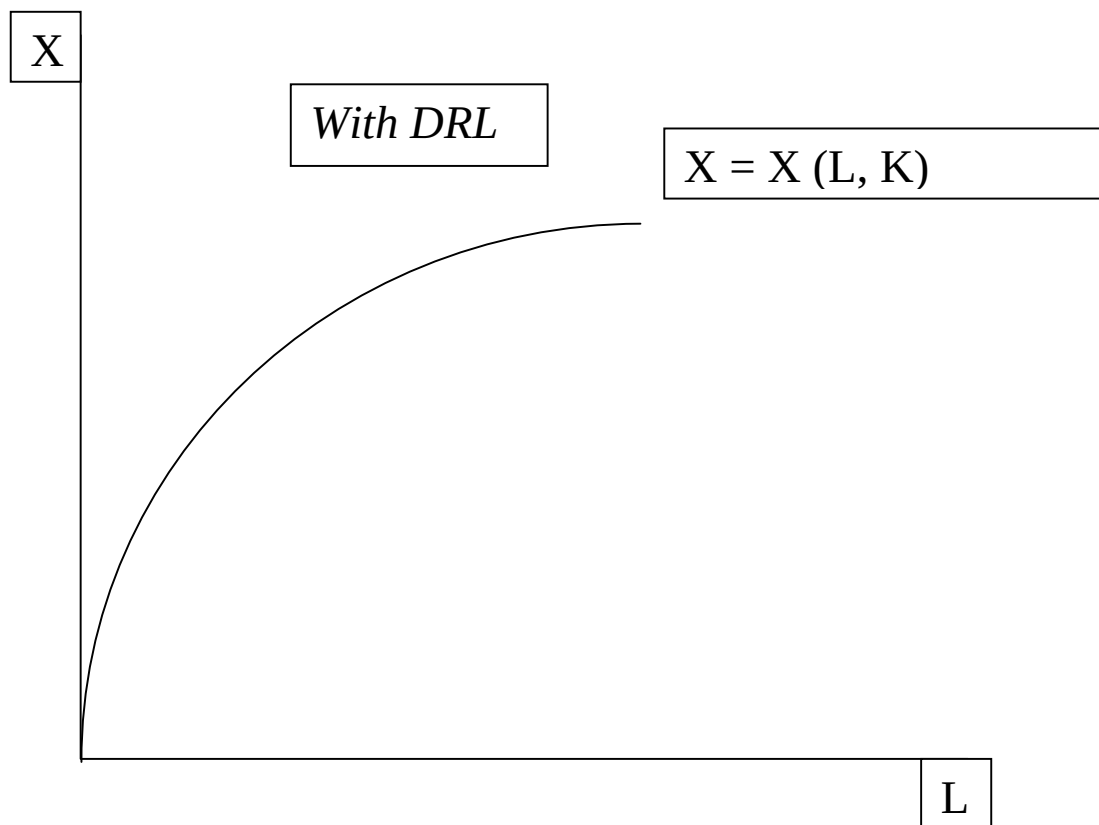
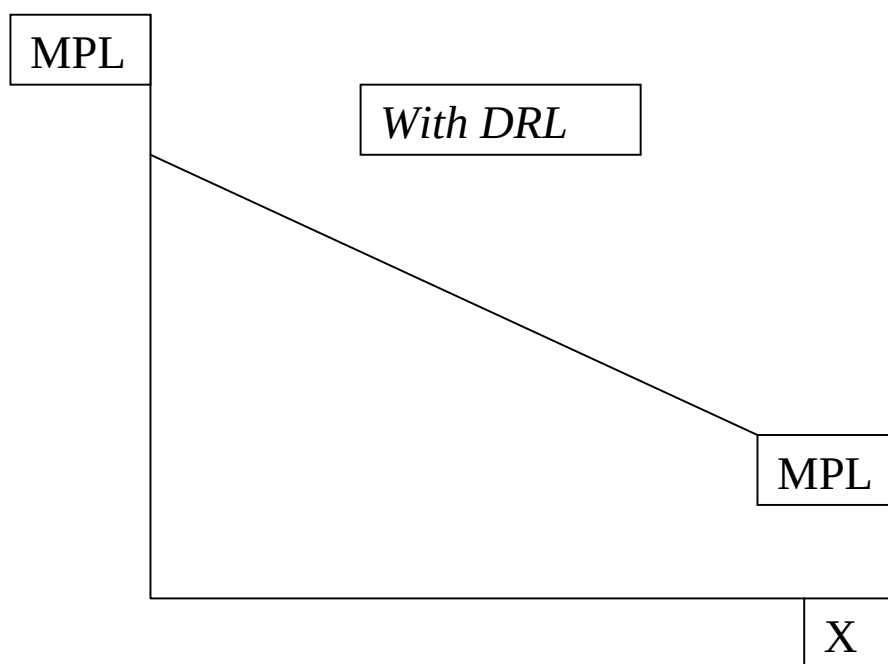


Figure 4.11



OHP58

Figure 4.12

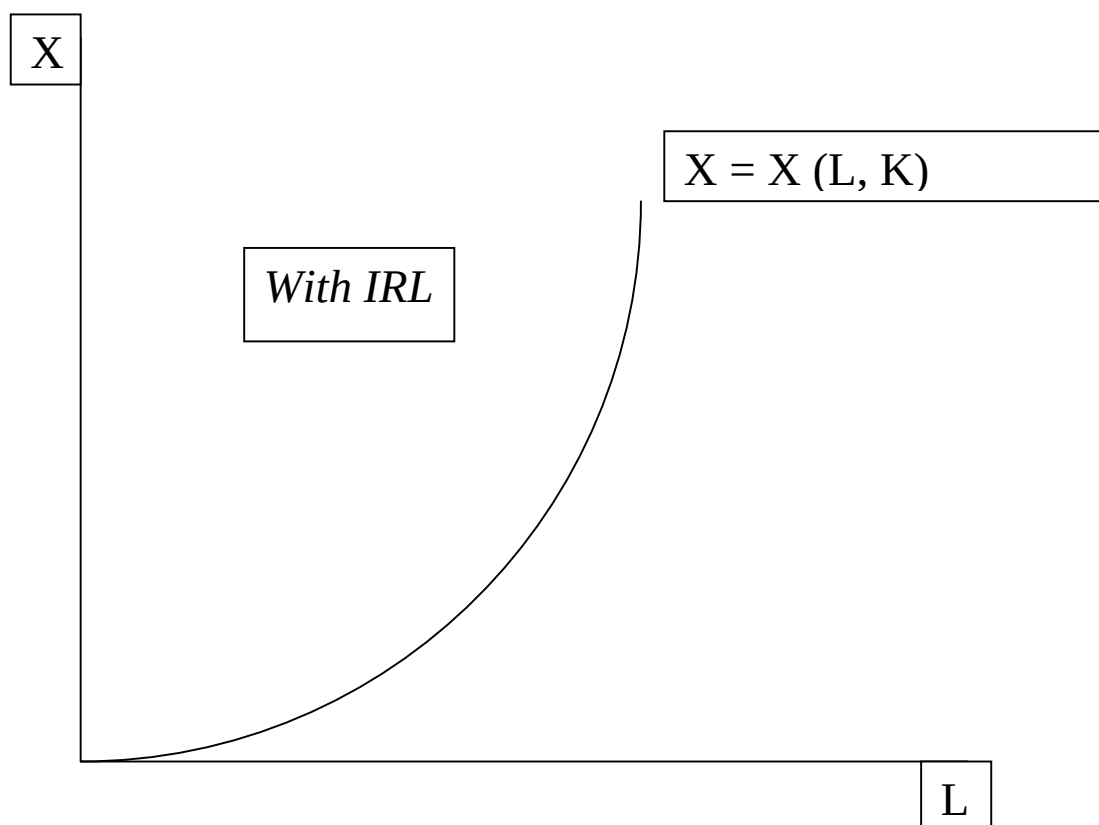
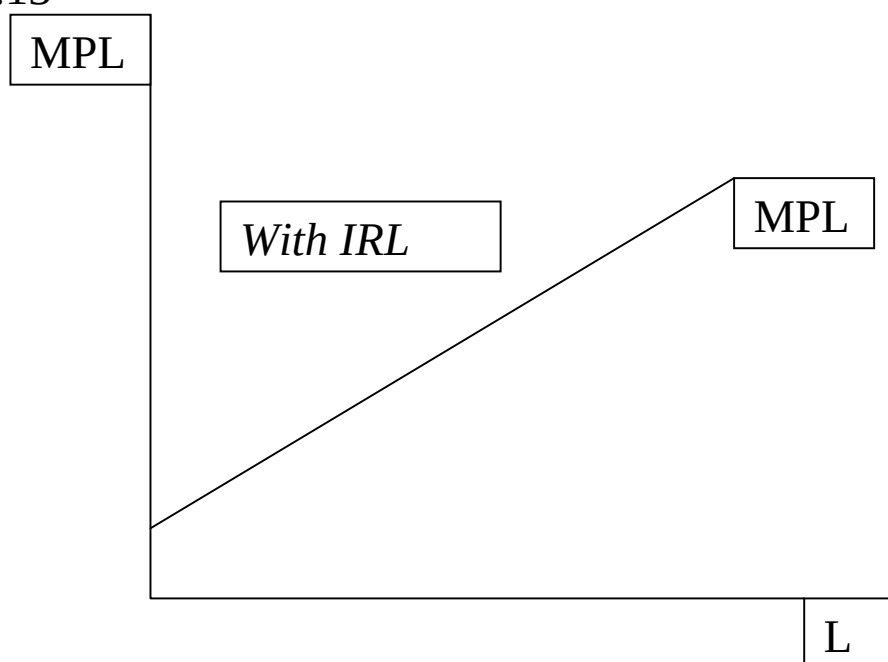


Figure 4.13



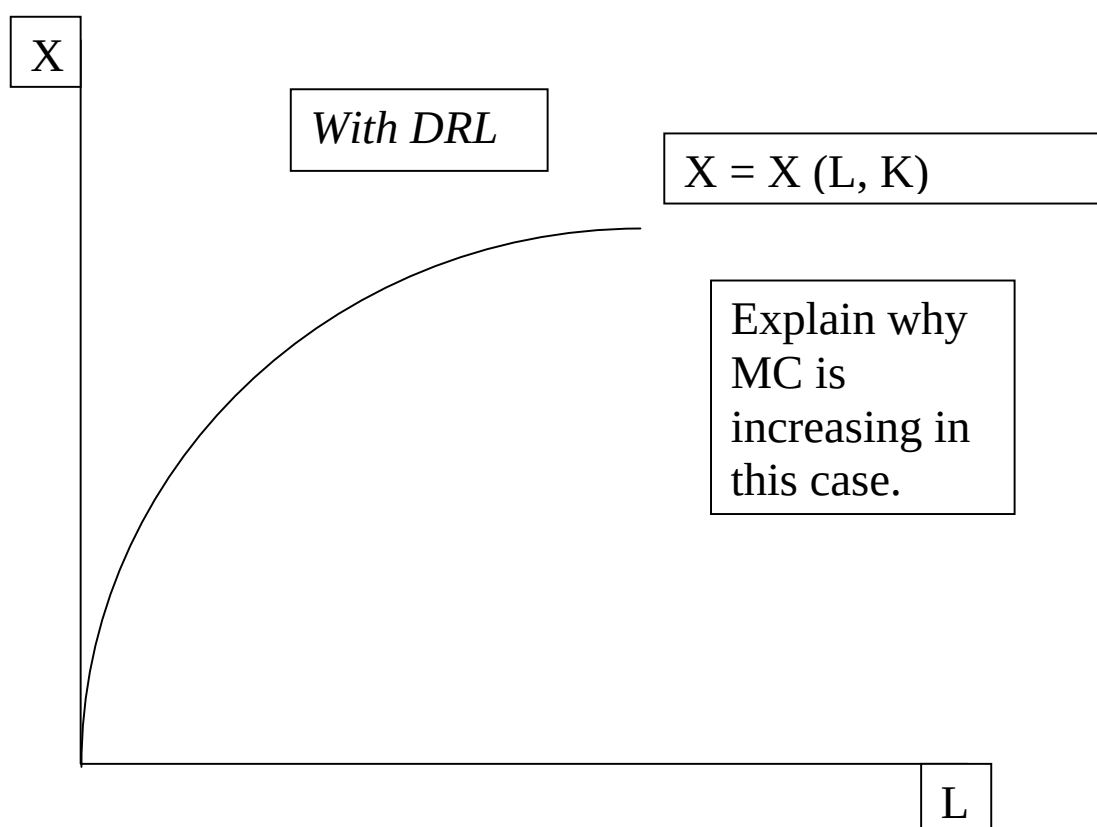
## OHP59

Why is the shape of the production function, and hence of the MPL, so important?

Because they determine the shape of the total cost and marginal cost curves that we met earlier, at least those which pertain to the short-run when the amount of capital,  $K$ , is fixed. Let's see why this is.

We are going to start by assuming DRL. Then the production function is as shown as in Figure 4.10 above: i.e.,

Figure 4.14



Suppose  $L$  (and hence  $X$ ) are initially at a low level and the firm wants to increase output by a given amount. How much extra  $L$  does it need? Show this in the figure.

Suppose now that  $L$  (and hence  $X$ ) are initially at a high level and the firm wants to increase output by the same given amount. Now how much extra  $L$  does it need? Show this in the figure also.

When output is low, the given increase in output, say  $dX_1$ , requires an increase in labour input measured by  $dL_1$ . By how much will this increase in output raise the total costs of the firm? I.e., what will be the Marginal Cost,  $MC$ , of the increase in output  $dX_1$ ? The  $MC$  will be given by the additional wage costs of employing the extra  $dL_1$  workers. Assume each extra worker costs the firm an amount given by ' $w$ ', the wage per worker. Then the  $MC$  will be equal to:

$$MC_1 = w \cdot dL_1.$$

Conversely, the Marginal Cost of raising output by  $dX_1$  when output is already high will be given by  $w$ , the wage per worker, multiplied by the increase in labour employment needed to raise output by  $dX_1$ . The increase in labour needed is given by  $dL_2$ : *show this in the figure*. Hence, the  $MC$  is given by  $w \cdot dL_2$ . I.e.,

$$MC_2 = w \cdot dL_2,$$

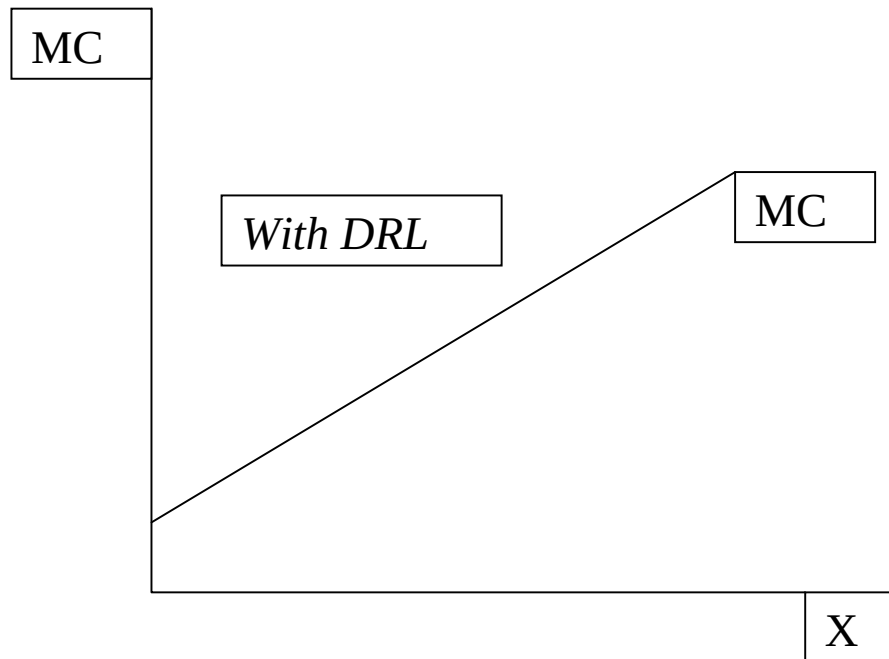
Where  $dL_2$  is greater than  $dL_1$ . (Why? Because of the assumption of DRL.)

As  $dL_2$  is greater than  $dL_1$ , it follows that  $MC_2$  is greater than  $MC_1$ . Hence, the MC is rising as output rises, under the assumption of DRL.

Thus, the MC curve is positively sloped under DRL, as shown in Figure 4.15,

## OHP60

Figure 4.15



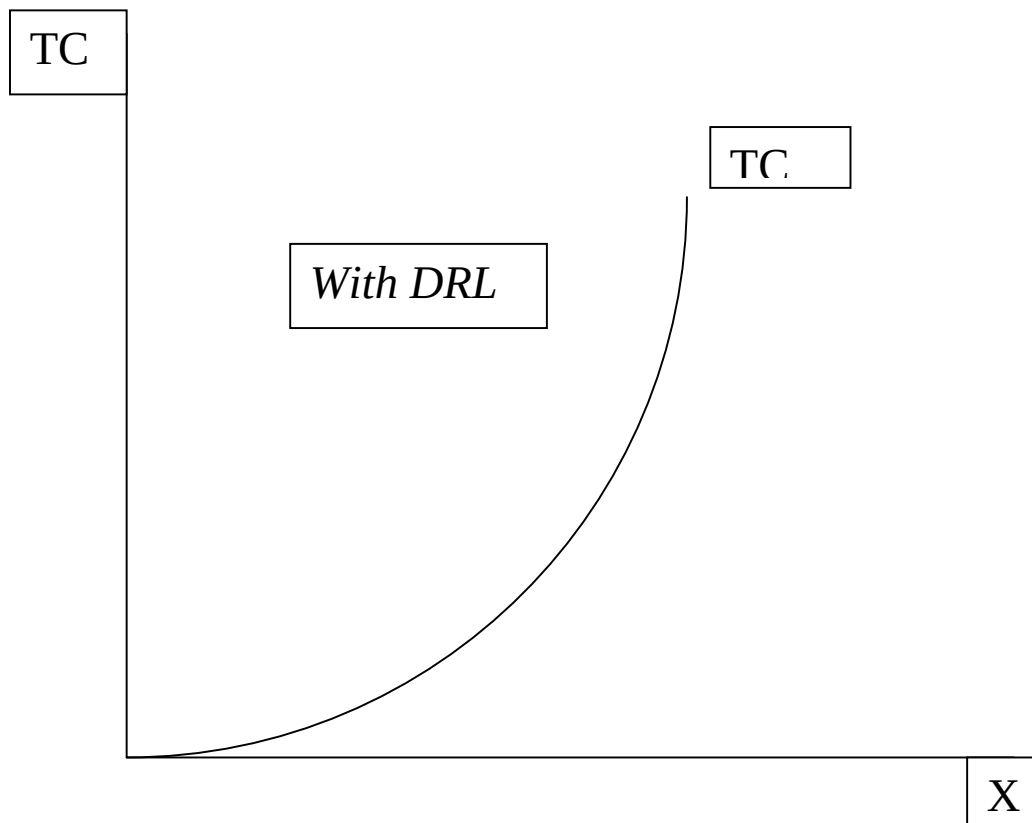
Notice the relationship between the shape of MC in Figure 4.15 and the shape of the MPL in Figure 4.11, both drawn under the assumption of DRL. Think about this and be able to explain it.

### OHP61

In Figure 4.15, the MC curve is upward-sloping. What does this tell us about the slope of the Total Cost curve (TC)?

The MC represents the slope of the TC curve (we've seen this already). Thus, if MC is rising, it means that the slope of the TC is getting steeper as X increases. This is shown in Figure 4.16.

Figure 4.16



Notice the relationship between the shape of TC in Figure 4.16 and the shape of the production function in Figure 4.10, both drawn under the assumption of DRL. Think about this too and be able to explain it. Essentially, the shape of the production function depends upon the returns to labour (MPL) and these also determine the shapes of MC and TC. Be sure you understand this.

In lectures, we'll discuss this further, and consider the shapes of  $X(L,K)$ , MPL, MC and TC under the assumptions of DRL and CRL.

We now consider the firm's short-run **Average** Cost curves.

## OHP62

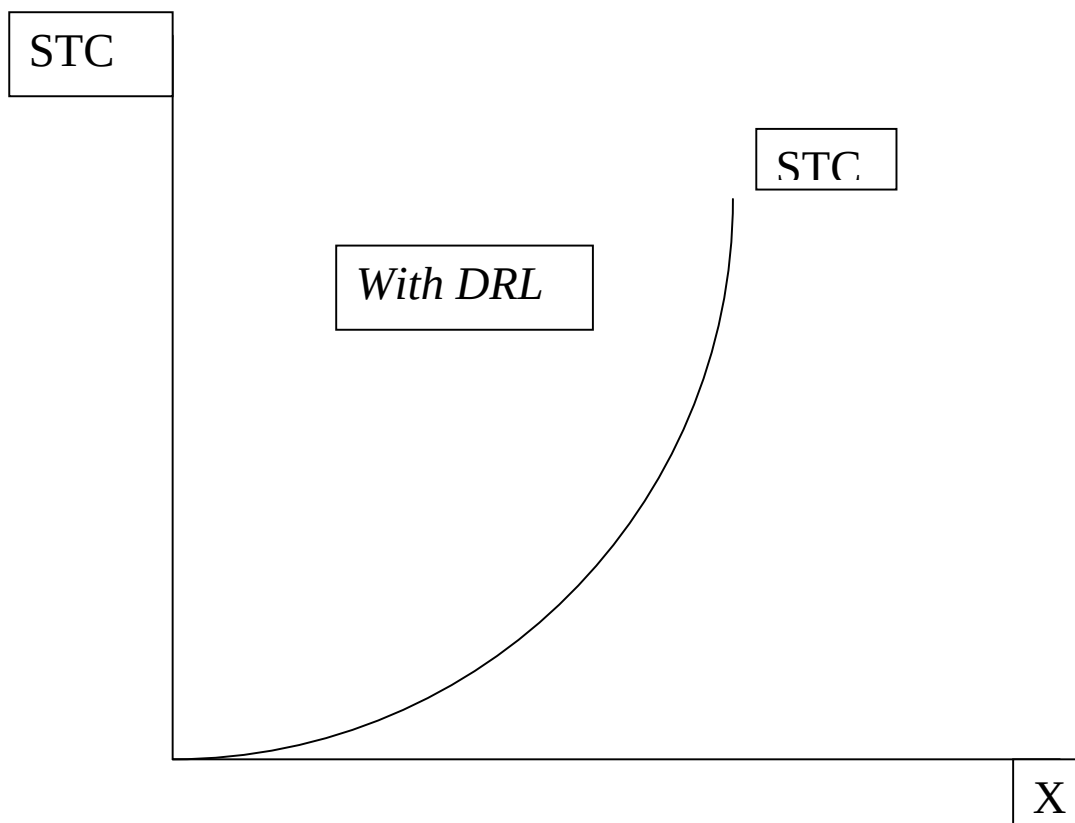
The firm's short-run ***Average*** Cost curves.

These are derived from the short-run Total Cost curve (STC) curve.

(Soon, we are going to distinguish between short-run and long-run costs. So let's start putting an 'S' for 'Short-run' in front of cost curve name labels.)

Consider the STC curve shown in Figure 4.17.

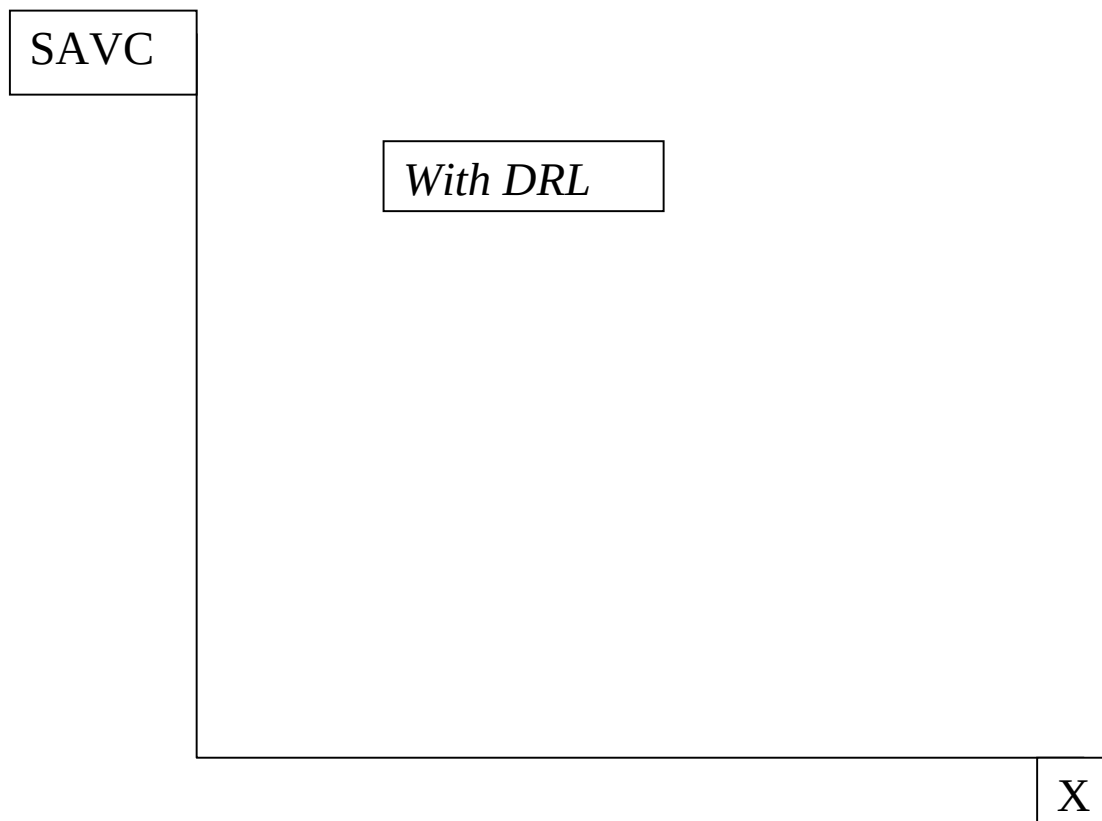
Figure 4.17



## OHP63

Notice in Figure 4.17 that when  $X = 0$ ,  $STC = 0$ . This means that in this particular example we are assuming that there are no Fixed Costs of production (we'll define these in the lectures). Essentially, the firm is employing no capital, just labour. Thus, the  $STC$  curve in this example represents just the total variable *labour* costs. We can divide these by the amount of labour employed. This gives us the Short-run Average Variable Cost. *Show it in the diagram.* And transpose  $SAVC$  into Figure 4.18.

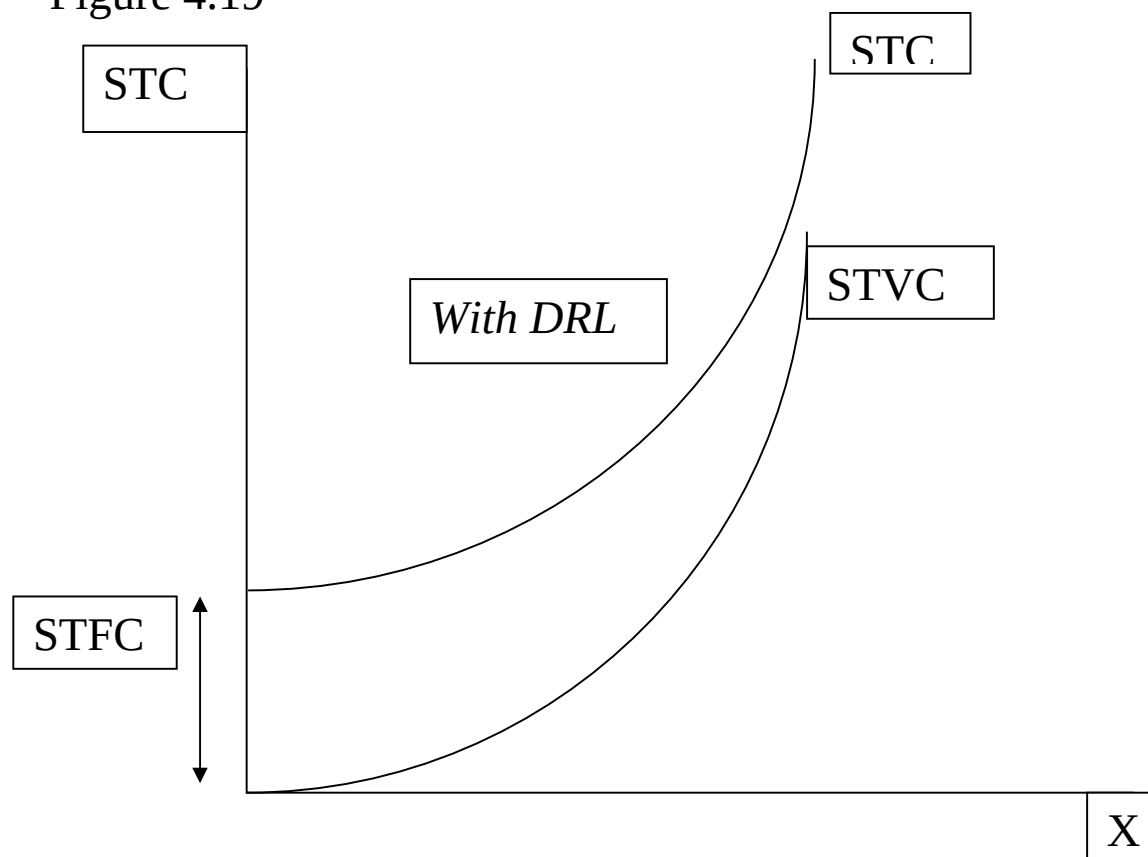
Figure 4.18



### OHP64

Suppose now that there are also Fixed Costs of production. These are shown in Figure 4.19.

Figure 4.19



### OHP65

Think just about Short-Run Total Fixed Costs, STFC. If we divide these by  $X$ , we get Short-run Average Fixed Costs SAFC: *draw these onto Figure 4.19.*

$$\text{SAFC} = \text{STFC}/X$$

and

$$\text{SAVC} = \text{STVC}/X$$

and

$$\text{SAC} = \text{STC}/X = (\text{STFC} + \text{STVC})/X = \text{SAFC} + \text{SAVC}$$

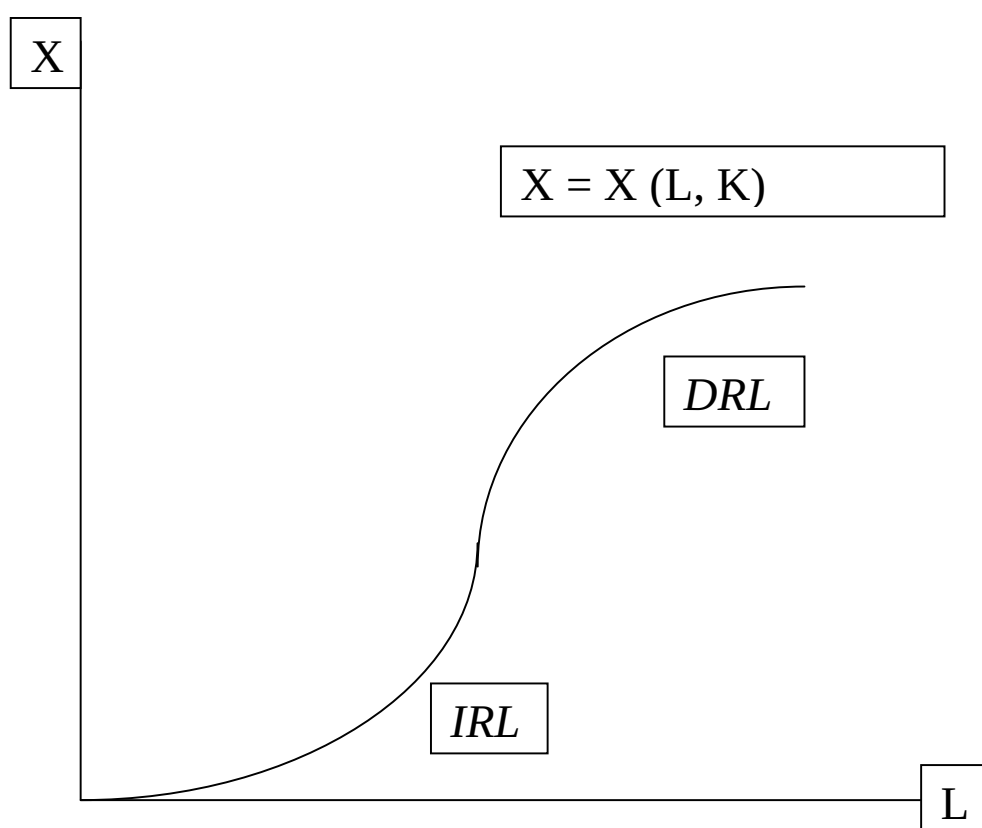
*(Draw all these average cost curves onto Figure 4.18.)*

## OHP66

The shapes of SAC, SAVC, SMC when the production function exhibits first IRL and then DRL.

Suppose the production function is as shown in Figure 4.20

Figure 4.20

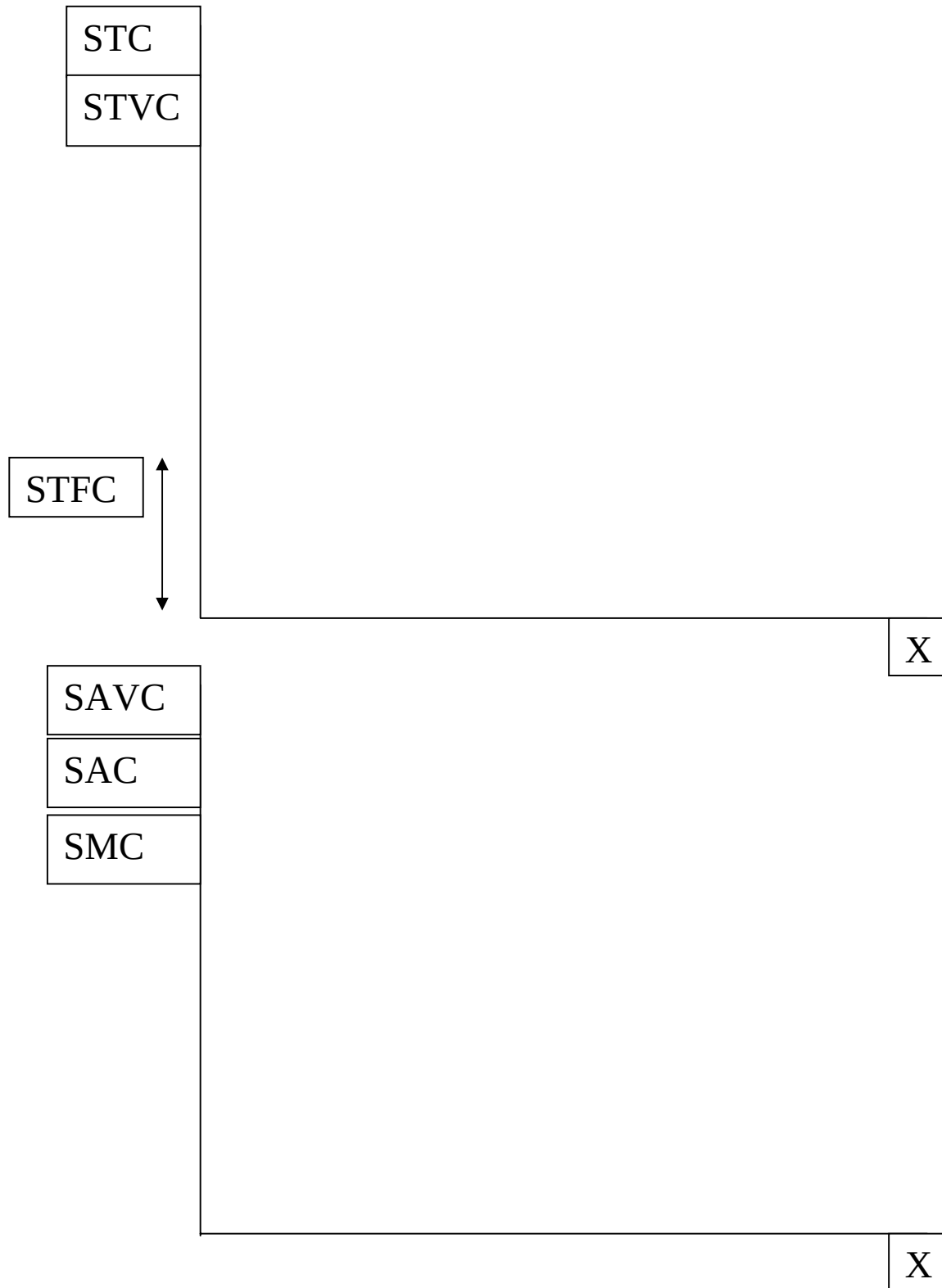


Derive the STVC curve from this production function and plot it in Figure 4.21. Show the STC too for some given level of  $K$ .

From Figure 4.21, then derive the SAVC, SAC, and SMC curves in Figure 4.22.

OHP67

Figures 4.22, 23

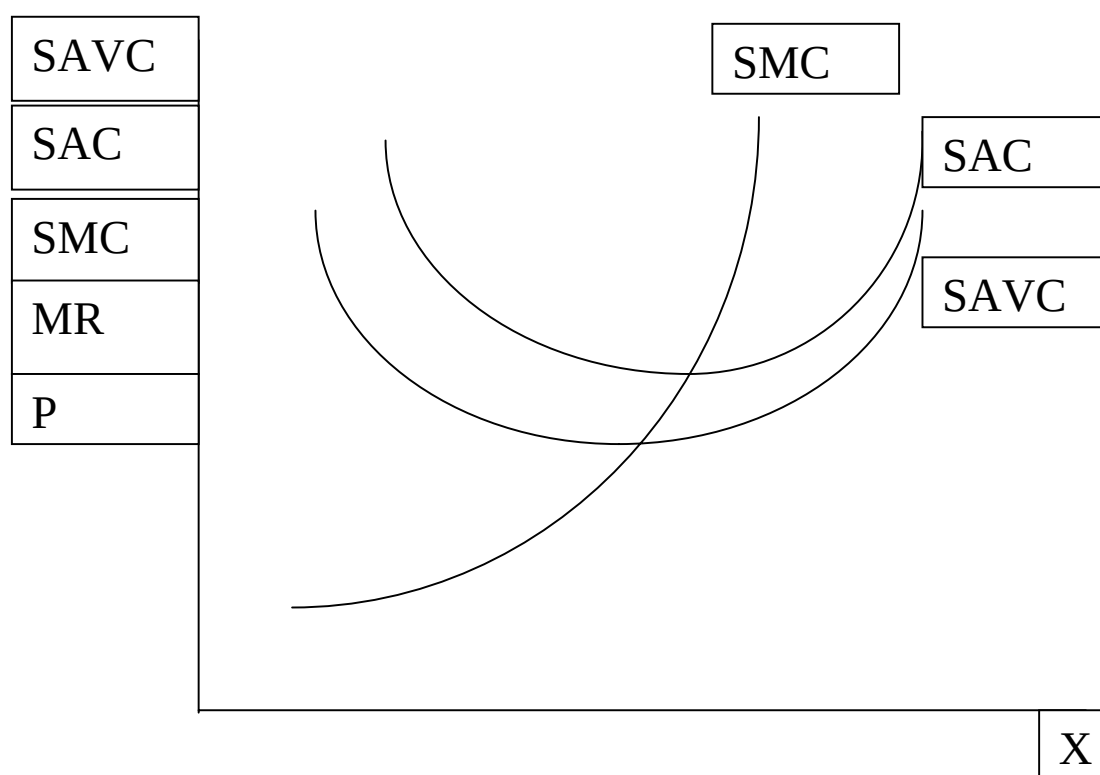


## OHP68

We are now in a position to analyse the firm's short-run profit-maximising decisions with respect to price and quantities, by bringing together in a single diagram the revenue and cost curves we have been deriving.

We know from before that profit-maximising occurs when  $MC = MR$ . In the short-run, this means when  $SMC = MR$ .

Consider Figure 4.24:



Add the demand and MR curves to this figure.  
Derive the shut-down price from this figure.

## OHP69

So far, we have talked exclusively about the short-run analysis of profit-maximisation.

In the long-run, the firm is able to vary the amount of capital it employs.

In the lectures, we shall see how this affects the firm's cost curves and how we can derive the LAC curve from the SAC curves. We can then analyse long-run profit maximisation.