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Have Inflation Expectations become dislodged from the Bank of England's target after the financial crisis?

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Abstract

This study investigates whether the largest financial crisis for a generation and unprecedented monetary policy has caused a shift in inflation expectations, away from the target level set by the Bank of England. It examines whether there has been a change in the sentiment of agents in the UK economy to either not believing the validity of the inflation target or the ability of the central bank to meet it. Using a method previously reserved for predicting equity prices and currency rates, this paper will examine risk-neutral probability density functions of inflation expectations at various horizons, and lay out the method of calculating such results for further studies to use. The results found expected inflation in the short and medium run to be unattached to the long run expectation, suggesting agents do not believe the central bank is willing and/or able to achieve the steady level of inflation it desires. However, the study also highlights many limitations with this new method applied to zero coupon inflation options, which could question the accuracy of its results, and outlines areas for improvement in the method that could establish it as a key part of the toolkit used by central banks and research institutions across the world.

Word Count: 4990

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Contents

I. Introduction

Inflation expectations are one of the most important indicators in the UK economy, relevant to financial markets, firms with future cash flow and the Government. However, they are arguably most relevant to the Bank of England, which must consider them when setting the base rate of interest and, recently, the level of quantitative easing to achieve its inflation goals.

Stable inflation expectations rely on transparent monetary policy aimed at achieving the Bank of England's 2% target rate of inflation. However, due to the financial crisis and the subsequent economic downturn, the Bank of England has maintained the interest rate at a record low of 0.5% since March 2002, on top of which it has initiated an unprecedented scheme of quantitative easing. These two expansionary policies are clearly inflationary, and coupled with rising food and energy prices, an increase in VAT and an increase in import prices after a depreciation of the pound, the consumer price index measure of inflation has averaged 3.35% since December 2009.

With the relentless consistency in monetary policy, one may worry that agents in the UK economy will start to question both the credibility of the Bank of England's target rate of inflation and the ability of the bank to reach it. This change in sentiment may well be observed in a change in inflation expectations, which is of great concern to the Bank of England. An increase in inflation expectations would work against the Bank's current strategy, as it would increase real rates of interest in the financial markets, it would make the exports less competitive and a change in expected inflation may even affect current inflation through price and wage setting mechanisms.

The study of inflation expectations and the models used to compute them is extremely popular in academia. The literature review following this section highlights a selection of studies and methods from the plethora available. However, this paper will use a fairly new method, so far only used sparingly by the Bank of England to evaluate the following hypothesis:

Hypothesis 1: *Inflation Expectations in the short term have moved away from the long term Bank of England target. The same is true to a lesser extent in the medium term, however, expectations in the long term are still anchored to the target.*

The results produced in this paper used to evaluate this and other hypotheses, will be conditional on the production of probability density functions formed from market data on options on inflation. The method for doing this is well documented, but it has rarely been applied to produce inflation expectations. Further information on the methodology shall be detailed in the section with this name and expanded upon in the appendix.

Due to the shortage of papers documenting the technique of applying this common method to inflation derivatives, the intention of this work is to lay the foundations for further papers to both refine the method and expand upon the results used in this paper.

Furthermore, this paper will examine whether a in change inflation expectations can be viewed through the variance of the probability density functions. An increase in the variance could be caused by an increase in uncertainty of inflation, suggesting agents are questioning both the credibility and ability of the Bank of England, or an increase in disagreement between agents, suggesting the Bank's target is being misinterpreted. Additionally, inflation expectations should not be sensitive to macroeconomic news, as they should be based purely on the strategy of the Bank of England, which through forward guidance and inflation targeting has attempted to remove shocks. The following hypotheses demonstrate the author's predictions prior to the results, of the change in the aforementioned variables.

Hypothesis 2: *Variance of inflation expectations has increased post financial crisis, and the probability of extreme values of inflation has increased.*

Hypothesis 3: *Inflation expectations in the short term have become sensitive to macroeconomic news and data.*

II. Literature Review

Inflation Expectations are a key part to setting monetary policy and their importance has been amplified since the financial crisis. The state of inflation expectations greatly influences actual inflation and thus the central bank's ability to achieve price stability, Bernanke (2007).

In theory, inflation expectations are assumed to be constant, as the central bank's objective function is known and constant, so the rational expectations hypothesis implies long-run expectations are constant. However, there are new models concerning how agents form long-run expectations. Demertzis and Viegi (2009), proposed an information game, where agents form expectations on either the central bank's target or three factors which are; monetary authorities objectives and policy decisions, shocks occurring after these decisions and the average of individuals' inflation expectations. They derived a time varying parameter to capture the perceived credibility of the central bank's target; such that, if the target were credible, then agents would set the long run inflation expectations in line with the central bank's target. This could explain expectations on average but the time varying parameter will be unique to each individual and hard to calculate. Brazier et al (2008) also proposed a model suggesting that agents varied between the central bank's target and lagged inflation, based on an imperfect assessment on which was performing better, to set expectations. Clark and Davig (2008) showed, through VAR analysis, that inflation expectations in the short and long term were affected by shock news on important macroeconomic variables and some commodity price shocks. All of these papers show that expectations cannot be assumed constant, so measuring them is extremely important.

There are two distinct ways of calculating inflation expectations, using survey data or inflation-indexed financial products. Surveys come in a variety of forms, and are usually targeted at professionals and academics, for example the Survey of Professional forecasters, Livingston Survey and the Bank of England's Survey of External Forecasters; but they often use opinions from members of the general public, for example the Michigan Survey. Surveys are thought to produce good data on inflation expectations across different time horizons, mainly because the data is not skewed by uncontrollable variables.

However, there are a few problems with surveys. Firstly, they are often only made quarterly, making it hard to measure the impact of shocks. Furthermore, there is no risk for the agents to 'put their money where their mouth is' so the answers provided may not be a true reflection of their opinion. Bruine de Bruin, et al (2012) showed that the wording of the questions could lead to different answers, producing conflict between surveys. Mankiw, Reis and Wolfers (2003) found disagreement over 50 years between consumers and professionals, concerning the level of inflation, the absolute change of inflation and the relative price variability. This will lead to a lot of noise in the overall expected inflation rate. Cross sectional dispersion of expectations can also arise, as the rate which media news reaches economic agents varies, with professionals considered to be more up to date than members of the public, Mankiw and Reis (2002).

Furthermore, since the new millennium it is thought that there has been a change in the way that expectations should be formed. Trehan (2010) showed that expectations should place less weight on recent inflation data, and households continue to overvalue this information. This has led them to drop from the most to the least accurate forecasters in the Michigan Survey. Professionals have adjusted their expectations, however, they place too much weight on recent core inflation.

Inflation-indexed financial instruments can also be used. These are available at extremely high (almost continuous) frequency, and thus can be very useful in measuring the impact of shocks. The first step in computing inflation expectations in this way is to compute the difference between nominal and inflation-linked bonds. This removes the affect of variances in liquidity across bond and equity markets. However, Hördahl (2009) explained that the difference is not just expected inflation and includes the following, an inflation risk premia, a liquidity premia and technical factors, so expectations appear different when the underlying inflation rate hasn't changed.

There are a few studies considering whether inflation expectations have changed recently. Galati, Heemeijer, Moessner (2011) considered the Euro zone countries, and used a more high frequency survey. Unfortunately, this is impractical for this project. They were able to observe whether long-run expectations had become more responsive to news, such as the Greek debt crisis or Lehman Brothers collapse. They found that long-term expectations remained anchored, however short-term expectations drifted and reacted to the Greek crisis. The survey also found that participants changed their expectations frequently, although the frequency fell as the time horizon widened.

Galati, Poelhekke, Zhou (2011) used survey-based measures and inflation-swaps to see whether or not expectations had become less anchored in the EU, US and UK. The survey showed that expectations had drifted up in the US and UK, but not the EU. It also showed that disagreement increased in the US and UK. The swap results showed that volatility was up in all areas and sensitivity to news increased in the US, but not significantly in the UK and EU. The reasons for the difference across the areas could well be due to the use of explicit inflation targeting.

Levin, Natalucci and Piger (2004) concur with this hypothesis, showing that targets do have anchoring properties for inflation expectations. Gürkaynak, Sack and Swanson (2005) showed that long run expectations in the US are sensitive to news, and not perfectly anchored, but in a further paper, they showed that inflation expectations are more anchored in the UK and other inflation targeting countries, Gürkaynak, Levin and Swanson (2010) and Gürkaynak et al. (2007).

There has also been work done by members of the Bank of England, on whether inflation expectations have changed in each of the past three years, Maule and Pugh (2013), Harimohan (2012) and Macallan, Taylor and O'Grady (2011). All found that inflation expectations remained on target in the long-run, but with slight deviations in the short and medium run, with volatility and responsiveness to news increasing slightly.

III. Data

III.a. Outline of the data

The main data set used in this paper was provided by *Tullett Prebon*, which includes both prices and implied volatilities for zero coupon inflation options from 18th December 2012 to the 26th February 2014. The data covers strike prices from -2% to 6% at 1% intervals and was taken at the close of trading (17:00 GMT) every trading day in the period mentioned above.

Unfortunately, considering the relative immaturity of this derivative and the small size of the market, data was only available from a small number of institutions, most of which do not make it accessible for non-clients. Hence the time period covered is far smaller than the study would have hoped, and unfortunately this limits the paper to considering only the post crisis years. Future studies may benefit from groundwork done by this paper to produce more comprehensive results.

Furthermore, data provided on the implied volatility of each option was often found to be too far away from the trend or other similar data points. Though the author is thankful for all the data provided by *Tullett Prebon*, they were unable to provide their measure of implied volatility for examination. The anomalous results were omitted from the study.

Additional information used in the production of the results was obtained from various sources. The retail prices index used for the current inflation rate at different points during the period examined was obtained from the *ONS (Office for National Statistics)*. The risk free rate, also used in the method, was the US 30 year Government Bond rate taken from the *US Department of the Treasury*.

III.b. Inflation Options

The data provided by *Tullett Prebon* gave this study the choice of two different derivatives to produce results. These were inflation caps and zero coupon options. The former give the buyer a payoff every year, up to maturity, equal to the difference between the strike rate and the realised rate at that point in time multiplied by a pre-agreed nominal. This derivative would require a method called caplet stripping, which would isolate the price at each maturity. The latter looks at the difference between strike rate and the realised rate only at maturity. Hence the price of the option will be purely representative of the expectation of inflation at this maturity. This paper shall only be considering the latter due to issues with the implied volatility alluded to previously, which shall be elaborated in the next section.

IV. Methodology

Using option prices to compute a probability density function (PDF) of where the market expects the price of the underlying to be at varying points in time is a commonly used tool. As such there is a lot of literature on the method first established by Breedon and Litzenberger (1978), where it was discovered that the PDF of an underlying asset can be recovered by calculating the second partial derivative of the call price function with respect to the strike price. However, the application of this method to inflation options and hence inflation expectations is not documented at all, barring a few results from the Bank of England. One of the main aims of this paper is to set out the precise methodology for applying this procedure to inflation options, which this section aims to do.

Inflation options come in the form of caps and floors, which are identical to standard calls and puts respectively, with a pre-agreed nominal for the payoff, since the payoff cannot be in percentage points. The data provided for this paper gave floors with strikes from -2% to 3% and caps with strikes from 0% to 6%. To make full use of the spectrum of strike rates, the put-call parity is used to convert the inflation floors into inflation caps. Using this transformation preserves implied volatility as the underlying is not changed, hence each floor can simply be written as a cap with the same implied volatility.

Theoretically, the first place to start from, using zero coupon inflation caps and their corresponding prices, is to convert the prices into volatility space. The objective is to produce a continuous call price function. However, considering the relatively small number of strikes across a large interval, to find the prices for the intermediate strikes it is more accurate to smooth in the volatility space; this result is shown in Shimko (1993). The first and largest problem with this methodology is that there appears to be no implied volatility formula designed for the exotic nature of inflation options, even when strike rates are indexed to avoid negative strikes. Using the standard iterative Black Scholes approach, results often tend to infinity or do not converge. Estimation methods such as the one formulated by Corrado and Miller (1996) and others documented in a paper by Chambers and Nawalkha (2001), do not provide results that match the implied volatilities provided in the data at discrete strike rates. As previously mentioned, *Tullett Prebon* were unable to provide their method for calculating implied volatility for this paper, though anomalies suggest their method is not perfectly fitted to these options. Overall, this paper will omit the first conversion from price to volatility space and consider the values of implied volatility provided in the data.

In the Volatility-Strike space the next stage is to smooth and thus find intermediate values. The fit used in this paper is a SABR (Stochastic Alpha Beta Rho) model, first produced by Hagan et al. (2002), the details of which shall be expanded upon in the Appendix. Figure 1 shows the fit used here is more appropriate than the standard quadratic model used in Shimko (1993), as the option in question has much higher volatility at the tails than most standard options.

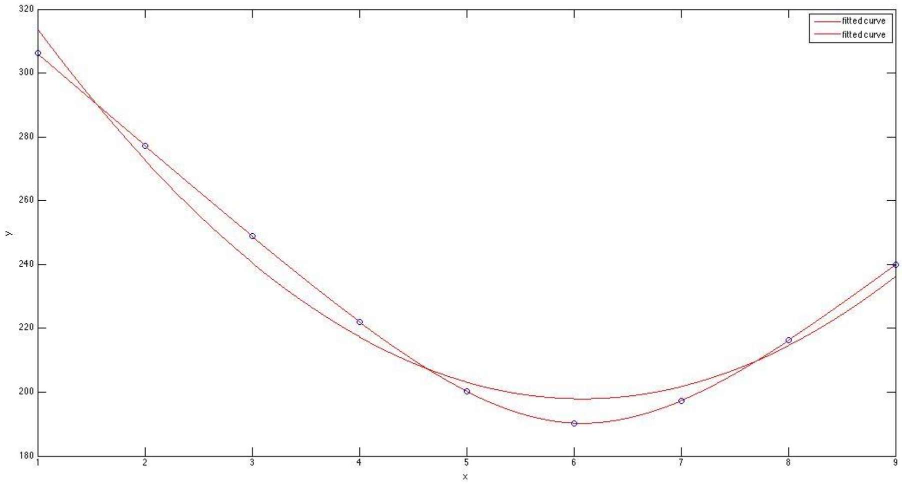


Figure 1 SABR fit vs quadratic fit

Once the smoothed curve is fitted the next step is to convert from the volatility-strike space back to the call price-strike space, providing a continuum of prices ready for differentiation. To do this it is sufficient to use the Black Scholes call price formula. However, one cannot insert the formula for the volatility smile found previously, into the Black Scholes formula, as two of its components involve a cumulative normal distribution function. To circumvent this problem 100,000 points were taken on the volatility smile at equally distanced points between -2% and 6% strikes, then each of these points were inputted into the Black Scholes formula, to produce 100,000 call prices. The number of points taken was chosen arbitrarily; although more points would improve the curve fit, the improvement would be negligible.

Once into the call price space, we must again find a smooth curve to fit the points, so that we can differentiate twice to find the pdf. The result by Breedon and Litzenberger (1978) is provided below:

$$\frac{\partial^2 C}{\partial X^2} = e^{-r\tau} g(S_T)$$

$C = call\ option\ price, X = Option\ strike\ price, r = risk\ free\ rate,$
$\tau = T - t\ which\ is\ the\ maturity\ of\ the\ option,$
$S_T\ terminal\ underlying\ price, g(S_T)\ is\ the\ risk\ neutral\ pdf.$

However, simply smoothing these points and computing the second partial derivative with respect to strike price often leads to discontinuities in the final function, due to the nature of the software (MatLab) used. A few examples of this are provided below.

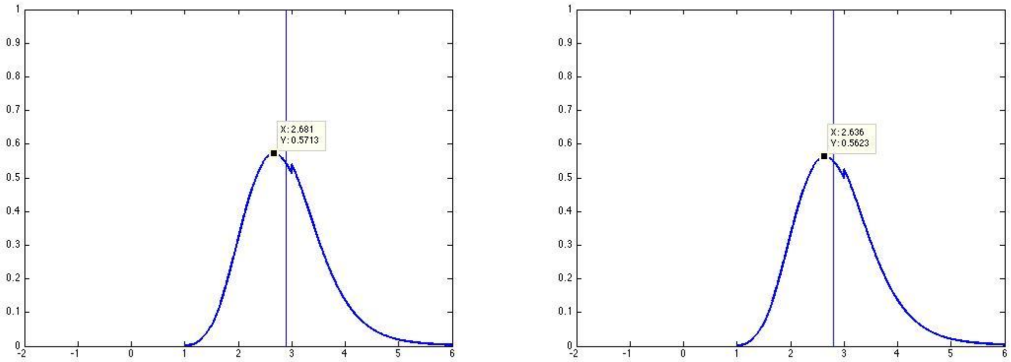


Figure 2 Discontinuities of second derivative

Hence this paper has applied a curve fit at the call price space and each derivative after. Each curve fit is a piecewise cubic polynomial with a smoothing parameter chosen to minimise the Root-Mean-Square Error (RSME) whilst preserving the curve at the tails. The parameters were chosen by analysis of the RSME and the trend of the curve outside of the range of strikes. The following are the average test statistics across all dates and maturities tested.

	R-Square	Adjusted R-Square	Root-Mean-Square Error
Average:			
Call Price Space	1	1	0.0127843
First Derivative	1	1	0.0492157
PDF space	1	1	0.0971200

These statistics clearly show that each stage was a good approximation and hence the error in the results will be minimal; a more detailed breakdown of the results is available in the appendix.

Once the second derivative is obtained, and the data is smoothed, the last stage is simply to inverse the discount factor shown in the formula above to obtain the risk-neutral probability density function for the inflation rate.

An extension of this method would be to fit the tails of the distribution, as evidenced in the results section, as the upper tail doesn't tend to zero due to the restriction of the 6% strike. Though this method would produce more accurate results, this is beyond the scope of this paper and would not have changed the absolute values of the mean or the relative values of the variance.

V. Results

(Please note that all PDF graphs have a vertical line from 0 to 1 at the headline rate of inflation on that date)

V.a. Selecting Results

As the data was provided for every trading day over a period of 15 months there are far too many results to present and examine each one graphically. The selection has been limited to the dates after inflation rate announcements. The reason for such a selection is that the majority of significant changes in the mean and the shape of the PDF occur just after the announcement of a change in the RPI headline rate. Figure 3 clearly shows that major changes in the mean expectation of inflation occur after the announcement of a change in the headline rate (highlighted by the vertical red lines).

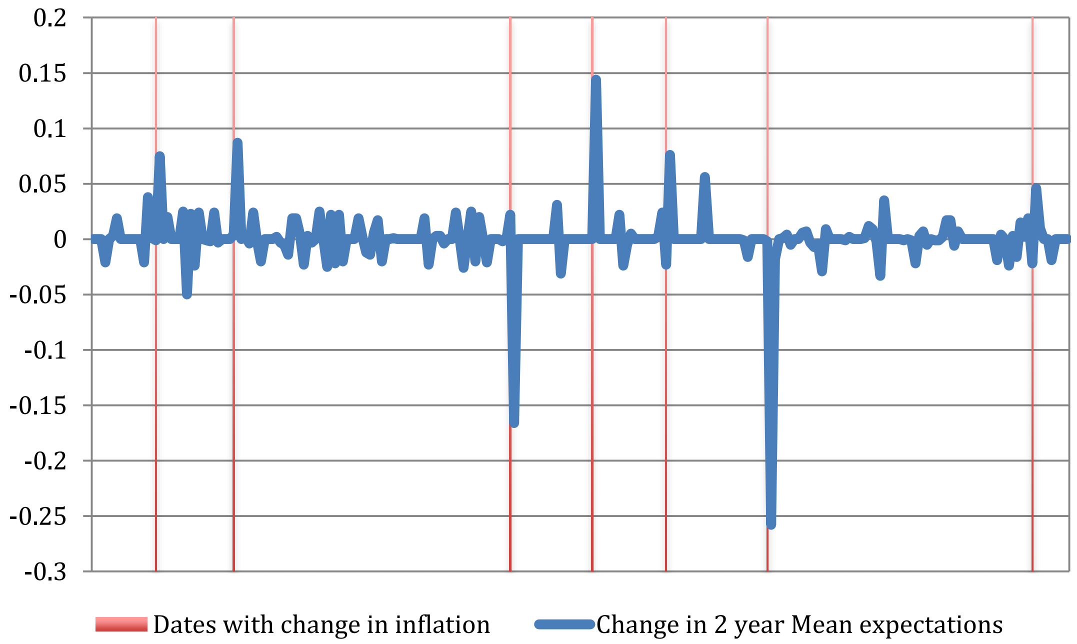


Figure 3

This graph can also be used to evaluate *Hypothesis 3* from the introduction, that short-term inflation expectations have become more responsive to macroeconomic news. The graph suggests that inflation expectations do change when a new headline rate of inflation is announced. However, this encapsulates the majority of major shifts in expectations, implying that other macroeconomic variable releases have little to no affect on mean inflation expectations in the short run. Furthermore, it is possible that the shift is due to the design of inflation option contracts and any hesitation of the market to adopt a new equilibrium, not a change in the expectations of agents in the economy. A more thorough event study of this data could be conducted to test this hypothesis more robustly, however this is not the main purpose of this paper and hence shall not be considered in any more detail.

V.b. Main Results

The probability density functions from each trading date appear to evolve in a similar way, and provide an interesting view on the progression of inflation over the next thirty years. The graphs below represent outlooks for inflation, at each maturity provided in the data (2, 5, 7, 10, 12, 15, 20 and 30 year horizons) and at three dates taken selected after the announcement of the RPI rate during the period of this study.

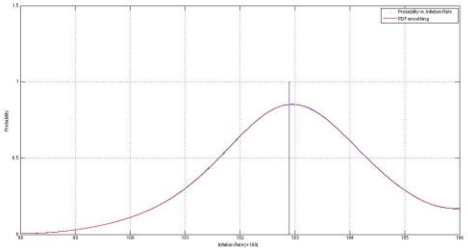


Figure 4: 18/12/2012, 2 year horizon

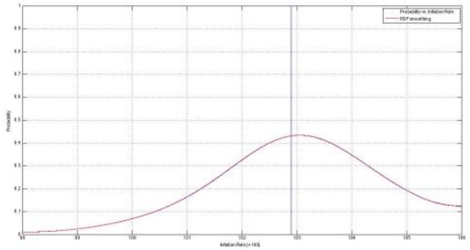


Figure 5: 18/12/2012, 5 year horizon

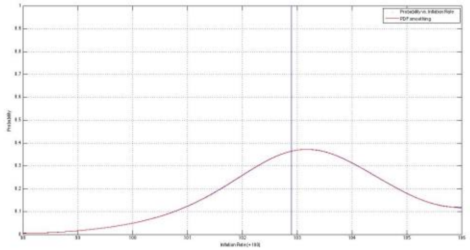


Figure 6: 18/12/2012, 7 year horizon

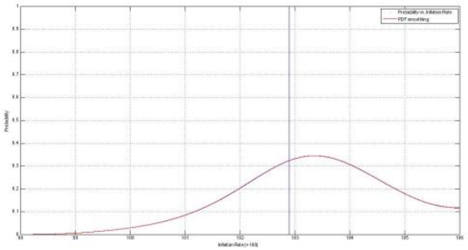


Figure 7: 18/12/2012, 10 year horizon

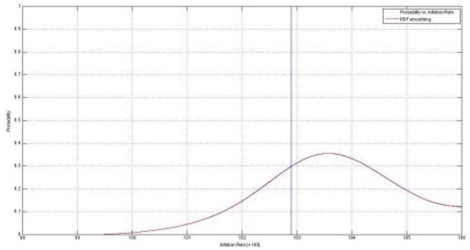


Figure 8: 18/12/2012, 15 year horizon

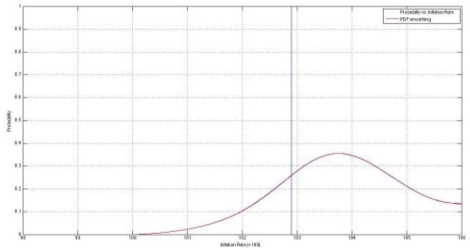


Figure 9: 18/12/2012, 20 year horizon

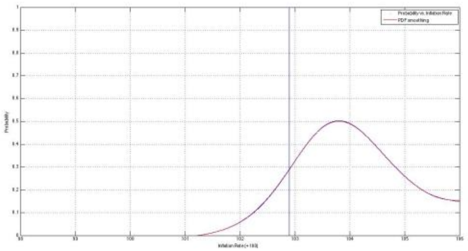


Figure 10: 18/12/2012, 30 year horizon

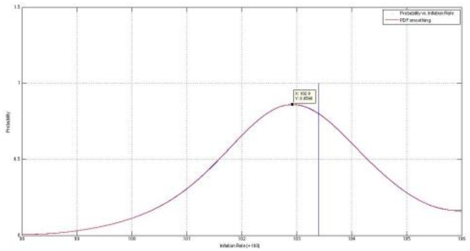


Figure 11: 16/7/2013, 2 year horizon

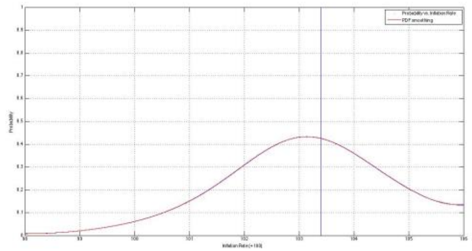


Figure 12: 16/7/2013, 5 year horizon

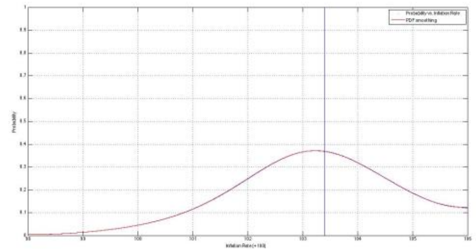


Figure 13: 16/7/2013, 7 year horizon

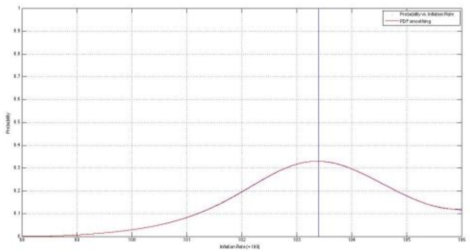


Figure 14: 16/7/2013, 10 year horizon

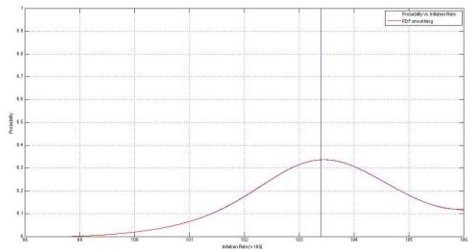


Figure 15: 16/7/2013, 12 year horizon

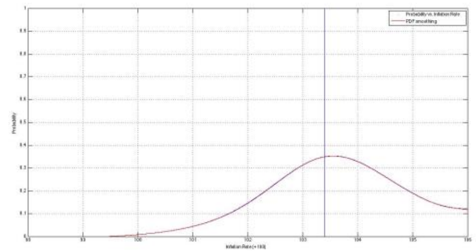


Figure 16: 16/7/2013, 15 year horizon

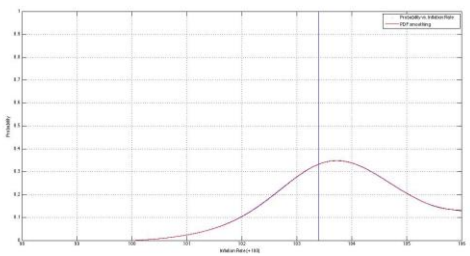


Figure 17: 16/7/2013, 20 year horizon

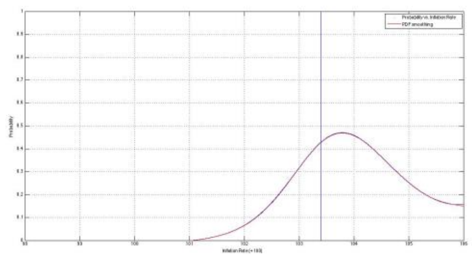


Figure 18: 16/7/2013, 30 year horizon

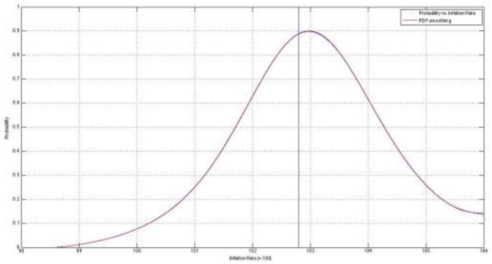


Figure 19: 18/02/2014, 2 year horizon

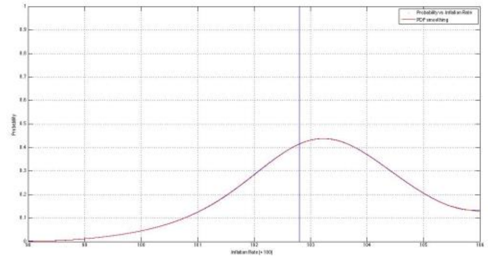


Figure 20: 18/02/2014, 5 year horizon

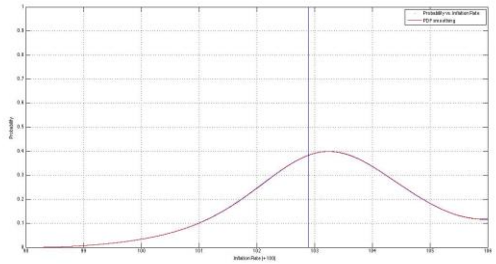


Figure 21: 18/02/2014, 7 year horizon

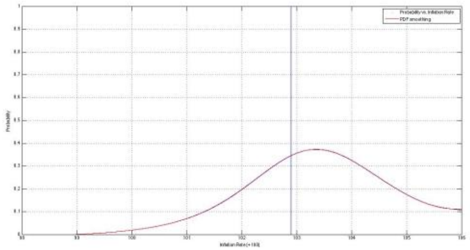


Figure 22: 18/02/2014, 10 year horizon

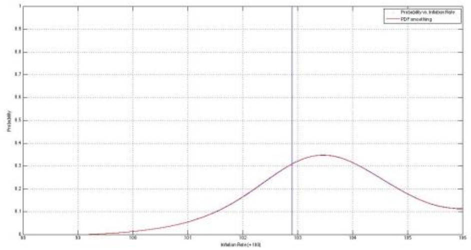


Figure 23: 18/02/2014, 12 year horizon

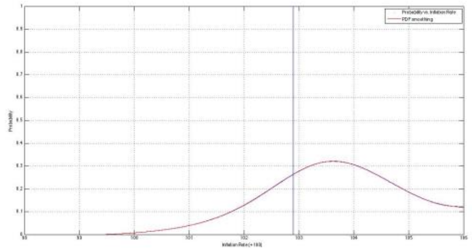


Figure 24: 18/02/2014, 15 year horizon

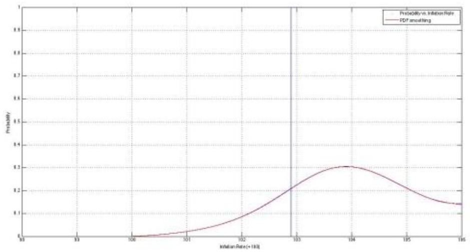


Figure 25: 18/02/2014, 20 year horizon

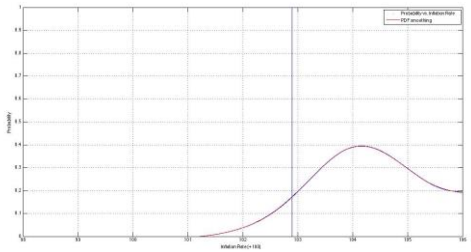


Figure 26: 18/02/2014, 30 year horizon

The graphs above seem to demonstrate that inflation expectations initially start close to the headline rate of inflation, and much lower than the long run rate of inflation expectations, suggesting that short term expectations are missing the target rate of inflation. The spread of inflation expectations in the short term seems to be quite large, with greater probability given to deflation than at any other time period, highlighting a high level of disagreement between agents or a higher level of uncertainty about inflation, which seems to support *Hypothesis 2*.

However it should be noted that probabilities (not the mean) provided in the two year horizon are incorrect, the cause of which is likely to be that agents are protecting against deflation as we come out of recession, creating a small bubble in the market around the tail ends which in turn is affecting the slope of the call price function and hence the PDF.

As the graphs progress to the medium term we see inflation expectations slowly moving upwards. Moreover, whilst uncertainty around the mean appears to be increasing, the probabilities attached to the extremities appear to be decreasing. This seems to imply that in seven to fifteen years agents expect the Bank of England to have more control over the rate of inflation, which slightly contradicts *Hypothesis 2*, though these expectations are still not in line with the long-term rates. The increase in uncertainty around the mean rate does bring into question the Bank's target slightly, although this measure is on RPI not CPI.

Finally, long run expectations show the inflation rate rising further and the probability of the mean rate occurring increasing. Furthermore, the probabilities of extreme values of inflation are at their lowest rate and all of these observations seem to point to increased confidence in the Bank of England's ability to maintain price stability. Twenty and thirty year rates, seen in the appendix, are similar, suggesting this is the target rate that agents believe that Bank of England is aiming for.

Considering the variation of expectations across constant time horizons is also important when evaluating hypotheses 1 and 2, to do this we shall examine the short (two year), medium (ten year) and long-term (thirty year) maturities.

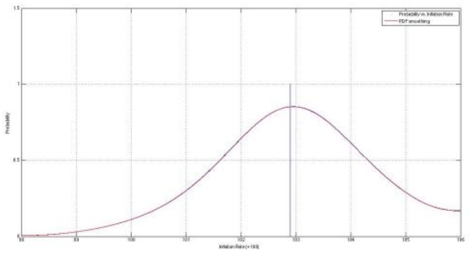


Figure 27: 18/12/2012 2 year horizon

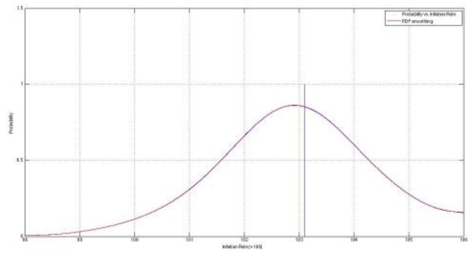


Figure 28: 15/1/2013 2 year horizon

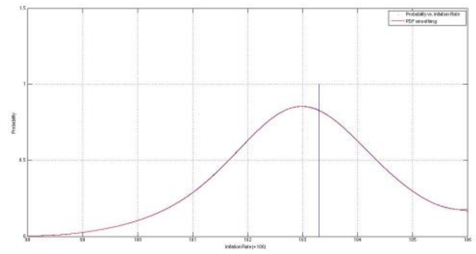


Figure 29: 12/2/2013 2 year horizon

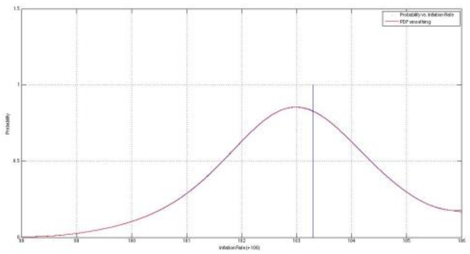


Figure 30: 19/3/2013 2 year horizon

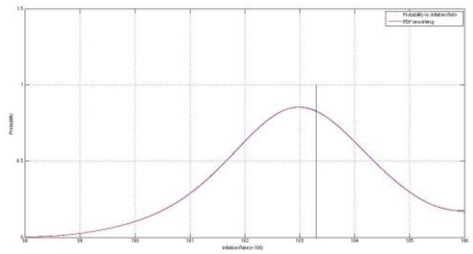


Figure 31: 16/4/2013 2 year horizon

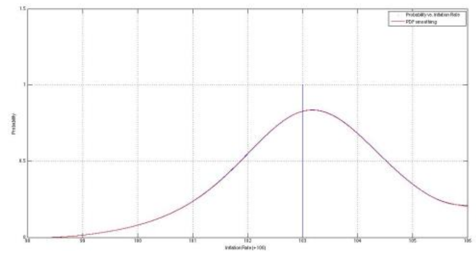
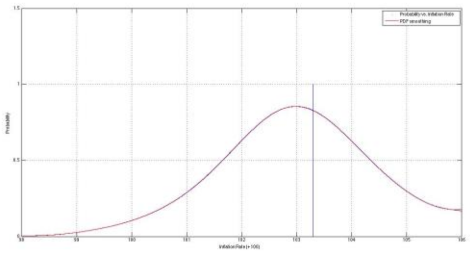


Figure 32: 21/5/2013 2 year horizon



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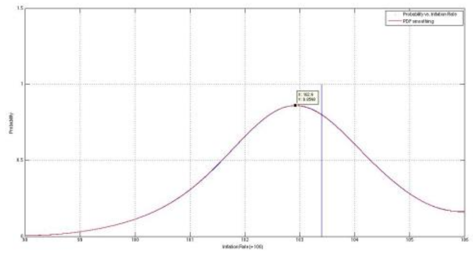


Figure 34: 16/7/2013 2 year horizon

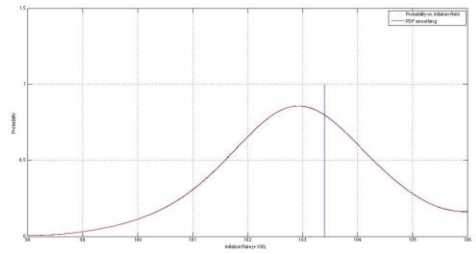
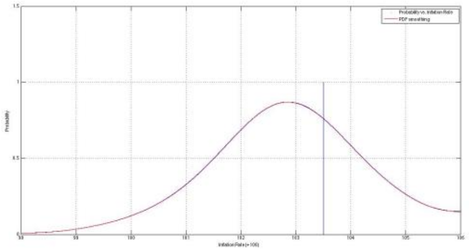


Figure 35: 13/8/2013 2 year horizon



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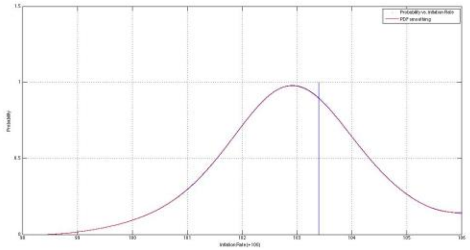


Figure 37: 17/10/2013 2 year horizon

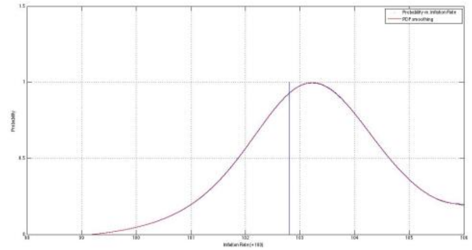


Figure 38: 12/11/2013 2 year horizon

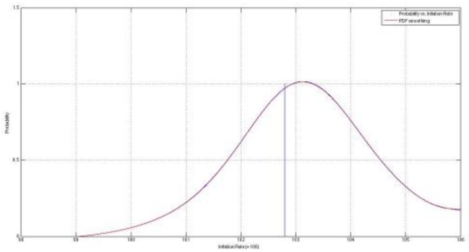


Figure 39: 17/12/2013 2 year horizon

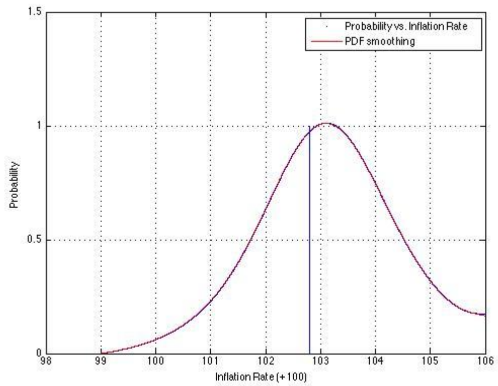


Figure 40: 14/1/2014 2 year horizon

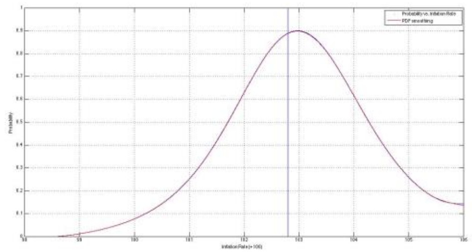


Figure 41: 18/2/2014 2 year horizon

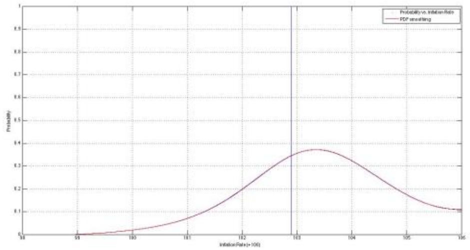


Figure 42: 26/2/2014 10 year horizon

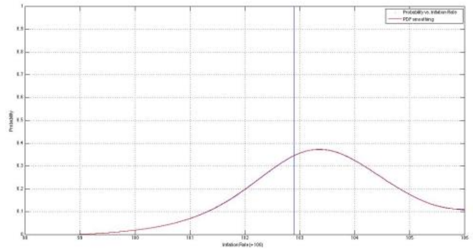


Figure 43: 18/2/2014 10 year horizon

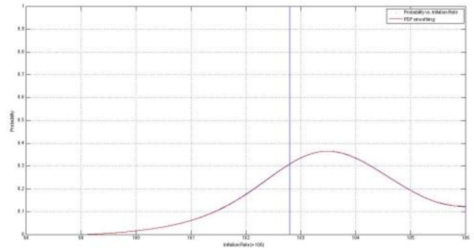


Figure 44: 14/1/2014 10 year horizon

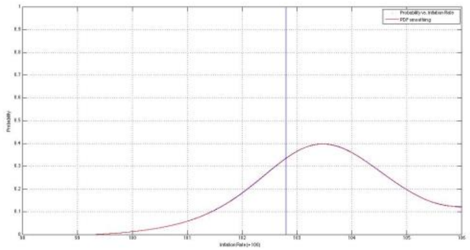
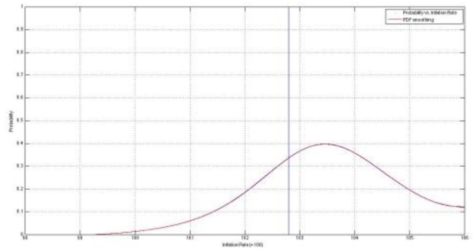


Figure 45: 17/12/2013 10 year horizon



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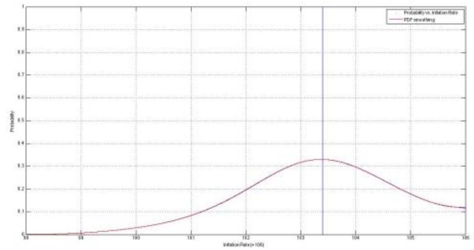
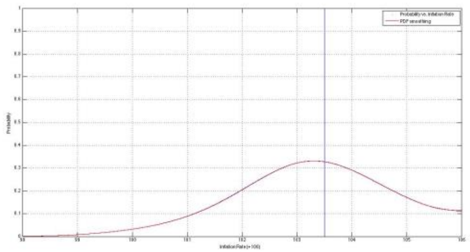


Figure 47: 15/10/2013 10 year horizon



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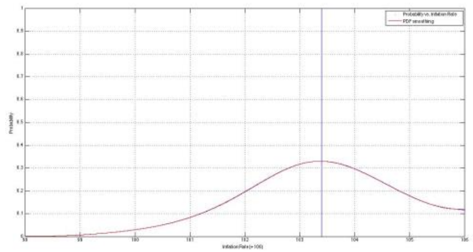
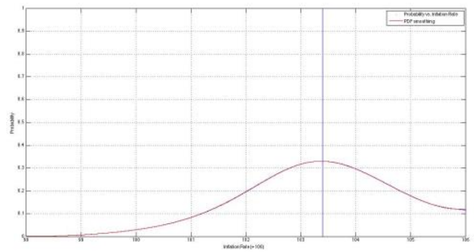


Figure 49: 13/8/2013 10 year horizon



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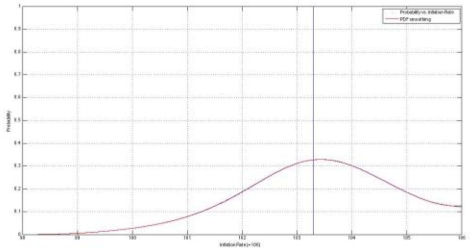

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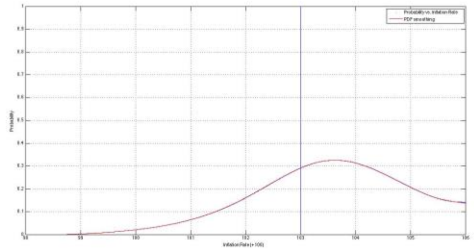


Figure 52: 21/5/2013 10 year horizon

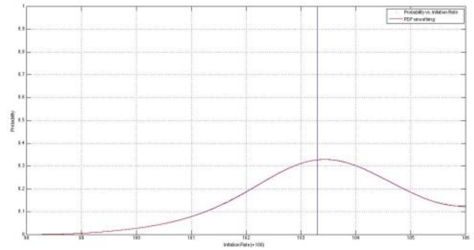


Figure 53: 16/4/2013 10 year horizon

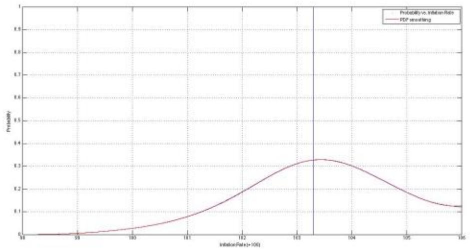


Figure 54: 19/3/2013 10 year horizon

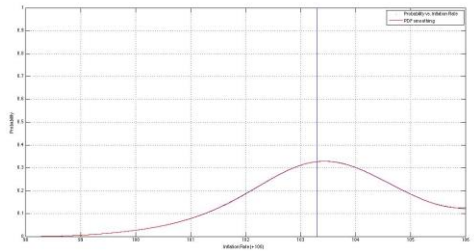


Figure 55: 12/2/2013 10 year horizon

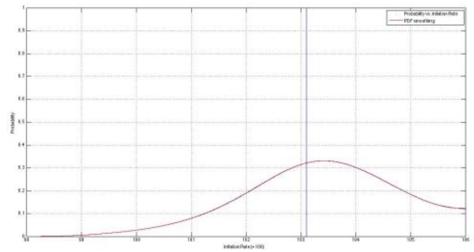


Figure 56: 15/1/2013 10 year horizon

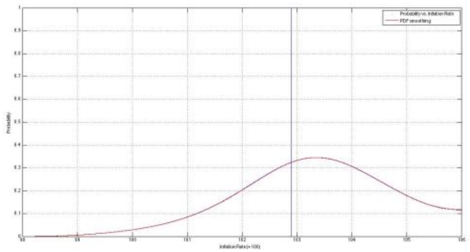


Figure 57: 18/12/2012 10 year horizon

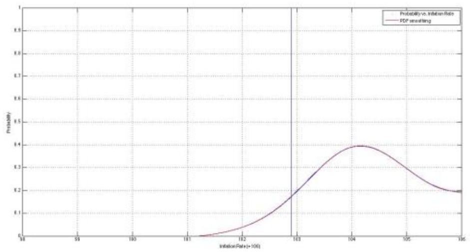
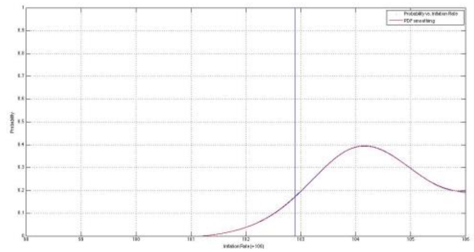


Figure 58: 26/2/2014 30 year horizon



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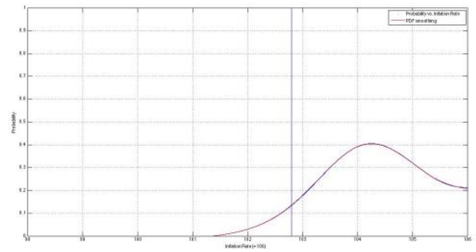

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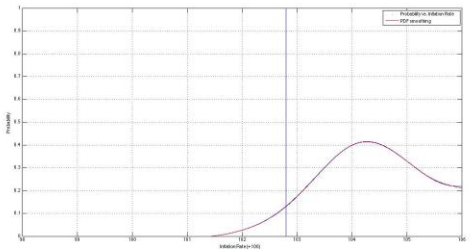

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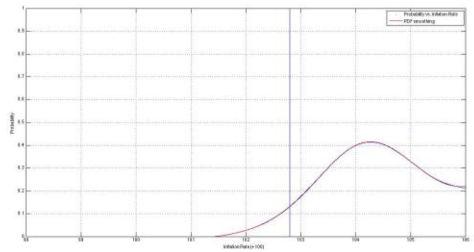
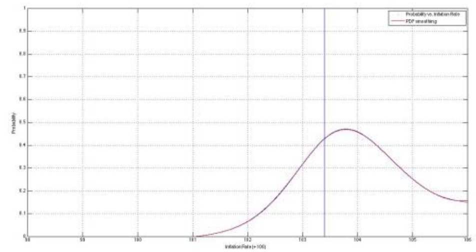


Figure 63: 12/11/2013 30 year horizon



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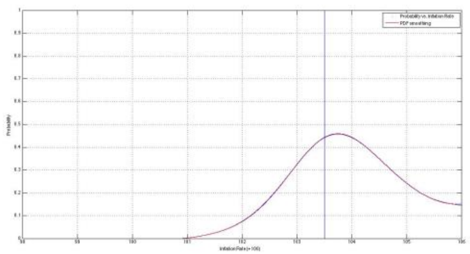

Figure 65: 17/9/2013 30 year horizon

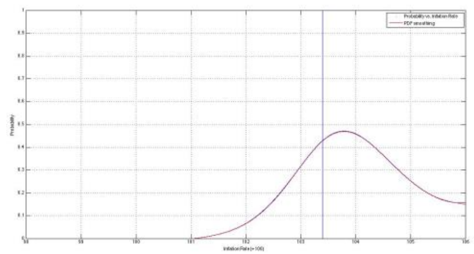

Figure 66: 13/8/2013 30 year horizon

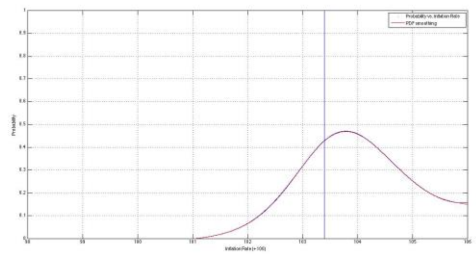

Figure 67: 16/7/2013 30 year horizon

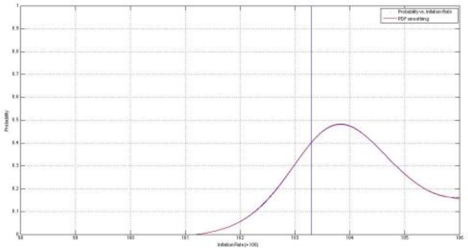

Figure 68: 18/6/2013 30 year horizon

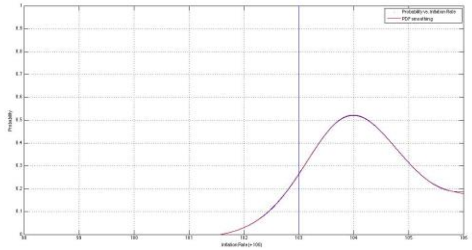

Figure 69: 21/5/2013 30 year horizon

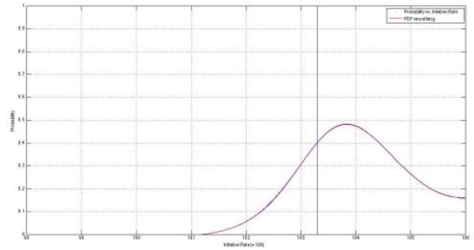

Figure 70: 16/4/2013 30 year horizon

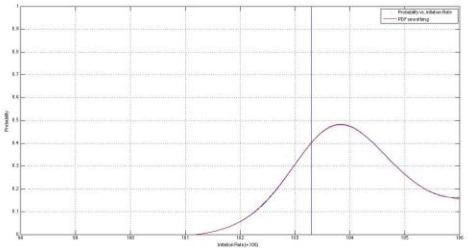

Figure 71: 19/3/2013 30 year horizon

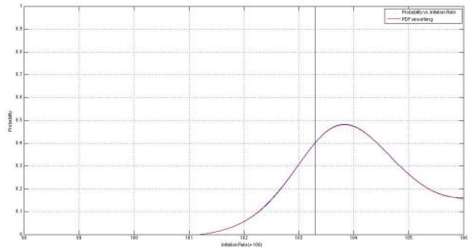

Figure 72: 12/2/2013 30 year horizon

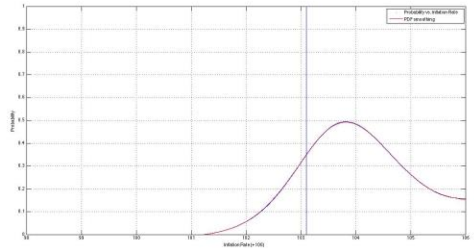

Figure 73: 15/1/2013 30 year horizon

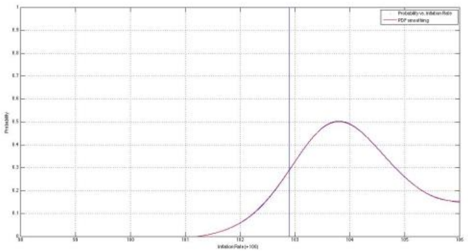

Figure 74: 18/12/2012 30 year horizon

Evidence from the two-year graphs and their statistics contradict *Hypothesis 1*, which suggests that expectations have become less grounded. The average of mean expectations over this period is 3%, which appears to be a good level for RPI. However, it does contradict the level of 4% from the long run results, which may be the correct level due to influences of premia that I will discuss later. Overall, expectations may be lower than the Bank of England's target but with a variance of the mean at only 1.434%, they are fairly stable.

At first glance the results from the medium term appear to contradict *Hypothesis 1* again. The mean is consistently 3.4%, a reasonable level for RPI, and the variance of the mean is only 0.809%. However, again noting this mean is below the twenty and thirty year rates, covered in the appendix, suggests that agents are expecting less than the Bank of England's target in the medium term, which confirms *Hypothesis 1*.

In the long-term the statistics appear to disagree with *Hypothesis 1*, which stated that the long-term expected inflation rate should attached to the Bank of England's target. The average of the mean expectation is 3.93% and the variance of this mean is 4.125%, higher than any other period. However, looking at the specific results we can observe consistency in two equilibria of 3.8% and 4.0%. The variance of the mean around these two points is negligible, implying that the long-term rate was stable. The sudden change between the equilibria is likely to be caused by a change in the market for inflation options, rather than a change in public opinion of inflation as no unpredictable economic event occurred at the time of the change; this highlights the impact the size of the market for inflation options has on the results produced.

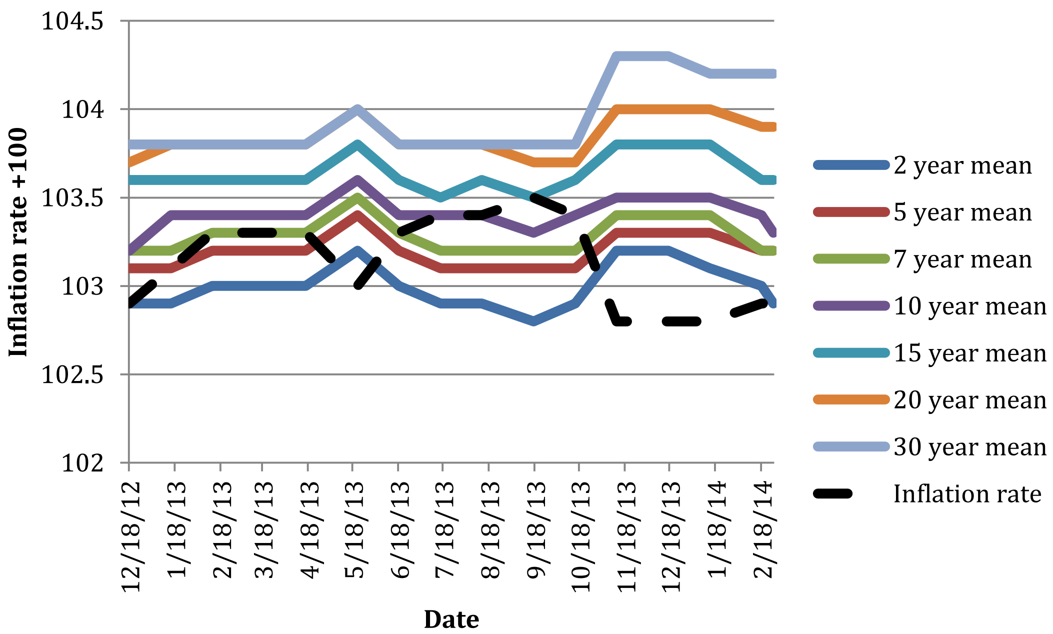

Figure 75

This final graph (figure 72) on mean expectations summarises the majority of the results from this paper. Here we can see a spread in expected inflation over all the maturities, though the longest appear to be the closest, supporting *Hypothesis 1*. Furthermore, there does not appear to be a strong relationship between the movements in inflation and expected inflation, suggesting that expectations are formed from other variables. This graph also highlights periods where inflation is expected to drop then rise. Most recently it shows that inflation is due to rise from 2 years onwards, possibly due to the UK's recent improvement in economic conditions. The Bank of England are probably pleased that expectations appear to be predicting that inflation will stay within the bounds of relative price stability.

The variance of the PDFs (not the mean) appears to be more reactive in shorter maturities (figure 76), representing a greater range of disagreement. Additionally, though the lack of tail fitting has limited the ability to compare variance across maturities, there appears to be a clear fall in variance as time increases, confirming *Hypothesis 2* for the short and medium term.

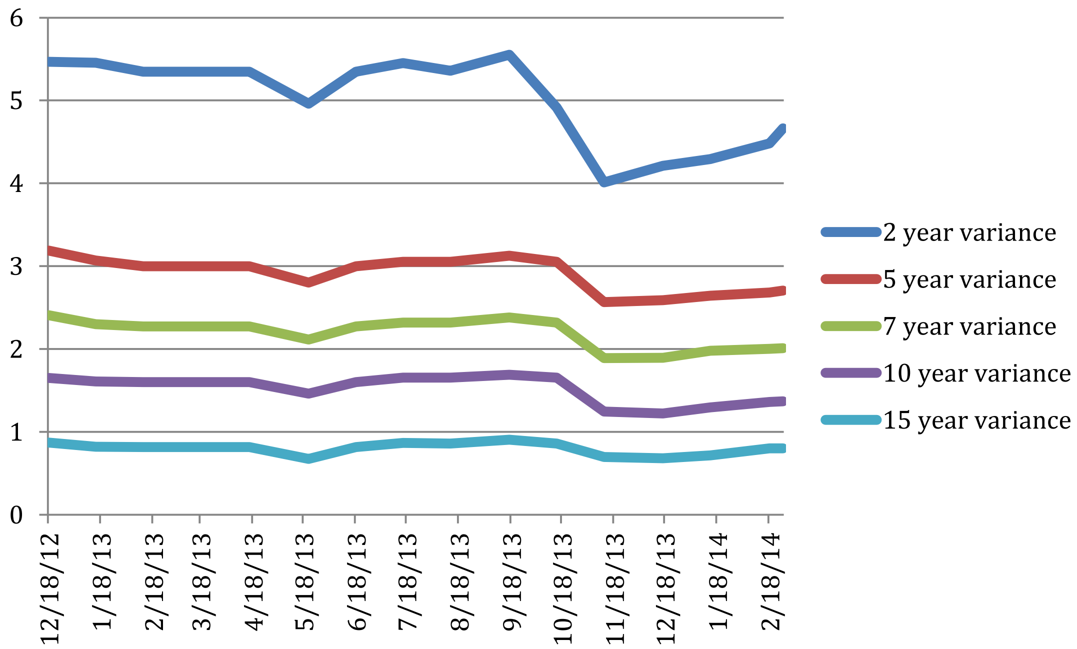

Figure 76

VI. Limitations and Extensions

Many of the limitations and extensions have been alluded to previously in this paper; this section shall elaborate on these and introduce others.

Firstly, as mentioned in the data section, this study would have benefitted from a longer time horizon, allowing a comparison of pre-crisis, banking crisis, Government debt crisis and post-crisis inflation expectations. Additionally, more maturities either from the data or by caplet stripping inflation options, would have allowed better tracking of the progression of inflation.

The method itself could have been improved in a number of ways. An implied volatility model appropriate for inflation options would have opened up more data. Furthermore, fitting the tails of the PDFs that were produced would have generated more accurate results and statistics. However, these are both beyond the scope of this paper.

Though the method followed the SABR model, which was picked by the Bank of England for fitting inflation data well, one could argue that it produces curves that are too smooth, hence creates more symmetric PDFs. Using a cubic piecewise polynomial at the volatility stage provided results that seemed to produce a more detailed picture of the market. Nevertheless this method is not tested, and the data set available from this paper is too small to do so.

As mentioned in the results section, all of the PDFs are inflated versions of the true values of expectations. The market is small and hence subject to a liquidity premia, moreover the extremities of inflation are likely to be more heavily traded due to their use in hedging. Furthermore, the method derived by Breedon and Litzenberger (1978) relies on all agents being risk neutral, where it is well known that most agents are risk averse. All three of these factors are likely to cause expectations to be higher than their true rate, which explains why the long run expectation of inflation was around 4%.

The final extension of the work done in this paper would be to apply this method to other markets. This could easily be done in the EU and its major countries, however it would be difficult to do the same in the US. This is because the main operators in inflation options are pension funds, minimising inflation risk, and the set up of this industry in the US negates this risk.

VII. Conclusion

An examination of the results provided by this unique method, has revealed that inflation expectations in the short and medium term may well have moved away from the Bank of England's target rate for RPI, supporting *Hypothesis 1*. This could have been caused by the visible contrast between consistent monetary policy and inconsistent inflation that has persuaded agents to believe that the banks are not willing or able to combat price instability, as we move out of recession. This also appears to have impacted the variance in expectations, which is much higher in the short term than any other maturity, supporting *Hypothesis 2*. However, expectations do appear to be fairly unresponsive to macroeconomic news other than inflation releases, contradicting *Hypothesis 3*. Overall, expectations appear to have become less grounded than an efficient market would suggest. However, the extent to which they have moved away from the target rate does not appear to suggest that there isn't confidence in the central bank, despite it's consistent monetary policy.

Throughout this study limitations of the model have made some of the results hard to interpret, and the lack of data has meant that certain conclusions cannot be reached, though the results presented in this paper do support the work done by the Bank of England, in all three of the hypotheses.

The paper has successfully created a working model for producing inflation expectations from inflation options and this groundwork can only help further studies to produce information that is extremely relevant to the Economy. Many extensions have been laid out that will solve the limiting factors in these results, and hopefully make this method a leading tool for central banks in the UK and Europe.

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IX. Appendix

IX.a. SABR- Stochastic Alpha Beta Rho model

First introduced by Hagan et al. (2002) the SABR model is an extension of both the Black Scholes model and the Local Volatility model, the former produces identical volatilities across all strike prices and the later produces dynamics which work in opposition to changes in the underlying price, hence the need for a new model.

The SABR model bases its formula on the Black Scholes model but uses a two-factor model, representing volatility as a random function of time, modelled using a stochastic process.

$$s_X = f(F, X, t, s_{ATM}, b, r, u)$$

Where F is the forward rate, X is the strike rate, τ is the time to maturity, σ_{ATM} is the 'at the money' volatility, β is a market variable (set at 0.7 for this paper, which is a level suggested by Hagan et al. (2002)), ρ is the correlation between the underlying and the volatility and υ is the volatility of the volatility. To find a better approximation of beta more data would have been required for this study.

The model itself is defined by the following formulas, which are proved in Hagan's paper:

$$\begin{aligned}
&\boldsymbol{\sigma_B(K,f)} \\
&= \frac{\boldsymbol{\alpha}}{(\boldsymbol{fK})^{\frac{1-\beta}{2}}\left\{\mathbf{1} + \frac{(\mathbf{1}-\boldsymbol{\beta})^2}{\mathbf{24}}\boldsymbol{log}^2\left(\frac{\boldsymbol{f}}{\boldsymbol{K}}\right) + \frac{(\mathbf{1}-\boldsymbol{\beta})^2}{\mathbf{1920}}\boldsymbol{log}^4\left(\frac{\boldsymbol{f}}{\boldsymbol{K}}\right) + \cdots\right\}}\left(\frac{\boldsymbol{z}}{\boldsymbol{x(z)}}\right)\left\{\mathbf{1}\right. \\
&\left. + \left[\frac{(\mathbf{1}-\boldsymbol{\beta})^2\boldsymbol{\alpha}^2}{\mathbf{24}(\boldsymbol{fK})^{\mathbf{1}-\boldsymbol{\beta}}} + \frac{\boldsymbol{\rho\beta\upsilon\alpha}}{\mathbf{4}(\boldsymbol{fK})^{\frac{1-\beta}{2}}} + \frac{(\mathbf{2}-\mathbf{3}\boldsymbol{\rho}^2)\boldsymbol{\upsilon}^2}{\mathbf{24}}\right]\boldsymbol{t_{ex}} + \cdots\right\} \\
&\boldsymbol{x(z)} = \boldsymbol{log}\left\{\frac{\sqrt{\mathbf{1}-\mathbf{2}\boldsymbol{\rho z}+\boldsymbol{z}^2}+\boldsymbol{z}-\boldsymbol{\rho}}{\mathbf{1}-\boldsymbol{\rho}}\right\} \\
&\boldsymbol{z} = \left(\frac{\boldsymbol{\upsilon}}{\boldsymbol{\alpha}}\right)(\boldsymbol{fK})^{\frac{1-\beta}{2}}\boldsymbol{log}\left(\frac{\boldsymbol{f}}{\boldsymbol{K}}\right)
\end{aligned}$$

For use in this paper, it was necessary to collaborate all variables to find a satisfactory smile over the data in the Volatility-Strike price space. The first step is to find alpha using the ATM volatility, given the cubic piecewise polynomial and the SABR curve are close at strikes near the ATM rate this paper found an approximation of the ATM volatility using the smoothing spline and then used this to find alpha, which is simply a root of the equation above. Once alpha had been computed ρ and υ were found by minimising the standard square error between the smile and implied volatilities provided in the data.

The Matlab codes are provided below for this process and the method, special thanks has to be given to Anton Weigardh for his template of the codes for the SABR model.

Caplet Stripping Method

As mentioned earlier, this paper could have followed a method called caplet stripping to us inflation caps and floors. This would have required taking each price and forming it into annual premiums, paid as if the option was a credit default swap. Then the difference between consecutive maturities would have given the premium of an option paying of at just one maturity. Converting this back into a price would have provided the same data as the zero coupon options, but at more maturities.

IX.b. Tables of curve fitting errors:

Two-year horizon:

		Call Price Space			First Order			PDF		
Date	Mean	R-Square	Adjusted R-Square	Root-Mean-Square Error	R-Square	Adjusted R-Square	Root-Mean-Square Error	R-Square	Adjusted R-Square	Root-Mean-Square Error
26/02/2014	102.9	1	1	2.36E-05	1	1	0.0031	1	1	0.13597164
18/02/2014	103	1	1	2.54E-05	1	1	0.0034	1	1	0.150186407
14/01/2014	103.1	1	1	3.10E-05	1	1	0.004	1	1	0.182438416
17/12/2013	103.2	1	1	3.20E-05	1	1	0.0041	1	1	0.001
12/11/2013	103.2	1	1	3.61E-05	1	1	0.0041	1	1	0.0011
15/10/2013	102.9	1	1	2.55E-05	1	1	0.0036	1	1	0.158521855
17/09/2013	102.8	1	1	2.72E-05	1	1	0.0028	1	1	0.13444961
13/08/2013	102.9	1	1	2.94E-05	1	1	0.0029	1	1	0.137640195
16/07/2013	102.9	1	1	2.94E-05	1	1	0.0029	1	1	0.137929582
18/06/2013	103	1	1	3.15E-05	1	1	0.0029	1	1	0.142171484
21/05/2013	103.2	1	1	3.85E-05	1	1	0.003	1	1	0.156347788
16/04/2013	103	1	1	3.15E-05	1	1	0.0029	1	1	0.142171484
19/03/2013	103	1	1	3.15E-05	1	1	0.0029	1	1	0.142171484
12/02/2013	103	1	1	3.15E-05	1	1	0.0029	1	1	0.142171484
15/01/2013	102.9	1	1	2.89E-05	1	1	0.0029	1	1	0.137528469
18/12/2012	102.9	1	1	3.05E-05	1	1	0.0028	1	1	0.139169551
Average	103			0.020359			0.0032			0.12756059
Variance	0.014335937									

Five-year horizon:

		Call Price Space			First Order			PDF		
Date	Mean	R-Square	Adjusted R-Square	Root-Mean-Square Error	R-Square	Adjusted R-Square	Root-Mean-Square Error	R-Square	Adjusted R-Square	Root-Mean-Square Error
26/02/2014	103.2	1	1	2.15E-05	1	1	0.0014	1	1	0.078477018
18/02/2014	103.2	1	1	2.15E-05	1	1	0.0014	1	1	0.078477018
14/01/2014	103.3	1	1	2.44E-05	1	1	0.0016	1	1	0.089900482
17/12/2013	103.3	1	1	2.46E-05	1	1	0.0016	1	1	0.093382285
12/11/2013	103.3	1	1	2.50E-05	1	1	0.0017	1	1	0.094290741
15/10/2013	103.1	1	1	2.31E-05	1	1	0.0014	1	1	0.078704132
17/09/2013	103.1	1	1	2.20E-05	1	1	0.0013	1	1	0.076348741
13/08/2013	103.1	1	1	2.31E-05	1	1	0.0014	1	1	0.078704132
16/07/2013	103.1	1	1	2.31E-05	1	1	0.0014	1	1	0.078704132
18/06/2013	103.2	1	1	2.43E-05	1	1	0.0014	1	1	0.081127291
21/05/2013	103.4	1	1	2.80E-05	1	1	0.0015	1	1	0.088521314
16/04/2013	103.2	1	1	2.43E-05	1	1	0.0014	1	1	0.081127291
19/03/2013	103.2	1	1	2.43E-05	1	1	0.0014	1	1	0.081127291
12/02/2013	103.2	1	1	2.43E-05	1	1	0.0014	1	1	0.081127291
15/01/2013	103.1	1	1	2.27E-05	1	1	0.0014	1	1	0.078416576
18/12/2012	103.1	1	1	2.15E-05	1	1	0.0013	1	1	0.07543479
Average	103.1933333			0.0159016			0.0014375			0.082116908
Variance	0.008085938									

Ten-year horizon:

		Call Price Space			First Order			PDF		
Date	Mean	R-Square	Adjusted R-Square	Root-Mean-Square Error	R-Square	Adjusted R-Square	Root-Mean-Square Error	R-Square	Adjusted R-Square	Root-Mean-Square Error
26/02/2014	103.3	1	1	1.60E-05	1	1	0.0012	1	1	4.07E-04
18/02/2014	103.4	1	1	1.61E-05	1	1	0.0012	1	1	4.10E-04
14/01/2014	103.5	1	1	1.81E-05	1	1	0.0012	1	1	4.33E-04
17/12/2013	103.5	1	1	1.78E-05	1	1	0.0013	1	1	4.73E-04
12/11/2013	103.5	1	1	1.77E-05	1	1	0.0013	1	1	4.69E-04
15/10/2013	103.4	1	1	1.76E-05	1	1	9.94E-04	1	1	3.62E-04
17/09/2013	103.3	1	1	1.68E-05	1	1	9.69E-04	1	1	3.51E-04
13/08/2013	103.4	1	1	1.76E-05	1	1	9.94E-04	1	1	3.62E-04
16/07/2013	103.4	1	1	1.76E-05	1	1	9.94E-04	1	1	3.62E-04
18/06/2013	103.4	1	1	1.84E-05	1	1	0.001	1	1	3.74E-04
21/05/2013	103.6	1	1	2.10E-05	1	1	0.0011	1	1	4.09E-04
16/04/2013	103.4	1	1	1.84E-05	1	1	0.001	1	1	3.74E-04
19/03/2013	103.4	1	1	1.84E-05	1	1	0.001	1	1	3.74E-04
12/02/2013	103.4	1	1	1.84E-05	1	1	0.001	1	1	3.74E-04
15/01/2013	103.4	1	1	1.81E-05	1	1	0.001	1	1	3.74E-04
18/12/2012	103.2	1	1	1.72E-05	1	1	0.001	1	1	3.77E-04
Average	103.4133333			0.01199355			0.05			0.071980
Variance	0.008085937									

Twelve-year horizon:

		Call Price Space			First Order			PDF		
Date	Mean	R-Square	Adjusted R-Square	Root-Mean-Square Error	R-Square	Adjusted R-Square	Root-Mean-Square Error	R-Square	Adjusted R-Square	Root-Mean-Square Error
26/02/2014	103.4	1	1	1.56E-05	1	1	0.0011	1	1	4.03E-04
18/02/2014	103.5	1	1	1.56E-05	1	1	0.0011	1	1	4.05E-04
14/01/2014	103	1	1	1.74E-05	1	1	0.0011	1	1	4.32E-04
17/12/2013	103.6	1	1	1.73E-05	1	1	0.0012	1	1	4.63E-04
12/11/2013	103.6	1	1	1.71E-05	1	1	0.0012	1	1	4.56E-04
15/10/2013	103.5	1	1	1.64E-05	1	1	0.001	1	1	3.86E-04
17/09/2013	103.4	1	1	1.56E-05	1	1	9.80E-04	1	1	3.73E-04
13/08/2013	103.4	1	1	1.64E-05	1	1	0.001	1	1	3.86E-04
16/07/2013	103.4	1	1	1.71E-05	1	1	0.001	1	1	3.99E-04
18/06/2013	103.5	1	1	1.97E-05	1	1	0.0011	1	1	4.40E-04
21/05/2013	103.7	1	1	1.71E-05	1	1	0.001	1	1	3.99E-04
16/04/2013	103.5	1	1	1.71E-05	1	1	0.001	1	1	3.99E-04
19/03/2013	103.5	1	1	1.71E-05	1	1	0.001	1	1	3.99E-04
12/02/2013										
15/01/2013										
18/12/2012										
Average	103.4666667			0.011387			0.05			0.08
Variance	0.025443787									

Fifteen-year horizon:

Date	Mean	Call Price Space R-Square	Adjusted R-Square	Root-Mean-Square Error	First Order R-Square	Adjusted R-Square	Root-Mean-Square Error	PDF R-Square	Adjusted R-Square	Root-Mean-Square Error
26/02/2014	103.6	1	1	1.53E-05	1	1	9.64E-04	1	1	4.06E-04
18/02/2014	103.6	1	1	1.52E-05	1	1	9.68E-04	1	1	4.07E-04
14/01/2014	103.8	1	1	1.69E-05	1	1	0.001	1	1	4.36E-04
17/12/2013	103.8	1	1	1.70E-05	1	1	0.0011	1	1	4.58E-04
12/11/2013	103.8	1	1	1.68E-05	1	1	0.0011	1	1	4.54E-04
15/10/2013	103.6	1	1	1.52E-05	1	1	0.001	1	1	4.34E-04
17/09/2013	103.5	1	1	1.45E-05	1	1	0.001	1	1	4.19E-04
13/08/2013	103.6	1	1	1.52E-05	1	1	0.001	1	1	4.34E-04
16/07/2013	103.5	1	1	1.52E-05	1	1	0.001	1	1	4.34E-04
18/06/2013	103.6	1	1	1.59E-05	1	1	0.0011	1	1	4.50E-04
21/05/2013	103.8	1	1	1.84E-05	1	1	0.0012	1	1	5.03E-04
16/04/2013	103.6	1	1	1.59E-05	1	1	0.0011	1	1	4.50E-04
19/03/2013	103.6	1	1	1.59E-05	1	1	0.0011	1	1	4.50E-04
12/02/2013	103.6	1	1	1.59E-05	1	1	0.0011	1	1	4.50E-04
15/01/2013	103.6	1	1	1.56E-05	1	1	0.0011	1	1	4.53E-04
18/12/2012	103.6	1	1	1.53E-05	1	1	0.0011	1	1	4.44E-04
Average	103.64			0.010713			0.05			0.08
Variance	0.00984375									

Twenty-year horizon:

		Call Price Space			First Order			PDF		
Date	Mean	R-Square	Adjusted R-Square	Root-Mean-Square Error	R-Square	Adjusted R-Square	Root-Mean-Square Error	R-Square	Adjusted R-Square	Root-Mean-Square Error
26/02/2014	103.9	1	1	1.52E-05	1	1	8.80E-04	1	1	4.41E-04
18/02/2014	103.9	1	1	1.53E-05	1	1	8.86E-04	1	1	4.45E-04
14/01/2014	104	1	1	1.66E-05	1	1	9.34E-04	1	1	4.72E-04
17/12/2013	104	1	1	1.69E-05	1	1	9.83E-04	1	1	4.98E-04
12/11/2013	104	1	1	1.66E-05	1	1	9.72E-04	1	1	4.92E-04
15/10/2013	103.7	1	1	1.41E-05	1	1	9.86E-04	1	1	4.80E-04
17/09/2013	103.7	1	1	1.36E-05	1	1	9.59E-04	1	1	4.61E-04
13/08/2013	103.8	1	1	1.41E-05	1	1	9.86E-04	1	1	4.80E-04
16/07/2013	103.8	1	1	1.41E-05	1	1	9.86E-04	1	1	4.80E-04
18/06/2013	103.8	1	1	1.48E-05	1	1	0.001	1	1	4.99E-04
21/05/2013	104	1	1	1.70E-05	1	1	0.0011	1	1	5.67E-04
16/04/2013	103.8	1	1	1.48E-05	1	1	0.001	1	1	4.99E-04
19/03/2013	103.8	1	1	1.48E-05	1	1	0.001	1	1	4.99E-04
12/02/2013	103.8	1	1	1.48E-05	1	1	0.001	1	1	4.99E-04
15/01/2013	103.8	1	1	1.45E-05	1	1	0.001	1	1	5.05E-04
18/12/2012	103.7	1	1	1.43E-05	1	1	0.001	1	1	5.01E-04
Average	103.84			0.0101743			0.18			0.09
Variance	0.011210938									

Thirty-year horizon:

		Call Price Space			First Order			PDF		
Date	Mean	R-Square	Adjusted R-Square	Root-Mean-Square Error	R-Square	Adjusted R-Square	Root-Mean-Square Error	R-Square	Adjusted R-Square	Root-Mean-Square Error
26/02/2014	104.2	1	1	1.48E-05	1	1	0.001	1	1	7.32E-04
18/02/2014	104.2	1	1	1.48E-05	1	1	0.001	1	1	7.34E-04
14/01/2014	104.2	1	1	1.60E-05	1	1	0.0011	1	1	8.00E-04
17/12/2013	104.3	1	1	1.63E-05	1	1	0.0012	1	1	8.38E-04
12/11/2013	104.3	1	1	1.63E-05	1	1	0.0012	1	1	8.37E-04
15/10/2013	103.8	1	1	1.20E-05	1	1	0.0012	1	1	8.00E-04
17/09/2013	103.8	1	1	1.16E-05	1	1	0.0012	1	1	7.53E-04
13/08/2013	103.8	1	1	1.20E-05	1	1	0.0012	1	1	8.00E-04
16/07/2013	103.8	1	1	1.20E-05	1	1	0.0012	1	1	8.00E-04
18/06/2013	103.8	1	1	1.24E-05	1	1	0.0013	1	1	8.52E-04
21/05/2013	104	1	1	1.40E-05	1	1	0.0016	1	1	0.001
16/04/2013	103.8	1	1	1.24E-05	1	1	0.0013	1	1	8.53E-04
19/03/2013	103.8	1	1	1.24E-05	1	1	0.0013	1	1	8.53E-04
12/02/2013	103.8	1	1	1.24E-05	1	1	0.0013	1	1	8.53E-04
15/01/2013	103.8	1	1	1.21E-05	1	1	0.0014	1	1	8.84E-04
18/12/2012	103.8	1	1	1.21E-05	1	1	0.0014	1	1	9.01E-04
Average	103.9333333			0.0089615			0.0012			0.15
Variance	0.04125									

IX.c. Matlab Code:

```matlab
function [pp_2]=impliedpdfSABRcurvefit(IMPLIEDVOLATILITY,InflationRate,Maturity,RISKFREE)
Volatility=IMPLIEDVOLATILITY/1000;
Strike=[98 99 100 101 102 103 104 105 106]';
yy1=csaps(Strike,Volatility);
ATMVol=fnval(yy1,InflationRate);
start = [0.3,0.3];
options = optimset('MaxFunEvals', 1e5, 'TolFun', 1e-8, 'TolX', 1e-10);
Beta=0.7;
[param, feval]=fminsearch(@(par)EstimateRhoAndVol(par,Strike,Volatility,ATMVol,InflationRate,Maturity,Beta),start,options);
r1=param(1);
v1=param(2);
a1 = findAlpha(InflationRate,103,Maturity,ATMVol,Beta,r1,v1);
STRIKE=linspace(98,106,100000);
for j=1:length(STRIKE);
SABRVolatility(j) = SABRvol(a1,Beta,r1,v1,102.9,STRIKE(j),2);
end
d_1=((sqrt(Maturity)*SABRVolatility).^-1).*(log(InflationRate*(STRIKE.^-1))+2*(RISKFREE+((SABRVolatility.^2)/2)));
d_2=((sqrt(Maturity)*SABRVolatility).^-1).*(log(InflationRate*(STRIKE.^-1))+2*(RISKFREE-((SABRVolatility.^2)/2)));
d1=double(d_1);
d2=double(d_2);
c=InflationRate*normcdf(d1')-(STRIKE'*exp(-RISKFREE*2)).*normcdf(d2');
[call, gof]=callpricesmooth(STRIKE,c);
dcalldx=differentiate(call,STRIKE);
[firstorder, gof1]=firstordersmooth(STRIKE,dcalldx);
d2calldx2=differentiate(firstorder,STRIKE);
[pdf, gof2]=pdfsmooth(STRIKE,(exp(Maturity*RISKFREE).*d2calldx2));
hold
y=[0 1];
x=[InflationRate InflationRate];
plot(x,y)
```

```matlab
function y = findAlpha(F,K,T,ATMvol,b,r,v)
% F = spot price
% K = strike price
% T = maturity
% ATMvol = ATM market volatility
% b = beta parameter
% r = rho parameter
% v = vol of vol parameter

C0 = -ATMvol*F^(1-b);
C1 = (1 + (2-3*r^2)*v^2*T/24);
C2 = r*b*v*T/4/F^(1-b);
C3 = (1-b)^2*T/24/F^(2-2*b);

AlphaVector = roots([C3 C2 C1 C0]);

index = find(AlphaVector>0);
Alpha = AlphaVector(index);
y = min(Alpha);
```

```matlab
function y = EstimateRhoAndVol(params,MktStrike,MktVol,ATMVol,F,T,b)

% Parameters:
% r = rho
% v = vol-of-vol
% Inputs:
% MktStrike = Vector of Strikes
% MktVol    = Vector of corresponding volatilities
% ATMVol = ATM volatility
% F = spot price
% T = maturity
% b = beta parameter

r = params(1);
v = params(2);
a = findAlpha(F,F,T,ATMVol,b,r,v);
N = length(MktVol);

for i=1:N
    ModelVol(i) = SABRvol(a,b,r,v,F,MktStrike(i),T);
    error(i) = (ModelVol(i) - MktVol(i))^2;
end;

y = sum(error);

if abs(r)>1
    y = 1e100;
end
```

```matlab
function y = SABRvol(a,b,r,v,F,K,T);
% a = alpha parameter
% b = beta parameter
% r = rho parameter
% v = vol of vol parameter
% F = spot price
% K = strike price
% T = maturity

if abs(F-K) <= 0.001

    Term1 = a/F^(1-b);
    Term2 = ((1-b)^2/24*a^2/F^(2-2*b) + r*b*a*v/4/F^(1-b) + (2-3*r^2)*v^2/24);
    y = Term1*(1 + Term2*T);

else

    z = v/a*(F*K)^((1-b)/2)*log(F/K);
    x = log((sqrt(1 - 2*r*z + z^2) + z - r)/(1-r));
    Term1 = a / (F*K)^((1-b)/2) / (1 + (1-b)^2/24*log(F/K)^2 + (1-b)^4/1920*log(F/K)^4);
    if abs(x-z) < 1e-10
        Term2 = 1;
    else
        Term2 = z / x;
    end
    Term3 = 1 + ((1-b)^2/24*a^2/(F*K)^(1-b) + r*b*v*a/4/(F*K)^((1-b)/2) + (2-3*r^2)/24*v^2)*T;
    y = Term1*Term2*Term3;

end
```