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Does Negative Interest Rate Policy work? A theoretical model of negative interest rate and deposit rate rigidity

Abstract:

This paper studies the transmission mechanism of monetary policy once policy rate fall below zero, particularly under a situation when deposit rate is rigid. It is being done so by constructing and solving an IS---MP model with the specification of equilibrium banking condition. The paper concludes that the effectiveness of negative interest rate policy depends on deposit rate rigidity. Moreover, the Government Spending multiplier is larger under negative interest rate, given deposit rate rigidity.

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Content

Introduction

The Zero Lower Bound has been known as the constraint on monetary policy, once policy rate reaches zero, monetary policy will become ineffective. As what Bernanke (2015) commented on the Taylor rule's prediction on Fed Fund rate "*If the Taylor rule predicts a sharply negative funds rate, which of course is not feasible ……*" With the consensus in the past that rate cannot fall below zero, Central Banks could only use unconventional monetary policy if they were to boost economic activities, such as Credit Easing and Quantitative Easing. However, deflationary pressure in some countries has pushed Central Banks such as the ECB, BOJ, Denmark National Bank to test the limit of our macro economy by making interest rate into the negative territory. In addition, Fed Chair Yellen said the negative interest rate remains as one of the potential option. The appearance of such an unorthodox monetary policy leads to a large debate on its feasibility and potential consequences. Therefore, the aim of this project is to build a theoretical model to study the transmission mechanism under negative interest rate policy, particularly under deposit rate rigidity.

The Negative interest rate

According to Goodfriend (2015), the mechanism of negative interest rate is to charge on commercial banks' reserve balance in the Central bank. Banks will then lend to each other to avoid being charged on excess reserves, driving the interbank rate to negative as well. It works because the other alternative, storing cash in physical term, has some operational cost. E.g. The operational cost handling huge amount of cash. Commercial banks might rather pay a little fee to the CB in order to avoid handling large amount of cash balance, and that is what make negative rate feasible. In theory, it also put downward pressure on other sources of loanable fund such as saving deposit caused by market force. Afterwards, the lower cost of loanable fund encourages banks to lend or buy assets and reduce rates on those assets as well. As a result, the negative rate stimulate spending and investment.

However, when a small negative interest rate takes place, commercial banks are reluctant to pass the negative rate to depositors, as they are afraid of losing customers, according to Goodfriend (2015) and Alsterlind (2015). It is somehow true if we look at the interest rate spread in Japan and Switzerland, deposit rate is being stuck at zero while policy rate is negative, see figure 1 and 2. I will then define a term called "deposit rate rigidity" as a situation when deposit rate cannot fall below a certain value. Given deposit rate is rigid and being kept above zero, it is reasonable to argue that banks' profit margins would be reduced due to the extra charge of negative reserve interest, that potentially lead to higher lending rates and dis-incentivise lending, according to Kane (2016), Halligan (2016). From their argument, it seems that the rigidity of deposit rate is playing a crucial role in determining the outcome of a negative policy rate.

The question then arises is **whether the effectiveness of negative interest rate depends on the rigidity of deposit rate**. But since the negative rate experiment have just started few years ago, very few observations could be used to conduct econometrics analysis, and along with the lagging effect of monetary policy, the empirical outcome remains very unclear. The alternative is to look at predictions by theoretical models in the literature. As far as I am concerned, there is little discussion regarding this aspect, it is mainly being focused on the feature of financial friction. Hence, the lead aim of this project is to answer this question by making an extension base on one of those financial friction models.

Existing models on financial friction

Carlin & Soskice (2015) proposed a simple model to model the banking mark-up. They use a commercial bank profit function and the loan demand equation to construct a system of equation for the loan market. This model is pretty intuitive in the way that it relates risk to the banking mark-up, basically by adding a risk premium on the policy rate. However, this model has two major drawbacks. 1) It is a model for a particular commercial bank and so do the risk, if we were to consider the banking sector as a whole, idiosyncratic risk should be diversified away and systematic risk is what left over, which this model tell us little about. 2) We have to assume that any extra unit of loan will have to be fully financed by lending from the CB, which is not realistic as there is no role for deposit.

Christiano and Ikeda (2012) have shown a two period model that I believe will not have the two issues above. In their model, households are lending to banks, and banks are lending to firms. Firms return are being drawn by a same distribution while some firms will be able to repay the loan upon realisation while some of them will not. Most importantly, there is some monitoring cost arises when banks are going to collect assets from default firms. Under a competitive market, p=mc. Therefore, the rate of lending have to be higher, generating a spread. This model is more useful when we are trying to capture the spread for the whole banking sector, rather than an individual bank. It includes the fact that there is real "cost" of making loans (monitoring cost). However, this is an endowment economy model, which means there is no production and price level and we cannot formulate a policy rule of CB that is responding to output gap and inflation. It is not very sensible to simply assume that the interest rate is exogenous.

Goodfriend and McCallum (2007) proposed a two sectors model, which are the household and the banking sector. It follows more or less a baseline New Keynesian model, but the differentiated part is how they formulate the banking sector. They specified that deposit is needed for transaction to take place, as a medium of exchange, and they have assumed that households borrow from banks in order to fund deposits. These assumptions allow households to demand lending and deposit and therefore generating rates that are associated with demand, it is the reason for the existence of different rates. In summary, there are five rates in their model, which are lending rate, deposit rate, bond rate, capital return and a return of a shadow riskless bond.

In my paper, the model follows more or less the one from Goodfriend and McCallum (2007). However, I have decided to adjust few elements that would make the model fits the topic more.

- Include Rate on reserves. The mechanism of negative interest rate is to charge on reserves, therefore I have added a feature of paying interest payment on central bank reserves for commercial banks. While such reserves rate can be positive or negative.
- Allow Deposit rate rigidity. Which means deposit rate cannot change according to policy rate, the deposit rate is being fixed after reaching certain low level.
- I excluded the feature of collateral service. The lead aim of this project is to analyse the effect of a negative policy rate but not the effect of having various rates, so it might be useful to cancel rates that only would have been occurred if there is collateral services, as it helps only little when dealing with the topic of this paper.
- I have taken away the dynamics feature for the sake of simplicity and feasibility. So the model in this paper will be a basic IS-MP model with the specification of money and banking.

In following sections, I will discuss equations that are representing different part of the economy and I will first solve the model treating monetary policy as exogenous. After that, I will endogenise the monetary rule and discuss the Government spending multiplier. Lastly, I will discuss about the limitation and further scope of research and conclude the paper.

Core model outline

Household

In this model, I separate household into two categories, which are depositors and lenders. Each categories will be represented by the following household equations:

(1) $C_D = cY_D - \gamma_D(i_D - E\pi)$

(2) $C_L = cY_L - \gamma_L(i_L - E\pi)$

where $C_{D,L}$ is the consumption by (depositors, borrowers) and $Y_{D,L}$ is the income that the (depositors, borrowers) receive and c is the marginal propensity to consume, with i_D is the deposit rate that depositors save, i_L is the lending rate at which borrowers borrow and $E\pi$ as expected inflation which make rates in real term. γ_L and γ_D are positive parameters.

In both functions, consumption is linearly related to income. As we can see from the functional from, it is similar to the Keynesian's consumption function, the slightly different part is how I relate consumption with various rates. In this case, I assume that consumption amongst depositors is negatively related to deposit rate, as we can see from (1). If we put it in the simple two period consumer's optimization problem, it would mean that the substitution effect dominates the wealth effect, given that the household is a net saver (depositor). This assumption is being used to create the unambiguous relationship between interest rate and consumption. I.e. If the central bank can decrease deposit rate, current consumption has to increase. On the other hand, for borrowers, substitution effect and wealth effect go in the same way, an decrease in lending rate would unambiguously make them consume more due to the lower cost[2].

Aggregate demand

(3) $Y = C_L + C_D + \varepsilon_d$

Y is the total income and ε_d is any exogenous spending or shock.

This is an AD equation while it takes into account two types of consumptions, exogenous spending and shocks. Since I assume other spending is exogenous e.g. Government Spending, I group such spending with exogenous shock so that they are all reflected by one parameter, ε_d.

The sum of two groups income is the total income, $Y_D + Y_L = Y$. Therefore, we can substitute this in to the combination of (1)-(3) to get the following expression[3].

(4) $Y = \dfrac{\varepsilon_d - \gamma_D(i_D - E\pi) - \gamma_L(i_L - E\pi)}{1-c}$

This equation summarize the demand side of the economy, which looks very much like an normal IS curve as real income is depends on real interest rates. This is a major difference from Goodfriend's (2007) model, as he was using dynamics household optimization problem as the foundation of the economy, while here it is an statics IS curve.

Supply side: The Phillips Curve

(5) $\pi = E\pi + \beta(Y - \overline{Y}) + \varepsilon_\pi$

This is a standard form of Phillips Curve, where π is inflation, $\overline{Y}$ is natural output, ε_π is inflation shock and β is a positive parameter.

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[2] A Full demonstration can be found in the appendix, page 22.
[3] A Full derivation can be found in the appendix, page 23.

Following what have been imposed by Goodfriend (2007), part of the firms in the economy cannot freely set price at each period, which creates price rigidity. So firms with rigidity will have to set price according to their expectation, and firms with flexible pricing will have to set price according to economic activity at current period. As a reflection of this, inflation would be a function of expected inflation, output and some exogenous shocks.

The monetary rule

(6) $i_{IB} = \theta_Y (Y - \bar{Y}) + \theta_\pi (\pi - \pi^T) + \rho + E\pi$

i_{IB} is the interbank rate, ρ is the natural rate of interest, π^T is the inflation target that central bank set and $\theta_{\pi,Y}$ are positive parameters. This is a Taylor rule type of monetary rule where interest rate is adjusting according to output deviation from potential level and the inflation deviation from its target. When there is a positive output gap and/or positive inflation deviation, central may increase interest rate to damp economic activities in order to drive the economy back to its potential. Here I assume the Central Bank has perfect control of interbank rate by changing the target policy rate and by using various kinds of facilities. For example, short term repurchase agreement. Moreover, some may refer this condition as the first order condition of minimizing a central bank loss function, see Carlin and Soskice (2015). But I will just present the rule arbitrarily that identifies the central bank behavior for the sake of simplicity.

Commercial Banks

The banking equilibrium conditions:

(7) $i_L = \mu + CORF$

i_L is the rate on loan, μ is the marginal effort/monitoring cost of producing an extra unit of loan. CORF is cost of raising a unit of fund

I assume a competitive banking market, therefore we would expect to see price equals to marginal cost. In the banking context, price per unit is the interest that a bank receives per unit of loan, which is i_L. The marginal cost can be defined as the sum of Monitoring Cost μ and CORF.

Monitoring cost is the effort that the bank put in order to produce the unit of loan. In Goodfriend (2007) paper, this marginal monitoring cost is endogenous which is determined by a loan production function equation. It is because loan is costly to produce, commercial banks need workers to do massive research about borrowers' risk, therefore it takes some real resources for bank to produce loans. But here I assume it is an exogenous mark up in order to remain the focus on deposit rate rigidity.

We now consider a bank j that raises all its fund from depositors:

(8) $CORF = i_D \frac{1}{(1-rr)} - i_{IB} \frac{rr}{(1-rr)}$

rr is an exogenous reserve ratio, we can interpret it as the reserve requirement that the Government need banks to keep with the central bank or it is the minimum reserve that banks has to keep in order to deal with transactions between banks.

CORF is the Cost of raising a unit of fund, which is the funding cost by borrowing from depositors minus the interest payment from the central bank on reserve.
I divided the deposit rate by (1-rr) because for every unit of loan, we have to raised 1/(1-rr) of deposit as some part of the deposit has to go to reserve according to the reserve ratio. For example, if rr = 0.2, we need to raised 1/1-0.2 = 1.25 units of deposit in order to lend out 1 unit of loan and keep 0.25 unit of reserve to keep the reserve ratio constant.
i_{IB} is the interest payment on 1 unit of reserve. For 1/(1-rr) unit of deposit raised, rr proportion of it will have to be saved in the Central Bank which receive interest. Using the example above, 0.2/0.8 = 0.25 unit has to be saved as reserve. It is arguable that i_{IB} might be different from reserve interest in the reality. But in fact, as an example, the Bank of England has suspended the channel system that drives the interbank rate down to the rate of reserve after the financial crisis, because too much reserves has been supplied, see Bank of England (2015). Therefore, I have chosen this abnormal time's set up because this paper is based on abnormal period. Such remuneration from the central bank reduces the cost of raising fund, so I deduct it away from the deposit raising cost.

No arbitrage condition

According to Goodfriend (2004), a commercial bank can raise fund from household or raise fund from the interbank market in rate i_{IB}. The cost of raising fund from two sources has to be the same in order to have no arbitrary in equilibrium, shown by the following equation.

(9) $CORF: i_{IB} = i_D \frac{1}{(1-rr)} - i_{IB} \frac{rr}{(1-rr)}$

The left hand side is the cost of raising fund as if the commercial bank borrows a unit from the interbank market. The right hand side is the cost of raising fund as if the commercial bank borrows from depositors. If we rearrange the equation,

$$i_{IB} = i_D \frac{1}{(1-rr)} - i_{IB} \frac{rr}{(1-rr)}$$

$$i_{IB} + i_{IB} \frac{rr}{(1-rr)} = i_D \frac{1}{(1-rr)}$$

$$i_{IB} = i_D$$

(9') $i_{IB} = i_D$

We will notice that i_D has to be adjusted according to i_{IB} in order to equalize two cost of funding. For example, when $i_{IB} < i_D$, banks will try to switch fund source to interbank market and it drives down i_D by market force.

If the deposit rate is flexible, the cost of two loanable fund source equalize and the equilibrium banking condition will be:

(10) $i_L = \mu + \underbrace{i_{IB}}_{CORF}$

We can see how monetary policy will be effective. If Central Bank decrease policy rate, the no arbitrage condition ensures deposit rate keep inline. It reduces the CORF for banks and lending rate decrease as well which leads to a successful expansionary monetary policy. But in contrast, what if the deposit rate is rigid and cannot fall below a certain value ω, which means commercial banks are "reluctant" to pass the low rate to depositors, we might see a different equilibrium conditions.

Deposit rate rigidity

In the following section I will model how deposit rate is going to be rigid and therefore derive the equilibrium banking condition under rigidity. I will model deposit rate rigidity by imposing banks a marginal "switching cost (SC)" when they are trying to switch to raise fund from depositors to interbank market. The SC can be seen as the opportunity cost of giving up a unit of deposit. When a commercial bank reduces the deposit rate that is low enough to induce cash withdrawal, there is a cost of losing customers. Moreover, I define a certain value ω as the minimum deposit rate that depositors accept and commercial banks know about this.

If $i_D > \omega$:

In a point of view of a particular commercial bank j, I know that depositors are still willing to deposit with me, and the market will always keep $i_{IB} = i_D$. So if the Central Bank decrease i_{IB}, the market will react and drive i_D down as well, and of course my depositors will also accept if I decrease the deposit rate at my bank, because they cannot find any better rates outside. So I will decrease i_D.

If $i_D = \omega$:

And if the CB decreases i_{IB} below ω, I will have two choices. I can either keep raising fund from depositors or I can raise fund from the interbank market at the lower rate. But I know that if I decrease i_D to i_{IB} and switch to raise fund from the interbank market, I will potentially lose all customers as no one would want to save at that rate. This is costly and I know I would be worst off if I turn down all my loyal customers. For example, given that $i_D = \omega$ =3, i_{IB} =3 and SC = 2. As a commercial bank, I originally raised 1 unit of money from depositors that cost me 3, but suddenly i_{IB} drop to 2. I would have been better off if I switch to raise fund from i_{IB} that just cost 2. But with the SC, the total cost would be 2+2=4. I would rather stay with paying depositor the interest of 3.

I assume all the banks will have the same analogy as above, the deposit rate will then be flexible above ω and rigid at ω . i.e. i_D stop following i_{IB} once it drops to ω . Moreover, i_D is rigid only if the SC is large enough to satisfy the following condition:

$$SC > \omega - i_{IB}$$

If i_{IB} is slightly below ω, banks will keep raise fund from depositors at the higher rate to avoid the high switching cost SC[4]. But if i_{IB} is low enough to violate the above condition, banks will find that SC is now comparatively a small loss comparing to the relatively high deposit rate that they are paying. All banks **have incentive** to switch to interbank market. But due to arbitrary, the market equilibrium should ensure i_D will go align with i_{IB} again at the very negative rate. This somehow agrees with Goodfriend's point that when policy rate is slightly negative, commercial bank is reluctant to pass the negative rate to depositors but once policy rate become very negative, commercial banks will have the incentive to do so. But this should be rare in reality as there will be no deposit demand at that rate and functions of banks break down.

The modeling above is used to derive the following condition, by combining (7), (8), $i_D = \omega$ and the assumption that all banks are identical[5].

(11) $$i_L = \mu + \underbrace{\omega \frac{1}{(1-rr)} - i_{IB} \frac{rr}{(1-rr)}}_{CORF}$$

Noted that the CORF specification suggests that banks keep borrowing from depositors at the lowest rate ω and have no incentive to switch to borrow from the interbank market. Otherwise, CORF = i_{IB}. Moreover, the i_{IB} term reflects the "cost" of putting reserve at the Central Bank, because i_{IB} is negative.

From the expression above, we can see **if we decrease i_{IB} while ω is fixed, we would increase the cost of raising fund and therefore the price of loan i_L in equilibrium.** During recession, when the Central Bank is trying to decrease

[4] In fact, almost all countries that have imposed negative interest rate did not be able to make it far negative than -1%, it is a reasonably little amount.

[5] Banks raise their fund entirely from depositors for the sake of simplicity, although it weakens the no arbitrage condition under flexible deposit rate.

policy rate but deposit rate is rigid, all it is doing is to increase the cost of raising fund, and it would potentially be harmful to the economy.

Solving the model

Does the effectiveness of negative interest rate depend on deposit rate rigidity?

In the next section, I will consider the consequence of decreasing the policy rate down to the negative territory. Firstly, I will treat i_{IB} as exogenous and see the consequences in scenario A) Flexible deposit rate and B) Rigid deposit rate.

Scenario A) Flexible deposit rate

Solving equilibrium conditions (4) , (5) , (9') and (10)[6]:

$$Y = \frac{\varepsilon_d - \gamma_D(i_D - E\pi) - \gamma_L(i_L - E\pi)}{1-c}$$

$$\pi = E\pi + \beta(Y - \overline{Y}) + \varepsilon_\pi$$

$$i_L = i_{IB} + \mu$$

$$i_D = i_{IB}$$

given $\varepsilon_d, \gamma_D, c, \beta, \overline{Y}, i_{IB}$, $\varepsilon_\pi = 0$ and $E\pi = \pi^T$[7] exogenously.

$$Y^* = \frac{\varepsilon_d - \gamma_D(i_{IB} - \pi^T) - \gamma_L(i_{IB} + \mu - \pi^T)}{1-c}$$

$$\frac{dY^*}{di_{IB}} = -\frac{\gamma_D + \gamma_L}{1-c}$$

I solve for Y* and represent it by all exogenous variables and find the derivatives with respect to policy rate. The expression is unambiguously negative, which means a decrease in policy rate would increase output. The model predicts that if

6 Full work out can be found in appendix, page 24.
7 I assume rational expectation and credible Central Bank.

the deposit rate is flexible, monetary policy works perfectly fine, even when interest rate goes below negative. We can imagine that it is a scenario when the lowest deposit rate that depositors accept is negative, i.e. ω <0. In that case, deposit rate will still be flexible even it is below 0 and above ω .

Scenario B) Rigid deposit rate

Solving equilibrium conditions (4) , (5) and (11):

$$Y=\frac{\varepsilon_d-\gamma_D(i_D-E\pi)-\gamma_L(i_L-E\pi)}{1-c}$$

$$\pi=E\pi+\beta(Y-\overline{Y})+\varepsilon_\pi$$

$$i_L=\mu+\omega\frac{1}{(1-rr)}-i_{IB}\frac{rr}{(1-rr)}$$

given $\varepsilon_d,\gamma_D,c,\beta,\overline{Y},i_{IB}$, $\varepsilon_\pi=0$, $E\pi=\pi^T$ and $i_D=\omega$ exogenously[8].

$$Y^*=\frac{\varepsilon_d-\gamma_D(\omega-\pi^T)-\gamma_L(\mu+\frac{\omega}{1-rr}-i_{IB}\frac{rr}{1-rr}-\pi^T)}{1-c}$$

$$\frac{dY^*}{di_{IB}}=\frac{\gamma_L rr}{(1-c)(1-rr)}>0$$

The derivative of output with respect to policy rate is unambiguously positive. Suppose the economy is now at an equilibrium where $i_D=\omega$ =0, a further decrease in policy rate down to negative will cause deposit rate to be rigid. Further suppose a negative shock hits the economy and the Central Bank decrease policy rate to counteract the shock, the monetary policy will be ineffective as all it is doing is to increase the cost of funding for commercial bank, which in turn increase the lending rate that will discourage net borrower's consumption and therefore impose a negative effect on output. However, the magnitude of the effect should not be expected to be huge. Because 1) In reality, we are observing slightly negative rates amongst those who turned negative, which should brings a small effect. i.e. di_{IB} is small. 2) rr is reasonably small in many countries. In fact, the reserve ratio in Japan is about 1%, it further limits the negative effect.

[8] Full work out can be found in appendix, page 25.

The role of rr

$$d\left(\frac{dY^*}{di_{IB}}\right)\Big/ drr = \frac{\gamma_L}{(1-rr)}\left(\frac{1}{(1-c)} + \frac{rr}{1-rr}\right) > 0$$

The derivative above is strictly positive, which means the larger the rr, the larger the cost of negative interest rate. If rr is higher, a higher proportion of deposit will have to be saved for one unit of loan raised, lending rate is then much more sensitive to policy rate change, i.e. the change of i_L is larger due to a unit change of i_{IB}, creating a larger impact on the change of output by a unit change of policy rate.

Government Spending Multiplier under negative interest rate.

Referring to Christiano, Eichenbaum and Rebelo (2009), they found that the Government Spending multiplier will be larger at the zero lower bound, as the nominal interest rate does not response to the increase in Government spending. In the following section, I am going to show that the Government spending multiplier is also going to be larger under negative interest rate (given rigidity), comparing to an scenario where the interest rate is positive (flexible). To do that, I will endogenise the policy rate by putting the monetary rule into the system, and solve the system of equations in two scenarios, C) Negative interest rate (Rigid deposit rate) and D) Positive interest rate (Flexible deposit rate). Here I will assume that deposit rate has to be rigid if policy rate is negative, i.e. ω =0.

Scenario C) Negative interest rate[9] (therefore Rigid deposit rate). Solving equilibrium conditions (4) , (5) , (6) and (11)[10], given $\varepsilon_d, \gamma_D, c, \beta, \overline{Y}$, $\varepsilon_\pi = 0$, $E\pi = \pi^T$ and $i_D = \omega$ exogenously. I will treat the exogenous Government spending a positive demand shock, and we will have to find the derivatives of output with respect to ε_d to find the multiplier.

[9] I assume the economy has already entered Negative interest rate territory by some previous negative shock.

[10] Full work out can be found in appendix, page 26-27.

$$\frac{dY_c*}{d\varepsilon_d}=\frac{1}{1-c-\dfrac{rr\gamma_L(\theta_Y+\theta_\pi\beta)}{1-rr}}$$

where $\dfrac{rr\gamma_L(\theta_Y+\theta_\pi\beta)}{1-rr}>0$, as all of the parameter is positive and rr<1. But we need to have $\dfrac{rr\gamma_L(\theta_Y+\theta_\pi\beta)}{1-rr}<1-c$ to avoid non-sensible solution. This should be a reasonable specification given that rr is reasonably small.

Scenario D) Positive interest rate (Flexible deposit rate)

Solving equilibrium conditions (4) , (5) , (6), (9') and (10)[11], given $\varepsilon_d,\gamma_D,c,\beta,\overline{Y}$, $\varepsilon_\pi=0$ and $E\pi=\pi^T$ exogenously.

$$\frac{dY_d*}{d\varepsilon_d}=\frac{1}{1-c+(\gamma_D+\gamma_L)(\theta_Y+\theta_\pi\beta)}$$

where $(\gamma_D+\gamma_L)(\theta_Y+\theta_\pi\beta)>0$

$$\therefore\frac{dY_c*}{d\varepsilon_d}>\frac{dY_d*}{d\varepsilon_d}$$

The multiplier c is greater than d. In c, the additional Government spending provokes the CB to increase rate, which make the rate "less negative". The smaller negative rate reduces the cost of negative interest and therefore promotes economic activity, co-operating with the additional Government spending. In contrast, the d multiplier is smaller because Central Bank is trying to limit the deviation of output by increasing interest rate. This result is some how similar to what Christiano, Eichenbaum and Rebelo have found, but in the prediction of this model, the multiplier is larger not because nominal interest does not react to additional spending, but rather being "less negative" that co-operates with Government Spending and further promote economic activities.

[11] Full work out can be found in appendix, page 28.

Limitations & Further scope of research

Unlike the general New Keynesian type of model, the specification of consumption in this model is a linear equation that depends only on current income and interest rates. It did not take into account the dynamics of income and expectations, and therefore is not a maximization problem subject to certain constraints, that indeed makes the model less intuitive and less comprehensive. Perhaps the next step of research in this area is to build a similar model in the New Keynesian context, we will need to use log linearization in order to find the impulses response function. While this technique rely on a particular steady state, we then have to be careful when finding steady states, as different rates in general will have different steady states, that might cause problem when doing analysis. Furthermore, there is no role for production to take place in this model, so it might be less convincing to present a Phillips curve where it represents how firms set price.

I slightly discussed the reserve ratio rr is this paper, but I would think that the reserve ratio topic might be an interesting area to explore. Here I have just assumed that it is exogenous, i.e. it is being set by the Government or by the operational requirement. But in reality, many commercial banks keep a lot more reserve than they needed to do so, see Keister and McAndrews (2009). The reserve ratio is not binding. It might somehow be related to the risk aversion of commercial banks and the rr ratio could have become more endogenous in further research.

A potential issue of Negative interest

In this paper, I have discussed that the effectiveness of negative interest rate did depends on deposit rate rigidity. At the case when ω =0, deposit rate is rigid at 0, and negative interest will be ineffective and costly. But even when the case is ω <0, i.e. depositors accept paying a little cost of saving to avoid handling large amount of cash. So that deposit rates keep flexible for a little negative range (for which negative interest rate is effective under that range), there might be a problem. There is always going to be a "lower bound" for which deposit rate cannot fall below, there will be no deposit demand below that rate and commercial banks are not willing to offer lower rates than that. The "negative interest rate" policy has to stop working once it falls below the lowest value ω. Same problem occurs between the CB and commercial banks as well. I assume reserve ratio is fixed and exogenous in this paper, but in reality, a commercial bank can in theory not putting any reserve in the CB (if there is no reserve requirement) and if interest rate is so negative that induce huge cost. The commercial bank can do that by storing physical cash that enable it to operate normally, it gives them in definition a 0% interest rate, although it bring additional cost of handling physical cash. The effectiveness of negative interest become very limited due to this fact.

Goodfriend (2015) suggests some methods that could overcome such situation. e.g. eliminate physical cash. It should in theory get rid of the "best alternative" of saving deposit. But it would be very hard to materialize because it is implicitly "charging" consumers and banks to possess wealth in monetary term and might be very hard to be approved.

Conclusion

After the 2008 financial crisis, ease monetary policy was being made in many countries, we have seen that nominal interests are being dragged to very low levels comparing to those during pre-crisis period. In the presence of deflationary pressure in many countries, CBs are struggling to find an effective way to stimulate the economy given rates are approaching zero. Following Goodfriend's idea on un-encumbering interest rate policy at the zero lower bound, CBs are trying to test the limit of the economy by making rate into the negative territory, and I believe many others CBs may follow if another crisis were to arrive in the future.

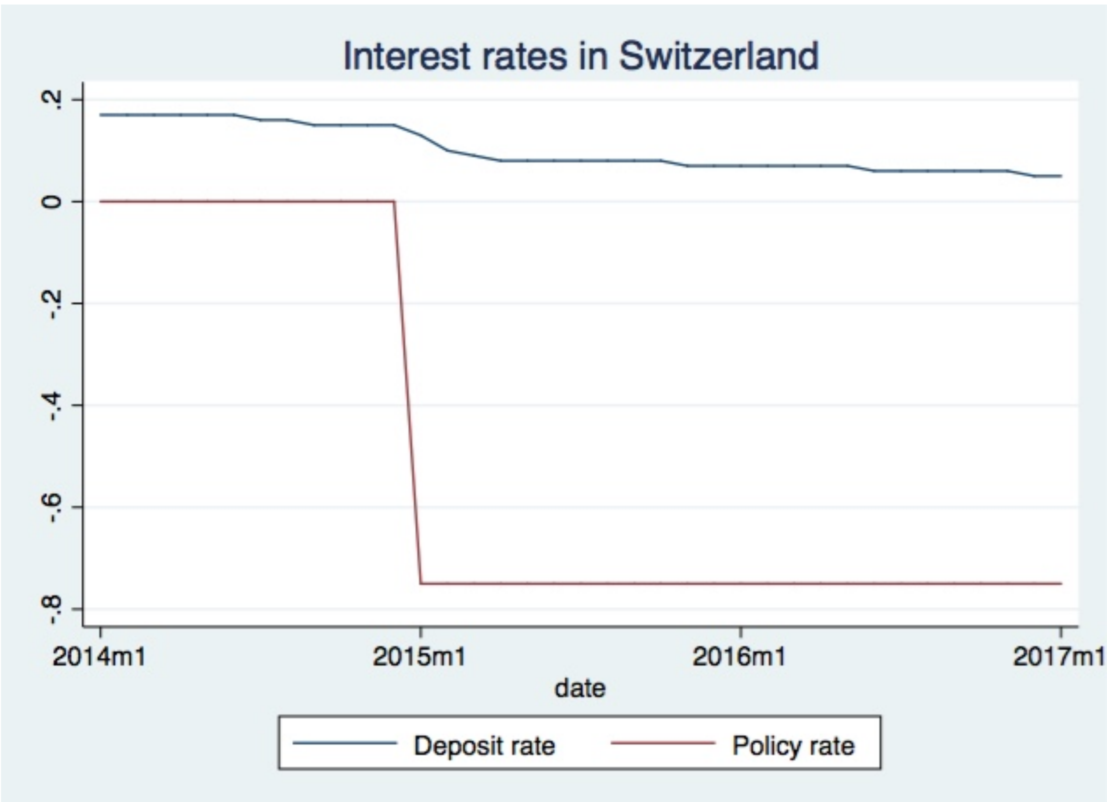
This project outlines one of the potential consequences of the negative interest rate in a theoretical point of view. If deposit rate is rigid, negative interest rate could be ineffective and costly. In contrast, if deposit rate is flexible, negative interest rate should works perfectly fine, similar to a conventional monetary policy. I have also found that the Government spending multiplier is larger when deposit rate is rigid, that similar to the result found by Christiano, Eichenbaum and Rebelo (2009). However, even there is a negative range that deposit rate is flexible, there is always going to be a lower bound that interest rate can not fall below, which limit the effectiveness of negative interest rate. When we are considering negative interest rate as a potential option of monetary policy, we should fully assess the economy and avoid imposing costs that would depress economic activities.

Appendix

Fig 1, Rates in Japan. (Green line as deposit rate and orange line as policy)

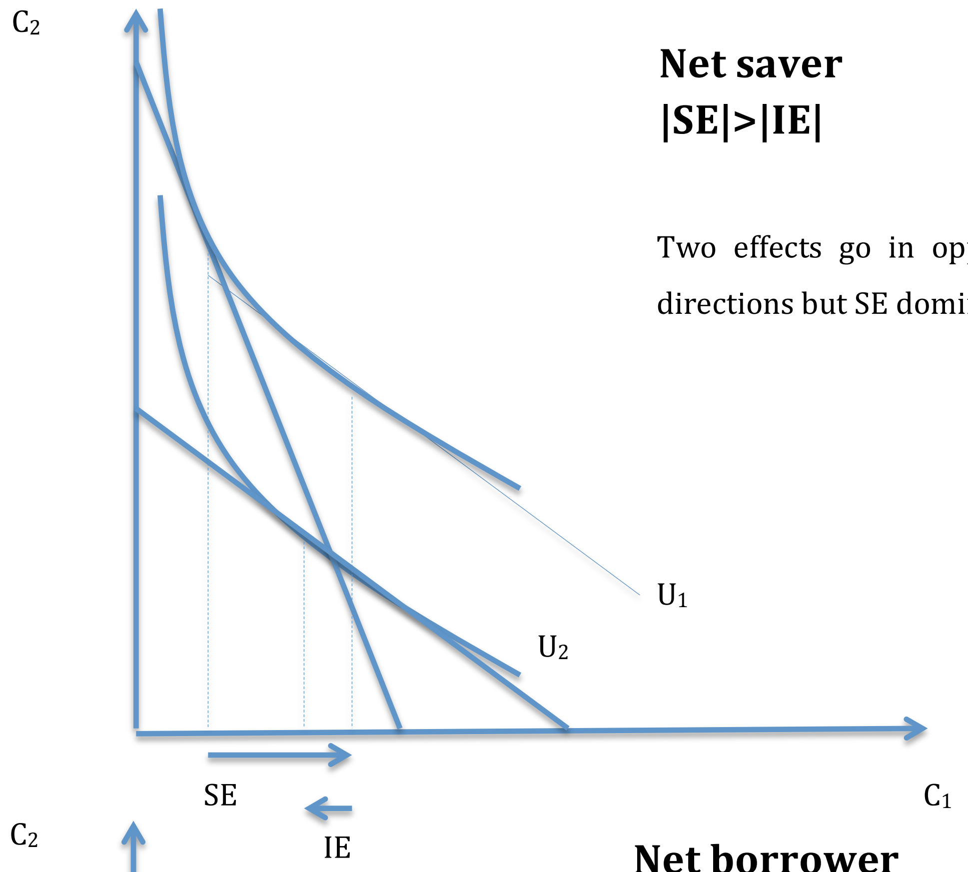
Source: Bank of Japan.

Fig 2, Interest rates in Switzerland.



Source: Swiss National Bank

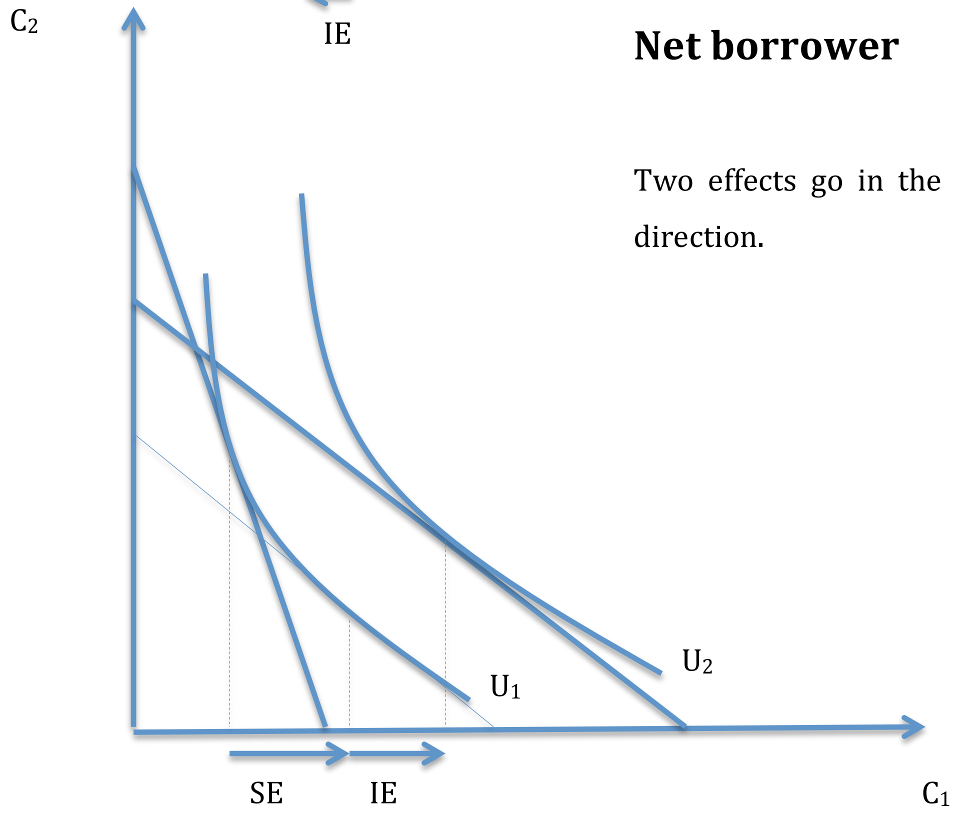
Two period Optimization Problem: A fall in interest rate.



Net saver

|SE|>|IE|

Two effects go in opposite directions but SE dominates.



Net borrower

Two effects go in the same direction.

Deriving the expression of real income

Combine these three equations:

$$C_D = cY_D - \gamma_D(i_D - E\pi)$$

$$C_L = cY_L - \gamma_L(i_L - E\pi)$$

$$Y = C_L + C_D + \varepsilon_d$$

We get,

$$Y = cY_L - \gamma_L(i_L - E\pi) + cY_D - \gamma_D(i_D - E\pi) + \varepsilon_d$$

$$Y - cY_L - cY_D = -\gamma_L(i_L - E\pi) - \gamma_D(i_D - E\pi) + \varepsilon_d$$

Include the fact that : $Y_D + Y_L = Y$ and the fraction remains constant

$$\frac{Y_D}{Y} = \eta$$

$$\frac{Y_L}{Y} = 1 - \eta$$

We will get:

$$Y - c(1-\eta)Y - c\eta Y = -\gamma_L(i_L - E\pi) - \gamma_D(i_D - E\pi) + \varepsilon_d$$

$$Y(1-c) = \varepsilon_d - \gamma_L(i_L - E\pi) - \gamma_D(i_D - E\pi)$$

$$\therefore Y = \frac{\varepsilon_d - \gamma_D(i_D - E\pi) - \gamma_L(i_L - E\pi)}{1-c}$$

Scenario A

(1) $Y = \frac{\varepsilon_d - \gamma_D(i_D - E\pi) - \gamma_L(i_L - E\pi)}{1-c}$

(2) $\pi = E\pi + \beta(Y - \overline{Y}) + \varepsilon_\pi$

(3) $i_L = i_{IB} + \mu$

(4) $i_D = i_{IB}$

(5) $E\pi = \pi^T$

(6) $\varepsilon_\pi = 0$

Combine 1,3-5 and get,

$$Y^* = \frac{\varepsilon_d - \gamma_D(i_{IB} - \pi^T) - \gamma_L(i_{IB} + \mu - \pi^T)}{1-c}$$

thus,

$$\pi^* = \pi^T + \beta\left(\frac{\varepsilon_d - \gamma_D(i_{IB} - \pi^T) - \gamma_L(i_{IB} + \mu - \pi^T)}{1-c} - \overline{Y}\right)$$

and

$$\frac{dY^*}{di_{IB}} = -\frac{\gamma_D + \gamma_L}{1-c}$$

Scenario B

(1) $Y = \frac{\varepsilon_d - \gamma_D (i_D - E\pi) - \gamma_L (i_L - E\pi)}{1-c}$

(2) $\pi = E\pi + \beta(Y - \overline{Y}) + \varepsilon_\pi$

(3) $i_L = \mu + \omega \frac{1}{(1-rr)} - i_{IB} \frac{rr}{(1-rr)}$

(4) $E\pi = \pi^T$

(5) $i_D = \omega$

(6) $\varepsilon_\pi = 0$

Combine 1,3-4 and get,

$$Y^* = \frac{\varepsilon_d - \gamma_D(\omega - \pi^T) - \gamma_L(\mu + \frac{\omega}{1-rr} - i_{IB}\frac{rr}{1-rr} - \pi^T)}{1-c}$$

Combine Y*,4 and 6

$$\pi^* = \pi^T + \beta(\frac{\varepsilon_d - \gamma_D(\omega - \pi^T) - \gamma_L(\mu + \frac{\omega}{1-rr} - i_{IB}\frac{rr}{1-rr} - \pi^T)}{1-c} - \overline{Y})$$

and

$$\frac{dY^*}{di_{IB}} = \frac{\gamma_L rr}{(1-c)(1-rr)}$$

Scenario C

(1) $Y = \frac{\varepsilon_d - \gamma_D(i_D - E\pi) - \gamma_L(i_L - E\pi)}{1-c}$

(2) $\pi = E\pi + \beta(Y - \overline{Y}) + \varepsilon_\pi$

(3) $i_{IB} = \theta_Y(Y - \overline{Y}) + \theta_\pi(\pi - \pi^T) + \rho + E\pi$

(4) $i_L = \mu + \omega \frac{1}{(1-rr)} - i_{IB} \frac{rr}{(1-rr)}$

(5) $i_D = \omega$

(6) $E\pi = \pi^T$

(7) $\varepsilon_\pi = 0$

Combine 2-4 and 6-7

$$i_L = \mu + \omega \frac{1}{(1-rr)} - \left\{\theta_Y(Y-\overline{Y}) + \theta_\pi(\pi^T + \beta(Y-\overline{Y}) - \pi^T) + \rho + \pi^T\right\} \frac{rr}{(1-rr)}$$

$$i_L = \mu + \omega \frac{1}{(1-rr)} - \frac{rr\theta_Y(Y-\overline{Y})}{(1-rr)} - \frac{rr\theta_\pi}{(1-rr)}(\pi^T + \beta(Y-\overline{Y}) - \pi^T) - \frac{rr\rho}{(1-rr)} - \frac{rr\pi^T}{(1-rr)}$$

$$i_L = \mu + \omega \frac{1}{(1-rr)} - \frac{rr\theta_Y(Y-\overline{Y})}{(1-rr)} - \cancel{\frac{rr\theta_\pi}{(1-rr)}\pi^T} - \frac{rr\theta_\pi}{(1-rr)}\beta(Y-\overline{Y}) + \cancel{\frac{rr\theta_\pi}{(1-rr)}\pi^T} - \frac{rr\rho}{(1-rr)} - \frac{rr\pi^T}{(1-rr)}$$

$$i_L = \mu + \omega \frac{1}{(1-rr)} - \frac{rr\rho}{(1-rr)} - \frac{rr\pi^T}{(1-rr)} - \frac{rr\theta_Y}{(1-rr)}(Y-\overline{Y}) - \frac{rr\theta_\pi\beta}{(1-rr)}(Y-\overline{Y})$$

$$i_L = \mu + \omega \frac{1}{(1-rr)} - \frac{rr\rho}{(1-rr)} - \frac{rr\pi^T}{(1-rr)} - \frac{rr\theta_Y}{(1-rr)}Y + \frac{rr\theta_Y}{(1-rr)}\overline{Y} - \frac{rr\theta_\pi\beta}{(1-rr)}Y + \frac{rr\theta_\pi\beta}{(1-rr)}\overline{Y}$$

$$i_L = \underbrace{\mu + \omega \frac{1}{(1-rr)} - \frac{rr\rho}{(1-rr)} - \frac{rr\pi^T}{(1-rr)} + \frac{rr\theta_Y}{(1-rr)}\overline{Y} + \frac{rr\theta_\pi\beta}{(1-rr)}\overline{Y}}_{\overline{A}} - Y\frac{rr(\theta_Y + \theta_\pi\beta)}{(1-rr)}$$

(8) $i_L = \overline{A} - Y\frac{rr(\theta_Y + \theta_\pi\beta)}{(1-rr)}$

Combine 1,5,6 and 8

$$Y(1-c) = \varepsilon_d - \gamma_D(\omega - \pi^T) - \gamma_L(\overline{A} - Y\frac{rr(\theta_Y + \theta_\pi\beta)}{(1-rr)} - \pi^T)$$

$$Y(1-c) = \varepsilon_d - \gamma_D(\omega - \pi^T) - \gamma_L\overline{A} + \gamma_L\pi^T + Y\frac{rr\gamma_L(\theta_Y + \theta_\pi\beta)}{(1-rr)}$$

$$Y(1-c-\frac{rr\gamma_L(\theta_Y + \theta_\pi\beta)}{(1-rr)}) = \varepsilon_d \underbrace{-\gamma_D(\omega - \pi^T) - \gamma_L\overline{A} + \gamma_L\pi^T}_{\overline{B}}$$

$$Y(1-c-\frac{rr\gamma_L(\theta_Y + \theta_\pi\beta)}{(1-rr)}) = \varepsilon_d + \overline{B}$$

$$Y^* = \frac{\varepsilon_d + \overline{B}}{1 - c - \frac{rr\gamma_L(\theta_Y + \theta_\pi \beta)}{(1-rr)}}$$

thus

$$\pi^* = \pi^T + \frac{\beta(\varepsilon_d + \overline{B})}{1 - c - \frac{rr\gamma_L(\theta_Y + \theta_\pi \beta)}{(1-rr)}} - \overline{Y}$$

and

$$\frac{dY_c^*}{d\varepsilon_d} = \frac{1}{1 - c - \frac{rr\gamma_L(\theta_Y + \theta_\pi \beta)}{1-rr}}$$

Scenario D

(1) $Y = \dfrac{\varepsilon_d - \gamma_D(i_D - E\pi) - \gamma_L(i_L - E\pi)}{1-c}$

(2) $\pi = E\pi + \beta(Y - \overline{Y}) + \varepsilon_\pi$

(3) $i_{IB} = \theta_Y(Y - \overline{Y}) + \theta_\pi(\pi - \pi^T) + \rho + E\pi$

(4) $i_L = i_{IB} + \mu$

(5) $i_D = i_{IB}$

(6) $E\pi = \pi^T$

(7) $\varepsilon_\pi = 0$

Combine 1,5-6

$$Y(1-c) = \varepsilon_d - \gamma_D(i_{IB} - \pi^T) - \gamma_L(i_{IB} + \mu - \pi^T)$$

$$Y(1-c) = \varepsilon_d - \gamma_D i_{IB} + \gamma_D \pi^T - \gamma_L i_{IB} - \gamma_L \mu + \gamma_L \pi^T$$

$$Y(1-c) = \varepsilon_d + \underbrace{\gamma_D \pi^T - \gamma_L \mu + \gamma_L \pi^T}_{\overline{C}} - i_{IB}(\gamma_D + \gamma_L)$$

(8) $Y(1-c) = \varepsilon_d + \overline{C} - i_{IB}(\gamma_D + \gamma_L)$

Combine 2,3,6 and 7

$$i_{IB} = \theta_Y(Y - \overline{Y}) + \theta_\pi(\pi^T + \beta(Y - \overline{Y}) - \pi^T) + \rho + \pi^T$$

$$i_{IB} = \underbrace{\rho + \pi^T - \theta_Y \overline{Y} - \theta_\pi \beta \overline{Y}}_{\overline{D}} + Y(\theta_Y + \theta_\pi \beta)$$

(9) $i_{IB} = \overline{D} + Y(\theta_Y + \theta_\pi \beta)$

Combine 8 and 9

$$Y(1-c) = \varepsilon_d + \overline{C} - (\gamma_D + \gamma_L)(\overline{D} + Y(\theta_Y + \theta_\pi \beta))$$

$$Y(1-c) = \varepsilon_d + \overline{C} - (\gamma_D + \gamma_L)\overline{D} - (\gamma_D + \gamma_L)(\theta_Y + \theta_\pi \beta)Y$$

$$Y(1 - c + (\gamma_D + \gamma_L)(\theta_Y + \theta_\pi \beta)) = \varepsilon_d + \overline{C} - (\gamma_D + \gamma_L)\overline{D}$$

$$Y^* = \frac{\varepsilon_d + \overline{C} - (\gamma_D + \gamma_L)\overline{D}}{1 - c + (\gamma_D + \gamma_L)(\theta_Y + \theta_\pi \beta)}$$

thus

$$\frac{dY_d{}^*}{d\varepsilon_d} = \frac{1}{1 - c + (\gamma_D + \gamma_L)(\theta_Y + \theta_\pi \beta)}$$

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