Lecture 1: Basic Models of Growth

Eugenio Proto

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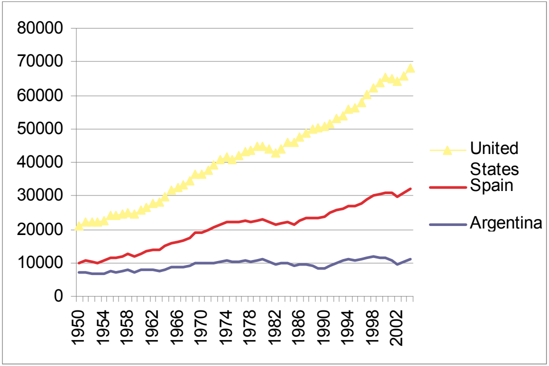
Some Kaldor's Fact

1. Per Capita output grows over time, and its growth rate does not tend to diminish
2. Physical Capital per worker grows over time
3. The growth rate of output per worker differs substantially across countries

Different paths of Growth



- Kaldor facts do not apply to stagnating countries
- Macroeconomic growth consider growing countries
- Development and growth proceed ed separately
- Generating a unique model for growth and development is still far away

Ramsey-Samuelson model

- Household behavior
  - size: $L(t) = e^{nt}$
  - Utility $U = \int_0^\infty u[c(t)]L(t)e^{-\rho t}dt$
  - per cap. wealth acc.: $\dot{a} = w + ra - c - na$
  - no ponzi game: $\lim_{t\to\infty}\{a(t)\exp[-\int_0^t [r(v) - n]dv\} \geq 0$
- Firms' Behavior
  - $Y = F(K, \hat{L})$ with $\hat{L} = L(t)T(t)$
  - increasing and concave in K and $\hat{L}$,
  - Constant Return to scale:

$$F(\lambda K, \lambda \hat{L}) = \lambda F(K, \hat{L})$$

Static Equilibria

- Household Optimal choice:

$$\max_c U(c)$$
$$w + ra - c - na \geq 0$$
$$\lim\{a(t)\exp[-\int_0^t [r(v) - n]dv\} \geq 0$$

  - Euler Condition
$$r = \rho + [\frac{-u''(c)c}{u'(c)}] * \dot{\hat{c}}/\hat{c}$$
  - with $u(c) = \frac{c^{1-\theta}-1}{1-\theta}$ (CIES):
$$\dot{\hat{c}}/\hat{c} = (1/\theta)(r - \rho)$$

Static Equilibrium (cont'd)

- Firms' optimal Choice

$$\max F(K, \hat{L}) - (r+\delta)K - wL =$$
$$\max f(\hat{k}) - (r+\delta)\hat{k} - \frac{w}{T(t)}$$

- FOCs

$$f'(\hat{k}) = r + \delta$$
$$[f(\hat{k}) - \hat{k}f'(\hat{k})]e^{xt} = w$$

Steady state

- Dynamics ($k = a$)

$$\dot{\hat{k}} = f(\hat{k}) - \hat{c} - (x + n + \delta)\hat{k}$$
$$\dot{\hat{c}}/\hat{c} = (1/\theta)(f'(k) - \delta - \rho - \theta x)$$
$$\lim\{a(t)\exp[-\int_0^t [r(v) - n]dv\} = 0$$

- Equilibrium

$$\begin{aligned} \dot{\hat{c}} &= 0 \rightarrow f'(k^*) = \delta + \rho + \theta x \\ \dot{\hat{k}} &= 0 \rightarrow \hat{c}^* = f(\hat{k}^*) - (x + n + \delta)\hat{k}^* \end{aligned}$$

with

$$\frac{\dot{\hat{y}}}{\hat{y}} = \alpha\frac{\dot{\hat{k}}}{\hat{k}} = 0$$

Steady state analysis

- Growth in steady state

$$\frac{\dot{\hat{c}}}{\hat{c}} = \frac{\partial\left(\frac{C(t)}{e^{(n+x)t}}\right)}{\partial t} / \frac{C(t)}{e^{(n+x)t}} = \dot{c} - x = 0$$

$$\frac{\dot{\hat{k}}}{\hat{k}} = \frac{\partial\left(\frac{K(t)}{e^{(n+x)t}}\right)}{\partial t} / \frac{K(t)}{e^{(n+x)t}} = \dot{k} - x = 0$$

$$\frac{\dot{\hat{y}}}{\hat{y}} = \dot{y} - x = \alpha\frac{\dot{\hat{k}}}{\hat{k}} = 0$$

- y (per capita income) in steady state grows with $T(t) = e^{xt}$
  GROWTH IS EXOGENOUS

AK model

- Consumer behavior exactly as in Ramsey
- Firms behavior and static equilibrium

$$\begin{aligned} Y &= AK \\ y &= f(k) = Ak \end{aligned}$$

  - capital = human capital, knowledge, public good...
  - no raw labour , $w = 0$
  - $r = A - \delta$

AK model

Steady state

- Dynamics ($k = a$)

$$\dot{k} = (A - \delta - n) - c/k$$
$$\dot{c}/c = (1/\theta)(A - \delta - \rho)$$
$$\lim\{k(t)e^{-(A-\delta-\rho)t}\} = 0$$

- Equilibrium

$$\begin{aligned} \dot{c}/c &= cons \\ \dot{k}/k &= \dot{c}/c \\ \dot{y}/y &= \dot{k}/k = cons \end{aligned}$$

Model with Human capital

- Firms produce $Y = F(H, K)$,
  - let $Y = Kf(H/K)$
- Market determines R_H, R_K
- Depreciation rates δ_H, δ_K
- In equilibrium

$$f(H/K) - f'(H/K)(1 + H/K) = \delta_K - \delta_H$$

unique value for H/K.

- Define $A = f(H/K)$ and we obtain a AK model

Model with learning by doing and spillover

- $Y_i = F(K_i, KL_i) = L_i F(k_i, K)$
  - K is the aggregate (physical or human) capital, since $k_i = k$ then $K = kL$
- $F(k, K)/k = f(K/k) = f(L)$ and

$$F_1(k, K) = f(K/k) - f'(K/k)\frac{K}{k_i^2}k_i = f(L) - f'(L)L$$

private marginal product of capital is non decreasing in k..

- $\dot{c}/c = (1/\theta)[f(L) - Lf'(L) - \delta - \rho]$ constant
- Generates long run growth