Introductory Mathematics and Statistics Syllabus and Readings

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This syllabus provides an outline of the four broad topics which we intend to cover this September: multivariate calculus, matrix algebra, dynamics and statistics. Please bear in mind that this list is indicative and may be subject to minor changes. Stars (*) denote non-examinable material.

1 Indicative Syllabus

There are four broad topic areas which will be covered in this course and they are:

1.1 Linear Algebra

Which has within it the following 6 areas:

1.1.1 Vectors

Column, row, n–tuple forms. Vector addition and scalar multiplication. Properties of $\mathbb{R}^n$ as a vector space. Norm and inner (dot) product. Zero vector $\mathbf{0}$. Unit vectors. Standard unit vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$. Orthogonal vectors.

1.1.2 Matrices

Matrix addition. Matrix multiplication. Scalar multiplication. Transposition. Square, diagonal and triangular matrices. The identity matrix. Symmetric matrices. Matrices acting on vectors. Linear maps $f : \mathbb{R}^n \to \mathbb{R}^m$ as $m \times n$ matrices. Determinants. Interpretation as area/volume scaling. Traces. Singular/non-singular matrices. Matrix inversion. Orthogonal matrices.

1.1.3 $\mathbb{R}^n$ Linear Equations

Systems of simultaneous linear equations. Coefficient and augmented matrices. Elementary row operations. Row echelon form, reduced row echelon form. Rank. Solubility of equations. Kernel (null space), column space (image) and row space of a matrix.* The dimension formula.*

1.1.4 Coordinate systems

Representing vectors in coordinate systems. Spanning. Linear independence. Basis vectors. Bases as coordinate systems. Dimension. Change of basis. Sub-spaces of $\mathbb{R}^n$.*

1.1.5 Eigenvalues and Eigenvectors

Eigenvalues. Eigenvectors. Characteristic equation. Diagonal and triangular matrices. Matrix diagonalisation. Applications of matrix diagonalisation: simultaneous difference equations, second-order difference equations, transition matrices.* Long-term behaviour.*

1.1.6 Quadratic Forms

Quadratic forms. Classification using definiteness. Coefficient matrix. Diagonalisation with orthogonal matrices. Tests for definiteness: eigenvalue test, Sylvester's Criteria (using principal minors).

1.2 Calculus and Dynamics

Which has within it the following 7 areas:

1.2.1 Necessary and Sufficient Conditions

Sufficient condition, necessary condition, logical equivalence, positive and negative conclusions from an implication, contrapositive statements.

1.2.2 Exponentiation and Compounding

Number e as a limit, existence of a limit, sandwich lemma, exponential function, discrete and continuous compounding, inverse functions, logarithms and their properties.

1.2.3 Derivatives and Elasticity

Continuity and differentiation of single-variable functions, first and higher derivatives and their properties, extreme points (minima and maxima), first and second order conditions, elasticity, logarithmic derivatives.

1.2.4 Taylor Series

Taylor's Theorem, calculation of Taylor-Maclaurin series, examples, convergence, the Ratio Test.

1.2.5 Analytical Theorems

Open and closed sets, compact sets, the Intermediate Value Theorem, Brouwer's Fixed Point Theorem.

1.2.6 Single Difference Equations

First order linear equations, particular solution, homogeneous equations, solving first-order inhomogeneous equations, examples, stability of solutions

1.2.7 Concave and Quasi-concave Functions

Multivariate notation, convex sets, concave and convex functions, definition of a graph, level curve, level set, links between concavity and level sets, quasi-concavity, strict quasi-concavity, relationship of basic concepts, quasi-convexity, strict quasi-convexity, basic properties.

1.3 Multivariable Calculus and Optimisation

Which has within it the following 4 areas:

1.3.1 Multivariate Calculus

Partial derivatives, continuous differentiability, gradient chain rule, total derivative, homogeneous functions, Euler's Theorem, total differential, Hessian matrix, Young's Theorem, stationary points, local minima and maxima, second order Taylor expansion, second order conditions, definiteness of the Hessian, convexity and concavity.

1.3.2 Constrained Optimisation

Objective function, constraint functions, Extreme Value Theorem, the Lagrangian, Lagrange multipliers, KKT (Karush-Kuhn-Tucker) conditions, exogenous parameters, maximisation with parameters, the Envelope Theorem.

1.3.3 Systems of Difference Equations

Matrix notation for systems of linear difference equations, coefficient matrix, homogeneous systems, diagonal and diagonalisable cases, inhomogeneous systems and particular solutions, stability, solving higher order equations.

1.3.4 Differential Equations

Terminology, linear and nonlinear equations, first order separable equations, initial/boundary conditions, first order linear equations, solution via integrating factor, second order linear equations, characteristic equation, complementary solution, particular solution, solving inhomogeneous equations,

1.4 Probability and Statistics

Which has within it the following 6 areas:

1.4.1 Probability

Sample space, events, axioms of probability, conditional probability, Bayes theorem*, independent events

1.4.2 Random Variables and Probability Distributions

Definition of a random variable, discrete random variable, bivariate discrete random variables, joint probability distribution, continuous and bivariate random variable and their probability density functions, conditional densities, cumulative distribution function, change of variables

1.4.3 Moments

Expected value and variance of discrete and continuous random variables, covariance, conditional mean and variance, Law of Iterated Expectation (LIE)

1.4.4 Some frequently used probability distributions

Bernoulli, binomial, normal, uniform, chi-square, Student's t and F-distribution.

1.4.5 Statistical Inference

Sample statistics, maximum likelihood*, and central limit theorem

1.4.6 Hypothesis testing

Null and alternative hypothesis, significance level, and power of tests, basic tests of a single mean and difference in means.

2 Readings

2.1 Elementary reading

When our students struggle with this course, in many instances it is not because the course is "too difficult", but rather because they lack foundations. If you just need to refresh few basic concepts, it is often best to use whichever books you are already familiar with (since then you will be able to use if efficiently), or even browse Wikipedia or YouTube for explanations or tutorials. In many cases this approach will work sufficiently well. If you problems are more persistent, try:

- *Essential mathematics for economic analysis* by Knut Sydsæter, Peter Hammond, Arne Strøm and Andreés Carvajal (published by Pearson, further abbreviates as: SHSC).

On arrival, you will find multiple copies of this book available in the Warwick University Library (edition does not matter much). In any instance remember that you can always ask your lecturers and tutors for help (also regarding the material which is not covered in this course) and we will be there if you are going to need us.

2.2 Core reading

Nothing. You do not need to read anything apart from lecture slides or course notes and you will not have time for it. The course will be self-contained.

2.3 Additional reading

While you do not need to read any particular books for this course, you might need to reference or learn certain techniques either during the course of your programme, or to use them in your MSc dissertation. For this purpose you could turn to books such as the following:

- *Schaum's Outline of Linear Algebra* by Seymour Lipschutz and Marc Lipson, 2012.
- *Introduction to Statistics and Econometrics* by Takeshi Amemiya, 2006
- *Fundamental methods of mathematical economics* by Alpha C. Chiang, Kevin Wainwright, 2005.
- *Mathematics for Economists: An Introductory Textbook* by Malcolm Pemberton and Nich¬olas Rau, 2015.
- *Further Mathematics for Economic Analysis* by Knut Sydsæter, Peter Hammond, Atle Seierstad and Arne Strøm, 2008 (further abbreviates as: SHSS).
- *Mathematics for Economists* by Carl P. Simon and Lawrence Blume, 1994.