Introductory Mathematics and Statistics

Syllabus

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This syllabus provides an outline of the four broad topics which we intend to cover this September: multivariate calculus, matrix algebra, dynamics and statistics. Please bear in mind that this list is indicative and may be subject to minor changes. Stars (*) denote non-examinable material.

1 Indicative Syllabus and Optional Readings

1.1 Linear Algebra

1. **Vectors**: Column, row, *n*–tuple forms. Vector addition and scalar multiplication. Properties of R^n as a vector space. Norm and inner (dot) product. Zero vector **0**. Unit vectors. Standard unit vectors **i**, **j**, **k**. Orthogonal vectors.

2. **Matrices**: Matrix addition. Matrix multiplication. Scalar multiplication. Transposition. Square, diagonal and triangular matrices. The identity matrix. Symmetric matrices. Matrices acting on vectors. Linear maps *f*: $R^n \to R$ $f:\mathbb{R}^n \to \mathbb{R}^m$ $m\times n$ $m\times n$ matrices. Determinants. Interpretation as area/volume scaling. Traces. Singular/nonsingular matrices. Matrix inversion. Orthogonal matrices.

3. $\mathbb{R}^n$ **Linear equations**: Systems of simultaneous linear equations. Coefficient and augmented matrices. Elementary row operations. Row echelon form, reduced row echelon form. Rank. Solubility of equations. Kernel (null space), column space (image) and row space of a matrix.* The dimension formula.*

4. **Coordinate systems**: Representing vectors in coordinate systems. Spanning. Linear independence. Basis vectors. Bases as coordinate systems. Dimension. Change of basis. Subspaces of R^n.*

5. **Eigenvalues and eigenvectors**: Eigenvalues. Eigenvectors. Characteristic equation. Diagonal and triangular matrices. Matrix diagonalisation. Applications of matrix diagonalisation: simultaneous difference equations, second-order difference equations, transition matrices.* Long-term behaviour.*

6. **Quadratic forms**: Quadratic forms. Classification using definiteness. Coefficient matrix. Diagonalisation with orthogonal matrices. Tests for definiteness: eigenvalue test, Sylvester's Criteria (using principal minors).

1.2 Calculus and Dynamics

1. **Necessary and Sufficient Conditions**: sufficient condition, necessary condition, logical equivalence, positive and negative conclusions from an implication, contrapositive statements.

2. **Exponentiation and Compounding**: number *e* as a limit, existence of a limit, sandwich lemma, exponential function, discrete and continuous compounding, inverse functions, logarithms and their properties.

3. **Derivatives and Elasticity**: continuity and differentiation of single-variable functions, first and higher derivatives and their properties, extreme points (minima and maxima), first and second order conditions, elasticity, logarithmic derivatives.

4. **Taylor Series**: Taylor's Theorem, calculation of Taylor-Maclaurin series, examples, convergence, the Ratio Test.

5. **Analytical Theorems**: open and closed sets, compact sets, the Intermediate Value Theorem, Brouwer's Fixed Point Theorem.

6. **Single Difference Equations**: first order linear equations, particular solution, homogeneous equations, solving first-order inhomogeneous equations, examples, stability of solutions

7. **Concave and Quasiconcave Functions**: multivariate notation, convex sets, concave and convex functions, definition of a graph, level curve, level set, links between concavity and level sets, quasiconcavity, strict quasiconcavity, relationship of basic concepts, quasiconvexity, strict quasiconvexity, basic properties.

1.3 Multivariable Calculus and Optimisation

1. **Multivariate Calculus:** partial derivatives, continuous differentiability, gradient chain rule, total derivative, homogeneous functions, Euler's Theorem, total differential, Hessian matrix, Young's Theorem, stationary points, local minima and maxima, second order Taylor expansion, second order conditions, definiteness of the Hessian, convexity and concavity.

2. **Constrained Optimisation**: objective function, constraint functions, Extreme Value Theorem, the Lagrangian, Lagrange multipliers, KKT (Karush-Kuhn-Tucker) conditions, exogenous parameters, maximisation with parameters, the Envelope Theorem.

3. **Systems of Difference Equations**: matrix notation for systems of linear difference equations, coefficient matrix, homogeneous systems, diagonal and diagonalisable cases, inhomogeneous systems and particular solutions, stability, solving higher order equations.

4. **Differential Equations**: terminology, linear and nonlinear equations, first order separable equations, initial/boundary conditions, first order linear equations, solution via integrating factor, second order linear equations, characteristic equation, complementary solution, particular solution, solving inhomogeneous equations,

1.4 Probability and Statistics

1. **Probability**: sample space, events, axioms of probability, conditional probability, Bayes theorem*, independent events
2. **Random Variables and Probability Distributions**: definition of a random variable, discrete random variable, bivariate discrete random variables, joint probability distribution, continuous and bivariate random variable and their probability density functions, conditional densities, cumulative distribution function, change of variables
3. **Moments**: expected value and variance of discrete and continuous random variables, covariance, conditional mean and variance, Law of Iterated Expectation (LIE)
4. **Some frequently used probability distributions**: Bernoulli, binomial, normal, uniform, chi-square, Student's t and F-distribution.
5. **Statistical Inference**: Sample statistics, maximum likelihood, and central limit theorem
6. **Hypothesis testing**: null and alternative hypothesis, significance level, and power of tests, basic tests of a single mean and difference in means.

2 Optional Readings

2.1 Elementary reading

When our students struggle with this course, in many instances it is not because the course is *"too difficult"*, but rather because they lack foundations. If you just need to refresh few basic concepts, it is often best to use whichever books you are *already familiar with* (since then you will be able to use if efficiently), or even browse wikipedia or youtube for explanations or tutorials. In many cases this approach will work sufficiently well. If you problems are more persistent, try:

- *Essential mathematics for economic analysis* by Knut Sydsæter, Peter Hammond, Arne Strøm and Andreés Carvajal (published by Pearson, further abbreviates as: SHSC).

On arrival, you will find multiple copies of this book available in the Warwick University Library (edition does not matter much). In any instance remember that you can always ask your lecturers and tutors for help (also regarding the material which is not covered in this course) and we will be there if you are going to need us.

2.2 Core reading

Nothing. You do not need to read anything apart from lecture slides or course notes and you will not have time for it. The course will be self-contained.

2.3 Additional reading

While you do not need to read any particular books for this course, you might need to reference or learn certain techniques either during the course of your programme, or to use them in your MSc dissertation. For this purpose you could turn to books such as the following:

- *Schaum's Outline of Linear Algebra* by Seymour Lipschutz and Marc Lipson, 2012
- *Introduction to Statistics and Econometrics* by Takeshi Amemiya, 2006
- *Fundamental methods of mathematical economics* by Alpha C. Chiang, Kevin Wainwright, 2005
- *Mathematics for Economists: An Introductory Textbook* by Malcolm Pemberton and Nicholas Rau, 2015
- *Further Mathematics for Economic Analysis* by Knut Sydsæter, Peter Hammond, Atle Seierstad and Arne Strøm, 2008 (further abbreviates as: SHSS)
- *Mathematics for Economists* by Carl P. Simon and Lawrence Blume, 1994.