Introductory Mathematics and Statistics

Syllabus

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This syllabus provides an outline of the four broad topics which we intend to cover this September: multivariate calculus, matrix algebra, dynamics and statistics. Please bear in mind that this list is indicative and may be subject to minor changes. Stars (*) denote non-examinable material.

1 Indicative Syllabus and Optional Readings

1.1 Matrix Algebra

1. **Vectors**: Column, row, *n*–tuple forms. Vector addition and scalar multiplication. Properties of R^n as a vector space. Norm and inner (dot) product. Zero vector **0**. Unit vectors. Standard unit vectors **i**, **j**, **k**. Orthogonal vectors.

2. **Matrices**: Matrix addition. Matrix multiplication. Scalar multiplication. Transposition. Square, diagonal and triangular matrices. The identity matrix. Symmetric matrices. Matrices acting on vectors. Linear maps $f:\mathbb{R}^n \to \mathbb{R}^m$ as $m \times n$ matrices. Determinants. Interpretation as area/volume scaling. Traces. Singular/nonsingular matrices. Matrix inversion. Orthogonal matrices.

3. **Coordinate systems**: Representing vectors in coordinate systems. Spanning. Linear independence. Basis vectors. Bases as coordinate systems. Dimension. Change of basis. Subspaces of $\mathbb{R}^n$.*

4. **Linear equations**: Systems of simultaneous linear equations. Coefficient and augmented matrices. Elementary row operations. Row echelon form, reduced row echelon form. Rank. Solubility of equations. Kernel (null space), column space (image) and row space of a matrix.* The dimension formula.*

5. **Eigenvalues and eigenvectors**: Eigenvalues. Eigenvectors. Characteristic equation. Diagonal and triangular matrices. Matrix diagonalisation. Applications of matrix diagonalisation: simultaneous difference equations, second-order difference equations, transition matrices.* Long-term behaviour.*

6. **Quadratic forms**: Quadratic forms. Classification using definiteness. Coefficient matrix. Diagonalisation with orthogonal matrices. Tests for definiteness: eigenvalue test, Sylvester's Criteria (using principal minors).

1.2 Multivariate Calculus

1. **Sufficient and Necessary Conditions**: sufficient condition, necessary condition, logical equivalence, positive and negative conclusions from an implication, truth tables
2. **Exponential Function and Approximation**: number *e* as a limit, existence of a limit, sandwich lemma, exponential function, continuous compounding, Taylor series of a uni-variate function, Taylor expansion of a univariate function, approximation of *e*, elasticity, log-derivative
3. **Concave and Quasiconcave Functions**: multivariate notation, convex set, concave and convex functions, definition of a graph, level curve, level set, links between concavity and level sets, quasiconcavity, strict quasiconcavity, how are basic concepts nested one within another, quasiconvexity, strict quasiconvexity, basic properties
4. **Multivariate Calculus**: partial derivative, chain rules of differentiation, gradient, homogeneous function, total differential, Hessian, Young Theorem
5. **Stationary Points**: local maximum or minimum, local extreme point, examples, Taylor expansion of multivariate functions, links between 2nd order expansion and extreme points, definiteness of a Hessian, links between: 2nd order Taylor expansion, definiteness of a Hessian and shape (convexity/concavity)
6. **Constrained Optimization**: Extreme Value Theorem, formulation of a standard maximisation problem, Lagrangian, Karush-Kuhn-Tucker (KKT) conditions, geometric interpretation, role of concavity, KKT conditions when all the variables are non-negative
7. **Comparative Statics**: value function, Envelope Theorem (ET), proof and application of the Envelope Theorem

1.3 Difference and Differential Equations

1. **Single Difference Equation**: solving single linear difference equation via substitution, decomposition of the general solution to: particular solution and the general solution to homogeneous equation, stability of solutions, examples
2. **Systems of Difference Equations (SDE)**: system of two first-order linear difference equations, SDE in matrix notation, system of unrelated homogeneous linear difference equations (**A** is diagonal), homogeneous system when **A** is diagonalizable, non-homogeneous system (when **A** is diagonalizable), its general solution, solution to SDE obtained thorough matrix algebra, global asymptotic stability (convergence) of a solution, further characterization of stability
3. **Higher Order Difference Equations**: change of variables, higher order difference equations as systems of difference equations
4. **Single Differential Equation**: particular solution, general solution to homogeneous equation, its general solution, stability (convergence) of this solutions, solving differential equations on a computer*

1.4 Probability and Statistics

1. **Probability**: sample space, events, axioms of probability, conditional probability, Bayes theorem, independent events.
2. **Random Variables and Probability Distributions**: definition of a random variable, discrete random variable, bivariate discrete random variables, joint probability distribution, continuous and bivariate random variable and their probability density functions, conditional densities, cumulative distribution function, change of variables.
3. **Moments**: expected value of discrete and continuous random variables, expectation of a random function with respect to discrete or continuous probability distribution, moments, central moments, variance, standard deviation, properties of expectation and variance, covariance, conditional mean and variance, Law of Iterated Expectation (LIE).
4. **Special Distributions**: Bernoulli, binomial, uniform, normal, chi-square, Student's t, F distribution.
5. **Properties of Statistics**: Moments, unbiasedness, efficiency, consistency, maximum likelihood, Central Limit Theorem (CLT).
6. **Hypothesis testing**: Types of errors, power of a test, hypothesis testing for single mean, hypothesis testing for a difference in means, hypothesis testing for sample variance(s), ANOVA, confidence intervals

2 Optional Readings

2.1 Elementary reading

When our students struggle with this course, in many instances it is not because the course is *"too difficult"*, but rather because they lack foundations. If you just need to refresh few basic concepts, it is often best to use whichever books you are *already familiar with* (since then you will be able to use if efficiently), or even browse wikipedia or youtube for explanations or tutorials. In many cases this approach will work sufficiently well. If you problems are more persistent, try:

- *Essential mathematics for economic analysis* by Knut Sydsæter, Peter Hammond, Arne Strøm and Andreés Carvajal (published by Pearson, further abbreviates as: SHSC).

On arrival, you will find multiple copies of this book available in the Warwick University Library (edition does not matter much). In any instance remember that you can always ask your lecturers and tutors for help (also regarding the material which is not covered in this course) and we will be there if you are going to need us.

2.2 Core reading

Nothing. You do not need to read anything apart from lecture slides or course notes and you will not have time for it. The course will be self-contained.

2.3 Additional reading

While you do not need to read any particular books for this course, you might need to reference or learn certain techniques either during the course of your programme, or

to use them in your MSc dissertation. For this purpose you could turn to books such as the following:

- *Schaum's Outline of Linear Algebra* by Seymour Lipschut and Marc Lipson, 2012
- *Introduction to Statistics and Econometrics* by Takeshi Amemiya, 2006
- *Fundamental methods of mathematical economics* by Alpha C. Chiang, Kevin Wainwright, 2005
- *Mathematics for Economists: An Introductory Textbook* by Malcolm Pemberton and Nicholas Rau, 2015
- *Further Mathematics for Economic Analysis* by Knut Sydsæter, Peter Hammond, Atle Seierstad and Arne Strøm, 2008 (further abbreviates as: SHSS)
- *Mathematics for Economists* by Carl P. Simon and Lawrence Blume, 1994.