

EC124 Statistical Techniques B: Formula Sheet

Descriptive Statistics

	Sample (n observations)	Population (N observations)
Mean	$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$	$\mu = \frac{\sum_{i=1}^N x_i}{N}$
Variance	$s_x^2 = \frac{\sum_{i=1}^n x_i^2 - n\bar{x}^2}{n-1}$	$\sigma_x^2 = \frac{\sum_{i=1}^N x_i^2 - N\mu_x^2}{N}$
Covariance	$s_{xy} = \frac{\sum_{i=1}^n x_i y_i - n\bar{x}\bar{y}}{n-1}$	$\sigma_{xy} = \frac{\sum_{i=1}^N x_i y_i - N\mu_x\mu_y}{N}$
Correlation	$r_{xy} = \frac{s_{xy}}{s_x s_y}$	$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$

Probability and Expectations

Bayes' Theorem: $P(E_i|A) = \frac{P(A|E_i) \times P(E_i)}{\sum_i P(A|E_i) \times P(E_i)}$

Distribution	Mean	Variance
Discrete	$E(X) = \sum_i x_i P_X(x_i)$	$V(X) = E(X^2) - E(X)^2$ and $E(X^2) = \sum_i x_i^2 P_X(x_i)$
Continuous	$E(X) = \int_x x f(x) dx$	$V(X) = E(X^2) - E(X)^2$ and $E(X^2) = \int_x x^2 f(x) dx$

Probability distributions

	Probability (density) functions	Mean	Variance
Binomial	$P_X(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$	np	$np(1-p)$
Poisson	$P_X(x) = \frac{e^{-\lambda} \lambda^x}{x!}$	λ	λ
Normal	$f_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$	μ	σ^2

Non-Parametric Tests

Sign Test: $W \sim B(n, p)$

Signed Rank Test: $T = \min(W_+, W_-), E(T) = \frac{n(n+1)}{4}, V(T) = \frac{n(n+1)(2n+1)}{24}$

Mann-Whitney Test: $U = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1, E(U) = \frac{n_1 n_2}{2}, V(U) = \frac{n_1 n_2 (n_1 + n_2 + 1)}{12}$

Goodness of Fit Test: $\sum_{i=1}^K \frac{(O_i - E_i)^2}{E_i} = \sum_{i=1}^K \frac{O_i^2}{E_i} - n \sim \chi_{K-1}^2$

Contingency Tables: $\sum_{i=1}^K \sum_{j=1}^H \frac{(O_{ij} - E_{ij})^2}{E_{ij}} = \sum_{i=1}^K \sum_{j=1}^H \frac{O_{ij}^2}{E_{ij}} - n \sim \chi_{(K-1)(H-1)}^2$

Hypothesis Testing

Sample	X_i	H_0	σ^2 Known	σ^2 Not Known
Any	Normal	$\mu = \mu_0$	$\frac{(\bar{x} - \mu_0)}{\sqrt{\frac{\sigma_x^2}{n}}} \sim N$	$\frac{(\bar{x} - \mu_0)}{\sqrt{\frac{s_x^2}{n}}} \sim t_{n-1}$
>30	Any	$\mu = \mu_0$	$\frac{(\bar{x} - \mu_0)}{\sqrt{\frac{\sigma_x^2}{n}}} \sim^a N$	$\frac{(\bar{x} - \mu_0)}{\sqrt{\frac{s_x^2}{n}}} \sim^a N$
>30	Bernoulli	$\pi = \pi_0$	$\frac{(\bar{x} - \pi_0)}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}} \sim^a N$	
>30	Any	$\mu_1 - \mu_2 = \delta$	$\frac{(\bar{x}_1 - \bar{x}_2 - \delta)}{\sqrt{\frac{\sigma_{x_1}^2}{n_1} + \frac{\sigma_{x_2}^2}{n_2}}} \sim^a N$	$\frac{(\bar{x}_1 - \bar{x}_2 - \delta)}{\sqrt{\frac{s_{x_1}^2}{n_1} + \frac{s_{x_2}^2}{n_2}}} \sim^a N$
Any	Normal	$\mu_1 - \mu_2 = \delta$	$\frac{(\bar{x}_1 - \bar{x}_1 - \delta)}{\sqrt{\frac{\sigma_{x_1}^2}{n_1} + \frac{\sigma_{x_2}^2}{n_2}}} \sim N$	$\frac{(\bar{x}_1 - \bar{x}_2 - \delta)}{\sqrt{\frac{s_{x_1}^2}{n_1} + \frac{s_{x_2}^2}{n_2}}} \sim t_{n_1+n_2-2}$ $s_x^2 = \frac{(n_1-1)s_{x_1}^2 + (n_2-1)s_{x_2}^2}{(n_1+n_2-2)}$
Any	Normal	$\mu_1 - \mu_2 = \delta$		$\frac{(\bar{x}_1 - \bar{x}_2 - \delta)}{\sqrt{\frac{s_{x_1}^2}{n_1} + \frac{s_{x_2}^2}{n_2}}} \sim t_{DoF}$ $DoF = \frac{\left[\frac{s_{x_1}^2}{n_1} + \frac{s_{x_2}^2}{n_2} \right]^2}{\left(\frac{s_{x_1}^2}{n_1} \right)^2 + \left(\frac{s_{x_2}^2}{n_2} \right)^2}$
>30	Bernoulli	$\pi_1 - \pi_2 = 0$		$\frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{p(1-p)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim^a N$ $p = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$
Any	Normal	$\sigma^2 = \sigma_0^2$		$\frac{(n-1)s_x^2}{\sigma_0^2} \sim \chi_{n-1}^2$
Any	Normal	$\sigma_1^2 = \sigma_2^2$		$\frac{s_{x_1}^2}{s_{x_2}^2} \sim F_{(n_1-1, n_2-1)}$
Any	Normal	$\rho(X, Y) = 0$		$\frac{r_{xy}}{\sqrt{\frac{1-r_{xy}^2}{n-2}}} \sim t_{n-2}$
Any	Normal	$\mu^1 = \dots = \mu^k$		$\frac{\sum_{j=1}^k n_j (\bar{X}^j - \bar{X})^2 / (k-1)}{\sum_{j=1}^k \sum_{i=1}^{n_j} (X_i^j - \bar{X}^j)^2 / (n-k)} \sim F_{k-1, n-k}$

NOTE: 1. $\sim N$: critical value is from a $N(0,1)$; 2. $\sim^a N$: critical value is approximately from a $N(0,1)$; 3. $\sim t_d$: critical value is from a t-distribution with d degrees of freedom; 4. $\sim \chi_d^2$: critical value is from a χ^2 distribution with d degrees of freedom; 5. $\sim F_{d,k}$: critical value is from an F-distribution with d and k degrees of freedom.

Critical values of the Wilcoxon Signed Rank Test ($n < 30$)

1-tailed	$\alpha = 0.05$	$\alpha = 0.025$	$\alpha = 0.01$	$\alpha = 0.005$
2-tailed	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.02$	$\alpha = 0.01$
n				
6	2	0	-	-
7	3	2	0	-
8	5	3	1	0
9	8	5	3	1
10	10	8	5	3
11	13	10	7	5
12	17	13	9	7
13	21	17	12	9
14	25	21	15	12
15	30	25	19	15
16	35	29	23	19
17	41	34	27	23
18	47	40	32	27
19	53	46	37	32
20	60	52	43	37
21	67	58	49	42
22	75	65	55	48
23	83	73	62	54
24	91	81	69	61
25	100	89	76	68

Mann-Whitney 1% Critical Values

		n_2															
		5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
n_1	5	1	2	3	3	4	5	6	7	8	8	10	10	11	12	13	13
	6	2	3	4	5	6	7	8	9	10	12	13	14	16	17	18	19
	7	3	4	5	7	8	9	11	12	14	15	17	18	20	21	23	24
	8	3	5	7	9	10	12	14	15	17	19	21	23	24	26	28	30
	9	4	6	8	10	13	14	16	19	21	23	25	28	30	32	34	36
	10	5	7	9	12	14	17	19	21	25	27	29	32	35	38	40	42
	11	6	8	11	14	16	19	22	24	28	30	32	36	39	42	45	48
	12	7	9	12	15	19	17	24	28	30	34	37	41	44	47	51	55
	13	8	10	14	17	21	21	28	30	35	38	42	46	50	53	57	61
	14	8	12	15	19	23	25	30	34	38	43	47	51	56	59	63	66
	15	10	13	17	21	25	27	32	37	42	47	51	56	61	64	68	72
	16	10	14	18	23	28	29	36	41	46	51	56	61	66	70	74	78
	17	11	16	20	24	30	32	39	44	50	56	61	66	71	76	82	86
	18	12	17	21	26	32	35	42	47	53	59	64	70	76	83	88	93
	19	13	18	23	28	34	40	45	51	57	63	68	74	82	88	94	99
	20	13	19	24	30	36	42	48	55	61	66	72	78	86	93	99	107

Mann-Whitney 5% Critical Values

		n_2															
		5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
n_1	5	3	4	5	7	8	9	10	11	13	14	15	16	18	19	20	21
	6	4	6	7	9	10	12	14	15	17	18	20	22	23	25	26	28
	7	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35
	8	7	9	11	14	16	18	21	23	25	28	30	32	35	37	40	42
	9	8	10	13	16	18	21	24	26	29	32	35	37	40	42	46	48
	10	9	12	15	18	21	24	27	30	34	37	39	43	46	49	52	55
	11	10	14	17	21	24	27	30	34	37	41	44	48	51	55	59	62
	12	11	15	19	23	26	24	34	38	41	45	49	53	57	61	65	69
	13	13	17	21	25	29	30	37	41	46	50	54	58	62	67	71	75
	14	14	18	23	28	32	34	41	45	50	55	59	64	68	74	78	84
	15	15	20	25	30	35	37	44	49	54	59	64	69	75	80	86	91
	16	16	22	27	32	37	39	48	53	58	64	69	76	81	87	93	99
	17	18	23	29	35	40	43	51	57	62	68	75	81	87	94	100	106
	18	19	25	31	37	42	46	55	61	67	74	80	87	94	99	106	113
	19	20	26	33	40	46	52	59	65	71	78	86	93	100	106	113	120
	20	21	28	35	42	48	55	62	69	75	84	91	99	106	113	120	128