Linear Algebra

FORMULA SHEET

Axioms for a field

A1	$a + (b + c) = (a + b) + c$	Associative law for addition
A2	$a + 0 = 0 + a = a$	Existence of an additive identity
A3	$a + (-a) = (-a) + a = 0$	Existence of an additive inverse
A4	$a + b = b + a$	Commutative law for addition
M1	$a \cdot (b \cdot c) = (a \cdot b) \cdot c$	Associative law for multiplication
M2	$a \cdot 1 = 1 \cdot a = a,\ a \neq 0$	Existence of a multiplicative identity
M3	$a \cdot a^{-1} = a^{-1} \cdot a = 1,\ a \neq 0$	Existence of a multiplicative inverse
M4	$a \cdot b = b \cdot a$	Commutative law for multiplication
D1	$a \cdot (b + c) = a \cdot b + a \cdot c$	Distributive law

Axioms for a vector space

V0	$\mathbf{u} + \mathbf{v} \in V$	Closure under vector addition
V1	$(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$	Associative law for vector addition
V2	$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$	Commutative law for vector addition
V3	$\mathbf{0} + \mathbf{v} = \mathbf{v} + \mathbf{0} = \mathbf{v}$	Existence of zero vector (additive identity)
V4	$\mathbf{v} + (-\mathbf{v}) = (-\mathbf{v}) + \mathbf{v} = \mathbf{0}$	Existence of negative vectors (additive inverses)
S0	$k\mathbf{v} \in V$	Closure under scalar multiplication
S1	$h(k\mathbf{v}) = (hk)\mathbf{v}$	Associative law for scalar multiplication
S2	$1\mathbf{v} = \mathbf{v}$	Multiplicative identity for scalar multiplication
S3	$(h + k)\mathbf{v} = h\mathbf{v} + k\mathbf{v}$	First distributive law
S4	$k(\mathbf{u} + \mathbf{v}) = k\mathbf{u} + k\mathbf{v}$	Second distributive law

Properties of matrix operations

$$\operatorname{tr}(A+B) = \operatorname{tr}(A) + \operatorname{tr}(B) \qquad \operatorname{tr}(AB) = \operatorname{tr}(BA)$$

$$\operatorname{tr}(A^T) = \operatorname{tr}(A) \qquad \operatorname{tr}(kA) = k \operatorname{tr}(A)$$

$$\det(AB) = \det(A)\det(B) \qquad \det(A^T) = \det(A)$$

$$\det(A^{-1}) = \det(A)^{-1} \qquad \det(kA) = k \det(A)$$

$$(A + B)^T = A^T + B^T \qquad (AB)^T = B^T A^T$$

(End)