EC124 Statistical Techniques B: Formula Sheet

Descriptive Statistics

	Sample (n observations)	Population (N observations)
Mean	$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$	$\mu = \frac{\sum_{i=1}^{N} x_i}{N}$
Variance	$s_x^2 = \frac{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}{n-1}$	$\sigma_x^2 = \frac{\sum_{i=1}^{N} x_i^2 - N\mu_x^2}{N}$
Covariance	$s_{xy} = \frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}{n-1}$	$\sigma_{xy} = \frac{\sum_{i=1}^{N} x_i y_i - N\mu_x\mu_y}{N}$
Correlation	$r_{xy} = \frac{s_{xy}}{s_x s_y}$	$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$

Probability and Expectations

Bayes' Theorem: $P(E_i|A) = \frac{P(A|E_i) \times P(E_i)}{\sum_i P(A|E_i) \times P(E_i)}$

Distribution	Mean	Variance
Discrete	$E(X) = \sum_i x_i P_X(x_i)$	$V(X) = E(X^2) - E(X)^2$ and $E(X^2) = \sum_i x_i^2 P_X(x_i)$
Continuous	$E(X) = \int_x x f(x) dx$	$V(X) = E(X^2) - E(X)^2$ and $E(X^2) = \int_x x^2 f(x) dx$

Probability distributions

	Probability (density) functions	Mean	Variance
Binomial	$P_X(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$	np	$np(1-p)$
Poisson	$P_X(x) = \frac{e^{-\lambda}\lambda^x}{x!}$	λ	λ
Normal	$f_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}} exp(-\frac{(x-\mu)^2}{2\sigma^2})$	μ	σ^2

Non-Parametric Tests

Sign Test: $W \sim B(n, p)$

Signed Rank Test: $T = min(W_+, W_-)$,$E(T) = \frac{n(n+1)}{4}$,$V(T) = \frac{n(n+1)(2n+1)}{24}$

Mann-Whitney Test: $U = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1$, $E(U) = \frac{n_1 n_2}{2}$,$V(U) = \frac{n_1 n_2 (n_1+n_2+1)}{12}$

Goodness of Fit Test: $\sum_{i=1}^{K} \frac{(O_i - E_i)^2}{E_i} = \sum_{i=1}^{K} \frac{O_i^2}{E_i} - n \sim \chi^2_{K-1}$

Contingency Tables: $\sum_{i=1}^{K} \sum_{j=1}^{H} \frac{(O_{ij} - E_{ij})^2}{E_{ij}} = \sum_{i=1}^{K} \sum_{j=1}^{H} \frac{O_{ij}^2}{E_{ij}} - n \sim \chi^2_{(K-1)(H-1)}$

Hypothesis Testing

Sample	X_i	H_0	σ^2 Known	σ^2 Not Known
Any	Normal	$\mu = \mu_0$	$\frac{(\bar{x}-\mu_0)}{\sqrt{\frac{\sigma_x^2}{n}}} \sim N$	$\frac{(\bar{x}-\mu_0)}{\sqrt{\frac{s_x^2}{n}}} \sim t_{n-1}$
$\geq$25	Any	$\mu = \mu_0$	$\frac{(\bar{x}-\mu_0)}{\sqrt{\frac{\sigma_x^2}{n}}} \sim^a N$	$\frac{(\bar{x}-\mu_0)}{\sqrt{\frac{s_x^2}{n}}} \sim^a N$
$\geq$25	Bernoulli	$\pi = \pi_0$	$\frac{(\bar{x}-\pi_0)}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}} \sim^a N$	
$\geq$25	Any	$\mu_1 - \mu_2 = \delta$	$\frac{(\bar{x}_1-\bar{x}_2-\delta)}{\sqrt{\frac{\sigma_{x_1}^2}{n_1}+\frac{\sigma_{x_2}^2}{n_2}}} \sim^a N$	$\frac{(\bar{x}_1-\bar{x}_2-\delta)}{\sqrt{\frac{s_{x_1}^2}{n_1}+\frac{s_{x_2}^2}{n_2}}} \sim^a N$
Any	Normal	$\mu_1 - \mu_2 = \delta$	$\frac{(\bar{x}_1-\bar{x}_1-\delta)}{\sqrt{\frac{\sigma_{x_1}^2}{n_1}+\frac{\sigma_{x_2}^2}{n_2}}} \sim N$	$\frac{(\bar{x}_1-\bar{x}_2-\delta)}{\sqrt{\frac{s_x^2}{n_1}+\frac{s_x^2}{n_2}}} \sim t_{n_1+n_2-2}$ $s_x^2 = \frac{(n_1-1)s_{x_1}^2+(n_2-1)s_{x_2}^2}{(n_1+n_2-2)}$
Any	Normal	$\mu_1 - \mu_2 = \delta$		$\frac{(\bar{x}_1-\bar{x}_2-\delta)}{\sqrt{\frac{s_{x_1}^2}{n_1}+\frac{s_{x_2}^2}{n_2}}} \sim t_{DoF}$ $DoF = \frac{\left[\frac{s_{x_1}^2}{n_1}+\frac{s_{x_2}^2}{n_2}\right]^2}{\frac{\left(\frac{s_{x_1}^2}{n_1}\right)^2}{n_1-1}+\frac{\left(\frac{s_{x_2}^2}{n_2}\right)^2}{n_2-1}}$
$\geq$25	Bernoulli	$\pi_1 - \pi_2 = 0$		$\frac{(\bar{x}_1-\bar{x}_2)}{\sqrt{p(1-p)(\frac{1}{n_1}+\frac{1}{n_2})}} \sim^a N$ $p = \frac{n_1\bar{x}_1+n_2\bar{x}_2}{n_1+n_2}$
Any	Normal	$\sigma^2 = \sigma_0^2$		$\frac{(n-1)s_x^2}{\sigma_0^2} \sim \chi_{n-1}^2$
Any	Normal	$\sigma_1^2 = \sigma_2^2$		$\frac{s_{x_1}^2}{s_{x_2}^2} \sim F_{(n_1-1,n_2-1)}$
Any	Normal	$\rho(X,Y) = 0$		$\frac{r_{xy}}{\sqrt{\frac{1-r_{xy}^2}{n-2}}} \sim t_{n-2}$
Any	Normal	$\mu^1 = .. = \mu^k$		$\frac{\sum_{j=1}^{k} n_j(\bar{X}^j-\bar{X})^2/(k-1)}{\sum_{j=1}^{k}\sum_{i=1}^{n_j}(X_i^j-\bar{X}^j)^2/(n-k)} \sim F_{k-1,n-k}$

NOTE: 1. $\sim N$: critical value is from a N(0,1); 2. $\sim^a N$: critical value is approximately from a N(0,1); 3. $\sim t_d$: critical value is from a t-distribution with d degrees of freedom; 4. $\sim \chi_d^2$: critical value is from a χ^2 distribution with d degrees of freedom; 5. $\sim F_{d,k}$: critical value is from an F-distribution with d and k degrees of freedom.

Critical values of the Wilcoxon Signed Rank Test ($n < 30$)

1-tailed 2-tailed n	$\alpha = 0.05$ $\alpha = 0.10$	$\alpha = 0.025$ $\alpha = 0.05$	$\alpha = 0.01$ $\alpha = 0.02$	$\alpha = 0.005$ $\alpha = 0.01$
6	2	0	-	-
7	3	2	0	-
8	5	3	1	0
9	8	5	3	1
10	10	8	5	3
11	13	10	7	5
12	17	13	9	7
13	21	17	12	9
14	25	21	15	12
15	30	25	19	15
16	35	29	23	19
17	41	34	27	23
18	47	40	32	27
19	53	46	37	32
20	60	52	43	37
21	67	58	49	42
22	75	65	55	48
23	83	73	62	54
24	91	81	69	61
25	100	89	76	68

Mann-Whitney 1% Critical Values

		n_2															
		5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	5	1	2	3	3	4	5	6	7	8	8	10	10	11	12	13	13
	6	2	3	4	5	6	7	8	9	10	12	13	14	16	17	18	19
	7	3	4	5	7	8	9	11	12	14	15	17	18	20	21	23	24
	8	3	5	7	9	10	12	14	15	17	19	21	23	24	26	28	30
	9	4	6	8	10	13	14	16	19	21	23	25	28	30	32	34	36
	10	5	7	9	12	14	17	19	21	25	27	29	32	35	38	40	42
	11	6	8	11	14	16	19	22	24	28	30	32	36	39	42	45	48
n_1	12	7	9	12	15	19	17	24	28	30	34	37	41	44	47	51	55
	13	8	10	14	17	21	21	28	30	35	38	42	46	50	53	57	61
	14	8	12	15	19	23	25	30	34	38	43	47	51	56	59	63	66
	15	10	13	17	21	25	27	32	37	42	47	51	56	61	64	68	72
	16	10	14	18	23	28	29	36	41	46	51	56	61	66	70	74	78
	17	11	16	20	24	30	32	39	44	50	56	61	66	71	76	82	86
	18	12	17	21	26	32	35	42	47	53	59	64	70	76	83	88	93
	19	13	18	23	28	34	40	45	51	57	63	68	74	82	88	94	99
	20	13	19	24	30	36	42	48	55	61	66	72	78	86	93	99	107

Mann-Whitney 5% Critical Values

		n_2															
		5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	5	3	4	5	7	8	9	10	11	13	14	15	16	18	19	20	21
	6	4	6	7	9	10	12	14	15	17	18	20	22	23	25	26	28
	7	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35
	8	7	9	11	14	16	18	21	23	25	28	30	32	35	37	40	42
	9	8	10	13	16	18	21	24	26	29	32	35	37	40	42	46	48
	10	9	12	15	18	21	24	27	30	34	37	39	43	46	49	52	55
	11	10	14	17	21	24	27	30	34	37	41	44	48	51	55	59	62
n_1	12	11	15	19	23	26	24	34	38	41	45	49	53	57	61	65	69
	13	13	17	21	25	29	30	37	41	46	50	54	58	62	67	71	75
	14	14	18	23	28	32	34	41	45	50	55	59	64	68	74	78	84
	15	15	20	25	30	35	37	44	49	54	59	64	69	75	80	86	91
	16	16	22	27	32	37	39	48	53	58	64	69	76	81	87	93	99
	17	18	23	29	35	40	43	51	57	62	68	75	81	87	94	100	106
	18	19	25	31	37	42	46	55	61	67	74	80	87	94	99	106	113
	19	20	26	33	40	46	52	59	65	71	78	86	93	100	106	113	120
	20	21	28	35	42	48	55	62	69	75	84	91	99	106	113	120	128