

Formula Sheet

1. Call Value at expiration: $C = \max(S - K, 0)$
2. Put value at expiration: $P = \max(K - S, 0)$
3. Put-Call Parity: $S + P = C + PV(K)$

Solution of Mean-Variance optimization

Let Σ denote the variance/covariance matrix, \mathbf{w} the portfolio vector, and $\bar{\mathbf{R}}$ the vector of expected returns. The solution to the following optimization problem:

$$\begin{aligned} \min_{\mathbf{w}} \quad & \frac{1}{2} \mathbf{w}' \Sigma \mathbf{w} \\ \text{s.t.} \quad & \mathbf{w}' \bar{\mathbf{R}} = R_p \\ & \mathbf{w}' \mathbf{e} = 1 \end{aligned}$$

Is given by $\mathbf{w} = \lambda_1 \Sigma^{-1} \bar{\mathbf{R}} + \lambda_2 \Sigma^{-1} \mathbf{e}$. where λ_1, λ_2 are Lagrange multipliers,

In the case when there is a risk free asset we consider excess returns (i.e., subtract the risk free rate). We also focus first on the weights on the risky assets so remove the second constraint. The solution to the following optimization problem:

$$\begin{aligned} \min_{\mathbf{w}} \quad & \frac{1}{2} \mathbf{w}' \Sigma \mathbf{w} \\ \text{s.t.} \quad & \mathbf{w}' \bar{\mathbf{R}}^e = R_p^e \end{aligned}$$

4.

Is given by $\mathbf{w} = \lambda \Sigma^{-1} \bar{\mathbf{R}}^e$