Sovereign debt and Growth Expectations*

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Abstract

This paper shows empirical evidence and theory consistent with the US government using debt to adjust public consumption to news about long-term growth. First, using historical forecasts from the Congressional Budget Office (CBO) starting in 1984, I find that government purchases and deficits are positively correlated with expectations about long-term tax revenue and GDP growth. I also document that these facts are robust to controlling for current growth and to using à-la-Kalman estimated forecast values for a longer time span. Second, I present a simple open economy RBC model with stochastic productivity trend and endogenous public purchases decided by a forward-looking government. Calibrated to the US economy, the model produces moments similar to those observed in the data. Finally, the role of imperfect information is found negligible to explain these facts.

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1 Introduction

This paper studies the relation between government purchases and expectations about the long-term growth rate of output, and it derives implications of that relation for the dynamics of sovereign debt and deficits. This analysis is currently of special interest as the financial crisis has led to a downward revision of US long-term growth forecasts. My paper makes two main contributions: (i) it derives and discusses a new set of stylized facts about long-term growth expectations of tax revenues and GDP, and their impact on fiscal policy; (ii) it develops a novel dynamic macro model with shocks to long-run (trend) growth rates and endogenous fiscal policy. The paper shows that the model explains the new set of stylized facts.

The US Congressional Budget Office (CBO) publishes fiscal year projections of federal revenues and outlays, since 1984. Analysing data for 1984-2012, I find that higher growth expectations (of both fiscal revenue and GDP) are positively correlated with federal purchases and deficits (divided by GDP). This suggests that the government, searching to adjust its consumption, borrows if expectations on future revenues improve; or lends when facing gloomier prospects. These features of government behaviour have not been previously identified in the fiscal policy literature, as that literature usually assumes that fiscal variables are linked to current real activity, through mechanical feedback rules that ignore the role of expectations about future GDP growth. However, simple regression analysis shows that long-term GDP growth expectations are as important as current GDP growth to explain fiscal purchases and savings dynamics.

My paper also documents that these correlations hold for a longer time span, 1971-2012, making use of estimated long-term productivity growth expectations to approximate long-term real GDP growth forecasts. This approximation is realistic because, for instance, there is 0.93 correlation between the 10-year-ahead forecasts on real GDP growth and productivity from the US.
Survey of Professional Forecasters. I apply Kalman filtering techniques to real-time labour productivity quarterly data since Edge et al. (2007) showed that the resulting forecasts replicate successfully those collected by the Survey of Professional Forecasters.

These findings motivate the second main contribution of my paper, namely the development of an open economy Real Business Cycle (RBC) model with stochastic productivity trends and a forward-looking government. More specifically, the model assumes that the government is a utility-maximizing agent that decides on its purchases subject to the standard budget constraint, while tax rates are determined according to simple mechanical rules. Remarkably, this simple model can replicate the above-mentioned stylized facts. It produces a high positive correlation of long-term growth expectations with government purchases (divided by output), while negative with fiscal surpluses (also as a share of output). Moreover, the model explains well the time series properties of other macroeconomic variables.

This paper highlights the fact that optimal decisions of a forward-looking government on purchases and savings depend on the type of shocks to the productivity growth rate and their different effects on the present-discounted value of government’s income. In this respect, the response of output to a 1% purely transitory increase in productivity growth is a transition to a new balance growth path where output is permanently 1% higher. However, if the 1% increase is permanent, the subsequent increase of future productivity and output is more than one order of magnitude larger. Consequently, in response to a positive transitory increase in productivity growth, government purchases increase less than revenue and debt decreases. However, if the shock is permanent, the increase of public purchases is much larger than that of revenue, deteriorating the primary fiscal balance. As a result the government increases its debt because higher future revenues will be available at the time of repayment.

Current US long-run real GDP growth expectations are -not as low as in the 80’s- but much lower than in the decade before the crisis. The IMF
World Economic Outlook provides output growth projections for the group of advanced economies from 2013 to 2018 that are 1.5% lower than their average rate from 1980 to 2007. As a result, there is a renewed interest in the effects of shocks to long-run growth in the economy. Several papers have studied their impact on asset prices, international financial flows, and investment dynamics, however no previous paper has studied the effect of trend changes in the context of fiscal policy and public finance. This is surprising since, for instance, GDP growth has been shown to be the largest contributor to holding down the US debt-to-GDP ratio (see Hall and Sargent (2011)). This paper contributes to the study of the effect of trend shocks showing evidence that the US government have used sovereign debt to adjust public consumption to news about long-term growth.

2 Review of related literature

This paper shows and explains that US government purchases and fiscal deficits comove with changes in long-term GDP growth forecasts. Conceptually, these forecast variations can be interpreted as news about long-term growth implied by movements in the productivity growth trend. Since the 90’s—a decade of particularly high GDP and productivity growth—stochastic productivity trend movements have been found useful in explaining some features of macro data. For example, Pakko (2002) shows that sharp declines in capital stock growth in the mid-1980s and early 1990s are associated with positive technology growth shocks. Using a New Keynesian model, Gilchrist and Leahy (2002) show that asset prices may rise in response to a persistent increase in the growth rate of technology and Gilchrist and Saito (2008) derive implications of trend changes for monetary policy. More recently, Hoffmann et al. (2012) show that increasing housing prices in the US until 2005 can be explained by rational expectations models that allow for productivity trend changes.
Stochastic trend models have also been useful in the international macro literature. Aguiar and Gopinath (2007) document that some particular features of emerging economies (strongly countercyclical current accounts, consumption volatility that exceeds income volatility, and sudden stops in capital inflows) are mainly caused by perceived shocks to trend growth rather than transitory fluctuations around a stable trend. More recently, Hoffmann et al. (2013) presents a two-country RBC model with stochastic productivity trends and shows that it successfully produces the increasing US current account deficits of the late 90’s and early 2000’s, as well as their reduction in the years before crisis. In this period US long-term growth expectations became more optimistic (relative to the rest of the world) due to bigger increases in the US productivity trend (again, relative to that of the rest of the world). As suggested by the intertemporal model of the current account, higher (relative) long-term growth expectations encouraged the US economy as a whole to borrow from the rest of the world -running current account deficits- and to adjust, in this way, its consumption\(^1\).

No previous paper has studied the effect of trend changes in the context of fiscal policy and public finance. This is surprising since long-term GDP growth have a direct effect in long-term government revenue expectations and potentially on the sustainability of sovereign debt. For example, Hall and Sargent (2011) show that GDP growth has been the largest contributor to holding down the debt-to-GDP ratio. My paper focuses on the link between public purchases (and savings) and expectations about future output and public revenue growth. Moreover the facts presented in the next section are explained with a model that emphasizes the forward-looking behaviour of the government. In this sense, my model departs from the usual way in which fiscal policy is modelled where fiscal variables are linked to current real activity through mechanical feedback rules that ignore the role of trend

\(^1\)Hoffmann et al. (2013) build on Engel and Rogers (2006) "world output shares" model, contributing by -among other ways- modelling their intuition in a two-country DSGE framework.
expectations\(^2\).

Consequently, my paper is related with Craig et al. (2013) where a buffer stock model is fitted to data from the U.S. state Unemployment Insurance programs (1976-2008) capturing, as a result, forward-looking government savings behaviour. Mendoza and Oviedo (2005) includes a forward-looking government in an open economy setup to reproduce the procyclicality of government consumption in developing countries. Also –and broadly– this paper is related with the literature on optimal fiscal policy where endogenous fiscal variables are determined while maximizing the sum of the household’s discounted expected utility. In particular, the present work is linked to Ambler and Paquet (1996), Ambler and Cardia (1997) or -more recently- Gali and Monacelli (2008), where optimal purchases are determined subject to exogenously given productivity growth and tax rates\(^3\).

Finally, the literature including both productivity growth trend shocks and forward-looking agents have stressed the role gradual learning about the nature of the productivity shocks (transitory or permanent). Generally this is done by specifying that the agents formulate (rational) expectations about future growth according to the Kalman filter estimates of the permanent component of the productivity growth rate. Empirically, Edge et al. (2007) show that using Kalman filtering techniques on real-time (historical) labour productivity data yields model forecasts of long-term productivity growth similar to those collected in the US Survey of Professional Forecasters. The role of learning is also studied in the last section of the paper.

\(^2\)See, for example, Andrés and Doménech (2006), Leeper et al. (2010a) or Kollmann et al. (2012).

\(^3\)There is an older literature on optimal taxation -Lucas and Stokey (1983) or Chari et al. (1994)- that investigates optimal tax rates to finance an exogenously determined string of government purchases.
3 Empirical evidence

The Congressional Budget Office publishes annual reports—recently, twice a year—on the budget and economic outlook since 1975\(^4\). Each report contains the observed results for the previous fiscal year, an estimate of the outcome for the current year, and projections up to 5 years ahead\(^5\) for the federal budget and general economic variables, like nominal GDP, inflation or the unemployment rate. Moreover, the CBO has reported fiscal year projections of baseline federal revenues and outlays on a National Income Accounts basis since 1984. Consequently, they are suitable for comparison with the data on federal purchases and fiscal balances provided in Table 3 of the NIPA. For information on the definition of each variable, see Appendix A; and for a detailed explanation on the data from the CBO reports, see Appendix B.

Table 1: Correlation of federal purchases and fiscal surpluses (to GDP) for 1984-2012 with CBO long-run revenue and GDP growth projections

<table>
<thead>
<tr>
<th>Fraction of GDP</th>
<th>GDP growth forecast</th>
<th>Net Revenue growth forecast</th>
<th>GDP growth forecast</th>
<th>Net Revenue growth forecast</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purchases</td>
<td>0.44</td>
<td>0.85</td>
<td>0.21</td>
<td>0.79</td>
</tr>
<tr>
<td>prim. Surplus</td>
<td>-0.45</td>
<td>-0.89</td>
<td>-0.21</td>
<td>-0.79</td>
</tr>
<tr>
<td>Surplus</td>
<td>-0.30</td>
<td>-0.79</td>
<td>-0.21</td>
<td>-0.70</td>
</tr>
</tbody>
</table>

only non-defense purchases | all purchases included

Table 1 shows correlations between government purchases divided by GDP and expected growth for real output and real federal net revenue. The same correlations are reported for ratios corresponding to federal fiscal balances. As described in the appendices, federal net revenue is equal to federal revenue minus net transfers payments. In the first two columns only non-defense purchases are considered (and consequently revenue is also net of defense purchases). The CBO was created in July 12, 1974; but official operations did not begin until February 24, 1975.

\(^4\)Since 1996 up to 10-years-ahead projections are reported.
purchases), while in the third and fourth columns the variable \textit{Purchases} includes those with both defense and non-defense destination. The focus of the paper is on the endogenous part of the government purchasing and saving behaviour and, therefore, I prefer to limit the study to non-defense purchases\textsuperscript{6}. However, correlations calculated with all public purchases provide similar results, as can be seen in the third and fourth columns.

The correlations shown in Table 1 correspond to the fiscal years 1984-2012\textsuperscript{7}. Data for calculating fiscal ratios are taken from the NIPA tables\textsuperscript{8}, although using real-time data from the reports provide similar results. In particular, the correlations presented correspond to the actual ratio of year \( t \) (for example, 1993) and the annualized growth rate from \( t \) to \( t + 6 \) (i.e. from 1993 to 1999). This expectation can be calculated using budget projections shown in the report of year \( t + 1 \) (that is, published in early 1994)\textsuperscript{9}.

Notably, revenue growth expectations are positively correlated with purchases ratios, while negatively correlated with fiscal balances. These correlations are consistent with the behaviour of a government adjusting public consumption: when expected revenue growth is relatively high (low), it will run primary deficits (surpluses) and borrow from (save for) the future. Regarding real GDP growth expectations (which are inbedded in revenue growth forecasts), this relation is also observed: higher (lower) GDP growth forecasts coincide with higher (lower) purchases ratios and lower (higher) surpluses\textsuperscript{10}.

Table 2 provides complementary results to Table 1 showing these correla-

\textsuperscript{6}Several other papers also focus on non-defense purchases: as, for instance, those studing optimal fiscal policy –Ambler and Paquet (1996) or Ambler and Cardia (1997)– or political economy drivers of endogenous purchases, as in Bachmann and Bai (2012). In the empirical literature, Ramey (2011) use precisely war dates to identify exogenous public spending shocks.

\textsuperscript{7}To avoid considering growth rates related to negative federal net revenue quantities, like the ones observed during the recent crisis, expected changes in net revenue are shown relative to current GDP.

\textsuperscript{8}Fiscal year values are the corresponding averages of quarterly values reported in the NIPA tables.

\textsuperscript{9}Comparing ratios of year \( t \) with forecasts published in year \( t \) yields similar correlations.

\textsuperscript{10}Figure B.1 in Appendix B shows both growth forecasts for GDP and net federal revenue.
tions with expected long-term real GDP growth after controlling for current growth. The estimated models are:

\[ \text{FiscalRatio}_t = \alpha + \beta \cdot \text{GDPgrowth}_{t-1,t} + \gamma \cdot E_t \text{GDPgrowth}_{t,t+6} + \epsilon_t \]

where the regressors are the ratios of the primary fiscal surplus and non-defense purchases to GDP and the regressants are real GDP growth in the current fiscal year and the expected 6-years-ahead real GDP growth forecasts from the CBO reports. The regressants have been normalized so that their standard deviation are equal to one.

Table 2: Regression results for current GDP growth vs. CBO forecasts

<table>
<thead>
<tr>
<th>Regressor</th>
<th>( \hat{\alpha} )</th>
<th>( \hat{\beta} )</th>
<th>( \hat{\gamma} )</th>
<th>( R^2_{adj} )</th>
<th>( F - \text{test} )</th>
<th>( p - \text{value} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>prim. Surplus</td>
<td>-2.36**</td>
<td>0.78***</td>
<td>0.22</td>
<td>0.0062</td>
<td></td>
<td></td>
</tr>
<tr>
<td>gov'n't Purchases</td>
<td>2.11***</td>
<td>-0.05***</td>
<td>0.25</td>
<td>0.0036</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.65***</td>
<td>-0.09***</td>
<td>0.07**</td>
<td>0.40</td>
<td>0.0005</td>
<td></td>
</tr>
</tbody>
</table>

Table 2 shows that the models including long-term growth forecasts have higher adjusted \( R^2 \) coefficient meaning that they explain a larger fraction of the variability in the fiscal ratios. Moreover, the coefficients for both current and expected future growth rates have a similar size suggesting that both variables are equally important to predict changes in federal purchases and deficits\(^{11}\). Consequently, forward-looking behaviour is a non-negligible component of fiscal policy.

It would be desirable to confirm that these relations hold for a longer time span but, unfortunately, data on tax revenue and GDP growth expectations for previous years are not available. In the literature, Kalman filter techniques have been used to successfully reproduce market expectations on productivity growth –see, for instance, Gilchrist and Saito (2008) or Pakko

\(^{11}\)Similar results are obtained for when comparing current vs. expected net revenue growth, when controlling instead for GDP growth for current and past fiscal years, and for the model with total (not primary) fiscal balances.
(2002)–, and are found suitable to include in models where real GDP growth expectations play a crucial role, as in Hoffmann et al. (2013). Interestingly, Edge et al. (2007) shows that Kalman filtered data on labour productivity growth follow very closely productivity growth expectations reported in the Survey of Professional Forecasters (SPF). Since the correlation between the 10-year-ahead forecasts on real GDP growth and productivity from the SPF is 0.93, estimates of productivity growth expectations are used in this paper to approximate long-term GDP growth expectations.

The data filtered is the (quarterly) series of business sector output per hour from the Bureau of Labor Statistics (BLS) from 1971Q1 to 2012Q2. Edge et al. (2007) stress the importance of using real-time (meaning non-revised historical) data on labour productivity to successfully reproduce the long-term productivity growth expectations provided by the Survey of Professional Forecasters. As a result, I also apply the Kalman filter to real-time data retrieved from the reports available in the BLS website after 1992, while for previous years revised data is used instead.

In this setting, the Kalman filter basically extracts the permanent component assumed by the model in equations (1) and (2) for the growth rate the labour productivity $Z_t$ (denoted with $\Delta z_t$ since $z_t = \ln(Z_t)$). Therefore, this exogenous variable is assumed to have a permanent and a transitory component:

$$\Delta z_t = \Delta z^P_t + \epsilon^z_t$$  \hspace{1cm} (1)

where the trend, $\Delta z^P_t$, follows the stationary process

$$\Delta z^P_t = \rho_z \Delta z^P_{t-1} + (1 - \rho_z) \Delta z + \epsilon^P_t$$  \hspace{1cm} (2)

where both $\epsilon^z_t$ and $\epsilon^P_t$ are i.i.d. (0,1) shocks implying, respectively, permanent changes in the level of productivity or persistent variations in its growth rate.

In the implementation of the Kalman filter, the signal-to-noise ratio, $\chi$, which measures the importance of persistent innovations to labour productivity growth relative to purely transitory shocks, is calibrated to 0.01 as in Gilchrist
and Saito (2008). The mean $\Delta z$ is set equal to 0.42% (1.7% annualized), the average quarterly growth rate of total factor productivity in the United States between 1959 and 2002. Since $\rho_z = 0.975$, the gain $\kappa$

$$
\kappa = \frac{\chi - (1 - \rho_z^2) + \chi \sqrt{\left(\frac{(1 - \rho_z^2)}{\chi}\right)^2 + 1 + 2(1 + \rho_z^2)/\chi}}{2 + \chi - (1 - \rho_z^2) + \chi \sqrt{\left(\frac{(1 - \rho_z^2)}{\chi}\right)^2 + 1 + 2(1 + \rho_z^2)/\chi}}
$$

equal to 0.076 is used in the filtering equation:

$$
E_t \Delta z_t^p = E_{t-1} \Delta z_t + \kappa \left( \Delta z_t - E_{t-1} \Delta z_t \right) = (1 - \kappa) E_{t-1} \Delta z_t + \kappa \Delta z_t \quad (3)
$$

Figure 1 shows the 10-years-ahead productivity growth forecasts produced by applying the Kalman filter to labour productivity data (right-hand side axis) and the quarterly series of the total government primary fiscal balance (divided by GDP)$^{12}$. It is also illustrative to obtain estimates of expected long-term net revenue growth. An easy way is to find a reasonable statistical model for the (persistent or long-term movements in the) ratio of net revenue to GDP and approximate long-term net revenue growth as the sum of the expected growth of the ratio and the expected growth of GDP (or productivity for long-term growth rates). An AR(1) process with with mean 3.5 and autocorrelation coefficient equal to 0.975 is used for this purpose.

$^{12}$Figure B.2 compares the forecasts produced by Kalman filtering quarterly data with the mean value of 10-year-ahead productivity growth forecasts from the Survey of Professional Forecasters.
Table 3 displays correlations between the estimated growth expectations and (total) government consumption, investment, purchases (consumption + investment) and fiscal surpluses divided by GDP. It is, in fact, an extension of Table 1 for total government variables using quarterly data for a longer time.
span. The result is that again the GDP shares of public purchases are positively correlated with GDP and net revenue growth expectations suggesting that the government increases its consumption and investment when it faces more optimistic prospects. Consistently with this interpretation, public surpluses are negatively correlated with these projections. The shares of net revenue provide similar results.

Finally, Table 4 shows the estimation results for the model

$$FiscalRatio_t = \alpha + \beta \cdot GDPgrowth_{t-4,t} + \gamma \cdot E_t PRODgrowth_{t,t+40} + \epsilon_t$$

where quarterly primary fiscal surplus and government purchases data is regressed against the current annual growth rate and the 10-years-ahead (40-quarters-ahead) productivity growth forecast produced with the Kalman filter. Clearly, the regression results confirm the role of long-term growth expectations in fiscal policy for quarterly data and a longer time span.

<table>
<thead>
<tr>
<th>Regressor</th>
<th>$\hat{\alpha}$</th>
<th>$\hat{\beta}$</th>
<th>$\hat{\gamma}$</th>
<th>$R^2_{adj}$</th>
<th>$F - test$</th>
<th>$p - value$</th>
</tr>
</thead>
<tbody>
<tr>
<td>prim.</td>
<td>$-1.72^{**}$</td>
<td>$1.34^{***}$</td>
<td>0.18</td>
<td>0.0000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surplus</td>
<td>5.93***</td>
<td>$1.61^{***}$</td>
<td>$-1.40^{***}$</td>
<td>0.38</td>
<td></td>
<td></td>
</tr>
<tr>
<td>gov'n't</td>
<td>13.06***</td>
<td>$-0.29^{***}$</td>
<td>0.28</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Purchases</td>
<td>11.79***</td>
<td>$-0.33^{***}$</td>
<td>0.23</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4 Model

The model in this paper is based on the two-country model in Hoffmann et al. (2013). It is natural to specify a model of a (big) open economy, like the US, where both private and public sector can borrow from the rest of the world. The two countries will be home (the US) and foreign (the rest of the world), the latter denoted with a star *. The population size of the home
country, $\mathcal{P}$, is normalized to 1 and the relative population size of the foreign economy to $\mathcal{P}^*$, so that $1/(1 + \mathcal{P}^*)$ is the fraction of home population in the world.

Each country is inhabited by a large number of identical households that live infinitely and a competitive firm that exploits a constant returns to scale production technology on private inputs. In addition, the home country has a government that raises taxes, issues debt, decides on its level of public consumption, and builds a stock of capital that increases the productivity of the economy. For simplicity, the government for the foreign country is not formally included in the model. Therefore firms produce a single good which can be used for private consumption and investment in both countries and public consumption and investment in the home country.

Households in the home economy maximize the expected present value of their instantaneous utility, discounted with a factor $\beta$. Preferences are of the type described in King et al. (1988). Both consumption $C_t$ and leisure $(1 - N_t)$ give utility, but there is some degree of external habit formation on consumption, determined by the parameter $h$. The parameter $\nu$ -together with the number of hours supplied along the trend- determines the Frisch elasticity of labour supply and $\sigma$ is the inverse of the elasticity of intertemporal substitution. Thus a representative households maximizes

\[
E_0 \sum_{t=0}^{\infty} \frac{\beta^t}{1 - \sigma} \left[ \left( C_t - h\bar{C}_{t-1} \right) \exp \left( \theta \frac{(1 - N_t)^{1-\nu}}{1 - \nu} \right) \right]^{1-\sigma} \tag{4}
\]

subject to the budget constraint:

\[
C_t + I_t + B_t = (1 - \tau_t^K)\bar{K}_tK_{t-1} + \tau_t^K [\delta K_{t-1} + \Phi(\frac{I_t}{I_{t-1}})I_t]
+ (1 - \tau_t^N)W_tN_t + S_t + (1 + r_t)B_{t-1}
\]

\[
(5)
\]

Income before transfers, the latter denoted with $S_t$, and taxes in period $t$
consists of real labour income $W_t N_t$; as well as the return on capital supply to the firm, $r_t K_{t-1}$, and the net return on a single non-contingent real bond, $r_t B_{t-1}$, respectively. The interest rate paid at time $t$ is set in the previous period in the international bond markets.

After-tax income is used to buy consumption $C_t$, investment $I_t$, and to accumulate assets $B_t$. Financial markets are incomplete because households cannot insure against all possible contingencies. Capital is accumulated according to the equation:

$$K_t = (1 - \delta) K_{t-1} + I_t \left[1 - \Phi\left(\frac{I_t}{I_{t-1}}\right)\right]$$ (6)

where $\Phi(\cdot)$ is an investment adjustment costs function such that $\Phi(1) = 0$, $\Phi'(1) = 0$, and $\Phi''(1) > 0$ at the steady state. In particular, I choose the quadratic cost\(^\text{13}\) formulation:

$$\Phi\left(\frac{I_t}{I_{t-1}}\right) = \frac{\phi}{2} \left(\frac{I_t}{I_{t-1}} - (1 + \Delta z)\right)^2$$ (7)

where $\Delta z$ is the long-run growth rate of labour-augmenting productivity.

Solving the utility maximization problem of the household implies a first order condition that determines labour supply,

$$(1 - \tau_t^N)W_t = \theta(1 - N_t)^{-\nu}(C_t - hC_{t-1})$$ (8)

Euler equations for both financial assets and private capital,

$$1 = (1 + r_t) E_t MRS_{t,t+1}$$ (9)

$$q_t = E_t MRS_{t,t+1} \left[ (1 - \tau_{t+1}^K) r_{t+1}^K + \tau_{t+1}^K \delta + q_{t+1}(1 - \delta) \right]$$ (10)

where $MRS_{t,t+1}$ is the marginal rate of substitution of consumption between periods $t$ and $t+1$. Finally the marginal value of an installed private capital.

\(^{13}\)Also used in Jaimovich and Rebelo (2009) or Christiano et al. (2005), for instance.
unit $q_t$ follows

$$1 = q_t \left[ 1 - \Phi \left( \frac{I_t}{I_{t-1}} \right) - \Phi' \left( \frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] + \tau_t K \left[ \Phi' \left( \frac{I_t}{I_{t-1}} \right) + \Phi \left( \frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right]$$

$$+ E_t MRS_{t,t+1} \Phi' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \left[ q_{t+1} - \tau_{t+1} K \right]$$

(11)

The firm in home country can use a constant returns to scale production technology in private inputs $K_{t-1}$ –private capital used in production at time t– and $N_t$ –hours of labour– to produce output $Y_t$:

$$Y_t = \left( \hat{K}_G^{t-1} \right)^{\alpha G} \left( K_{t-1} \right)^{\alpha} \left( Z_t N_t \right)^{1-\alpha}$$

(12)

where $0 < \alpha, \alpha_G < 1$, with $\alpha + \alpha_G < 1$ and $\hat{K}_G^{t-1}$ is the level of public capital normalized by the long-term balance growth path $\Delta z$. As presented above, assuming $z_t = \ln(Z_t)$, the growth rate of labour productivity (i.e. $\Delta z_t$) follows a process characterized by a stochastic and stationary trend, $\Delta z_t^P$, and a one-period i.i.d. shock, $\epsilon_t^z$, as in equations (2) and (3).

A competitive firm can exploit the production technology in each country and maximizes profits

$$Y_t - r_t^k K_{t-1} - W_t N_t$$

(13)

subject to (12) and taking input prices, public capital $\hat{K}_G^{t-1}$, and productivity $Z_t$ as given. The usual first order conditions imply that the marginal productivity of inputs should be equal to their marginal cost, i.e. the rental price of capital $r_t^k$ and wages $W_t$.

$$r_t^k = \alpha Y_t / K_{t-1}$$

(14)

$$W_t = (1 - \alpha) Y_t / N_t$$

(15)

With constant returns to scale in private inputs, factor payments exhaust
output and profit maximization yields zero profits.

The forward-looking government maximizes expected utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( C_t^G - h^G \bar{C}_{t-1}^G \right)^{1-\sigma}$$  \hspace{1cm} (16)

subject to the budget constraint:

$$C_t^G + I_t^G + (1 + r_t)D_{t-1} = T_t + D_t$$ \hspace{1cm} (17)

where $T_t$ is tax revenue net of transfers and military spending at time $t$, that is,

$$T_t = \tau_t^K \left[ r_t^K K_{t-1} - \delta K_{t-1} + \Phi \left( \frac{I_t}{I_{t-1}} \right) I_t \right] + \tau_t^N W_t N_t - s_t Y_t - m_t Y_t$$ \hspace{1cm} (18)

In other words, the government raises distorting taxes, $\tau_t^K$ and $\tau_t^N$; makes transfers and military spending equal to $s_t$ and $m_t$ fractions of output; and issues new debt, $D_t - D_{t-1}$, to finance debt interest payments, $r_t D_{t-1}$, and purchases $C_t^G$ and $I_t^G$. Through public investment, the government manages the stock of public capital that, as private capital, accumulates according to

$$K_t^G = (1 - \delta^G) K_{t-1}^G + I_t^G \left[ 1 - \Phi^G \left( \frac{I_t}{I_{t-1}} \right) \right]$$ \hspace{1cm} (19)

where $\Phi^G$ denotes the presence of public investment adjustment costs

$$\Phi^G \left( \frac{I_t}{I_{t-1}} \right) = \frac{\phi^G}{2} \left( \frac{I_t}{I_{t-1}} - (1 + \Delta z) \right)^2$$ \hspace{1cm} (20)

Importantly, public consumption yields utility for the government. Equivalently, this could be interpreted as if public consumption gives utility to the
household -without having an effect on its decisions, as in Baxter and King (1993) or, more recently, Andrés and Doménech (2006) - but it is determined by a benevolent government that focuses in maximizing the utility of the household coming exclusively from public good consumption.

Sovereign debt, interest payments and public purchases are entirely endogenously determined, while the tax rates and transfers and military spending shares of GDP follow the rules:

\[(\tau_t - \tau) = \rho_t(\tau_{t-1} - \tau) + \eta_t \left( \frac{B_{t-1}}{Y_{t-1}} - \frac{B}{Y} \right) + \epsilon_t^\tau \] (21)

and

\[(x_t - x) = \rho_x(x_{t-1} - x) + \epsilon_t^x \] (22)

where \(\tau = \{\tau_K, \tau_N\}\) and \(x = \{s, m\}\) and \(\epsilon_t\) is in all cases i.i.d.(0,1). Therefore, the process for revenue net of transfers and military spending, \(T_t\), is broadly equivalent to the one described in section 3.

Given the specification of the government’s problem, the resulting first order conditions resemble those of the household. There is an Euler equation for sovereign debt

\[1 = (1 + r_t)E_t MRS_{t,t+1}^G = (1 + r_t)E_t \beta \left[ \frac{C_{t+1}^G - h^G C_{t+1}^G}{C_t^G - h^G C_{t-1}^G} \right]^{-\sigma} \] (23)

and for public capital

\[q_t^G = E_t MRS_{t,t+1}^G \left[ \left( \alpha \tau_{t+1}^K + (1 - \alpha) \tau_{t+1}^N \right) \alpha_G \frac{Y_{t+1}}{K_t^G} + q_{t+1}^G (1 - \delta^G) \right] \] (24)

while the marginal value of installed public capital unit \(q_t^G\) follows

\[1 = q_t^G \left[ 1 - \Phi \left( \frac{I_t^G}{I_{t-1}^G} \right) - \Phi' \left( \frac{I_t^G}{I_{t-1}^G} \right) \frac{I_t^G}{I_{t-1}^G} \right] + E_t MRS_{t,t+1}^G q_{t+1}^G \Phi' \left( \frac{I_{t+1}^G}{I_t^G} \right) \left( \frac{I_{t+1}^G}{I_t^G} \right)^2 \] (25)

Hoffmann et al. (2013) includes a premium for financial intermediation. This
premium depends on the ratio of net foreign asset position to GDP, \( \frac{NFA_t}{Y_t} \), and on the real interest rate in the rest of the world, \( r^*_t \), as in

\[
r_t = r^*_t - \eta \left( \frac{NFA_t}{Y_t} - \frac{NFA}{Y} \right)
\]

where \( NFA/Y \) is the corresponding steady-state ratio\(^{15}\).

Finally, clearing conditions in the goods and assets markets are necessary to close the model\(^{16}\). Regarding the first,

\[
\frac{1}{1 + \bar{p}_*} (C_t + I_t + C^G_t + I^G_t) + \frac{\bar{p}_*}{1 + \bar{p}_*} (C^*_t + I^*_t) = \frac{1}{1 + \bar{p}_*} Y_t + \frac{\bar{p}_*}{1 + \bar{p}_*} Y^*_t
\]

(27)

Given that, the net foreign asset position, \( NFA_t = A_t - D_t \); then asset markets will clear if the following equality holds:

\[
\frac{1}{1 + \bar{p}_*} NFA_t + \frac{\bar{p}_*}{1 + \bar{p}_*} NFA^*_t = 0
\]

(28)

An equilibrium in this economy can now be defined given productivity processes \( \{\Delta z_t, \Delta z^*_t\}, t = 0, 1, ... \) following (2) and (3); fiscal variables \( \{\tau^K_t, \tau^N_t, S_t, M_t\}, t = 0, 1, ... \) following equations (21) and (22); and the initial values \( \{K_{-1}, K^*_t, K^G_{-1}, B_{-1}, B^*_t, r_{-1}, r^*_{-1}, D_{-1}\} \).

A competitive equilibrium is a collection of sequences \( t = 0, 1, ... \) of prices \( \{r^K_t, W_t, r_t\} \) (and the foreign economy counterparts), quantities \( \{C_t, I_t, K_t, Y_t, B_t, N_t\} \) (and foreign economy counterparts), and fiscal variables \( \{C^G_t, I^G_t, K^G_t, D_t\} \) such that

- the household maximizes (4) subject to (5)-(7), i.e. equations (8)-(11) hold for both countries

\(^{15}\)Schmitt-Grohé and Uribe (2003) explain that, by introducing a debt-elastic interest-rate premium, the net foreign asset position in an open economy model with incomplete asset markets becomes stationary. However, since my results do not depend on the specification of this premium, I do not include it to keep the model as simple as possible.

\(^{16}\)Although specifying asset market clearing would be sufficient because if all budget constraints hold then, by Walras law, goods market also clears.
• firm profits (13) are maximized subject to (12), i.e. equations (14) and (15) –and foreign economy counterparts– hold

• the home government’s utility (16) is maximized subject to (17)-(22), i.e. (23)-(25) hold

• and the international interest rates (equation 26) clear the markets, i.e. (27)-(28) hold.

5 Calibration and data

Table 5 provides the list of necessary calibrated values in addition to the ones described above ($\Delta z, \chi$). The values for $\delta, \alpha, N$ are standard for quarterly data in the RBC literature (see, for example, King and Rebelo (1999)).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>2</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.995</td>
</tr>
<tr>
<td>$\nu$</td>
<td>2</td>
</tr>
<tr>
<td>$\theta$</td>
<td>10.1 (11.1)</td>
</tr>
<tr>
<td>$\delta, \delta_G$</td>
<td>0.025, 0.1</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.36</td>
</tr>
<tr>
<td>$\phi, \phi_G$</td>
<td>7, 10</td>
</tr>
<tr>
<td>$h, h_G$</td>
<td>0.75, 0.9</td>
</tr>
<tr>
<td>$\mathcal{P}^*$</td>
<td>3</td>
</tr>
<tr>
<td>$\alpha_M$</td>
<td>0.075</td>
</tr>
<tr>
<td>$\eta_r$</td>
<td>0.0002</td>
</tr>
<tr>
<td>$\eta_T$</td>
<td>0.00032</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>0.95</td>
</tr>
<tr>
<td>$\rho^x$</td>
<td>0.975</td>
</tr>
</tbody>
</table>

The value of $\sigma = 2$ is often used in open economy models (see again, Schmitt-Grohè and Uribe (2003) or Mendoza and Oviedo (2005)). The time discount factor $\beta$ is chosen so that, together with $\Delta z$, the steady-state interest rate is
1.35% quarterly, i.e. 5.5% annually. The Frisch elasticity of labour supply (equal to \((1-N)/(N \nu)\)) is equal to 2, pinning down the values of \(\nu\) and \(\theta\).

The value of the investment adjustment costs and external habit formation parameters are taken from Smets and Wouters (2007) and Hoffmann et al. (2013). Finally, \(P^*\) is set equal to 3—as in Hoffmann et al. (2013)—because the US economy represents approximately one fourth of world GDP. The persistence parameter \(\rho_z\) is set equal to 0.95, as in Gilchrist and Saito (2008).

The steady state ratios that refer to the government replicate the ratios observed in US data, except for public investment that in reality is close to 3%. In the model this steady state ratio of public investment to output is determined jointly by the parameters \(\delta_G\) and \(\alpha_G\), though. Leeper et al. (2010b) gives values to \(\alpha_G\) between 0.05 and 0.1; so the parameter is calibrated to the average between the two. Depreciation of public capital is assumed to be higher than for private capital, and public investment costs and habit formation parameters are slightly bigger than for the private counterparts since it seems logical to think of public variables as reacting more sluggishly on impact. The values for \(\tau^K\) and \(\tau^N\) are similar to their empirical counterparts (based on the definitions of Jones (2002)). The parameter \(\eta_r\) is set to control for explosive sovereign debt and the AR(1) process parameters are put equal to 0.975. Finally, model moments are obtained with simulations generated with the (plausible) shock volatilities reported in Table 6.

| \(\epsilon^z\) | 0.01 | \(\epsilon^K\) | 0.006 |
| \(\epsilon^P\) | 0.001 | \(\epsilon^N\) | 0.003 |
| \(\epsilon^s, \epsilon^m\) | 0.001 |

(standard deviations)
6 Model results

This section shows the results of the model in terms of impulse-responses and the moments that it generates for artificial simulations. Also the role of imperfect information, modelled as the gradual adjustment of agents’ expectations to trend changes, is studied.

6.1 Impulse-response analysis

Figure 2: Responses of productivity, output and net tax revenue

The model impulse-responses highlight that optimal decisions of a forward-looking government on purchases and savings depend on the type of shocks to the productivity growth rate and their different effects on the present-discounted value of current and future income. All graphs in this subsection show responses to 1 standard deviation transitory and permanent shock under the assumption of perfect knowledge of the nature of each shock.
The graphs on the left in Figure 2 show responses for productivity and the perceived permanent component of its growth rate. Clearly, a purely transitory shock increases the level of productivity 1%, while a persistent shock to the growth rate implies an increase of more than 2%. The trend change caused by a transitory shock is understood to be null, while a permanent shock produces a persistent trend shift in productivity. The graphs on the right show the responses of output and government tax revenue (net of transfers and military purchases). In response to a purely transitory shock, output and net public revenue move to a balance growth path where both become permanently slightly over 1% higher (due to the increase of both productivity and public capital). However, if the shock is permanent, a wealth effect causes a small fall of output and net revenue on impact but, in the long-run, the effect is positive and much larger than in the case of transitory shocks.

Figure 3: Responses of govn’t purchases, primary fiscal balance and debt

Figure 3 show the responses of public purchases, primary surpluses and debt to both types of shocks. My model predicts that in response to a transitory increase in productivity growth public purchases become slightly above a 1%
higher than their level if there have been no change, like in the case of output and net revenue. Given the strong sluggishness imposed on public purchases (high habit formation and investment adjustment costs), the transition to the new balance growth path implies a series of decreasing positive primary surpluses that tend to disappear in the long-run but cause sovereign debt to fall\textsuperscript{17}.

If the shock is permanent, the increase of public consumption is much larger than for transitory shocks, even higher than that of revenue, deteriorating the primary fiscal balance. The long-run effects on public investment are also notably higher than for transitory shocks, although not in the first years. The result is that the government increases its debt because higher future revenues (due to higher future productivity and GDP growth) will be available at the time of repayment.

The intuition behind these responses is that the government, searching to maximize the utility derived from public consumption, smooths public consumption over time. In this setting, the presence of trend shocks implies that the government runs fiscal deficits to adjust the level of government consumption to news about higher future net revenue.

### 6.2 Moments for simulations

In this section, the performance of the model is first analysed in terms of the moments generally discussed in the business cycle literature and then in comparison with the stylized facts described in section 3.

Table 7 the statistical moments of HP filtered data from 1971q2 to 2012q2 and equivalent moments generated by the model\textsuperscript{18}. Public consumption, that

\textsuperscript{17}In fact, lower parameter values for government habit formation and public investment adjustment costs produced almost negligible primary surpluses.

\textsuperscript{18}Model generated moments are the average of those obtained by creating 10000 artificial simulations of the same size as the original sample.
represents approximately 15% of output (with around 5% being devoted to Defence) is less volatile than output, but more than its private counterpart, that represents approximately 60% of output. Its (positive) correlation with output has decreased to the extent of becoming slightly negative when the last two decades are also considered. The share of public investment on output is 3% while private investment moves around 22%. It is positively correlated with output and is more than twice as volatile, however it is more stable than private investment that triples output volatility. The moments generated by the model are aligned with those in the data, although the ratio of net exports to output is more volatile and becomes procyclical (instead of countercyclical, as in the data). The volatility of public consumption becomes easily larger than the volatility of output for lower values of government habit formation.

<table>
<thead>
<tr>
<th>Variable</th>
<th>sd(X)/sd(Y)</th>
<th>Cor(X,Y)</th>
<th>Cor(Xt,Xt-1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>data/model</td>
<td>data/model data/model</td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td>1.00/1.00</td>
<td>1.00/1.00</td>
<td>0.88/0.75</td>
</tr>
<tr>
<td>C</td>
<td>0.59/0.77</td>
<td>0.85/0.54</td>
<td>0.86/0.92</td>
</tr>
<tr>
<td>I</td>
<td>3.99/4.08</td>
<td>0.94/0.13</td>
<td>0.89/0.90</td>
</tr>
<tr>
<td>C^G</td>
<td>0.84/0.94</td>
<td>-0.02/0.38</td>
<td>0.76/0.94</td>
</tr>
<tr>
<td>I^G</td>
<td>2.41/1.47</td>
<td>0.11/0.53</td>
<td>0.71/0.93</td>
</tr>
<tr>
<td>G</td>
<td>0.93/0.91</td>
<td>0.04/0.46</td>
<td>0.80/0.94</td>
</tr>
<tr>
<td>pSpt/Y</td>
<td>1.96/0.74</td>
<td>0.40/0.19</td>
<td>0.98/0.95</td>
</tr>
<tr>
<td>Spt/Y</td>
<td>1.85/1.77</td>
<td>0.44/0.10</td>
<td>0.97/0.53</td>
</tr>
<tr>
<td>NX/Y</td>
<td>1.14/4.15</td>
<td>-0.20/0.17</td>
<td>0.98/0.97</td>
</tr>
</tbody>
</table>

More importantly, Table 8 shows the correlations of the ratios of these variables to output with the 10-year-ahead (40-quarters-ahead) expectations for productivity, output and net revenue growth and can be easily compared with the stylized facts of Tables 1 and 3.

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19See Bachmann and Bai (2011) for another updated and more detailed description of the facts regarding public consumption. For investment, see for instance Leeper et al. (2010b).
Table 8: Correlations with 10-year-ahead growth expectations

<table>
<thead>
<tr>
<th>Productivity growth forecast</th>
<th>Output growth forecast</th>
<th>Net Revenue growth forecast</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_t[\Delta z_{t+40}]$</td>
<td>$E_t[\Delta \ln(Y_{t+40})]$</td>
<td>$E_t[\Delta \ln(T_{t+40})]$</td>
</tr>
<tr>
<td>$C$ 0.60</td>
<td>0.64</td>
<td>0.61</td>
</tr>
<tr>
<td>$I$ 0.53</td>
<td>0.75</td>
<td>0.55</td>
</tr>
<tr>
<td>$CG$ 0.68</td>
<td>0.73</td>
<td>0.68</td>
</tr>
<tr>
<td>$IG$ 0.05</td>
<td>0.18</td>
<td>0.04</td>
</tr>
<tr>
<td>$G$ 0.64</td>
<td>0.71</td>
<td>0.64</td>
</tr>
<tr>
<td>$pSpI$ -0.35</td>
<td>-0.32</td>
<td>-0.38</td>
</tr>
<tr>
<td>$SpI$ -0.24</td>
<td>-0.19</td>
<td>-0.26</td>
</tr>
<tr>
<td>$NX$ -0.63</td>
<td>-0.80</td>
<td>-0.65</td>
</tr>
</tbody>
</table>

Notably, the model replicates the positive correlations for the ratios of government purchases and fiscal deficits that motivated my paper. In particular, the model delivers public consumption ratios that are highly positively correlated with net revenue and productivity long-term growth expectations. As I explained previously, the model exploits the consumption smoothing behaviour of the forward-looking government to fit these stylized facts. To achieve this goal, both public investment and assets are used to spread income available for consumption over time. The correlations of fiscal surpluses ratios with the long-term productivity growth expectations generated by the model are very similar. Public investment is positively correlated with growth expectations although less than public consumption: exactly as in the data.

The model also gives correlations close to those observed in the data for private consumption and net exports divided by output. The former is positively correlated with productivity growth expectations while the latter is negatively correlated. The intuition behind is that the US economy as a
whole also looks at changes on the present value of current and future wealth and spreads its effects on consumption over time optimally—as explained by Hoffmann et al. (2013) and, in general, the literature on the intertemporal approach to the current account. As a result, international bond markets are used to achieve consumption smoothing. My model has shown evidence of the same type of behaviour for the US government.

6.3 The role of imperfect information

Some of the stylized facts in section 3 used productivity growth forecasts produced with the Kalman filter. This technique has also been introduced in DSGE models to incorporate imperfect information or gradual learning about the nature of productivity shocks (permanent or transitory); see, for example, Hoffmann et al. (2012) and (2013) or Gilchrist and Saito (2008). Given that the Kalman filter was used in some of the motivating facts, this section studies the consequences of introducing imperfect information in the model presented in this paper. Recalling the specification for the productivity process in equations (1) and (2), that $\rho_z$ equal 0.95%, and the signal-to-noise ratio is 0.1, the Kalman gain $\kappa$ is put equal to 6.14%.

![Figure 4: True effect and perceived effect on productivity growth trend](image)

Figure 4: True effect and perceived effect on productivity growth trend
Figure 4 shows the true effect and the perceived effect of transitory and permanent shocks in productivity trend growth, that is, in the permanent component of this growth rate. Notably, under imperfect information economic agents judge the increase of productivity resulting from a transitory shock as partly due to a trend shock, although they gradually learn that the shock is purely transitory. In a similar way, persistent shocks are initially perceived to be partly transitory but its true nature is understood in the subsequent periods.

Table 9: Correlations with 10-year-ahead growth expectations

<table>
<thead>
<tr>
<th></th>
<th>Productivity growth forecast</th>
<th>Output growth forecast</th>
<th>Net Revenue growth forecast</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( E_t[\Delta z_{t+40}] )</td>
<td>( E_t[\Delta \ln(Y_{t+40})] )</td>
<td>( E_t[\Delta \ln(T_{t+40})] )</td>
</tr>
<tr>
<td></td>
<td>full</td>
<td>learn</td>
<td>full</td>
</tr>
<tr>
<td>( C )</td>
<td>0.60</td>
<td>0.36</td>
<td>0.64</td>
</tr>
<tr>
<td>( I )</td>
<td>0.53</td>
<td>0.63</td>
<td>0.75</td>
</tr>
<tr>
<td>( C^G )</td>
<td>0.68</td>
<td>0.67</td>
<td>0.73</td>
</tr>
<tr>
<td>( I^G )</td>
<td>0.05</td>
<td>0.14</td>
<td>0.18</td>
</tr>
<tr>
<td>( G )</td>
<td>0.64</td>
<td>0.64</td>
<td>0.71</td>
</tr>
<tr>
<td>( pSpl )</td>
<td>-0.35</td>
<td>-0.15</td>
<td>-0.32</td>
</tr>
<tr>
<td>( Spl )</td>
<td>-0.24</td>
<td>-0.09</td>
<td>-0.19</td>
</tr>
<tr>
<td>( NX )</td>
<td>-0.63</td>
<td>-0.62</td>
<td>-0.80</td>
</tr>
</tbody>
</table>

Table 9 shows that introducing learning does not change the ability of the model to match the motivating facts. Basically, the positive correlation of purchases and deficits ratios with long-term growth forecasts is very similar. However, under imperfect information, the model produces higher volatilities for all variables, including both types of consumption that become more volatile than output.
7 Conclusion

The financial crisis has led to a downward revision of long-term real GDP growth forecasts for advanced economies and raise renewed interests on the implications long-run risk. Although forecasts for the US have been revised moderately, previous papers have shown the important role that shocks hitting the long-term (trend) growth rate have had in asset prices and international capital flows. This paper have firstly investigated some of their effects in the context of fiscal policy and public finance.

First, I presented a set of novel stylized facts relating, precisely, fiscal variables with expectations about real GDP (and net revenue) long-term growth using CBO forecasts data. Remarkably, government purchases and fiscal deficits (divided by GDP) are positively correlated with growth expectations. Moreover, simple regression analysis has shown that long-term growth forecast are as important as current GDP growth for fiscal policy. I also document that these facts hold for a longer time span (1971-2012) and total government data using à-la-Kalman estimates of productivity forecasts previously used in the literature as a proxy for long-term GDP growth expectations. These stylized facts suggest that the government, searching to adjust public consumption to news about long-term growth, borrows if it expects higher future wealth, or lends when facing gloomier prospects.

Second, I show that a simple open economy RBC model successfully reproduces these correlations. The first important element in the model is that the productivity growth rate is subject to both purely transitory and permanent (trend) shocks. A second essential ingredient is a forward-looking government that decides endogenously on purchases and savings, while tax rates follow mechanical rules. As shown by the impulse-responses, these two features identify the government’s effort to smooth the effect of trend changes on public consumption as the main driver of the presented correlations. Model derived moments show that allowing for agents to learn gradually about the nature of the shock does not change the ability of the model to match the
motivating stylized facts.

My work leaves aside interesting questions beyond the scope of the present paper. For example, the political process and its own dynamics is clearly an important factor determining government purchases and its financing; however, I admittedly ignore political economy considerations to focus instead in the novel feature of public consumption smoothing behaviour of the government over periods of higher or lower trend growth.
References


Edge, R., Laubach, T., and Williams, J. (2007). Learning and shifts in long-


A  Data definitions

Output (GDP): table 1.1.5 line 1

Consumption (C): private consumption of non-durable goods (table 1.1.5 line 5) + services (line 6)

Investment (I): gross private domestic investment (table 1.1.5 line 7) + private consumption of durable goods (line 4)

Net exports (NX): of goods and services (table 1.1.5 line 14)

Average personal income tax rate: \( \tau_p = \frac{IT}{W + PRI/2 + CI} \)
where, capital income, \( CI = PRI/2 + RI + CP + NI \)
and IT = personal tax income (table 3.1 line 3)
\( W = \) wages (table 1.12 line 3)
\( PRI = \) propietor’s income (table 1.12 line 9)
\( RI = \) rental incom (table 1.12 line 12)
\( CP = \) corporate profits (table 1.12 line 13)
\( NI = \) net interest (table 1.12 line 18)

Labour income tax rate: \( \tau_N = \frac{\tau_p(W + PRI/2) + CSI}{EC + PRI/2} \)
where, CSI = contributions to government social insurance (table 3.1 line 7)
\( EC = \) total compensation of employees (table 1.12 line 2)

Capital income tax rate: \( \tau_K = \frac{\tau_p CI + CT + PT}{CI + PT} \)
where, CT = corporate taxes (table 3.1 line 5)
\( PT = \) property taxes (table 3.3 line 8)

Total government revenue: \( IT + CSI + CT + PT \)
Transfers: Current transfer payments (table 3.1 line 17) + Subsidies (line 25) - Current transfer receipts (line 11) - Current surplus of government enterprises (line 14) + Capital transfer payments (line 36) - Capital transfer receipts (line 32)

Military (or Defence) expenditures: Defence govn’t consumption expenditures and Gross investment (table 3.9.5 line 11) - Defence consumption of general govn’t fixed capital (table 3.10.5 line 27)

Net revenue \( (T) \) = Total govn’t revenue - Transfers - Military Expenditures

Govn’t (non-defence) consumption of fixed capital = government consumption of fixed capital (table 3.1 line 38) - defence consumption of general government fixed capital (table 3.10.5 line 27)

Govn’t (non-defence) consumption \( (C^G) \) = federal non-defence consumption expenditures (table 3.9.5 line 17) + state and local non-defence consumption expenditures (line 21) - (non-defence) government consumption of fixed capital

Govn’t (non-defence) investment \( (I^G) \) = federal non-defence gross investment (table 3.9.5 line 18) + state and local gross investment (line 23)

Primary govn’t fiscal surplus \( (pSpl) \) = net lending or net borrowing (table 3.9.5 line 39) + net purchases of nonproduced assets (line 37) - interest payments (table 3.1 line 22)

For turning variables into real and per-capita terms, I used:

price Indexes for GDP (table 1.1.4 line 23)
population (St.Louis Fed, database)

In order to compare these variables with the net revenue forecasts constructed with the CBO budget projections, equally defined federal counterparts for non-defence consumption, investment and have been calculated.
B Data treatment of CBO projections

An example of the tables provided in the CBO reports is shown in Table B.1. It corresponds to the report published in January 1994. Clearly, the report corresponding to year \( t \) (1994, in the example) shows realized values for fiscal year \( t - 1 \) (1993) and forecasts for fiscal years \( t \) to \( t + 5 \) (1994 to 1999). In addition, a similar table reports forecasts for the economic outlook: for instance, nominal GDP, inflation, unemployment or interest rates.

In order to keep variable definitions close to their counterparts for the general government, federal variables are constructed as:

- **Revenue** = Personal Tax and Nontax Receipts + Corporate Profits Tax Accruals + Indirect Business Tax and Nontax Accruals + Contributions for Government Social
- **Transfers** = Current Transfer Payments + Subsidies Less Current Surplus of Government Enterprises - Current Transfer Receipts
- **Defence expenditures** = Defence consumption + Defence consumption of fixed capital
- **Net revenue** = Revenue - Transfers - Defence expenditures.
- **Non-defence consumption** = non-defence consumption expenditures - non-defence consumption of fixed capital
- **Non-defence investment** = non-defence consumption of fixed capital
- **Primary surplus** = Net federal saving + Interest payments

*Note that, differently from before, I am not considering all gross investment, but only consumption of fixed capital.

Table 1 shows correlations between the expected growth rates for real net revenue and GDP and the share of non-defence federal consumption, investment and primary surpluses on GDP or net revenue. In other words, I look at the comovement between non-defence federal consumption to GDP for any year \( t \) (for instance, 1993) and the annualized growth rate of real net revenue from years \( t - 1 \) to \( t + 5 \) (1992 to 1998). The annualized gross growth rate is
calculated as:

\[
\left( E_{t-1} \left[ \frac{T_{t+5} Defl_{t-1}}{T_{t-1} Defl_{t+5}} \right] \right)^{\frac{1}{6}}
\]  \hspace{1cm} (29)

where \( Defl_t \) is the forecasted GDP deflator corresponding to year \( t \). The growth rate for GDP is calculated in a similar way. Notice that

\[
\left( E_{t-1} \left[ \frac{T_{t+5}}{T_{t-1}} - \frac{GDP_{t+5}}{GDP_{t-1}} \right] \right)^{\frac{1}{6}}
\]  \hspace{1cm} (30)

is the expected (annualized) growth rate at \( t - 1 \) of the fraction of net revenue to GDP from period \( t - 1 \) to \( t + 5 \).
Table B.1: Summary of projections for Fiscal Year 1994

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td><strong>Current Receipts</strong></td>
<td>1,245</td>
<td>1,334</td>
<td>1,426</td>
<td>1,502</td>
<td>1,569</td>
<td>1,651</td>
<td>1,731</td>
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<tr>
<td>Personal Tax and Nontax Receipts</td>
<td>515</td>
<td>556</td>
<td>606</td>
<td>646</td>
<td>679</td>
<td>720</td>
<td>760</td>
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<tr>
<td>Contributions for Gov. Social Insurance</td>
<td>511</td>
<td>547</td>
<td>581</td>
<td>613</td>
<td>645</td>
<td>677</td>
<td>708</td>
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<tr>
<td>Corporate Profits Tax Accruals</td>
<td>135</td>
<td>141</td>
<td>142</td>
<td>147</td>
<td>152</td>
<td>158</td>
<td>165</td>
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<tr>
<td>Indirect Business Tax and Nontax Accruals</td>
<td>84</td>
<td>90</td>
<td>97</td>
<td>96</td>
<td>93</td>
<td>96</td>
<td>98</td>
</tr>
<tr>
<td><strong>Current Expenditures</strong></td>
<td>1,485</td>
<td>1,536</td>
<td>1,592</td>
<td>1,671</td>
<td>1,744</td>
<td>1,823</td>
<td>1,928</td>
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<tr>
<td>Consumption Expenditures</td>
<td>446</td>
<td>439</td>
<td>444</td>
<td>455</td>
<td>469</td>
<td>484</td>
<td>501</td>
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<tr>
<td>Defense</td>
<td>307</td>
<td>293</td>
<td>291</td>
<td>297</td>
<td>305</td>
<td>314</td>
<td>325</td>
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<tr>
<td>Nondefense</td>
<td>139</td>
<td>146</td>
<td>153</td>
<td>158</td>
<td>164</td>
<td>170</td>
<td>176</td>
</tr>
<tr>
<td>Current Transfer Payments</td>
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<td>878</td>
<td>923</td>
<td>976</td>
<td>1023</td>
<td>1075</td>
<td>1153</td>
</tr>
<tr>
<td>Required Reductions in Discretionary Spending</td>
<td>-11</td>
<td>-19</td>
<td>-35</td>
<td>-51</td>
<td>-50</td>
<td></td>
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<tr>
<td>Interest Payments</td>
<td>181</td>
<td>186</td>
<td>196</td>
<td>209</td>
<td>219</td>
<td>229</td>
<td>239</td>
</tr>
<tr>
<td>Subsidies Less Current Surplus of Gov. Enterprises</td>
<td>32</td>
<td>33</td>
<td>29</td>
<td>31</td>
<td>33</td>
<td>35</td>
<td>35</td>
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<tr>
<td><strong>Net Federal Gov. Saving</strong></td>
<td>-240</td>
<td>-202</td>
<td>-166</td>
<td>-169</td>
<td>-175</td>
<td>-172</td>
<td>-197</td>
</tr>
</tbody>
</table>
Figure B.2: Comparison of SPF 10-years-ahead productivity growth forecasts with those obtained with the Kalman filter