Unemployment Risk and the Distribution of Assets*

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Abstract

How does the distribution of assets affect job search decisions? We analyze unemployed workers and how their asset holdings affect the allocation to jobs of different productivity. In the absence of insurance, workers with low asset holdings direct their search to low productivity jobs because they offer a low wage and low risk. We show that this occurs under a condition closely related to Decreasing Relative Risk Aversion. There is perfect segregation of asset holders into job productivities even when assets holdings are private. We also find that for a given worker, the productivity of jobs she applies for is decreasing in the duration of unemployment. As assets gradually deplete, she takes more secure, low wage jobs. We also use the model to evaluate a tax financed unemployment insurance scheme in a dynamic framework. Higher benefits are beneficial for consumption smoothing but deter the entry of low productive firms and change the probability of job finding by affecting the distribution of workers and their allocation to firms. In our quantitative analysis, we find the optimal level of UI and we also demonstrate that in the presence of two sided heterogeneity and assortative matching between workers and firms, productivity changes generate the unemployment fluctuations we see in business cycles.

Keywords. Unemployment Risk. Precautionary Savings. Sorting. Mismatch. Optimal Unemployment Insurance.

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1 Introduction

In this paper we analyze how the distribution of assets affect job search decisions, and in particular how the workers' asset holdings affect the allocation to jobs of different productivity. In the absence of insurance, workers with low asset holdings direct their search to low productivity jobs because they offer a low wage and low risk. We show that this occurs under a condition closely related to Decreasing Relative Risk Aversion. There is perfect segregation of asset holders into job productivities even when assets holdings are private. We also find that for a given worker, the productivity of jobs she applies for is decreasing in the duration of unemployment. As assets gradually deplete, she takes more secure, low wage jobs. When workers are heterogeneous in skills, there is a trade off between wages and insurance. The skilled but poor worker will necessarily go for the less ambitious, low wage job in order to hedge risk.

Unemployment risk is, arguably, one of the biggest causes of income uncertainty. In this model, we analyze labor market equilibrium in the presence of three sources of market incompleteness: uninsurable unemployment risk, job search and private information about assets. The key aspect that we focus on is heterogeneity in asset holdings. How do workers with different asset holdings insure against unemployment risk in the absence of a formal insurance market? While our focus is on incomplete markets, we do allow for complete capital markets with consumption smoothing. However, there is no full insurance and therefore there is a role for precautionary savings.

Our main finding is that in equilibrium there is full separation in the job search decision under a condition closely related to Decreasing Absolute Risk Aversion. Workers with more asset holdings apply to more productive jobs even though they are not more productive. This is due to the fact that high productive firms set higher wages because their opportunity cost of not filling the vacancy is higher. On the other side of the market, high asset holders are more willing to take risk. Therefore they apply for high paying, high risk jobs.

In the second part of this paper, we use the model to evaluate a tax financed unemployment insurance scheme in a dynamic framework. The unemployed search until they find a job, and once employed they hold the job unless they are hit by idiosyncratic employment shocks. The labour market has a directed search structure: firms enter by posting vacancies and workers can apply for different jobs and form the submarkets. The market is perfectly segmented and unemployed workers match with firms in each submarket bilaterally. The probability of match in each submarket depends on market tightness (the ratio of vacancies to unemployed in each submarket) and it is determined by a matching function.

Higher benefits are beneficial for consumption smoothing but deter the entry of low productive firms and change the probability of job finding by affecting the distribution of workers and their allocation to firms. In our quantitative analysis, we find the level of UI above which the distribution and allocation effects are more potent for welfare outcomes and below which consumption smoothing effect is stronger. We also demonstrate that in the presence of two sided heterogeneity and assortative matching between workers and firms, productivity changes generate the unemployment fluctuations we see in business cycles.

This paper is related to a large literature on unemployment risk and risk averse agents. Danforth (1979) is one of the first to analyze optimal search in a partial equilibrium setting. Hopenhayn and Nicolini (1997), Shimer and Werning (2007) and Shimer and Werning (2008) analyze optimal unemployment insurance in a similar setting. Our paper is a general equilibrium search model with risk averse agents and closely related to Acemoglu and Shimer (1999). They analyze workers with identical asset holdings and focus on the incentives for firms to create jobs. Golosov, Maziero, and Menzio (2012) consider the setup in Acemoglu and Shimer (1999) and analyze private job search decisions by identical agents, driven by the participation constraint. Here, we focus on private assets and where the distribution of those assets is non-degenerate.

Also Guerrieri, Shimer, and Wright (2010) and Eeckhout and Kircher (2010) have private information, but there the source of complementarities and hence the separation is technological, namely the complementarity between worker skills and job productivity or the single crossing between worker skill and effort. Here instead, the complementarity derives entirely from preferences and how the risk attitude changes with assets. This is an entirely novel approach to matching since it basically involves two-sided matching with non-linear pairwise Pareto frontiers.

Unemployment insurance (UI) is an important policy tool to help families insure against the risk of job loss. In 2010, the US federal government spent $162 billion on UI. Since the onset of the great recession, UI payments have gone up fivefold, up from $33 billion in 2007. This is mainly due to the extension of the eligibility period from 26 weeks to 99 weeks. One of the central policy questions in economics is how those UI payments affect aggregate labor market variables such as the unemployment rate, job creation and productivity. The focus of attention of the literature is on the incentive effects of the presence or absence of UI.[1] How does UI affect workers' reservation wage? How much effort are they willing to put in to search for a job when generous benefits are available?

In this paper, we analyze the job search decision by workers who are heterogenous in their asset holdings. Since the needs to smooth consumption by the rich are very different from those of the poor, asset holdings will affect workers' job search behavior. In particular, workers with high asset holdings will apply for the more productive jobs that pay higher wages. For the workers with low asset holding those jobs have too much unemployment risk and they prefer to apply for low productivity jobs with lower wages yet higher employment chances. Because workers are homogeneous in productivity, this allocation of asset holdings to job productivities is inefficient, i.e., there is mismatch. When UI benefits are not conditioned on assets/income, there are two reasons why the high asset holders get too much benefits. First, because they do not need the smoothing since they can rely on their assets. Second, due to the fact that they choose jobs with higher unemployment risk they tend to receive unemployment

[1] There may of course also be aggregate demand effects of transferring consumption to the unemployed. For a recent analysis of such a mechanism, see Kaplan and Menzio (2012).

benefits more often.

2 The Model

Timing. This is a two-period economy in which agents make a joint consumption-savings and job search decision. Endowed with assets, in there first period they choose their consumption-savings level and in the second period they choose the optimal job search. In section **??** we extend the model to an infinite horizon setting.

Agents. There is a measure one of workers, indexed by their heterogeneous asset holdings $a \in \mathcal{A} = [\underline{a}, \overline{a}] \subset \mathbb{R}_+$. Let $G(a)$ denote the measure of workers with asset levels weakly below $a \in \mathcal{A}$. We assume a is private information. Each worker supplies her labor and can only apply to one job at a time. Firms are heterogeneous in their productivities y and each have one job. Let $y \in \mathcal{Y} = [\underline{y}, \overline{y}] \subset \mathbb{R}_+$ and assume the firm type is observable. $F(y)$ denotes the measure of firms with a type weakly below y. The total measure of firms is $F(\overline{y})$. F and G are C^2 with strictly positive derivatives f and g.

Preferences and Technology. Workers are risk averse and their preferences are represented by the Bernouilli utility function $u(c)$ over consumption level c, where $u : \mathbb{R}_+ \to \mathbb{R}_{++}$. We assume that u is increasing and concave: $u' > 0, u'' < 0$. Agents discount utility with factor $\beta < 1$. Savings can be invested in a risk free bond at a fixed rate $R > 1$. We assume that firms are owned by entrepreneurs who are risk neutral and who do not participate in the labor market.[2] Output produced at a firm of type y is $v(y)$.

Search Technology. Job search is directed and there is a search technology that governs those frictions. The frictions crucially depend on the degree of competition for jobs, as captured by ratio of workers to firms, denoted by $\lambda \in [0, \infty]$. This represents the relative supply and demand for jobs, as it determines the probability of a match $m(\lambda)$, where $m : [0, \infty] \to [0, 1]$: the higher the value of λ, the easier it is for a firm to fill it's vacancy, so m is a strictly increasing function: $m' > 0$. In contrast, the higher the ratio of workers to firms, the harder it is for the worker to find a match. We denote the probability that a worker gets matched by $q(\lambda)$, where $q : [0, \infty] \to [0, 1]$ is a strictly decreasing function, $q' < 0$. Since matching is always in pairs, matching probability of workers must be consistent with those of firms, in particular, it must be the case that $q(\lambda) = m(\lambda)/\lambda$. We also require the standard assumptions hold: m is twice continuously differentiable, strictly concave and has a strictly decreasing elasticity. The fact that we express the matching probability in terms of the ratio of workers to firms λ and not the number of workers and firms effectively means that we assume a matching technology that is constant

[2] We could also have assumed that profits are distributed as the risk free dividend of a mutual fund owned by all workers and that has all firms in its portfolio as in Golosov, Maziero, and Menzio (2012). This approach does not affect any of the results since the dividend deterministically increases the workers' asset holdings and merely shifts the asset distribution.

returns. As the number of workers and firms doubles, the number of matches doubles, yet the matching probabilities remain unchanged.

Actions. In the first period, agents choose their consumption-savings bundle for both periods. Given assets a, each worker chooses consumption $c_1 = a - a'$ where a' are the assets saved, and second period consumption is contingent on the labor market outcome, $c_{2,e} = Ra' + w$ when employed and $c_{2,u} = Ra'$ when unemployed. Within the second period, a two-stage labor search extensive form game determines the labor market outcome. Firms first simultaneously announce wages $w \in \mathcal{W} = [\underline{w}, \overline{w}] \subset \mathbb{R}_+$. After observing wage-firm type pairs (w, y), the workers then choose which pair to apply to. A worker always has the option not to apply for any job, the choice of which is denoted by $\emptyset$. Due to the presence of market frictions, not all applications are successful. Denote by $P(y, w)$ and $Q(a, a', y, w)$ the distribution of actions by firms and workers: $P(y, w)$ is the measure of firms that offers a productivity-wage pair below (y, w) and $Q(a, a', y, w)$ is the measure of workers with assets below a who save less than a' and who apply for productivity-wage pairs below (y, w). We impose that those distributions of actions are consistent with the initial distributions of types $F(y)$ and $G(a)$, i.e., that there is market clearing. In particular, it must be the case that $P_{\mathcal{Y}}(\cdot) = F(\cdot)$ and $Q_{\mathcal{A}} = G(\cdot)$, where $P_{\mathcal{Y}}$ and $Q_{\mathcal{A}}$ are the marginal distributions. This ensures that the allocation is measure preserving.

Equilibrium. We adopt the equilibrium concept used by Acemoglu and Shimer (1999). To accommodate the two sided heterogeneity of firm productivity and worker assets, we will use the version of their equilibrium adjusted by Eeckhout and Kircher (2010) to allow for two-sided heterogeneity and a continuum of agents. They consider the Acemoglu and Shimer (1999) setup as a large game where each individual's payoff is determined only by her own action and the distribution of actions in the economy, which consists of the optimal choices of each of the individuals in the distribution. Denote by the function $\lambda_{PQ} : \mathcal{Y} \times \mathcal{W} \to [0, \infty]$ the expected queue length at each productivity-wage combination (y, w).[3] The expected payoff of the workers is given by:

$$\begin{aligned} U(a, a', y, w, P, Q) &= u(c_1) + \beta \mathbb{E} u(c_2) \\ &= u(c_1) + \beta q(\lambda_{PQ}) u(c_{2,e}) + (1 - q(\lambda_{PQ})) u(c_{2,u}), \end{aligned} \tag{1}$$

Since $c_1 = a - a', c_{2,e} = Ra' + w, c_{2,u} = Ra'$, the consumption is completely pinned down by the choice of a'. The expected profits of the firm are

$$\pi(y, w, P, Q) = m(\lambda_{PQ}) \left(v(y) - w \right). \tag{2}$$

In line with the literature on directed search (see for example McAfee (1993), Acemoglu and Shimer (1999)), we impose restrictions on the beliefs about off equilibrium path behavior. In the current setup,

[3] Then along the support of the firms' wage setting distribution, $\lambda_{PQ} = dQ_{\mathcal{YW}}/dP$ is given by the Radon-Nikodym derivative, where $Q_{\mathcal{YW}}$ is the marginal distribution of Q with respect to $\mathcal{Y}$ and $\mathcal{W}$.

beliefs about the queue length corresponding to firm or worker choices that do not occur in equilibrium are not defined. Therefore, we define those off equilibrium path beliefs corresponding to the notion of subgame perfection.[4] Firms expect workers to queue up for jobs as long as it is profitable for them to do so given the options they have on the equilibrium path. Formally, this defines the queue length over the entire domain as: $\lambda_{PQ}(a, w) = \sup\{\lambda \in \mathbb{R}_+ : \exists a, q(\lambda)[y - w \geq \max_{y,w \in \text{supp} P} U(a, y, w, P, Q)]\}$. In all other cases, the queue length is zero. This now permits us to formally define equilibrium:

Definition 1 *An equilibrium is a pair of distributions (P, Q) such that the following conditions hold:*

1. *Worker optimality: $(a, a', y, w) \in$ supp Q only if (y, p) maximizes (1) for a;*
2. *Firm optimality: $(y, w) \in$ supp P only if w maximizes (2) for y;*

This is a matching problem with a non-linear pairwise Pareto frontier. Existence is established in Legros and Newman (2007) and Kaneko (1982). Jerez (2012) establishes the existence of an equilibrium in a directed search model with a continuum of agents and a general matching technology.

The (measure preserving) market clearing condition is particularly simple when matching is monotone. Then there is one-to-one matching of a to y, which we represent by a a function $\mu : \mathcal{Y} \to \mathcal{A}$. Under positive assortative matching (PAM), $\mu'(y)$ is positive and it is negative under negative assortative matching (NAM). Under PAM high asset workers match with high productivity jobs, and the market clearing condition can be written as:

$$\int_a^{\overline{a}} f(a)da = \int_{\mu(a)}^{\overline{y}} \lambda(y)g(y)dy.$$

3 The Decentralized Equilibrium Allocation

The firm problem is to set wages to maximize expected profits $\pi(y) = \max_w m\,(v(y) - w)$. The consumer's problem is to maximize expected utility from consumption while simultaneously making an optimal search decision. The worker's consumption–labor choice problem is:

$$\max_{a',\lambda} u(a - a') + \beta\left(q' u(Ra' + w) + (1 - q)\, u(Ra')\right),$$

We can therefore write the equilibrium worker and firm optimization as:

$$\max_{a',\lambda} u(a - a') + \beta\left(q' u(Ra' + w) + (1 - q)\, u(Ra')\right)$$
$$\text{s.t. } \pi = \max_w m\,(v(y) - w)\,.$$

[4]Peters (1997) and Peters (2000) provide micro foundations for a version of this model where this assumption is indeed justified as the limit of deviations in a finite game.

Given $w = v(y) - \frac{\pi}{m}$ and the optimal choice of wages follows from the optimal choice of queue length λ, we can write this problem as a matching problem with non-linear Pareto frontier denoted by $\Phi(a, y, \pi)$:

$$\Phi(a, y, \pi) = \max_{a', \lambda} u(a - a') + \beta \left[qu\left(c_e\right) + (1-q)\, u(Ra') \right]$$

where $c_e = Ra' + v(y) - \frac{\pi}{m}$. Then the solution to the maximization problem is $a'^{\star}(a, y, \pi), \lambda^{\star}(a, y, \pi)$ and satisfies:

$$\begin{aligned} -u'(a - a') + \beta R \left[qu'(c_e) + (1-q)\, u'(Ra') \right] &= 0 && (3) \\ \beta q' \left[u\left(c_e\right) - u(Ra') \right] + \beta u'\left(c_e\right) \frac{m'\pi}{\lambda m} &= 0 && (4) \end{aligned}$$

The optimal savings behavior and optimal job search simultaneously implies a matching decision. That is a worker a chooses a firm y. Her optimal decision given a schedule $\pi(y)$ is to choose the firm type y that maximizes her expected utility. The optimal y therefore satisfies $\Phi_y + \Phi_\pi \frac{\partial \pi}{\partial y} = 0$. This implies:

$$\beta q u'(c_e) \left(v_y - \frac{\pi'}{m} \right) = 0. \tag{5}$$

where the effect of y and π on π through a' and λ is zero from the envelope theorem: $\Phi_{a'} = 0, \Phi_\lambda = 0$. The details of the derivation of the partial derivatives are in the Appendix.

We want to ascertain under which circumstances there is monotone matching of asset holdings a in job productivities y. This is now a matching problem $\Phi(a, y, \pi)$ where a type a chooses the optimal y, given optimizing behavior. The first order condition to this problem is $\Phi_y + \Phi_\pi \frac{\partial \pi}{\partial y} = 0$. An allocation $a = \mu(y)$. The total cross derivative is positive provided

$$\frac{d^2}{dady}\Phi = \Phi_{ay} + \Phi_{\pi y}\frac{\partial \pi}{\partial y} = \Phi_{ay} - \frac{\Phi_y}{\Phi_\pi}\Phi_{\pi a},$$

where we use the first order condition to substitute for $\frac{\partial \pi}{\partial y}$. Therefore, there will be Positive Assortative Matching in types a, y provided $\Phi_{ay} > \frac{\Phi_y}{\Phi_\pi}\Phi_{a\pi}$. Then $\Phi_{ay} > \frac{\Phi_y}{\Phi_\pi}\Phi_{a\pi}$ provided (where the partial derivatives of Φ are derived in the Appendix):

$$\begin{aligned} -u''(a - a')a'_y &> \frac{\beta q u'(c_e) v_y}{\beta q u'(c_e) \frac{-1}{m}} (-u''(a - a')a'_\pi) \\ a'_y &> -m v_y a'_\pi \end{aligned}$$

We obtain the expressions for a'_y and a'_π from the first order conditions (in Appendix). Denote the maximand by $\phi(a', \lambda) = u(a - a') + \beta \left[qu\left(c_e\right) + (1-q)\, u(Ra') \right]$, i.e., the objective function that is maximized with respect to a', λ. Then the condition for positive sorting of a on y becomes:

$$\begin{aligned} a'_y &> -mv_y a'_\pi \\ \left(\phi_{a'y} + mv_y\phi_{a'\pi}\right)\phi_{\lambda\lambda} &< \left(\phi_{\lambda y} + mv_y\phi_{\lambda\pi}\right)\phi_{a'\lambda} \end{aligned}$$

Observe that from the first order conditions to the maximization problem, we obtain the cross partials on ϕ. First, note that $\phi_{a'y} = -v_y m\phi_{a'\pi} = v_y\beta Rqu''(c_e)$ so that the LHS is zero. Then we derive the following:

$$\begin{aligned} \phi_{\lambda y} &= \beta q'u'(c_e)v_y + \beta u''(c_e)v_y\frac{m'\pi}{\lambda m} \\ \phi_{\lambda\pi} &= \beta q'u'(c_e)\frac{-1}{m} + \beta u'(c_e)\frac{m'}{\lambda m} + \beta u''(c_e)\frac{-1}{m}\frac{m'\pi}{\lambda m} \\ &= \frac{-1}{v_y m}\phi_{\lambda y} + \beta u'(c_e)\frac{m'}{\lambda m} \end{aligned}$$

Therefore, the inequality can be written as:

$$0 < \beta u'(c_e)\frac{m'}{\lambda}v_y\phi_{a'\lambda}$$

The condition for positive sorting of a on y is $\phi_{a'\lambda} > 0$ is thus,

$$\beta R\left(q'[u'(c_e) - u'(Ra')] + u''(c_e)\frac{\pi m'}{\lambda m}\right) > 0$$

This now allows us to derive the first result:

Proposition 1 *Workers with higher initial asset levels a will apply for higher wage jobs provided*

$$\frac{u'(c_e) - u'(Ra')}{u(c_e) - u(Ra')} < \frac{u''(c_e)}{u'(c_e)}. \tag{\textbf{U}}$$

Proof. From the first order condition $\phi_\lambda = 0$ we obtain:

$$\frac{m'\pi}{\lambda m} = -q'\frac{u(c_e) - u(Ra')}{u'(c_e)}.$$

Substituting in the condition $\phi_{a'\lambda} > 0$:

$$q'[u'(c_e) - u'(Ra')] - u''(c_e)q'\frac{u(c_e) - u(Ra')}{u'(c_e)} > 0,$$

or, noting that $q' < 0$,

$$u'(c_e)[u'(c_e) - u'(Ra')] < u''(c_e)\left[u(c_e) - u(Ra')\right].$$

or alternatively

$$\frac{u'(c_e) - u'(Ra')}{u(c_e) - u(Ra')} < \frac{u''(c_e)}{u'(c_e)}.$$

■

This Proposition establishes under what conditions of the utility function agents with higher levels of assets will choose more risky jobs. The does not immediately allow for a straightforward interpretation, and in the next two results we characterize the properties. First, we show that within the class of Hyperbolic Absolute Risk Aversion (HARA) utility functions, the condition is satisfied whenever absolute risk aversion is decreasing (DARA).

Proposition 2 *Consider the class of utility functions with Hyperbolic Absolute Risk Aversion (HARA):*

$$u(c) = \frac{1-\gamma}{\gamma}\left(\frac{\alpha c}{1-\gamma} + \beta\right)^{\gamma} \qquad \textit{where } \alpha > 0, \beta > \frac{\alpha c}{1-\gamma}.$$

*Then condition (**U**) holds whenever there is Decreasing Absolute Risk Aversion (DARA): $\gamma < 1$. It holds with opposite inequality when there is Increasing Absolute Risk Aversion (IARA): $\gamma > 1$.*

Proof. In Appendix. ■

A number of results for special cases of the HARA preferences immediately follow, including CRRA, logarithmic, CARA, risk neutrality and the quadratic.

Proposition 3 *Consider the class of HARA utility functions. Condition (**U**) holds:*

1. *under CRRA $u(c) = \frac{1-\gamma}{\gamma}c^{\gamma}$ $(\alpha = 1-\gamma, \gamma < 1, \beta = 0)$ and Log utility: : $u(c) = \log c$ $(CRRA, \gamma \to 0)$;*
2. *with equality under CARA $u(c) = 1 - e^{-\alpha c}$ $(\beta = 1, \gamma \to -\infty)$ and Risk Neutral $u(c) = \alpha c$ $(\gamma = 1)$;*
3. *with opposite inequality under Quadratic utility: $u(c) = -\frac{1}{2}\left(-\alpha c + \beta\right)^2$ $(\gamma = 2)$.*

Proof. In Appendix. ■

The results for HARA may indicate that condition (**U**) holds more generally. The answer is partially true. For small differences between the level of consumption when a job is obtained and the consumption of unemployment ($c_e - Ra' = w$ small), we can indeed completely generalize the characterization: when there is DARA, condition (**U**) is satisfied and high asset types choose high productivity jobs. This is proven in Proposition 4. However, for general utility functions beyond HARA and with wages w large, this characterization does not hold. In Example 1 in the Appendix, we show by counterexample that for w large, Decreasing Absolute Risk Aversion (DARA) is not sufficient for the condition to hold.

Proposition 4 *When w is small, condition* (**U**) *is satisfied for any utility function that exhibits Decreasing Absolute Risk Aversion (DARA),* $-\frac{u''}{u'} < 0$*, and thus has positive risk prudence,* $u''' > 0$*. Likewise, it holds with opposite inequality under IARA.*

Proof. In Appendix. ■

Characterization. Condition (**U**) establishes that there are complementarities in the match value between a firm type y and a worker with assets a. In other words, the match value $\Phi(a, y, \pi)$ between types a and y is supermodular, and therefore the equilibrium expected payoff for the worker is increasing in a, and so is the equilibrium expected payoff to the firm in y. While there are no technological complementarities (all workers are identically skilled), the preferences generate a complementarity between assets and job productivity.

The implication of this condition is that high asset workers:

1. apply for high productivity jobs ($y \uparrow$)
2. earn higher wages ($w \uparrow$)
3. have higher unemployment ($\lambda \uparrow \Rightarrow q(\lambda) \downarrow$)
4. have higher expected consumption ($c \uparrow$)
5. have higher expected utility ($U \uparrow$)

And likewise, high productivity firms:

1. post higher wages ($w \uparrow$)
2. attract higher asset workers ($a \uparrow$)
3. have higher expected profits ($\pi \uparrow$)
4. fill vacancies faster ($\lambda \uparrow \Rightarrow m(\lambda) \uparrow$)

4 Private Assets

Participation Constraint. So far, we have considered a fixed population. With private information, we first introduce a participation constraint. When there is a cost of applying for a job, and the mechanism does not have the commitment power to make the worker apply, then there is a constraint on the payoff. In particular, the expected benefit of application must be no lower than the application cost. Otherwise, the worker will not apply. Then mechanism's optimal solution needs to satisfy the following participation constraint, where c denotes the ex ante cost for a job application:

$$\beta q(\lambda) u \left(Ra' + v(\mu) - \frac{\pi(\mu)}{m(\lambda)} \right) + (1 - q(\lambda)) \beta u(Ra') \geq c + \beta u(Ra')$$

and hence

$$\beta q(\lambda)\left[u\left(Ra' + v(\mu) - \frac{\pi(\mu)}{m(\lambda)}\right) - u(Ra')\right] \geq c.$$

If the payoff is increasing in assets – this is the case for example in the decentralized equilibrium allocation, irrespective of condition (**U**) – then this constraint is binding only for the lowest type $\underline{a}$:

$$\beta q(\lambda(\underline{a}))\left[u\left(Ra'(\underline{a}) + v(\mu(\underline{a})) - \frac{\pi(\mu(\underline{a}))}{m(\lambda(\underline{a}))}\right) - u(Ra'(\underline{a}))\right] \geq c.$$

Truthtelling Constraint. Now we analyze what happens when asset holdings of the workers are private. Consider a mechanism (the planner's or the market) in which workers participate and that consists of an allocation $\mu(\hat{a})$ and a payoff when matched $t(\hat{a})$. Because the mechanism will use all the pairwise surplus, that is equivalent to specifying a payoff to the firm. Moreover, since we take the matching technology as a constraint of the mechanism, we can express the mechanism in terms of the expected payoff to the firm, denoted by $\pi(\mu(\hat{a}))$.

Truthtelling satisfies:

$$\begin{aligned}
& \Phi(a, \mu(a), \pi(\mu(a)) \geq \Phi(a, \mu(\hat{a}), \pi(\mu(\hat{a})) \\
\iff \quad & \hat{a} \in \arg\max_{\hat{a}} \Phi(a, \mu(\hat{a}), \pi(\mu(\hat{a}))) \\
\iff \quad & \Phi_y \mu'(a) + \Phi_\pi \pi' \mu'(a) = 0 \\
\iff \quad & \Phi_y + \Phi_\pi \pi' = 0.
\end{aligned}$$

Observe that this implies:

$$\begin{aligned}
& \left(-u'(a-a') + \beta R\left[qu'(c_e) + (1-q)\,u'(Ra')\right]\right)\frac{\partial a'}{\partial \hat{a}} \\
& + \left(\beta q'\left[u\left(c_e\right) - u(Ra')\right] + \beta u'\left(c_e\right)\frac{m'\pi}{\lambda m}\right)\frac{\partial \lambda}{\partial \hat{a}} \\
& + \left(\beta q u'(c_e)\left(v_y - \frac{\pi'}{m}\right)\right)\frac{\partial \mu}{\partial \hat{a}} = 0. \qquad (6)
\end{aligned}$$

So far, in the decentralized economy, we have assumed observable assets. The following result establishes that the decentralized outcome will be achieved even if asset holdings are private.

Proposition 5 *Let condition (***U***) hold. Then the complete information decentralized equilibrium outcome is incentive compatible and can therefore be attained under private asset holdings.*

Proof. Inspection of the incentive constraint (6) immediately reveals that each of the terms in brackets coincides with the first order conditions of the solution to the decentralized equilibrium allocation (3), (4), (5). ■

This is a feature of the the matching model. A planner with a social welfare function will in general have a different objective than to implement the equilibrium allocation in the presence of private information. We can also establish the following:

Proposition 6 *Let condition (***U***) hold. Then the private information decentralized equilibrium outcome is unique.*

Proof. The condition for monotone matching – that Φ is supermodular in a and y – is equivalent to uniqueness of the equilibrium allocation (see Legros and Newman (2007)). Condition (**U**) is equivalent to supermodularity of Φ, thus establishing the result. ■

Efficiency. The planner's problem, with weights ω_1, ω_2 (possibly functions of a) on worker utility and firm profits, is:

$$\max_{\mu,\pi(\cdot),\lambda(\cdot)} \int_0^{\overline{a}} \left\{ \omega_1 \Phi(a, \mu(a), \pi) + \omega_2 \frac{\pi(\mu(a))}{\lambda(a)} \right\} f(a) da$$

$$\text{s.t.} \quad \mu'(a) = \frac{f(a)}{g(\mu(a))} \frac{1}{\lambda(a)}$$

$$\pi'(a) = -\frac{\Phi_y}{\Phi_\pi}$$

We can write the Hamiltonian as:

$$\mathcal{H}(\mu, \pi) = \left\{ \omega_1 \Phi(a, \mu(a), \pi) + \omega_2 \frac{\pi(\mu(a))}{\lambda(a)} \right\} f(a) + \delta_1 \frac{f(a)}{g(\mu(a))} \frac{1}{\lambda(a)} - \delta_2 \frac{\Phi_y}{\Phi_\pi}.$$

which implies the first order conditions

$$\begin{aligned} \frac{\partial \mathcal{H}}{\partial a'} &= \omega_1 \Phi_{a'} f(a) - \delta_2 \frac{\partial}{\partial a'} \frac{\Phi_y}{\Phi_\pi} = 0 \\ \frac{\partial \mathcal{H}}{\partial \lambda} &= \omega_1 \Phi_\lambda f(a) - \omega_2 \beta \frac{\pi(\mu)}{\lambda^2} f(a) - \frac{\delta_1}{g(\mu)\lambda^2} - \delta_2 \frac{\partial}{\partial \lambda} \frac{\Phi_y}{\Phi_\pi} = 0 \end{aligned}$$

as well as the costate equations:

$$\begin{aligned} \frac{\partial \mathcal{H}}{\partial \mu} &= \left(\omega_1 \Phi_y + \omega_2 \frac{\pi'}{\lambda} \right) f(a) - \delta_1 \frac{f(a) g'}{g^2} - \delta_2 \frac{\partial}{\partial \mu} \frac{\Phi_y}{\Phi_\pi} = -\delta_1' \\ \frac{\partial \mathcal{H}}{\partial \pi} &= \left(\omega_1 \Phi_\pi + \omega_2 \frac{1}{\pi} \right) f(a) - \delta_2 \frac{\partial}{\partial \pi} \frac{\Phi_y}{\Phi_\pi} = -\delta_2' \end{aligned}$$

Compared with the equilibrium allocation, where we have first-order conditions:

$$\begin{aligned}\frac{\partial \mathcal{H}}{\partial a'} &= \Phi_{a'} f(a) = 0 \\ \frac{\partial \mathcal{H}}{\partial \lambda} &= \Phi_{\lambda} f(a) - \frac{\delta_1}{g(\mu)\lambda^2} = 0,\end{aligned}$$

and costate equations:

$$\begin{aligned}\frac{\partial \mathcal{H}}{\partial \mu} &= \Phi_y f(a) - \delta_1 \frac{f(a)g'}{g^2} = -\delta_1' \\ \frac{\partial \mathcal{H}}{\partial \pi} &= \Phi_{\pi} f(a) = -\delta_2'\end{aligned}$$

Observe that even with a Utilitarian Planner, i.e. when $\omega_1 = \omega_2 = 1$, the equilibrium solution does not coincide with the planner's solution.

5 Infinite Horizon

In this section, we develop a model without aggregate shocks, and our focus is on a stationary equilibrium. Using the model, we then evaluate tax-financed UI.

5.1 Population, preferences and technology

Time is discrete. There are new unemployed workesr and firms with vacancies who enter the market every period. Production is decentralised; each worker produces f(y), where y represents the productivity level of firm. Workers have standard time-additive preferences with discount factor and do not value leisure. Output is either consumed or invested.The remainder of the model description focuses on the decentralised equilibrium.

5.2 Matching

There are many firms (one for each job) and many unemployed workers with different levels of productivities and assets respectively. These firms all act competitively. unemployed and firms meet and produce output in a frictional labor market. The labor market is organised along a continuum of submarkets that differ in the wage contract that is offered. More specifically, we allow only for fixed-wage contracts. When a firm meets a worker in submarket θ, the firm offers the worker an employment contract that pays the worker a wage of w every period until the match ends exogenously with probability λ. Firms and workers fully commit to this contract. Consequently, each submarket can be indexed by the market tightness θ (he ratio of the number of vacancies created by firms in that submarket to the number of workers). Search in the labor market is directed: firms choose the wage to offer and each vacancy requires the payment of a posting cost, k. Similarly, unemployed decide in which submarket to look for jobs. We denote by $\theta(a)$ the market tightness of submarket where the unemployed with asset level a are looking for jobs. Once in a submarket, workers find jobs with probability $q(\theta(a))$ and firms find workers with probability $p(\theta(a)) = \frac{q(\theta(a))}{\theta(a)}$.

5.3 Workers and firms

We consider a job market where the unemployed search until they find a job, and once employed they hold the job forever unless they die with probability λ . Denote by $U(a)$ the value of being unemployed with asset level a and by $E(a)$ the value of being employed. Every period new firms and new unemployed workers enter the market at rate δ. We assume that every period there are new entries of unemployed and firms with vacancies to the market. Then the Bellman equations for workers are:

$$U(a) = \max_{a',\theta} \left\{ u(c_u) + \beta(1-\lambda)\left[q(\theta)E(a') + (1-q(\theta))U(a')\right] \right\}$$
$$\text{s.t:} c_u + a' = (1+r)a + b$$

$$E(a) = \max_{a'} \left\{ u(c_e) + \beta(1-\lambda)E(a') \right\}$$
$$\text{s.t:} c_e + a' = (1+r)a + (1-\tau)w$$

Since the employment-consumption problem $E(a)$ is stationary and independent of $U(a)$, we can solve it separately.With $\beta R = 1$ the solution is $c_e = ra + (1-\tau)w$ and we can explicitly write the value for employment:

$$E(a) = \frac{u(ra + (1-\tau)w)}{1-\beta(1-\lambda)}$$

We can then write the problem of the unemployed as:

$$U(a) = \max_{a',\theta} \left\{ u((1+r)a + b - a') + \beta(1-\lambda)\left[q(\theta)\left(\frac{u(ra+(1-\tau)w)}{1-\beta(1-\lambda)}\right) + (1-q(\theta))U(a')\right]\right\}$$

with the problem of the firms with vacancy and filled jobs in infinite horizon and in a stationary equilibrium:

$$V(y) = -k + \max_{w} \{p(\theta)(f(y) - w + \beta(1-\lambda)J(y,a)) + (1-p(\theta))\beta(1-\lambda)V(y)\}$$
$$J(y,w) = f(y) - w + \beta(1-\lambda)J(y,a)$$

Which is equivalent to:

$$V(y) = \max_{w} \left\{ \frac{1}{1-\beta(1-p(\theta))(1-\lambda)} \left[-k + \frac{p(\theta)}{1-\beta(1-\lambda)}(f(y)-w) \right] \right\}$$

We can then write the problem as:

$$U(a) = \max_{a',\theta} \left\{ u((1+r)a + b - a') + \beta(1-\lambda) \left[q(\theta) \left(\frac{u(ra + (1-\tau)w)}{1-\beta(1-\lambda)} \right) + (1-q(\theta))U(a') \right] \right\}$$
$$\text{s.t:} V(y) = \max_{w} \left\{ \frac{1}{1-\beta(1-p(\theta))(1-\lambda)} \left[-k + \frac{p(\theta)}{1-\beta(1-\lambda)}(f(y)-w) \right] \right\}$$

But now we can substitute for wages from the firm's problem and write the matching problem as:

$$U(a) = \max_{a',\theta,y} \left\{ u((1+r)a + b - a') + \beta(1-\lambda) \left[\frac{q(\theta)}{1-\beta(1-\lambda)} u \left(ra + (1-\tau) \left(f(y) - \frac{(1-\beta(1-\lambda))}{p(\theta)} \left[(1-\beta(1-p(\theta)(1-\lambda)))V(y) + k \right] \right) \right.\right.\right.$$

Which means:

$$U(a) = \max_{a',\theta,y} \frac{1}{1-\beta(1-q(\theta))(1-\lambda)} \left\{ u((1+r)a + b - a') + \beta(1-\lambda) \left[\frac{q(\theta)}{1-\beta(1-\lambda)} u \left(ra + (1-\tau) \left(f(y) - \frac{(1-\beta(1-\lambda))}{p(\theta)} [(1-\beta(1-p\right.\right.\right.\right.$$

FOC:

$$-u'((1+r)a + b - a') + \beta(1-\lambda) \left[\frac{q(\theta)}{1-\beta(1-\lambda)} r u' \left(ra + (1-\tau) \left(f(y) - \frac{(1-\beta(1-\lambda))}{p(\theta)} \left[(1-\beta(1-p(\theta)(1-\lambda)))V(y) + k \right] \right) \right) \right] = 0$$
$$\frac{\beta(1-\lambda)q'(\theta)}{1-\beta(1-q(\theta))(1-\lambda)} [u(c_e) - u(c_u)] + \beta(1-\lambda)(1-\tau) [(1-\beta(1-\lambda))V(y) + k] u'(c_e) \left(\frac{q'(\theta)\theta}{q(\theta)} - 1 \right) = 0$$
$$f_y - \frac{(1-\beta(1-\lambda))(1-\beta(1-p(\theta))(1-\lambda))}{p(\theta)} \frac{\delta V(y)}{\delta y} = 0$$

Note that the third FOC would be different if k was a function of y.

where:

$$c_e = ra + (1-\tau) \left(f(y) - \frac{(1-\beta(1-\lambda))}{p(\theta)} \left[(1-\beta(1-p(\theta)(1-\lambda)))V(y) + k \right] \right)$$
$$w = f(y) - \frac{(1-\beta(1-\lambda))}{p(\theta)} \left[(1-\beta(1-p(\theta)(1-\lambda)))V(y) + k \right]$$
$$c_u = (1+r)a + b - a'$$

5.4 Law of Motios

Law of motions for workers are derived from the policy functions of workers for saving and their decisions in labour market. Likewise, law of motions for firms are derived from their policy function in labour market.

$$
\begin{aligned}
v(y_{t+1}) &= (1-\lambda)(1-\frac{q(\theta)}{\theta})v(y_t) + \delta_f \eta(y) \\
j(y_{t+1}) &= (1-\lambda)j(y_t) + (1-\lambda)\frac{q(\theta)}{\theta}v(y_t) \\
u(a_{t+1}) &= (1-\lambda)(1-q(\theta))\sum_a \mathbb{1}\{a' = G(a)\}u(a_t) + \delta_w \mu(a) \\
e(a_{t+1}) &= (1-\lambda)e(a_t) + (1-\lambda)q(\theta)u(a_t)
\end{aligned}
$$

At stationary equilibrium we can find the invariant distribution of workers and firms over assets and productivities:

$$
\begin{aligned}
v(y_{t+1}) &= \frac{\delta_f \eta(y)}{\lambda + p(\theta) - \lambda p(\theta)} \\
j(y_{t+1}) &= \frac{(1-\lambda)p(\theta)}{\lambda} v(y_t) \\
u(a'_{t+1}) &= \frac{\delta_w \mu(a)}{(\lambda + q(\theta) - \lambda q(\theta))\sum_a \mathbb{1}\{a' = G(a)\}} \\
e(a_{t+1}) &= \frac{(1-\lambda)q(\theta)}{\lambda} u(a_t)
\end{aligned}
$$

so we can write:

$$
\begin{aligned}
j(y_{t+1}) &= \frac{(1-\lambda)p(\theta)}{\lambda} \frac{\delta_f \eta(y)}{\lambda + p(\theta) - \lambda p(\theta)} \\
e(a_{t+1}) &= \frac{(1-\lambda)q(\theta)}{\lambda} \frac{\delta_w \mu(a)}{(\lambda + q(\theta) - \lambda q(\theta))\sum_a \mathbb{1}\{a' = G(a)\}}
\end{aligned}
$$

Proposition 7 *Workers with higher initial asset levels a will apply for higher wage jobs provided*

$$
\frac{u'(c_e) - \Phi'(Ra')}{\frac{1}{1-\beta(1-\lambda)}u(c_e) - \Phi(Ra')} < \frac{u''(c_e)}{u'(c_e)} \qquad (\mathbf{U}_\infty)
$$

Proposition 8 *Under condition ($\mathbf{U}_\infty$) and for a given worker with assets a, the job productivity y decreases in the duration of unemployment.*

5.5 Calibration

We use our model economy for two main purposes: we analyse UI and we examine how changes in productivity and job destruction rate represented by a change in the probability of death (λ) affect labour-market outcomes. We therefore first calibrate the model and report its basic properties. We then look at UI policy and the effects of productivity change.

5.5.1 Benchmark Calibration

One period is set to be 6 weeks.The production function is $f(y) = y$. The borrowing constraint is set at 0 and $\beta = 0.9881$. We use the utility function $u(c) = \log(c)$. Following Shimer(2005), we set the value of the household production (benefit) ,b, to be 40% of the average wage. We thus set b to 40 which, in equilibrium, turns out to be about 40% of the average wage. The probability of death (λ) is set to be 0.02 on the basis of the observation by Shimer that the average duration of employment at the same job is 5.5 year. Following Menzio and Shi (2011) and Schaal (2010), we pick the CES contact rate functions, $q(\theta) = \theta(1 + \theta^{\gamma})^{\frac{1}{\gamma}}$. To calibrate the elasticity of matching function (γ) we target the unemployment rate to be 5% at equilibrium and set the γ to match the job-finding probability. We compute the cost of posting a vacancy as 35% of the average productivity which is 50% of the lowest prodictive firms and 25% of highest. The new firms are assumed to have a uniform distribution over productivities and new unemployed are assumed to a have a log normal distribution with mean !!! and standard deviation 1.9. The asset domain is $a \in \mathcal{A} = [100, 300]$ and the productivity domain is $y \in \mathcal{Y} = [100, 200]$.

Parameter	Definition	Value
r	Interest rate	0.005
β	discount factor	0.995
b	unemployment benefit	40
k	cost of vacancy	50
λ	Probability of dying	0.02
γ	elasticity of matching fn	1.5

5.6 Characterization of the Steady State

We first display some key model features and then discuss our experiments. Table 1 presents the summary statistics for the calibrations.

u(%)	v(%)	$avg(\theta)$	$avg(w)$
5.2%		0.387	100.236

Having positive assortative matching in workers and firms implies that at equilibrium workers with higher level of asset are matched with more productive firms. Figure 1 shows the allocation in labour market. The market clearing condition implies that all workers are allocated to submarkets while some firms below a productivity threshold are staying out of the market. This threshold is sensitive to different parameterization of model. For instance, a higher vacancy cost or unemployment benefit results in a higher threshold where more firms are staying out of market.

Figure 2 shows how the value of unemployed workers and firms change with their asset and productivity respectively. Workers with higher initial asset levels will apply for more productive firms offering them in average higher wages and lower probability of job finding. Intuitively, the workers who can maintain a higher level of consumption during unemployment due to their asset stock prefer to apply for higher wages job though their decision are associated with longer duration of unemployment.

Figure 1: Allocation

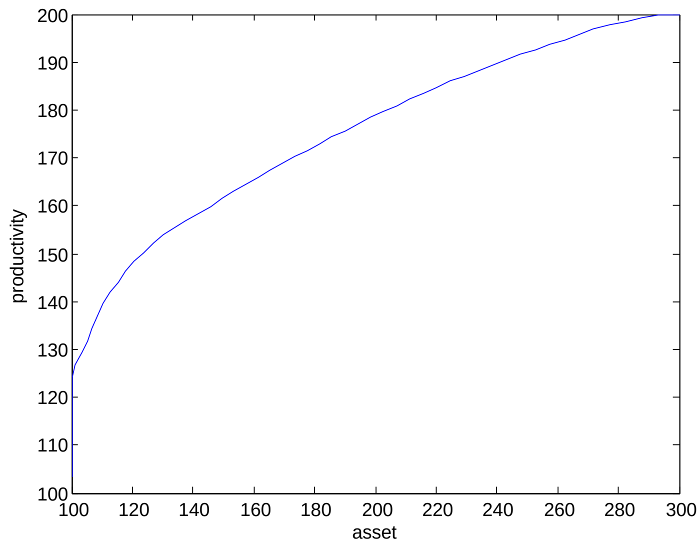


Figure 2: Value of unemployment for workers and vacancy for firms

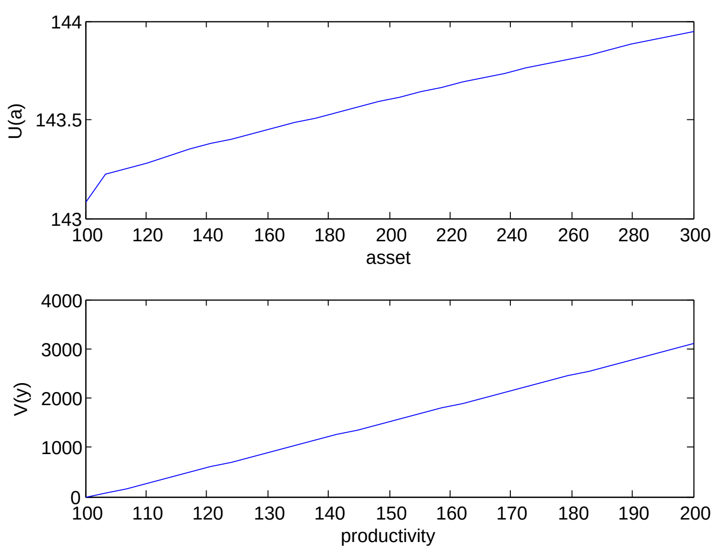


Figure 3 and 4 show the wage and job finding functions. The observed concavity of wage and convexity of job finding functions follow this feature, intuitively: the wage and job finding functions being increasing and decreasing respectively, which are due to the outside option being worse for consumers with a low stock of assets (because their buffer against unemployment shocks is lower). Although the wages clearly change with asset levels but most of the variation in terms of wage-probability is absorbed by probability of job finding. the probability of job finding for the worker with lowest asset level is %14 higher than the unemployed with highest asset level, though his wage is only %0.2 lower. In contrast to KMS that wage function is only increasing for very low asset workers and flat for the rest of asset domain, this model can generate much higher wage dispersion. Intuitively, having directed search with assortative matching implies that the offered wage and probability of job finding is different in each submarket, while in DMP or KMS search is not submarket specific but it happens randomly in the whole market. Therefore, in those models although unemployed with different level of assets might have different surplus for match but since all firms are homogenous the wage offered to workers does not change meaningfully with their asset level. While in our framework, having two sided heterogeneity in the model changes the value of unemployment and posting vacancy in the market sharply such that it results to a considerably higher wage and probability of job finding dispersion.

Figure 3: wage function

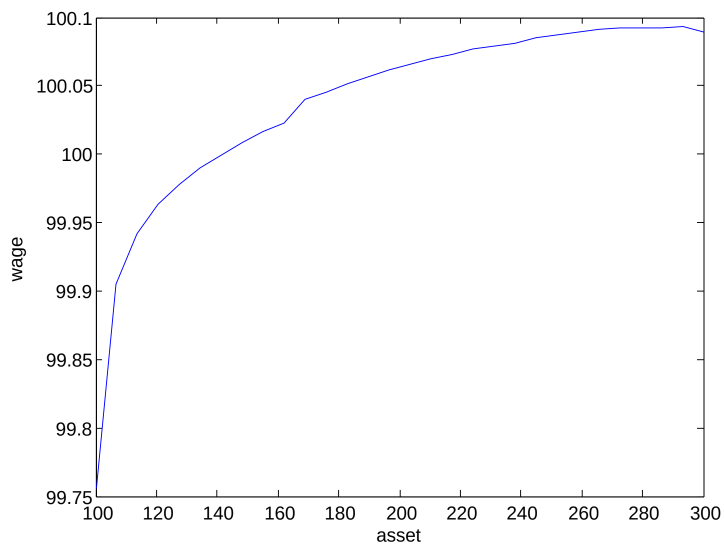


Figure 5 shows the saving decision for different asset levels. Interestingly, the dynamic nature of the problem now implies a time variant job choice decision. A worker who fails to become employed sees her assets gradually deplete (since, trivially, $a < a$). But the optimal search decision dictates application to less productive, lower wage jobs when assets are lower. As a result, over the duration of unemployment and as assets deplete, workers will apply for less productive, lower wage jobs.

The depletion of asset during the unemployment together with a frictional labour market and different job finding rates for workers with different asset levels results in a distribution of workers with

Figure 4: probability of job finding function

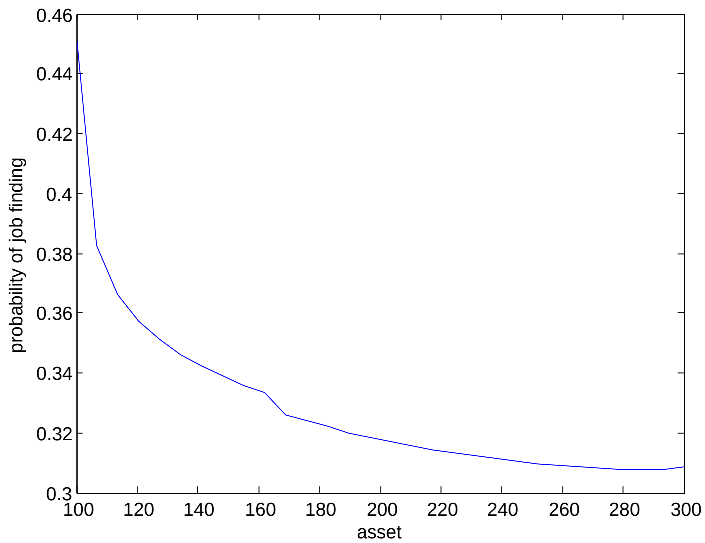
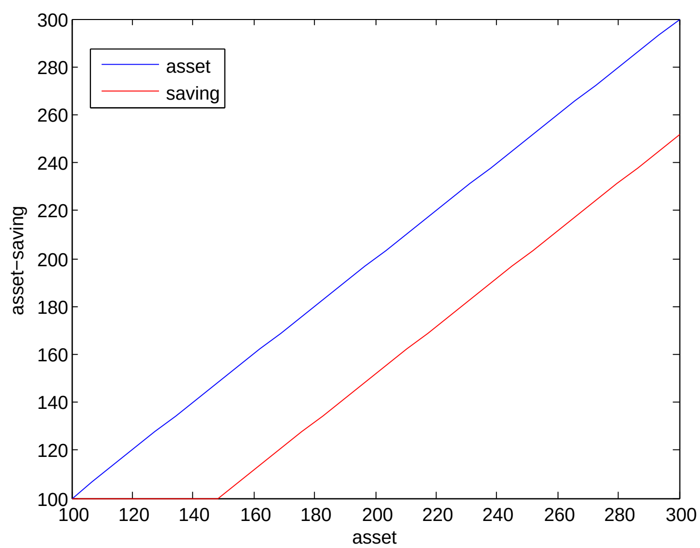


Figure 5: saving decision



a much fatter left tail at steady state. Although, the probability of job finding is significantly lower for high asset workers but to smooth the consumption during the unemployment, they reduce their stock of assets by negative saving and climb down the asset ladder. This is depicted in figure 6. On the firm side, we also observe a fatter left tail for the distribution of firms at steady state. Since the forgone profit of more productive firms are higher during the vacancy posting time, they increase the probability of filling the vacancy by offering higher wages to unemployed. Therefore more productive firms can find matchs faster than less productive ones and this is shown in figure 7.

Figure 6: Distribution of workers

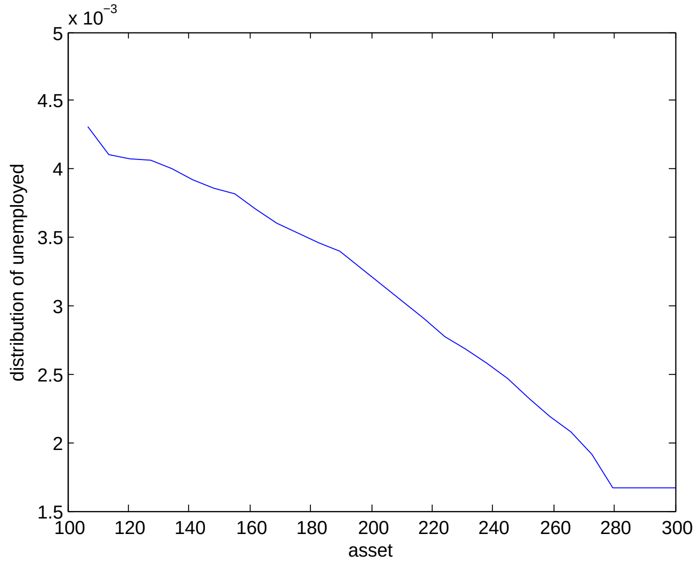
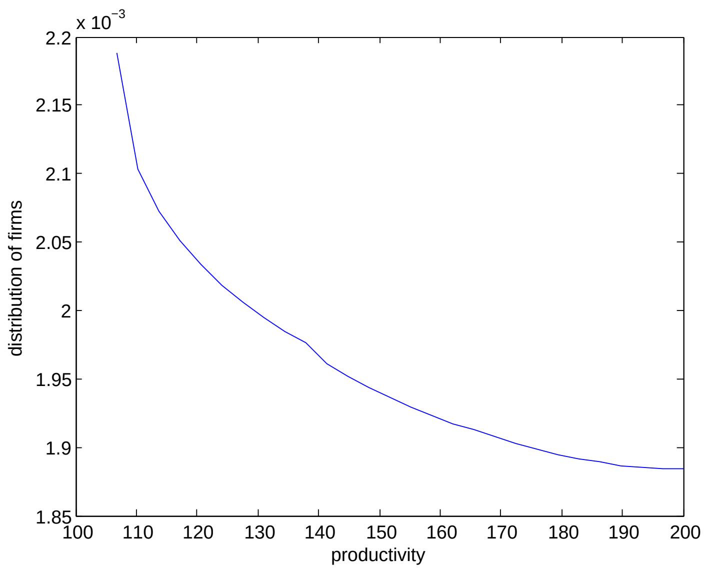


Figure 7: Distribution of firms



5.7 Welfare effects of UI

From the perspective of the precautionary-savings literaturethe BHA modeldirect insurance against unemployment is clearly welfare improving from the workers point of view by increasing the consumption of workers when unemployed. But taking into account the general equilibrium effects, the KMS model has a different answer. They show that UI will affect wage formation by influencing the firms incentives to enter the market. A higher unemployment benefit discourages firm entry and reduces the probability of employment for workers. However, in our framework direct insurance against unemployment not only affect the entry decision of firms by changing the threshold above which firms post vacancies, but also it affects the distribution of unemployed by influencing their saving decisions and therefore their job finding probability. Since the workers with different asset levels enter different submarkets and have different probability of job finding, the change in distribution of workers can have a significant impact on workers welfare. We now study how these there effects interact with each others.

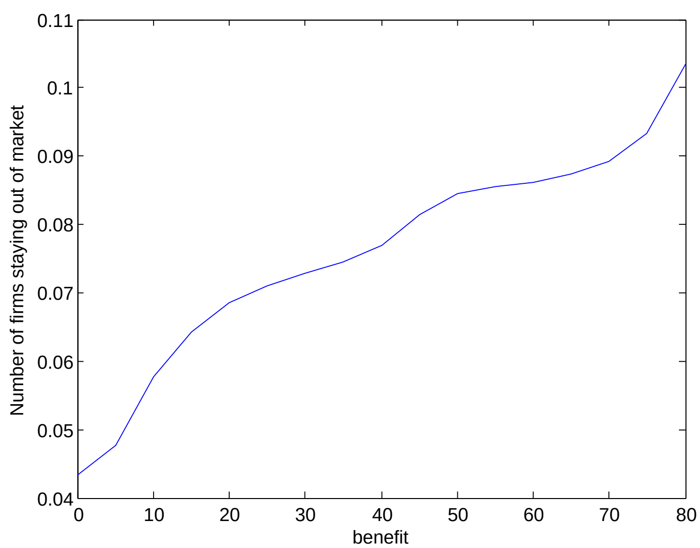
5.7.1 Aggregate economy

We assume that UI is financed using taxes that are proportional to wages. We also assume that all the income for the unemployed comes from the UI. For a given b, the government sets τ to balance its period-by-period budget constraint:

$$ub = \tau \int \omega(a) f_e(a) da$$

Figure 8 shows how the number of firms staying out of the market increases with UI. A higher b increases the relative value of the outside option for the workers and reduces the firms incentive to post vacancies, therefore more firms with low productivities stay out of the market.

Figure 8: The measure of firms staying out of the market as a function of b



Moreover, higher insurance affects consumption smoothing decision of unemployed. A higher level of benefit reduces the risk of unemployment by increasing the outside option of workers. This results in a higher level of saving and workers try to pick submarkets where they can get a higher wage though with a lower probability.

Figure 9 shows how the saving decision of a worker with asset level a changes as unemployment benefit increases. This increase in saving shifts the distribution of workers to the right. In figure 10 we plotted the distribution of unemployed workers for several different benefit levels and one can clearly see that a higher saving rate as a result of higher income during unemployment results in a fatter right tail at steady state.

Figure 9: Saving for asset level a as a function of b

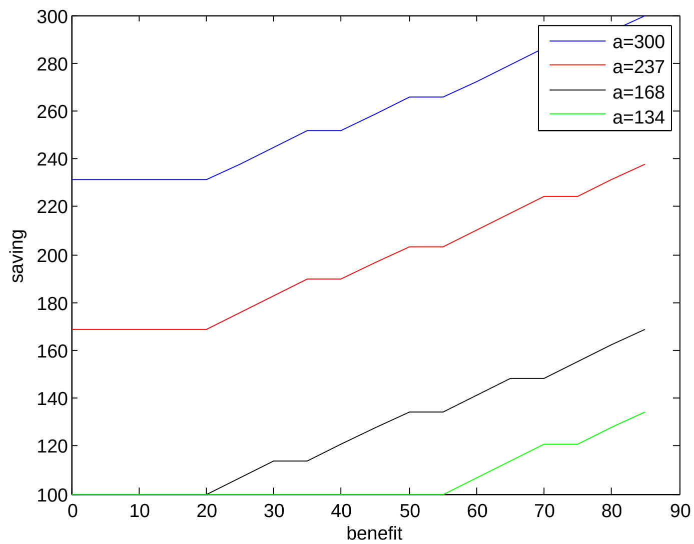
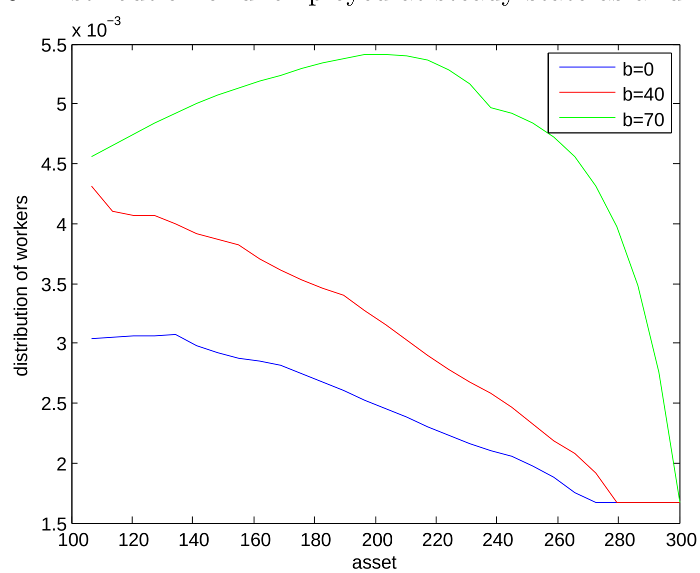
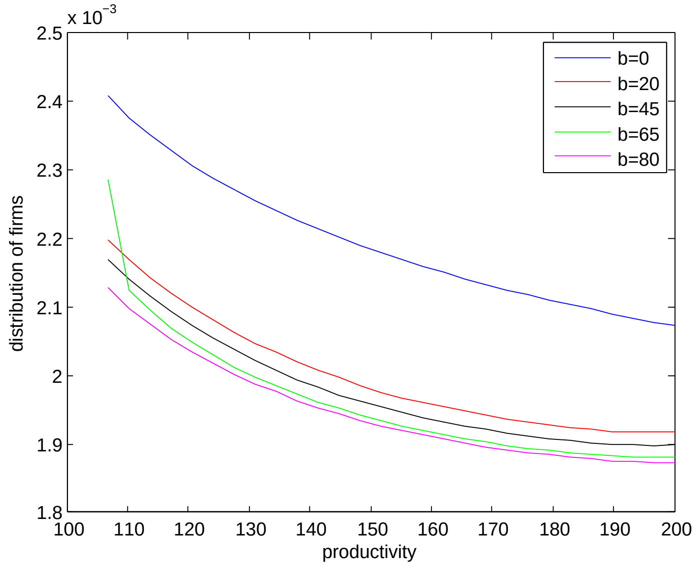


Figure 10: Distribution of unemployed at steady state as a function of b



Furthermore, a shift in the distribution of workers as a result of higher unemployment insurance reduces the probability of job finding for workers with high asset levels and has a non-monotone impact on the workers with low asset levels. An increase in the number of wealthier workers with higher outside option who are trying to be matched with more productive firms decreases their chance of employment.

Figure 11: Distribution of firms at steady state as a function of b



In contrast, the probability of job finding reduces first for unemployed with lower level of asset as benefit increases. This is mainly caused by an increase in the outside option of those workers which deters the entry decision of low productive firms. However further increases in unemployment insurance rise the concentration of workers in the right tail of distribution such that workers with low level of asset can find jobs faster (CHECK THIS!!!) This is depicted in Figure 11.

A higher outside option for workers changes the allocation of workers-firms non monotonically. (EXPLANATION!!!!)

Figure 12 illustrates how unemployment rate increase as a result of rise in UI. In contrast to KMP, A higher level of unemployment insurance not only raises the unemployment through deterring the entry of low productive firms to the market, but also it changes the distribution of workers and their allocation with firms. This is one of our key results; a change in distribution of workers affects the allocation of workers with firms and their probability of job finding. Wealthier workers choose submarkets which offer them a higher wage with lower likelihood of finding jobs which results in higher unemployment.

5.7.2 Welfare

Rather than compute full transition dynamics as a result of a change in the level of UI, we compare steady states with different levels of UI. Figure 14 shows a straight comparison of, say, total weighted utility across steady states.

The total weighted utility in the total population of unemployed is maximised at b equal to 40. Recall that as we lower b further to a laissez faire economy, this induces more vacancy posting and

Figure 12: Probability of job finding for asset level a workers as a function of b

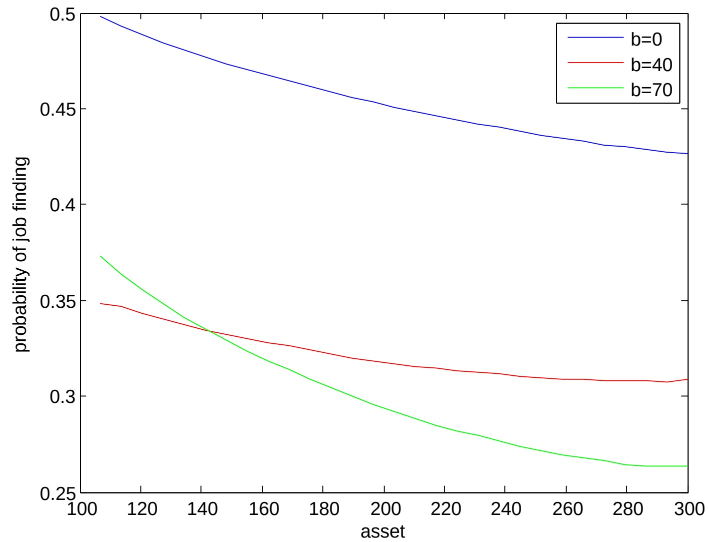
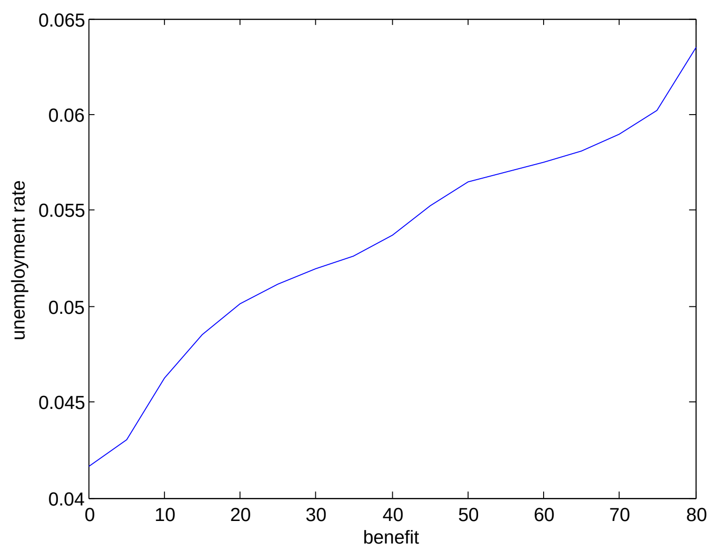


Figure 13: Total unemploymnet rate as a function of b



reduces unemployment. This has been depicted in Figure 15 we can see how the probability of finding a job decreases with b for different asset levels a. However, there are three negative welfare effects coming from a lower b. The first is that the before-tax wage rate declines with lower b. Figure 16 shows the second one which is a direct effect of lowering b: a lower b means less insurance for unemployed workers and that considerably decrease the consumption during unemployment. Moreover, the third one is a leftward shift in the distribution of unemployed which increases the inequality between poor and reach. Figure 16,17 and 18 show how the difference between the probability of job finding, wage and value of unemployment increases if b goes down.

Figure 14: Total weighted utility across steady states as a function of b

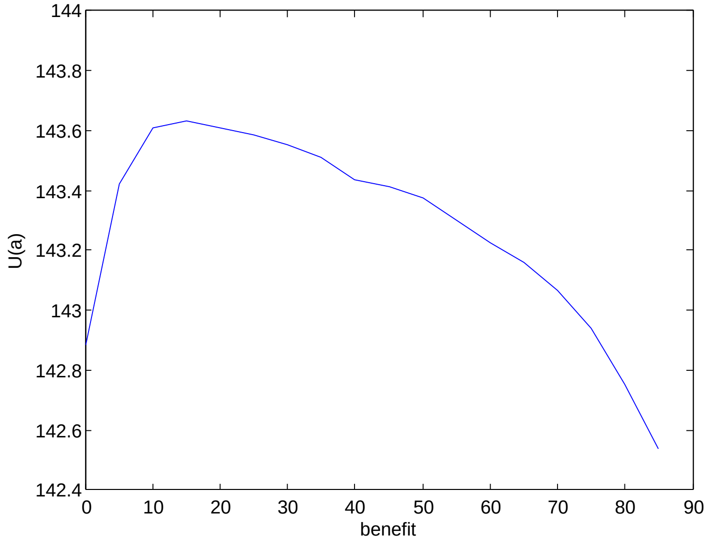
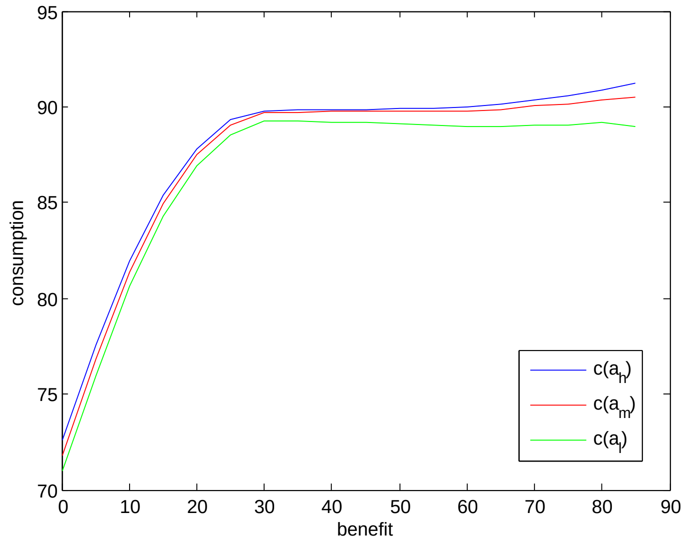


Figure 15: consumption for asset level a as a function of b



Almost for all values of b below 20, the negative effects of lowering b dominate, and welfare decreases as we lower b. And the adverse is true for all values of b above 20.

Figure 16: Difference in probability of job finding for two asset levels as a function of b

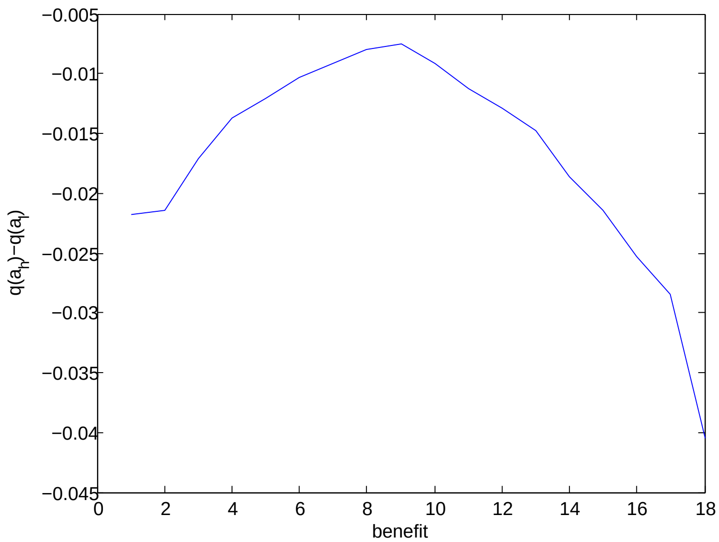


Figure 17: Difference in wage for two asset levels as a function of b

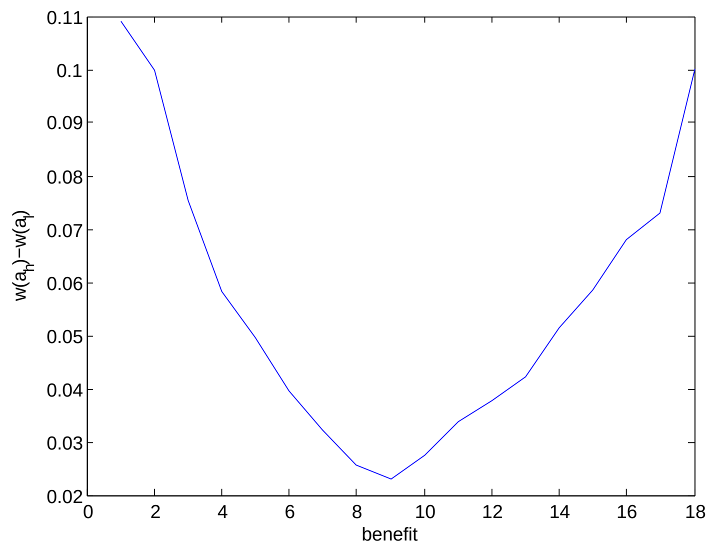
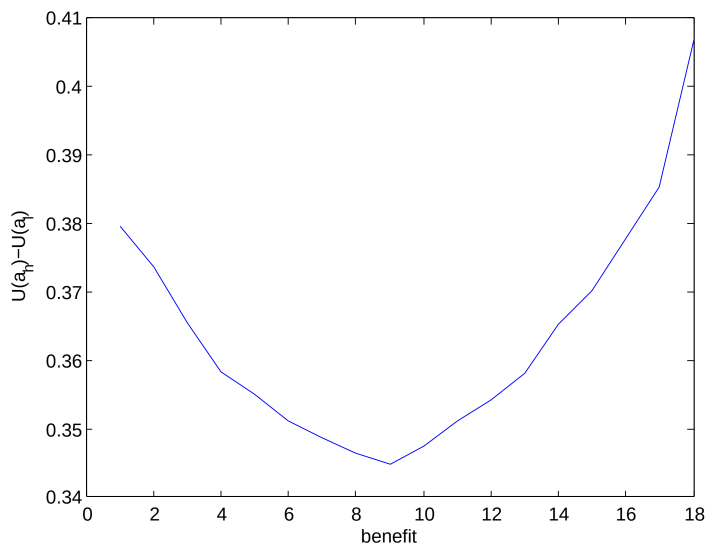


Figure 18: Difference in value of unemploymnet for two asset levels as a function of b



Appendix

Derivatives of Φ and a'

$$\begin{aligned}
\Phi_y &= \beta q u'(c_e) v_y + \Phi_{a'} a'_y + \Phi_\lambda \lambda_y = \beta q u'(c_e) v_y \\
\Phi_a &= u'(a-a') + \Phi_{a'} a'_a + \Phi_\lambda \lambda_a = u'(a-a') \\
\Phi_\pi &= \beta q u'(c_e) \frac{-1}{m} + \Phi_{a'} a'_\pi + \Phi_\lambda \lambda_\pi = \beta q u'(c_e) \frac{-1}{m} \\
\Phi_{ay} &= -u''(a-a') a'_y (= \partial / \partial a \Phi_y) \\
\Phi_{a\pi} &= -u''(a-a') a'_\pi
\end{aligned}$$

where $\Phi_{a'} = 0$ and $\Phi_\lambda = 0$ from the envelope theorem.

We calculate the derivative of a' using the implicit function theorem. For the problem to have a maximum, we require that the Hessian of the maximand is positive $|\mathbf{H}| > 0$ (recall that $\phi_{\lambda\lambda}$ is assumed negative), where:

$$|\mathbf{H}| = \begin{vmatrix} \phi_{a'a'} & \phi_{a'\lambda} \\ \phi_{\lambda a'} & \phi_{\lambda\lambda} \end{vmatrix}$$

Applying the implicit function theorem,

$$a'_y = \frac{\partial a'}{\partial y} = -\frac{\begin{vmatrix} \phi_{a'y} & \phi_{a'\lambda} \\ \phi_{\lambda y} & \phi_{\lambda\lambda} \end{vmatrix}}{|\mathbf{H}|} = \frac{\phi_{a'y}\phi_{\lambda\lambda} - \phi_{\lambda y}\phi_{a'\lambda}}{|\mathbf{H}|} \quad \text{and} \quad a'_\pi = \frac{\partial a'}{\partial \pi} = -\frac{\begin{vmatrix} \phi_{a'\pi} & \phi_{a'\lambda} \\ \phi_{\lambda \pi} & \phi_{\lambda\lambda} \end{vmatrix}}{|\mathbf{H}|} = \frac{\phi_{a'\pi}\phi_{\lambda\lambda} - \phi_{\lambda \pi}\phi_{a'\lambda}}{|\mathbf{H}|}$$

Proof of Proposition 3

Proof. We can calculate the derivatives:

$$\begin{aligned}
u'(c) &= \alpha \left(\frac{\alpha c}{1-\gamma} + \beta \right)^{\gamma-1} \\
u''(c) &= -\alpha^2 \left(\frac{\alpha c}{1-\gamma} + \beta \right)^{\gamma-2}
\end{aligned}$$

and condition ($\mathbf{U}$) becomes (where $c = Ra'$):

$$\alpha \left(\frac{\alpha c_e}{1-\gamma} + \beta \right)^{\gamma-1} \left[\alpha \left(\frac{\alpha c_e}{1-\gamma} + \beta \right)^{\gamma-1} - \alpha \left(\frac{\alpha c}{1-\gamma} + \beta \right)^{\gamma-1} \right] <$$
$$-\alpha^2 \left(\frac{\alpha c_e}{1-\gamma} + \beta \right)^{\gamma-2} \left[\frac{1-\gamma}{\gamma} \left(\frac{\alpha c_e}{1-\gamma} + \beta \right)^{\gamma} - \frac{1-\gamma}{\gamma} \left(\frac{\alpha c}{1-\gamma} + \beta \right)^{\gamma} \right]$$

and after dividing by α^2 and by $\left(\frac{\alpha c_e}{1-\gamma}+\beta\right)^{2\gamma-2}$, which under our assumptions are both positive, this implies:

$$1-\left(\frac{\frac{\alpha c}{1-\gamma}+\beta}{\frac{\alpha c_e}{1-\gamma}+\beta}\right)^{\gamma-1} < -\frac{1-\gamma}{\gamma}\left[1-\left(\frac{\frac{\alpha c}{1-\gamma}+\beta}{\frac{\alpha c_e}{1-\gamma}+\beta}\right)^{\gamma}\right],$$

or

$$1-x^{\gamma-1} < -\frac{1-\gamma}{\gamma}\left[1-x^{\gamma}\right] \quad \text{where} \quad x=\frac{\frac{\alpha c}{1-\gamma}+\beta}{\frac{\alpha c_e}{1-\gamma}+\beta}\in(0,1).$$

First consider $\gamma>0$. After rearranging and multiplying by $\gamma x^{1-\gamma}$, which is positive for $\gamma>0$:

$$\begin{aligned} x^{1-\gamma}-(\gamma+(1-\gamma)x) &< 0 \\ G(\gamma)-H(\gamma) &< 0. \end{aligned}$$

At $\gamma=0$ and $\gamma=1$ the expression is exactly zero, i.e., G and H cross at 0 and 1. Now, $G'(\gamma)=-x^{1-\gamma}\log x, H'(\gamma)=1-x$, and $G''(\gamma)=x^{1-\gamma}(\log x)^2>0, H''(\gamma)=0$. Observe that $G(\gamma)$ is convex, $G''(\gamma)=x^{1-\gamma}(\log x)^2>0$, while $H(\gamma)$ is linear. As a result, for $\gamma\in(0,1)$ condition (**U**) holds with strict inequality. For $\gamma=1$, (**U**) holds with equality and for $\gamma>1$ it holds with opposite inequality.

Now consider $\gamma<0$. Since we multiplied by $\gamma<0$, condition (**U**) now implies that $G(\gamma)-H(\gamma)>0$. Using the same logic, we establish that condition (**U**) holds for $\gamma<0$.

This establishes that for a risk averse worker with HARA utility function, condition (**U**) holds strictly if and only if $\gamma<1$, i.e., there is DARA. The condition holds with opposite inequality when there is IARA and $\gamma>1$. ■

Proof of Proposition 3

Proof. All the cases can immediately be verified from Proposition 3, except for the case of CARA. There, $u'(c)=\alpha e^{-\alpha c}, u''(c)=-\alpha^2 e^{-\alpha c}$, so that condition (**U**) becomes:

$$\begin{aligned} \alpha e^{-\alpha c_e}\left(\alpha e^{-\alpha c_e}-\alpha e^{-\alpha c}\right) &< -\alpha^2 e^{-\alpha c_e}\left(1-e^{-\alpha c_e}-1+e^{-\alpha c}\right) \\ e^{-\alpha c_e}-e^{-\alpha c} &< -\left(-e^{-\alpha c_e}+e^{-\alpha c}\right) \end{aligned}$$

which holds with equality. ■

Proof of Proposition 4

Proof. It is immediate that this condition is not satisfied when $u'''=0$. To see this, observe that then $u'(c_e)-u'(Ra')=wu''(c_e)$ and the condition (**U**) can be written as $u'(c_e)wu''(c_e)<$

$u''(c_e)\left[u(c_e) - u(Ra')\right]$, or $u'(c_e)w > u(c_e) - u(Ra')$. This condition only holds under convexity of u, and therefore is never satisfied for risk averse agents.

When $u''' < 0$, we have instead that $u'(c_e) - u'(Ra') > wu''(c_e)$, so the left hand side is even smaller, and again, condition (**U**) implies $u'(c_e)w > u(c_e) - u(Ra')$, which is not satisfied for risk averse agents.

Now consider $u''' > 0$. Then we can write the utility function and its derivative as

$$u(c) = u(c_e) + u'(c_e)(c - c_e) + \frac{u''(c_e)}{2}(c - c_e)^2 + \cdots$$
$$u'(c) = u'(c_e) + u''(c_e)(c - c_e) + \frac{u'''(c_e)}{2}(c - c_e)^2 + \cdots$$

and therefore condition (**U**) becomes:

$$u'(c_e)\left[u''(c_e)(c_e - c) - \frac{u'''(c_e)}{2}(c_e - c)^2 + \frac{u''''(c_e)}{6}(c_e - c)^3 - \cdots\right] <$$
$$u''(c_e)\left[u'(c_e)(c_e - c) - \frac{u''(c_e)}{2}(c_e - c)^2 + \frac{u'''(c_e)}{6}(c_e - c)^3 - \cdots\right].$$

Canceling terms and dividing by $(c_e - c)^2$, this condition implies that at least for small $c_e - c = w$ implies

$$u'''(c_e) > \frac{u''(c_e)^2}{u'(c_e)}.$$

This is equivalent to requiring that the coefficient of risk aversion $A(c) = -\frac{u''}{u'}$ is decreasing, i.e., $A' = -\frac{u'''u' - (^2u'')}{u'^2}$ or $u''' > \frac{(u'')^2}{u'} = -u''A(c)$. ■

A Counter Example

Example 1 *Let w be large enough and find a u-function with u'''' suitably chosen such that the condition is not satisfied. Let $u(c)$ be defined as:*

$$u(c) = u(c_e) + u'(c_e)(c - c_e) + \frac{u''(c_e)}{2}(c - c_e)^2 + \frac{u'''(c_e)}{6}(c - c_e)^3 + \frac{u''''(c_e)}{24}(c - c_e)^4.$$

Evaluating u at $c = Ra'$ and observing that $c_e - Ra' = w$ we can then write

$$\begin{aligned} u(c_e) - u(Ra') &= u'(c_e)w - \frac{1}{2}u''(c_e)w^2 + \frac{1}{6}u'''(c_e)w^3 - \frac{1}{24}u''''(c_e)w^4 \\ u'(c_e) - u'(Ra') &= u''(c_e)w - \frac{1}{2}u'''(c_e)w^2 + \frac{1}{6}u''''(c_e)w^3. \end{aligned}$$

*Now we can write condition (**U**) as (where u denotes $u(c_e)$):*

$$\begin{aligned} u'\left[u''w - \frac{1}{2}u'''w^2 + \frac{1}{6}u''''w^3\right] &< u''\left[u'w - \frac{1}{2}u''w^2 + \frac{1}{6}u'''w^3 - \frac{1}{24}u''''w^4\right] \\ u'u''' &> u''^2 - \frac{1}{3}u''u'''w + \frac{1}{3}u''''w\left[u' + \frac{1}{4}u''w\right] \end{aligned}$$

Observe that $u'u''' > u''^2$ is the standard condition for Decreasing Absolute Risk Aversion. But for any $u''' > 0$, however large, we can find a utility function with $\frac{1}{3}u''''w\left[u' + \frac{1}{4}u''w\right]$ sufficiently large such that the inequality is not satisfied. For example, if $u' + \frac{1}{4}u''w > 0$ we can choose u'''' positive and large. Conversely, if $u' + \frac{1}{4}u''w < 0$ we can choose u'''' sufficiently negative such that the inequality does not hold.

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