Transparency in contests

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Abstract

A contestant exerts more effort the more his rival is of similar type. Hence, a rent-extracting contest-administrator would like contestants (not) to know their rival's types when types are the same (different). If the administrator commits to disclose or conceal players' types before getting to know them, we show that the incentive to disclose (conceal) is stronger if the distribution of types is non-degenerate and strictly-negatively (positively) skewed. If instead the administrator observes types before choosing whether to disclose or conceal, the disclosure choice is a game à la Milgrom (1981) where concealment affects contestants' beliefs on their rival's opponent. We show that, even if Milgrom's assumptions do not hold, the same full-disclosure result is obtained.

JEL Classification: C72 - D72 - D82

1 Introduction

Jack Welch, former General Electric CEO, designed the competition for his succession 6 years before resigning, by short-listing 3 candidates inside General Electric and letting them know not only of being short-listed, but also the identity of the other candidates.[1] Why did he choose to make them aware of who they were competing against?

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[1] See Konrad (2012) and the references therein.

A contest is a game where contestants (the General Electric candidates in the above example) exert efforts in order to increase their probability of winning a prize (CEO successorship). We analyse contests where the contest administrator (Jack Welch) decides whether to disclose or conceal publicly contestants' types in order to maximise the efforts they exert.[2] Disclosure makes contestants aware of each others' type, whereas concealment makes contestants' exerted efforts be based only on their beliefs of the opponent's type. Is the administrator better-off disclosing or concealing contestants' types?

We answer this question by means of a model with 2 contestants of stochastic types who compete for a prize by simultaneously exerting effort. The administrator's goal is to maximise the sum of contestants' efforts. The choice of time-line is the main fork in spelling out the model: the administrator's disclosure choice is made either before (*blind disclosure*) or after (*aware disclosure*) observing contestants' types. We analyse both time-lines.

Blind disclosure. First we analyse the case when the administrator decides the disclosure policy *before* (or equivalently, without) getting to know the contestants' types. This seems natural in a number of possible applications of which we give these examples:

- the administrator of a research contest has to specify the contest rules already in the tender notice, hence before getting to know who the participants are - i.e., their types -, and among the contest rules the information that will be disclosed to the contestants has to be ex-ante transparent.
- the government has to decide the transparency policy for procurement contests run in its country. The government is not aware of contestants' types when choosing the policy, and yet it might oblige contest organizers to disclose contestants' type publicly by means of a law.

We gave these two particular examples in order to pave the way for the following remark. In the first example it is the administrator who personally discloses contestants' types by releasing for example the list of participants on the contest's webpage, whereas in the second example the administrator

[2]Notice that in many contests the contestants are exogenously aware of their rival's type, and the administrator has no possibility of concealing this information. A tennis match is an example of such a contest: tennis players see their opponents.

forces contestants' types to be publicly released by means of a law, and she does not necessarily get to know the types. As long as information is hard (i.e., no lies are allowed), there are no actual differences between those two frameworks. For simplicity, henceforward we say that the administrator actively discloses or conceals the types.

In this "blind disclosure" setting, we assume that types are drawn from a commonly-known dichotomic distribution (h for high-type with probability p, and l for low-type with probability $1-p$) to avoid technical exposition, and we find that the optimal disclosure choice is to disclose (conceal) if $p \in (0.5, 1)$ (if $p \in (0, 0.5)$), and indifference between disclosure and concealment arises when $p = 0$, $p = 0.5$, or $p = 1$. First of all notice that if $p = 0$ or $p = 1$ information disclosure does not play any role as contestants already know who they are competing against, and therefore the administrator is trivially indifferent between disclosing and concealing. In the remainder of this paragraph we give a stylized intuition of the result, and for a more complete intuition the reader can (already) see the end of Section 3. The administrator evaluates the difference between expected sum of efforts under concealment and under disclosure weighting each possible contingency - $\{h,h\}$, $\{l,l\}$, and $\{h,l\}$ - to its probability of occurrence - respectively, p^2, $(1-p)^2$, and $2p(1-p)$. A contestant competing with a rival of the same type (symmetric contest) exerts greater effort than if competing with a stronger or weaker opponent (asymmetric contest) since competition for the prize is fiercer.[3] Hence, in contingencies $\{h,h\}$ and $\{l,l\}$ the administrator prefers to disclose, whereas in $\{h,l\}$ she prefers to conceal. Which of these two incentives is ex-ante stronger? Let us start by analysing the incentive to conceal if $\{h,l\}$, and consider either high or low p, where we choose 0.25 and 0.75 in order to abract away differences in the probability of contingency $\{h,l\}$. If $p = 0.25$, the low-types are highly likely, and hence they exert high effort as they are likely to be competing in a symmetric contest. This is known by the high-type who is indeed likely to be competing against a low-type. Hence, the high effort of the low-type **increases** the effort of the high-type (strategic complementarity of the effort of the high-type in the effort of the low-type). On the contrary, if $p = 0.75$ - the high-types are highly likely, and hence exert high effort as they are likely to be competing in a symmetric contest. This is known by the low-type who is indeed likely to be competing against a

[3] This result is well-known in the contest literature. The intuition can be seen for example in Konrad (2012), and it will be also given in Remark 4.

high-type. Hence, the high effort of the high-type **decreases** the effort of the low-type (strategic substitution of the effort of the low-type in the effort of the high-type). In other words, in $\{h,l\}$ and concealment there is an upward or downward (according to p) distortion in efforts due to the awareness of being in an asymmetric contest with a rival thinking of being in a symmetric contest. Lastly, notice that both under disclosure as well as in the remaining two contingencies - i.e., $\{h,h\}$ and $\{l,l\}$ - these is no such a distortion in efforts, and this makes the incentive to disclose symmetric in p. Therefore, for low (high) p concealment leads to an upward (downward) distortion in efforts that makes the administrator better-off concealing (disclosing). For the same reason, the administrator is indifferent between disclosure and concealment when $p = 0.5$, as these two (upward and downward) distortions in efforts under concealment balance out.

Aware disclosure. In the second part of the paper, we analyse the case when the administrator decides whether to disclose or conceal contestants' types *after* getting to know them. Beside promotion contests like Jack Welch's successorship contest, we mention other possible scenarios where our results could be applied:

- traders compete for a prize raffled off by the CEO who does - or does not - make them aware of their rival trader's skills. Traders' effort is work effort, and the CEO benefits from the overall exerted effort.

- a policy maker is lobbied by interest groups not aware of the identity of the other lobbyists. High aggregate lobbying activity is in the interests of the policy maker.

- administrators of online games do - or do not - make players aware of the rival's type - e.g., the position in the game overall ranking. The administrator maximises the aggregate effort as she benefits from competition and time spent playing by players.

In this "aware disclosure" setting, if we keep the dichotomicity of types - h and l -, we would get the trivial result that if types are symmetric (asymmetric) the administrator discloses (conceals) types. This makes contestants always aware of their rival's type since only in the contingency $\{h,l\}$ concealment occurs. We prove that this full-awareness by contestants is robust to any finite set of contestants' types, and we find the administrator discloses unless the lowest and highest types show up. The intuition is given in what follows for three types - h, m, and l -, but it is easily extendable to any finite set of types. Notice first that contestants are aware of the fact that the administrator makes her disclosure choice knowing types, and hence they update their beliefs on the rival's type upon observing concealment. Again, individual effort is maximized if a type is certain of being in a symmetric contest, and this makes the administrator better-off disclosing in any contingency with two symmetric types - i.e., $\{h,h\}$, $\{m,m\}$, and $\{l,l\}$ - for any belief of contestants. Also, individual effort is minimized if a type meets a rival of the most different type, and this makes the administrator better-off concealing in contingency $\{h,l\}$ for any belief of contestants. We are then left analysing the disclosure policy under two contingencies - $\{h,m\}$ and $\{m,l\}$ -, which we do in the following paragraph.

In $\{h,m\}$, disclosure is optimal if the skills of the medium type are sufficiently close to the ones of the high type, because in this case the administrator does not want to make high and medium types think that they are competing against such a different type (the low). Conversely, concealment in $\{h,m\}$ is optimal if medium is sufficiently close to low. Consider this last case, and the opposite case will follow symmetrically. Since low and medium type have now similar skills, if the two of them show up, the administrator is better-off disclosing their types, in order not to let them think of being against a high type, whose skills are much higher than the ones of the medium and low. Hence, concealment occurs in $\{h,m\}$ and $\{h,l\}$. This makes medium and low types sure of their rival's type under concealment (and hence their efforts are not affected by the disclosure policy), whereas the high type would have a belief on his rival's type that assigns positive weight to both medium and low type rival. The high types exerts more effort competing against a more similar type, and hence in contingency $\{h,m\}$ concealment is not sustainable in equilibrium. This makes any perfect Bayesian Nash equilibrium have the administrator disclosing types unless the highest and lowest types show up (in this case h and l), and hence the full-awareness result carries over to any finite set of contestants' types.

Related literature

It has been emphasized by many the fact that information plays a relevant role in contests, and this is why several works are heading towards a better understanding of the role of information in contests. In particular, information in contests strongly affects exerted efforts as well as the probability of winning the prize, see Warneryd (2003). Examples of contest design regarding information are Aoyagi (2010), who analyses the optimal disclosure of contestants' performance in a dynamic setting, Lim and Matros (2009) and Fu, Jiao and Lu (2011b), who analyse the disclosure of the number of contestants.

While we analyse the administrator's disclosure choice, other papers analyse the incentive of players themselves to disclose their own types, either in all-pay auctions - see Kovenock, Morath, and Munster (2010) and Szech (2011) -, or in imperfectly discriminating contests - see Denter, Morgan, and Sisak (2011), and Yildirim (2005).

We are aware of one paper only which analyses the administrator's disclosure choice: Fu, Jiao and Lu (2011a). Their analysis is in an all-pay auction setting with an administrator aware of contestants' types. They find that the administrator always conceals contestants' types. Our work can be seen as a two-fold extention of their results. We extend their results first to imperfectly discriminating contests, and second to an administrator not aware of contestant's types.

In Section 2 we spell out the model and solve for the equilibrium efforts under disclosure, which are not affected by the administrator awareness of contestants' types. Prior to the contest, we first let the administrator choose and commit to a disclosure policy before getting to know the contestants' types (Section 3), and then we make the administrator aware of contestants' types when deciding the disclosure policy (Section 4). In Section 5 we discuss our results. All Proofs which are not in the main text are in the Appendix.

2 Model

Consider a contest with 2 risk-neutral contestants indexed by $i \in \{1, 2\}$ competing for a prize whose value we normalise wlog to 1. Each contestant i chooses effort level e_i, and has a probability of winning the prize equal to

$$p_i(e_i, e_j) = \begin{cases} 0 & \text{if } e_i = e_j = 0 \\ \frac{e_i^r}{e_i^r + e_j^r} & \text{otherwise} \end{cases} \tag{1}$$

with $i, j = 1, 2,\ j \neq i,\ r \in [0,1]$ and $p_i : \mathbb{R}^2_+ \to [0,1]$. The cost of effort is linear, and the marginal cost[4] depends on a stochastic contestant-idiosyncratic parameter θ_i drawn from the following commonly-known distribution of types:

$$\theta_i = \begin{cases} h & \text{with prob } p \\ l & \text{with prob } 1-p \end{cases} \tag{2}$$

with $h > l > 0$,[5] and $p \in [0,1]$ representing the probability of a high type. Hence, the types are independent. The expected utility of contestant i is:

$$E\left[u_i(e_i, e_j)\,|I_i\right] = E\left[\frac{e_i^r}{e_i^r + e_j^r}\,|I_i\right] - \frac{e_i}{\theta_i} \tag{3}$$

The administrator chooses between disclosure ($\mathcal{D}$) of types such that $I_i = \{\theta_1, \theta_2\}$, or concealment ($\mathcal{C}$) of types such that $I_i = \{\theta_i\}$. If the administrator discloses, she discloses truthfully.[6] We also assume that disclosure is costless, although results hold under fixed or arbitrarily small disclosure costs.[7] The administrator's choice is driven by maximisation of expected sum of efforts, and she either knows (*aware disclosure*) or ignores (*blind disclosure*) the realizations of types according to the choice of timeline for the model. Formally,

Blind Disclosure (BD):

1. Nature draws (θ_1, θ_2) from (2).
2. The administrator chooses $\mathcal{D}$ or $\mathcal{C}$
3. Contest.

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[4]Notice that an equivalent specification would be to model types as a contestant-idiosyncratic prize valuation and keep the marginal cost constant.

[5]We rule out from the beginning the possibility of $h = l$ or $l = 0$ in order to avoid engaging in those trivial cases later on.

[6]We provide a two-fold interpretation of this assumption. The first interpretation is that she cannot misreport types *exogenously*, for example because non-truthful disclosures are illegal, and/or they gravely undermine future credibility of the administrator. The second interpretation is that she *endogenously* does not lie on types because the only reports that affect the contestants' beliefs are the one certifiable by means of a deed or warranty.

[7]Examples of costless disclosures are: public releasing of the contestants list on the contest webpage or thtough a mailing list, or a preliminary event which gathers contestants.

Aware Disclosure (AD):

1. Nature draws (θ_1, θ_2) from (2).
2. The administrator observes (θ_1, θ_2)
3. The administrator chooses $\mathcal{D}$ or $\mathcal{C}$
4. Contest.

In the last stage of both timelines contestants 1 and 2 simultaneously choose efforts e_1 and e_2, and the winner of the contest is determined according to the probabilities in (1). In BD the administrator's choice unravels to choosing whether maximum expected sum of efforts is expected in a private or public information contest. Notice also that in BD the administrator needs not to get to know the types before the contest takes place, as she might just force contestants to disclose their types as already discussed in the introduction. In AD, the administrator has more information than in BD, but contestants might retrieve some information from the chosen disclosure policy in order to update their beliefs on the opponents' type. This update will take place in $\mathcal{C}$ only, since in $\mathcal{D}$ types are simply revealed, and information is assumed to be hard.

Notation for equilibrium efforts. In $\mathcal{D}$, we denote by $e_{\theta_i\theta_j}$ the equilibrium effort of type θ_i (first subindex) who knows to be competing with a type θ_j (second subindex). In $\mathcal{C}$, we denote as e_{θ_i} the equilibrium effort of type θ_i who is not told the type of her rival. Hence, e_{hl} is the effort of an h-type who plays under $\mathcal{D}$ against an l-type, and e_h is the effort of an h-type who plays under $\mathcal{C}$ and has a certain belief on the opponent's type.

Prior to discussing the optimal disclosure policy, we solve for the equilibrium efforts in $\mathcal{D}$, which are the same for AD and BD. In subgame $\mathcal{D}$ contestants play a well-known Tullock-contest under public information. Hence, for given (θ_1, θ_2) the solution is characterized by the following system of FOCs:

$$\begin{cases} \frac{r e^r_{\theta_2\theta_1} e^{r-1}_{\theta_1\theta_2}}{(e^r_{\theta_1\theta_2}+e^r_{\theta_2\theta_1})^2} = \frac{1}{\theta_1} \\ \frac{r e^{r-1}_{\theta_2\theta_1} e^r_{\theta_1\theta_2}}{(e^r_{\theta_1\theta_2}+e^r_{\theta_2\theta_1})^2} = \frac{1}{\theta_2} \end{cases} \tag{4}$$

Dividing the two equations, we get:

$$\frac{e_{\theta_1\theta_2}}{e_{\theta_2\theta_1}} = \frac{\theta_1}{\theta_2} \tag{5}$$

We substitute (5) into (4) and we get:

$$e_{\theta_i\theta_j} = r\frac{\theta_i^{r+1}\theta_j^r}{(\theta_i^r + \theta_j^r)^2} \text{ with } i = 1, 2 \text{ and } i \neq j \tag{6}$$

As (3) is concave in e_i, (6) is the unique equilibrium.

3 Blind Disclosure

In the subgame $\mathcal{C}$ the Tullock-contest is played under private information of types. This makes the problem's tractability falls severely. We do not solve for equilibrium efforts. We state properties that hold in every equilibrium. Together with a proof of the existence of an equilibrium (see Theorem 1 in Einy et al., 2013), this suffices for our purpose.

In $\mathcal{C}$, a pure-strategy Bayesian equilibrium (henceforth simply equilibrium) is given by a mapping from a contestant's type to effort. As already mentioned in the end of Section 2, we denote this function by e_h (e_l) for the h-type (l-type). We show the following property of equilibrium efforts in $\mathcal{C}$.

Proposition 1 *In an equilibrium of $\mathcal{C}$ the following holds:*

$$(1-p)e_l = \frac{pl}{h}e_h + \frac{l(1-2p)}{4}r \tag{7}$$

Proof. The existence of an equilibrium is a special case of Theorem 1 in Einy et al., 2013. Hence, we focus on showing that (7) holds in every equilibrium.

The expected utility of player i is given by (3), whose FOC depends on i's type:

$$p\frac{re_h^r e_{\theta_i}^{r-1}}{(e_h^r + e_{\theta_i}^r)^2} + (1-p)\frac{re_l^r e_{\theta_i}^{r-1}}{(e_{\theta_i} + e_l)^2} = \frac{1}{\theta_i} \text{ for } \theta_i = h, l$$

Therefore, the system of equations characterizing the equilibrium is:

$$\begin{cases} p\frac{r}{4e_h} + (1-p)\frac{re_l^r e_h^{r-1}}{(e_h^r+e_l^r)^2} = \frac{1}{h} \\ (1-p)\frac{r}{4e_l} + p\frac{re_h^r e_l^{r-1}}{(e_h^r+e_l^r)^2} = \frac{1}{l} \end{cases} \tag{8}$$

We divide the second addend of the first equation by the one of the second equation.

$$\frac{(1-p)e_l}{pe_h} = \frac{\frac{4e_h - prh}{4e_h h}}{\frac{4e_l-(1-p)rl}{4e_l l}}$$

We simplify and get:

$$4(1-p)he_l - (1-p)^2 rhl = 4ple_h - p^2 rhl$$

which leads to the result. ■

Proposition 1 will suffice for our purpose of finding the optimal disclosure policy for the administrator. In the remainder of this section we analyse in more depth the equilibrium efforts in order to pave the way for a more thorough understanding of the intuition behind the main result.

Proposition 2 *$\frac{e_h}{e_l} > \frac{h}{l}$ iff $p > \frac{1}{2}$. Also $\frac{e_h}{e_l} = \frac{h}{l}$ iff $p = \frac{1}{2}$.*

Proof. We rewrite (8) in the following way,

$$\begin{cases} p\frac{r}{4} + (1-p)\frac{re_l^r e_h^r}{(e_h^r + e_l^r)^2} = \frac{e_h}{h} \\ (1-p)\frac{r}{4} + p\frac{re_h^r e_l^r}{(e_h^r + e_l^r)^2} = \frac{e_l}{l} \end{cases}$$

and we consider the ratio of the right-hand sides to get,

$$e = \frac{h\left[p(e^r+1)^2 + 4(1-p)e^r\right]}{l\left[(1-p)(e^r+1)^2 + 4pe^r\right]}$$

where we defined $e = \frac{e_h}{e_l}$. Hence the inequality $e > \frac{h}{l}$ is equivalent to,

$$(e^r+1)^2(2p-1) > 4e^r(2p-1)$$

which holds iff $p > \frac{1}{2}$. The rest of the Proposition follows trivially. ■

While in $\mathcal{D}$ the ratio of equilibrium efforts equals the ratio of marginal costs for all values of p - see (5), which is known property when the contest success function is homogeneous of degree zero -, in $\mathcal{C}$ this holds only if $p = \frac{1}{2}$. This is due to the fact that competing with someone of the same (different) type increases (decreases) effort, and this is likely to happen to the l-types (h-types) if $p < \frac{1}{2}$.

For a better understanding of the upcoming comments on equilibrium efforts we plot[8] in Figure 1 the equilibrium efforts in $\mathcal{C}$ and $\mathcal{D}$ as functions of p when $r = 1$, $l = 1$ and $h = 2$. The thick lines are the efforts in $\mathcal{D}$, whereas the thin ones are in $\mathcal{C}$.

Remark 3 *$e_{\theta_i\theta_j}$ does not depend on p, whereas e_{θ_i} does, as in $\mathcal{C}$ contestants equilibrium efforts are affected by the distribution of types (2).*[9]

From (6) it immediately follows that:

Remark 4 *$e_{hh} \geq e_{hl}$ and $e_{ll} \geq e_{lh}$. These inequalities are strict iff $r > 0$.*

The intuition is as follows. When a contestant competes with someone of the same type, competition is tough, and contestants' efforts are greater than in case of competing with a stronger or weaker contestant. Competing with a stronger contestant makes the contestant *lose hope* and fight mildly. Competing with a weaker contestant makes the contestant *take advantage* of her stronger position, as she needs a lower effort to achieve the same probability of winning.

Remark 5 *$e_h \leq e_{hh}$ and $e_l \leq e_{ll}$.*

A contestant aware of being competing with someone of the same type exerts more effort than a contestant of the same type who has a belief (2) over the type of her opponent. The reason is the same behind the previous Remark.

Remark 6 *If $p = 1$, $e_h = e_{hh}$. If $p = 0$, $e_l = e_{ll}$.*

[8] All the plots are made using the command *Root* of *Mathematica*® 8.0 on the polynomial in e_h-only obtained from (8).

[9] We make a simplifying abuse of notation by omitting the dependence on p of the equilibrium efforts in $\mathcal{C}$. Hence, we write e_{θ_i} instead of $e_{\theta_i}(p)$.

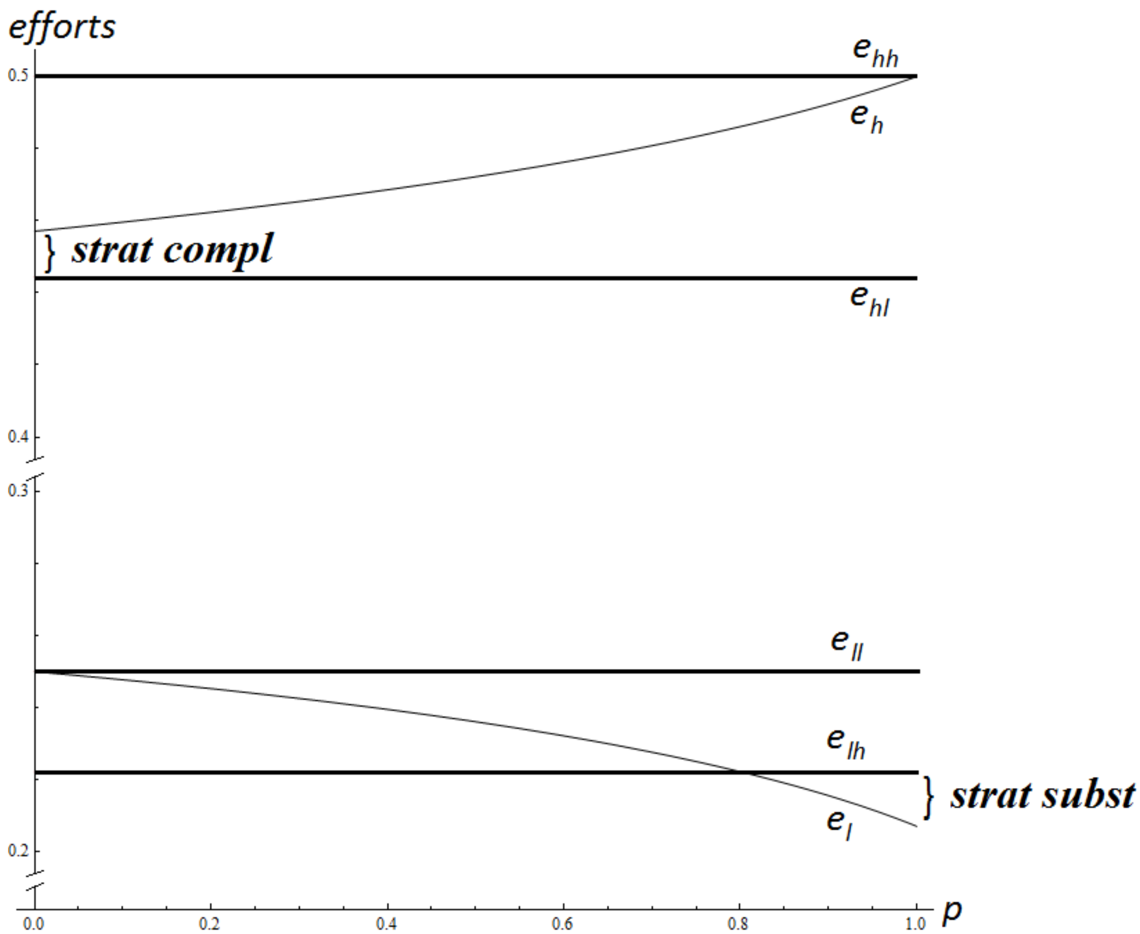

Figure 1: Equilibrium efforts as functions of p when $r = 1$, $l = 1$ and $h = 2$.

A high (low) type who knows that the probability of high types tends to 1 (0) is sure of being competing with a contestant of her same type, and therefore she exerts the same level of effort in $\mathcal{C}$ and $\mathcal{D}$.

Remark 7 *If $p \to 1$, $e_l < e_{lh}$ ($e_l \to e_{lh}$ iff $r = 0$). If $p \to 0$, $e_h > e_{hl}$ ($e_h \to e_{hl}$ iff $r = 0$).*

If $p \to 0$ (1), a high (low) type - even though she has almost 0 probability of being drawn - knows that she is competing with a low (high) type who believes she is against a low (high) type. Hence, this low (high) type exerts effort $e_l \to e_{ll} > e_{lh}$ ($e_h \to e_{hh} > e_{hl}$). This makes the high (low) type compete more (less) intensely in $\mathcal{C}$ than in $\mathcal{D}$.

Now, in order to decide between inducing $\mathcal{C}$ or $\mathcal{D}$, the administrator compares the expected sum of efforts in both cases. We know the equilibrium efforts in $\mathcal{D}$ by (6). As for equilibrium efforts in $\mathcal{C}$, Proposition 1 suffices for our purposes. We get the following:

Theorem 8 *The administrator is indifferent between disclosing or concealing contestants' types iff $p \in \{0, \frac{1}{2}, 1\}$. The administrator prefers concealment if $p \in (0, \frac{1}{2})$. The administrator prefers disclosure if $p \in (\frac{1}{2}, 1)$.*

Notice that Theorem 8 holds regardless of the values of r, h and l.

We plot the function $\pi^{\mathcal{D}-\mathcal{C}}$ (the difference between the expected payoff under disclosure and under concealment) when $r = 1$, $h = 2$ and $l = 1$ in Figure 2 for a visual interpretation of Theorem 8. We give the intuition behind this result in the remainder of this section, and we suggest the reader to keep an eye on Figure 1 and on the following table contingencies/likelihood.

Likelihood	Contingency	Incentive to
p^2	$\{h,h\}$	$\mathcal{D}$
$(1-p)^2$	$\{l,l\}$	$\mathcal{D}$
$2p(1-p)$	$\{h,l\}$	$\mathcal{C}$

Intuition behind Theorem 8

First of all notice that if $p = 0$ or $p = 1$ information does not play any role, as the equilibrium efforts (and hence the expected payoffs) are the same under

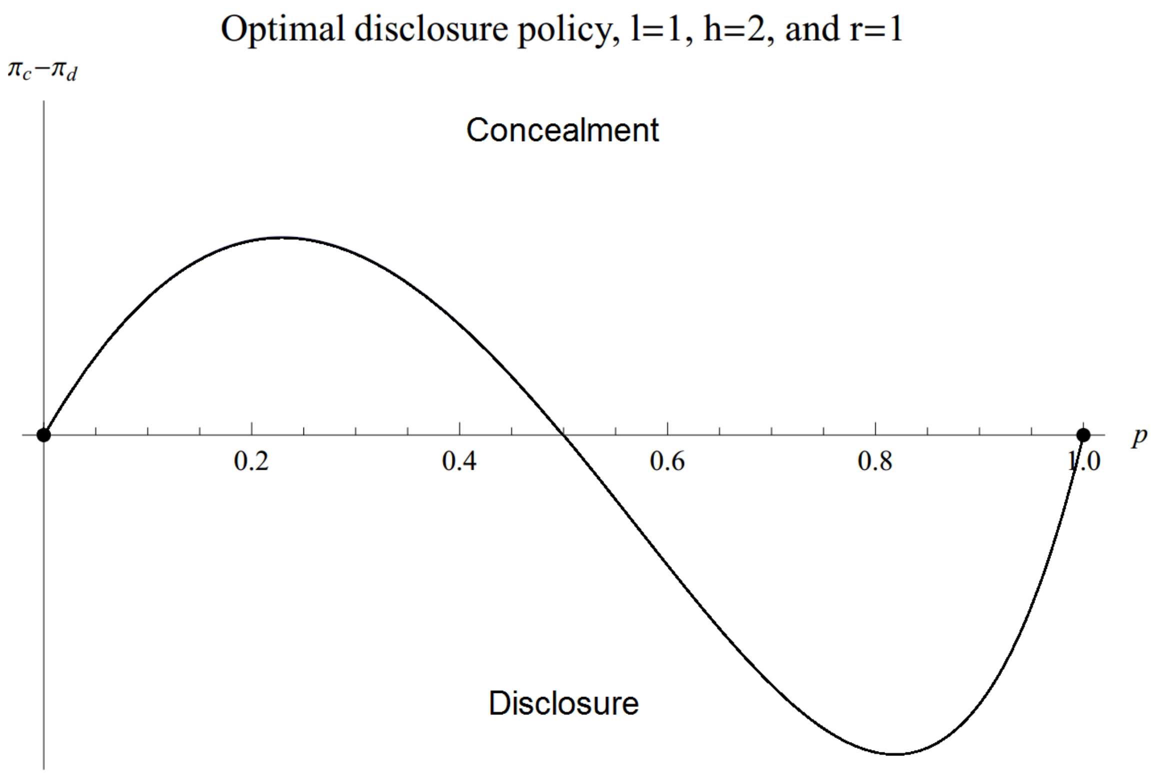

Figure 2: Optimal disclosure policy as a function of p when $r = 1$, $l = 1$ and $h = 2$.

concealment and disclosure. A contestant competing with a rival of the same type exerts greater effort than if competing with a stronger or weaker opponent. The administrator evaluates the difference between expected payoffs under concealment and under disclosure weighting each possible contingency - $\{h,h\}$, $\{h,l\}$, and $\{l,l\}$ - to its probability of occurrence. Let us start discussing the incentive to disclose, which is when two contestants of the same type show up (either $\{h,h\}$ or $\{l,l\}$). If p is low, in $\{h,h\}$ the incentive to disclose is strong (as a high type under concealment would believe she is likely to be against a low type, and hence exerts low effort) with low weight (as $\{h,h\}$ is unlikely under low p). On the contrary, the case $\{l,l\}$ is very likely with a low p, but if so the two low-types will exert a similar effort under disclosure and concealment, as the low p makes them believe under concealment that they are very likely to be competing against another low type. Conversely, if p is high, the case $\{h,h\}$ has weak incentive to disclose with high likelihood of occurrence, and the case $\{l,l\}$ has strong incentive to disclose with low weight. This shows that the incentive to disclose are somehow symmetric in p.

How about the incentive to conceal? The contingencies where concealment is preferred are the ones where the players are asymmetric - i.e., $\{h,l\}$. If p is low or high - say 0.25 or 0.75 - the likelihood of the contingency $\{h,l\}$ is the same, but we show in what follows that the incentive to conceal is asymmetric in p, in particular it is stronger under low p.

Consider low p and $\{h,l\}$. The low type exerts high effort under concealment (since she is quite confident of being against another low), and this is known by the high, who in turn exerts a significantly **higher** effort than in case of disclosure, because she is likely to be competing against a low type who is likely of being against another low.

Consider now a high p and $\{h,l\}$. The high type exerts high effort under concealment (as she is quite confident of being against another high), and this is known by the low, who in turn exerts a significantly **lower** effort than in case of disclosure, because she is likely to be is competing against a high type who is likely of being against another high.

Hence, to summarize, the main mechanism driving the result can be seen in Remark 7, or equivalently in Figure 1: concealment is optimal for low p because of the strategic complementarity of effort of the high-type together with the misperception on the rival's type of the low type in case of asymmetric types and unskilled population.

4 Aware Disclosure

In this section the administrator is assumed to know the contestants' types when choosing the disclosure policy. Therefore, keeping the dichotomic prior (2) would lead to the trivial result that the administrator discloses if two high or two low types show up, and conceals if a high and a low type show up. This makes contestants fully aware of their rival's type since upon observing concealment a high (low) type is certain of being competing against a low (high) type. Hence, the question we address in this section is whether this result of full-awareness of the rival's type carries over for more than 2-types priors. We define the following prior:

$$\theta_i = \begin{cases} \theta_1 & \text{with prob } p_1 \\ \theta_2 & \text{with prob } p_2 \\ .... & .... \\ \theta_n & \text{with prob } p_n \end{cases} \tag{9}$$

with $\theta_1 > \theta_2 > ... > \theta_n > 0$,[10] $p_i \in [0,1]$, $\sum_{i=1}^{n} p_i = 1$, and $n \geq 2$. We keep the same probability of winning in (1), assuming that $r = 1$ in order to avoid technical exposition due to the more general prior.

Contestants. A strategy for a contestant of type θ_i is an effort under disclosure for each possible rival's type - i.e., $e_{\theta_i \theta_j}$-, and an effort under concealment - i.e., e_{θ_i} - which is associated to a belief on the opponent's type. The beliefs after $\mathcal{D}$ are trivial. In $\mathcal{C}$ players update their beliefs on the rival's type according to Bayes' rule. We denote contestants' beliefs of being in contingency $\{\theta_i, \theta_j\}$ after observing $\mathcal{C}$ by $\mu(\{\theta_i, \theta_j\} | \mathcal{C})$.[11] As a consequence, the equilibrium efforts under disclosure are the same of the previous section (6), whereas the equilibrium efforts under concealments are possibly different since the administrator's concealment might change the contestants' beliefs on the rival's type. In particular, posterior beliefs and prior coincide only if the administrator conceals regardless of the types she observes.

[10] Notice that the strict ordering of types is assumed wlog as if $\theta_k = \theta_{k+1}$, we could redifine the prior without θ_{k+1} and with a probability of θ_k equal to $p_k + p_{k+1}$. All results would carry over.

[11] Notice that $\mu(\{\theta_i, \theta_j\} | \mathcal{C})$ should formally be $\mu(\{\theta_i, \theta_j\} \vee \{\theta_j, \theta_i\} | \mathcal{C})$, but the two events $\{\theta_i, \theta_j\}$ and $\{\theta_j, \theta_i\}$ have the same strategical implications for administrator and contestants.

Administrator. A strategy for the administrator is a plan of action - $\mathcal{D}$ or $\mathcal{C}$ - covering every contingency that might arise. The set of contingencies is the power set of cardinality 2 of the set of possible types, $\Theta = \{\mathcal{P}(\{\theta_1, ..., \theta_n\}) : |\mathcal{P}(\{\theta_1, ..., \theta_n\}| = 2\}$, and hence a strategy for the administrator is a mapping $S : \Theta \to \{\mathcal{D}, \mathcal{C}\}^{\frac{n(n+1)}{2}}$. As an example, if prior is (2), strategy $\{\mathcal{C}, \mathcal{C}, \mathcal{D}\}$ means that the administrator conceals if she observes $\{h, h\}$ or $\{h, l\}$, and discloses if she observes $\{l, l\}$. Notice that contingencies $\{\theta_i, \theta_j\}$ and $\{\theta_j, \theta_i\}$ are equivalent, therefore no distinction is needed. We focus on pure-strategies.

We look for a Perfect Bayesian Equilibrium (PBE) of this game, and we get the following intermediate result:

Proposition 9 *In any PBE:*

9.1 $e_{\theta_1} > ... > e_{\theta_n} > 0$.

9.2 $e_{\theta_i} < e_{\theta_i \theta_i} = \frac{\theta_i}{4}$

9.3 If $\{\theta_i,\theta_i\}$ *is observed, the administrator chooses* $\mathcal{D}$.

9.4 If $\{\theta_1,\theta_n\}$ *is observed, the administrator chooses* $\mathcal{C}$ *unless* $\mu(\{\theta_1, \theta_n\} \,|\mathcal{C}) = 1$.

9.5 If $\{\theta_i,\theta_j\}$ *with* $i \neq j$ *and* $\{i,j\} \neq \{1,n\}$ *is observed, the administrator chooses* $\mathcal{D}$.

We provide the intuition of each result in Proposition 9 in what follows, and notice that all these properties (except 9.4) hold regardless of the contestants' beliefs on the rival's type.

9.1 trivially says that more skilled players exert higher effort than less skilled players. 9.2 says that every type exerts less effort under concealment than under certainty of being against an identical type. 9.3 says that if two symmetric types show up, the administrator is better-off disclosing the types, and the reasoning is the same behind 9.2. 9.4 says that if the types observed are the highest and lowest, the administrator conceals since under disclosure they would exert the lowest possible efforts. 9.5 tops off the contingencies not analysed in previous points, and its intuition is explained in details for three-type priors in the end of the Introduction.

Since there is only one contingency which might make the administrator conceal, a Corollary which is straightforward from Proposition 9 is that either the administrator always discloses, or she conceals only if the highest and lowest types show up.[12]

Corollary 10 *There are two PBE:*

- $\{\mathcal{D}\}^{\frac{n(n+1)}{2}}$*, sustained by off-the-equilibrium beliefs* $\mu(\{\theta_1, \theta_n\} | \mathcal{C}) = 1$.
- $\{\mathcal{D}, ..., \mathcal{D}, \mathcal{C}, \mathcal{D}, ...\mathcal{D}\}$ *where* $\mathcal{C}$ *is in the* n^{th} *position, sustained by off-the-equilibrium beliefs* $\mu(\{\theta_1, \theta_n\} | \mathcal{C}) < 1$.

Also, since at most in one contingency the administrator plays $\mathcal{C}$, contestants are always certain of their rival's type, and hence:

Corollary 11 *In any PBE contestants have no uncertainty on their rival's type.*

The full disclosure result of Corollary 11 is related to Okuno-Fujiwara et al. (1990). They analyse a general strategic information revelation game and they obtain sufficient conditions for full disclosure of private information. The condition they provide is that equilibrium expected payoffs are weakly positive monotone in beliefs. .....[explain]... . This condition is granted in our setting if the types space is dichotomic, whereas if the number of types is 3 or more, contingencies are interdependent and disclosure signals are not rankable in terms of informativeness - using the terminology in Migrom (1981)

[12] For completeness, we remark that uniqueness of equilibrium could be achieved by means of PBE's refinements. The Intuitive Criterion - proposed by Cho and Kreps (1987) - pins down as a unique equilibrium the one where the administrator conceals only in $\{\theta_1, \theta_n\}$. Trivially, any other stronger refinement based on consistency of beliefs would detect the same unique PBE (Divinity-Criterion by Banks and Sobel (1987), Never-a-Weak-Best-Response by Kohlberg and Mertens (1986)).

Also, assuming that contestants are boundedly rational in that they do not update their beliefs on the rival's type upon observing the administrator's disclosure choice immediately leads to the same unique equilibrium pinned down by any PBE's refinement.

5 Discussion

We have studied the disclosure choice of the administrator of an imperfectly-discriminating contest by mean of a model with independent and stochastic types of contestant under different assumptions on the information held by the administrator. We found that (i) an administrator not aware of contestants' types chooses disclosure if the probability of high type contestant is sufficiently high, and (ii) an administrator aware of contestants' types makes contestants fully aware of their rival's type.

The implication of our result (i) is that, for example, among the rules of a research contest the administrator should state clearly that information on the participants will be made public if the set of contestants is sufficiently skilled. This founding is in line with the successorship competition designed by Jack Welch and mentioned in the Introduction, as the short-listed candidates knew of being all most likely high types. In this direction, it seems natural to endow the model with an entry fee capable of sifting out bad applicants. Another sensible extention of (i) is to have a pool of contestants whose application to the contest depends on the declared disclosure policy. Model-wise this would make p depend on the administrator's disclosure policy. In this extention we conjecture that concealment deters applications, especially the low-type ones, and this might result in a benefit for the administrator.

An implication of findings (ii) could be the following. In assessing PhD candidates for a certain position, university recruiters should release publicly the names of the short-listed candidates (unless the asymmetry is maximum).

These two results together - (i) and (ii) - make a possible bottom-line of this work read as follows. If the administrator could choose whether to write the disclosure policy ex-ante among the contest's rules or not to commit to any disclosure policy and decide on a case-by-case basis, she would have no strict preference if the population is sufficiently skilled, but she would rather commit ex-ante to conceal if the population is not skilled enough. This is consistent with the fact that in many early stages of research contests the list of candidates (possibly including many low types) is ex-ante claimed that it will not be released, whereas the list of short-listed finalist will be released.[13]

Besides detecting the administrator's disclosure policy, our findings provide equilibrium properties of private information contests that might be

[13] See for example the Amazon Breakthrough Novel Award Contest, where the ex-ante specified rules state that the list of participants will not be released, whereas the list of the 5 finalists will be publicly released on the contest's webpage.

useful for answering other questions related to imperfectly-discriminating contests.

The results of this paper are just a first step meant to pave the way for future works investigating the robustness of our results to, e.g., continuous priors, partial disclosure, more than 2 contestants, and several others.

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A Appendix - Proofs

Proofs of Remark 5 and Remark 6 are special cases of the proof of **9.2** below. In order to prove Remark 7, we first establish the following:

Lemma 12 *e_h and e_l are continuous.*

Proof. The result directly follows from the Maximum Theorem applied to (3), noticing that (3) is strictly concave and continuous in e_i. ■

Proof of Remark 7. We want to show (i) If $p \to 1$, $e_l < e_{lh}$ ($e_l \to e_{lh}$ iff $r = 0$), (ii) If $p \to 0$, $e_h > e_{hl}$ ($e_h \to e_{hl}$ iff $r = 0$).

We prove (i) for $p = 1$ and make use of Lemma 12. The proof of (ii) will be symmetric.

If $p = 1$, the following holds

$$\frac{re_h^r e_l^r}{(e_h^r + e_l^r)^2} = \frac{e_l}{l}$$

and given that e_{lh} is given by (6), $e_l < e_{lh}$ is equivalent to,

$$\frac{re_h^r e_l^r}{(e_h^r + e_l^r)^2} < \frac{rh^r l^r}{(h^r + l^r)^2}$$

which proves that $e_l \to e_{lh}$ iff $r = 0$. Also, $\forall r > 0$, it simplifies to,

$$(l^r e_h^r - h^r e_l^r)(h^r e_h^r - l^r e_l^r) > 0$$

From Proposition 2 we know $\frac{e_h}{e_l} > \frac{h}{l}$ since $p = 1$, and the result follows.

■

Proof of Theorem 8. Disclosure dominates concealment if and only if the difference between the sum of efforts under disclosure is higher than under concealment. We denote this difference by $\pi^{\mathcal{D}-\mathcal{C}}$ and we consider the cases highlighted in Table 1. We prove the result in 2 steps, and we suggest the reader to see Figure 2 to follow the upcoming proof. In the first part of the proof (**Step 1**), we show that the function $\pi^{\mathcal{D}-\mathcal{C}}$ has exactly 3 roots corresponding to three values of p (0, $\frac{1}{2}$, and 1). In the second part of the proof (**Step 2**) we show that the derivative of $\pi^{\mathcal{D}-\mathcal{C}}$ with respect to p in $p = \frac{1}{2}$ is strictly positive. These two results together with continuity of $\pi^{\mathcal{D}-\mathcal{C}}$ - which directly follow from Lemma 12 - necessarily lead to the signs of $\pi^{\mathcal{D}-\mathcal{C}}$ in Figure 2, and hence the Theorem 8 follows.

Step 1. First, we analyse when the administrator is indifferent between $\mathcal{D}$ and $\mathcal{C}$, i.e.:

$$\pi^{\mathcal{D}-\mathcal{C}} = p^2[2e_{hh} - 2e_h] + 2p(1-p)[e_{hl} + e_{lh} - e_h - e_l] + (1-p)^2[2e_{ll} - 2e_l] = 0$$

which simplifies to:

$$p^2 e_{hh} + (1-p)^2 e_{ll} + p(1-p)[e_{hl} + e_{lh}] - pe_h - (1-p)e_l = 0$$

We now substitute (6) and (7) and get:

$$p^2\frac{rh}{4} + (1-p)^2\frac{rl}{4} + p(1-p)\frac{r(h+l)h^r l^r}{(h^r + l^r)^2} - p\frac{h+l}{h}e_h - \frac{1-2p}{4}rl = 0$$

Or equivalently:

$$p^2\frac{rh}{4} + p^2\frac{rl}{4} + p(1-p)\frac{r(h+l)h^r l^r}{(h^r + l^r)^2} - p\frac{h+l}{h}e_h = 0$$

A trivial solution is $p = 0$, when information disclosure does not play any role as (2) collapses to certainty of high type. We rearrange terms and get:

$$pr\frac{h+l}{4} + (1-p)\frac{r(h+l)h^r l^r}{(h^r + l^r)^2} = \frac{h+l}{h}e_h$$

which is equivalent to:

$$pe_{hh} + (1-p)e_{hl} = e_h \tag{10}$$

Hence, (10) is a condition for the administrator's indifference between $\mathcal{D}$ and $\mathcal{C}$ written in terms of efforts exerted by the h-type only. We notice from (10) that there is a second trivial solution that leads to indifference which follows from Remark 6: $p = 1$. With a similar procedure we can get the value of e_l for which the administrator is indifferent between $\mathcal{D}$ and $\mathcal{C}$, which symmetrically to (10) is:

$$(1-p)e_{ll} + pe_{lh} = e_l \tag{11}$$

In order to see if there are other values of p besides 0 and 1 leading to administrator's indifference between $\mathcal{D}$ and $\mathcal{C}$, we plug (10) and (11) into the top-equation of (8), and see if any $p \in (0,1)$ solves that resulting equation.

First, we rewrite the expressions (10) and (11) for e_l and e_h as:

$$e_h = rh\frac{p(h^r + l^r)^2 + 4(1-p)h^r l^r}{4(h^r + l^r)^2} \tag{12}$$

$$e_l = rl\frac{(1-p)(h^r + l^r)^2 + 4ph^r l^r}{4(h^r + l^r)^2} \tag{13}$$

These efforts are the ones that, if exerted in $\mathcal{C}$, make the administrator indifferent between $\mathcal{C}$ and $\mathcal{D}$. Now, by (8) we check if these effort levels are reached for some parameter values. Hence, we rewrite the top-equation of (8) as:

$$pr\frac{h}{4} + (1-p)r\frac{he_h^r e_l^r}{(e_h^r + e_l^r)^2} = e_h \tag{14}$$

Plugging (12) into (14), we get the following simplified expression:

$$\frac{e_h^r e_l^r}{(e_h^r + e_l^r)^2} = \frac{h^r l^r}{(h^r + l^r)^2}$$

Finally, we plug (12) and (13) in the above expression, simplify, and get,

$$\begin{aligned} p(h^r + l^r)^2 + 4(1-p)h^r l^r &= (1-p)(h^r + l^r)^2 + 4ph^r l^r \\ 4(1-2p)h^r l^r &= (1-2p)(h^r + l^r)^2 \end{aligned}$$

and this leads to a third and last solution in p: $p = \frac{1}{2}$. Similar algebra shows that $p = \frac{1}{2}$, (12) and (13) satisfy also the second equation of (8). Hence, we proved the first statement of Proposition 8.

Step 2. The missing step to finish the proof of Proposition 8 is to show the second and third statement, i.e. concealment is optimal if $p \in (0, \frac{1}{2})$ and disclosure is optimal if $p \in (\frac{1}{2}, 1)$. In order to do so, we write the system (8) as a unique equation in terms of e_h and parameters only. Then we make use of the implicit function theorem to evaluate the derivative of $\pi^{\mathcal{D}-\mathcal{C}}$ in $p = \frac{1}{2}$, and prove that it is positive.

From above we know $\frac{\partial \pi^{\mathcal{D}-\mathcal{C}}}{\partial p} > 0$ is equivalent to $\frac{\partial}{\partial p}(p^2 e_{hh} + p(1-p)e_{hl} - pe_h) > 0$ (see (10) where we simplified a p). Therefore, in order to prove the claim it now suffices to show that:

$$e_{hh} - e_{hl} > \left.\frac{\partial e_h(p)}{\partial p}\right|_{p=\frac{1}{2}} \tag{15}$$

With respect to (15), the left-hand side is known by (6). The right-hand side is trickier. We write the system (8) as a unique equation in p and e_h by substituting (7) into the top equation of (8), and we get:

$$f(e_h, p) = \frac{pr}{4e_h} - \frac{1}{h} + 4^r h^r l^r r \frac{(1-p)^{r+1} e_h^{r-1}[h(1-2p) + 4pe_h]^r}{[4^r h^r (1-p)^r e_h^r + (4ple_h + hl(1-2p))^r]^2} = 0$$

By the implicit function theorem,

$$\left.\frac{\partial e_h(p)}{\partial p}\right|_{p=\frac{1}{2}} = -\frac{\left.\frac{\partial f(e_h,p)}{\partial p}\right|_{p=\frac{1}{2}}}{\left.\frac{\partial f(e_h,p)}{\partial e_h}\right|_{p=\frac{1}{2}}} \tag{16}$$

Notice that when $p = \frac{1}{2}$, the solution to (8) is $e_h = rh\frac{h^{2r}+l^{2r}+6h^r l^r}{8(h^r+l^r)^2}$. We use this fact to evaluate numerator and denominator of (16). We start with the denominator:

$$\begin{aligned}
\left.\frac{\partial f(e_h, p)}{\partial e_h}\right|_{p=\frac{1}{2}, e_h = rh\frac{h^{2r}+l^{2r}+6h^r l^r}{8(h^r+l^r)^2}} &= -\frac{r}{8e_h^2} - \frac{rh^r l^r}{2(h^r+l^r)^2 e_h^2} \\
&= -\frac{1}{he_h}
\end{aligned}$$

Hence, expression (16) yields:

$$
\begin{aligned}
\left.\frac{\partial e_h(p)}{\partial p}\right|_{p=\frac{1}{2}} &= -\frac{\left.\frac{\partial f(e_h,p)}{\partial p}\right|_{p=\frac{1}{2}}}{\left.\frac{\partial f(e_h,p)}{\partial e_h}\right|_{p=\frac{1}{2}}} \\
&= he_h\left(\left.\frac{\partial f(e_h,p)}{\partial p}\right|_{p=\frac{1}{2}}\right) \\
&= \frac{rh}{4} + 4^r h^r l^r r\left[\frac{\partial}{\partial p}\left.\left[\frac{(1-p)^{r+1}e_h^{r-1}[h(1-2p)+4pe_h]^r}{[4^r h^r(1-p)^r e_h^r + (4ple_h + hl(1-2p))^r]^2}\right]\right|_{p=\frac{1}{2}}\right] \\
&= \frac{rh}{4} + 4^r h^{r+1} l^r r e_h^r \frac{\left[re_h^{r-1}(e_h - h/2) + e_h^r(r+1)\right] 2^r e_h^r(h^r+l^r)}{\left[2^r e_h^r(h^r+l^r)\right]^3} + \\
&\quad -4^r h^{r+1} l^r r e_h^r \frac{re_h^r 2^{r+1}\left[l^r e_h^{r-1}(e_h-h/2) - h^r e_h^r\right]}{\left[2^r e_h^r(h^r+l^r)\right]^3} \\
&= \frac{rh}{4} + h^{r+2} l^r r^2 \frac{2l^r - (h^r+l^r)}{2e_h(h^r+l^r)^3} - h^{r+1} l^r r \frac{l^r(1+2r) - h^r(1-2r)}{(h^r+l^r)^3} \\
&= \frac{rh}{4} + 4h^{r+1} l^r r \frac{l^r - h^r}{(h^r+l^r)\left[(h^r+l^r)^2 + 4h^r l^r\right]} - h^{r+1} l^r r \frac{l^r(1+2r) - h^r(1-2r)}{(h^r+l^r)^3}
\end{aligned}
$$

Where in the last step we used $e_h = rh\frac{h^{2r}+l^{2r}+6h^r l^r}{8(h^r+l^r)^2}$.
Therefore we can eventually evaluate expression (15),

$$
\begin{aligned}
\frac{rh}{4} - rh\frac{h^r l^r}{(h^r+l^r)^2} &> \frac{rh}{4} + 4h^{r+1} l^r r \frac{l^r - h^r}{(h^r+l^r)\left[(h^r+l^r)^2 + 4h^r l^r\right]} + \\
&\quad -h^{r+1} l^r r \frac{l^r(1+2r) - h^r(1-2r)}{(h^r+l^r)^3} \\
4\frac{l^r - h^r}{\left[(h^r+l^r)^2 + 4h^r l^r\right]} &< \frac{l^r(1+2r) - h^r(1-2r) - h^r - l^r}{(h^r+l^r)^2} \\
4\frac{l^r - h^r}{\left[(h^r+l^r)^2 + 4h^r l^r\right]} &< 2r\frac{l^r - h^r}{(h^r+l^r)^2} \\
2(h^r+l^r)^2 &> r\left[(h^r+l^r)^2 + 4h^r l^r\right]
\end{aligned}
$$

By $r \leq 1$, it suffices to show that,

$$
\begin{aligned}
2(h^r+l^r)^2 &> \left[(h^r+l^r)^2 + 4h^r l^r\right] \\
(h^r - l^r)^2 &> 0
\end{aligned}
$$

and the result follows. ■

Proof of Proposition 9. In what follows we use e_i to refer to e_{θ_i}, and the system of FOCs characterizing the equilibrium is:

$$\begin{cases} p_{11}\frac{e_1}{(e_1+e_1)^2}+p_{12}\frac{e_2}{(e_1+e_2)^2}+...+p_{1n}\frac{e_n}{(e_1+e_n)^2}=\frac{1}{\theta_1} \\ p_{21}\frac{e_1}{(e_2+e_1)^2}+p_{22}\frac{e_2}{(e_2+e_2)^2}+...+p_{2n}\frac{e_n}{(e_2+e_n)^2}=\frac{1}{\theta_2} \\ \quad\quad\quad\quad .... \\ p_{n1}\frac{e_1}{(e_n+e_1)^2}+p_{n2}\frac{e_2}{(e_n+e_2)^2}+...+p_{nn}\frac{e_n}{(e_n+e_n)^2}=\frac{1}{\theta_n} \end{cases} \tag{17}$$

where the p's have to be interpreted as contestants' beliefs over contingencies. Hence, p_{ij} is the belief of player of type i to be against a player of type j, i.e. $p_{ij}=\mu(\{\theta_i,\theta_j\}\,|\mathcal{C})$.

9.1. $e_1>...>e_n>0$.

By $\theta_1>\theta_2$, $\frac{1}{\theta_2}>\frac{1}{\theta_1}$, and hence using the two corresponding equations in (17), we get:

$$\frac{p_2e_2-p_1e_1}{(e_1+e_2)^2}+\frac{p_1}{4e_1}-\frac{p_2}{4e_2}<\sum_{j=3}^{n}p_j\left(\frac{e_j}{(e_2+e_j)^2}-\frac{e_j}{(e_1+e_j)^2}\right) \tag{18}$$

where the right-hand side of (18) is positive if and only if $e_1>e_2$. Notice that $e_1\neq e_2$ directly by $\theta_1\neq\theta_2$ and (17).

Assume by contradiction that $e_1<e_2$. Then the left-hand side of (18) is negative, or equivalently,

$$\frac{p_1e_2-p_2e_1}{4e_1e_2}<\frac{p_1e_1-p_2e_2}{(e_1+e_2)^2}$$

By $(e_1+e_2)^2>4e_1e_2$, it implies that,

$$(p_1+p_2)(e_1-e_2)>0$$

which contradicts $e_1<e_2$.

The inequalities for the ranking of other efforts follow in the same way.

9.2. $e_{\theta_i}<\frac{\theta_i}{4}$.

Let us analyse left-hand side and right-hand side of the FOC for player θ_1 from (17). The right-hand side is constant, whereas the left-hand side is a convex combination of addends of the form $\frac{e_j}{(e_1+e_j)^2}$ which have two properties:

1. they are decreasing in e_1
2. reach a unique maximum in $e_1=e_j$

Hence, the highest e_1 is achieved only if $p_1 = 1$, in which case the FOC reads: $\frac{1}{4e_1} = \frac{1}{\theta_1}$, and hence $e_1 = \frac{\theta_1}{4}$. Given that all p's are interior, the result follows.

9.3. If $\{\theta_i,\theta_i\}$ is observed, the administrator chooses $\mathcal{D}$.

The proof follows directly from **9.2** and from $e_{\theta_i\theta_i} = \frac{\theta_i}{4}$ (if $r = 1$ in (6)).

9.4. If $\{\theta_1,\theta_n\}$ is observed, the administrator chooses $\mathcal{C}$.

The payoff under concealment is $e_1 + e_n$, and the payoff under disclosure is $\frac{\theta_1\theta_n}{\theta_1+\theta_n}$ from (6). The condition for concealment being better than disclosure can be written as,

$$\frac{1}{\theta_1} + \frac{1}{\theta_n} > \frac{1}{e_1 + e_n}$$

and using the first and last equations of (17), it is equivalent to,

$$\begin{aligned}
\frac{p_1e_1 + p_ne_n}{(e_1+e_n)^2} + \frac{p_1}{4e_1} + \frac{p_n}{4e_n} + \sum_{j=2}^{n-1} p_j\left(\frac{e_j}{(e_1+e_j)^2} + \frac{e_j}{(e_n+e_j)^2}\right) &> \frac{1}{e_1+e_n} \\
\frac{p_1e_n + p_ne_1}{4e_1e_n} + \sum_{j=2}^{n-1} p_j\left(\frac{e_j}{(e_1+e_j)^2} + \frac{e_j}{(e_n+e_j)^2}\right) &> \frac{(1-p_1)e_1 + (1-p_n)e_n}{(e_1+e_n)^2} \\
\frac{p_1e_n + p_ne_1}{4e_1e_n} + \sum_{j=2}^{n-1} p_j\left(\frac{e_j}{(e_1+e_j)^2} + \frac{e_j}{(e_n+e_j)^2}\right) &> \frac{1}{e_1+e_n}\left(\sum_{j=2}^{n-1} p_j\right) + \frac{p_1e_n + p_ne_1}{(e_1+e_n)^2} \qquad (19)
\end{aligned}$$

Notice that by 9.1,

$$\begin{aligned}
\frac{e_j}{(e_1+e_j)^2} &> \frac{e_n}{(e_1+e_n)^2} \quad \forall j = 2, ..., n-1 \\
\frac{e_j}{(e_n+e_j)^2} &> \frac{e_1}{(e_1+e_n)^2} \quad \forall j = 2, ..., n-1
\end{aligned}$$

and hence,

$$\sum_{j=2}^{n-1} p_j\left(\frac{e_j}{(e_1+e_j)^2} + \frac{e_j}{(e_n+e_j)^2}\right) > \frac{1}{e_1+e_n}\left(\sum_{j=2}^{n-1} p_j\right)$$

We use this last inequality in (19) to get,

$$\frac{p_1e_n + p_ne_1}{4e_1e_n} > \frac{p_1e_n + p_ne_1}{(e_1+e_n)^2}$$

and by $(e_1 + e_n)^2 > 4e_1e_n$ the result follows.

9.5. If $\{\theta_i,\theta_j\}$ with $i \neq j$ and $\{i,j\}\neq\{1,n\}$ is observed, the administrator chooses $\mathcal{D}$.

By **9.2** we know that in (17) $p_{ii} = 0$. For n even or odd the proof of **9.5** is different. Here for the sake of space we prove it for $n = 3$ and for $n = 4$, and the same procedure applies to finer type-spaces.

$n = 3$

We first show that if $\{\theta_1, \theta_2\}$ is observed, concealment is chosen. We write (17) for types θ_1 and θ_2 as:

$$\begin{cases} (1-p_{13})\frac{e_2}{(e_1+e_2)^2} + p_{13}\frac{e_3}{(e_1+e_3)^2} = \frac{1}{\theta_1} \\ (1-p_{23})\frac{e_1}{(e_1+e_2)^2} + p_{23}\frac{e_3}{(e_1+e_3)^2} = \frac{1}{\theta_2} \end{cases} \tag{20}$$

and the administrator discloses iff,

$$\begin{aligned} \frac{\theta_1\theta_2}{\theta_1+\theta_2} &> e_1 + e_2 \\ \frac{1}{e_1+e_2} &> \frac{1}{\theta_1} + \frac{1}{\theta_n} \\ \frac{e_1}{(e_1+e_2)^2} + \frac{e_2}{(e_1+e_2)^2} &> \frac{1}{\theta_1} + \frac{1}{\theta_n} \end{aligned}$$

and using (20) this is equivalent to,

$$p_{13}\left(\frac{e_3}{(e_1+e_3)^2} - \frac{e_2}{(e_1+e_2)^2}\right) + p_{23}\left(\frac{e_3}{(e_1+e_3)^2} - \frac{e_1}{(e_1+e_2)^2}\right) < 0$$

where both terms are negative by **9.1**. This proves that if $\{\theta_1, \theta_2\}$ is observed, concealment is chosen.

Now, let us consider the case when $\{\theta_2, \theta_3\}$ is observed. Using the finding above, $p_{12} = p_{21} = 0$, and hence the FOCs for types θ_2 and θ_3 can be written as:

$$\begin{cases} \frac{e_3}{(e_2+e_3)^2} = \frac{1}{\theta_2} \\ p_{31}\frac{e_1}{(e_1+e_3)^2} + (1-p_{31})\frac{e_2}{(e_2+e_3)^2} = \frac{1}{\theta_3} \end{cases} \tag{21}$$

and disclosure is then optimal if,

$$\begin{aligned} \frac{\theta_2\theta_3}{\theta_2+\theta_3} &> e_2 + e_3 \\ \frac{e_3}{(e_2+e_3)^2} + \frac{e_2}{(e_2+e_3)^2} &> \frac{1}{\theta_2} + \frac{1}{\theta_3} \\ p_{31}\left(\frac{e_1}{(e_1+e_3)^2} - \frac{e_2}{(e_2+e_3)^2}\right) &< 0 \end{aligned}$$

which holds by **9.1**. This concludes the proof for $n = 3$.

$n = 4$

If n is even, the starting point is to show that if the two central types show up disclosure is chosen. In this case, we show that $\{\theta_2, \theta_3\}$ implies disclosure. Using the same manipulation of FOCs for $n = 3$ above gives us the following condition for disclosure upon observing $\{\theta_2, \theta_3\}$,

$$p_{21}\underbrace{\left(\frac{e_1}{(e_1+e_2)^2} - \frac{e_3}{(e_2+e_3)^2}\right)}_{A} + p_{24}\underbrace{\left(\frac{e_4}{(e_4+e_2)^2} - \frac{e_3}{(e_2+e_3)^2}\right)}_{B} +$$
$$+p_{31}\underbrace{\left(\frac{e_1}{(e_1+e_3)^2} - \frac{e_2}{(e_2+e_3)^2}\right)}_{C} + p_{34}\underbrace{\left(\frac{e_4}{(e_3+e_4)^2} - \frac{e_2}{(e_2+e_3)^2}\right)}_{D} < 0 \quad (22)$$

where $B < 0$ and $C < 0$ by **9.1**. Now, let us focus on A. We know that $A > 0$ iff $e_2^2 > e_1 e_3$, but if this is the case the condition for disclosure in $\{\theta_1, \theta_2\}$ holds as it reads,

$$p_{14}\left(\frac{e_4}{(e_1+e_4)^2} - \frac{e_2}{(e_1+e_2)^2}\right) + p_{13}\left(\frac{e_3}{(e_1+e_3)^2} - \frac{e_2}{(e_1+e_2)^2}\right) +$$
$$+p_{23}\left(\frac{e_3}{(e_2+e_3)^2} - \frac{e_1}{(e_1+e_2)^2}\right) + p_{24}\left(\frac{e_4}{(e_2+e_4)^2} - \frac{e_1}{(e_1+e_2)^2}\right) < 0 \quad (23)$$

and all terms are negative (the last two using $e_2^2 > e_1 e_3$). Therefore in (22) $p_{21} = 0$. This proves that $p_{21}A$ in (22) is never positive.

We now do the same for D in (22). $D > 0$ iff $e_3^2 < e_2 e_4$, and if this is the case the condition for disclosure in $\{\theta_3, \theta_4\}$ holds as it reads,

$$p_{31}\left(\frac{e_1}{(e_1+e_3)^2} - \frac{e_4}{(e_3+e_4)^2}\right) + p_{32}\left(\frac{e_2}{(e_2+e_3)^2} - \frac{e_4}{(e_3+e_4)^2}\right) +$$
$$+p_{41}\left(\frac{e_1}{(e_1+e_4)^2} - \frac{e_3}{(e_3+e_4)^2}\right) + p_{43}\left(\frac{e_2}{(e_2+e_4)^2} - \frac{e_3}{(e_3+e_4)^2}\right) < 0 \quad (24)$$

and all terms are negative (the last two using $e_3^2 < e_2 e_4$). Therefore in (22) $p_{34} = 0$. This proves that $p_{34}D$ in (22) is never positive.

The above proves the optimality of disclosure upon observing $\{\theta_2, \theta_3\}$. Now, let us consider two cases:

Case 1. $e_3^2 > e_1 e_4$.

This implies $e_2^2 > e_1e_4$ and from (23) with $p_{23} = 0$ it implies disclosure upon observing $\{\theta_1, \theta_2\}$. Now, from the disclosure condition for $\{\theta_1, \theta_3\}$ (which we do not do henceforth as it is constructed symmetrically to the above) we know that $e_3^2 > e_1e_4$ implies disclosure upon observing $\{\theta_1, \theta_3\}$.

Case 2. $e_2^2 < e_1e_4$.

This implies $e_3^2 < e_1e_4$ and from (24) with $p_{32} = 0$ it implies disclosure upon observing $\{\theta_3, \theta_4\}$. From the disclosure condition for $\{\theta_2, \theta_4\}$ we know that $e_2^2 < e_1e_4$ implies disclosure upon observing $\{\theta_2, \theta_4\}$.

Now, Case 1 and Case 2 cover the all possibilities since $e_2 > e_3$, and hence we get either disclosure if $\{\theta_1, \theta_2\}$ and $\{\theta_1, \theta_3\}$ (Case 1), or disclosure if $\{\theta_3, \theta_4\}$ and $\{\theta_2, \theta_4\}$ (Case 2). The former implies disclosure if $\{\theta_3, \theta_4\}$ from (24), and hence disclosure if $\{\theta_2, \theta_4\}$ from its corresponding condition, and the latter implies disclosure if $\{\theta_1, \theta_2\}$ from (23), and hence disclosure if $\{\theta_1, \theta_3\}$ from its corresponding condition.

This concludes the proof. ■

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