Factional Conflict and Territorial Rents*

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Abstract

I study a model of factional conflict over territories from which rents are endogenously generated through the exercise of market power. Factionalization leads to more frequent, but less intense, conflict. As a result, factionalization is associated with a decrease in both the variability of violence and the stability of the configuration of territorial control, but has a non-monotone relationship to expected violence. Consistent with standard intuitions, changes to economic conditions that increase market power or market size at all territories lead to a positive association between rents and conflict. However, contrary to these same intuitions, changes in local economic conditions at a territory under dispute lead to a negative association between rents and conflict. Moreover, because local economic shocks have spillover effects at neighboring territories, the model predicts that standard identification strategies used to study the effect of economic shocks on conflict yield biased estimates—some types of shocks yield systematic overestimates and other types of shocks yield a bias whose sign cannot be known by the econometrician.

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In many settings, armed factions compete for control of territory that can be used to extract economic rents.[1]

From March through June of 2009 violence in the Mexican state of Michoacán more than tripled, reaching an average 67 drug-related homicides per month. The proximate cause was a factional war over territory. Los Zetas, in the midst of splitting from the Gulf Cartel, sought to wrest control over Michoacán and its valuable transshipment routes from La Familia. Such territorial conflicts reached their apex in 2011 and 2012 when two of the largest Mexican drug trafficking organizations—Los Zetas and the Sinaloa Cartel—fought an extraordinarily bloody battle over transshipment routes and cultivation territories from Veracruz, to Guadalajara, to Nuevo Laredo. During this two-year period, the Mexican drug war claimed well over 10,000 lives per year (Rios, 2013).

Some of the most significant factional violence over the last decade in Afghanistan occurred in Helmand Province, a Taliban stronghold and Afghanistan's leading poppy producer. The United Nations Office of Drugs and Crime reports that local factional leaders use control over these territories and transshipment routes to extract economic rents by levying taxes on drug traffickers, poppy farmers, and owners of heroin laboratories.[2]

Over 500 people were murdered in Chicago in 2012. "Most of Chicago's violent crime," according to he head of the DEA for the five-state region that includes Illinois, "comes from gangs trying to maintain control of drug-selling territories."[3] Investigative journalist John Lippert reports that the gangs are motivated precisely by access to territorial rents: "[i]f you want to expand your sales, you have to expand your street corners. You know, you have to physically take street corners, which is a violent act."[4]

Such competition over territorial rents characterizes many other conflicts as well, from Colombian rebels fighting for territory where they can extract protection money from oil companies or tax and control the illicit drug trade to ISIL rebels seeking control over oil producing territories in Iraq. Somewhat more speculatively, one could argue that this sort

[1] For theoretical models of conflict over economic rents (though typically not local rents), see, for example, Hirshleifer (1991); Grossman (1999); Hafer (2006); Fearon (2008); Besley and Persson (2011); Dal Bó and Dal Bó (2011).

[2] United Nations Office on Drugs and Crime. "The Global Afghan Opium Trade: A Threat Assessment." July, 2011.
http://www.unodc.org/documents/data-and-analysis/Studies/Global_Afghan_Opium_Trade_2011-web.pdf

[3] John Lippert. "Heroin Pushed on Chicago by Cartel Fueling Gang Murders." *Bloomberg Markets Magazine*. September 16, 2013. http://www.bloomberg.com/news/2013-09-17/heroin-pushed-on-chicago-by-cartel-fueling-gang-murders.html

[4] "Probing Ties Between Mexican Cartel And Chicago's Violence." *National Public Radio*. http://www.npr.org/2013/09/17/223309103/probing-ties-between-mexican-drug-cartel-and-chicagos-violence

of conflict over territorial rents typified the wars that led to the rise of the modern state in early modern Europe (Bean, 1973; Tilly, 1992; Besley and Persson, 2009). A recent empirical literature is increasingly interested in the relationship between territorial control, local rents, and violent conflict. (See, for example, Angrist and Kugler (2008); Castillo, Mejia and Restrepo (2013); Mejia and Restrepo (2013); Dube, García-Ponce and Thom (2014); Dell (Forthcoming).)

I propose a model to investigate the relationship between factionalization, local economic conditions, the exercise of market power, and territorial conflict. The model makes three types of contributions. First, it yields testable hypotheses about the effects of factionalization, market power, and market size on a variety of conflict outcomes. Second, it suggests a theoretical critique of standard identification strategies used in the empirical literature on economic shocks and conflict. Third, it highlights the conceptual importance of endogenizing the economic returns to territorial conquest in models of conflict.

A common logic drives all three contributions. Factions fight over control of territories that allow for the exercise of market power.[5] The number of factions, transportation costs, and population density affect the endogenously determined returns to taking new territory. Changes to different factions' incremental returns to winning control over a given territory have different effects on the distribution of violence. An increase in the incremental return to winning for the faction that values winning the most tends to decrease conflict by scaring-off other factions. But an increase in the incremental return to winning for the faction that values winning the second most tends to increase conflict by reducing scare-off and increasing incentives to invest in violence. Often some change in the environment increases (or decreases) both incremental returns, resulting in competing effects. Because the incremental return of the faction that values winning the second most affects violence through two mechanisms, while the incremental return of the faction that values winning the most affects violence through only one mechanism, the effect on the second faction's incremental return typically dominates. (This is made precise in Section 2.)

Factionalization Qualitative accounts and conventional wisdom often suggest that increased factionalization causes an increase in violence.[6] The analysis here finds matters are more subtle.

[5] The economic model is in the spirit of Salop (1979), but with fixed locations, as in Alesina and Spolaore's (2005) model of state formation.

[6] See, for example, Beittel (2013) or Jeremy Garner. Gang factions lead to spike in city violence. *The Chicago Tribune*, October 3, 2012. Available: http://articles.chicagotribune.com/2012-10-03/news/ct-met-street-gang-bloodshed-20121003_1_gang-violence-gangster-disciples-black-p-stones.

In the model, highly factionalized environments are characterized by frequent, but low-intensity, conflict and instability of the pattern of territorial control. Consolidated environments are characterized by infrequent, but high-intensity, conflict and stability of the pattern of territorial control. The overall expected amount of violence is non-monotone in the number of factions. Three factors drive these relationships. First, factionalization diminishes market power and, thus, decreases incentives for violence—explaining the diminution in the intensity of conflict. Second, factionalization decreases scare-off by diminishing the difference in the incremental returns of the factions involved in conflict. Third, factionalization is associated with the disappearance of "safe territories"—territories that are insulated from attack by virtue of being surrounded entirely by other territories controlled by the same faction. The diminution in both scare-off and safe territory associated with increased factionalization leads to an increase in the frequency of conflict and a decrease in the stability of the pattern of territorial control. The non-monotone relationship between factionalization and expected violence is a consequence of the interaction between increased frequency and decreased intensity of conflict.

Market Size and Market Power I study two types of comparative statics—global and local—regarding the effects of market power (transportation costs) and market size (population density). The global comparative statics analyze what happens to the distribution of violence when market conditions change at all territories. Because global increases in market size or power are associated with an increase in the stakes of conflict, but no change in scare-off, such changes increase both expected violence and the variability of violence.

The local comparative statics are both more interesting and more relevant for empirical research exploiting local variation. Here I ask how the distribution of violence is affected by variation in local economic conditions. Several findings emerge from this analysis.

First, local economic shocks at a disputed territory create a negative association between rents and violence, exactly the reverse of the global comparative statics. To see why, consider the case of a shock to the population density (market size) surrounding some territory under dispute. The increase in local population density increases the marginal costs to raising prices (in terms of foregone demand) for the factions that control surrounding territories. Hence, as local population density at some territory increases, prices at the surrounding territories decrease, which spills over into lower prices at all territories. While this price decline tends to reduce both factions' rents, the rents decrease more slowly for whichever faction ends up with control over the shocked territory, since increased population density also has a direct positive effect on demand at that territory. As a consequence, both factions' incremental returns to winning the conflict are increasing in local population density—the attacker's rents are decreasing less quickly if it wins the shocked territory and the defender's rents are decreasing less quickly if it holds the shocked territory. As emphasized above, the effect on the smaller incremental return (here, the defender's) dominates. This means that, even when the shock decreases both factions' rents, it increases expected violence. (A similar intuition holds in reverse for shocks to local transportation costs.)

Relationship to Empirical Literature The results of the model highlight at least two ways in which a dialogue between theoretical and empirical scholarship is essential to understanding causal relationships in areas as strategically complex as conflict.

The first comes from the contrast between the local and global comparative statics. Often, empirical work assumes that territorial conflict is expected to increase with rents. But that intuition comes from thinking about changes akin to my global comparative statics. The model here shows that exactly the opposite is true for local changes at the territory under dispute. As the empirical literature becomes increasingly concerned with identification, this is precisely the kind of variation being studied.

Second, and more importantly, the model predicts that local shocks at one territory affect violence at neighboring territories in subtle ways. For instance, shocks to local transportation costs at some territory have non-monotone effects on expected violence at neighboring territories, while shocks to local population density at a territory cause a decrease in expected violence at neighboring territories. In addition to being testable hypotheses in their own right, these results suggest a potential problem for the standard difference-in-differences identification strategy used in studies estimating the effect of economic shocks on conflict.[7] Such studies exploit an economic shock—from weather, a natural disaster, world commodity prices, etc.—that affects locality i at time t. The treatment effect of interest is the change in violence at locality i following the shock at time t. However, there is concern that other factors that affect violence may have changed at the same time that locality i experienced the shock. To isolate the effect of the shock, the researcher studies the change in violence from t to $t+1$ at territory i relative to the change in violence from t to $t+1$ at nearby localities. Under a parallel trends assumption, this difference-in-differences strategy identifies the causal effect of the shock at i (Angrist and Pischke, 2008).

[7]See, among others, Deininger (2003); Angrist and Kugler (2008); Brückner and Ciccone (2010); Hidalgo et al. (2010); Besley and Persson (2011); Dube and Vargas (2013); Bazzi and Blattman (2014); Dube, García-Ponce and Thom (2014); Maystadt and Ecker (2014); Mitra and Ray (2014). A related literature looks at the effect of local development aid on local conflict (e.g., Berman, Shapiro and Felter, 2011; Crost, Felter and Johnston, 2014).

My results suggest that the parallel trends assumption may not hold, since economic shocks at i affect the distribution of violence at nearby localities. Furthermore, the model offers predictions about the sign of the bias. In the case of local population shocks (i.e., shocks to market size), the model predicts that difference-in-differences yields a systematic over estimate—the shock increases violence at the shocked territory and decreases violence at its neighbors. In the case of local transportation shocks (i.e., shocks to local market power), the model predicts that the sign of the bias depends on the magnitude of the shock and is, thus, probably unknowable by the econometrician. This is because the shock decreases violence at the shocked territory and has a non-monotone effect at its neighbors, so the difference-in-differences may be too large or too small relative to the true effect.

Relationship to the Theoretical Literature The theoretical conflict literature is vast and I do not attempt to summarize it here. But it is worth noting that most of that literature, focusing on providing an account of the underlying causes of conflict, assumes that groups compete over a prize of fixed value (see Jackson and Morelli, 2011, for an overview). By contrast, in my model, all of the predicted relationships are driven by the fact that the value of controlling a particular territory is determined endogenously by future economic behavior, which, in turn, depends on transportation costs, population density, and the number of factions. Hence, the model highlights, in one setting, the value of endogenizing the economic returns to conflict for understanding how conflict plays out. (Fearon, 2008, studies a model in which a victorious state sets tax policy in the conquered state following conflict.)

The remainder of the paper is organized as follows. Section 1 specifies the formal model. Section 2 characterizes the distribution of conflict for arbitrary incremental returns. Section 3 characterizes the rents associated with the economic equilibrium. Section 4 uses the results from the previous two sections to characterize equilibrium and then analyzes the effects of factionalization. Sections 5 and 6 develop global and local comparative statics, respectively. Section 7 concludes.

1 The Model

There are six fixed territories, labeled $A-F$, located at equal intervals on the perimeter of a circle.[8] Two territories are *contiguous* if they are located next to each other. The territories

[8] Six territories is the smallest number needed for the comparisons in Section 4.

are arrayed in alphabetical order (so territory F is contiguous with territories A and E). The territories are controlled by factions. I consider variants of the game with between two and six factions. Let $\mathcal{F}$ be the set of factions. A population of mass N is located uniformly on the perimeter of the circle.

The game is played as follows.

(i) At the beginning of the game, there is some configuration of factional control of the territories described by a partition of $\{A, B, C, D, E, F\}$.

(ii) Nature chooses one territory to become *vulnerable*. Any faction, $i \in \mathcal{F}$, that controls either the vulnerable territory or a territory contiguous with it chooses an amount, $a_i \in \mathbb{R}_+$, to invest in fighting for control of the vulnerable territory.

(iii) At the end of the conflict either the territory is still controlled by its original owner or has changed hands. Factions then set prices for the single good traded in the economy. A faction can set a different price at each territory it controls. The price at territory j is $p_j \in [0, 1]$.

(iv) Each population member decides whether and from which territory to buy the good and the game ends.

Conflict is modeled as an all-pay auction. Call the initial holder of a vulnerable territory the *defender* and all factions with contiguous territories *attackers*. If one of the factions involved in fighting invests strictly more than any other faction, it wins the territory. If the defender is involved in a tie, she wins. If two attackers are involved in a tie, they win with equal probability.[9]

Each population member gets a benefit of 1 from consuming the good. Population members bear linear transportation costs, t. If a population member buys the good for price p from a territory at distance x from her location, her payoff is:

$$1 - p - tx.$$

If she doesn't buy the good, her payoff is zero. To insure interior solutions and that the full market is served, I assume that transportation costs are not too large: $0 < t \leq 1$.

The factions bear costs for investing in conflict and make profits from selling the good. If a faction makes revenues r and invests a in conflict, its payoff is

$$r - a.$$

[9] Since ties never occur in equilibrium, the tie breaking rule is irrelevant.

I will primarily be interested in the amount of *observed violence*. Say that violence is observed if at least two factions make a positive investment. If violence is observed, it is the sum of the investments:

$$v = \begin{cases} \sum_{i \in \mathcal{F}} a_i & \text{if } \big|\{i \in \mathcal{F} : a_i > 0\}\big| \geq 2 \\ 0 & \text{else,} \end{cases}$$

where a_i is constrained to be zero for factions that are neither attackers nor the defender.

The solution concept is subgame perfect Nash equilibrium.

1.1 Comments on the Model

Before turning to the analysis, I briefly discuss some assumptions and matters of interpretation.

Most important is the interpretation of transportation costs. For some natural applications, transportation costs can be taken pretty literally, but for other applications they should be viewed more metaphorically. Consider a few examples.

In the case of U.S. street gangs, transportation costs can be understood as a model of consumers' search and travel costs for finding alternative suppliers.

One of the ways that Mexican drug trafficking organizations and Afghan warlords extract territorial rents is by acting as monopsonists relative to marijuana or poppy farmers on territories they control. For instance, consider the following description, from a Mexican marijuana farmer, of the local market power exerted by the Sinaloa cartel: "The thing is that no one else can buy marijuana around here, only the lord."[10] In such situations, the extent of market power depends on those farmers' costs for seeking out alternative buyers, which are reasonably modeled as transportation costs.

Those same organizations also extract territorial rents directly from the drug trade. Afghan factions often tax travel on roads they control and charge for protection services.[11] The associated market power depends on the availability of alternative routes, which are also reasonably modeled as transportation costs.

[10] Translation of, "La cosa es que nadie más puede comprar mariguana por acá, sólo el Señor." From Río Doce. "Mariguana, la únical inversión seguar." http://www.marinonavegante.net/2012/09/mariguana-la-unica-inversion-segura.html. I learned about this account from Dube, García-Ponce and Thom (2014), who discuss the idea of Mexican drug trafficking organizations acting as monopsonists.

[11] United Nations Office on Drugs and Crime. "The Global Afghan Opium Trade: A Threat Assessment." July, 2011.
http://www.unodc.org/documents/data-and-analysis/Studies/Global_Afghan_Opium_Trade_2011-web.pdf

For Mexican drug transshipment, and in some other potential applications, market power derives from sources that are perhaps more distant from transportation costs. The model maps less cleanly onto such cases, but even so may provide some insight if we think of transportation costs here as a metaphor for market power more generally.

Of course, one could also ask a variety of interesting questions about the spillover effects of violence onto transportation costs or about factions' strategic use of violence to manipulate market power. Those questions are left for future research, as are potentially important dynamic considerations.

It is also worth highlighting a few important assumptions. First, because total rents are increasing in market concentration, the factions would benefit from forming a cartel. The same is true of any particular dispute over a territory—the attackers and defender would be best off not fighting, ceding control of the territory to the largest faction, and sharing the rents. Hence, the model implicitly assumes a commitment problem among the factions that prevents such agreements (Fearon, 1995; Powell, 2004).

Second, violence has no negative welfare consequences for consumers. Since I do not provide a welfare analysis, no results would be changed by allowing such externalities.

Third, only one territory is vulnerable at any one time and only contiguous factions can attack that territory. These assumptions aid with tractability. (Absent them, expected payoffs have to be calculated over a large set of possible permutations of territorial control). But they also capture a substantively plausible intuition. As factions consolidate, more territories are located in the interior of factions' overall area of control. These internal territories are harder to attack and, thus, consolidation reduces opportunities for conflict. (See Papachristos, Hureau and Braga (2013) and Dell (Forthcoming) for evidence on the lower frequency of attacks against non-contiguous and interior territories, respectively.) I point out when this effect plays a role in the analysis.

Fourth, a faction bears the costs of investment in conflict even if the other factions do not invest (ceding the territory). Since preparing for conflict involves converting valuable resources into training and weapons, it is natural to think that factions bear some costs in such circumstances. A somewhat more satisfying assumption might be that the costs of investment are lower when it turns out that no actual fighting is required. However, the benefits of such an assumption, in terms of verisimilitude, come at a significant cost in terms of tractability.

2 Conflict for General Incremental Returns

In the conflict game, a faction deciding how much to invest in fighting for control over the vulnerable territory compares its expected payoff in the economic equilibrium should it win versus lose the fight. Label a specific configuration of territorial control and vulnerability ξ. Call the difference in faction j's expected equilibrium payoff should it win versus lose its *incremental return to winning* at ξ, IR_j^ξ.

At most three factions can be involved in conflict. As we will see later, in the configurations of interest, it turns out that, even when three factions can fight, the defender's incremental return is strictly lower than the attackers'. As such, the following results from the literature on all-pay auctions are key for the analysis:

Theorem 2.1 *(Hillman and Riley, 1989; Baye, Kovenock and De Vries, 1996) In an all-pay auction with linear costs, let* IR_i *be player i's expected incremental return from winning the auction instead of losing the auction. If either there are two players with* $\mathrm{IR}_1 \geq \mathrm{IR}_2$ *or there are three players with* $\mathrm{IR}_1 \geq \mathrm{IR}_2 > \mathrm{IR}_3$*, then there is a unique equilibrium. In it, Player 1 bids the realization of a random variable drawn from the uniform distribution on* $[0, \mathrm{IR}_2]$ *and Players 2 bids 0 with probability* $\frac{\mathrm{IR}_1 - \mathrm{IR}_2}{\mathrm{IR}_1}$ *and with the complementary probability bids the realizations of an independent random variable drawn from the uniform distribution on* $[0, \mathrm{IR}_2]$*. Player 3 (if she exists) bids zero.*

There is a subtlety associated with calculating the incremental returns in my model that does not exist in the standard auction setting. In an all-pay auction, a player's incremental return is exogenous to the bids of the other players—it is simply the value of the asset. Here, payoffs in the economic equilibrium are sensitive to the configuration of territorial control. Hence, in a conflict potentially involving three factions, a faction's expected payoff should it lose depends on its beliefs about the likelihood that each other faction wins, which depend on those factions' strategies. I attend to this issue when providing a complete characterization of equilibrium in Section 4. In this section, I focus on building intuitions for configurations of territorial control and vulnerability in which only two factions will actively fight, which turn out to be the cases of interest.

Suppose there is some pattern of territorial control and vulnerability, ξ, such that either two or three factions $(1, 2$ and $3)$ can fight over the vulnerable territory. Further, assume $\mathrm{IR}_1^\xi \geq \mathrm{IR}_2^\xi (> \mathrm{IR}_3^\xi)$, so faction 1 values winning at least as much as does faction 2, and both factions value winning more than faction 3. Theorem 2.1 indicates that only factions 1 and 2 will invest in conflict with positive probability. Faction 1 invests the realization of a uniform random variable on $[0, \text{IR}_2^\xi]$. With probability $\frac{\text{IR}_2^\xi}{\text{IR}_1^\xi}$, faction 2 does likewise and with the complementary probability, faction 2 remains inactive, ceding the territory to faction 1.

Because faction 2 cedes with positive probability when $\text{IR}_2^\xi < \text{IR}_1^\xi$, a conflict does not always result in observed violence.[12] From an ex ante perspective, the amount of observed violence is a random variable. With probability $1 - \frac{\text{IR}_2^\xi}{\text{IR}_1^\xi}$, v takes the value 0. And with probability $\frac{\text{IR}_2^\xi}{\text{IR}_1^\xi}$, v is the sum of two uniform random variables on $[0, \text{IR}_2^\xi]$ and, so, has a symmetric triangular distribution on $[0, 2\text{IR}_2^\xi]$. Hence, v has a CDF given by

$$\Phi^\xi(v) = \begin{cases} 1 - \frac{\text{IR}_2^\xi}{\text{IR}_1^\xi} + \frac{\text{IR}_2^\xi}{\text{IR}_1^\xi}\left(\frac{v^2}{2(\text{IR}_2^\xi)^2}\right) & \text{if } v \in [0, \text{IR}_2^\xi] \\ \\ 1 - \frac{\text{IR}_2^\xi}{\text{IR}_1^\xi} + \frac{\text{IR}_2^\xi}{\text{IR}_1^\xi}\left(1 - \frac{(2\text{IR}_2^\xi - v)^2}{2(\text{IR}_2^\xi)^2}\right) & \text{if } v \in [\text{IR}_2^\xi, 2\text{IR}_2^\xi]. \end{cases} \tag{1}$$

It is straightforward to calculate expected observed violence and its comparative statics.

Proposition 2.1 *Given a configuration of vulnerability and territorial control, ξ, such that $\text{IR}_1^\xi \geq \text{IR}_2^\xi$ and IR_2^ξ is larger than any other players' incremental return, expected observed violence is*

$$E[v|\xi] = \int_0^{2\text{IR}_2^\xi} v \, d\Phi^\xi(v) = \frac{(\text{IR}_2^\xi)^2}{\text{IR}_1^\xi},$$

which is increasing in IR_2^ξ and decreasing in IR_1^ξ.

Proof. Follows from the analysis in the text. ■

Let's unpack the intuition behind the relationships recorded in Proposition 2.1. As faction 1's incremental return to winning increases, faction 1 becomes more willing to invest in conflict. Were it to do so, this would make the second faction unwilling to fight at all, since it would be so likely to lose. But if the first faction is certain the second faction will not fight, then it has no incentive to invest. To maintain equilibrium, as IR_1^ξ increases, faction 1's increased willingness to invest leads faction 2 to cede more often, which then establishes equilibrium by decreasing faction 1's incentive to invest. Thus, this *scare-off* effect of an increase in IR_1^ξ tends to reduce the expected amount of observed violence by increasing the probability that the territory is ceded.

[12] This is a difference between the application of all-pay auctions to conflict and its application to, say, competition for political influence, where the player that values winning the most makes observable donations regardless of whether any other players doe so (Becker, 1983; Baye, Kovenock and De Vries, 1996; Krishna and Morgan, 1997).

An increase in IR_2^ξ has two effects, both of which tend to increase observed violence. First, as faction 2's incremental return to winning increases, faction 2 becomes less willing to cede the territory. This *anti-scare-off* effect increases expected observed violence by increasing the probability that both factions are active. Second, as faction 2's incremental return increases, faction 2 becomes willing to invest more. This *stakes* effect increases both factions' expected investment and, thus, increases expected observed violence.

Often some factor will simultaneously increase both IR_1^ξ and IR_2^ξ. Such a change can increase or decrease expected observed violence, depending on the relative size of the effects on the two incremental returns. Note, however, that IR_2^ξ increases expected observed violence through two mechanisms—anti-scare-off and stakes—while IR_1^ξ decreases expected observed violence through only one mechanism—scare-off. Hence, if some factor were to change both incremental returns by similar amounts, the effect on IR_2^ξ would dominate. Indeed, in order for the effect on IR_1^ξ to dominate, it must be more than twice as large. To see this, suppose that both incremental returns are strictly increasing, differentiable functions of some parameter θ. Then expected observed violence is decreasing in θ if and only if:

$$\frac{\partial \text{IR}_2^\xi(\theta)/\partial\theta}{\partial \text{IR}_1^\xi(\theta)/\partial\theta} < \frac{\text{IR}_2^\xi(\theta)}{2\text{IR}_1^\xi(\theta)} \leq \frac{1}{2}. \tag{2}$$

3 Economic Equilibrium

The results in Section 2 show how changes to the factions' incremental returns to winning affect the distribution of observed violence. I calculate these incremental returns by computing the rents captured by each faction in the economic equilibrium that follows conflict.

Consider two contiguous territories, i and j, charging prices p_i and p_j. A population member located between i and j at distance x from i will purchase from i rather than purchasing from j or staying home if:

$$p_i + tx \leq p_j + t\left(\frac{1}{6} - x\right) \qquad \text{and} \qquad 1 - p_i - tx \geq 0.$$

The population member who is indifferent between purchasing from i and j is located at distance $x^*_{i,j}$ from i, given by:

$$x^*_{i,j} = \frac{1}{12} + \frac{p_j - p_i}{2t}.$$

Plugging this in and rearranging, this population member will purchase if

$$p_i \leq 2 - p_j - \frac{t}{6}. \tag{3}$$

As long as Condition 3 holds, demand at territory i from population members located between i and j is:[13]

$$D_i(p_i, p_j) = \begin{cases} \frac{N}{6} & \text{if } p_i < p_j - \frac{t}{6} \\ N\left(\frac{1}{12} + \frac{p_j - p_i}{2t}\right) & \text{if } p_i \in \left[p_j - \frac{t}{6}, p_j + \frac{t}{6}\right] \\ 0 & \text{if } p_i > p_j + \frac{t}{6}. \end{cases} \tag{4}$$

Since such games are standard, I relegate the characterization of the economic equilibrium for all relevant configurations to Supplemental Appendix D. To fix ideas, I illustrate two cases. It will be useful to adopt the notation that territory $i+1$ is the territory one letter higher in the alphabet than i, except in the case of F, where $F + 1 = A$.

First suppose there are two symmetric factions, one controlling territories A, B, C and the other controlling territories D, E, F. (I will notate this $3, 3$, since there are two factions, each controlling 3 territories.) If demand is characterized by Equation 4 at some vector of prices, the factions' rents are:

$$\sum_{i=A}^{C} p_i \left[D_i(p_i, p_{i+1}) + D_i(p_i, p_{i-1})\right] \qquad \text{and} \qquad \sum_{i=D}^{F} p_i \left[D_i(p_i, p_{i+1}) + D_i(p_i, p_{i-1})\right].$$

From the first-order conditions, equilibrium prices are

$$p_A^* = p_C^* = p_D^* = p_F^* = \frac{t}{2} \qquad \text{and} \qquad p_B^* = p_E^* = \frac{7t}{12}.$$

Notice several facts. First, prices are higher at interior territories (B and E), reflecting greater market power. Second, for all $i, j \in \{A, B, C, D, E, F\}$, we have $p_i \leq 2 - p_j - \frac{t}{6}$ and each consumer purchases from one of the two territories to which she is closest, so demand is in fact described by Equation 4. Equilibrium rents for each faction are

$$u^{3,3} = \frac{37Nt}{144}.$$

Now suppose there are two factions, one controlling territories A, B, C, D and the other

[13] As shown in the Supplemental Appendix D, Condition 3 always holds in equilibrium.

controlling territories E, F (I denote this $4, 2$). If demand is characterized by Equation 4 at some vector of prices, the large and small factions' rents, respectively, are:

$$\sum_{i=A}^{D} p_i \left[D_i(p_i, p_{i+1}) + D_i(p_i, p_{i-1})\right] \qquad \text{and} \qquad \sum_{i=E}^{F} p_i \left[D_i(p_i, p_{i+1}) + D_i(p_i, p_{i-1})\right].$$

From the first-order conditions, equilibrium prices are

$$p_A^* = p_D^* = \frac{5t}{9} \qquad p_B^* = p_C^* = \frac{13t}{18} \qquad p_E^* = p_F^* = \frac{4t}{9}.$$

Again, for all $i, j \in \{A, B, C, D, E, F\}$, we have $p_i \leq 2 - p_j - \frac{t}{6}$ and each consumer purchases from one of the two territories to which she is closest, so demand is in fact described by Equation 4. The large and small factions' equilibrium rents, respectively, are:

$$u^{\mathbf{4},2} = \frac{109Nt}{324} \qquad u^{4,\mathbf{2}} = \frac{16Nt}{81}.$$

A few points are worth emphasizing. First, prices again are higher at interior territories (B and C). Second, the large faction charges higher prices than the small faction at border territories, reflecting its greater market power. Third, because consolidation leads to higher prices, total rents are higher with unequal factions than with equal factions. That is,

$$u^{\mathbf{4},2} + u^{4,\mathbf{2}} = \frac{173Nt}{324} > \frac{74Nt}{144} = u^{3,3} + u^{3,3}. \tag{5}$$

Table 3 summarizes economic rents for all relevant configurations of territorial control (details in Supplemental Appendix D). Configurations are notated according to the number of territories controlled by each faction. So, for example, $3, 2, 1$ is a configuration with three factions controlling three, two, and one territories, respectively. For payoffs, the relevant faction's number is in bold, so those factions' rents are $u^{\mathbf{3},2,1}$, $u^{3,\mathbf{2},1}$, and $u^{3,2,\mathbf{1}}$, respectively.

4 Number of Factions and Conflict

I now query the effect of the number of factions on violent outcomes. In order to hold all else equal while changing the number of factions, I compare what I refer to as the three *symmetric-connected configurations.* That is, I consider a configuration of territorial control with six factions each of which controls one territory, a configuration with three factions each of which controls two contiguous territories, and a configuration with two factions each

Configuration	Highest Payoff	2nd Highest Payoff	3rd Highest Payoff
$1,1,1,1,1,1$	$u^{1,1,1,1,1,1} = \frac{Nt}{36}$		
$2,1,1,1,1$	$u^{\mathbf{2},1,1,1,1} = \frac{145Nt}{2166}$	$u^{2,\mathbf{1},1,1,1} = \frac{40Nt}{1083}$	$u^{2,1,\mathbf{1},1,1} = \frac{100Nt}{3249}$
$2,2,2$	$u^{2,2,2} = \frac{Nt}{9}$		
$3,2,1$	$u^{\mathbf{3},2,1} = \frac{447{,}343Nt}{2{,}643{,}878}$	$u^{3,\mathbf{2},1} = \frac{298{,}831Nt}{2{,}643{,}876}$	$u^{3,2,\mathbf{1}} = \frac{5041Nt}{73{,}441}$
$3,3$	$u^{3,3} = \frac{37Nt}{144}$		
$4,2$	$u^{\mathbf{4},2} = \frac{109Nt}{324}$	$u^{4,\mathbf{2}} = \frac{16Nt}{81}$	

Table 3.1: Economic rents associated with different configurations of territorial control.

of which controls three contiguous territories.

Six Symmetric-Connected Factions If there are six factions and the defender wins the conflict, the outcome is the status quo. If the defender loses the conflict, then it is eliminated, making a payoff of zero. Hence, the defender's incremental return is

$$\text{IR}_{\text{def}}^{1,1,1,1,1,1} = u^{1,1,1,1,1,1} - 0.$$

There are two attackers—the factions in control of territories on either side of the vulnerable territory. If an attacker faction, j, wins the conflict, it becomes the large faction in a five faction configuration. If it loses the conflict, then one of two things happens: either the defender wins (and j remains in a six faction configuration) or the other attacker wins (and j becomes a small faction bordering the large faction in a five faction configuration). Let π be j's belief about the probability that the defender invests more than the other attacker. Then faction j's incremental return is

$$\text{IR}_{\text{att}}^{1,1,1,1,1,1}(\pi) = u^{\mathbf{2},1,1,1,1} - \pi u^{1,1,1,1,1,1} - (1-\pi)u^{2,\mathbf{1},1,1,1}.$$

Lemma 4.1 shows that the two attackers have strictly higher incremental returns to winning than does the defender, which is important for characterizing equilibrium.

Lemma 4.1 $\text{IR}_{\text{att}}^{1,1,1,1,1,1}(\pi) > \text{IR}_{\text{def}}^{1,1,1,1,1,1}$ *for any* $\pi \in [0,1]$.

Proof. See Appendix A ■

As stated in Theorem 2.1, given that the two attackers' incremental returns are larger than the defender's, the defender invests zero and loses for certain, which implies that $\pi = 0$

in equilibrium. At $\pi = 0$, we have

$$\text{IR}_{\text{att}}^{1,1,1,1,1,1}(0) = u^{\mathbf{2},1,1,1,1} - u^{2,\mathbf{1},1,1,1} = \frac{65Nt}{2166}.$$

Given this, we have the following result.

Proposition 4.1 *When the initial configuration involves six factions, regardless of which territory becomes vulnerable, equilibrium play at the conflict stage is as follows:*

- *The defender invests zero and loses for certain.*
- *The attackers' investments are drawn independently from a uniform distribution on* $\left[0, \frac{65Nt}{2166}\right]$.

The ex ante distribution of observed violence is as in Equation 1, with $\text{IR}_1^{1,1,1,1,1,1} = \text{IR}_2^{1,1,1,1,1,1} = \text{IR}_{\text{att}}^{1,1,1,1,1,1}(0) = \frac{65Nt}{2166}$.

Proof. Follows from Theorem 2.1, Equation 1, and the analysis in the text. ■

Three Symmetric-Connected Factions Now suppose there are three factions, each of which controls two contiguous territories. For any vulnerable territory, only two factions can fight—an attacker and a defender. The defender's incremental return is

$$\text{IR}_{\text{def}}^{2,2,2} = u^{2,2,2} - u^{3,2,\mathbf{1}} = \frac{28,072Nt}{660,969}$$

and the attacker's incremental return is

$$\text{IR}_{\text{att}}^{2,2,2} = u^{\mathbf{3},2,1} - u^{2,2,2} = \frac{51,193Nt}{881,292}.$$

Hence, the equilibrium outcomes follow from the analysis in Section 2.

Proposition 4.2 *When the initial configuration involves three symmetric-connected factions, regardless of which territory becomes vulnerable, equilibrium at the conflict stage is as follows:*

- *With probability* $\frac{112,288}{153,579}$, *the defender's investment is drawn from a uniform distribution on* $\left[0, \frac{28,072Nt}{660,969}\right]$ *and with complementary probability the defender invests zero.*
- *The attacker's investment is drawn independently from a uniform distribution on* $\left[0, \frac{28,072Nt}{660,969}\right]$.

The ex ante distribution of observed violence is as in Equation 1, with $\mathrm{IR}_1^{2,2,2} = \mathrm{IR}_{\text{att}}^{2,2,2} = \frac{51{,}193Nt}{881{,}292}$ *and* $\mathrm{IR}_2^{2,2,2} = \mathrm{IR}_{\text{def}}^{2,2,2} = \frac{28{,}072Nt}{660{,}969}$.

Proof. Follows from Theorem 2.1, Equation 1, and the analysis in the text. ■

Two Symmetric-Connected Factions Now suppose there are two factions, each of which controls three contiguous territories. There are two cases: the vulnerable territory is on a border or is interior. In the latter case, there is no conflict, so consider the former.

As we saw in Condition 5, there are more total rents in the event that the attacker wins. This implies that the attacker's incremental return is larger than the defender's:

$$u^{\mathbf{4},2} + u^{4,\mathbf{2}} > u^{3,3} + u^{3,3} \Rightarrow u^{\mathbf{4},2} - u^{3,3} > u^{3,3} - u^{4,\mathbf{2}}.$$

We can calculate these incremental returns directly. The defender's incremental return is

$$\mathrm{IR}_{\text{def}}^{3,3} = u^{3,3} - u^{4,\mathbf{2}} = \frac{77Nt}{1296},$$

while the attacker's incremental return is

$$\mathrm{IR}_{\text{att}}^{3,3} = u^{\mathbf{4},2} - u^{3,3} = \frac{103Nt}{1296}.$$

Given this, conditional on a border territory being vulnerable, the equilibrium outcome follows from the analysis in Section 2, with the attacker having the higher incremental return. Hence, the ex ante distribution of observed violence is a mixture of an atom on zero with probability mass $\frac{1}{3}$ (the probability an interior territory is vulnerable) and the distribution described in Equation 1 with probability $\frac{2}{3}$.

Proposition 4.3 *When the initial configuration involves two symmetric-connected factions, if the vulnerable territory is interior, there is no conflict. If the vulnerable territory is a border territory, then equilibrium play at the conflict stage is as follows:*

- *With probability* $\frac{77}{103}$ *the defender's investment is drawn from a uniform distribution on* $\left[0, \frac{77Nt}{1296}\right]$ *and with complementary probability the defender invests zero.*

- *The attacker's investment is drawn independently from a uniform distribution on* $\left[0, \frac{77Nt}{1296}\right]$.

Consequently, the ex ante distribution of observed violence is given by the following CDF:

$$\Phi^{3,3}(v) = \begin{cases} \frac{1}{3} + \frac{2}{3} \cdot \frac{26}{103} + \frac{2}{3} \cdot \frac{77}{103} \left(\frac{1296^2 v^2}{77 \cdot 154 N^2 t^2} \right) & \text{if } v \in \left[0, \frac{77Nt}{1296}\right] \\ \frac{1}{3} + \frac{2}{3} \cdot \frac{26}{103} + \frac{2}{3} \cdot \frac{77}{103} \left(1 - \frac{1296^2 \left(\frac{154Nt}{1296} - v \right)^2}{77 \cdot 154 N^2 t^2} \right) & \text{if } v \in \left[\frac{77Nt}{1296}, \frac{154Nt}{1296}\right]. \end{cases}$$

Proof. Follows from the argument in the text. ■

4.1 Factionalization and Violent Outcomes

Given these characterizations, I can now assess the effect of a change in the number of factions on violent outcomes. I decompose the analysis into four parts:

(i) The frequency of observed violence—i.e., the probability that the realization of v is positive.

(ii) The expected intensity of observed violence—i.e., the expectation of v conditional on its realization being positive.

(iii) The variability of observed violence—i.e., the variance of v.

(iv) Expected observed violence—i..e, the unconditional expectation of v.

Frequency of Violence Two factors affect how frequently violence is observed: (*i*) how often the vulnerable territory is ceded by all but one faction and (*ii*) the number of safe territories.

First, consider the case of six factions. In this setting, there are no safe territories. Further, because control is diffuse, there are always two attackers, one on each side of the vulnerable territory. While the defender cedes the territory with certainty, neither attacker is willing to cede, so there is always observed violence.

This is not the case when the factions further consolidate. Comparing the configuration with three symmetric-connected factions to the configuration with two symmetric-connected factions, two things change. On the one hand, Condition 6 below shows that, due to differential changes in attacker's and defender's incremental returns, the defender cedes more often when there are more factions. This tends to increase the frequency of observed violence in more consolidated environments.

$$1 - \frac{\text{IR}^{2,2,2}_{\text{def}}}{\text{IR}^{2,2,2}_{\text{att}}} = \frac{41,291}{153,579} > \frac{26}{103} = 1 - \frac{\text{IR}^{3,3}_{\text{def}}}{\text{IR}^{3,3}_{\text{att}}}. \tag{6}$$

On the other hand, consolidation creates safe territories, reducing opportunities for conflict.

These effects pull in competing directions. The net effect, as formalized in Proposition 4.4, is that factionalization is associated with more frequent observed violence.

Proposition 4.4 *Assume symmetric-connected factions. The frequency with which violence is observed is increasing in the number of factions.*

Proof. See Appendix A. ■

Intensity of Observed Violence Define the expected intensity of observed violence as the expectation of v, conditional on at least two factions making positive investments. While the frequency of observing violence is increasing in the number of factions, the opposite is true of the expected intensity of that violence. Because the economic model has increasing returns, greater factionalization is associated with smaller incremental returns (conflict is lower stakes) and, thus, a lower expected intensity of observed violence.

Proposition 4.5 *Assume symmetric-connected factions. The expected intensity of observed violence—i.e.,* $\mathbb{E}[v|v > 0]$*—is decreasing in the number of factions.*

Proof. See Appendix A. ■

Variance of Observed Violence Increased factionalization leads to more frequent, but less intense, observed violence. This straightforwardly leads to a prediction about the variability of observed violence, i.e., $\text{var}(v)$. The more factions, the less likely are both very low (zero) and very high levels of observed violence. Consequently, factionalization is associated with decreased variance.

Proposition 4.6 *Assume symmetric-connected factions. The variance of observed violence is decreasing in the number of factions.*

Proof. See Appendix A. ■

Expected Observed Violence Given all of these effects, how does the overall expected level of observed violence respond to factionalization? As summarized in Table 4.1, expected observed violence is non-monotone in the number of factions. The reasons are subtle, reflecting the competing effects of factionalization on intensity and frequency. Two key points are worth noting.

Configuration	IR_1	IR_2	Expected Observed Violence
$1,1,1,1,1,1$	$\frac{65Nt}{2166} \approx 0.0300Nt$	$\frac{65Nt}{2166} \approx 0.0300Nt$	$\frac{65Nt}{2166} \approx 0.0300Nt$
$2,2,2$	$\frac{51{,}193}{881{,}292} \approx 0.0581Nt$	$\frac{28{,}072Nt}{660{,}969} \approx 0.0425Nt$	$\frac{3{,}152{,}148{,}736Nt}{101{,}510{,}958{,}051} \approx 0.0311Nt$
$3,3$ (border vulnerable)	$\frac{103Nt}{1296} \approx 0.0795Nt$	$\frac{77Nt}{1296} \approx 0.059Nt$	$\frac{5929Nt}{133{,}488} \approx 0.0444Nt$
$3,3$ (interior vulnerable)	N/A	N/A	0
$3,3$ (ex ante)	N/A	N/A	$\frac{2}{3} \cdot \frac{5929Nt}{133{,}488} \approx 0.0296Nt$

Table 4.1: Expected observed violence as a function of the number of factions.

First, expected observed violence, conditional on a border territory being vulnerable, is monotonically decreasing in the number of factions. Because factionalization reduces market power, it reduces both the attacker's and defender's incremental returns to winning. While this generates competing effects, as suggested by Equation 2, the effect on the defender's incremental return dominates—decreasing the incremental returns leads to a decrease in expected observed violence at border territories.

But now consider the expected observed violence without conditioning on a border territory being vulnerable. There is an additional effect that leads to a non-monotonicity. The consolidation to two factions creates two safe territories so that one-third of the time there is no opportunity for conflict.

Overall, then, equilibrium incentives for investing in conflict decrease as the number of factions increases, but opportunities for conflict increase. As a consequence, from an ex ante perspective, the scenario with only two factions has the lowest expected observed violence even though, conditional on a border territory becoming vulnerable, it has the highest expected observed violence.

Proposition 4.7 *Assume symmetric-connected factions.*

(i) Conditional on a border territory being vulnerable, expected observed violence is decreasing in the number of factions.

(ii) Unconditionally (i.e., allowing for the possibility of interior territories being vulnerable), expected observed violence is non-monotone in the number of factions:

$$\mathbb{E}[v|2,2,2] > \mathbb{E}[v|1,1,1,1,1,1] > \mathbb{E}[v|3,3].$$

Proof. Follows from the argument in the text and calculations reported in Table 4.1. ■

Configuration	Transition Probability if Border Vulnerable	Overall Transition Probability
$1, 1, 1, 1, 1, 1$	1	1
$2, 2, 2$	$\frac{97{,}435}{153{,}579} \approx 0.634$	$\frac{97{,}435}{153{,}579} \approx 0.634$
$3, 3$	$\frac{129}{206} \approx 0.626$	$\frac{2}{3} \cdot \frac{129}{206} \approx 0.417$

Table 4.2: Probability a territory changes hands as a function of the number of factions and vulnerability.

4.2 Factionalization and Stability

Because factionalization affects conflict, it affects the stability of the configuration of territorial control. Here I explore this relationship.

As we have already seen, because there are attackers on both sides in the most highly factionalized environment (i.e., six factions), the vulnerable territory always changes hands—the defender cedes the territory to the two attackers, each of whom wins control with probability one-half. Hence, the most highly factionalized configuration is completely unstable.

In more consolidated configurations, there are two forces at work. First, as was shown in Condition 6, moving from three to two factions decreases scare-off—i.e., conditional on a border territory being vulnerable, the defender is less likely to cede when there are only two factions. This effect is recorded in the second column of Table 4.2, which shows the probability that an attacker wins a conflict, given that a border territory is vulnerable. This decrease in scare-off tends to increase stability under more consolidated configurations.

Second, more highly concentrated territorial control creates safe territories that are not subject to capture. This also tends to increase the stability of more consolidated configurations.

These two effects pull in the same direction. And so, as shown in the third column of Table 4.2 (and formalized in the next result), factionalization tends to decrease stability.

Proposition 4.8 *Assume symmetric-connected factions. The stability of the configuration of territorial control is decreasing in the number of factions.*

Proof. Follows from the argument in the text and calculations reported in Table 4.2. ■

5 Global Comparative Statics

In this section I explore how violent outcomes change as population density or transportation costs change at all territories. In the next section I consider local comparative statics—i.e., changes to population density and transportation costs near one particular territory.

In all three symmetric-connected configurations, both factions' incremental returns are linearly increasing in both transportation costs and population density. Hence, a change to either of those parameters has no effect on scare-off, which is determined by the ratio of the two incremental returns. A change to global transportation costs or population density affects observed violence only through the stakes effect. The value of a territory, and thus the size of the stakes effect, is increasing in both t and N.

These facts have several implications. First, an increase in global transportation costs or global population density is associated with higher expected observed violence. This is consistent with the standard intuition, discussed in the introduction, that an increase in rents leads to an increase in observed violence (Grossman, 1999). Second, an increase in global transportation costs or global population density is associated with an increase in the variance of observed violence because it has no effect on scare-off, but increases the upper bound of the support of the distribution of investments in conflict. Third, an increase in global transportation costs or global population density has no effect on the stability of a factional configuration, since it has no effect on scare-off.

Proposition 5.1 *In any symmetric-connected configuration, expected observed violence and the variance of observed violence are increasing in both N and t. The stability of the configuration is constant in N and t.*

Proof. See Appendix B ■

6 Local Comparative Statics

Now I turn to comparative statics when the change occurs locally at a particular territory. To focus the analysis, I restrict attention to a configuration with two factions, each of which controls three contiguous territories. I ask what happens to observed violence when there are changes to:

(i) the population density surrounding one particular territory, and

(ii) the transportation costs associated with getting to one particular territory.

I consider how observed violence changes when the territory experiencing the population density or transportation cost shock is vulnerable and also what happens when its nearest neighbor controlled by the other faction is vulnerable.

For concreteness, I discuss the case where one faction controls ABC, the other faction controls DEF, and the economic shock is at territory F. I analyze the effects of such shocks when F is vulnerable and when A is vulnerable. I refer to F as the *shocked territory*.

6.1 Local Population Density

Consider a situation in which the population in the sixth of the circle surrounding territory F increases to $\frac{\eta N}{6}$, for some $\eta \in [1,2]$, while the population elsewhere stays as it was. (At $\eta = 1$, this is the baseline model.) I first consider the case where F is vulnerable and then turn to the case where A is vulnerable.

6.1.1 Shocked Territory (F) is Vulnerable

To compute the incremental returns, I need the equilibrium rents from the economic game (as a function of η) in two scenarios: ABC, DEF and $ABCF, DE$.

For a given vector of prices, demand is the same as in Equation 4 at territories B, C, and D but may be changed at A, E, and F. In particular, assuming $p_A \leq 2 - p_j - \frac{t}{6}$, for $j \in \{A, E\}$, demand at territory F from the part of the population between F and j is:

$$D_F(p_F, p_j) = \begin{cases} \frac{(1+\eta)N}{6} & \text{if } p_F \leq p_j - \frac{t}{6} \\ \frac{\eta N}{12} + N\left(\frac{p_j - p_F}{2t}\right) & \text{if } p_F \in \left(p_j - \frac{t}{6}, p_j\right) \\ \eta N\left(\frac{1}{12} + \frac{p_j - p_F}{2t}\right) & \text{if } p_F \in \left(p_j, p_j + \frac{t}{6}\right) \\ 0 & \text{if } p_F \geq p_j + \frac{t}{6}. \end{cases} \tag{7}$$

For territory $j \in \{A, E\}$, demand from the part of the population between j and F is the complement. The economic equilibria are characterized in Supplemental Appendix E.1.

The key facts are illustrated in Figure 6.1. An increase in local population density has two effects on rents. First, there is a direct effect that tends to increase the rents of the faction that ends up with control of F—for a fixed vector of prices, demand at F increases in η. Second, there is an indirect effect—when local population density around F increases, the marginal cost (in terms of foregone demand) associated with a price increase at A or E increases. Consequently, prices at A and E decrease. Since the economic game has complementarities, this results in price decreases at all territories, as illustrated in the upper

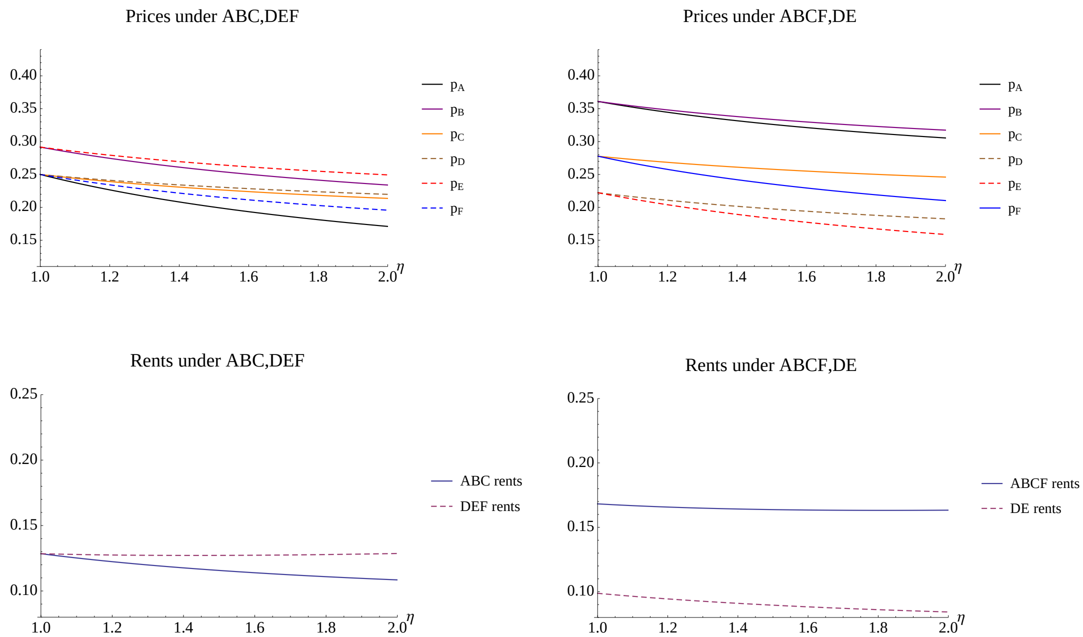

Figure 6.1: Prices and rents as a function of local population density at F (η) for both the ABC, DEF and $ABCF, DE$ configurations. The figures are drawn for $t = 1/2$.

panels of Figure 6.1, which show that, in both configurations, all prices are decreasing in η. This indirect effect tends to decrease both factions' rents in both configurations.

The lower panels of Figure 6.1 illustrate the net effect on rents. Since the faction that does not control territory F at the end of the conflict experiences only the indirect effect, its rents are decreasing in local population density at F. However, for the faction that ends up in control of territory F there are competing effects. Consequently, its rents are non-monotone in local population density at F. (These facts are recorded in Proposition 6.1.)

To understand the effect of local population density on observed violence, we need to understand the incremental returns, which are the difference between the rents associated with each of the configurations above.

For a given value of η, the attacker's incremental return to winning territory F is

$$\mathrm{IR}^{\mathrm{pop}}_{\mathrm{att,\ F}}(\eta) = u^{\mathbf{ABCF},DE}(\eta) - u^{\mathbf{ABC},DEF}(\eta)$$

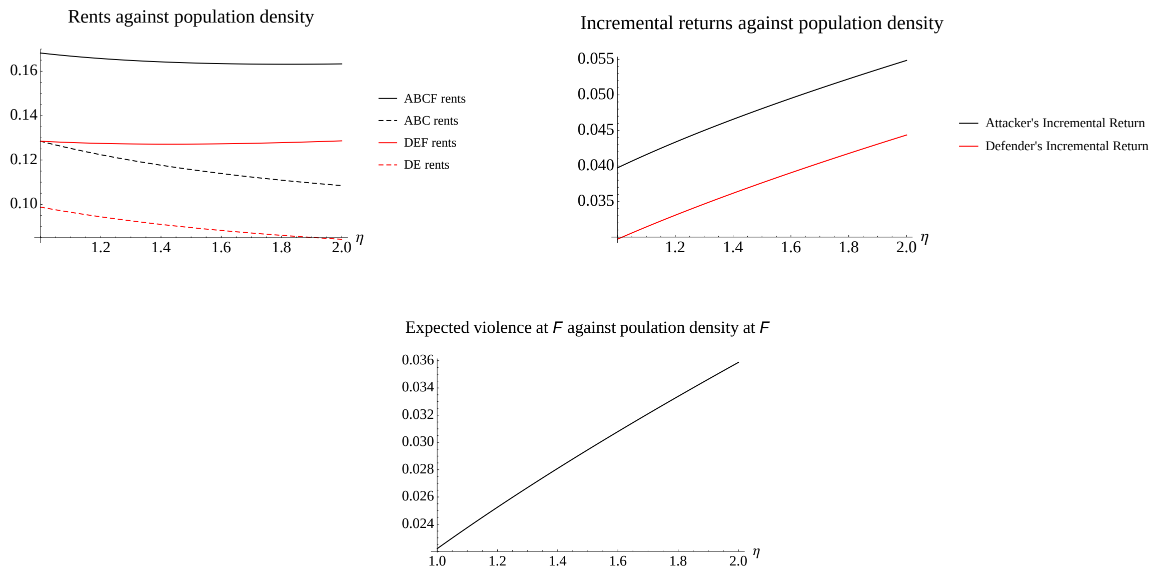

Figure 6.2: Rents are decreasing more slowly in population density at F for the faction that controls F. Hence, when F is vulnerable, incremental returns and expected observed violence at F are increasing in local population density at F. The figures are drawn for $t = 1/2$.

and the defender's incremental return to winning is

$$\mathrm{IR}^{\mathrm{pop}}_{\mathrm{def,\ F}}(\eta) = u^{ABC,\mathbf{DEF}}(\eta) - u^{ABCF,\mathbf{DE}}(\eta).$$

Because of the direct effect, controlling territory F tends to decrease the rate at which a faction's rents decrease in local population density at F (indeed, for high enough η, rents can be increasing). That is, $u^{\mathbf{ABC},DEF}(\eta)$ is decreasing faster in η than is $u^{\mathbf{ABCF},DE}(\eta)$ (indeed, the latter is sometimes increasing) and $u^{ABCF,\mathbf{DE}}(\eta)$ is decreasing faster in η than is $u^{ABC,\mathbf{DEF}}(\eta)$ (again, the latter is sometimes increasing). Hence, both factions' incremental returns to winning the conflict over F are increasing in η. This fact is illustrated in the upper panels of Figure 6.2 (and formalized in Proposition 6.1), where the left-hand panel shows the two components of each faction's incremental return and the right-hand panel shows the incremental returns themselves.

Since both factions' incremental returns are increasing in η, there are competing effects on expected observed violence. As shown in Equation 2, the effect on the smaller incremental return (here the defender's) dominates unless the larger incremental return changes a lot more, which is not the case here. Hence, when conflict is over F, an increase in local population density at F increases incremental returns and expected observed violence, even when it decreases all factions' rents. These facts are illustrated in the lower panel of Figure 6.2 and formalized in the next result.

Proposition 6.1 *Suppose there are two symmetric-connected factions. Moreover, suppose the population on the sixth of the circle with the vulnerable territory at its center is of mass* $\frac{\eta N}{6}$ *for some* $\eta \in [1, 2]$*, while population elsewhere on the circle remains fixed:*

(i) Regardless of what happens at the conflict stage:

- *Rents for the faction that does not end up with control of the vulnerable territory are decreasing in* η*.*
- *Rents for the faction that ends up with control of the vulnerable territory are decreasing in* η *at* $\eta = 1$*, increasing in* η *at* $\eta = 2$*, and strictly convex in* η*.*

(ii) Both factions' incremental returns to winning the conflict over a border territory are increasing in η*.*

(iii) Expected observed violence is increasing in η*.*

Proof. See Appendix C.1. ■

The model returns two noteworthy results here. First, an increase in the size of a local market can be associated with a decrease in profits for all factions. Second, even a change in local population density that is associated with a decrease in both factions' rents is associated with an increase in expected observed violence. Hence, contrary to the standard intuitions discussed in Section 5, rents and observed violence need not be positively associated when the source of variation is a local economic shock.[14]

6.1.2 Shocked Territory's Neighbor (A) is Vulnerable

Continue to consider a shock to population density at F, but suppose territory A is vulnerable. To compute the incremental returns, I need the equilibrium rents from the economic game (as a function of η) in two scenarios: ABC, DEF and $BC, ADEF$. Again, I characterize the economic equilibria in Supplemental Appendix E.1.

Figure 6.3 illustrates the key facts. As we've already seen, when different factions control territories A and F (as in the left-hand panel of Figure 6.3), there is an important

[14] Of course, local rents extracted only from territory F are increasing in population density at F, but overall rents need not be.

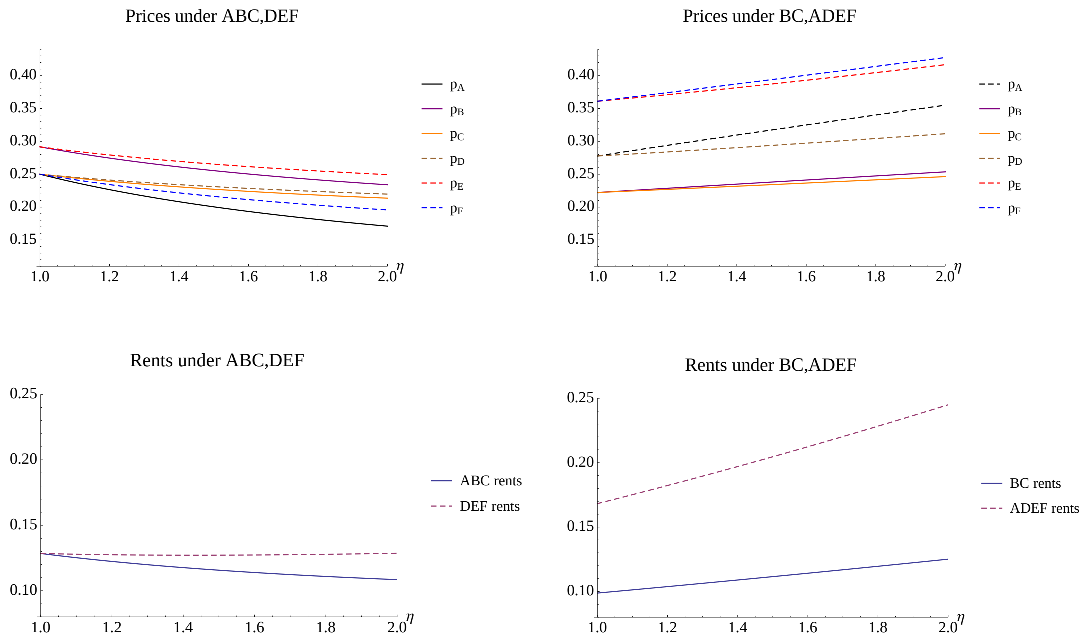

Figure 6.3: Prices and rents as a function of local population density at F (η) for both the ABC, DEF and $BC, ADEF$ configurations. The figures are drawn for $t = 1/2$.

indirect effect of the population shock—as population density at F increases, it becomes more tempting to lower prices at A, which leads to a diminution in prices at all territories. In contrast, when the faction that controls F also controls A and E (as in the right-hand panel of Figure 6.3) this indirect effect is no longer important because whichever territory the consumers surrounding F buy from, they are customers of the same faction. Consequently, when the attacker wins (so the same faction controls territories A, E, and F), the direct effect dominates—as population density at F increases, the marginal benefit of raising prices at F increases, and both factions' rents increase.

This intuition will help us understand the effect of a population density shock at F on the incremental returns to winning territory A. For a given value of η, the attacker's incremental return to winning territory A is

$$\text{IR}^{\text{pop}}_{\text{att, A}}(\eta) = u^{BC,\mathbf{ADEF}}(\eta) - u^{ABC,\mathbf{DEF}}(\eta),$$

and the defender's incremental return is

$$\text{IR}^{\text{pop}}_{\text{def, A}}(\eta) = u^{\mathbf{ABC},DEF}(\eta) - u^{\mathbf{BC},ADEF}(\eta).$$

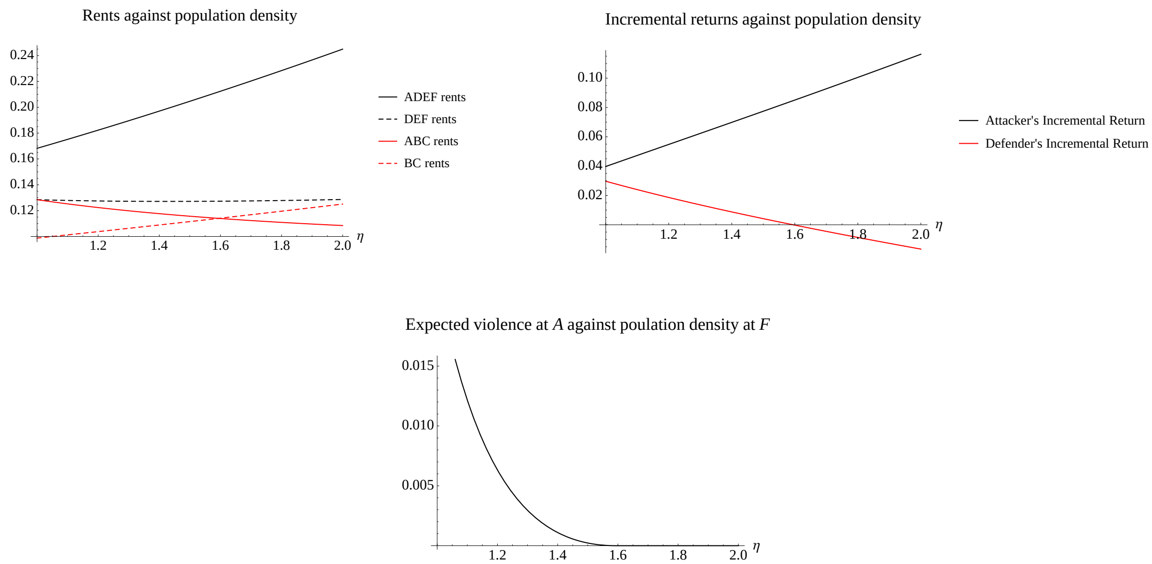

Figure 6.4: When conflict is over territory A, the attacker's incremental return is increasing in local population density at F and the defender's incremental return is decreasing in local population density at F. Moreover, for η sufficiently large, the defender's incremental return is negative. Hence, expected observed violence is decreasing in η and, for η sufficiently large, expected observed violence is zero. The figures are drawn for $t = 1/2$.

The reversal of the effect of η on prices and rents in the event that the attacker wins (so that A, E, and F are unified) has interesting implications for the incremental returns and observed violence. The defender's rents are decreasing in η if she wins the fight, but increasing in η if she loses the fight. As such, the defender's incremental return is decreasing in η. Similarly, the attacker's is increasing. Since the effect on the defender's incremental return dominates, expected observed violence is decreasing in η. Further, if η is sufficiently large, the defender's incremental return is negative—she would prefer to cede territory A to the other faction because doing so leads to increased prices and profits at her remaining territories that more than compensate for the loss of territory A—so observed violence drops to zero. These intuitions are illustrated in Figure 6.4 and made formal in the next result.

Proposition 6.2 *Suppose there are two symmetric-connected factions. Moreover, suppose the population on the sixth of the circle with territory j at its center is of mass $\frac{\eta N}{6}$ for some $\eta \in [1, 2]$, while the population elsewhere on the circle remains fixed. If the territory contiguous with j and controlled by the other faction is vulnerable:*

(i) The incremental returns to winning are increasing in η for the attacker and decreasing

in η for the defender.

(ii) There is a critical threshold $\hat{\eta} \in (1, 2)$ such that the defender's incremental return is positive for all $\eta \in [1, \hat{\eta})$ and negative for all $\eta \in (\hat{\eta}, 2)$.

(iii) Expected observed violence is strictly decreasing in η for $\eta < \hat{\eta}$ and is zero for $\eta \geq \hat{\eta}$.

Proof. See Appendix C.1. ∎

This spillover effect of local population density shocks at F on observed violence at A is a novel testable hypothesis in its own right. In addition, in Section 6.3 I explore its implications for the empirical literature on the causal effects of economic shocks on conflict.

6.2 Local Transportation Costs

Now consider a situation in which the the transportation costs for getting to territory F increase from t to τt for some $\tau \in [1, 2]$. I first consider the case where F is vulnerable and then turn to the case where A is vulnerable.

6.2.1 Shocked Territory (F) is Vulnerable

To compute the incremental returns, I need the equilibrium rents from the economic game (as a function of τ) in two scenarios: ABC, DEF and $ABCF, DE$.

For a given vector of prices, demand is the same as in Equation 4 at territories B, C, and D but it may be changed at A, E, and F. Fix a vector of prices. As long as $p_F \leq \frac{\tau+1}{\tau} - \frac{p_A}{\tau} - \frac{t}{6\tau}$, for $j \in \{A, E\}$, demand at territory F from the part of the population between F and j is:

$$D_F(p_F, p_j) = \begin{cases} \frac{N}{6} & \text{if } p_F \leq p_j - \frac{\tau t}{6} \\ N\left(\frac{1}{6(\tau+1)} + \frac{p_j - p_F}{t(\tau+1)}\right) & \text{if } p_F \in \left(p_j - \frac{\tau t}{6}, p_j + \frac{\tau t}{6}\right) \\ 0 & \text{if } p_F \geq p_j + \frac{\tau t}{6}. \end{cases} \tag{8}$$

For territory $j \in \{A, E\}$, demand from the population between j and F is the complement. The economic equilibria are characterized in Supplemental Appendix E.2.

The key facts are illustrated in Figure 6.5. An increase in local transportation costs at F has two effects. First, there is a direct effect that tends to reduce the rents of the faction that ends up with control over F—for a fixed vector of prices, when local transportation costs at F go up, demand at F goes down. Second, there is an indirect effect—when local transportation costs at F go up, the marginal cost (in terms of foregone demand) associated

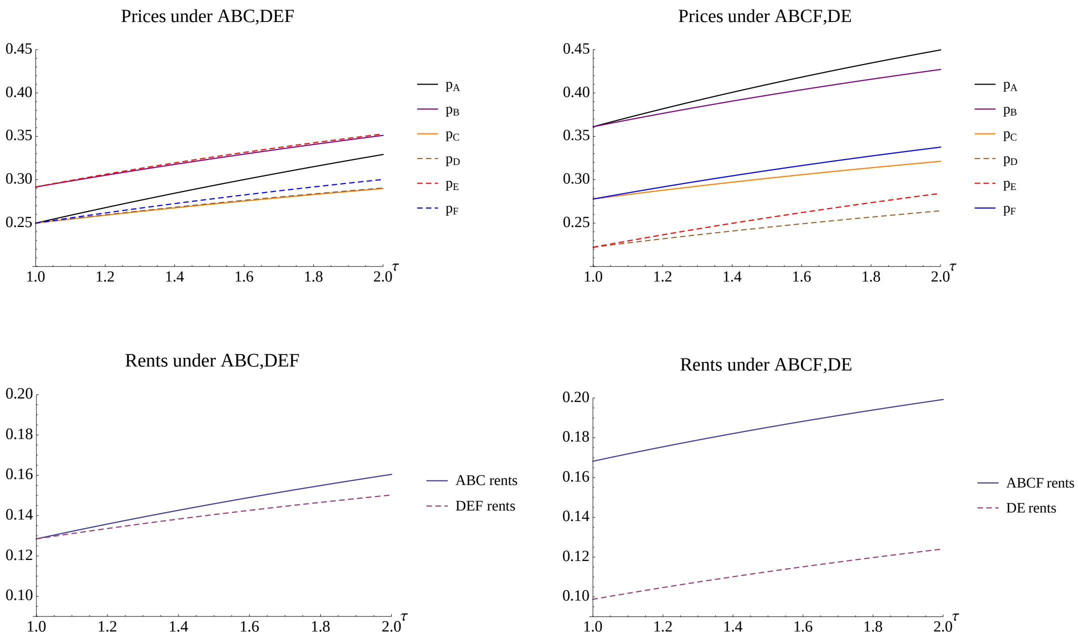

Figure 6.5: When conflict is over territory F, prices and rents are increasing in local transportation costs at F (τ) for both the ABC, DEF and $ABCF, DE$ configurations. The figures are drawn for $t = 1/2$.

with a price increase at A or E goes down. Consequently, prices at A and E increase. Since the economic game has complementarities, this results in price increases at all territories, as illustrated in the upper two panels of Figure 6.5, which show that in both configurations, all prices are increasing in τ. This indirect effect on prices tends to increase both factions' rents in both configurations. Moreover, as illustrated in the two lower panels of Figure 6.5 (and formalized in Proposition 6.3), the indirect effect dominates—on net, both factions' rents are increasing in local transportation costs at F in both configurations.

The fact that rents are increasing in local transportation costs does not tell us what happens to expected observed violence. For that, we need to understand the incremental returns. For a given τ, the attacker's incremental return to winning territory F is

$$\mathrm{IR}^{\mathrm{trans}}_{\mathrm{att,\ F}}(\tau) = u^{\mathbf{ABCF},DE}(\tau) - u^{\mathbf{ABC},DEF}(\tau)$$

and the defender's incremental return is

$$\mathrm{IR}^{\mathrm{trans}}_{\mathrm{def,\ F}}(\tau) = u^{ABC,\mathbf{DEF}}(\tau) - u^{ABCF,\mathbf{DE}}(\tau).$$

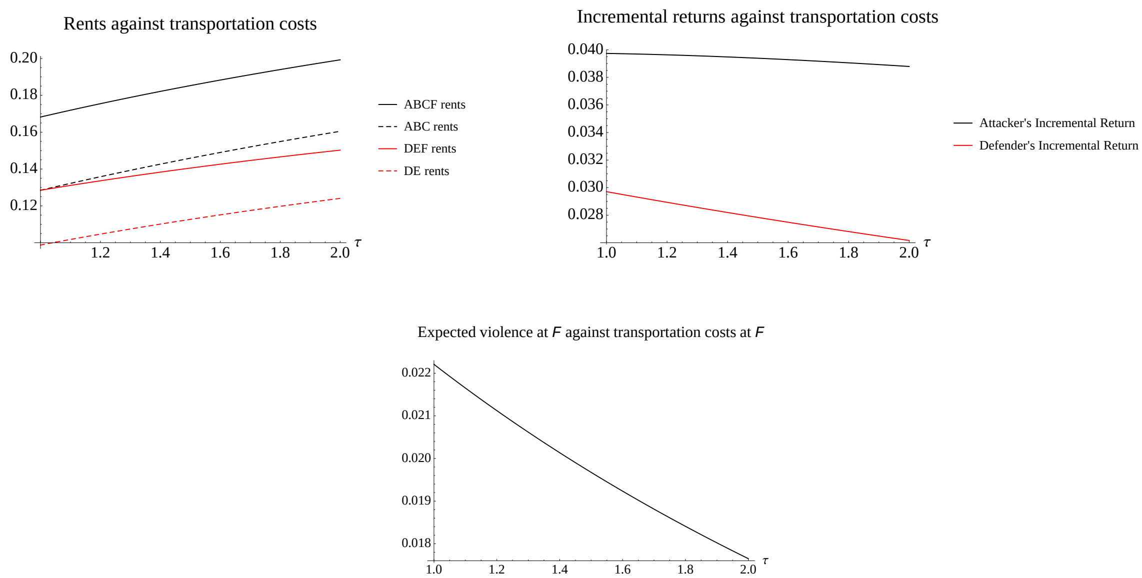

Figure 6.6: Rents are increasing more slowly in transportation costs at F for the faction that controls F. Hence, when F is vulnerable, incremental returns and expected observed violence are decreasing in local transportation costs at F. The figures are drawn for $t = 1/2$.

These incremental returns reveal a key intuition. Because of the direct effect, controlling territory F decreases the rate at which a faction's rents increase in local transportation costs. That is, $u^{\mathbf{ABC},DEF}(\tau)$ is increasing faster in τ than is $u^{\mathbf{ABCF},DE}(\tau)$ and $u^{ABCF,\mathbf{DE}}(\tau)$ is increasing faster in τ than is $u^{ABC,\mathbf{DEF}}(\tau)$. Hence, although both factions' rents are increasing in τ, their incremental returns to winning the conflict over F are decreasing in τ. This fact is illustrated in Figure 6.6 (and formalized in Proposition 6.3), where the upper left-hand panel shows the two components of each faction's incremental return and the upper right-hand panel shows the incremental returns themselves.

Since both factions' incremental returns are increasing in τ, as shown in Equation 2, the effect on the smaller incremental return (here the defender's) dominates unless the larger incremental return changes a lot more, which is not the case here. Hence, as illustrated in Figure 6.6, the effect of an increase in local transportation costs at the vulnerable territory is to increase rents, but decrease incremental returns and expected observed violence. These facts are formalized in the next result.

Proposition 6.3 *Suppose there are two symmetric-connected factions. Moreover, suppose the transportation costs associated with the vulnerable territory are τt for some $\tau \in [1, 2]$:*

(i) Regardless of what happens at the conflict stage, both factions' rents at the economic stage are increasing in τ.

(ii) When the vulnerable territory is a border territory, both factions' incremental returns to winning the conflict are decreasing in τ.

(iii) Expected observed violence is decreasing in τ.

Proof. See Appendix C.2. ■

This result is surprising in light of the hypothesized positive association between rents and observed violence (discussed in Section 5) which motivates much of the empirical literature. Here, with respect to local transportation costs, exactly the opposite holds—an economic shock that increases overall rents is associated with decreased incremental returns and decreased expected observed violence at the shocked territory.[15]

6.2.2 Shocked Territory's Neighbor (A) is Vulnerable

Continue to consider a shock to transportation costs at F, but now suppose territory A is vulnerable. To compute the incremental returns, I need the equilibrium rents from the economic game (as a function of τ) in two scenarios: ABC, DEF and $BC, ADEF$. Again, the economic equilibrium is characterized in Supplemental Appendix E.2.

The key facts are illustrated in Figure 6.7. The intuition when the defender wins is the same as above. But the case where the attacker wins reveals an important contrast between the case where F is vulnerable and the case where A is vulnerable. As shown in the right-hand panels of Figure 6.7, if the attacker wins control of territory A, both prices and rents become significantly less responsive to changes in transportation costs at F. Further, some prices are now decreasing in τ. Both of these facts are intuitive. When the attacking faction wins control of A, territory F no longer faces competition from any territory controlled by the other faction. Consequently, a change to transportation costs at F has little effect because the faction in control of F is relatively unconcerned about whether the consumers surrounding F transact at F or at one of its neighbors.

What does this imply for the incremental returns? For a given value of τ, the attacker's incremental return to winning territory A is

$$\text{IR}^{\text{trans}}_{\text{att, A}}(\tau) = u^{BC,\mathbf{ADEF}}(\tau) - u^{ABC,\mathbf{DEF}}(\tau)$$

[15] Similarly to the case of local population density, local rents extracted only from territory F are decreasing in transportation costs at F, but overall rents are not.

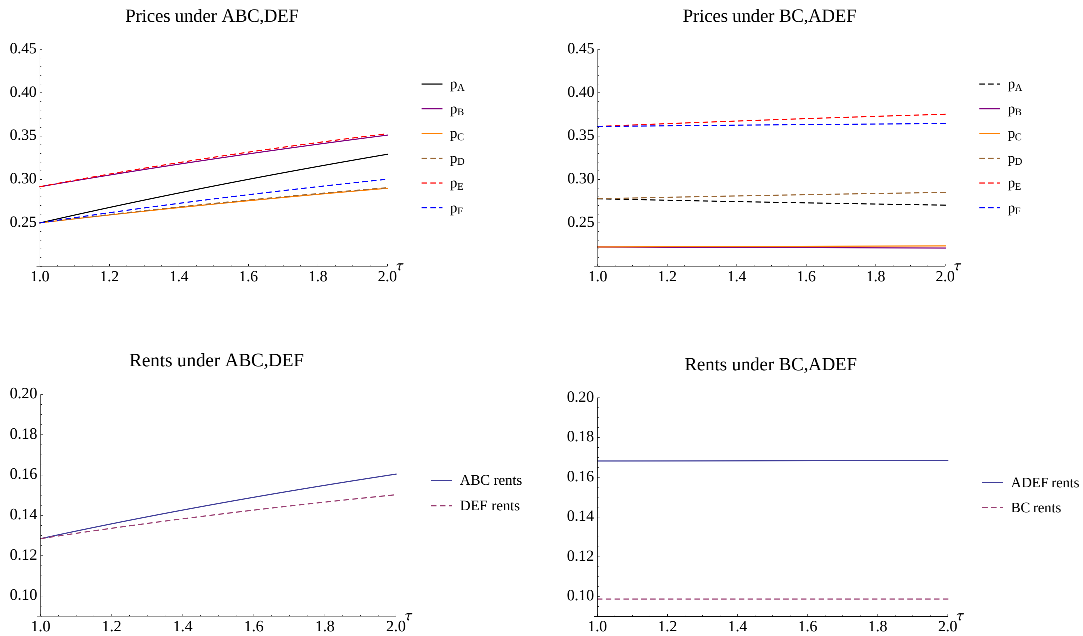

Figure 6.7: Prices and rents, as a function of local transportation costs (τ) at F, for both the ABC, DEF and $BC, ADEF$ configurations. The figures are drawn for the case of $t = 1/2$.

and the defender's incremental return is

$$\mathrm{IR}^{\text{trans}}_{\text{def, A}}(\tau) = u^{\mathbf{ABC},DEF}(\tau) - u^{\mathbf{BC},ADEF}(\tau).$$

The fact that rents are relatively unresponsive to transportation costs if the attacker wins ($BC, ADEF$) means that the effect of transportation costs at F on incremental returns is driven by their effect when the defender wins (i.e., ABC, DEF). As we've already seen, in that scenario, both factions' rents are increasing in transportation costs at F. Hence, the attacker's incremental return is decreasing in transportation costs at F, while the defender's incremental return is increasing in transportation costs at F. These intuitions are illustrated in the upper two panels of Figure 6.8.

What this implies about expected observed violence depends on whether the attacker's or defender's incremental return is larger. If the attacker's incremental return is larger, expected observed violence is increasing in local transportation costs at F. If the defender's incremental return is larger, expected observed violence is decreasing in local transportation costs at F. The upper right-hand panel of Figure 6.8 shows that the incremental returns cross. For low values of τ, the attacker has the higher incremental return and for high

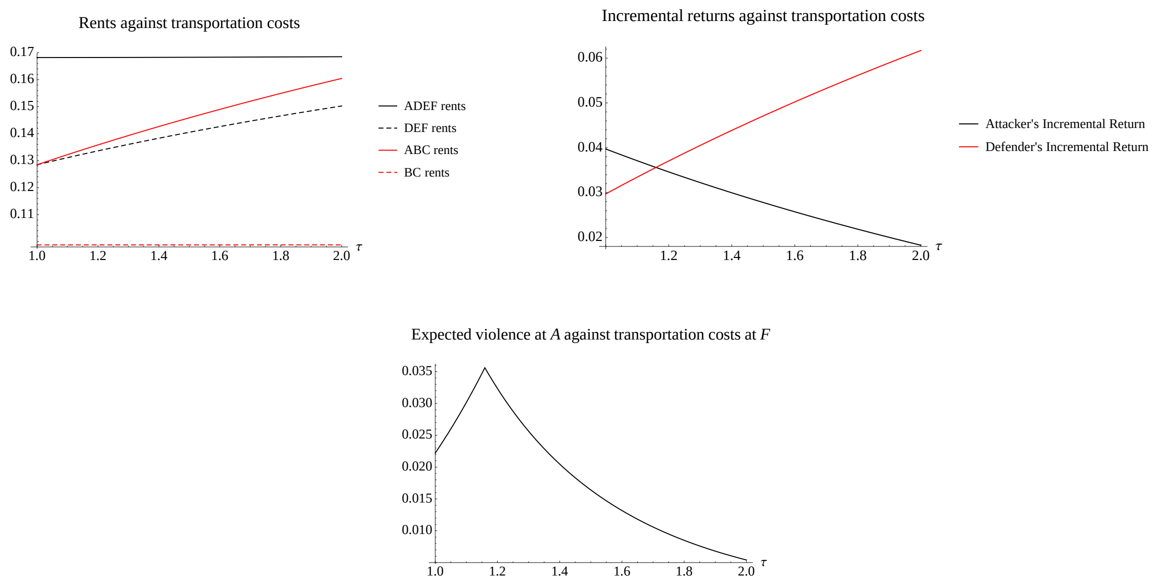

Figure 6.8: When conflict is over territory A, the attacker's incremental return is decreasing in local transportation costs at F and the defender's incremental return is increasing in local transportation costs at F. As a result, expected observed violence is non-monotone in local transportation costs at F. The figures are drawn for $t = 1/2$.

values of τ, the defender has the higher incremental return. This creates a non-monotonic relationship between local transportation costs at F and expected observed violence at A which is illustrated in the lower panel of Figure 6.8 and formalized in the next result.

Proposition 6.4 *Suppose there are two symmetric-connected factions. Let the transportation costs associated with a border territory j be τt for some $\tau \in [1, 2]$. If the territory contiguous with j and controlled by the other faction is vulnerable:*

(i) The incremental returns to winning are increasing in τ for the defender and decreasing in τ for the attacker.

(ii) Expected observed violence is non-monotone in τ. In particular, there is a critical threshold $\hat{\tau} \in (1, 2)$ such that expected observed violence is increasing in τ for $\tau \in [1, \hat{\tau})$ and decreasing in τ for $\tau \in (\hat{\tau}, 2]$.

Proof. See Appendix C.2. ■

This non-monotonic effect of a shock to transportation costs at territory F on observed violence at A is a novel testable hypothesis in its own right. In addition, in Section 6.3 I explore its implications for the empirical literature on the causal effects of economic shocks on conflict.

6.3 Implications for Empirical Work

These results on the effects of local economic shocks on observed violence have a variety of implications for empirical scholarship. Of course, each comparative static can be interpreted as a testable hypothesis regarding the effects of changes to local market conditions on observed violence. And I have already emphasized the contrast between the global and local comparative statics. But there are also implications that come from thinking about the effects of an economic shock at F on observed violence at F versus on observed violence at A.

A common identification strategy in the empirical literature is to estimate the effects of a local economic shock on observed violence within a difference-in-differences framework.[16] The idea is as follows. The investigator identifies a shock—from weather, a natural disaster, world commodity prices, etc.—to the economy of territory i at time t. The treatment effect of interest is the change in observed violence at territory i following the shock at time t. There is concern that other factors that affect observed violence may have changed at the same time that territory i experienced the shock. To isolate the effect of the shock, the researcher studies the change in observed violence at territory i from t to $t+1$ relative to the change in observed violence at nearby localities from t to $t+1$. Under a parallel trends assumption, this difference-in-differences strategy identifies the causal effect of the economic shock at territory i on observed violence at territory i (Angrist and Pischke, 2008).

The results in this section suggest that, in a political economy like the one modeled here, the parallel trends assumption does not hold. Moreover, the direction of the resulting bias depends on the type and magnitude of the shock. Why is this the case?

In the model, the true effect of a shock at F on observed violence at F is given by the comparative statics illustrated in the lower panels of Figures 6.2 and 6.6 (and formalized in Propositions 6.1 and 6.3). But, as illustrated in the lower panels of Figures 6.4 and 6.8 (and formalized in Propositions 6.2 and 6.4) there is also an effect of a shock at F on observed violence at A. A difference-in-differences approach that uses changes in observed violence at A as the baseline against which to compare the effects of a shock at F on observed violence

[16] See, among others, Deininger (2003); Angrist and Kugler (2008); Brückner and Ciccone (2010); Hidalgo et al. (2010); Besley and Persson (2011); Berman, Shapiro and Felter (2011); Dube and Vargas (2013); Bazzi and Blattman (2014); Dube, García-Ponce and Thom (2014); Maystadt and Ecker (2014); Mitra and Ray (2014).

at F is estimating the difference between these two effects of the shock. This quantity is not the true effect of the shock at F on observed violence at F.

We can use the theoretical model to learn about the resulting bias. Since the model has different implications for the two types of shocks, I take them in turn.

6.3.1 Transportation Cost Shocks

Write the expected level of observed violence at territory i given a shock of size τ to the transportation costs at territory j as:

$$\mathbb{E}[v_i|j,\tau].$$

The true effect on expected observed violence at i of a shock of size τ to transportation costs at j is

$$\delta_i(j,\tau) = \mathbb{E}[v_i|j,\tau] - \mathbb{E}[v_i|j,1].$$

Difference-in-differences estimates the difference between the change in expected observed violence at F and the change in expected observed violence at A following a shock of size τ at F:

$$\Delta_{F,A}(F,\tau) = \delta_F(F,\tau) - \delta_A(F,\tau).$$

As we've seen, a shock to transportation costs at F decreases expected observed violence at F, but has a non-monotone effect at A. Because of this violation of parallel trends, difference-in-differences does not recover the true effect of the shock at F on expected observed violence at F. More disturbingly, the direction of the bias depends on the size of the shock. For small shocks (so the effect of τ at A is positive), difference-in-differences underestimates the effect. For larger shocks (so the effect of τ at A is negative), difference-in-differences overestimates the effect. Thus, not only does difference-in-differences not estimate the causal effect, the sign of the bias cannot be known by the econometrician.

These facts are illustrated in Figure 6.9. The left-hand panel shows the effect of a shock of size τ to transportation costs at F on expected observed violence at F ($\delta_F(F,\tau)$), on expected observed violence at A ($\delta_A(F,\tau)$), and on the difference-in-differences ($\Delta_{F,A}(F,\tau)$). The right-hand side shows that the sign of the bias ($\Delta(\tau)-\delta_F(\tau)$) can be positive or negative, depending on the size of the shock.

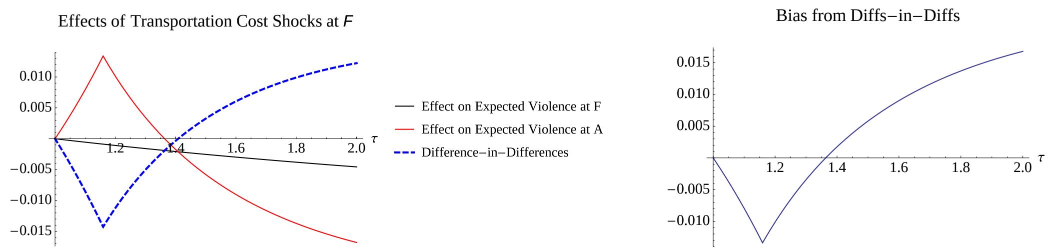

Figure 6.9: Difference-in-differences is biased for the effect of a local transportation cost shock on observed local violence. Both the sign and magnitude of the bias depend on the size of the shock. The figure is drawn for $t = 1/2$.

6.3.2 Population Density

Write the expected level of observed violence at territory i given a shock of size η to the population density at j as

$$\mathbb{E}[v_i|j,\eta].$$

The effect on expected observed violence at i of a shock of size η to population density at j is

$$\delta_i(j,\eta) = \mathbb{E}[v_i|j,\eta] - \mathbb{E}[v_i|j,1].$$

Again, difference-in-differences estimates the difference between the change at F and the change at A following a shock of size η at F:

$$\Delta_{F,A}(F,\eta) = \delta_F(F,\eta) - \delta_A(F,\eta).$$

As we've already seen, a shock to population density at F increases expected observed violence at F and decreases expected observed violence at A. Again, because of the violation of parallel trends, difference-in-differences does not recover the true effect of a shock at F on expected observed violence at F. But, unlike the case of transportation cost shocks, here we know the sign of the bias. Unfortunately, because shocks at F decrease expected observed violence at A, the model predicts that difference-in-differences overestimates the effect of a shock at F on expected observed violence at F.

This fact is illustrated in Figure 6.10. The left-hand panel shows the effect of a shock to population density at F of size η on expected observed violence at F ($\delta_F(F,\eta)$), on expected observed violence at A ($\delta_A(F,\eta)$), and on the difference-in-differences ($\Delta_{F,A}(F,\eta)$). The right-hand side shows that the bias associated with the difference-in-differences ($\Delta_{F,A}(F,\eta)-$

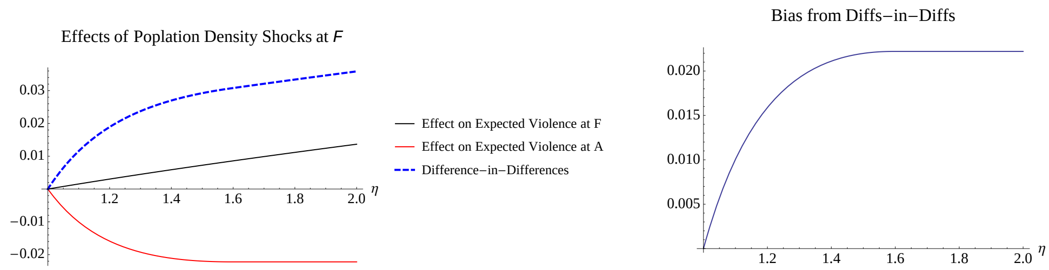

Figure 6.10: Difference-in-differences overestimates the effect of a local population density shock on local observed violence. The figure is drawn for $t = 1/2$.

$\delta_F(F, \eta))$ is positive and its magnitude is increasing in the size of the shock (until the shock gets large enough that there is no longer any fighting over A at which point the bias is flat).

7 Conclusion

I study a model of armed factions fighting over control of territories from which they endogenously extract economic rents. The analysis, building on canonical models of both conflict and spatial price competition, yields several results worth reemphasizing.

First, the effect of changes to market conditions on the distribution of observed violence depends on whether those changes are local or global. Most of the modern empirical literature exploits local variation. Yet, the model's predictions about the effects of local changes are different from the conventional hypotheses (which are more similar to the model's predictions regarding global changes). In particular, the model predicts that changes to local economic conditions (transportation costs or population density) that increase factions' overall rents are associated with a decrease in expected observed violence.

Second, local economic shocks have spillover effects on observed violence at neighboring territories. Consequently, the model suggests that a difference-in-differences research design, which has become a workhorse in the empirical literature, produces biased estimates of the effect of economic shocks on observed violence. In the case of shocks to local population density (market size), the bias is always positive, so that difference-in-differences leads to systematic overestimates of the true effect. In the case of shocks to local transportation costs (market power), the sign of the bias depends on the magnitude of the shock and, thus, probably cannot be known by the econometrician.

Both the divergence between the local and the global comparative statics, and the implications of the local comparative statics for difference-in-differences approaches, highlight an important complementarity between identification-oriented, micro-empirical scholarship on conflict and theoretical models within which we can think about the sources of variation used in such studies.

Third, qualitative accounts and conventional wisdom suggest that an increase in the number of armed factions leads to an increase in observed violence. Here, the predicted relationship is more nuanced. An increase in the number of factions does lead to an increase in the frequency of observed violence. However, when violence occurs, the more factions, the less intense it is. Highly factionalized environments, then, are characterized by frequent, low-level conflict and instability of the pattern of territorial control. Consolidated environments are characterized by infrequent, high-level conflict and stability of the pattern of territorial control. The overall expected amount of observed violence is non-monotone in the number of factions.

Finally, the model highlights a conceptual point. Typically, models of conflict have taken the returns to winning to be exogenous to conflict outcomes. The results here only arise because conflict outcomes feed back into economic behavior, which affects the returns to winning the conflict. Hence, the model demonstrates the importance of a political economy approach to the study of conflict that takes seriously this endogenous interaction between economic and conflict behavior.

A Factionalization

Proof of Lemma 4.1. First note that

$$u^{\mathbf{2},1,1,1,1} = \frac{145t}{2166} > \frac{t}{36} = u^{1,1,1,1,1,1}.$$

Hence, $\text{IR}_{\text{att}}^{1,1,1,1,1,1}(\pi)$ is minimized at $\pi = 0$, where we have

$$\text{IR}_{\text{att}}^{1,1,1,1,1,1}(0) = \frac{65t}{2166} > \frac{t}{36} = \text{IR}_{\text{def}}^{1,1,1,1,1,1}.$$

■

Proof of Proposition 4.4. It follows from Proposition 4.1 that the probability of observing violence with six factions is 1.

With three symmetric-connected factions, there are no safe territories. Thus, from Proposition 4.2, the probability of observing violence is

$$\frac{\text{IR}_{\text{def}}^{2,2,2}}{\text{IR}_{\text{att}}^{2,2,2}} = \frac{112,288}{153,579} < 1.$$

With two symmetric-connected factions, conditional on a border territory being vulnerable, the probability of observing violence is

$$\frac{\text{IR}_{\text{def}}^{3,3}}{\text{IR}_{\text{att}}^{3,3}} = \frac{77}{103}.$$

A border territory is vulnerable with probability 2/3. In the other 1/3 of cases, the probability of observing violence is zero. Hence, the overall probability of observing violence with two symmetric-connected factions is

$$\frac{2}{3} \cdot \frac{77}{103} = \frac{154}{309} < \frac{112,288}{153,579}.$$

■

Proof of Proposition 4.5. In all symmetric-connected configurations, $\mathbb{E}[v|v > 0]$ is the incremental return of the faction that values winning the second most. From Propositions 4.1, 4.2, and 4.3, those incremental returns are:

$$\text{IR}_{\text{att}}^{1,1,1,1,1,1}(0) = \frac{65Nt}{2166} < \text{IR}_{\text{def}}^{2,2,2} = \frac{28,072Nt}{660,969} < \text{IR}_{\text{def}}^{3,3} = \frac{77Nt}{1296}.$$

■

Proof of Proposition 4.6.

A random variable whose distribution places mass α on zero and mass $1-\alpha$ on a symmetric triangular distribution on $[0,b]$ has variance:

$$\left[\int_0^{\frac{b}{2}} x^2 \cdot \frac{(1-\alpha)x}{b^2}\,dx + \int_{\frac{b}{2}}^{b} x^2 \cdot \frac{(1-\alpha)(b-x)}{b^2}\,dx\right] - \left[\int_0^{\frac{b}{2}} x \cdot \frac{(1-\alpha)x}{b^2}\,dx + \int_{\frac{b}{2}}^{b} x \cdot \frac{(1-\alpha)(b-x)}{b^2}\,dx\right]^2 = \frac{(1+5\alpha-6\alpha^2)b^2}{24}. \quad (9)$$

In the case of 6 factions, observed violence is such a random variable with $\alpha = 0$ and $b = 2\text{IR}^{1,1,1,1,1,1}_{\text{att}}(0)$. Plugging these into Equation 9 yields

$$\text{var}[v|1,1,1,1,1,1] = \frac{4225N^2t^2}{28,149,336} \approx 0.00015N^2t^2.$$

In the case of 3 symmetric-connected factions, observed violence is such a random variable with $\alpha = 1 - \frac{\text{IR}^{2,2,2}_{\text{def}}}{\text{IR}^{2,2,2}_{\text{att}}}$ and $b = 2\text{IR}^{2,2,2}_{\text{def}}$. Plugging these into Equation 9 yields

$$\text{var}[v|2,2,2] = \frac{5,918,682,193,315,302,400N^2t^2}{10,304,474,604,431,881,718,601} \approx 0.00057N^2t^2$$

In the case of 2 symmetric-connected factions, observed violence is such a random variable with $\alpha = \frac{1}{3} + \frac{2}{3}\left(1 - \frac{\text{IR}^{3,3}_{\text{def}}}{\text{IR}^{3,3}_{\text{att}}}\right)$ and $b = 2\text{IR}^{3,3}_{\text{def}}$. Plugging these into Equation 9 yields

$$\text{var}[v|3,3] = \frac{188,548,129N^2t^2}{160,371,415,296} \approx 0.00118N^2t^2.$$

■

B Global Comparative Statics

Proof of Proposition 5.1.

The results on expected observed violence follow directly from calculations reported in Table 4.1. The results on variance follow direction from calculations in the proof of Proposition 4.6.

The probability of transitioning from 6 factions to 5 factions is 1, which is constant in t and N. The probability of transitioning from $2, 2, 2$ to $3, 2, 1$ is

$$\frac{\mathrm{IR}^{2,2,2}_{\text{def}}}{\mathrm{IR}^{2,2,2}_{\text{att}}} \cdot \frac{1}{2} = \frac{\frac{28{,}072Nt}{660{,}969}}{\frac{51{,}193Nt}{881{,}292}} \cdot \frac{1}{2},$$

which is constant in N and t. The probability of transitioning from $3, 3$ to $4, 2$ is

$$\frac{2}{3} \cdot \frac{\mathrm{IR}^{3,3}_{\text{def}}}{\mathrm{IR}^{3,3}_{\text{att}}} \cdot \frac{1}{2} = \frac{2}{3} \cdot \frac{77}{103} \cdot \frac{1}{2},$$

which is constant in t and N. ■

C Local Comparative Statics

In this appendix I provide proofs of results for the local comparative statics. Characterization of the economic equilibria are in Supplemental Appendix E.

C.1 Proofs for Local Population Density Shocks

Proof of Proposition 6.1. From Supplemental Appendix E.1, rents under ABC, DEF and under $ABCF, DE$ are:

$$u^{\mathbf{ABC},DEF}(\eta) = \frac{(602 + 6408\eta + 23371\eta^2 + 32308\eta^3 + 11724\eta^4 + 512\eta^5)Nt}{1296(1 + 6\eta + 8\eta^2)^2} \tag{10}$$

$$u^{ABC,\mathbf{DEF}}(\eta) = \frac{(410 + 4752\eta + 19315\eta^2 + 31492\eta^3 + 16908\eta^4 + 2048\eta^5)Nt}{1296(1 + 6\eta + 8\eta^2)^2}, \tag{11}$$

$$u^{\mathbf{ABCF},DE}(\eta) = \frac{(2408 + 26576\eta + 100262\eta^2 + 146966\eta^3 + 71201\eta^4 + 6728\eta^5)t}{324(4 + 24\eta + 29\eta^2)^2}$$

$$u^{ABCF,\mathbf{DE}}(\eta) = \frac{(820 + 9208\eta + 34069\eta^2 + 44527\eta^3 + 14503\eta^4 + 841\eta^5)t}{162(4 + 24\eta + 29\eta^2)^2}.$$

Hence, if conflict is over F, the incremental returns are:

$$\mathrm{IR}^{\text{pop}}_{\text{att},F}(\eta) = u^{\mathbf{ABCF},DE}(\eta) - u^{\mathbf{ABC},DEF}(\eta)$$
$$= \frac{\left(1291776\eta^9 + 10238420\eta^8 + 22459372\eta^7 + 23035725\eta^6 + 13031928\eta^5 + 4314594\eta^4 + 833112\eta^3 + 86872\eta^2 + 3776\eta\right) Nt}{1296\left(8\eta^2 + 6\eta + 1\right)^2 \left(29\eta^2 + 24\eta + 4\right)^2}$$

and

$$\mathrm{IR}^{\mathrm{pop}}_{\mathrm{def},F}(\eta) = u^{ABC,\mathbf{DEF}}(\eta) - u^{ABCF,\mathbf{DE}}(\eta)$$
$$= \frac{\left(1291776\eta^9 + 8999020\eta^8 + 17389508\eta^7 + 16381611\eta^6 + 8805816\eta^5 + 2828202\eta^4 + 535560\eta^3 + 55064\eta^2 + 2368\eta\right)Nt}{1296\left(8\eta^2+6\eta+1\right)^2\left(29\eta^2+24\eta+4\right)^2}.$$

(i) Differentiating the rents, we have

$$\frac{\partial u^{\mathbf{ABC},DEF}(\eta)}{\partial \eta} = \frac{\left(2048\eta^6 + 4608\eta^5 - 57608\eta^4 - 66596\eta^3 - 28434\eta^2 - 5485\eta - 408\right)Nt}{648\left(8\eta^2+6\eta+1\right)^3}.$$

This is negative if the numerator is negative. To see that this is the case, notice that for any $\eta \in [1,2]$, $2048\eta^6 < 57608\eta^4$ and $4608\eta^5 < 66596\eta^3$, so the positive terms are more than off-set by the negative terms.

$$\frac{\partial u^{ABC,\mathbf{DEF}}(\eta)}{\partial \eta} = \frac{\left(8192\eta^6 + 18432\eta^5 - 19400\eta^4 - 26228\eta^3 - 9786\eta^2 - 1501\eta - 84\right)Nt}{648\left(8\eta^2+6\eta+1\right)^3}$$

To see that rents are decreasing and then increasing, note that at $\eta = 1$ this derivative is $-\frac{Nt}{72} < 0$ and at $\eta = 2$ it is $\frac{91943Nt}{9841500} > 0$. To see that rents are convex, differentiate again:

$$\frac{\partial^2 u^{ABC,\mathbf{DEF}}(\eta)}{\partial \eta^2} = \frac{\left(580736\eta^5 + 605232\eta^4 + 235552\eta^3 + 40072\eta^2 + 2472\eta + 11\right)t}{648\left(8\eta^2+6\eta+1\right)^4},$$

which is clearly positive for $\eta \in [1,2]$.

$$\frac{\partial u^{\mathbf{ABCF},DE}(\eta)}{\partial \eta} = \frac{\left(97556\eta^6 + 242208\eta^5 - 354903\eta^4 - 574398\eta^3 - 274260\eta^2 - 57528\eta - 4640\right)Nt}{162\left(29\eta^2+24\eta+4\right)^3}$$

To see that rents decreasing and then increasing, note that at $\eta = 1$ this derivative is $-\frac{5Nt}{162} < 0$ and at $\eta = 2$ it is $\frac{23Nt}{7056} > 0$. To see that it rents are convex, differentiate again:

$$\frac{\partial^2 u^{\mathbf{ABCF},DE}(\eta)}{\partial \eta^2} = \frac{\left(1919539\eta^5 + 2572173\eta^4 + 1451984\eta^3 + 446168\eta^2 + 76368\eta + 5776\right)t}{9\left(29\eta^2+24\eta+4\right)^4},$$

which is clearly positive for $\eta \in [1,2]$.

$$\frac{\partial u^{ABCF,\mathbf{DE}}(\eta)}{\partial \eta} = \frac{\left(24389\eta^6 + 60552\eta^5 - 578319\eta^4 - 675306\eta^3 - 266772\eta^2 - 43560\eta - 2528\right)Nt}{162\left(29\eta^2 + 24\eta + 4\right)^3}$$

To see that this is negative, notice that for any $\eta \in [1,2]$, $24389\eta^6 < 578319\eta^4$ and $60552\eta^5 < 675306\eta^3$, so the positive terms are more than off-set by the negative terms.

(ii) Differentiating the incremental returns, we have:

$$\frac{\partial \mathrm{IR}^{\mathrm{pop}}_{\mathrm{att},F}(\eta)}{\partial \eta} = \frac{Nt}{648\left(8\eta^2+6\eta+1\right)^3\left(29\eta^2+24\eta+4\right)^3}$$

$$\Big[149846016\eta^{12}+709185024\eta^{11}+1804009768\eta^{10}+3180547444\eta^{9}+3868730394\eta^{8}+3194448545\eta^{7}$$

$$+1799287064\eta^{6}+696886116\eta^{5}+185369396\eta^{4}+33247640\eta^{3}+3837552\eta^{2}+256864\eta+7552\Big]$$

and

$$\frac{\partial \mathrm{IR}^{\mathrm{pop}}_{\mathrm{def},F}(\eta)}{\partial \eta} = \frac{Nt}{648\left(8\eta^2+6\eta+1\right)^3\left(29\eta^2+24\eta+4\right)^3}$$

$$\Big[149846016\eta^{12}+709185024\eta^{11}+1938493592\eta^{10}+3288362780\eta^{9}+3601960590\eta^{8}+2665205479\eta^{7}$$

$$+1372937176\eta^{6}+498251100\eta^{5}+126687292\eta^{4}+22024216\eta^{3}+2485200\eta^{2}+163424\eta+4736\Big],$$

both of which are clearly positive for any $\eta \in [1,2]$.

(iii) In the event that an interior territory is vulnerable, observed violence is zero. Hence, it suffices to focus on the case of a border territory being vulnerable.

First, let's see that the attacker's incremental return is larger than the defender's. Subtracting, we have:

$$\mathrm{IR}^{\mathrm{pop}}_{\mathrm{att},F}(\eta) - \mathrm{IR}^{\mathrm{pop}}_{\mathrm{def},F}(\eta) = \frac{\left(619700\eta^8 + 2534932\eta^7 + 3327057\eta^6 + 2113056\eta^5 + 743196\eta^4 + 148776\eta^3 + 15904\eta^2 + 704\eta\right)Nt}{648\left(8\eta^2+6\eta+1\right)^2\left(29\eta^2+24\eta+4\right)^2},$$

which is clearly positive for any $\eta \in [1,2]$.

Thus, expected observed violence is

$$\frac{\mathrm{IR}^{\mathrm{pop}}_{\mathrm{def},F}(\eta)^2}{\mathrm{IR}^{\mathrm{pop}}_{\mathrm{att},F}(\eta)} = \frac{\left(1291776\eta^9+8999020\eta^8+17389508\eta^7+16381611\eta^6+8805816\eta^5+2828202\eta^4+535560\eta^3+55064\eta^2+2368\eta\right)^2 Nt}{1296(8\eta^2+6\eta+1)^2(29\eta^2+24\eta+4)^2(1291776\eta^8+10238420\eta^7+22459372\eta^6+23035725\eta^5+13031928\eta^4+4314594\eta^3+833112\eta^2+86872\eta+3776)}.$$

Differentiating, we have:

$$\frac{\partial}{\partial\eta}\frac{\mathrm{IR}^{\mathrm{trans}}_{\mathrm{def},F}(\eta)^2}{\mathrm{IR}^{\mathrm{pop}}_{\mathrm{att},F}(\eta)} = \frac{\left(1291776\eta^8+8999020\eta^7+17389508\eta^6+16381611\eta^5+8805816\eta^4+2828202\eta^3+535560\eta^2+55064\eta+2368\right)Nt}{648(8\eta^2+6\eta+1)^3(29\eta^2+24\eta+4)^3(1291776\eta^8+10238420\eta^7+22459372\eta^6+23035725\eta^5+13031928\eta^4+4314594\eta^3+833112\eta^2+86872\eta+3776)^2}$$

$$\times\Big[193567487164416\eta^{20}+2636013792927744\eta^{19}+14942866864822272\eta^{18}+51819230507149024\eta^{17}+ 122367280695000336\eta^{16}+206967166643804864\eta^{15}+259890474116763824\eta^{14}+249193190122341378\eta^{13}+ 186403803800274835\eta^{12}+110473424142844948\eta^{11}+52399600963659870\eta^{10}+19995986823887684\eta^9+ 6143372086092296\eta^8+1513744764870168\eta^7+296519405242384\eta^6+45491642345344\eta^5+ 5339965611648\eta^4+462318253184\eta^3+27777575168\eta^2+1032932352\eta+17883136\Big],$$

which is clearly positive for any $\eta \in [1, 2]$.

■

Proof of Proposition 6.2. From Supplemental Appendix E.1, rents under $BC, ADEF$ are:

$$u^{\mathbf{BC},ADEF}(\eta) = \frac{\left(313\eta^4+4686\eta^3+20497\eta^2+20532\eta+5956\right)Nt}{81(35\eta+22)^2}$$

$$u^{BC,\mathbf{ADEF}}(\eta) = \frac{\left(11744\eta^4+103041\eta^3+278285\eta^2+246312\eta+68900\right)Nt}{648(35\eta+22)^2},$$

and rents under ABC, DEF are reported in Equations 10 and 11.

Hence, if conflict is over A, the incremental returns are:

$$\begin{aligned}\mathrm{IR}^{\mathrm{pop}}_{\mathrm{att},A}(\eta) &= u^{BC,\mathbf{ADEF}}(\eta) - u^{ABC,\mathbf{DEF}}(\eta)\\ &= \frac{\left(1503232\eta^8+12935296\eta^7+32759508\eta^6+30349524\eta^5+7207189\eta^4-5078258\eta^3-3535132\eta^2-785144\eta-60640\right)Nt}{1296(35\eta+22)^2(8\eta^2+6\eta+1)^2}\end{aligned}$$

and

$$\begin{aligned}\mathrm{IR}^{\mathrm{pop}}_{\mathrm{def},A}(\eta) &= u^{\mathbf{ABC},DEF}(\eta) - u^{\mathbf{BC},ADEF}(\eta)\\ &= -\frac{\left(320512\eta^8+4652032\eta^7+13296660\eta^6-1413060\eta^5-28463891\eta^4-29236772\eta^3-12691846\eta^2-2556488\eta-196072\right)Nt}{1296(35\eta+22)^2(8\eta^2+6\eta+1)^2}.\end{aligned}$$

(i) Differentiating the incremental returns, we have:

$$\frac{\partial \mathrm{IR}^{\text{pop}}_{\text{att},A}(\eta)}{\partial \eta} = \frac{Nt}{648(35\eta+22)^3\left(8\eta^2+6\eta+1\right)^3}$$
$$\Big[420904960\eta^{10}+2971436544\eta^9+8242655616\eta^8+13929007224\eta^7+15777138676\eta^6+11897551650\eta^5$$
$$+5832602097\eta^4+1807243525\eta^3+337953462\eta^2+34600700\eta+1490296\Big],$$

which is clearly positive for any $\eta \in [1,2]$, and

$$\frac{\partial \mathrm{IR}^{\text{pop}}_{\text{def},A}(\eta)}{\partial \eta} = \frac{-Nt}{648(35\eta+22)^3\left(8\eta^2+6\eta+1\right)^3}$$
$$\Big[89743360\eta^{10}+898719744\eta^9+2854103040\eta^8+7300767576\eta^7+12496459804\eta^6+12803052270\eta^5$$
$$+7918942659\eta^4+2994912490\eta^3+680093556\eta^2+85613720\eta+4622656\Big],$$

which is clearly negative for any $\eta \in [1,2]$.

(ii) Point (i) of this proposition shows the defender's incremental return is strictly decreasing. Thus, to show that an $\hat{\eta} \in (1,2)$ exists, it suffices to show that the defender's incremental return is positive at $\eta = 1$ and negative at $\eta = 2$. At $\eta = 1$, the defender's incremental return is $\frac{77Nt}{1296} > 0$. At $\eta = 2$ the defender's incremental return is $-\frac{7686319Nt}{231384600} < 0$.

(iii) Given the previous results in this proposition, it now suffices to show that the defender's incremental return is less than the attacker's. Subtracting, this is the case if:

$$\mathrm{IR}^{\text{pop}}_{\text{att, A}}(\eta) - \mathrm{IR}^{\text{pop}}_{\text{def, A}}(\eta) =$$
$$\frac{\left(911872\eta^8+8793664\eta^7+23028084\eta^6+14468232\eta^5-10628351\eta^4-17157515\eta^3-8113489\eta^2-1670816\eta-128356\right)Nt}{648(35\eta+22)^2\left(8\eta^2+6\eta+1\right)^2} > 0.$$

Since the defender's incremental return is decreasing in η and the attacker's is increasing in η, the left-hand side is minimized at $\eta = 1$. Thus, it suffices to show that the inequality holds at $\eta = 1$. At $\eta = 1$, the inequality reduces to $\frac{13t}{648} > 0$.

■

C.2 Proofs of for Local Transportation Cost Shocks

Proof of Proposition 6.3. From Supplemental Appendix E.2, the rents under ABC, DEF and under $ABCF, DE$ are:

$$u^{\mathbf{ABC},DEF}(\tau) = \frac{\left(3500\tau^4 + 46780\tau^3 + 190407\tau^2 + 252436\tau + 106277\right) Nt}{1296(\tau+1)(\tau+3)(2\tau+13)^2} \tag{12}$$

$$u^{ABC,\mathbf{DEF}}(\tau) = \frac{\left(4948\tau^4 + 75452\tau^3 + 351465\tau^2 + 520802\tau + 246133\right) Nt}{2592(\tau+1)(\tau+3)(2\tau+13)^2}, \tag{13}$$

$$u^{\mathbf{ABCF},DE}(\tau) = \frac{\left(14000\tau^4 + 158266\tau^3 + 582603\tau^2 + 782964\tau + 350919\right) Nt}{1296(\tau+1)(\tau+5)(4\tau+15)^2}$$

$$u^{ABCF,\mathbf{DE}}(\tau) = \frac{\left(2474\tau^4 + 26854\tau^3 + 93111\tau^2 + 111528\tau + 43281\right) Nt}{324(\tau+1)(\tau+5)(4\tau+15)^2}.$$

Hence, when a border territory is vulnerable, the incremental returns are:

$$\begin{aligned}\mathrm{IR}^{\text{trans}}_{\text{att, F}}(\tau) &= u^{\mathbf{ABCF},DE}(\tau) - u^{\mathbf{ABC},DEF}(\tau)\\ &= \frac{\left(40292\tau^6+859712\tau^5+7150761\tau^4+29599651\tau^3+64289431\tau^2+69671199\tau+29177154\right)Nt}{648(\tau+1)(\tau+3)(\tau+5)(2\tau+13)^2(4\tau+15)^2}\end{aligned}$$

and

$$\begin{aligned}\mathrm{IR}^{\text{trans}}_{\text{def, F}}(\tau) &= u^{ABC,\mathbf{DEF}}(\tau) - u^{ABCF,\mathbf{DE}}(\tau)\\ &= \frac{\left(70816\tau^6+1634740\tau^5+15343560\tau^4+73444901\tau^3+184495487\tau^2+224073807\tau+101351889\right)Nt}{2592(\tau+1)(\tau+3)(\tau+5)(2\tau+13)^2(4\tau+15)^2}.\end{aligned}$$

(i) Differentiating the rents, we have

$$\frac{\partial u^{\mathbf{ABC},DEF}(\tau)}{\partial\tau} = \frac{\left(6360\tau^5 + 108628\tau^4 + 663482\tau^3 + 1800891\tau^2 + 2005820\tau + 760819\right) Nt}{324(\tau+1)^2(\tau+3)^2(2\tau+13)^3}$$

$$\frac{\partial u^{ABC,\mathbf{DEF}}(\tau)}{\partial\tau} = \frac{\left(8664\tau^5 + 203140\tau^4 + 1567538\tau^3 + 5015871\tau^2 + 5991404\tau + 2279383\right) Nt}{1296(\tau+1)^2(\tau+3)^2(2\tau+13)^3}$$

$$\frac{\partial u^{\mathbf{ABCF},DE}(\tau)}{\partial\tau} = \frac{\left(61468\tau^5 + 1026583\tau^4 + 6237580\tau^3 + 16551342\tau^2 + 17968716\tau + 6551415\right) Nt}{648(\tau+1)^2(\tau+5)^2(4\tau+15)^3}$$

$$\frac{\partial u^{ABCF,\mathbf{DE}}(\tau)}{\partial\tau} = \frac{\left(13090\tau^5 + 212431\tau^4 + 1270000\tau^3 + 3351690\tau^2 + 3660714\tau + 1369035\right) Nt}{162(\tau+1)^2(\tau+5)^2(4\tau+15)^3},$$

all of which are clearly positive for any $\tau \in [1, 2]$.

(ii) Differentiating the incremental returns, we have:

$$\frac{\partial \mathrm{IR}^{\mathrm{trans}}_{\mathrm{att,F}}(\tau)}{\partial \tau} = \frac{-Nt}{648(\tau+1)^2(\tau+3)^2(\tau+5)^2(2\tau+13)^3(4\tau+15)^3}$$

$$\Big[322336\tau^{10}+10451448\tau^9+143061988\tau^8+1073013626\tau^7+4766618725\tau^6+12523786196\tau^5$$

$$+17710031949\tau^4+8367954734\tau^3-7878111669\tau^2-8896414788\tau-1152922545\Big]$$

and

$$\frac{\partial \mathrm{IR}^{\mathrm{trans}}_{\mathrm{def,F}}(\tau)}{\partial \tau} = \frac{-Nt}{1296(\tau+1)^2(\tau+3)^2(\tau+5)^2(2\tau+13)^3(4\tau+15)^3}$$

$$\Big[283264\tau^{10}+10174464\tau^9+163482400\tau^8+1529546792\tau^7+9107162500\tau^6+35555048270\tau^5$$

$$+90894354783\tau^4+148662284540\tau^3+149453302806\tau^2+87077604366\tau+24236491815\Big].$$

The incremental returns are decreasing if the arguments in square brackets are positive. This is clearly the case for the defender for any $\tau \in [1,2]$. Now consider the attacker. To see that the term in the square brackets is positive in this case, note that for any $\tau \in [1,2]$, $8367954734\tau^3 > 1152922545$, $17710031949\tau^4 > 8896414788\tau$, and $12523786196\tau^5 > 7878111669\tau^2$, so each negative terms is more than off-set by a separate positive term.

(iii) In the event that an interior territory is vulnerable, observed violence is zero. Hence, it suffices to focus on the case of a border territory being vulnerable.

First, let's see that the attacker's incremental return is larger than the defender's. Subtracting, this is the case if:

$$\mathrm{IR}^{\mathrm{trans}}_{\mathrm{att,\ F}}(\tau)-\mathrm{IR}^{\mathrm{trans}}_{\mathrm{def,\ F}}(\tau) = \frac{\left(90352\tau^5+1713756\tau^4+11545728\tau^3+33407975\tau^2+39254262\tau+15356727\right)Nt}{2592(\tau+3)(\tau+5)(2\tau+13)^2(4\tau+15)^2} > 0,$$

which holds for any $\tau \in [1,2]$.

Thus, expected observed violence is

$$\frac{\mathrm{IR}^{\mathrm{trans}}_{\mathrm{def,\ F}}(\tau)^2}{\mathrm{IR}^{\mathrm{trans}}_{\mathrm{att,\ F}}(\tau)} = \frac{\left(70816\tau^6+1634740\tau^5+15343560\tau^4+73444901\tau^3+184495487\tau^2+224073807\tau+101351889\right)^2 Nt}{10368(\tau+1)(\tau+3)(\tau+5)(2\tau+13)^2(4\tau+15)^2(40292\tau^6+859712\tau^5+7150761\tau^4+29599651\tau^3+64289431\tau^2+69671199\tau+29177154)}.$$

Differentiating, we have:

$$\frac{\partial}{\partial \tau}\frac{\text{IR}^{\text{trans}}_{\text{def, F}}(\tau)^2}{\text{IR}^{\text{trans}}_{\text{att, F}}(\tau)} = \frac{-\left(70816\tau^6+1634740\tau^5+15343560\tau^4+73444901\tau^3+184495487\tau^2+224073807\tau+101351889\right)Nt}{10368(\tau+1)^2(\tau+3)^2(\tau+5)^2(2\tau+13)^3(4\tau+15)^3(40292\tau^6+859712\tau^5+7150761\tau^4+29599651\tau^3+64289431\tau^2+69671199\tau+29177154)^2}$$

$$\times \Big[22826546176\tau^{16}+1346834559616\tau^{15}+37276519674400\tau^{14}+639371782994576\tau^{13}+ 7567435768222208\tau^{12}+65199087795895376\tau^{11}+420975308247002594\tau^{10}+2069002610638570577\tau^9+ 7793137617277828811\tau^8 + 22498728719469456958\tau^7 + 49489440661438539010\tau^6 + 81914683489662021400\tau^5 + 99928825843407467628\tau^4 + 86905973146295199618\tau^3 + 50946917644932029964\tau^2 + 18077056655295975543\tau + 2945458294230415545\Big]$$

which is clearly negative for any $\tau \in [1, 2]$.

■

Proof of Proposition 6.4.

From Supplemental Appendix E.2, rents under $BC, ADEF$ are:

$$u^{BC,\mathbf{ADEF}}(\tau) = \frac{19\left(409\tau^2+3406\tau+7129\right)Nt}{324(11\tau+46)^2}$$

$$u^{\mathbf{BC},ADEF}(\tau) = \frac{\left(109193\tau^3+995320\tau^2+2701885\tau+1859858\right)Nt}{2592(\tau+1)(11\tau+46)^2},$$

and rents under ABC, DEF are reported in Equations 12 and 13.

Hence, when the neighboring territory is vulnerable, incremental returns are:

$$\begin{aligned}\text{IR}^{\text{trans}}_{\text{att, A}}(\tau) &= u^{BC,\mathbf{ADEF}}(\tau) - u^{ABC,\mathbf{DEF}}(\tau)\\ &= -\frac{\left(161936\tau^6+3167436\tau^5+19358912\tau^4+19156131\tau^3-201237120\tau^2-623205917\tau-422130578\right)Nt}{2592(\tau+1)(\tau+3)(2\tau+13)^2(11\tau+46)^2}\end{aligned}$$

and

$$\begin{aligned}\text{IR}^{\text{trans}}_{\text{def, A}}(\tau) &= u^{\mathbf{ABC},DEF}(\tau) - u^{\mathbf{BC},ADEF}(\tau)\\ &= \frac{\left(299164\tau^6+6053244\tau^5+45925507\tau^4+158823576\tau^3+229336425\tau^2+59685980\tau-49812496\right)Nt}{1296(\tau+1)(\tau+3)(2\tau+13)^2(11\tau+46)^2}.\end{aligned}$$

Note that the attacker's incremental return is positive because $422130578 > 161936\tau^6 + 3167436\tau^5 + 19358912\tau^4 + 19156131\tau^3$ for any $\tau \in [1, 2]$.

(i) Differentiating the incremental returns, we have:

$$\frac{\partial \mathrm{IR}^{\text{trans}}_{\text{att},A}(\tau)}{\partial \tau} = \frac{-Nt}{2592(\tau+1)^2(\tau+3)^2(2\tau+13)^3(11\tau+46)^3}$$
$$\Big[20676696\tau^8+757941316\tau^7+11271735818\tau^6+89934566643\tau^5+421079277994\tau^4+1174243638776\tau^3 + 1871978337534\tau^2 + 1524040258765\tau + 486909042458\Big],$$

which is clearly negative for any $\tau \in [1,2]$, and

$$\frac{\partial \mathrm{IR}^{\text{trans}}_{\text{def},A}(\tau)}{\partial \tau} = \frac{Nt}{324(\tau+1)^2(\tau+3)^2(2\tau+13)^3(11\tau+46)^3}$$
$$\Big[8440536\tau^8+250130612\tau^7+3134243962\tau^6+21644759691\tau^5+89552437616\tau^4+224745641977\tau^3 + 328095694158\tau^2 + 248494489970\tau + 74115939478\Big],$$

which is clearly positive for any $\tau \in [1,2]$.

(ii) Point (i) of this proposition implies that both incremental returns are monotone in τ and that observed violence is increasing in τ if $\mathrm{IR}^{\text{trans}}_{\text{def},A}(\tau) < \mathrm{IR}^{\text{trans}}_{\text{att},A}(\tau)$ and decreasing in τ if $\mathrm{IR}^{\text{trans}}_{\text{def},A}(\tau) > \mathrm{IR}^{\text{trans}}_{\text{att},A}(\tau)$. Hence, to show that a $\hat{\tau} \in (1,2)$ exists, it suffices to show that

$$\mathrm{IR}^{\text{trans}}_{\text{att},A}(1) - \mathrm{IR}^{\text{trans}}_{\text{def},A}(1) > 0 \qquad \text{and} \qquad \mathrm{IR}^{\text{trans}}_{\text{att},A}(2) - \mathrm{IR}^{\text{trans}}_{\text{def},A}(2) < 0.$$

Subtracting, we have:

$$\mathrm{IR}^{\text{trans}}_{\text{att},A}(\tau) - \mathrm{IR}^{\text{trans}}_{\text{def},A}(\tau) = \frac{-\big(760264\tau^6+15273924\tau^5+111209926\tau^4+336803283\tau^3+257435730\tau^2-503833957\tau-521755570\big)Nt}{2592(\tau+1)(\tau+3)(2\tau+13)^2(11\tau+46)^2}.$$

At $\tau = 1$ this reduces to $\frac{13t}{648} > 0$ and at $\tau = 2$ it reduces to $-\frac{780541t}{8989056} < 0$, as required.

■

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Supplemental Appendices
"Factional Conflict and Territorial Rents"

D Supplemental Appendix: Economic Equilibrium

D.1 Six Factions: $1, 1, 1, 1, 1, 1$

Suppose there are six factions, each of which controls one territory. If demand is characterized by Equation 4 at some vector of prices, profits from territory i are:

$$p_i \left[D_i(p_i, p_{i+1}) + D_i(p_i, p_{i-1})\right].$$

Given the symmetry of the factions, equilibrium prices are characterized by the following condition:

$$N\left[\frac{2p^* - 2p^*}{2t} + \frac{1}{6}\right] - \frac{Np^*}{t} = 0.$$

This implies that in equilibrium the common price is

$$p^*_{1,1,1,1,1,1} = \frac{t}{6}$$

Note that for any $t \leq 1$, we have $p_i \leq 2 - p_j - \frac{t}{6}$ for all i, j, so demand is in fact characterized by Equation 4. Each faction's equilibrium rents are

$$u^{1,1,1,1,1,1} = \frac{t}{6} \cdot \frac{N}{6} = \frac{Nt}{36}.$$

D.2 Five Factions: $2, 1, 1, 1, 1$

Suppose there are five factions—one controlling two contiguous territories and all the remaining factions controlling one. Without loss of generality, suppose the large faction controls territories A and B. The there are three kinds of factions to consider:

(i) Large faction (controls A and B)

(ii) Border faction (controls C or F)

(iii) Interior faction (controls D or E)

If demand is characterized by Equation 4 at some vector of prices, the large faction's profits are:

$$N\left[p_A\left(\frac{1}{6}+\frac{p_B+p_F-2p_A}{2t}\right)+p_B\left(\frac{1}{6}+\frac{p_A+p_C-2p_B}{2t}\right)\right],$$

the C-border faction's profits are (the F-border faction is symmetric):

$$Np_C\left(\frac{1}{6}+\frac{p_B+p_D-2p_C}{2t}\right),$$

and the $D-$interior faction's profits are (the E-interior faction is symmetric):

$$Np_D\left(\frac{1}{6}+\frac{p_C+p_E-2p_D}{2t}\right).$$

An equilibrium is described by the following first-order and symmetry conditions:

$$\frac{1}{6}+\frac{p_B^*+p_F^*-2p_A^*}{2t}-\frac{p_A^*}{t}+\frac{p_B^*}{2t}=0$$

$$\frac{1}{6}+\frac{p_B^*+p_D^*-2p_C^*}{2t}-\frac{p_C^*}{t}=0$$

$$\frac{1}{6}+\frac{p_C^*+p_E^*-2p_D^*}{2t}-\frac{p_D^*}{t}=0$$

$$p_A^*=p_B^*$$

$$p_C^*=p_F^*$$

$$p_D^*=p_E^*.$$

This implies that in equilibrium we have:

$$p_A^*=p_B^*=\frac{5t}{19}\qquad p_C^*=p_F^*=\frac{11t}{57}\qquad p_D^*=p_E^*=\frac{10t}{57}.$$

Note that for any $t\le 1$, we have $p_i\le 2-p_j-\frac{t}{6}$ for all i,j, so demand is in fact characterized by Equation 4.

Rents for the large faction, the border factions, and the interior factions, respectively, are

$$u^{\mathbf{2},1,1,1,1}=\frac{145Nt}{2166}\qquad u^{2,\mathbf{1},1,1,\mathbf{1}}=\frac{40Nt}{1083}\qquad u^{2,1,\mathbf{1},\mathbf{1},1}=\frac{100Nt}{3249}.$$

D.3 Three Symmetric-Connected Factions: $2, 2, 2$

Suppose there are three factions, each controlling two contiguous territories. If demand is characterized by Equation 4 at some vector of prices, then a faction controlling territories i and $i+1$ has profits:

$$p_i \left[D_i(p_i, p_{i+1}) + D_i(p_i, p_{i-1})\right] + p_{i+1} \left[D_{i+1}(p_{i+1}, p_{i+2}) + D_{i+1}(p_{i+1}, p_i)\right].$$

Given the symmetry, equilibrium prices are described by the following condition:

$$\frac{1}{6} - \frac{p^*}{t} + \frac{p^*}{2t} = 0,$$

which implies the following common price:

$$p^*_{2,2,2} = \frac{t}{3}$$

Notice $p^*_{2,2,2} > 2 - p^*_{2,2,2} - \frac{t}{6}$ for any $t \leq 1$, so demand is in fact characterized by Equation 4.

Equilibrium profits are:

$$u^{2,2,2} = \frac{t}{3} \cdot \frac{N}{3} = \frac{Nt}{9}.$$

D.4 Three Asymmetric Factions: $3, 2, 1$

Suppose there are three factions, one controlling three contiguous territories, one controlling two contiguous territories, and one controlling one territory. Without loss of generality, suppose the three factions are ABC, DE, F.

If demand is characterized by Equation 4 at some vector of prices, then the large faction's payoffs are

$$N\left[p_A\left(\frac{1}{6} + \frac{p_B + p_F - 2p_A}{2t}\right) + p_B\left(\frac{1}{6} + \frac{p_A + p_C - 2p_B}{2t}\right) + p_C\left(\frac{1}{6} + \frac{p_B + p_D - 2p_C}{2t}\right)\right],$$

the medium faction's payoffs are

$$N\left[p_D\left(\frac{1}{6} + \frac{p_C + p_E - 2p_D}{2t}\right) + p_E\left(\frac{1}{6} + \frac{p_D + p_F - 2p_E}{2t}\right)\right],$$

and the small faction's payoffs are

$$Np_F\left(\frac{1}{6}+\frac{p_E+p_A-2p_F}{2t}\right).$$

Prices satisfy the following six first-order conditions:

$$\frac{1}{6}+\frac{p_B^*+p_F^*-2p_A^*}{2t}-\frac{p_A^*}{t}+\frac{p_B^*}{2t}=0$$

$$\frac{1}{6}+\frac{p_C^*+p_A^*-2p_B^*}{2t}-\frac{p_B^*}{t}+\frac{p_A^*}{2t}+\frac{p_C^*}{2t}=0$$

$$\frac{1}{6}+\frac{p_B^*+p_D^*-2p_C^*}{2t}-\frac{p_C^*}{t}+\frac{p_B^*}{2t}=0$$

$$\frac{1}{6}+\frac{p_C^*+p_E^*-2p_D^*}{2t}-\frac{p_D^*}{t}+\frac{p_E^*}{2t}=0$$

$$\frac{1}{6}+\frac{p_D^*+p_F^*-2p_E^*}{2t}-\frac{p_E^*}{t}+\frac{p_D^*}{2t}=0$$

$$\frac{1}{6}+\frac{p_A^*+p_E^*-2p_F^*}{2t}-\frac{p_F^*}{t}=0$$

Solving, this implies the following equilibrium prices:

$$p_A^*=\frac{637t}{1626}\quad p_B^*=\frac{395t}{813}\quad p_C^*=\frac{112t}{271}$$

$$p_D^*=\frac{283t}{813}\quad p_E^*=\frac{175t}{542}$$

$$p_F^*=\frac{71t}{271}.$$

Note that for any $t\leq 1$, we have $p_i\leq 2-p_j-\frac{t}{6}$ for all i,j, so demand is in fact characterized by Equation 4.

These prices imply the following equilibrium rents:

$$u^{\mathbf{3},2,1}=\frac{447,343Nt}{2,643,878}\qquad u^{3,\mathbf{2},1}=\frac{298,831Nt}{2,643,876}\qquad u^{3,2,\mathbf{1}}=\frac{5041Nt}{73,441}.$$

D.5 Two Symmetric-Connected Factions: $3,3$

Suppose there are two factions, each controlling three contiguous territories. Without loss of generality, suppose the factions control A,B,C and D,E,F, respectively.

If demand is characterized by Equation 4 at some vector of prices, then a faction controlling territories $i-1$, i and $i+1$ has profits:

$$p_{i-1}\left[D_{i-1}(p_{i-1},p_i)+D_{i-1}(p_{i-1},p_{i-2})\right]+p_i\left[D_i(p_i,p_{i+1})+D_i(p_i,p_{i-1})\right]+p_{i+1}\left[D_{i+1}(p_{i+1},p_{i+2})+D_{i+1}(p_{i+1},p_i)\right].$$

Equilibrium prices are described by the following first-order and symmetry conditions:

$$\frac{1}{6}+\frac{p_B^*+p_F^*-2p_A^*}{2t}-\frac{p_A^*}{t}+\frac{p_B^*}{2t}=0$$

$$\frac{1}{6}+\frac{p_C^*+p_A^*-2p_B^*}{2t}-\frac{p_B^*}{t}+\frac{p_A^*}{2t}+\frac{p_C^*}{2t}=0$$

$$p_A^*=p_C^*=p_D^*=p_F^*$$

$$p_B^*=p_E^*.$$

Solving, this implies the following equilibrium prices:

$$p_A^*=p_C^*=p_D^*=p_F^*=\frac{t}{2}\qquad p_B^*=p_E^*=\frac{7t}{12}.$$

Note that for any $t\leq 1$, we have $p_i\leq 2-p_j-\frac{t}{6}$ for all i,j, so demand is in fact described by Equation 4.

Equilibrium profits for each faction are:

$$u^{3,3}=\frac{37Nt}{144}.$$

D.6 Two Asymmetric Factions: $4,2$

Suppose there are two factions, one controlling four contiguous territories and one controlling two contiguous territories. Without loss of generality, suppose the two factions control A,B,C,D and E,F.

If demand is characterized by Equation 4 at some vector of prices, then the large faction's payoffs are:

$$N\left(p_A\left(\frac{1}{6}+\frac{p_B+p_F-2p_A}{2t}\right)+p_B\left(\frac{1}{6}+\frac{p_A+p_C-2p_B}{2t}\right)+p_C\left(\frac{1}{6}+\frac{p_B+p_D-2p_C}{2t}\right)+p_D\left(\frac{1}{6}+\frac{p_C+p_E-2p_D}{2t}\right)\right),$$

and the small faction's payoffs are:

$$N\left(p_E\left(\frac{1}{6}+\frac{p_D+p_F-2p_E}{2t}\right)+p_F\left(\frac{1}{6}+\frac{p_E+p_A-2p_F}{2t}\right)\right).$$

In equilibrium, prices are described by the following first-order and symmetry conditions:

$$\frac{1}{6} + \frac{p_B^* + p_F^* - 2p_A^*}{2t} - \frac{p_A^*}{t} + \frac{p_B^*}{2t} = 0$$

$$\frac{1}{6} + \frac{p_C^* + p_A^* - 2p_B^*}{2t} - \frac{p_B^*}{t} + \frac{p_A^*}{2t} + \frac{p_C^*}{2t} = 0$$

$$\frac{1}{6} + \frac{p_A^* + p_E^* - 2p_F^*}{2t} - \frac{p_F^*}{t} + \frac{p_E^*}{2t} = 0$$

$$p_A^* = p_D^*$$

$$p_B^* = p_C^*$$

$$p_E^* = p_F^*.$$

Solving, this implies the following equilibrium prices:

$$p_A^* = p_D^* = \frac{5t}{9} \qquad p_B^* = p_C^* = \frac{13t}{18} \qquad p_E^* = p_F^* = \frac{4t}{9}.$$

Note, for any $t \leq 1$, we have $p_i \leq 2 - p_j - \frac{t}{6}$ for all i, j, so demand is in fact described by Equation 4.

These prices imply the following rents:

$$u^{\mathbf{4},2} = \frac{109Nt}{324} \qquad u^{4,\mathbf{2}} = \frac{16Nt}{81}.$$

E Supplemental Appendix: Economic Equilibria for Local Comparative Statics

E.1 Local Population Density

Without loss of generality, suppose the two factions start controlling A, B, C and D, E, F. To find the incremental returns, I start by characterizing equilibrium in the three scenarios: ABC, DEF, $ABCF, DE$, and $BC, ADEF$.

E.1.1 ABC, DEF

There are four cases to consider:

(i) Suppose $p_A < p_F$ and $p_E \geq p_F$. If demand is given by Equations 4 and 7, then taking

first-order conditions and solving yields:

$$p_A = \frac{\left(16\eta^2 + 94\eta + 25\right)t}{18\left(8\eta^2 + 6\eta + 1\right)}$$

$$p_B = \frac{\left(86\eta^2 + 185\eta + 44\right)t}{36\left(8\eta^2 + 6\eta + 1\right)}$$

$$p_C = \frac{\left(46\eta^2 + 73\eta + 16\right)t}{18\left(8\eta^2 + 6\eta + 1\right)}$$

$$p_D = \frac{\left(50\eta^2 + 71\eta + 14\right)t}{18\left(8\eta^2 + 6\eta + 1\right)}$$

$$p_E = \frac{\left(106\eta^2 + 175\eta + 34\right)t}{36\left(8\eta^2 + 6\eta + 1\right)}$$

$$p_F = \frac{\left(32\eta^2 + 86\eta + 17\right)t}{18\left(8\eta^2 + 6\eta + 1\right)}.$$

These prices are consistent with $p_A < p_F$ and $p_E \geq p_F$ for any $\eta \in [1, 2]$. Hence, this case is a candidate for an equilibrium.

(ii) Suppose $p_A < p_F$ and $p_E < p_F$. If demand is given by Equations 4 and 7, then taking first-order conditions and solving yields:

$$p_A = \frac{\left(11\eta^2 + 74\eta + 50\right)t}{9\left(11\eta^2 + 15\eta + 4\right)}$$

$$p_B = \frac{\left(113\eta^2 + 341\eta + 176\right)t}{36(\eta + 1)(11\eta + 4)}$$

$$p_C = \frac{\left(29\eta^2 + 74\eta + 32\right)t}{9(\eta + 1)(11\eta + 4)}$$

$$p_D = \frac{\left(53\eta^2 + 161\eta + 56\right)t}{18(\eta + 1)(11\eta + 4)}$$

$$p_E = \frac{(4\eta + 17)t}{18(\eta + 1)}$$

$$p_F = \frac{\left(44\eta^2 + 161\eta + 65\right)t}{18(\eta + 1)(11\eta + 4)}.$$

These prices are inconsistent with $p_E < p_F$ for any $\eta \in [1, 2]$, so there is no such

equilibrium.

(iii) Suppose $p_A \geq p_F$ and $p_E < p_F$. If demand is given by Equations 4 and 7, then taking first-order conditions and solving yields:

$$p_A = \frac{\left(13\eta^2 + 97\eta + 25\right)t}{9(23\eta + 7)}$$

$$p_B = \frac{(2\eta + 19)t}{36}$$

$$p_C = \frac{\left(10\eta^2 + 94\eta + 31\right)t}{9(23\eta + 7)}$$

$$p_D = \frac{\left(34\eta^2 + 163\eta + 73\right)t}{18(23\eta + 7)}$$

$$p_E = \frac{\left(58\eta^2 + 163\eta + 94\right)t}{18(23\eta + 7)}$$

$$p_F = \frac{\left(58\eta^2 + 187\eta + 25\right)t}{18(23\eta + 7)}.$$

These prices are inconsistent with $p_E < p_F$ for any $\eta \in [1, 2]$, so there is no such equilibrium.

(iv) Suppose $p_A \geq p_F$ and $p_E \geq p_F$. If demand is given by Equations 4 and 7, then taking first-order conditions and solving yields:

$$p_A = \frac{(2\eta + 13)t}{30}$$

$$p_B = \frac{(2\eta + 19)t}{36}$$

$$p_C = \frac{(4\eta + 41)t}{90}$$

$$p_D = \frac{(13 + 2\eta)t}{30}$$

$$p_E = \frac{(4\eta + 17)t}{36}$$

$$p_F = \frac{(14\eta + 31)t}{90}.$$

These prices are inconsistent with $p_A \geq p_F$ for any $\eta \in [1, 2]$, so there is no such

equilibrium.

We have only one candidate for an equilibrium (case (i)). For this to be an equilibrium, it must be that the worst off citizen prefers to purchase the good, so that demand is in fact characterized by Equations 4 and 7. In this candidate profile of prices, the prices are ordered as follows:

$$p_E > p_B > p_D > p_C > p_F > p_A.$$

Hence, the worst-off population member is the one who is just indifferent between buying from D and E. This person is located at

$$x^*_{DE} = \frac{18\eta^2 + 23\eta + 4}{192\eta^2 + 144\eta + 24}.$$

This person prefers to buy the good as long as:

$$1 - p_D - x^*_{DE}t \geq 0.$$

This is true if and only if:

$$\frac{-254\eta^2 t + 576\eta^2 - 353\eta t + 432\eta - 68t + 72}{72\,(8\eta^2 + 6\eta + 1)} \geq 0.$$

The left-hand side is linearly decreasing in t, so it suffices to check $t = 1$. At $t = 1$, the inequality holds if $322\eta^2 + 79\eta + 4 \geq 0$, which is the case for any $\eta \in [0, 1]$.

Equilibrium rents are:

$$u^{\mathbf{ABC},DEF}(\eta) = \frac{(602 + 6408\eta + 23371\eta^2 + 32308\eta^3 + 11724\eta^4 + 512\eta^5)Nt}{1296(1 + 6\eta + 8\eta^2)^2}$$

and

$$u^{ABC,\mathbf{DEF}}(\eta) = \frac{(410 + 4752\eta + 19315\eta^2 + 31492\eta^3 + 16908\eta^4 + 2048\eta^5)Nt}{1296(1 + 6\eta + 8\eta^2)^2}.$$

E.1.2 $ABCF, DE$

There are four cases to consider:

(i) Suppose $p_A < p_F$ and $p_E \geq p_F$. If demand is given by Equations 4 and 7, then taking

first-order conditions and solving yields:

$$p_A = \frac{\left(128\eta^2 + 372\eta + 241\right) t}{18(46\eta + 11)}$$

$$p_B = \frac{\left(92\eta^2 + 441\eta + 208\right) t}{18(46\eta + 11)}$$

$$p_C = \frac{\left(28\eta^2 + 186\eta + 71\right) t}{9(46\eta + 11)}$$

$$p_D = \frac{\left(20\eta^2 + 165\eta + 43\right) t}{9(46\eta + 11)}$$

$$p_E = \frac{2\left(13\eta^2 + 84\eta + 17\right) t}{9(46\eta + 11)}$$

$$p_F = \frac{\left(64\eta^2 + 204\eta + 17\right) t}{9(46\eta + 11)}.$$

These prices are inconsistent with $p_E \geq p_F$ for any $\eta \in [1, 2]$, so there is no such equilibrium.

(ii) Suppose $p_A < p_F$ and $p_E < p_F$. If demand is given by Equations 4 and 7, then taking first-order conditions and solving yields:

$$p_A = \frac{\left(88\eta^2 + 453\eta + 200\right) t}{18\left(22\eta^2 + 27\eta + 8\right)}$$

$$p_B = \frac{\left(139\eta^2 + 426\eta + 176\right) t}{18\left(22\eta^2 + 27\eta + 8\right)}$$

$$p_C = \frac{(31\eta + 64)t}{9(11\eta + 8)}$$

$$p_D = \frac{\left(43\eta^2 + 129\eta + 56\right) t}{9\left(22\eta^2 + 27\eta + 8\right)}$$

$$p_E = \frac{2\left(11\eta^2 + 69\eta + 34\right) t}{9\left(22\eta^2 + 27\eta + 8\right)}$$

$$p_F = \frac{(22\eta + 73)t}{9(11\eta + 8)}.$$

These prices are inconsistent with $p_A < p_F$ for any $\eta \in [1, 2]$, so there is no such

equilibrium.

(iii) Suppose $p_A \geq p_F$ and $p_E < p_F$. If demand is given by Equations 4 and 7, then taking first-order conditions and solving yields:

$$p_A = \frac{\left(233\eta^2 + 408\eta + 100\right) t}{18\left(29\eta^2 + 24\eta + 4\right)}$$

$$p_B = \frac{\left(263\eta^2 + 390\eta + 88\right) t}{18\left(29\eta^2 + 24\eta + 4\right)}$$

$$p_C = \frac{\left(103\eta^2 + 150\eta + 32\right) t}{9\left(29\eta^2 + 24\eta + 4\right)}$$

$$p_D = \frac{2\left(31\eta^2 + 69\eta + 14\right) t}{9\left(29\eta^2 + 24\eta + 4\right)}$$

$$p_E = \frac{\left(29\eta^2 + 165\eta + 34\right) t}{9\left(29\eta^2 + 24\eta + 4\right)}$$

$$p_F = \frac{\left(58\eta^2 + 177\eta + 50\right) t}{9\left(29\eta^2 + 24\eta + 4\right)}.$$

These prices are consistent with $p_E < p_F < p_A$ for all $\eta \in [1, 2]$, so this case is a candidate for an equilibrium.

(iv) Suppose $p_A \geq p_F$ and $p_E \geq p_F$. If demand is given by Equations 4 and 7, then taking first-order conditions and solving yields:

$$p_A = \frac{(44\eta + 203)t}{342}$$

$$p_B = \frac{(32\eta t + 215t)}{342}$$

$$p_C = \frac{5(2\eta + 17)t}{171}$$

$$p_D = \frac{4(2\eta + 17)t}{171}$$

$$p_E = \frac{(11\eta + 65)t}{171}$$

$$p_F = \frac{(28\eta + 67)t}{171}.$$

These prices are inconsistent with $p_E \geq p_F$ for any $\eta \in [1,2]$, so there is no such equilibrium.

We have only one candidate for equilibrium (case (iii)). For this to be an equilibrium, it must be that the worst off citizen prefers to purchase the good, so that demand is in fact characterized by Equations 4 and 7. In this profile, prices are ordered as follows:

$$p_B > p_A > p_C > p_F > p_D > p_E.$$

Hence, the worst-off population member is the one who is just indifferent between A and B. This person's position is

$$x^*_{AB} = \frac{19\eta^2 + 30\eta + 8}{348\eta^2 + 288\eta + 48}.$$

Plugging this in, the person indifferent between A and B prefers to purchase the good if:

$$1 - p_B - x^*_{AB} t \geq 0$$

which is true if and only if:

$$\eta^2(1044 - 583t) + \eta(864 - 870t) - 200t + 144 \geq 0.$$

The left-hand side is linearly decreasing in t, so it suffices to check $t = 1$. At $t = 1$ the inequality holds if and only if $461\eta^2 - 6\eta - 56 \geq 0$, which is true for any $\eta \in [1,2]$.

The equilibrium rents are

$$u^{\mathbf{ABCF},DE}(\eta) = \frac{(2408 + 26576\eta + 100262\eta^2 + 146966\eta^3 + 71201\eta^4 + 6728\eta^5)t}{324(4 + 24\eta + 29\eta^2)^2}$$

$$u^{ABCF,\mathbf{DE}}(\eta) = \frac{(820 + 9208\eta + 34069\eta^2 + 44527\eta^3 + 14503\eta^4 + 841\eta^5)t}{162(4 + 24\eta + 29\eta^2)^2}.$$

E.1.3 $BC, ADEF$

There are four cases to consider:

(i) Suppose $p_A < p_F$ and $p_E \geq p_F$. If demand is given by Equations 4 and 7, then taking

first-order conditions and solving yields:

$$p_A = \frac{\left(64\eta^2 + 186\eta + 35\right) t}{9(46\eta + 11)}$$

$$p_B = \frac{\left(26\eta^2 + 165\eta + 37\right) t}{9(46\eta + 11)}$$

$$p_C = \frac{4\left(5\eta^2 + 42\eta + 10\right) t}{9(46\eta + 11)}$$

$$p_D = \frac{\left(28\eta^2 + 204\eta + 53\right) t}{9(46\eta + 11)}$$

$$p_E = \frac{\left(92\eta^2 + 510\eta + 139\right) t}{18(46\eta + 11)}$$

$$p_F = \frac{\left(128\eta^2 + 474\eta + 139\right) t}{18(46\eta + 11)}.$$

These prices are inconsistent with $p_E \geq p_F$ for any $\eta \in [1, 2]$, so there is no such equilibrium.

(ii) Suppose $p_A < p_F$ and $p_E < p_F$. If demand is given by Equations 4 and 7, then taking first-order conditions and solving yields:

$$p_A = \frac{\left(44\eta^2 + 171\eta + 70\right) t}{9(35\eta + 22)}$$

$$p_B = \frac{\left(19\eta^2 + 135\eta + 74\right) t}{9(35\eta + 22)}$$

$$p_C = \frac{4\left(4\eta^2 + 33\eta + 20\right) t}{9(35\eta + 22)}$$

$$p_D = \frac{\left(26\eta^2 + 153\eta + 106\right) t}{9(35\eta + 22)}$$

$$p_E = \frac{\left(88\eta^2 + 375\eta + 278\right) t}{18(35\eta + 22)}$$

$$p_F = \frac{\left(88\eta^2 + 411\eta + 242\right) t}{18(35\eta + 22)}.$$

These prices are consistent with $p_A < p_F$ and $p_E < p_F$ for all $\eta \in [1, 2]$, so this case is a candidate for an equilibrium.

(iii) Suppose $p_A \geq p_F$ and $p_E < p_F$. If demand is given by Equations 4 and 7, then taking first-order conditions and solving yields:

$$p_A = \frac{\left(46\eta^2 + 204\eta + 35\right) t}{9(46\eta + 11)}$$

$$p_B = \frac{\left(23\eta^2 + 168\eta + 37\right) t}{9(46\eta + 11)}$$

$$p_C = \frac{\left(23\eta^2 + 165\eta + 40\right) t}{9(46\eta + 11)}$$

$$p_D = \frac{\left(46\eta^2 + 186\eta + 53\right) t}{9(46\eta + 11)}$$

$$p_E = \frac{\left(161\eta^2 + 441\eta + 139\right) t}{18(46\eta + 11)}$$

$$p_F = \frac{\left(161\eta^2 + 510\eta + 70\right) t}{18(46\eta + 11)}.$$

These prices are inconsistent with $p_A \geq p_F$ for any $\eta \in [1, 2]$, so there is no such equilibrium.

(iv) Suppose $p_A \geq p_F$ and $p_E \geq p_F$. If demand is given by Equations 4 and 7, then taking first-order conditions and solving yields:

$$p_A = \frac{(22\eta + 73)t}{171}$$

$$p_B = \frac{(10\eta + 66)t}{171}$$

$$p_C = \frac{5(9\eta + 67)t}{171}$$

$$p_D = \frac{4(16\eta + 79)t}{171}$$

$$p_E = \frac{(55\eta + 192)t}{342}$$

$$p_F = \frac{(78\eta + 169)t}{342}.$$

These prices are inconsistent with $p_E \geq p_F$ or $p_A \geq p_F$ for any $\eta \in [1,2]$, so there is no such equilibrium.

We have only one candidate for equilibrium (case (ii)). For this to be an equilibrium, it must be that the worst off citizen prefers to purchase the good, so that demand is in fact characterized by Equations 4 and 7. In this profile, prices are ordered as follows:

$$p_F > p_E > p_A > p_D > p_B > p_C.$$

Hence, the worst-off population member is the one who is just indifferent between E and F. This consumer's position is

$$x^*_{EF} = \frac{47\eta + 10}{420\eta + 264}.$$

Plugging this in, the person indifferent between E and F prefers to purchase the good if:

$$1 - p_E - x^*_{EF}t \geq 0$$

which is true if and only if:

$$\frac{-176\eta^2 t - 891\eta t + 1260\eta - 586t + 792}{1260\eta + 792} \geq 0.$$

The left-hand side is linearly decreasing in t, so it suffices to check $t = 1$. At $t = 1$, the inequality holds if $-176\eta^2 + 369\eta + 206 \geq 0$, which is true for any $\eta \in [1,2]$.

The equilibrium rents are

$$u^{\mathbf{BC},ADEF}(\eta) = \frac{\left(313\eta^4 + 4686\eta^3 + 20497\eta^2 + 20532\eta + 5956\right)Nt}{81(35\eta + 22)^2}$$

$$u^{BC,\mathbf{ADEF}}(\eta) = \frac{\left(11744\eta^4 + 103041\eta^3 + 278285\eta^2 + 246312\eta + 68900\right)Nt}{648(35\eta + 22)^2}.$$

E.2 Local Transportation Costs

Without loss of generality, suppose the two factions start controlling A, B, C and D, E, F. To find the incremental returns, I first characterize equilibrium in the three scenarios: ABC, DEF, $ABCF, DE$, and $BC, ADEF$.

E.2.1 ABC, DEF

Assuming that demand is given by Equations 4 and 8, taking first-order conditions and solving gives the following prices:

$$p_A = \frac{\left(62\tau^2 + 281\tau + 197\right)t}{18\left(2\tau^2 + 19\tau + 39\right)}$$

$$p_B = \frac{\left(106\tau^2 + 571\tau + 583\right)t}{36(\tau + 3)(2\tau + 13)}$$

$$p_C = \frac{\left(38\tau^2 + 233\tau + 269\right)t}{18(\tau + 3)(2\tau + 13)}$$

$$p_D = \frac{\left(34\tau^2 + 247\tau + 259\right)t}{18(\tau + 3)(2\tau + 13)}$$

$$p_E = \frac{(43\tau + 41)t}{36(\tau + 3)}$$

$$p_F = \frac{\left(40\tau^2 + 259\tau + 241\right)t}{18(\tau + 3)(2\tau + 13)}.$$

For this to be an equilibrium, it must be that the worst off citizen prefers to purchase the good, so that demand is in fact characterized by Equations 4 and 8.

The order of prices is $p_E > p_B > p_A > p_F > p_D > p_C$. Hence, there are two candidates for the worst-off citizen: the citizen indifferent between buying from E and F and the citizen indifferent between buying from A and B.

The citizen indifferent between E and F is located at

$$x^*_{EF} = \frac{4\tau^3 + 36\tau^2 + 37\tau - 17}{24\tau^3 + 252\tau^2 + 696\tau + 468}.$$

We need the following:

$$1 - p_E - x^*_{EF}t \geq 0$$

which is true if

$$\frac{-98\tau^3 t + 72\tau^3 - 835\tau^2 t + 756\tau^2 - 1285\tau t + 2088\tau - 482t + 1404}{36(\tau + 3)\left(2\tau^2 + 15\tau + 13\right)} \geq 0$$

The left-hand side of this inequality is linearly decreasing in t, so it suffices to check $t = 1$. At $t = 1$, the inequality holds if and only if $-26\tau^3 - 79\tau^2 + 803\tau + 922 \geq 0$, which is true for any $\tau \in [1,2]$.

The citizen indifferent between A and B is located at

$$x^*_{AB} = \frac{47 - 2\tau}{48\tau + 312}.$$

We need the following:

$$1 - p_A - x^*_{AB} t \geq 0$$

which is true if

$$\frac{-242\tau^2 t + 144\tau^2 - 1247\tau t + 1368\tau - 1211t + 2808}{72(\tau + 3)(2\tau + 13)}.$$

The left-hand side of this inequality is linearly decreasing in t, so it suffices to check $t = 1$. At $t = 1$, the inequality holds if and only if $-98\tau^2 + 121\tau + 1597 \geq 0$, which is true for any $\tau \in [1,2]$.

The rents at these equilibrium prices are:

$$u^{\mathbf{ABC},DEF}(\tau) = \frac{\left(3500\tau^4 + 46780\tau^3 + 190407\tau^2 + 252436\tau + 106277\right) Nt}{1296(\tau + 1)(\tau + 3)(2\tau + 13)^2}$$

and

$$u^{ABC,\mathbf{DEF}}(\tau) = \frac{\left(4948\tau^4 + 75452\tau^3 + 351465\tau^2 + 520802\tau + 246133\right) Nt}{2592(\tau + 1)(\tau + 3)(2\tau + 13)^2}.$$

E.2.2 $ABCF, DE$

Assuming that demand is given by Equations 4 and 8, taking first-order conditions and solving gives the following prices:

$$p_A = \frac{\left(62\tau^2 + 376\tau + 303\right) t}{9\left(4\tau^2 + 35\tau + 75\right)}$$

$$p_B = \frac{\left(212\tau^2 + 1351\tau + 1401\right) t}{36\left(4\tau^2 + 35\tau + 75\right)}$$

$$p_C = \frac{19(2\tau + 3)t}{9(4\tau + 15)}$$

$$p_D = \frac{\left(68\tau^2 + 415\tau + 429\right) t}{18\left(4\tau^2 + 35\tau + 75\right)}$$

$$p_E = \frac{\left(43\tau^2 + 239\tau + 174\right) t}{9\left(4\tau^2 + 35\tau + 75\right)}$$

$$p_F = \frac{(179\tau + 201)t}{36(4\tau + 15)}.$$

For this to be an equilibrium, it must be that the worst off citizen prefers to purchase the good, so that demand is in fact characterized by Equations 4 and 8.

The order of prices is $p_A > p_B > p_F > p_C > p_E > p_D$. Hence, there are two candidates for the worst-off citizen: the citizen indifferent between buying from A and F and the citizen indifferent between buying from A and B.

The citizen indifferent between A and F is located at

$$x^*_{AF} = \frac{8\tau^3 + 47\tau^2 + 14\tau - 69}{48\tau^3 + 468\tau^2 + 1320\tau + 900}.$$

We need the following:

$$1 - p_A - x^*_{AF} t \geq 0$$

which is true if

$$\frac{-272\tau^3 t + 144\tau^3 - 1893\tau^2 t + 1404\tau^2 - 2758\tau t + 3960\tau - 1005t + 2700}{36(\tau + 1)\left(4\tau^2 + 35\tau + 75\right)} \geq 0.$$

The left-hand side of this inequality is linearly decreasing in t, so it suffices to check $t = 1$. At $t = 1$, the inequality holds if and only if $-128\tau^3 - 489\tau^2 + 1202\tau + 1695 \geq 0$, which is true for any $\tau \in [1, 2]$.

The citizen indifferent between A and B is located at

$$x^*_{AB} = \frac{-4\tau^2 + 19\tau + 213}{96\tau^2 + 840\tau + 1800}.$$

We need the following:

$$1 - p_A - x^*_{AB} t \geq 0$$

which is true if

$$\frac{-484\tau^2 t + 288\tau^2 - 3065\tau t + 2520\tau - 3063t + 5400}{72\left(4\tau^2 + 35\tau + 75\right)} \geq 0.$$

The left-hand side of this inequality is linearly decreasing in t, so it suffices to check $t = 1$. At $t = 1$, the inequality holds if and only if $-196\tau^2 - 545\tau + 2337 \geq 0$, which is true for any $\tau \in [1, 2]$.

The rents at these equilibrium prices are:

$$u^{\mathbf{ABCF},DE}(\tau) = \frac{\left(14000\tau^4 + 158266\tau^3 + 582603\tau^2 + 782964\tau + 350919\right)Nt}{1296(\tau+1)(\tau+5)(4\tau+15)^2}$$

and

$$u^{ABCF,\mathbf{DE}}(\tau) = \frac{\left(2474\tau^4 + 26854\tau^3 + 93111\tau^2 + 111528\tau + 43281\right)Nt}{324(\tau+1)(\tau+5)(4\tau+15)^2}.$$

E.2.3 $BC, ADEF$

Assuming that demand is given by Equations 4 and 8, taking first-order conditions and solving gives the following prices:

$$p_A = \frac{(46\tau + 239)t}{99\tau + 414}$$

$$p_B = \frac{(85\tau + 371)t}{198\tau + 828}$$

$$p_C = \frac{(91\tau + 365)t}{198\tau + 828}$$

$$p_D = \frac{(64\tau + 221)t}{99\tau + 414}$$

$$p_E = \frac{(355\tau + 1127)t}{36(11\tau + 46)}$$

$$p_F = \frac{(605\tau + 2359)t}{72(11\tau + 46)}.$$

For this to be an equilibrium, it must be that the worst off citizen prefers to purchase the good, so that demand is in fact characterized by Equations 4 and 8.

The order of prices is $p_E > p_F > p_D > p_A > p_C > p_B$. Hence, the worst-off consumer is the one indifferent between buying from E and F.

The consumer indifferent between E and F is located at

$$x^*_{EF} = \frac{44\tau^2 + 149\tau + 35}{264\tau^2 + 1368\tau + 1104}.$$

We need the following:

$$1 - p_E - x^*_{EF}t \geq 0$$

which is true if

$$\frac{\tau^2(792-842t)-9\tau(379t-456)-2359t+3312}{72(\tau+1)(11\tau+46)} \geq 0.$$

The left-hand side is linearly decreasing in t, so it suffices to check $t=1$. At $t=1$, this condition holds if and only if $-50\tau^2+693\tau+953 \geq 0$, which is true for any $\tau \in [1,2]$.

The rents at these equilibrium prices are:

$$u^{BC,\mathbf{ADEF}}(\tau) = \frac{19\left(409\tau^2+3406\tau+7129\right)t}{324(11\tau+46)^2}$$

and

$$u^{\mathbf{BC},ADEF}(\tau) = \frac{\left(109193\tau^3+995320\tau^2+2701885\tau+1859858\right)t}{2592(\tau+1)(11\tau+46)^2}.$$