Political Disagreement and Information in Elections

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March 2, 2015

Preliminary and incomplete draft, please do not circulate

Abstract

We study a probabilistic voting model where citizen candidates representing different groups of voters compete for office. Groups might differ in their payoff functions and their beliefs about how government policies map into outcomes. In equilibrium, the candidate from the majority group wins with a probability that increases in the degree of political disagreement — the difference in expected payoffs from the policies supported by the candidates. Prior to the election, an individual (information controller) who supports the majority candidate is able to influence voters' behavior by designing the information content of a public signal. We show when and how the controller can use this signal to increase political disagreement and hence the majority candidate's victory probability. Even when all voters share the same payoff function, so that political disagreement is solely due to belief disagreement, we show how the controller strategically uses information to increase disagreement.

JEL classification: D72, D83.

Keywords: Information control, Bayesian persuasion, voting.

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1 Introduction

In his classic paper, Stokes (1963) highlights some empirical regularities that should be present in electoral models. First, voters care about multiple issues, and hence may have to trade-off the candidate that offers the highest payoff in one issue for the competing candidate that offers the highest payoff in a different, more relevant issue. Second, it is important to differentiate position-issues and valence-issues.[1] Third, voter behavior only depends on voter's *perception* of parties' ideological positions. Fourth, information may change this perception, thus leading a voter to revise the relative importance of the different issues. To the extent that voters' perceptions can be shaped by information, interested parties have an incentive to affect voters' learning, and control the salience of position- and valence-issues (see Iyengar and Simon 2000 for a survey).

In this paper we incorporate these empirical regularities into a persuasion model. We focus on how information shapes political disagreement, and how disagreement steers politics. We consider an interest group (IG) that supports one candidate and is able to control voters' learning about a position-issue. In equilibrium, the IG supplies information to change voters' perception of the degree of political disagreement between the candidates, which endogenously shifts the relative salience of policy and valence issues. When a majority of voters share a similar policy view as the candidate supported by the IG, the IG discloses information with the sole purpose of *increasing* political disagreement, which benefits its supported candidate.[2]

To illustrate the role of political disagreement and information, consider the following example. Voters who care about a position-issue (policy) and a valence-issue (competence) are divided into two groups: a majority A and a minority B. One citizen-candidate from

[1]In position-issues, parties advocate certain actions, and voters might have heterogenous preferences over actions. In valence-issues, parties are linked to some condition that is positively or negatively valued by the electorate.

[2]Our interest group resembles the persuaders in Downs (1957, pg. 83), who "are not interested *per se* in helping people who are uncertain become less so; they want certainty to produce a decision which aids their cause." The strategic use of information to shift the salience of issues is highlighted by Stokes (1963, pg. 372), who says that "the skills of political leaders who must maneuver for public support in a democracy consist partly in knowing what issue dimensions [...] can be made salient by suitable propaganda."

each group runs for office. The elected official must choose a policy: whether to keep the status quo or to implement a risky economic reform (e.g., signing an international trade agreement). The untried reform is either good or bad for all voters (it either yields a higher or lower payoff than the status quo). Voters in group A receive a higher payoff than voters B if the reform is good, so they are more willing to implement it. Voters in group A prefer to implement the reform if the probability of success is higher than 0.5, while voters in group B only prefer the reform if the success probability is higher than 0.8.[3] Suppose that the common prior belief is 0.6. Candidates cannot commit to future policies, but voters can predict their behavior: candidate A will implement the reform, but candidate B won't. Candidate A has a *policy advantage* — a majority of voters believe that she will choose a better policy. Voters in majority A are nonetheless willing to vote for the minority candidate if during the campaign they learn that she is sufficiently more competent.[4] A key observation is that voters are less willing to trade off policy for valence if voters' perception of the degree of *political disagreement* is higher. That is, if the difference in voters' expected payoffs from the policies supported by the candidates is higher. For instance, if new information leads players to update their belief and assign probability 0.7 to the reform being successful, then from the point of view of voters in group A disagreement increases — they become more convinced that the reform is the best policy. A majority of voters is then less likely to vote for minority candidate B.

Consider an interest group that wants to maximize the victory probability of the majority candidate A. How can it persuade voters to elect A? The IG benefits from disclosing information about the economic reform that *increases* the degree of political disagreement. Disagreement shifts salience from valence to policy, which increases the victory probability of A, who has a policy-advantage.

To study the supply of information in elections, we consider a Bayesian persuasion game (see Kamenica and Gentzkow 2011, KG henceforth). Voters are uncertain about an underlying state that describes how different policies map into payoffs. One interest group

[3] For concreteness, suppose all voters receive a zero payoff from the status quo, and -1 from a bad reform. A good reform yields payoff $+1$ to each voter in group A and $+0.25$ to each voter in B.

[4] Voters for which the final vote goes in consonance with valence preferences, rather than policy preferences, are dubbed "Stokes voters" by Groseclose (2001).

("sender" or "information controller") can sway voters' decision by designing what voters can learn from a public signal, i.e. by specifying the statistical relation of the signal with the underlying state. For example, the Teachers Union can commission a study about the current situation of public schools in the state, to influence the next gubernatorial election. While the Teachers Union may not be in control of the study's final conclusion, it can nevertheless shape public learning by dictating which questions the report should answer or which angle it should consider. Throughout the paper we maintain the "interest group" reference for simplicity, but we have a broader interpretation in mind. For instance, the information controller can be the current Governor, who wants to influence the election of a Senator, or a local newspaper that wants to affect a mayoral election.

Our basic model features the following ingredients: (i) Electorate: Uninformed voters are divided into two groups, majority A and minority B, with differing preferences/beliefs about the optimal policy. (ii) Candidates: One citizen-candidate from each group competes in a simple-majority rule election without policy commitment. Besides their preferences/beliefs, candidates also differ probabilistically in a second dimension, competence. In this scenario candidate A holds an initial "policy advantage", since a majority of voters prefer his policies over that of candidate B. (iii) Election: After observing the signal provided by the interest group and its realization, candidates' revise their beliefs, and hence their implemented policies if elected, while voters update their evaluation of the candidate's policies. Before the election, voters also observe the valence of each candidate.

A voter chooses candidate A if she is expected to deliver a higher payoff than B. The relevant information for voters' decision is then the degree of political disagreement (the expected policy payoff from candidate A minus that from B) and the valence differential (the realized valence of candidate A minus that of B). From an ex ante perspective, valence differential is a random variable with a known mean. Increasing this mean by shifting the location of the distribution implies that candidate A is expected to be more competent, and hence more likely to win the election. We define conditions on the valence distribution[5] such that the following single-crossing result holds for every signal about the policy issue. If a signal does not increase the victory probability of candidate A when the expected valence

[5]Formally, the distribution of the valence differential has a "thin-tail", in the sense that it follows a log-concave probability measure, such as the normal distribution.

differential is μ, then this signal does not increase her victory probability if the expected valence differential is higher than μ (Lemma 1).

In our basic model, the interest group supports the majority candidate. In this case, the previous result implies that there exists a cutoff μ^* such that the controller does not benefit from persuading voters when the expected valence differential exceeds this cutoff (Propositions 1 and 2). Consequently, the IG prefers not to release information about the policy issue if candidate A has a sufficiently high expected valence. The IG will be (somewhat) transparent if and only if the majority candidate's expected valence is sufficiently low relative to the minority candidate's valence. In fact, when the state is binary, the informativeness of the controller's signal decreases monotonically as the expected valence of the majority candidate increases (Proposition 3). A more competent candidate A induces the IG to supply less information to voters.

In Section 4, we restrict attention to cases where political disagreement stems solely from belief disagreement about the optimal policies. That is, in the absence of uncertainty, all voters would agree on the optimal policy and candidates would be judged solely on their valence. One may conjecture that if public information creates consensus between voters, then the controller will seldom benefit from persuasion and thus belief disagreement will foster opaqueness. We show that this view is flawed as even when information always reduces average belief disagreement, the controller can still find realizations that point-wise increase disagreement and such signals can raise the majority candidate's victory probability. For example, if political disagreement is related to the distance between each group's expected state, the controller generically benefits from releasing some information, irrespective of the valence differential between candidates (Proposition 4). Moreover, even if informative signals *always* reduce average political disagreement, the controller would still supply a signal that increases political disagreement with some probability (Proposition 5). That is, the controller favors (partial) transparency by resorting to signals that sometimes foster dissent.

Our paper relates to the recent papers on Bayesian persuasion that follow KG. As in Alonso and Câmara (2014c), the goal of the interest group in our model is to sway elections in favor of its preferred alternative. However, in Alonso and Câmara (2014c) the interest group simply wants to convince a majority of voters that the proposal is better than the status quo. An important feature of our model is that the interest group would like to convince voters from the majority group not only that their candidate supports a good policy, but also that the minority candidate supports a bad policy. That is, the "relative" expected payoff from the policies (the degree of political disagreement) is crucial.

Our paper also relates to the literature on how access to information can increase polarization and disagreement (e.g., Dixit and Weibull 2007). In most papers, a higher disagreement is a somewhat unintended side-effect of the actions of individuals generating information, such as the media catering information to the demand of biased voters. In our model, the interest group generates information with the sole purpose of increasing disagreement and benefiting its supported candidate.

The rest of the paper is organized as follows. Section 2 presents the basic model. Section 3 characterizes the value of information control and the information content of optimal signals as a function of the valence differential. Section 4 focuses on the role of belief disagreement on the controller's behavior. Section 5 considers several extensions of the basic model and Section 6 concludes. All proofs are in the Appendix.

2 Model

Voters' Preferences: Consider an electorate divided into two groups, A and B, where group A is larger than group B. One citizen-candidate from each group competes in a simple-majority rule election. Voters care about the policy choice and the valence (i.e. competence) of the elected official. If elected, the candidate has to choose one policy x from the compact, convex set $X \subset \mathbb{R}^d$. For example, X can represent the set of feasible governmental budget allocations across d projects, or can represent the government's policy on a left-right Downsian model. Each citizen's payoff from policy x depends on an unknown state $\theta \in \Theta \equiv \{\theta_1, \ldots, \theta_N\}$, $N \geq 2$. Citizens within each group are homogeneous, but groups might differ in their policy preferences. Formally, each citizen in group $i \in \{A, B\}$ has preferences over policies characterized by the von Neumann-Morgenstern utility function $u^i(x, \theta)$, where u^i is a differentiable, strictly quasi-concave function of x. Each candidate is also endowed with a valence η, which we discuss momentarily. For a voter in group i, the total payoff from electing a politician with valence η who implements policy x when state θ

is realized is

$$U^i(\eta, x, \theta) = \eta + u^i(x, \theta).$$

Voters' Prior Beliefs: We assume voters in the same group share a common prior belief, but we allow voters in opposite groups to openly disagree over the likelihood of state θ. That is, voters in group i have a common prior belief $p^i = (p_1^i, \dots, p_N^i)$ in the interior of the simplex $\Delta(\Theta)$, but in some cases we will have $p^A \neq p^B$. We assume that preferences and prior beliefs are common knowledge, so that if voters have different priors then they "agree to disagree." If we interpret θ as describing the mapping between policy x and the payoff outcomes, then different prior beliefs represent differences in voters' views of which outcomes are produced by the different government policies.

Interest Group: One interest group (IG) seeks to maximize the probability the majority candidate wins the election — the IG receives payoff one if candidate A is elected, and zero otherwise.[6] The IG has the same prior belief as the members of group A. The IG can influence the outcome of the election through the design of a public signal that is correlated with the state. For example, the IG can commission a report from a consulting firm on the social payoff generated by the government projects, such as the construction of a high-speed rail. The IG's influence in this case does not come from specifying ex-ante the actual reported outcome, but rather from determining which aspects or factors the consulting firm should investigate, that is, which reported outcomes are feasible. We also refer to the IG as the information controller of simply controller.

Formally, prior to the election the controller chooses a signal π, consisting of a finite realization space S and a family of distributions over S, $\{\pi(\cdot|\theta)\}_{\theta\in\Theta}$, with $\pi(\cdot|\theta) \in \Delta(S)$. Signal π is "commonly understood": π is observed by all players who agree on the likelihood functions $\pi(\cdot|\theta), \theta \in \Theta$ (see Alonso and Câmara 2014a for a discussion of this assumption). Players process information according to Bayes rule, so that $q^i(s|\pi, p)$ is the updated posterior belief[7] of voters in group i with prior p after observing π and its realization s. To simplify

[6]For simplicity, we consider a purely office-motivated IG. For example, in order to implement its policies, the government needs to contract one of two existing private companies. Voters are indifferent between the two companies, but if elected, candidate A will contract the IG's company, while the opposing candidate will hire the competing company. In Section 5 we discuss the case when the IG supports the minority candidate.

[7]We use "posterior belief" to indicate the players' belief about the state after the signal realization but

notation, we use $q^i(s)$ or q^i as a shorthand for $q^i(s|\pi, p)$.

We make two important assumptions regarding the set of signals available to the controller. First, she can choose *any signal* that is correlated with the state. Thus, our setup provides an upper bound on the controller's benefit from information control in settings with more restricted spaces of signals. In particular, he controller will not engage in designing a signal when she faces additional constraints if there is no value of information control in our unrestricted setup. Second, signals are costless to the controller. This is not a serious limitation if each signal is equally costly, and would not affect the choice of signal if the controller decides to influence voters.[8]

Candidates' Policy: Since group identity of candidates and the signal are common knowledge, all voters can correctly infer the updated beliefs q^A and q^B of both candidates after observing the signal realization. There are no commitment devices available to politicians, so that a contender cannot commit to a particular action during the campaign process. However, in equilibrium voters can correctly anticipate the policy that would be chosen by each candidate: if elected, the politician from group i will implement his preferred policy $x^{i*}(q^i) \equiv \arg\max_{x\in X} \sum_{\theta\in\Theta} q^i_\theta u^i(x, \theta)$.

In our basic setup, information about the payoffs from different policies is generated only prior to the elections. Section 5.3 discusses the case where after the election the politician has access to additional information about the unknown state.

Candidates' Valence: Besides this policy dimension, candidates also differ in a valence dimension. Valence η^i of candidate i is stochastic and only observed by all voters after the IG has chosen the signal about θ but before the election. Valences η^A and η^B are draw from some joint probability distribution $\mathcal{F}$. It is useful to work instead with the valence differential,

$$\eta^A - \eta^B \equiv \mu - \xi, \tag{1}$$

where the mean differential μ is a known constant and ξ is a mean-zero random variable

before voting.

[8]However, the optimal signal may change if different signals have different costs. Gentzkow and Kamenica (2014) offer an initial exploration of persuasion with costly signals, where the cost of a signal is given by the expected relative entropy of the beliefs that it induces.

distributed according to F with pdf f. If $\mu > 0$ then candidate A has an expected valence advantage, while if $\mu < 0$ then candidate B has the advantage. Throughout the paper we maintain the following assumption:

(A1) ξ *has full support on the real numbers and follows a log-concave probability measure.*

Note that ξ follows a log-concave probability measure if and only if its density function f is log-concave (see Prékopa 1971). Condition **(A1)** holds, for example, for the normal, logistic, and extreme value distributions. See Bagnoli and Bergstrom (2005) for a discussion on the properties of log-concave density functions.

Although we say that voters observe the "true" valences of candidates, the model can be easily reinterpreted as voters observe a noisy, exogenous signal about valence. In this case, variables η^A and η^B are interpreted as the new expected valence of each candidate, after voters observe the implicit realization of the signal about valence (see Boleslavsky and Cotton 2015 for a model of noisy information about valence).

Election: At the time of the election, voters from group i have a belief q^i about the state and can correctly infer the candidates' beliefs, q^A and q^B. Therefore, voters can predict candidates' policies $x^{*A}(q^A)$ and $x^{*B}(q^B)$. Voters also observe the realized valences η^A and η^B. Thus, for a citizen in group i, the total expected payoff of electing candidate j is

$$\mathcal{U}^{ij}(q^A, q^B, \eta^A, \eta^B) = \eta^j + \sum_{\theta \in \Theta} q^i_\theta u^i(x^{*j}(q^j), \theta).$$

To rule out uninteresting equilibria, we eliminate weakly dominated voting strategies. This implies that each voter votes for the candidate that delivers to him the highest expected utility[9]. The candidate who wins the majority of votes is elected and then implements his preferred policy. Note that voters in group A are pivotal, since the group encompasses a majority of voters. That is, a candidate wins if and only if he receives the support of the majority group.

[9] In our model voters have no private information about the state, so there is no information aggregation problem. Hence, the strategic voting considerations related to the probability of being pivotal are not relevant in our setup.

2.1 Political Disagreement

The previous discussion implies that a voter from group i votes for the candidate from group A if and only if[10]

$$\mathcal{U}^{iA}(q^A, q^B, \eta^A, \eta^B) \geq \mathcal{U}^{iB}(q^A, q^B, \eta^A, \eta^B) \tag{2}$$

$$\iff \sum_{\theta \in \Theta} q^i_\theta \left[u^i(x^{*A}(q^A), \theta) - u^i(x^{*B}(q^B), \theta)\right] \geq -(\eta^A - \eta^B). \tag{3}$$

Using (1), the RHS of (2) captures the realized valence differential. The LHS of (2) captures the *degree of political disagreement* between the two groups. That is, it captures, from the point of view of a voter in group i, the expected policy-payoff difference from electing the different candidates,

$$\mathcal{D}^i(q^A, q^B) \equiv \sum_{\theta \in \Theta} q^i_\theta \left[u^i(x^{*A}(q^A), \theta) - u^i(x^{*B}(q^B), \theta)\right]. \tag{4}$$

Since group A forms a majority, the degree of disagreement from the point of view of voters in A decides the winner. That is, inequality (2) implies that candidate A wins the election if and only if

$$\mathcal{D}^A(q^A, q^B) \geq -\mu + \xi.$$

If the realized ξ is sufficiently high, then even voters from group A vote for candidate B, and vice-versa.

We now rewrite $\mathcal{D}^A$. Let $r_\theta \equiv \frac{p^B_\theta}{p^A_\theta}$ and $r \equiv \{r_\theta\}_{\theta \in \Theta}$ capture the likelihood ratio of prior beliefs. Alonso and Câmara (2014a, Proposition 1) show that independently of the signal π and its realization s, we can rewrite q^B solely as a function of the belief of voters in group A,

$$q^B_\theta = \frac{q^A_\theta r_\theta}{\langle q^A, r\rangle}. \tag{5}$$

Therefore, we can express $\mathcal{D}^A(q^A, q^B)$ as a function of q^A only by

$$D(q^A) \equiv \mathcal{D}^A\left(q^A, q^A \frac{r}{\langle q^A, r\rangle}\right). \tag{6}$$

In summary, political disagreement D measures the difference from the point of view of the majority group in the expected payoff derived from the different policies favored by each group as a result of their differences in preferences and beliefs.

[10] We abstract from abstentions. One could extend our model so that a citizen is less likely to abstain if his expected payoff difference between the candidates is higher, similar to Matsusaka (1995).

After a signal realization that induces q^A, the majority candidate wins if and only if $D(q^A) \geq -\mu + \xi$. Since $\xi \sim F$, the majority candidate wins with probability

$$v(q^A) \equiv F(D(q^A) + \mu). \tag{7}$$

Therefore, candidate A wins the election with a probability that increases in the degree of political disagreement — candidate A has a "policy advantage" because a majority of voters believe he has the "correct" belief and the "correct" preference, and hence he will implement the "correct" policy.

In order to guarantee the existence of an optimal signal and simplify notation in some of the proofs, throughout the paper we maintain the following assumption:

(A2) *Political disagreement D is continuous in $\Delta(\Theta)$, and differentiable at the prior belief.*

Condition **(A2)** holds for a large class of models, including the applications we study in Section 4.

2.2 Notational Conventions

For vectors $q, w \in \mathbb{R}^J$, we denote by $\langle q, w \rangle$ the standard inner product in $\mathbb{R}^J$, i.e. $\langle q, w \rangle = \sum_{j=1}^{J} q_j w_j$, and we denote by qw the component-wise product of vectors q and w, i.e. $(qw)_j = q_j w_j$. We denote $\ln(q) = \{\ln(q_j)\}_{j=1}^{J}$.

For an arbitrary real-valued function g define $\widetilde{g}$ as the concave closure of g,

$$\widetilde{g}(q) = \sup \{y | (q, y) \in co(g)\},$$

where $co(g)$ is the convex hull of the graph of g. Finally, $card(S)$ denotes the cardinality of the set S.

2.3 Controller's Expected Payoff

The controller's problem is to choose a signal π that maximizes, from her point of view, the expected victory probability $E_\pi[v(q^A)]$. Continuity of v ensures the existence of an optimal signal, and choosing an optimal signal is equivalent to choosing a probability distribution σ

over q^A that maximizes $E_\pi[v(q^A)]$, subject to the constraint $E_\sigma[q^A] = p^A$ (see KG). That is, the supremum of the information controller's expected victory probability is

$$V = \sup_\sigma E_\sigma[v(q^A)], \qquad \text{s.t. } E_\sigma[q^A] = p^A.$$

The following remarks follow immediately from KG:

(R1) An optimal signal exists;

(R2) There exists an optimal signal with $card(S) \leq N$;[11]

(R3) The information controller's maximum expected utility is

$$V = \widetilde{v}(p^A); \tag{8}$$

(R4) The value of information control is

$$V - v(p^A) = \tilde{v}(p^A) - v(p^A). \tag{9}$$

3 Political Disagreement, Valence and Information Control

In our model, the controller supports the majority candidate and seeks to maximize his victory probability. Following (7), the likelihood that candidate A wins the election increases in the degree of political disagreement - a larger D implies that, in the eyes of group A voters, the minority candidate B is expected to implement a much "worse policy" than A. As the information obtained from the controller's signal changes the policy championed by each candidate, as well as voters' expected payoff from these policies, it follows that information can change the degree of political disagreement. As a result, the controller's choice of a signal is driven by her desire to uncover information that increases political disagreement.

[11]Note that in the original setup of KG, there exists an optimal straightforward signal that directly recommends an action to the receiver. In our setup, the pivotal majority voter has a binary action space: vote for candidate A or B. However, when $N > 2$ in our model, an optimal signal might require more than two realizations. This is so because from the point of view of the controller, before the valence shock is realized, the voting behavior is probabilistic rather than binary. That is, voting behavior can be interpreted ex ante as a continuous "action" (probability of electing A) in the interval $[0, 1]$ rather than a binary choice.

Importantly, voters are willing to trade policy choices for valence. It then follows that victory probabilities depend fundamentally on both the degree of political disagreement and the distribution of candidates' valences. In this section we address two questions: when does the controller benefit from uncovering information about the state, and how does this gain depend on the candidates' valences? How does the average valence differential μ affect the informativeness of the controller's optimal signal?

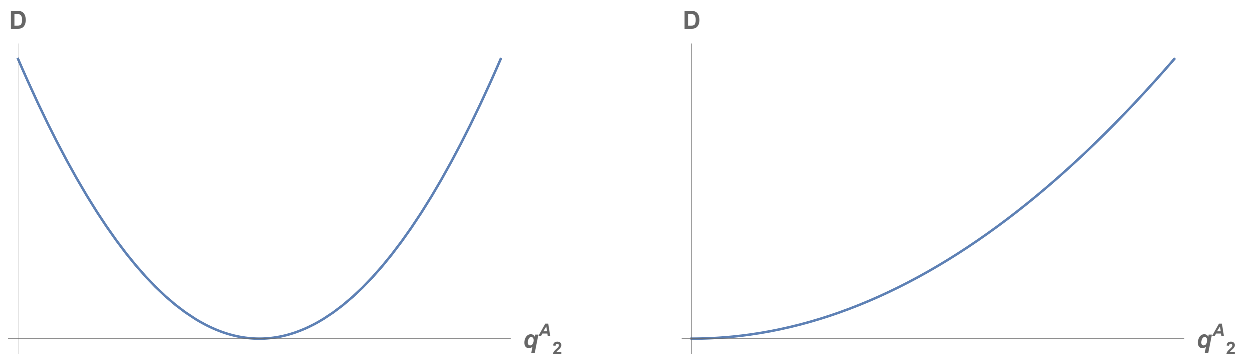
We first show that the gain from a given signal π has a single crossing property: if the gain is non-positive for an average valence differential μ', then it remains non-positive for any larger valence differential $\mu > \mu'$. This property allows us to establish two results on the monotonicity of the optimal signal with respect to μ. First, we show that if it is optimal for the controller to select a completely uninformative signal for μ', then she also finds it optimal to be completely opaque when $\mu > \mu'$. In essence, it is only optimal to uncover information when the majority candidate's expected valence is sufficiently low. Second, when the state space is binary, we show the stronger result that the informativeness of an optimal signal decreases with the average valence differential.

3.1 Using Information to Increase Disagreement

When does the controller benefit from uncovering information that affects political disagreement? In one extreme case, if disagreement is maximum at the prior belief, i.e. $D(q^A) \leq D(p^A)$ for all $q^A \in \Delta(\Theta)$, then a completely uninformative signal is optimal, independently of the expected valence differential μ and distribution F. In the other extreme case, if there exist a signal such that every signal realization increases disagreement, then the controller always benefits from choosing some informative signal, independently of μ and F.[12] Figure 1(a) illustrates such a case, using a binary-state example where political disagreement is maximized when voters know the true state. In this case, a fully informative signal increases disagreement with probability one.

The more interesting case occurs when every signal with realizations that strictly increase political disagreement must also have realizations that strictly decrease disagreement. Figure

[12] Formally, there exists such signal when there exists a set of beliefs $Q \subset \Delta(\Theta)$ such that (i) every belief $q^A \in Q$ increases prior disagreement, $D(q^A) \geq D(p^A)$, with at least one strict inequality, and (ii) the prior belief p belongs to the convex hull of Q.



(a) Controller can Increase D with Certainty

(b) Controller cannot Increase D with Certainty

Figure 1: Political Disagreement as a Function of Belief $q_2^A = Pr(\theta = \theta_2)$, with $\Theta = \{\theta_1, \theta_2\}$.

1(b) presents an example where political disagreement is strictly increasing and convex in q_2^A. In this case, Bayes Rule and strict monotonicity imply that a signal has realizations that increase disagreement if and only if it also has realizations that decrease disagreement. Moreover, convexity implies that any informative signal increases expected disagreement. Even in this case, however, it does not follow that the controller benefits from designing the signal. Indeed, the controller must also take into account how the degree of disagreement interacts with the valence shocks to affect electoral outcomes. We now turn to the study on how the average valence μ and the valence distribution F. affect the value of information control.

3.2 Valence and Information Control

Recall from (1) that a higher μ implies that the majority candidate draws, on average, a higher valence than the minority candidate. As the underlying state θ is independent of the valences of both candidates, the controller's choice of signal cannot affect the distribution of candidate's valences. Nevertheless, if the controller has access to a signal that on average increases disagreement, as in the example in Figure 1(b), it is then not clear why she would not gain from this signal independently from the expected valence μ. The next lemma shows that this gain actually satisfies a single-crossing condition: if a signal does not increase the expected victory probability for some μ', then the controller cannot gain from this signal for any $\mu > \mu'$.

Lemma 1 *Suppose that for some signal* π *and valence differential* μ' *we have* $E_\pi[v(q^A)|\mu'] \leq v(p^A)$. *Then, for any* $\mu > \mu'$ *we have* $E_\pi[v(q^A)|\mu] \leq v(p^A)$.

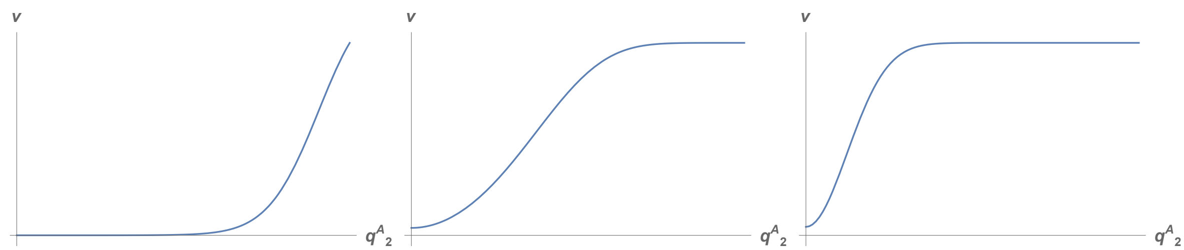
Following the lemma, improving the valence position of the majority candidate can never turn a given signal valuable to the controller. Conversely, if the controller gains from a given signal, she still does so as the valence position of the majority candidate worsens relative to the minority candidate. The intuition for this result is rooted in the fact that for log-concave probability measures the ratio $F(x + \Delta)/F(x)$, $\Delta > 0$, decreases in x. Therefore, and letting $x = D(p^A) + \mu$, the percentage increase in victory probability from a signal realization that increases political disagreement by an amount Δ, which equals $\left(F(x + \Delta) - F(x)\right)/F(x)$, decreases with the likelihood of the majority candidate's victory in the absence of the controller's signal (i.e. if $F(x)$ increases). Notice, in particular, that this property is satisfied irrespective of whether the majority candidate is expected to win the election ($F(x) > 1/2$) or the minority candidate is the frontrunner ($F(x) < 1/2$) in the absence of the controller's signal.

This single-crossing property for signals naturally translates to a single-crossing property for the value of information control (9), $V - v(p^A)$.

Proposition 1 *If* V *denotes the expected victory probability from an optimal signal and for* μ' *we have* $V - v(p^A) = 0$, *then* $V - v(p^A) = 0$ *for any* $\mu > \mu'$.

The fact that the value of information control $V - v(p^A)$ satisfies a single-crossing property implies that there exists a cutoff μ^* in the extended real line such that the controller chooses to be transparent and provides an informative signal when the majority candidate's expected valence is sufficiently low, that is when $\mu < \mu^*$, and chooses to be completely opaque when this expectation is sufficiently high, i.e. when $\mu > \mu^*$. Figure 2 illustrates how increasing μ can change the overall curvature of v, using the political disagreement D from Figure 1(b).

The single-crossing property of Proposition 1 does not guarantee that the cutoff μ^* is finite. In Proposition 2 we provide sufficient conditions to bound this cutoff from above or below. First, if there exists a signal for which every signal realization increases political disagreement, then the controller benefits from (some) transparency for every μ, and thus $\mu^* = +\infty$. Nevertheless, such signal does not exist when D is concave, which bounds μ^* from



(a) Low Values of μ (b) Intermediate Values of μ (c) High Values of v

Figure 2: Example of the Effects of Expected Valence μ on Victory Probability v.

above. Second, we provide conditions that bound μ^* from below. These conditions follow as negations of sufficient conditions for the controller to draw no value from designing the signal. This is the case when political disagreement is already at its maximum, $D(p^A) \geq D(q^A)$. It is also the case when v is globally concave for all finite values of μ.

Related to this last condition, we provide two alternative sufficient conditions that guarantee that v is not globally concave. First, if D is locally convex at the prior belief, then for any valence distribution satisfying **(A1)** v is locally convex at the prior for sufficiently small μ. Second, if D is locally concave at the prior belief, then we need distribution F to be "sufficiently convex" for low values of μ. More specifically, for a given bound on the curvature of D, the function v is locally convex at the prior if the limit of the reverse hazard rate $\frac{f(\xi)}{F(\xi)}$ as $\xi \to -\infty$ exceeds a related bound. It follows that if $\lim_{\xi \to -\infty} \frac{f(\xi)}{F(\xi)} = \infty$, then the function v is locally convex, and hence $\mu^* > -\infty$ regardless of the curvature of D. This condition on the limit of the reverse hazard rate holds, for example, for the normal distribution. We summarize these findings in the following proposition.

Proposition 2 *Suppose that **(A2)** holds, and let μ^* be such that $V - v(p^A) = 0$ iff $\mu \geq \mu^*$. Then,*

(a) if political disagreement D is concave, then $\mu^* < +\infty$;

(b) if political disagreement is not maximized at the prior belief, and either D is locally convex at the prior belief or $\lim_{\xi \to -\infty} \frac{f(\xi)}{F(\xi)} = \infty$, then $\mu^* > -\infty$.

3.3 The Informativeness of Optimal Signals

In the previous section we showed that improving the expected valence of the majority candidate cannot make the controller benefit from designing the signal. In fact, for any cutoff

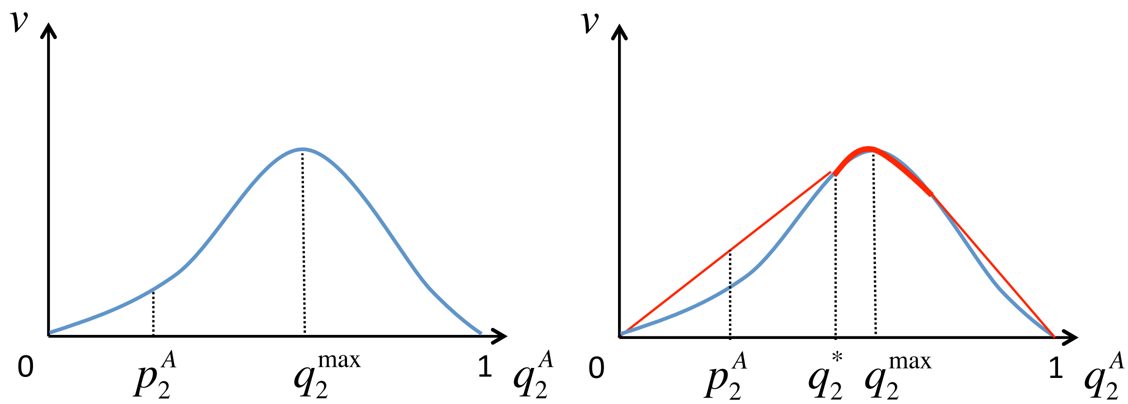
$\mu^* < \infty$ defined in Proposition 2, the controller supplies a completely uninformative signal whenever $\mu > \mu^*$. In this section we study the informativeness of the controller's optimal signal for different values of the expected valence. In order to provide a full characterization, we focus on the binary-state case, $\Theta = \{\theta_1, \theta_2\}$. For this case, we show that the controller always provides a less Blackwell-informative signal as the majority candidate has a higher expected valence.

Let q_2^A be the belief attached to state θ_2. Obviously, if the prior belief already maximizes political disagreement, then no information will be disclosed for any valence distribution. Hence, we focus on the relevant case where the prior p_2^A is less than the belief q_2^{max} that maximizes disagreement (the case $p_2^A > q_2^{max}$ is equivalent to a relabeling of the states). Moreover, to guarantee the existence of a unique class of optimal signals and derive sharper comparative statics, we need the concave closure of v in the range $[0, q_2^{max})$ to be "well behaved." To this end, in this range, we assume that payoff v is strictly increasing, and either concave or changes from convex to concave once. See the example in Figure 3(a). Under these conditions, there exists a cutoff $q_2^* \in [0, q_2^{max})$ such that the concave closure of v is a straight line in the range $[0, q_2^*]$, and v itself in the range $[q_2^*, q_2^{max}]$. See the example in Figure 3(b).

This characterization of the concave closure has two implications for optimal signals. First, if $q_2^* < p_2^A$, then a completely uninformative signal is optimal. Second, if $q_2^* > p_2^A$, then the optimal signal is partially informative and induces exactly two posterior beliefs, $q_2^A = 0$ and $q_2^A = q_2^*$. Posterior $q_2^A = 0$ arises after a signal realization that perfectly reveals that the state is θ_1 and decreases political disagreement to a local (possibly global) minimum. Posterior $q_2^A = q_2^*$ arises after a partially informative realization that increases disagreement and satisfies $q_2^* < q_2^{max}$. Interestingly, it is never optimal to increase disagreement to its maximum, but it is sometimes optimal to decrease disagreement to its minimum.

Finally, we show that the cutoff q_2^* decreases with μ. Since an optimal signal is supported on posteriors $q_2^A = 0$ and $q_2^A = q_2^*$, a lower valence advantage induces a more Blackwell-informative signal — equilibrium posterior beliefs move further away from the prior.

We now formally present these results. Since v is the composite of F and $(D + \mu)$, we state our assumptions in terms of these fundamental functions. First, in addition to **(A1)**, we follow Proposition 2(b) and assume that



(a) Payoff v

(b) Concave closure of v

Figure 3: Example of payoff v.

(A1′) *Density f is strictly log-concave, with* $\lim_{\xi \to -\infty} \frac{f(\xi)}{F(\xi)} = \infty$.

Second, and similarly to the strict log-concave assumption on f, we assume that D_2 (the derivative of D wrt q_2^A) is strictly log-concave in the range where it is positive. That is we extend (**A2**) in the following manner

(A2′) *Political disagreement D is twice differentiable. The derivative D_2 is strictly positive and strictly log-concave in the range $(0, q_2^{max})$, and negative in the range $(q_2^{max}, 1)$, where $0 < p_2 < q_2^{max} \leq 1$.*

Together assumptions **(A1′)** and **(A2′)** guarantee that v is strictly increasing in $[0, q_2^{max}]$, and either concave or changes from convex to concave once.

Proposition 3 *Suppose $\Theta = \{\theta_1, \theta_2\}$, and that **(A1′)** and **(A2′)** hold. Then transparency is a decreasing function of the majority candidate's expected valence, that is, signal π^* becomes less Blackwell-informative as μ increases.*

The optimal signal π^* takes the following form: There exixts a cutoff μ^* and belief $\overline{q}_2^A$ such that

(i) If $\mu \geq \mu^*$, then π^* is completely uninformative;

(ii) If $\mu < \mu^*$, then π^* induces two posterior beliefs $\underline{q} = (1, 0)$ and $\bar{q} = (1 - \overline{q}_2^A, \overline{q}_2^A)$, $\overline{q}_2^A \leq q_2^{max}$.

The proof of the proposition characterizes the cutoff μ^* and belief $\overline{q}_2^A$. Although there are multiple optimal signals, we show that in our model they all induce the same distribution over posterior beliefs: With probability $\frac{\overline{q}_2^A - p_2^A}{\overline{q}_2^A}$ players learn that the state is θ_1, and with probability $\frac{p_2^A}{\bar{q}_2^A}$ players from group A update their beliefs to $\bar{q}$. In the first case, players share the common posterior $\underline{q}$, which reduces political disagreement to $D(0) < D(p^A)$, while in the second case political disagreement is increased, $D(\overline{q}_2^A) > D(p^A)$. If disagreement is maximized when voters know that the state is θ_2, $q_2^{max} = 1$, then full information disclosure might be optimal (see Example 2 below). However, if $q_2^{max} < 1$, then full information disclosure is never optimal.

Interestingly, if $q_2^{max} < 1$, then the optimal signal does not induce the highest possible disagreement, $\overline{q}_2^A < q_2^{max}$. This is so because the controller faces a trade-off between increasing political disagreement by inducing a belief closer to q_2^{max} and increasing the likelihood that political disagreement is reduced. Hence, although the majority candidate derives his political advantage from disagreement, an optimal signal decreases disagreement with positive probability, but it does not increase disagreement to its maximum.

Finally, we show in the appendix that $\overline{q}_2^A$ decreases with $\mu < \mu^*$. That is, increasing the majority candidate's valence would draw $\overline{q}_2^A$ closer to the prior belief while keeping $\underline{q}$ constant. While the controller optimally selects a signal with a lower increase in disagreement, it is also less informative. This is so as this change increases the likelihood of posterior $\bar{q}$ (which is also closer to the prior), while it decreases the likelihood of posterior $\underline{q}$.

3.4 Example: Adaptation under Conflicting Interests

We illustrate our results in an application to information acquisition and adaptation, where citizens have conflicting preferences. To clarify the role of conflicting preferences, suppose players share a common prior p. Common priors and a commonly understood public signal imply that players share a common posterior q, so that political disagreement derives solely from preference disagreement. Consider a state space $\Theta = \{\theta_1, \theta_2\} \subset \mathbb{R}$. For voters in group $i \in \{A, B\}$ consider the quadratic policy payoff $u^i(x, \theta) = -(x - \beta^i \theta)^2$, where parameters $0 < \beta^B < \beta^A$ capture the misalignment in preferences. The optimal policy of each group is proportional to the expected value of the state, $x^{i*}(q) = \beta^i E[\theta|q]$. Preferences are partially aligned: when $E[\theta|q] \neq 0$, players agree on the direction (sign) of the optimal policy, but not on its magnitude. That is, since β^B is smaller, politician B "underreacts to information" relative to politician A, in the sense that he chooses actions closer to zero. Political disagreement (6) becomes

$$
\begin{aligned}
D(q) &= \mathcal{D}^A(q,q) \\
&= \sum_{\theta\in\Theta} q_\theta \left[-(x^{A*}(q)-\beta^A\theta)^2+(x^{B*}(q)-\beta^A\theta)^2\right] \\
&= (E[\theta|q])^2\left(\beta^A-\beta^B\right)^2. \qquad (10)
\end{aligned}
$$

Let $\beta^A = 2$ and $\beta^B = 1$, so that (10) simplifies to $D(q) = (E[\theta|q])^2$. We consider the following examples.

Example 1 — Extreme Preference Disagreement: Let $\Theta = \{-1, +1\}$, which corresponds to the case illustrated in Figure 1(a). Disagreement is maximized when voters know the true state. In this case, a fully informative signal is optimal independently of the prior belief p, the expected valence μ, or the valence distribution F. □

Example 2 — Partial Preference Disagreement: Let $\Theta = \{0, +1\}$, which corresponds to the case illustrated in Figure 1(b). Disagreement is maximized when voters know the state is +1, and minimized when voters know the state is 0 as in this case they all agree on the optimal policy. Disagreement is strictly increasing in the belief q_2^A, so the controller always faces a trade off: any signal that increases disagreement with positive probability must also decrease disagreement with positive probability. Consequently, whether or not the controller chooses an informative signal depends on the valence distribution. Following Propositions 1 and 3, if F satisfies **(A1′)** (e.g., if F is a normal distribution), then the controller prefers a fully informative signal if μ is sufficiently low, a partially informative signal if μ has intermediate values, and a completely uninformative signal if μ is sufficiently high. This is the case even though political disagreement is a convex function of beliefs, so that *any* informative signal increases expected disagreement. Figure 2 illustrates how μ affects the curvature of v. □

4 The Role of Belief Disagreement

In this Section we restrict attention to cases where political disagreement stems solely from belief disagreement. This is an important question since, as Callander (2011) points out, a large part of the difficulty in policy making is that policy makers may be uncertain about which policies produce which outcomes, and much political disagreement is over beliefs about this mapping. To this end, we focus on cases where all voters share the same preferences, so that political disagreement is zero when voters share a common belief. That is, voters would agree on the optimal policy if they all had the same beliefs about the state.

While our previous results on the interaction of valence and political disagreement still hold, restricting attention to belief disagreement clarifies the role of the public signal on affecting political disagreement. We first study a spatial policy model, where the degree of political disagreement arises from differing beliefs about the expected value of the state. We show that the IG generically benefits from information control in this case for any valence differential. We then study a budget allocation model where the degree of political disagreement is given by the relative entropy, which is a widely used measure of belief disagreement. In this case information always fosters consensus: every signal (weakly) reduces average political disagreement. Nevertheless, the IG may find it optimal to release (some) information. Finally, we contrast political disagreement and polarization.

4.1 Spatial Policy

Consider a spatial policy model where the state $\theta \in \Theta \subset \mathbb{R}$ represents the optimal policy in a left-right dimension. Let $\theta_1 < \cdots < \theta_N$ and $X = [\theta_1, \theta_N]$. Suppose voters share a common quadratic policy payoff $u(x, \theta) = -(x - \theta)^2$, so that the optimal policy equals the expected value of the state, $x^{i*}(q^i) = E[\theta|q^i]$. Political disagreement (4) translates naturally into the degree of belief disagreement over expectations,

$$\begin{aligned}
\mathcal{D}^A(q^A, q^B) &= \sum_{\theta\in\Theta} q^A_\theta \left[u^A(x^{*A}(q^A), \theta) - u^A(x^{*B}(q^B), \theta)\right] \\
&= \sum_{\theta\in\Theta} q^A_\theta \left[-(E[\theta|q^A] - \theta)^2 + (E[\theta|q^B] - \theta)^2\right] \\
&= (E[\theta|q^A] - E[\theta|q^B])^2.
\end{aligned}$$

Using (5) to rewrite q^B as a function of q^A, political disagreement becomes a quasilinear function of belief q^A,

$$D(q^A) = \left(\langle q^A, \theta\rangle - \langle q^A \frac{r}{\langle q^A, r\rangle}, \theta\rangle\right)^2 = \left(\langle q^A \left(1 - \frac{r}{\langle q^A, r\rangle}\right), \theta\rangle\right)^2.$$

Although voters share a common payoff function, we show in the next proposition that if the state space is rich enough ($N \geq 4$), then the controller can generically design a signal that increases political disagreement with probability one.

Proposition 4 *In the spatial policy model, if $N \geq 4$, then the controller can generically design a signal that increases political disagreement with probability one. Consequently, the value of information control is positive for each finite valence differential.*

Example 3 — Increasing Belief Disagreement: Let $\Theta = \{-2, -1, +1, +2\}$ and consider priors $p^A = (.4, .1, .1, .4)$ and $p^B = (.1, .4, .4, .1)$. Note that $E[\theta|p^A] = E[\theta|p^B] = 0$, hence initial political disagreement is zero, although prior beliefs are different. Now consider a binary signal $S = \{s_L, s_R\}$ so that the left states -2 and -1 induce signal s_L with probability one, while the right states $+1$ and $+2$ induce signal s_R with probability one. After observing signal s_L beliefs become $E[\theta|q^A] = -1.8$ and $E[\theta|q^B] = -1.2$, while s_R induces $E[\theta|q^A] = 1.2$ and $E[\theta|q^B] = 1.8$. Therefore, the signal induces a strictly higher belief disagreement for any of its realizations. $\square$

4.2 Budget Allocation

Consider the following budget allocation model. The government has one dollar to allocate among $N \geq 2$ different government projects. Let $x_n \geq 0$ represent the amount of money allocated to project n, such that the budget balances. Thus $X = \{x \in [0,1]^N | \sum_{n=1}^N x_n = 1\}$ and the vector $x = (x_1, \ldots, x_N) \in X$ represents a complete government budget.

There is uncertainty about the payoff derived from investing in each project. To simplify presentation, we consider the case where only one project is beneficial to voters — only one project can increase voters' payoff — while investment in any other project delivers a payoff of zero. Formally, there are N possible states, $\theta \in \Theta \equiv \{1, \ldots, N\}$, and citizens share a common payoff function: if the realized state is $\theta = n$ then voters receive a logarithmic payoff $\ln(x_n)$. In other words, $u(\theta, x) = \sum_{n=1}^N \mathbb{1}(n, \theta) \ln(x_n)$, where $\mathbb{1}(n, \theta) = 1$ if $\theta = n$, and $\mathbb{1}(n, \theta) = 0$ if $\theta \neq n$. If voter i has belief $q^i = (q_1^i, \ldots, q_N^i)$, then budget x delivers an expected policy payoff $\sum_{n=1}^N q_n^i \ln(x_n)$. The logarithmic utility implies that each voter prefers the budget to be allocated proportionally to his own beliefs — the preferred budget x^{i*} of voter i is simply $x_n^{i*} = q_n^i$ for all n, where we apply the convention $0\ln(0) = 0$.

In this case, the political disagreement (4) becomes

$$\mathcal{D}^A(q^A, q^B) = \sum_{n=1}^{N} q_n^A \left[\ln(q_n^A) - \ln(q_n^B)\right] = \sum_{n=1}^{N} q_n^A \ln\left(\frac{q_n^A}{q_n^B}\right) \equiv D_{KL}(q^A||q^B).$$

The relative entropy $D_{KL}(q^A||q^B)$, or Kullback-Leibler distance[13] between probability distributions q^A and q^B, is a measure of the belief disagreement between the two groups. Therefore, in our electoral model, the degree of of *political disagreement* is given directly by the level of *belief disagreement* as measured by the relative entropy: from the point of view of the majority group, D_{KL} measures the difference in the expected payoff derived from the different policies favored by each group. Political disagreement is zero if and only if both groups share common beliefs, and it is increasing in the extent of belief disagreement between the groups.

Remark 1: Information always decreases average political disagreement. Indeed, rewriting (6),

$$\begin{aligned} D(q^A) &= \mathcal{D}^A\left(q^A, q^A \frac{r}{\langle q^A, r\rangle}\right) \\ &= \sum_{n=1}^{N} q_n^A \ln\left(\frac{q_n^A \langle q^A, r\rangle}{q_n^A r_n}\right) \\ &= \ln\left(\langle q^A, r\rangle\right) - \langle q^A, \ln(r)\rangle. \end{aligned} \tag{11}$$

Thus $D(q^A)$ is concave as the sum of two concave functions, and Jensen's inequality implies that $E_\pi[D(q^A)] \leq D(p^A)$ for any signal π.

Nevertheless, it is possible to design a signal with at least one realization that strictly increases disagreement under the following condition:

Condition C1: Interior prior beliefs (p^A, p^B) are such that $r_\theta - \ln(r_\theta) \neq r_{\theta'} - \ln(r_{\theta'})$ for at least one pair of states $\theta, \theta' \in \Theta$.

Condition **C1** is violated if priors are common, in which case $r_\theta - \ln(r_\theta) = 1$ for all states. Nevertheless, condition **C1** holds generically, where genericity is interpreted over the space of

[13]Although it is not formally a distance measure, the relative entropy is a measure of the inefficiency of assuming that the probability distribution is q^B when the true distribution is q^A. See Cover and Thomas (2006, Chapter 2) for a discussion.

pairs of prior beliefs. If condition **C1** holds, then the IG can increase political disagreement with arbitrarily high probability.

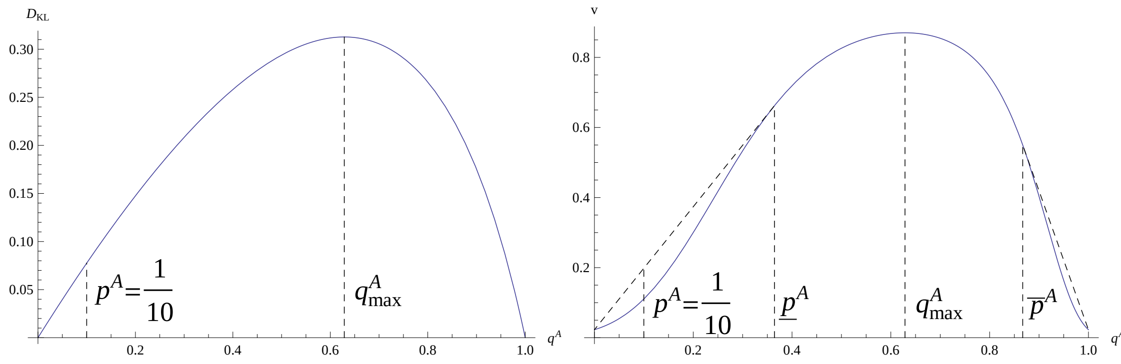
Lemma 2 *If condition **C1** holds, then for any $\delta \in (0,1)$ the information controller can design a signal π with realization s^+ such that: (i) s^+ strictly increases political disagreement, and (ii) s^+ occurs with probability δ.*

Full information disclosure always maximizes candidate B's victory probability, therefore the controller will never be totally transparent.

The policy advantage of candidate A derives solely from the belief disagreement among voters, and information disclosure always decreases the expected disagreement (Remark 1). In fact, a controller seeking to appoint the majority candidate but always strictly prefer complete opaqueness to full transparency. Does the controller ever benefit from disclosing some information? The answer is yes if the valence disadvantage of the majority candidate is sufficiently large. Following proposition 2, this is the case if $\lim_{\xi \to -\infty} \frac{f(\xi)}{F(\xi)} = \infty$ and condition **(C1)** holds (since Lemma 2 implies political disagreement can be increased).

Proposition 5 *In the budget allocation model, if $\lim_{\xi \to -\infty} \frac{f(\xi)}{F(\xi)} = \infty$ and condition **(C1)** holds, then there exists a finite cutoff μ^* such that a partially informative is optimal iff $\mu < \mu^*$.*

Example: Consider the example in Figure 4. Both groups believe that the state is $\theta = 1$ with high probability, hence the initial disagreement is small. Moreover, it is very likely that at the time of the election voters will view candidate B as more competent. Consequently, without information disclosure candidate A wins the election with probability 11% (from the point of view of the IG). The following optimal signal increase this victory probability to 19.8%. The IG chooses signal $s \in \{s_1, s_2\}$ where $Pr(s = s_2|\theta = 2) = 1$, $Pr(s = s_2|\theta = 1) \approx 0.19$. Signal s_1 eliminates disagreement and reduces the probability of winning to 2%; signal s_2 increases disagreement and the probability of winning to 66%. Overall, the IG expects an average probability of winning 19.8%. If the IG were to choose the more informative $Pr(s = s_2|\theta = 1) \approx 0.065$, then following signal s_2 the political disagreement would be maximized and the probability of winning, conditional on this signal realization, would be 87%. However, the IG would then expect an average victory probability of 15.7%.



(a) Disagreement (b) Probability Candidate A Wins

Figure 4: Disagreement and Victory Probability, $\mu = \frac{2}{10}$, $\sigma = \frac{1}{10}$, $p^A = \frac{1}{10}$, $p^B = \frac{1}{46}$, $R = 5$.

4.3 Political Disagreement vs. Polarization

We conclude this Section by noting that our notion of "political disagreement" is different from the notion of "polarization" present in many papers in the literature.[14] That is, an increase in political disagreement does not imply an increase in polarization, and vice versa. To illustrate this point, consider the spatial policy model from Section 4.1, where the state θ represents the optimal policy in a left-right dimension. Moreover, all voters share the same Euclidean policy payoff $u(x, \theta) = -|x - \theta|$. In this case, the optimal policy equals the expected median of the state, $x^{i*}(q) = M[\theta|q]$. Political disagreement then becomes

$$D^A(q^A, q^B) = -\sum_{\theta' \in \Theta} q^A_{\theta'} \Big[\big| M[\theta|q^A] - \theta' \big| - \big| M[\theta|q^B] - \theta' \big| \Big].$$

To see that information might strictly increase polarization and strictly decrease political disagreement, let $\Theta = \{-2, -1, +1, +2\}$ and consider priors $p^A = (.06, .8, .1, .04)$ and $p^B = (.04, .1, .8, .06)$. At the prior belief, candidate A prefers policy -1 while candidate B prefers $+1$. The original degree of political disagreement is $D^A(p^A, p^B) = 36/25$. Initial political disagreement is high because voters in group A are very confident that the state is -1 and not $+1$. Consider a binary signal that simply reveals if the state is in the partition $\{-1, +1\}$ or $\{-2, +2\}$. When the signal reveals that the state is in partition $\{-2, +2\}$, updated beliefs become $q^A = (.6, 0, 0, .4)$ and $q^B = (.4, 0, 0, .6)$. Candidates' preferred policies change to -2 and $+2$. Hence, the information results in more polarized policies. However, there is now

[14] Although there are different definitions of polarization in the literature, here we define polarization as the Euclidean distance between the policies supported by the candidates as in Dixit and Weibull (2007).

a lower degree of political disagreement, $D^A(q^A, q^B) = 4/5$. Although policies are more polarized (farther away from each other), voters in group A now believe that there is a much higher chance that the opposing policy championed by candidate B might be the correct policy. In a nutshell, optimal policies are further apart, but voters suffer a smaller loss from appointing the rival candidate.

5 Extensions

5.1 Controller Supports the Minority

Suppose the information controller supports the minority candidate — she receives payoff one if candidate B is elected, and zero otherwise. It is straightforward to rewrite the basic model so that the controller's payoff v reflects the minority's victory probability. Since the political advantage of the majority candidate is solely due to political disagreement, the controller now benefits from *decreasing* political disagreement. Similarly to the original analysis, if the initial political disagreement is already at the minimum, then the controller cannot benefit from designing a signal, independently of μ and F. If there exists a signal such that every realization decreases political disagreement, then the controller benefits from selecting a signal, independently of the valence distribution. In the remaining case, the controller must trade off decreases and increases in political disagreement, in which case the whether or not the controller chooses an informative signal depends on the valence distribution. The converse of Proposition 1 holds: there exists a cutoff μ^* such that the controller chooses an uninformative signal when $\mu < \mu^*$, and an informative signal when $\mu > \mu^*$.

In particular, consider the models of Section 4, where citizens share the same payoff function but hold different prior beliefs. In these cases, regardless of prior beliefs, full information disclosure is always optimal for the minority candidate. Complete transparency eliminates political disagreement and the policy advantage of the majority candidate, increasing the chances of the minority candidate.

5.2 Independent Shocks

In the basic model we assume that all voters receive the same valence shock. Now suppose the opposite: there is no aggregate uncertainty over valence, only individual uncertainty. To this end, suppose there is a measure one of voters, where fraction $\alpha \in (1/2, 1)$ of voters form group A, and $(1-\alpha)$ form group B. Each voter draws an i.i.d. shock from F. given posteriors (q^A, q^B), a fraction $F(D^A(q^A, q^B) + \mu)$ of voters from group A vote for candidate A, while fraction $F(D^B(q^A, q^B) + \mu)$ of voters from group B vote for candidate A.

Trade-off: more political disagreement increases the chances A votes for A, but may (!) reduce the chances B votes for A (does not necessarily reduces, because it is the disagreement from the point of view of voters in B).

Consequently, candidate A wins if he receives at least half of the votes, $\alpha F(D^A(q^A, q^B) + \mu) + (1-\alpha)F(D^B(q^A, q^B) + \mu) \geq 0.5$, and loses otherwise. The expected payoff of a controller supporting candidate A is

$$v(q^A, q^B) = \begin{cases} 1 & \text{if} \quad \alpha F(D^A(q^A, q^B) + \mu) + (1-\alpha)F(D^B(q^A, q^B) + \mu) \geq 0.5 \\ 0 & \text{if} \quad \alpha F(D^A(q^A, q^B) + \mu) + (1-\alpha)F(D^B(q^A, q^B) + \mu) < 0.5 \end{cases}$$

In this case, one can show the following. Suppose that, at the prior belief, disagreement is no at its maximum. Then there exists two cutoffs μ^* and $\underline{\mu}$ such that: no information disclosure is optimal if $\mu > \mu^*$, and there exists an optimal informative signal if $\underline{\mu} < \mu < \mu^*$. If $\mu < \underline{\mu}$, then the expected valence is so low that the majority candidate cannot win, even if disagreement is increased to its feasible maximum. This holds in the i.i.d. case with a continuum of voters because there is no aggregate uncertainty over valence and disagreement is assumed to be finite.

5.3 The Role of Post-election Information

Our basic model posits that informative signals of the underlying state will only be available to voters and politicians before voters are called to a vote. This can be the case if there is ample time prior to an election to evaluate alternative policies, while the elected politician would need to quickly implement the chosen policy once in office. In other cases, however, politicians may have access to additional information after the election, for instance as uncertainty naturally resolves. The policy-motivated politician would then take this new information into account when selecting her policy. We now show which results extend to the case where politicians receive an additional signal about the state once in office.

Suppose that after the election, but before choosing a policy, the elected politician has access to additional information. Formally, the elected politician observes a signal τ that is correlated with the underlying state θ. Let Z be the set of possible signal realizations, with $z \in Z$. If at the time of the election voters and politicians have beliefs $\{q^A, q^B\}$, it is straightforward to compute updated beliefs $\{q'^A(z), q'^B(z)\}$ obtained through Bayes' rule after observing realization z of signal τ. It is also straightforward to compute policy $x^{*i}(q'^i(z))$ that would be chosen by candidate i. Consequently, at the time of the election, from the point of view of voters in majority group A, the expected policy-payoff difference (4) from electing the two candidates now takes the form

$$\mathcal{D}_\tau^A(q^A, q^B) \equiv E_\tau[u^A(x^{*A}(q'^A(z)), \theta)|q^A] - E_\tau[u^A(x^{*B}(q'^B(z)), \theta)|q^A], \tag{12}$$

where $E_\tau[\cdot|q^A]$ denotes the expectation over the distribution of posterior beliefs that is obtained through Bayes' rule from belief q^A and the signal τ. Using (5), we can then obtain the *expected* degree of political disagreement from the perspective of group A as

$$D_\tau(q^A) \equiv \mathcal{D}_\tau^A\left(q^A, q^A \frac{r}{\langle q^A, r\rangle}\right).$$

The results of Lemma 1 and Propositions 1, 2 and 3 are directly applicable to this definition of the degree of political disagreement. In essence, even if the elected politician has access to better information after the election, it is still the case that improving the majority's valence advantage makes it less likely for the IG to engage in persuasion. Notably, however, access to better information will affect our results in Section 4. Recall that in Section 4 voters have the same underlying preferences, hence they agree on the optimal policy if they know the true state. In this case, if the post-election signal τ is perfectly informative of the state, then the IG cannot benefit from disclosing information prior to the election. This is so because expected political disagreement is zero, $D_\tau(q^A) = 0$, as the same policy will be implemented regardless of the identity of winner. Consequently, Proposition 4 and 5 no longer hold. However, if the post-election signal is not fully informative, then one can redefine conditions on Proposition 4 and 5 such that the IG can benefit from disclosing some information prior to the election.

6 Conclusion

To be included.

A Appendix

Before we present the proofs of Propositions 1 and 2, we provide the following Lemma.

Lemma A. 1 *Define*

$$G(a,b,\mu) \equiv \frac{F(b+\mu) - F(a+\mu)}{f(a+\mu)}, \tag{13}$$

*where F and f satisfy **(A1)**. Fix any $a, b, c \in \mathbb{R}$. Then:*

(i) $G(a,b,\mu)$ is non-increasing in μ, and it is strictly decreasing if f is strictly log-concave and $a \neq b$;

(ii) There exists a $\mu' < +\infty$ such that for any $\mu \geq \mu'(a,b)$ we have $G(a,b,\mu) \leq b-a$; and a $\mu'' < +\infty$ such that for any $\mu \leq \mu''(a,b)$ we have $G(a,b,\mu) \geq b-a$

(iii) If $b > a$ and $\lim_{\xi \to -\infty} \frac{f(\xi)}{F(\xi)} = \infty$, then there exists a $\mu' > -\infty$ such that for any $\mu \leq \mu'$ we have $\frac{F(b+\mu)-F(a+\mu)}{F(a+\mu)} \geq c$;

Proof of Lemma A.1:

Part (i): We first rewrite the function G as

$$G(a,b,\mu) = \int_0^{b-a} \frac{f(a+\mu+s)}{f(a+\mu)} ds.$$

Since f is log-concave, it exhibits decreasing ratios in the sense that for every $s > 0$ and $\mu \geq \mu'$ we have

$$\frac{f(a+\mu'+s)}{f(a+\mu')} \geq \frac{f(a+\mu+s)}{f(a+\mu)}. \tag{14}$$

Suppose first that $b > a$. Then integrating both sides of (14) between 0 and $b-a$ shows that $G(a,b,\mu') \geq G(a,b,\mu)$. Now suppose that $a > b$. Then for any $s \in [0, a-b]$ we can rewrite (14) as

$$\frac{f(a+\mu-s)}{f(a+\mu)} \geq \frac{f(a+\mu'-s)}{f(a+\mu')}.$$

Integrating between 0 and $b-a$ we conclude that

$$-G(a,b,\mu)=\int_0^{a-b}\frac{f(a+\mu-s)}{f(a+\mu)}ds\geq\int_0^{a-b}\frac{f(a+\mu'-s)}{f(a+\mu')}ds=-G(a,b,\mu'),$$

or, in other words, $G(a,b,\mu')\geq G(a,b,\mu)$.

To show that strict monotonicity follows from strict log-concavity, we first note that the pdf f must be continuous in its support (as the exponential of a concave, and hence continuous, function). Strict log-concavity implies that (14) holds with strict inequality whenever $\mu>\mu'$; the continuity of f implies that if $\mu>\mu'$, then the minimum separation between the left hand side and right hand side in (14) is bounded away from zero for any $s\in(0,a-b]$. Therefore, the integral is also bounded away from zero and $G(a,b,\mu')>G(a,b,\mu)$.

Part (ii-iii): Fix any c and $b>a$. We now establish some general facts that will be used in the proofs of Parts ii-iv. Log-concavity of f and full support of F imply that F is also log-concave. Therefore, the ratio $\frac{f(\xi)}{F(\xi)}$ is everywhere decreasing. Log-concavity of f also implies that f is unimodal. Full support then ensures the existence of a finite μ^* such that f is weakly increasing for $\mu\leq\mu^*$ and weakly decreasing for all $\mu\geq\mu^*$.

Part (ii): Fix any $a,b\in\mathbb{R}$, and define $\mu'=\mu^*-\min\{a,b\}$. This implies that for any $\mu\geq\mu'$, f is weakly decreasing in $[\min(a+\mu,b+\mu),\max(a+\mu,b+\mu)]$. We now show that for any $\mu\geq\mu'$, $G(a,b,\mu)\leq b-a$. The results hold trivially if $a=b$. Suppose $b>a$. Then for any $\mu\geq\mu'$ we have that monotonicity of f implies that

$$G(a,b,\mu)=\frac{\int_{a+\mu}^{b+\mu}f(y)\,dy}{f(a+\mu)}\leq\frac{\int_{a+\mu}^{b+\mu}f(a+\mu)\,dy}{f(a+\mu)}=b-a.$$

Now suppose $b<a$. By the same argument, for any $\mu\geq\mu'$ we have

$$G(a,b,\mu)=\frac{\int_{a+\mu}^{b+\mu}f(y)\,dy}{f(a+\mu)}=\frac{-\int_{b+\mu}^{a+\mu}f(y)\,dy}{f(a+\mu)}\leq\frac{-\int_{b+\mu}^{a+\mu}f(a+\mu)\,dy}{f(a+\mu)}=b-a,$$

which concludes the proof.

Part (iii): Define $\mu'=\mu^*-b$ so that for any $\mu\leq\mu'$, f is weakly increasing in $[a+\mu,b+\mu]$. This implies that for any $\mu\leq\mu'$ we have

$$\frac{F(b+\mu)-F(a+\mu)}{F(a+\mu)}=\frac{\int_{a+\mu}^{b+\mu}f(y)\,dy}{F(a+\mu)}\geq\frac{\int_{a+\mu}^{b+\mu}f(a+\mu)\,dy}{F(a+\mu)}=\frac{f(a+\mu)(b-a)}{F(a+\mu)}.$$

Since $\lim_{\xi\to-\infty}\frac{f(\xi)}{F(\xi)}=\infty$, the right hand side is unbounded for fixed a and b. As $\frac{f(\xi)}{F(\xi)}$ is decreasing, it follows that there is a $\hat{\mu}>-\infty$ such that $\frac{f(a+\mu)(b-a)}{F(a+\mu)}\geq c$ for all $\mu\leq\hat{\mu}$, concluding the proof. ■

Proof of Lemma 1:

Consider a signal π that generates a distribution $\eta \in \Delta(\Theta)$ over posterior beliefs of the majority group. Note that this distribution is independent of the valence differential. For any q_A in the support of η the change in the victory probability of the majority candidate is

$$v(q_A) - v(p_A) = F(D(q_A) + \mu') - F(D(p_A) + \mu') = f(D(p_A) + \mu')G(D(p_A), D(q_A), \mu'),$$

where G is defined by (13). Therefore, the expected increase in victory probability from signal π can be written as

$$E_\pi[v(q^A) - v(p^A)|\mu'] = f(D(p_A) + \mu') \int_{q_A \in \sup p(\eta)} G(D(p_A), D(q_A), \mu')d\eta.$$

From Lemma 1-i we know that G is non-increasing in μ, which implies that

$$\int_{q_A \in \sup p(\eta)} G(D(p_A), D(q_A), \mu')d\eta \geq \int_{q_A \in \sup p(\eta)} G(D(p_A), D(q_A), \mu)d\eta \tag{15}$$

for any $\mu > \mu'$. Non-negativity of f implies that if $E_\pi[v(q^A)|\mu'] < v(p^A)$ for μ', then the left hand side of (15) must be negative and $E_\pi[v(q^A)|\mu] < v(p^A)$ for any $\mu > \mu'$. ■

Proof of Proposition 1:

If for some μ' we have $V - v(p^A) = 0$, this implies that $\sup_\pi E_\pi[v(q^A)|\mu'] - v(p^A) = 0$, so that for every signal π, $E_\pi[v(q^A)|\mu'] - v(p^A) \leq 0$. Lemma 1 then implies that $E_\pi[v(q^A)|\mu] - v(p^A) \leq 0$ for any $\mu > \mu'$ so that $\sup_\pi E_\pi[v(q^A)|\mu] - v(p^A) = 0$ for any $\mu > \mu'$. ■

Proof of Proposition 2:

To proof this Proposition we will make use of the following fact. Since v is differentiable at the prior, we can apply the second part of Corollary 1 from Alonso and Câmara (2014a). In our setup, it implies that there is no value of persuasion if and only if

$$\langle \nabla v(p^A), q^A - p^A \rangle \geq v(q^A) - v(p^A), \qquad q^A \in \Delta(\Theta).$$

Since $\nabla v(p^A) = f\left(D(p^A)+\mu\right)\nabla D(p^A)$, and $f > 0$, we can rewrite the previous condition as

$$\langle \nabla D(p^A), q^A - p^A\rangle - G\left(D(p^A), D(q^A), \mu\right) \geq 0, \qquad q^A \in \Delta(\Theta), \tag{16}$$

Part (a): From Lemma A.1(ii), we know that for each q^A there exists a $\mu'(q^A) < \infty$ such that for any $\mu \geq \mu'(q^A)$ we have $G(D(p^A), D(q^A), \mu) \leq D(q^A) - D(p^A)$. Hence, the LHS of (16) is weakly greater than $\langle \nabla D(p^A), q^A - p^A\rangle - D(q^A) + D(p^A)$. Since D is concave, this term is weakly positive. Therefore, for any $\mu \geq \check{\mu} \equiv \sup \mu'(q^A)$ the LHS of (16) is positive for all $q^A \in \Delta(\Theta)$ and there is no value of persuasion.

Part (b): Suppose that there exists belief q^+ such that $D(q^+) > D(p^A)$. We will show that under the conditions of the proposition there exists a $\mu > -\infty$ such that information control is valuable by constructing a signal that yields a payoff strictly higher than a completely uninformative signal.

Suppose first that D is locally strictly convex at the prior. Local strict convexity guarantees the existence of q^+ and q^- with $D(q^+) > D(p^A)$ and $\lambda q^+ + (1-\lambda)q^- = p^A$ such that

$$\lambda D(q^+) + (1-\lambda)D(q^-) > D(p^A). \tag{17}$$

We now show that there exists μ such that

$$\lambda F(D(q^+)+\mu) + (1-\lambda)F(D(q^-)+\mu) > F(D(p^A)+\mu), \tag{18}$$

so that this signal outperforms a completely uninformative signal. Lemma A.1(ii) then guarantees the existence of μ'' such that $G(a, b, \mu) \geq b - a$ for $\mu \leq \mu''$. For any $\mu \leq \mu''$ then we have

$$\begin{aligned}
& \lambda\left(F(D(q^+)+\mu) - F(D(p^A)+\mu)\right) + (1-\lambda)\left(F(D(q^-)+\mu) - F(D(p^A)+\mu)\right) \\
= \;& \lambda f(D(p^A)+\mu)G(D(p^A), D(q^+), \mu) + (1-\lambda)f(D(p^A)+\mu)G(D(p^A), D(q^-), \mu) \\
\geq \;& f(D(p^A)+\mu)\left(\lambda(D(q^+) - D(p^A)) + (1-\lambda)(D(q^-) - D(p^A))\right) > 0
\end{aligned}$$

where the last inequality follows from (17). This establishes (18).

Suppose now that $\lim_{\xi \to -\infty} \frac{f(\xi)}{F(\xi)} = \infty$. Select a belief q^+ such that $D(q^+) > D(p^A)$ and $\lambda \in (0,1)$ and q^- such that $\lambda q^+ + (1-\lambda)q^- = p^A$. We now study the value to the controller that with probability λ induces posterior q^+, and with probability $1-\lambda$ induces posterior q^- in the majority group. If $D(q^-) \geq D(p^A)$ then the signal outperforms an uninformative signal for all μ. Suppose $D(q^-) < D(p^A)$. Lemma A.1(iii), guarantees the existence of $\mu' > -\infty$ such that $\frac{F(D(q^+)+\mu)-F(D(p^A)+\mu)}{F(D(p^A)+\mu)} \geq c$ with $c = \frac{1-\lambda}{\lambda}$ holds for any $\mu \leq \mu'$. Then

$$
\begin{aligned}
& F(D(q^+)+\mu) - F(D(p^A)+\mu) + \frac{1-\lambda}{\lambda}\big(F(D(q^-)+\mu) - F(D(p^A)+\mu)\big) \\
\geq \; & F(D(q^+)+\mu) - F(D(p^A)+\mu) - \frac{1-\lambda}{\lambda}F(D(p^A)+\mu) \geq 0.
\end{aligned}
$$

Which shows that (18) holds for any $\mu \leq \mu'$. ■

Proof of Proposition 3:

To Simplify presentation we consider the interior case $0 < p_2 < q_2^{max} < 1$. We start by presenting some properties of (7). Taking the derivative of v with respect to q_2^A,

$$
\begin{aligned}
v(q^A) &= F\left(D(q^A)+\mu\right), \\
\frac{\partial v(q^A)}{\partial q_2^A} &= f\left(D(q^A)+\mu\right)\left[\frac{\partial D(q^A)}{\partial q_2^A}\right].
\end{aligned}
$$

Note that v strictly increases in the range $[0, q_2^{max})$, since the p.d.f. f is strictly positive and **(A2′)** states that $\frac{\partial D(q^A)}{\partial q_2^A} > 0$ in this range. Moreover, $p_2^A < q_2^{max}$ implies that the prior is at the increasing segment. Since q_2^{max} is a global maximum, then every point is the segment $q_2^A \in (q_{max}^A, 1]$ is below $v(q^{max})$. Therefore, the relevant concave closure of v at the prior will never be supported by points in the segment $q_2^A \in (q_{max}^A, 1]$. Consequently, without loss of generality we can focus on constructing the concave closure of v in the range $(0, q_2^{max})$.

Taking the second derivative:

$$
\frac{\partial^2 v(q^A)}{\partial (q_2^A)^2} = f'\left(D(q^A)+\mu\right)\left[\frac{\partial D(q^A)}{\partial q_2^A}\right]^2 + f\left(D(q^A)+\mu\right)\left[\frac{\partial^2 D(q^A)}{\partial (q_2^A)^2}\right].
$$

Since $\frac{\partial D(q^A)}{\partial q_2^A}\Big|_{q_2^A=q_2^{max}} = 0$ and D is strictly concave at q_2^{max}, the function v is always strictly concave at its maximum,

$$
\frac{\partial^2 v(q^A)}{\partial (q_2^A)^2}\Big|_{q_2^A=q_2^{max}} = 0 + f\left(D(q^A)+\mu\right)\left[\frac{\partial^2 D(q^A)}{\partial (q_2^A)^2}\right]\Big|_{q_2^A=q_2^{max}} < 0.
$$

For $q_2^A < q_2^{max}$, rewrite

$$
\frac{\partial^2 v(q^A)}{\partial (q_2^A)^2} = f\left(D(q^A)+\mu\right)\left[\frac{\partial D(q^A)}{\partial q_2^A}\right]^2 \left\{\frac{f'\left(D(q^A)+\mu\right)}{f\left(D(q^A)+\mu\right)} + \frac{\left[\frac{\partial^2 D(q^A)}{\partial (q_2^A)^2}\right]}{\left[\frac{\partial D(q^A)}{\partial q_2^A}\right]^2}\right\}. \quad (19)
$$

Since f is strictly log-concave and D strictly increases in $q_2^A \in (0, q_2^{max})$, the term $\frac{f'}{f}$ strictly decreases in this range. Moreover, $\left[\frac{\partial^2 D(q^A)}{\partial (q_2^A)^2}\right] / \left[\frac{\partial D(q^A)}{\partial q_2^A}\right]^2$ weakly decreases in this range because D_2 is log-concave. It follows that either v is concave in the range, or it changes from convex to concave.

We now construct the concave closure of v in the two-dimensional graph — where we draw q_2^A in the horizontal axis and $v(1-q_2^A, q_2^A)$ in the vertical axis. Note that if v is concave is the relevant range $q_2^A \in (0, q_2^{max})$, then the concave closure of v is v itself. If v is convex is some range $q_2^A \in (0, q_2')$, then we need to find the straight line that starts in the origin $(0, v(1,0))$ and is tangent to v at some point $q_2^* \in (q_2', q_2^{max})$. Note that v must be concave at q_2^*. The concave closure of v is then this straight line in the segment $q_2^A \in [0, q_2^*]$, and v itself in the segment $q_2^A \in (q_2^*, q_2^{max})$. Information control is valuable if and only if $q_2^* > p_2^A$. In this case, the optimal signal must be supported only in the two posterior beliefs $q_2^A = 0$ and $q_2^A = q_2^*$.

Let $\underline{q} = (1,0)$. For any $q_2^A \in (0, q_2^{max})$, we have $D(q^A) > D(\underline{q})$, and a straight line from the origin is tangent to v at q_2^A if and only if

$$K(q_2^A, \mu_q) \equiv \frac{F\left(D(q^A) + \mu_q\right) - F\left(D(\underline{q}) + \mu_q\right)}{f\left(D(q^A) + \mu_q\right)} - q_2^A \left[\frac{\partial D(q^A)}{\partial q_2^A}\right] = 0. \tag{20}$$

Step 1: For each $q_2^A \in (0, q_2^{max})$, there is a unique μ_q such that $K(q_2^A, \mu_q) = 0$. This follows because $K(q_2^A, \mu_q)$ is a strictly increasing continuous function of μ_q (see Lemma A.1(i)), strictly positive when μ_q is sufficiently large, and strictly negative if μ_q is sufficiently negative (using Lemma A.1(iv) and the fact that $-q_2^A \left[\frac{\partial D(q^A)}{\partial q_2^A}\right] < 0$).

Step 2: Take any pair (q_2^A, μ_q) such that $K(q_2^A, \mu_q) = 0$. We will show that if we marginally increase q_2^A, then we need to marginally decrease μ_q in order for the equality to continue to hold. Taking the derivative:

$$\begin{aligned}
\frac{\partial K(q_2^A,\mu_q)}{\partial q_2^A} &= \frac{f\left(D(q^A)+\mu_q\right)}{f\left(D(q^A)+\mu_q\right)}\left[\frac{\partial D(q^A)}{\partial q_2^A}\right] \\
&\quad -\left[\frac{F\left(D(q^A)+\mu_q\right)-F\left(D(\underline{q})+\mu_q\right)}{f\left(D(q^A)+\mu_q\right)}\right]\frac{f'\left(D(q^A)+\mu_q\right)}{f\left(D(q^A)+\mu_q\right)}\left[\frac{\partial D(q^A)}{\partial q_2^A}\right] \\
&\quad -\left[\frac{\partial D(q^A)}{\partial q_2^A}\right]-q_2^A\left[\frac{\partial^2 D(q^A)}{\partial (q_2^A)^2}\right] \\
&= -q_2^A\frac{f'\left(D(q^A)+\mu_q\right)}{f\left(D(q^A)+\mu_q\right)}\left[\frac{\partial D(q^A)}{\partial q_2^A}\right]^2-q_2^A\left[\frac{\partial^2 D(q^A)}{\partial (q_2^A)^2}\right] \\
&= -q_2^A\left[\frac{\partial D(q^A)}{\partial q_2^A}\right]^2\left\{\frac{f'\left(D(q^A)+\mu\right)}{f\left(D(q^A)+\mu\right)}+\frac{\left[\frac{\partial^2 D(q^A)}{\partial (q_2^A)^2}\right]}{\left[\frac{\partial D(q^A)}{\partial q_2^A}\right]^2}\right\},
\end{aligned}$$

where we used the fact that $K(q_2^A,\mu_q)=0$ to substitute the term $\left[\frac{F\left(D(q^A)+\mu_q\right)-F\left(D(\underline{q})+\mu_q\right)}{f(D(q^A)+\mu_q)}\right]$ by $q_2^A\left[\frac{\partial D(q^A)}{\partial q_2^A}\right]$. Therefore, $\frac{\partial K(q_2^A,\mu_q)}{\partial q_2^A}$ has the opposite sign of (19). Since v is strictly concave at q^A, we have that $\frac{\partial K(q_2^A,\mu_q)}{\partial q_2^A}>0$. We know $K(q_2^A,\mu_q)$ is always increasing in μ_q, hence an increase in q_2^A must be balanced by a decrease in μ_q. ■

The proof of Proposition 4 will be based on the following Lemma

Lemma A. 2 *Define the vector* $v=r\left(\theta-E[\theta|q^B]\right)$*, the linear subspaces* $W_1=\left\{x\in\mathbb{R}^{card(\Theta)}:\langle x,1\rangle=0\right\}$ *and* $W_{\theta-v}=\left\{x\in\mathbb{R}^{card(\Theta)}:\langle x,\theta-v\rangle=0\right\}$*. If the projections of* θ *and* r *are not negatively colinear with respect to* $W_1\cap W_{\theta-v}$*, then there exist a signal* π *where all signal realizations increase political disagreement, i.e.* $D(q_A)>D(p_A)$*.*

Proof of Lemma A.2: Define $q_A=\varepsilon\lambda+p_A$, with $\lambda\in W_1=\{x:\langle x,1\rangle=0\}$ and $\varepsilon\in\mathbb{R}$, and let

$$L(\varepsilon;\lambda)=\langle q_A,\theta\rangle-\frac{\langle q_A r,\theta\rangle}{\langle q_A,r\rangle}=\varepsilon\langle\lambda,\theta\rangle+E[\theta|q^A]-\frac{\varepsilon\langle\lambda,r\theta\rangle+E[\theta|q^B]}{\varepsilon\langle\lambda,r\rangle+1}$$

We will show that under the conditions of the proposition then one can always find a vector of "marginal beliefs" λ' such that L achieves a local minimum with respect to ε at $\varepsilon=0$. This means that along the line λ' and in a neighbourhood of 0, any belief $q_A=\varepsilon\lambda'+p_A$ with $\varepsilon>0$ increases L, and thus $D(q_A)>D(p_A)$, while any belief $q_A=\varepsilon\lambda'+p_A$ with $\varepsilon<0$ also increases L, yielding $D(q_A)>D(p_A)$. That is, we have found collinear beliefs that can average to the prior and that increase D.

First, we have

$$\frac{dL}{d\varepsilon} = \langle\lambda,\theta\rangle - \frac{\langle\lambda,r\theta\rangle - \langle\lambda,r\rangle E_B[\theta]}{(\varepsilon\langle\lambda,r\rangle+1)^2}$$
$$\frac{d^2L}{d\varepsilon^2} = \frac{2\langle\lambda,r\rangle[\langle\lambda,r\theta\rangle - \langle\lambda,r\rangle E_B[\theta]]}{(\varepsilon\langle\lambda,r\rangle+1)^3}$$

For $L(\varepsilon;\lambda)$ to achieve a local minimum at $\varepsilon=0$ it is sufficient that there exists $\lambda\in W$ such that

$$\left.\frac{dL}{d\varepsilon}\right|_{\varepsilon=0} = 0 \Rightarrow \langle\lambda,\theta\rangle = \langle\lambda, r(\theta - E_B[\theta])\rangle \tag{21}$$
$$\left.\frac{d^2L}{d\varepsilon^2}\right|_{\varepsilon=0} > 0 \Rightarrow \langle\lambda,r\rangle\langle\lambda, r(\theta - E_B[\theta])\rangle > 0 \tag{22}$$

Since θ and r are not negatively collinear with respect to $W_1\cap W_{\theta-v}$ then there exists $\lambda'\in W_1\cap W_{\theta-v}$ with $\langle\lambda',\theta\rangle\langle\lambda',r\rangle>0$ (see Alonso and Camara 2014a). Such λ' then verifies This implies the existence of We now show that if the projections $\theta_{\|W\|}$ and $r_{\|W\|}$ are not collinear then there exists λ' satisfying (21) and (22).

Proof of Proposition 4: Lemma A.2 above shows that if the projection of θ and r are not negatively colinear with respect to $W_1\cap W_{\theta-v}$ then information control is valuable. We now show that negative colinearity of θ and r with respect to $W_1\cap W_{\theta-v}$ is a non-generic property if $N\geq 4$. First note that $W_1\cap W_{\theta-v}$ has at least dimension $N-2$, and thus the projections of θ and r also have dimension $N-2\geq 2$ As colinearity is a non-generic property with vectors of dimension at least 2, this concludes the proof.

Proof of Lemma 2:

Suppose condition **C1** holds for priors p^A and p^B in the interior of the simplex $\Delta(\Theta)$. Then there are states $\theta_H,\theta_L\in\Theta$ such that $r_{\theta_H}+\ln(r_{\theta_H}) > r_{\theta_L}+\ln(r_{\theta_L})$. Fix any probability $\delta\in(0,1)$. For a sufficiently small ϵ, construct a signal π with realization space $\{s^+,s^-\}$ as follows:

$$Pr[s=s^+|\theta_H] = \delta\left(1+\frac{\epsilon}{p^A_{\theta_H}}\right),$$
$$Pr[s=s^+|\theta_L] = \delta\left(1-\frac{\epsilon}{p^A_{\theta_L}}\right),$$

and $Pr[s=s^+|\theta]=\delta$ for all other $\theta\in\Theta$. Note that signal s^+ occurs with probability δ and results in a posterior q^+ such that: $q^+_{\theta_H}=p^A_{\theta_H}+\epsilon$, $q^+_{\theta_L}=p^A_{\theta_L}-\epsilon$, and $q^+_\theta=p^A_\theta$ for all other states. Moreover,

$$\begin{aligned} D(q^+) &= \ln(\langle q^+, r\rangle) - \langle q^+, \ln(r)\rangle \\ &= \ln(\langle p^A, r\rangle + \epsilon(r_{\theta_H} - r_{\theta_L})) - \langle p^A, \ln(r)\rangle - \epsilon(\ln(r_{\theta_H}) - \ln(r_{\theta_L})) \\ &= \ln(1 + \epsilon(r_{\theta_H} - r_{\theta_L})) - \epsilon(\ln(r_{\theta_H}) - \ln(r_{\theta_L})) + D(p^A). \end{aligned}$$

First note that if $\epsilon = 0$, then $D(q^+) = D(p^A)$. Second, $\left.\frac{\partial D(q^+)}{\partial \epsilon}\right|_{\epsilon=0} = r_{\theta_H} - r_{\theta_L} - \ln(r_{\theta_H}) + \ln(r_{\theta_L}) > 0$. Therefore, $D(q^+) > D(p^A)$ for any $\epsilon > 0$ sufficiently small. Consequently, signal s^+ occurs with probability δ and strictly increases disagreement, concluding the proof.

Finally, note that since D is concave and differentiable, then for every $q^A \in \Delta(\Theta)$

$$\langle \nabla D(p^A), q^A - p^A\rangle \geq D(q^A) - D(p^A),$$

where $\nabla D(p^A) = r - \ln(r)$. Suppose condition **C1** fails, so that $r_\theta - \ln(r_\theta) = \lambda$ for all $\theta \in \Theta$. Then $\langle \nabla D(p^A), q^A - p^A\rangle = \lambda\langle \mathbf{1}, q^A - p^A\rangle = 0$, where the last equality follows since $\langle \mathbf{1}, q^A - p^A\rangle = \langle \mathbf{1}, q^A\rangle - \langle \mathbf{1}, p^A\rangle = 1 - 1$. This implies $D(p^A) \geq D(q^A)$ for all q^A, hence the controller cannot increase disagreement.

Proposition 5:. In the text. ■

References

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