Strategic Complexity

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Abstract

We build a principal-agent model, where the principal demands a product that is designed by the agent. Both the output of the product and its complexity are the agent's choice, and they are not observed by the principal, who decides whether to accept the product proposed by the agent. While a product's quality determines the benefits of the product to the principal, a product's complexity affects the information she receives about its quality. Examples include banks that design financial products that then later try to sell to retail investors, or policymakers who draft rules that need to be approved by a legislative body or voters. We find that the complexity of products, and their relation with product quality, depend critically on features of the environment. In particular, products will tend to be more complex when the overall demand for products is high, when trust in the agent is high, or when when competition among product designers is low. Moreover, the model predicts that the relationship between product complexity and quality across different environments will depend on the underlying drivers of the heterogeneity. Our findings have important implications for empirical work on the complexity of financial products and regulation.

Keywords: strategic complexity, information frictions, regulation design, financial product design.

JEL Codes: D82, G18, P16, D78.

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1 Introduction

In recent decades, the issue of complexity has received increased attention among both financial market participants and policymakers. In the financial industry, it is argued that the products sold to retail investors have become increasingly complex – with product descriptions that contain jargon and complicated or vague explanations (Carlin (2009), Carlin and Manso (2011), Célérier and Vallée (2015)). Similarly, in the regulatory sphere, the United States Code of Federal Regulations has expanded eightfold over the last half century, and the complexity of regulation has increased proportionally – including broadly worded statutes and ambiguous standards (Davis (2017)).[1]

The debate around the increase in complexity has brought forth two observations: first, that complexity is often a deliberate choice of those who create products and not just an exogenous pre-determined property of those products (where regulatory rules should be interpreted as the product of regulators).[2] Second, complexity is generally presented as a characteristic of worse quality products. This view is supported by empirical evidence from the financial (Célérier and Vallée (2015)) and the regulatory (Davis (2017)) industries. Common sense indicates that the demand for complex products should be low if these products were known to be bad. However, the proliferation of complex financial products and increasingly complex regulatory environments suggest that the relationship between complexity and quality is not as straightforward as one might think. Motivated by these observations, we pose the following question: If product complexity is a choice, then what are its drivers, and how does it relate to product quality?

To address this question, we build a model where complexity is a strategic choice, together with product type, of those who design products. While a product type determines the benefits or costs to the user, we view a product's complexity as mainly affecting the quality of the information the user obtains about the product. Examples of product designers include banks that design financial products, or policymakers who draft rules. We focus on cases in which the objective of the product designer is to have his product *accepted*. For example, the objective of a bank is to convince a retail investor to purchase its financial product while the objective of a policymaker is to pass his rule.[3] Our main result is that product complexity is strongly influenced by the features of the environment, and not necessarily by the choice of product type: namely, complexity is not always associated with worse quality products. We

[1] More data in http://www.hblr.org/2012/06/complexity-of-regulation/

[2] See Célérier and Vallée (2015), http://www.hblr.org/2012/06/complexity-of-regulation/.

[3] Our objective as paper designers is to have the Editor accepting our proposed paper for publication.

find that products tend to be more complex when trust in the product designer is high or when the overall demand for products is high. Finally, the model generates novel and testable implications for the relationship between product complexity and quality, as a function of measurable features of the environment.

We consider a principal-agent setting, where the principal demands a product that is designed by the agent. When designing a product, the agent takes two hidden-actions: he chooses product output and product complexity. The choice of output determines the payoff to the principal, which can be good or bad. The principal, however, cannot directly observe the output. Instead, she receives a noisy signal about it. The choice of product complexity affects the precision of this signal. In particular, the principal is more likely to receive noisier information about the output of a more complex product. We interpret this as the agent choosing to use jargon or overly complicated or confusing wording in the product brochure or drafted regulation. The agent can be of two types: either aligned with the principal or misaligned with the principal. That is, aligned agents receive a high payoff from good products, while the opposite is true for misaligned agents. Most importantly, the agent's type is his private information. This captures the fact that the incentives of those who design products may not be known by those who demand these products, and that their incentives need not be aligned. The timing of the game is as follows. First, the agent chooses product output and complexity, and he proposes the product to the principal. Then, the principal observes a signal about product output and decides whether to accept or reject the product. If the principal accepts, product payoffs are realized; otherwise, the agent and the principal get their outside options.

A key take-away from the paper is that the agent's incentives to complexity or simplify a product depend crucially on the principal's decision –to accept or to reject the proposed product,- in the absence of information. This has two important implications. First, incentives are not only driven by the agent's type or his choice of product output, since neither is observed by the principal in the absence of information. Second, and most importantly, this helps us understand the main drivers of product complexity. The principal's decision to accept or reject a product depends, in the absence of information, on the relation between the (unconditional) expected value of accepting a product relative to her outside option, which in turn depend on the principal's prior belief about the output of the proposed product, her payoffs from approving good vs bad products, and her outside option.

The agent has an incentive to design more complex products when the principal accepts products in the absence of information. We refer to such case as the acceptance region. This result, even though surprising at first, is very intuitive. In the acceptance region, information weakly decreases the chances of the product being accepted, since it is accepted with probability one in the absence of information. In this scenario, even the agent that designs a good product may benefit from complexification. In contrast, when the principal rejects products in the absence of information, i.e. the rejection region, the agent has an incentives to design simpler products. In the rejection region, information weakly increases the chances of a product being accepted, since it is rejected with probability one in the absence of information. In this scenario, even the agent who designs a bad product may benefit from simplification. As a result, the complexity of proposed products increases on the principal's prior belief about output being good (which can be interpreted as the trust the principal has on the agent) and decreases in the principal's outside option (where a high outside option can be interpreted as low demand for the product).

The average quality of product output is tightly linked to the behavior of the misaligned agent. While the aligned agent always designs a good output product, the misaligned agent does so with probability strictly lower than one. The misaligned agent trades off a higher probability of acceptance (obtained with the good output product) with a higher payoff conditional on acceptance (obtained with the bad output product). As a result, the misaligned agent is more likely to design a good output product when bad output products are more likely to be rejected, which occurs when the fraction of misaligned agents is large, and thus the principal's prior beliefs are low, or when the principal's outside option is relatively high. When the fraction of aligned agents is sufficiently large, however, each agent type chooses his unconditional preferred action.

As the fraction of aligned agents increases, the complexity of proposed products increases together with output in equilibrium. This suggests a positive relation between the complexity and quality of proposed products. On the other hand, as the principal's outside option increases, the complexity of proposed products decreases whereas product output increases. This suggests the opposite relationship between complexity and product quality. We conclude that the relationship between complexity and product quality depends on the underlying drivers of heterogeneity. We believe this result is of utmost importance when conducting empirical studies of complexity in financial products or regulations. It suggests that to find a robust relation between complexity and quality, it may be necessary to condition on the appropriate features of the environment.

Our results have wide implications for the relationship between complexity and quality in different settings. For example, in financial markets, there are agents, such as banks, who design financial products for retail investors. The design of a product consists of determining a set of cash flows for different states of the world (payments, returns, fees, etc.), the product price, and writing these contract terms down. The product is offered to the investor, who decides whether to invest her money or not. If she doesn't, she invests somewhere else. This would be the principal's outside option. In this setup, our model suggests that complex financial products should be observed in periods where investors have high demand for financial products, and where the trust in the financial industry is high. Both of these features were characteristics of financial markets prior to the 2008/09 crisis, and while the trust in the financial system may have fallen in response to the crisis, the high demand for certain financial products still persists.

In the regulatory sphere, there are agents, such as policymakers, in charge of drafting regulations such as Dodd-Frank, Tax and Health Care Reform. We can interpret the principal in such a setup as the collection of interest groups that represent the general public and the policymaker's constituents. Our model suggests that complexity of rules will increase if the public's outside option is low; that is, when the public has a high demand or has a urgency to pass a given regulation –for instance, due to crisis. Another driver of regulatory complexity would be a high perceived ideological alignment between the politicians or agencies involved in drafting the rules and the dominant interest groups in that subject matter. The results of this paper could therefore inform the current policy debates about the effectiveness and desirability of reducing regulatory complexity.

Our paper is most closely related to the literature on learning and obfuscation (Ellison and Ellison, 2009; Ellison and Wolitzky, 2012; Carlin and Manso, 2011). In our model, the sender can influence the receiver's ability to learn about the state of the world by adding complexity. Our approach to modelling the choice of complexity in information transmission is also employed in Perez-Richet and Prady (2011) and Bar-Isaac et al. (2010). We differ from these models along two key dimensions. First, the sender also chooses the rule's type along with its complexity. We show that this allows us to obtain a separating equilibrium where each agent chooses a different rule and may choose different complexities. Second, allow for complexity to potentially present a direct cost for the principal, not just an indirect cost through the choice of output. The interaction of costly complexity added by the sender and the receiver's learning from a noisy signal relates this model to Dewatripont and Tirole (2005); however, their focus in on moral hazard on the side of both agents, while in our model we have an adverse selection problem on the side of the agents.

Our paper is also connected to the broader literature on strategic information transmission

(Crawford and Sobel, 1982; Grossman, 1981; Milgrom, 1981; Kartik, 2009). Our model differs from this literature in that it does not feature cheap talk, only hard informatrion, and the sender must pay a cost to reduce the information the receiver gets. Moreover, this model is different from the Bayesian persuasion literature (Kamenica and Gentzkow, 2011), since the sender is fully informed when choosing complexity, so information transmission becomes a strategic decision.

The issue of complexity has also been examined in the applied setting of financial products along the dimension of pricing (Carlin, 2009). Our model complements this literature by explaning the strategic use of complexity in other to produce worse outputs. Empirically, Célérier and Vallée (2015) show financial institutions in Europe have used complex product descriptions in order to sell financial products to retail investors. Christoffersen and Musto (2002) show that financial institutions use distortions in transparency in order to price discriminate among investors of varied levels of sophistication.

Our model also has implications for the issue of regulatory complexity, which relates us to the political economy literature on this topic (McCarty, 2017; Ash et al., 2018). The seminal work of Kane (1977) proposed a model in which regulatory complexity emerges as more provisions are added to rules in response to firms learning how to avoid aspects of rules. We propose a different mechanism for complexity, namely the strategic use of complexity in order to increase the chance of a rule's approval.

The rest of the paper is organized as follows. In Section 2, we present the setup of the model. In Section 3, we illustrate our main results for the case of binary and costless complexity. In Section 4, we extend the model to incorporate a direct cost of complexity to the principal. All of the proofs are relegated to the Appendix, which is available upon request.

2 The Model

There is a principal and an agent. The principal needs a product, that only the agent can produce. The principal-agent interactions are as follows. The agent takes two actions $\{y, \kappa\}$, where $y \in \{\mathbf{G}ood, \mathbf{B}ad\}$ determines the product's output, and $\kappa \in \mathcal{K} \subset \mathbb{R}$ the product's complexity, which we define in detail below. After this, the agent proposes the product to the principal. The principal evaluates the proposed product, and can either accept it ($a = 1$) or take her outside option ($a = 0$). An agent can be of two types, $t \in \{\mathbf{H}igh, \mathbf{L}ow\}$, with probabilities p and $1-p$ respectively, where different types differ in their incentives to propose a given product. Most importantly, an agent's type, t, is his private information and his actions $\{y, \kappa\}$ are unobservable to the principal.

A Product's Output. By taking action y, the agent affects the output of the proposed product. We assume that the payoff to the principal of accepting a product with output y is $w(y)$ and that her payoff if no product is accepted is w_0, which we refer to as her outside option. A t-type agent that takes action y receives payoff $v_t(y)$ from having his product being accepted, and receives zero otherwise. We make the following assumptions on the payoffs from product output:

Assumption 1 *The payoffs satisfy the following properties:*

1. $w(G) > w_0 > w(B)$.

2. $v_H(G) = v_L(B) = \bar{v} > v_L(G) = v_H(B) = \underline{v}$.

The first assumption states that the principal prefers the G-product to the B-product. That is, the principal wants to accept only **G**ood products. The second assumption states that an H-type (L-type) agent receives a higher payoff when the G-product (B-product) is accepted. That is, an H-type agent's preferences are aligned with those of the principal, while an L-type agent's preferences are misaligned with those of the principal.

A Product's Complexity. By taking action κ, the agent can affect the information the principal receives about the product's output, y. If a product is more (less) complex, the principal is more (less) likely to receive noisy information about its output. We model this by assuming that the principal observes a symmetric binary signal $s \in \{g, b\}$ about the product's output, y. This signal has noise $z \equiv \mathbb{P}(y = G|s = b) = P(y = B|s = g)$, which is influenced by the complexity of the product, $z \sim f(\cdot|\kappa)$ with full support on $\left[0, \frac{1}{2}\right]$ for all $\kappa \in \mathcal{K}$. We assume that the monotone likelihood ratio property (MLRP) holds: $\frac{f(z|\kappa')}{f(z|\kappa)}$ is increasing in z if $\kappa' > \kappa$. That is, higher complexity products are more likely to generate noisier signals about their output.

In practice, a product can be made "more" complex by the use of complicated words and jargon in its brochure, and it can be "simplified" by using terminology that the principal easily understands. This can be done without affecting the underlying output of the product. This approach is one of the main contributions of our paper: the agent first takes an action that directly affects the principal's payoff, and then takes another action that affects the quality of the information the principal receives about the first chosen action. The assumption that an agent's choice of complexity maps imperfectly into the quality of the information the principal receives has the natural interpretation that agents are unable to perfectly control the principal's information set (e.g., because of alternative information sources such as media).[4]

The Principal's Problem. The principal has to decide whether to accept the agent's product or not. Before making her decision, the principal observes signal s with noise z, and forms her posterior beliefs about $\{y, \kappa\}$. Given her beliefs, the principal makes her approval decision to maximize her expected payoff,

$$W(a|s,z) \equiv a \cdot E\left[w(y)|s,z\right] + (1-a) \cdot w_0 \tag{1}$$

If indifferent, we suppose the principal approves the product.

The Agent's Problem. A t-type agent chooses the product's output and complexity to maximize her expected payoff,

$$V_t\left(y,\kappa\right) \equiv \mathbb{P}\left(a=1|y,\kappa\right) \cdot v_t(y) \tag{2}$$

where $\mathbb{P}\left(a=1|y,\kappa\right)$ denotes the probability that product $\{y,\kappa\}$ is accepted by the principal.

Equilibrium Concept. We use Perfect Bayesian Equilibrium (PBE) as our equilibrium concept. First, the principal's acceptance decision must maximize her expected payoff at every information set taking the agent's strategy as given (*Principal's Optimality*). Second, the agent's actions (output and complexity) must maximize his expected payoff taking the principal's acceptance strategy as given (*Agent's Optimality*). Third, given her information set, the principal's beliefs are updated according to Bayes' rule whenever possible (*Belief Consistency*).

3 Binary and Costless Complexity

We first characterize equilibria in the simplest version of our model, where the forces driving our main results can be crystallized. First, we suppose that the agent has a binary choice of complexity: $\mathcal{K} = \{\underline{\kappa}, \bar{\kappa}\}$ with $\underline{\kappa} < \bar{\kappa}$. Second, we suppose that complexity does not impose a direct cost on the principal nor on the agent: $c(\kappa) = 0$ for all κ.

The Principal's Problem. Let $\mu(s,z) \equiv \mathbb{P}(y = G|s,z)$ be the principal's posterior belief about the product's output being G after observing signal s with noise z. Since the product's complexity does not generate a direct cost to the principal, problem (1) can be re-stated as

[4] This assumption also eliminates the potential for multiple equilibria (typical of signaling games), which can arise due to freedom of specifying off-equilibrium beliefs.

follows:

$$\max_{a \in \{0,1\}} a \cdot [\mu(s,z) \cdot w(G) + (1-\mu(s,z)) \cdot w(B)] + (1-a) \cdot w_0 \tag{3}$$

The Agent's Problem. The agent chooses $y \in \{G, B\}$ and $\kappa \in \{\underline{\kappa}, \bar{\kappa}\}$ to maximize (2), where $C(\kappa) = 0$ for all κ. A t-type agent's strategy is denoted by $\{m_t, \sigma_t\} \in [0,1]^2$, where $m_t = \mathbb{P}(y_t = B)$ and $\sigma_t = \mathbb{P}(\kappa_t = \bar{\kappa})$ are the probability of a t-type choosing the bad output and of choosing the highest complexity level in equilibrium, respectively.

3.1 The Principal's Approval Decision

From equation (3), the principal's acceptance decision follows a threshold strategy: she accepts the product, $a(s,z) = 1$, if her posterior belief is sufficiently high: $\mu(s,z) \geq \omega$, where $\omega \equiv \frac{w_0 - w(B)}{w(G) - w(B)}$ captures the relative importance of the principal's outside option. The principal's problem is straightforward. Upon observing signal s with noise z, she updates her belief about the product's output using Bayes' rule. If the posterior belief about the output being **G**ood is sufficiently high, she accepts the product. Otherwise, she takes her outside option.

We have established that the principal approves the product if $\mu(s,z) \geq \omega$. We now analyze the determinants of the posterior belief. Let μ denote the principal's prior belief about the product being **G**ood in equilibrium. After the agent has proposed his product, the principal receives signal s with noise z about the output of the proposed product. Since $z \sim f(z|\kappa)$, the principal updates her beliefs about the agent's type after observing noise z, which is a signal itself of the chosen complexity κ. We denote such interim belief by $\mu(z)$, computed using Bayes rule as follows

$$\mu(z) = \frac{\mu}{\mu + (1-\mu)\,\ell(z)} \tag{4}$$

where

$$\mu = p \cdot (1 - m_H) + (1-p) \cdot (1 - m_L) \tag{5}$$

$$\ell(z) \equiv \frac{f(z|t=L)}{f(z|t=H)} = \frac{\sigma_L \cdot f(z|\bar{\kappa}) + (1-\sigma_L) \cdot f(z|\underline{\kappa})}{(\sigma_H + \sigma_L) \cdot f(z|\bar{\kappa}) + (2 - \sigma_H - \sigma_L) \cdot f(z|\underline{\kappa})} \tag{6}$$

and where equilibrium strategies $\{m_t, \sigma_t\}$ are taken as given. Finally, the principal forms posterior belief $\mu(s,z)$ that the product is **G**ood using Bayes Rule:

$$\mu(s,z) = \frac{\mathbb{P}(S=s|y=G) \cdot \mu(z)}{\mathbb{P}(S=s|y=G) \cdot \mu(z) + \mathbb{P}(S=s|y=B) \cdot (1-\mu(z))} \tag{7}$$

There are two constraints that are relevant for understanding the principal's approval decision. The principal's decision is contingent on the observed signal only if she accepts the product when the signal is good, $S = g$, and rejects it when the signal is bad, $S = b$. For this to be optimal, the signal has to be informative enough so that:

$$\mu(b, z) \leq \omega \leq \mu(g, z) \tag{8}$$

Next, consider the threshold noise level $\bar{z}$ at which $\mu(s, \bar{z}) = \omega$ for some $s \in \{b, g\}$. In principle, there could be multiple such $\bar{z}$'s. In the Appendix, we impose regularity conditions which ensure that $\mu(b, z)$ is increasing and $\mu(g, z)$ is decreasing in z. This ensures the existence of a unique $\bar{z}$, since $\mu(g, 0) = 1$, $\mu(b, 0) = 0$, and $\mu(g, 0.5) = \mu(b, 0.5)$. In other words, we assume that the information content of the signal, s, is greater than the information content of observing the realization of the noise itself, z. Although not essential for our results, we believe this assumption is natural in our setting, as we want to understand the role of complexity in hindering the inference problem of the principal rather than as a signaling device itself.

The following definition will be used in the characterization of the principal's problem, and the resulting equilibrium of the game.

Definition 1 *We say that we are in the* **acceptance region** *when the threashold $\bar{z}$ is given by condition $\omega = \mu(b, \bar{z})$, and that we are in the* **rejection region** *when $\bar{z}$ is given by condition $\omega = \mu(g, \bar{z})$.*

The threshold $\bar{z}$ determines the maximum noise level at which the principal makes her acceptance decision contingent on the signal. When the noise of the signal is relatively high ($z > \bar{z}$) the principal disregards the signal when making her approval decision. As a result, for any signal realization, the principal approves the product in the *acceptance region*, and rejects the product in the *rejection region*. On the other hand, when the signal is sufficiently informative ($z \leq \bar{z}$), the principal makes her approval decision contingent on the signal: she accepts the product after observing a **g**ood signal, and rejects it after observing a **b**ad signal. Intuitively, we are more likely to be in the acceptance (rejection) region when in the absence of information, the principal would accept (reject) the proposed product.

These results are formalized in the following Lemma.

Lemma 1 *In the acceptance region, the principal's acceptance rule is:*

$$a = \begin{cases} \mathcal{I}_{\{S=g\}} & \text{if} \quad z \leq \bar{z} \\ 1 & \text{if} \quad z > \bar{z} \end{cases} \tag{9}$$

In the rejection region, the principal's acceptance rule is:

$$a = \begin{cases} \mathcal{I}_{\{S=g\}} & \text{if} \quad z \leq \bar{z} \\ 0 & \text{if} \quad z > \bar{z} \end{cases} \tag{10}$$

where $\mathcal{I}_{\{S=s\}}$ *is the indicator equal to one when the signal is equal to* s*.*

It is important to highlight the role of information in each scenario. In the acceptance region, information (weakly) increases the chances that any product is rejected, since a product is always accepted when information is too noisy ($z > \bar{z}$). The converse is true in the rejection region, where information (weakly) increases the chances that any product is accepted, since a product is always rejected when information is too noisy. Importantly, this holds for any product output, and will be an important driver of our main results.

Before proceeding to the agent's problem, we want to highlight that these acceptance decisions depend, through interim belief $\mu(z)$, on the t-agent's equilibrium strategies $\{m_t, \sigma_t\}$, which the principal takes as given. For brevity, we highlight this dependence only when necessary.

3.2 The Agent's Choice of Complexity

We proceed to characterize the agent's choice of complexity. In the next section, we will use these results to study the agent's choice of output. We can analyze these actions separately because complexity only affects the product's probability of acceptance, which is independent of type. A t-type agent who chooses output y, also chooses low complexity, $\underline{\kappa}$, when

$$\begin{aligned} & \mathbb{P}(a = 1|y, \underline{\kappa}) \cdot v_t(y) \geq \mathbb{P}(a = 1|y, \bar{\kappa}) \cdot v_t(y) \\ \iff & \mathbb{P}(a = 1|y, \underline{\kappa}) \geq \mathbb{P}(a = 1|y, \bar{\kappa}). \end{aligned} \tag{11}$$

Otherwise, the agent chooses high complexity, $\bar{\kappa}$. As (11) shows, the choice of complexity only depends on the agent's choice of output, y, and does not vary with the agent's type.

Using the results from Lemma 1, we compute the probability of acceptance of a y-output product conditional on a signal noise level, z, which we denote by $\pi(y, z)$. In the acceptance

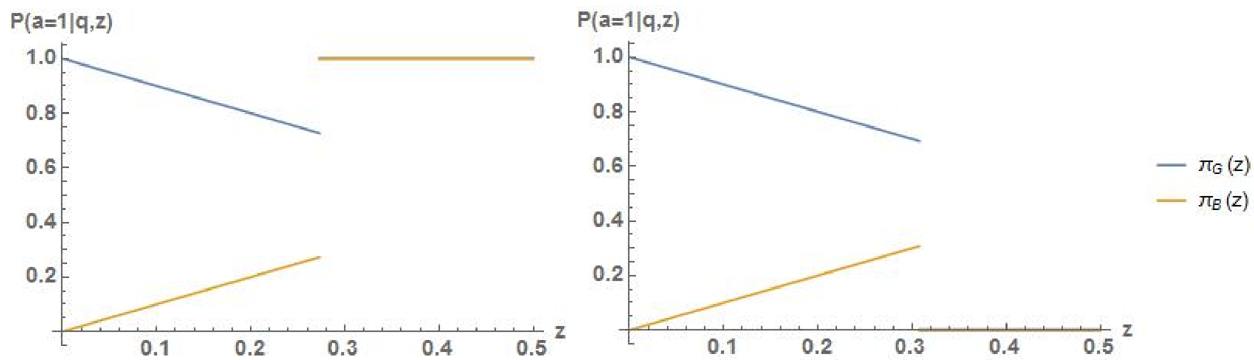

Figure 1: The probability of acceptance of a y-product as a function of the signal's noise, z.

For the plots we use $\omega = 0.6$. The left panel depicts the approval decision of an impatient principal, where $\mu = 0.7$. The right panel depicts the acceptance strategy of a patient principal, with $\mu = 0.4$

region (A), we have

$$\pi^A(G,z) = \begin{cases} 1-z & \text{if } z < \bar{z} \\ 1 & \text{if } z \geq \bar{z} \end{cases} \quad \text{and} \quad \pi^A(B,z) = \begin{cases} z & \text{if } z < \bar{z} \\ 1 & \text{if } z \geq \bar{z} \end{cases} \tag{12}$$

while in the rejection region (R), we have

$$\pi^R(G,z) = \begin{cases} 1-z & \text{if } z \leq \bar{z} \\ 0 & \text{if } z > \bar{z} \end{cases} \quad \text{and} \quad \pi^R(B,z) = \begin{cases} z & \text{if } z \leq \bar{z} \\ 0 & \text{if } z > \bar{z} \end{cases} \tag{13}$$

These acceptance probabilities are illustrated in Figure 1. Thus, the expected probability of product (y,κ) being accepted depends on the region $j \in \{A, R\}$ and is given by:

$$\mathbb{P}(a=1|y,\kappa) = \int_0^{\frac{1}{2}} \pi^j(y,z) \cdot f(z|\kappa) \cdot dz \tag{14}$$

The following Proposition formalizes the agent's choice of complexity for a given product output. We say that the agent *simplifies* when he chooses $\underline{\kappa}$ and that he *complexifies* when he chooses $\bar{\kappa}$.

Proposition 1 *Let $\hat{z}$ denote the unique solution to $\int_0^{\hat{z}} z \cdot f(z|\underline{\kappa})dz = \int_0^{\hat{z}} z \cdot f(z|\bar{\kappa})dz$. Then, in the acceptance region, the agent who proposes:*

- *B-product always complexifies.*

- *G-product complexifies if $\bar{z} \leq \hat{z}$, and simplifies otherwise (can mix when indifferent).*

In the rejection region, the agent who proposes:

- *B-product simplifies if $\bar{z} \leq \hat{z}$, and complexifies otherwise (can mix when indifferent).*

- *G-product always simplifies.*

The intuition behind these results can be obtained from inspection of Figure 1. First, note that the B-product's probability of acceptance increases in the noise of the signal in the acceptance region. As a result, the B-product benefits from complexity in such a scenario. Second, the G-product's probability of acceptance decreases in noise in the rejection region, consistent with the G-product benefiting from simplification in this region. Finally, and most importantly, the effect of noise on acceptance probability is non-monotonic for the B-product in the acceptance region and for the G-product in the rejection region. In these cases, the agent's choice of complexity depends on the principal's acceptance strategy, which is summarized by $\bar{z}$.

We conclude this section by highlighting that an agent's choice of complexity is not only determined by his choice of product output, but also by whether we are in the acceptance/rejection region. When the principal is more likely to accept products with noisy signals, an agent with a G-product may benefit from complexity; while when the principal is more likely to reject products with noisy signals, an agent with a B-product may choose simplicity. Thus, the relation between product output and complexity is determined by features of the environment, such as the principal's outside option and her unconditional expected value of accepting a product.

3.3 The Agent's Choice of Output

We now characterize the t-type agent's choice of output, which concludes the characterization of the agent's problem. Recall that a t-type agent receives payoff $\bar{v}$ when choosing the aligned product output, and receives payoff $\underline{v}$ otherwise.

The benefit for the L-type agent from choosing the action that the principal prefers, that is, the G-output, is given by:

$$F(\mu) \equiv \left\{ \max_{\kappa} \mathbb{P}_{\mu}(a = 1 | G, \kappa) \cdot \underline{v} \right\} - \left\{ \max_{\kappa} \mathbb{P}_{\mu}(a = 1 | B, \kappa) \cdot \bar{v} \right\}. \tag{15}$$

The first term is the expected payoff to the L-agent from choosing the G-output given the corresponding best choice of complexity, characterized in Proposition 1. The second term is the expected payoff to the L-agent from choosing the B-output. The probabilities of acceptance in each scenario are computed as in (14) given prior beliefs μ and corresponding equilibrium strategies $\{m_t, \sigma_t\}$, which affect the mapping $z \mapsto \mu(z)$. Since multiple equilibrium strategies

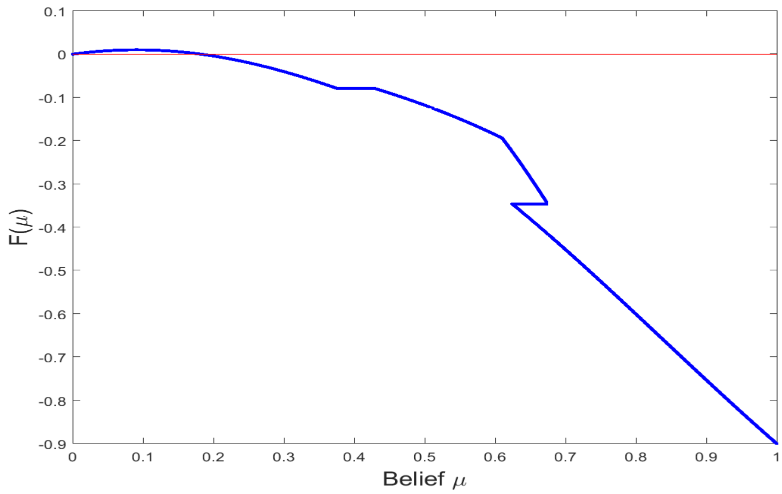

Figure 2: The extra payoff the L-type agent obtains from choosing the G-output instead of the B-output, given principal's belief, μ.

could be consistent with certain beliefs, F is a correspondence, as can be seen in Figure 2. To shorten notation, if $\forall f \in F(p), f < (>)0$ and we say that $F(\mu) < (>)0$, and if there exists $f \in F(\mu)$ such that $f = 0$ we say $F(\mu) = 0$. The following lemma characterizes the t-type agent's choice of output strategy in equilibrium.

Lemma 2 *For all $\mu > 0$, the H-type always chooses the G-product, i.e. $m_H = 0$, whereas the L-type chooses the B-product with probability* $m_L \begin{cases} 0 & \text{if } F(\mu) > 0 \\ \in [0,1] & \text{if } F(\mu) = 0 \\ = 1 & \text{if } F(\mu) > 0. \end{cases}$

Lemma 2 states that the H-type always chooses the product output that the principal desires. This is because, all else equal, the probability of a product being accepted is higher for a G than for a B-product. This can be seen from Figure 1. Thus, the G-type agent always has an incentive to choose output G, since he also obtains a higher payoff from such product being accepted. In contrast, the L-type agent faces a trade-off between increasing the probability of his product being accepted by choosing G-product and receiving a higher payoff conditional on acceptance by choosing B-product. Therefore, she will tend to choose the G-product when the difference in the probabilities of acceptance across the two products is sufficiently small.

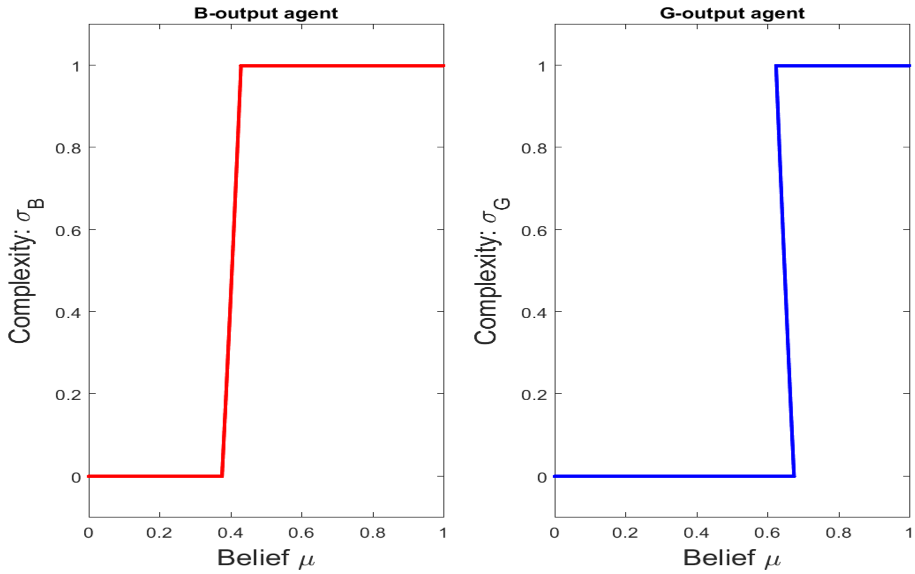

Figure 3: The choice of complexity of an agent that has chosen the y-product as a function of principal's belief, μ.

3.4 Characterization of Equilibria

We have characterized the principal's acceptance decision rule given the agent's equilibrium strategies $\{m_t, \sigma_t\}$ in Section 3.1, and the t-type agent's strategy given the principal's acceptance decision rule in Sections 3.2 and 3.3. In this section. we close the model by imposing belief consistency. That is, the beliefs of the principal at the approval stage need to be consistent with the t-type agent's strategies.

We proceed as follows. First, we characterize the choice of complexity for a y-product that is consistent with an equilibrium in which the principal's belief about the probability of an agent choosing G is μ. We then characterize the choice of output of a t-type agent that is consistent with an equilibrium. The following proposition describes the complexity strategy of an agent that has chosen output y, σ_y, when principal's belief is μ.

Proposition 2 *Let μ be the equilibrium belief that an agent has chosen the G-output. Then, the choice of complexity of an agent that has chosen the y-output is as follows,*

1. *If $\mu \leq \mu_1$, all agents simplify ($\sigma_B = \sigma_G = 0$).*

2. *If $\mu \in (\mu_1, \mu_2]$, the G-product agent simplifies ($\sigma_G = 0$), and the B-product agent com-*

plexifies with probability

$$\sigma_B = \frac{(1-\hat{z})\cdot\mu\cdot(1-\omega) - \hat{z}\cdot(1-\mu)\cdot\omega}{\hat{z}\cdot(1-\mu)\cdot\omega\cdot\left(\frac{f(\hat{z}|\bar{\kappa})}{f(\hat{z}|\underline{\kappa})}-1\right)}.$$

3. *If $\mu \in (\mu_2, \mu_3]$, the G-product agent simplifies ($\sigma_G = 0$), and the B-product agent complexifies ($\sigma_G = 1$).*

4. *If $\mu \in (\mu_3, \mu_4)$, the B-product agent complexifies ($\sigma_B = 1$), and the G-product agent complexifies with probability*

$$\sigma_G \in \left\{0,\ 1 - \frac{(1-\hat{z})\cdot(1-\mu)\cdot\omega - \hat{z}\cdot\mu\cdot(1-\omega)}{\hat{z}\cdot\mu\cdot(1-\omega)\cdot\left(\frac{f(\hat{z}|\underline{\kappa})}{f(\hat{z}|\bar{\kappa})}-1\right)},1\right\}.$$

5. *If $\mu \geq \mu_4$, all agents complexify ($\sigma_B = \sigma_G = 1$).*

where thresholds $\mu_1 - \mu_4$ are given in (24) − (27) and are a function of ω.

The results from Proposition 2 are illustrated in Figure 3. First, all agents choose high complexity when the principal's belief are sufficiently high, and they choose low complexity when beliefs are low. Since complexification is relatively more beneficial for the agent that chooses a B-product, such agents begin to complexity sooner than those with G-products. In sum, complexity choices not only depend on the agents' choice of product output, but most importantly, on the environment determined by $\{\mu, \omega, \hat{z}\}$.

In is worth noting that there is a region of beliefs, $\mu \in (\mu_3, \mu_4)$ in Proposition 2, where multiple choices of complexity may be consistent with an equilibrium. In what follows, we show that this feature may give rise to multiple equilibria. We proceed to analyze the choices of output consistent with an equilibrium belief μ. Recall from Section 3.3 that while the H-type always chooses the G-product, the values taken by correspondence $F(\mu)$ determine the L-type's incentives to choose output H ($F(\mu) > 0$) or B ($F(\mu) < 0$).

The next lemma establishes some properties of the correspondence F that are useful for the characterization of equilibria.

Lemma 3 *The set $\{\mu : F(\mu) = 0\}$ is non-empty and generically has only one strictly positive element, denoted by ψ. Furthermore, $\psi \in (\mu_3, \mu_4)$ and $F(\mu) > 0$ if $\mu < \mu_3$ and $F(\mu) < 0$ if $\mu > \mu_4$, with μ_3 and μ_4 given in (25) and (27).*

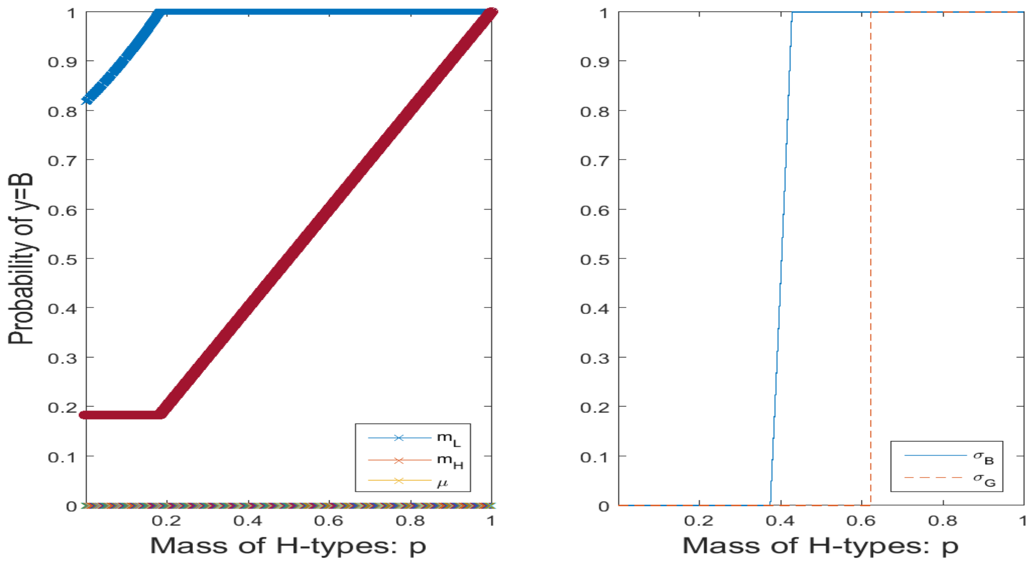

Figure 4: Equilibrium strategies as a function of p.

The following proposition concludes the characterization of equilibria by determining the t-type agent's choice of output, and the equilibrium belief level μ.

Proposition 3 *An equilibrium exists, and it falls within the following categories.*

1. *If $p \geq \psi$, in equilibrium the H-type agent chooses output G w.p.1, while the L-type agent chooses output B w.p.1., which implies $\mu = p$.*

2. *If $p \leq \mu_3$, in equilibrium the H-type agent chooses output G w.p.1, while the L-type agent chooses output B w.p.*
$$m_L = \frac{1-\psi}{1-p}$$
which implies $\mu = p + (1-p) \cdot (1 - m_L)$.

3. *If $p \in (\mu_3, \psi)$ and $\min\{F(\mu_3)\} > 0$, in equilibrium the H-type agent chooses output G w.p.1, while the L-type agent chooses output B w.p.*
$$m_L = \frac{1-\psi}{1-p}.$$
If instead $\min\{F(\mu_3)\} < 0$, there are two possible equilibria,
$$m_L \in \left\{ \frac{1-\psi}{1-p}\,,\ 1 \right\}.$$

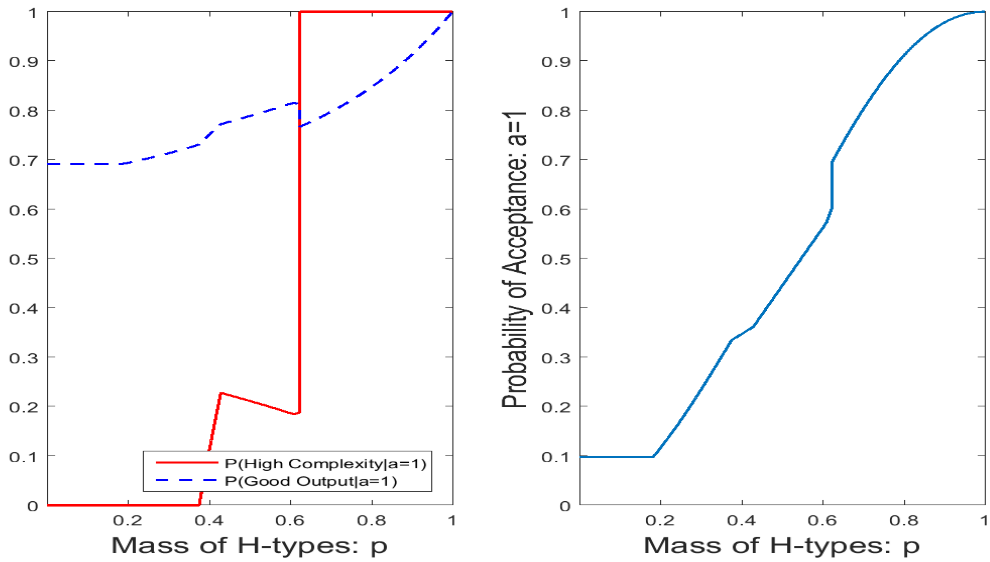

Figure 5: Quality of accepted products: output and complexity.

Both cases imply $\mu = p + (1-p) \cdot (1-m_L)$.

The choice of complexity is given in Proposition 2 for the corresponding μ *for each case.*

It is worth noting that multiple equilibria exist only when $p \in (\mu_3, \psi)$ and $F(\mu_3) < 0$. Otherwise, there are only two cases: the L-type chooses the B-product w.p.1. when $p > \psi$, and chooses the B-product with probability $m_L = \frac{1-\psi}{1-p} < 0$ otherwise. The results from Proposition 3 for such case are illustrated in Figure 4.

As the probability of an agent being H-type, p, increases, the L-type agent chooses worse output products in equilibrium: m_L weakly increases in p. This is intuitive, a higher p increases the incentives of the L-type to choose the B-product by increasing the relative probability of acceptance of B- vs. G-products. Furthermore, we learn that in any equilibrium the probability of a product being G-output is bounded below by μ_3: if p is below such level, the L-type has to mix so that principal's beliefs are higher in equilibrium. The reason is that when the principal's beliefs are relatively low, the L-type's expected payoff is higher when choosing the G-product ($F(p) > 0$). However, as discussed before, all agent choosing the G-product ($\mu = 1$) cannot be an equilibrium.

Figure 5 shows how the choice of output and complexity of accepted products vary in equilibrium with the measure of H-type agents, p. The right-hand side panel plots the probability of a product having G-output and high complexity, respectively, conditional on the product being accepted, as a function of p. Initially, as p increases, complexity and product output remain constant. This is because increases in p improve the incentives of the L-type to propose bad products. As a result, the measure of L-type agents proposing bad products increases to perfectly offset the reduction in measure of L-type agents. However, when p is high enough, agents begin to complexity their proposed products, and product output improves with the measure of H-type agents. The left-hand side panel shows the unconditional probability of accepting a product, which (weakly) increases in p.

Proposition 4 (Comparative Statics) *The equilibrium level of product complexity, σ_G and σ_B, is increasing in p and decreasing in ω. Moreover, $\sigma_G = \sigma_B = 1$ if either p is sufficiently large or ω is sufficiently low.*

Proposition 5 (Complexification) *If $\underline{v}$ is sufficiently close to $\bar{v}$, then in all equilibria $\sigma_G = \sigma_B = 1$.*

4 Binary and Costly Complexity

In this section, we incorporate a direct cost of accepting complex products. We suppose that the principal cost $c(\kappa)$ when a product with complexity κ is accepted. Thus, the principal's utility becomes:

$$W(a|s,z) \equiv a \cdot E\left[w(y) - c(\kappa)|s,z\right] + (1-a)\cdot w_0 \tag{16}$$

We continue to study the case of binary complexity and we make the following assumptions for payoffs.

Assumption 2 *The payoffs satisfy the following properties:*

1. $c(\underline{\kappa}) = 0$ *and* $c(\bar{\kappa}) = \bar{c} > 0$.
2. $w(G) - \bar{c} > w_0$.

The first assumption normalizes payoffs so that the cost of low complexity is zero. The second assumption states that the cost of complexity is not too large, so that a product is accepted if the principal is sufficiently confident that it is a Good.

The only difference with our baseline model of Section 3 is that the principal's approval decision is now modified to incorporate this direct cost of complexity. In particular, the principal's expected payoff, $W(a|s,z)$ as stated in (1), can be re-written as follows,

$$a \cdot \{\mu(s,z) \cdot [w(G) - \sigma_G(z)\cdot \bar{c}] + (1-\mu(s,z)) \cdot [w(B) - \sigma_B(z) \cdot \bar{c}]\} + (1-a)\cdot w_0 \tag{17}$$

where

$$\sigma_y(z) \equiv \mathbb{P}(\kappa = \bar{\kappa}|y, z) = \frac{\sigma_y f(z|\bar{\kappa})}{\sigma_y f(z|\bar{\kappa}) + (1-\sigma_y) f(z|\underline{\kappa})} \tag{18}$$

and $\sigma_y = \mathbb{P}(\kappa = \bar{\kappa}|y)$ is the probability that the agent who chooses y-output also makes it complex. Note that $\sigma_y(z)$ weakly increases in z for all y.

By inspection of (17), we see that the principal accepts the product, $a(s,z) = 1$, if:

$$\mu(s,z) \geq \frac{w_0 - w(B) + \sigma_B(z) \cdot \bar{c}}{w(G) - w(B) + (\sigma_B(z) - \sigma_G(z)) \cdot \bar{c}} \equiv \omega(z). \tag{19}$$

The left-hand side of condition (19) is the principal's posterior belief of the output being G, and is the same as in (7). The right-hand side incorporates the fact that complexity is costly to the principal. The new relative outside option, $\omega(z)$, differs from ω in Section 3 for two reasons. First, it is higher to reflect the fact that complex products are more costly to the principal. Second, its value weakly increases in the noise of the signal. In contrast to our baseline model, the principal now also has to update her beliefs about the complexity of the product after observing noise z, since $z \sim f(z|\kappa)$. Finally, note that when $c = 0$, the principal's problem becomes the same as in Section 3; that is, $\omega(z) = \omega$ for all z.

As in Section 3, let $\bar{z}$ be defined as the solution to (19) with equality for some s. Then, as before we say that:

Definition 2 *We are in the* **acceptance region** *when threshold $\bar{z}$ is given by condition $\omega(\bar{z}) = \mu(b, \bar{z})$; while we are in the* **rejection region** *when threshold $\bar{z}$ is given by condition $\omega(\bar{z}) = \mu(g, \bar{z})$.*

The following Lemma characterizes the principal's acceptance decision rule.

Lemma 4 *The threshold $\bar{z}$ is unique provided that $\bar{c}$ is not too large, and the principal's acceptance decision is as in Lemma 1. Furthermore, threshold $\bar{z}$ is increasing in the cost of complexity, $\bar{c}$, in the acceptance region, and decreasing in the rejection region.*

These acceptance decisions depend, through beliefs $\mu(z)$ and $\sigma_y(z)$, on the t-agent's equilibrium strategies $\{m_t, \sigma_t\}$, which the principal takes as given. As in the no cost model, the principal's acceptance decision rule is contingent on the signal only when the signal is sufficiently informative, i.e. $z < \bar{z}$. However, when complexity is costly, the principal's decision rule becomes more tight as she tends to reject products more often. In the acceptance region the principal relies more on the signal than in the no cost case making acceptance less likely to occur, while in the rejection region the principal relies less on the signal making rejection more likely to occur.

The equilibrium characterization follows analogously to the no cost case, and it is summarized in the following proposition.

Proposition 6 *The equilibrium characterization is as in Propositions 2 and 3, but with modified thresholds μ_1-μ_4 and ψ are given in* (**??**) $-$ 39*. In particular, the equilibrium level of product complexity is decreasing in the cost $\bar{c}$.*

Thus, the main qualitative results of our baseline model carry over to the case with a direct cost to complexity, with the not very surprising prediction that the equilibrium level of complexity will be lower if it is costlier.

5 Continuous and Costly Complexity

[Work in Progress.] In this section, we solve the most general version of our model, where we suppose $\kappa \in [\underline{\kappa}, \bar{\kappa}]$ and $c(\cdot)$ and $C(\cdot)$ are both increasing and convex. To be written.

6 Conclusion

This paper presents a model of complexity, in which complexity is a strategic choice made by agents. The model sheds light on the incentives of agents to add complexity to products or rules in order to increase their chance of approval by a principal decision maker. The model delivers a powerful insight: whether the agents add unnecessary complexity to products or over-simplify rules depends crucially on the principal's outside option when when she makes the approval decision. When the principal has a high demand for products, the agents have the incentive to complexify rules; however, if the principal has a low demand for the product, the agent will tend to simplify rules. This result emerges regardless of the biases that a agent has towards serving his or the principal's interest.

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A Appendix: Proofs and Complementary Lemmas

A.1 Proofs for Section 3

Proof of Proposition 1.

Case 1 (Acceptance Region): $\omega = \mu(b, \bar{z})$.

In the acceptance region,the Agent's problem is given by

$$\max_{\kappa \in \{\underline{\kappa}, \bar{\kappa}\}} \int_0^{\bar{z}} P(s = g|y) \cdot f(z|\kappa)dz + \int_{\bar{z}}^{1/2} f(z|\kappa)dz.$$

The agent with the B-product chooses action $\bar{\kappa}$ if

$$\int_0^{\bar{z}} z \cdot f(z|\bar{\kappa})dz + \int_{\bar{z}}^{1/2} f(z|\bar{\kappa})dz \geq \int_0^{\bar{z}} P(s = g|y = B) \cdot f(z|\underline{\kappa})dz + \int_{\bar{z}}^{1/2} f(z|\underline{\kappa})dz.$$

This reduces to

$$\int_0^{\bar{z}} (1 - z) \cdot (f(z|\underline{\kappa}) - f(z|\bar{\kappa}))dz \geq 0. \tag{20}$$

But

$$\int_0^{\bar{z}} (1 - z) \cdot (f(z|\underline{\kappa}) - f(z|\bar{\kappa}))dz > (1 - \bar{z})(F(\bar{z}|\underline{\kappa}) - F(\bar{z}|\bar{\kappa})) > 0.$$

Thus, condition (20) is satisfied for all $\bar{z}$, and the agent with the B-product always chooses $\bar{\kappa}$.

The agent with the G-product chooses action $\bar{\kappa}$ if

$$\int_0^{\bar{z}} z \cdot f(z|\bar{\kappa})dz + \int_{\bar{z}}^{1/2} f(z|\bar{\kappa})dz \geq \int_0^{\bar{z}} P(s = g|y = G) \cdot f(z|\underline{\kappa})dz + \int_{\bar{z}}^{1/2} f(z|\underline{\kappa})dz.$$

This reduces to

$$\int_0^{\bar{z}} z \cdot (f(z|\underline{\kappa}) - f(z|\bar{\kappa}))dz \geq 0. \tag{21}$$

Thus, (21)is satisfied and the agent with the G-product chooses $\bar{\kappa}$ if $\bar{z} \leq \hat{z}$. Otherwise, (21)is not satisfied and the agent with the G-product chooses $\underline{\kappa}$.

Case 2 (Rejection Region): $\omega = \mu(g, \bar{z})$.

In the rejection region, the Agent's problem is given by

$$\max_{\kappa \in \{\underline{\kappa}, \bar{\kappa}\}} \int_0^{\bar{z}} P(s = g|y) \cdot f(z|\kappa)dz.$$

The agent with the B-product chooses action $\underline{\kappa}$ if

$$\int_0^{\bar{z}} z \cdot f(z|\bar{\kappa})dz \leq \int_0^{\bar{z}} P(s = g|y = G) \cdot f(z|\underline{\kappa})dz.$$

This reduces to

$$\int_0^{\bar{z}} z \cdot (f(z|\underline{\kappa}) - f(z|\bar{\kappa}))dz \geq 0. \tag{22}$$

Thus, (22)is satisfied and the agent with the B-product chooses $\underline{\kappa}$ if $\bar{z} \leq \hat{z}$. Otherwise, (22)is not satisfied and the agent with the B-product chooses $\bar{\kappa}$.

The agent with the G-product chooses action $\underline{\kappa}$ if

$$\int_0^{\bar{z}} z \cdot f(z|\bar{\kappa})dz \leq \int_0^{\bar{z}} P(s = g|y = B) \cdot f(z|\underline{\kappa})dz.$$

This reduces to

$$\int_0^{\bar{z}} (1 - z) \cdot (f(z|\underline{\kappa}) - f(z|\bar{\kappa}))dz \geq 0. \tag{23}$$

But

$$\int_0^{\bar{z}} (f(z|\underline{\kappa}) - f(z|\bar{\kappa}))dz > \int_0^{\bar{z}} z \cdot (f(z|\underline{\kappa}) - f(z|\bar{\kappa}))dz$$

Thus, condition (23) is satisfied for all $\bar{z}$, and the agent with the G-product always chooses $\underline{\kappa}$. ■

Proof of Lemma 2. The H-type's net benefit from choosing output G is given by

$$F(\mu) = \left\{\max_{\kappa} \mathbb{P}_\mu(a = 1|G, \kappa) \cdot \bar{v}\right\} - \left\{\max_{\kappa} \mathbb{P}_\mu(a = 1|B, \kappa) \cdot \underline{v}\right\}.$$

Since $\mathbb{P}_\mu(a = 1|G, \kappa) \geq \mathbb{P}_\mu(a = 1|B, \kappa)$ for all κ, and $\bar{v} \geq \underline{v}$, it follows that the net benefit for the H-type is always positive. This type will therefore always get a higher expected payoff from producing the G-product over the B-product.

The L-type's net benefit from choosing output G is given by

$$F(\mu) = \left\{\max_{\kappa} \mathbb{P}_\mu(a = 1|G, \kappa) \cdot \underline{v}\right\} - \left\{\max_{\kappa} \mathbb{P}_\mu(a = 1|B, \kappa) \cdot \bar{v}\right\}.$$

Therefore, the L-type chooses the G-product whenever $F(\mu) > 0$, chooses the B-product whenever $F(\mu) < 0$, and is indifferent whenever $F(\mu) = 0$. In this latter case, the agent may mix between the two outputs, so $m_H \in [0, 1]$. ■

Proof of Proposition 2.

We begin by characterizing all equilbiria of in the subgame in which an agent with product output y choses strategy σ_y, where the principal starts from prior belief μ about the probability of an agent choosing $y = G$.

Consider first the **pooling equilibrium** in which choose $\sigma_B = \sigma_G = 0$. This requires that $\mu < \omega$. On the equilibrium path, there is no updating from observation of z and thus $\bar{z}$ is given implicitly by

$$\frac{(1 - \bar{z}) \cdot \mu}{(1 - \bar{z}) \cdot \mu + \bar{z} \cdot (1 - \mu)} = \omega.$$

Thus,

$$\bar{z} = \frac{(1 - \omega) \cdot \mu}{(1 - \omega) \cdot \mu + \omega \cdot (1 - \mu)}.$$

This is an equilibrium if and only if $\bar{z} \leq \hat{z}$. This is equivalent to

$$\mu_1 \equiv \frac{\omega}{\omega + (1-\omega)\frac{1-\hat{z}}{\hat{z}}} \geq \mu. \tag{24}$$

Consider next the **pooling equilibrium** in which $\sigma_B = \sigma_G = 1$. This requires that $\mu > \omega$. On the equilibrium path, there is no updating from observation of z and thus $\bar{z}$ is given by

$$\frac{\bar{z} \cdot \mu}{\bar{z} \cdot \mu + (1-\bar{z}) \cdot (1-\mu)} = \omega.$$

Thus,

$$\bar{z} = \frac{\omega \cdot (1-\mu)}{(1-\omega) \cdot \mu + \omega \cdot (1-\mu)}.$$

This is an equilibrium if and only if $\bar{z} \leq \hat{z}$. This implies

$$\mu_3 \equiv \frac{\omega \cdot \mu}{(\omega \cdot \mu + (1-\omega) \cdot \frac{\hat{z}}{1-\hat{z}}} \leq \mu \tag{25}$$

Let us construct a **separating equilibrium** in which $\sigma_B = 1$ and $\sigma_G = 0$. There are two cases to consider, depending on whether the principal rejects or accepts complex regulation, which depends on beliefs at $z = 1/2$. Suppose that

$$\frac{l(1/2) \cdot \mu}{l(1/2) \cdot \mu + (1-\mu)} < \omega.$$

Then, in the separating equilibrium, complex products are rejected. There is now updating from the observation of z and thus $\bar{z}$ is given by

$$\frac{(1-\bar{z}) \cdot \mu}{(1-\bar{z}) \cdot \mu + \bar{z} \cdot (1-\mu) \cdot \frac{1}{l(\bar{z})}} = \omega.$$

This is an equilibrium if and only if $\bar{z} \geq \hat{z}$, so

$$\frac{\omega}{\omega + (1-\omega) \cdot l(\hat{z}) \cdot \frac{1-\hat{z}}{\hat{z}}} \leq \mu \leq \frac{\omega}{\omega + (1-\omega) \cdot l(0.5)} \equiv \tilde{\mu}. \tag{26}$$

Suppose that

$$\frac{l(1/2) \cdot \mu}{l(1/2) \cdot \mu + (1-\mu)} \geq \omega.$$

Then, in the **separating equilibrium**, complex products are accepted. There is updating from the observation of z and thus $\bar{z}$ is given by

$$\frac{\bar{z} \cdot \mu}{\bar{z} \cdot \mu + (1-\bar{z}) \cdot (1-\mu) \cdot \frac{1}{l(\bar{z})}} = \omega.$$

This is an equilibrium if and only if $\bar{z} \geq \hat{z}$, so

$$\frac{\omega}{\omega+(1-\omega)\cdot l(0.5)} \leq \mu \leq \frac{\omega}{\omega+(1-\omega)\cdot l(\hat{z})\frac{\hat{z}}{1-\hat{z}}} \equiv \mu_4. \tag{27}$$

Let us construct a **semi-separating equilibrium** in which $sigma_B \in (0,1)$. Such an equilibrium requires $mu < \omega$, so $sigma_G = 0$.. When complexifying regulation with some probability σ_B, the posterior beliefs at $\hat{z}$ are:

$$\mu(G,\bar{z}) = \frac{(1-\bar{z})\cdot\mu}{((1-\bar{z})\cdot\mu+\bar{z}\cdot(1-\mu)\cdot\left(1-\sigma_B+\sigma_B\cdot\frac{1}{l(\bar{z})}\right)} = \omega.$$

This means

$$\sigma_B = \frac{(1-\hat{z})\cdot\mu\cdot(1-\omega)-\hat{z}\cdot(1-\mu)\cdot\omega}{\hat{z}\cdot(1-\mu)\cdot\omega\cdot\left(\frac{f(\hat{z}|\bar{\kappa})}{f(\hat{z}|\underline{\kappa})}-1\right)}.$$

The belief $\mu(G,\bar{z})$ is decreasing in σ_b since $\frac{1}{l(\bar{z})} > 1$. Therefore, an equilibrium in which $\sigma_B \in (0,1)$ exists if and only if:

$$\mu^U(G,\hat{z}) \leq \omega \leq \mu^N(G,\hat{z})$$

where

$$\mu^U(G,z) \equiv \frac{(1-z)\cdot\mu}{(1-z)\cdot\mu+z\cdot(1-\mu)\cdot\frac{1}{l(\bar{z})}}.$$

and

$$\mu^N(G,z) \equiv \frac{(1-z)\cdot\mu}{(1-z)\cdot\mu+z\cdot(1-\mu)}.$$

This is equivalent to

$$\frac{\omega}{\omega+(1-\omega)\cdot\frac{1-\hat{z}}{\hat{z}}} \leq \mu \leq \frac{\omega}{\omega+(1-\omega)\cdot\frac{1-\hat{z}}{\hat{z}}\cdot l(\hat{z})} \equiv \mu_2. \tag{28}$$

Finally, let us construct a **semi-separating equilibrium** in which $sigma_G \in (0,1)$. Such an equilibrium requires $mu \geq \omega$, so $sigma_B = 1$.. When complexifying regulation with some probability σ_G, the posterior beliefs at $\hat{z}$ are:

$$\mu(B,\bar{z}) = \frac{\bar{z}\cdot\mu}{(\bar{z}\cdot\mu+(1-\bar{z})\cdot(1-\mu)\cdot\frac{1}{1-\sigma_G+\sigma_G\cdot l(\bar{z})}} = \omega.$$

This implies

$$\sigma_G = 1 - \frac{(1-\hat{z})\cdot(1-\mu)\cdot\omega-\hat{z}\cdot\mu\cdot(1-\omega)}{\hat{z}\cdot\mu\cdot(1-\omega)\cdot\left(\frac{f(\hat{z}|\underline{\kappa})}{f(\hat{z}|\bar{\kappa})}-1\right)}.$$

The belief $\mu(B,\bar{z})$ is decreasing in σ_b since $\frac{1}{l(\bar{z})} > 1$. Therefore, an equilibrium in which

$\sigma_G \in (0,1)$ exists if and only if:

$$\mu^U(B,\hat{z}) \leq \omega \leq \mu^N(B,\hat{z}).$$

where

$$\mu^U(B,z) \equiv \frac{z\cdot\mu}{z\cdot\mu + (1-z)\cdot(1-\mu)\cdot\frac{1}{l(\bar{z})}}.$$

and

$$\mu^N(B,z) \equiv \frac{z\cdot\mu}{z\cdot\mu + (1-z)\cdot(1-\mu)}.$$

This is equivalent to

$$\frac{\omega}{\omega + (1-\omega)\cdot\frac{\hat{z}}{1-\hat{z}}} \leq \mu \leq \frac{\omega}{\omega + (1-\omega)\cdot\frac{\hat{z}}{1-\hat{z}}\cdot l(\hat{z})}.$$

Summing up the above results, we obtain that

1. If $\mu \leq \mu_1$, the equilibrium is $\sigma_B = \sigma_G = 0$.
2. If $\mu \in (\mu_1, \mu_2]$, then $\sigma_G = 0$,

$$\sigma_B = \frac{(1-\hat{z})\cdot\mu\cdot(1-\omega) - \hat{z}\cdot(1-\mu)\cdot\omega}{\hat{z}\cdot(1-\mu)\cdot\omega\cdot\left(\frac{f(\hat{z}|\bar{\kappa})}{f(\hat{z}|\underline{\kappa})} - 1\right)}.$$

3. If $\mu \in (\mu_2, \mu_4]$, then $\sigma_G = 0$ and $\sigma_B = 1$.
4. If $\mu \in (\mu_3, \mu_4)$, $\sigma_B = 1$ and

$$\sigma_G \in \left\{0,\ 1 - \frac{(1-\hat{z})\cdot(1-\mu)\cdot\omega - \hat{z}\cdot\mu\cdot(1-\omega)}{\hat{z}\cdot\mu\cdot(1-\omega)\cdot\left(\frac{f(\hat{z}|\underline{\kappa})}{f(\hat{z}|\bar{\kappa})} - 1\right)}\ , 1\right\}.$$

5. If $\mu \geq \mu_3$, then $\sigma_B = \sigma_G = 1$.

■

Proof of Lemma 3. For the H-type agent, we have

$$F(\mu) = \left\{\max_{\kappa} \mathbb{P}_\mu(a=1|G,\kappa)\cdot\bar{v}\right\} - \left\{\max_{\kappa} \mathbb{P}_\mu(a=1|B,\kappa)\cdot\underline{v}\right\}.$$

Notice that, $\forall\mu$, $\max_\kappa \mathbb{P}_\mu(a=1|G,\kappa) \geq \max_\kappa \mathbb{P}_\mu(a=1|B,\kappa)$. It then follows that $F(0)=0$ and all values $f \in F(\mu)$ satisfy $f > 0 \forall \mu \in (0,1]$.

For the L-type agent, we have

$$F(\mu) = \left\{\max_{\kappa} \mathbb{P}_\mu(a=1|G,\kappa)\cdot\underline{v}\right\} - \left\{\max_{\kappa} \mathbb{P}_\mu(a=1|B,\kappa)\cdot\bar{v}\right\}.$$

First, notice that $F(0) = 0$. Consider in what follows the values $\mu in\,(0,1]$. The values $\max_\kappa \mathbb{P}_\mu(a=1|G,\kappa)$ and $\max_\kappa \mathbb{P}_\mu(a=1|B,\kappa)$ depend on the equilibrium in the subgame in which σ_t is chosen. We therefore consider each case from the equilibria described in Proposition 2. As will become clear, $F(\mu)$ is a function when we restrict attention to only one type of equilibrium on complexity. Therefore, for each $\mu \in (\mu_3, \mu_4]$, for which we have multiple equilibria, we denote values of $F(\mu)$ in the three possible equilibria by $F^S(\mu)$ for the separating equilibrium with $\sigma_G = 0$, $F^P(\mu)$ for the pooling equilibrium with $\sigma_G = 1$, $F^M(\mu)$ for the mixing equilibrium with $\sigma_G \in (0,1)$.

Case 1. The equilibrium with $\sigma_G = \sigma_B = 0$. In this equilibrium, we have

$$\bar{z} = \frac{\mu \cdot (1-\omega)}{\mu \cdot (1-\omega) + (1-\mu) \cdot \omega}.$$

Thus

$$F(\mu) = \underline{v} \cdot \int_0^{\bar{z}(\mu)} (1-z) f(z|\underline{\kappa}) \, dz - \bar{v} \cdot \int_0^{\bar{z}(\mu)} z f(z|\underline{\kappa}) \, dz,$$

which implies

$$F'(\mu) = [\underline{v} - (\underline{v} + \bar{v}) \cdot \bar{z}] \cdot f(z|\underline{\kappa}) \cdot \frac{d\bar{z}}{d\mu}.$$

It is easy to check that $\frac{d\bar{z}}{d\mu} > 0$. In addition, notice that $\bar{z}(0) = 0$. For this to be an equilibrium,it must be that $\bar{z}(\mu) \leq \hat{z}$. Thus, this equilibrium exists for $\mu \in [0, \mu_1]$. Let μ_v be the value of μ at which

$$\bar{z}(\mu_v) = \frac{\underline{v}}{(\underline{v} + \bar{v})},$$

and notice that

$$\bar{z}(\mu_1) = \hat{z}.$$

Thus, μ_v is given by

$$\frac{1-\mu_v}{\mu_v} \frac{\omega}{1-\omega} = \frac{\bar{v}}{\underline{v}}. \tag{29}$$

So , comparing μ_v to μ_1, we infer that:

- If $\mu_v > \mu_1$, so $\frac{\bar{v}}{\underline{v}} < \frac{1-\hat{z}}{\hat{z}}$. then $F'(\mu) > 0$ for all $\mu \in [0, \mu_1]$;
- If $\mu_v \leq \mu_1$, so $\frac{\bar{v}}{\underline{v}} \geq \frac{1-\hat{z}}{\hat{z}}$, then $F'(\mu) > 0$ for $\mu \in (0, \mu_v)$ and $F'(\mu) < 0$ for $\mu \in (\mu_v, \mu_1]$.

Case 2. The equilibrium with $\sigma_G = 0$ and $\sigma_B \in (0,1)$.
In any equilibrium in which $\sigma_B \in (0,1)$, it must be the case that $\hat{z} = \bar{z}$ and therefore

$$\sigma_B = \frac{\hat{z}(1-\mu)\omega - (1-z)\mu(1-\omega)}{z(1-\mu)\omega} \frac{l(\hat{z})}{l(\hat{z})-1}$$

. So

$$F(\mu) = \underline{v} \cdot \int_0^{\hat{z}} (1-z) f(z|\underline{\kappa}) \, dz - \bar{v} \cdot \int_0^{\hat{z}} z f(z|\underline{\kappa}) \, dz = F(\mu_1).$$

Then

$$F'(\mu) = 0.$$

Thus, $F(\mu)$ is constant over this interval of μ values.

Case 3. The equilibrium with $\sigma_G = 0$ and $\sigma_B = 1$. This equilibrium exists in two different regions. **Part 3a.** First, consider the rejection region. In this region,

$$\bar{z}(\mu) = \frac{(1-\omega)\cdot\mu}{(1-\omega)\cdot\mu + (1-\mu)\cdot\omega\cdot\frac{f(\bar{z}|\bar{\kappa})}{f(\bar{z}|\underline{\kappa})}}$$

and this is an equilibrium when $\bar{z} \geq \hat{z}$. Notice that $\frac{d\bar{z}}{d\mu} > 0$, and so the minimum of $\hat{z}$ is reached at μ_2:

$$\frac{(1-\omega)\cdot\mu_2}{(1-\omega)\cdot\mu_2 + (1-\mu_2)\cdot\omega\cdot\frac{1}{l(\hat{z})}} = \hat{z},$$

and the maximum of $\frac{1}{2}$ is reached at $\tilde{\mu}$:

$$\frac{(1-\omega)\cdot\tilde{\mu}}{(1-\omega)\cdot\tilde{\mu} + (1-\tilde{\mu})\cdot\omega\cdot\frac{1}{l(0.5)}} = \frac{1}{2}.$$

Then,

$$F(\mu) = \underline{v}\cdot\int_0^{\bar{z}(\mu)}(1-z)\, f(z|\underline{\kappa})\,dz - \bar{v}\cdot\int_0^{\bar{z}(\mu)} z f(z|\bar{\kappa})\,dz$$

and

$$F'(\mu) = \left[\underline{v}\cdot(1-\bar{z})\cdot f(\bar{z}|\underline{\kappa}) - \bar{v}\cdot\bar{z}\cdot f(\bar{z}|\bar{\kappa})\right]\frac{d\bar{z}}{d\mu}.$$

So, $F'(\mu) \geq 0$ if

$$\bar{z}(\mu) \leq \frac{1}{1+\frac{\bar{v}}{\underline{v}}\cdot\frac{1}{l(\bar{z})}},$$

which reduces to

$$\frac{(1-\mu)\cdot\omega}{\mu\cdot(1-\omega)} \geq \frac{\bar{v}}{\underline{v}}. \tag{30}$$

Notice that the above expression holds with equality at $\mu = \mu_v$, with μ_v defined in (29). Notice that the left-hand side of (30) is decreasing in μ, and at $\tilde{\mu}$, it takes the value $l(0.5)$. Since $l(0.5) < 1$ and $1 < \frac{\bar{v}}{\underline{v}}$, it follows that $F'(\tilde{\mu}) < 0$.

At μ_2, the left-hand side of (30) becomes $\frac{(1-\hat{z})}{\hat{z}}\cdot l(\hat{z})$, so:

- If $\frac{(1-\hat{z})}{\hat{z}}\cdot l(\hat{z}) < \frac{\bar{v}}{\underline{v}}$, then $F'(\mu_2) < 0$ and $F'(\mu) < 0 \forall\mu \in (\mu_2, \tilde{\mu}]$. Also, $\mu_v \in (\mu_1, \mu_2)$.
- If $\frac{(1-\hat{z})}{\hat{z}}\cdot l(\hat{z}) \geq \frac{\bar{v}}{\underline{v}}$, then $F'(\mu_2) \geq 0$. Since μ_v we have $F'(\mu_v) = 0$, this implies $\tilde{\mu} > \mu_v > \mu_2$ and $F'(\mu) \geq 0\,\forall\mu \in (\mu_2, \mu_v)$ and $F'(\mu) < 0\forall\mu \in (\mu_v, \tilde{\mu}]$.

Part 3b. Second, consider the acceptance region. In this region, it must be the case that $\bar{z} \geq \hat{z}$, where

$$\bar{z}(\mu) = \frac{1}{1+\frac{\mu\cdot(1-\omega)}{(1-\mu)\cdot\omega}\cdot l(\bar{z})},$$

and thus

$$\frac{d\bar{z}}{d\mu} < 0.$$

This implies that the maximum $\bar{z} = 0.5$ is obtained at $\tilde{\mu}$, and the minimum $\bar{z}$ of $\hat{z}$ is obtained at μ_4. Then, for this equilibrium,

$$F(\mu) = \underline{v} \cdot \left[\int_0^{\bar{z}(\mu)} (1-z) f(z|\underline{\kappa})\, dz + (1 - F(\bar{z}|\underline{\kappa}))\right] - \bar{v} \cdot \left[\int_0^{\bar{z}(\mu)} z f(z|\bar{\kappa})\, dz + (1 - F(\bar{z}|\bar{\kappa}))\right]$$
$$= \underline{v} - \underline{v} \cdot \int_0^{\bar{z}(\mu)} z f(z|\underline{\kappa})\, dz - \bar{v} \cdot \left[\int_0^{\bar{z}(\mu)} z f(z|\bar{\kappa})\, dz + (1 - F(\bar{z}|\bar{\kappa}))\right]$$

Then,

$$F'(\mu) = \{\underline{v} \cdot [(1-\bar{z}) \cdot f(\bar{z}|\underline{\kappa}) - f(\bar{z}|\underline{\kappa})] - \bar{v} \cdot [\bar{z} \cdot f(\bar{z}|\bar{\kappa}) - f(\bar{z}|\bar{\kappa})]\} \frac{d\bar{z}}{d\mu}$$
$$= [\bar{v} \cdot (1-\bar{z}) \cdot f(\bar{z}|\bar{\kappa}) - \underline{v} \cdot \bar{z} \cdot f(\bar{z}|\underline{\kappa})] \frac{d\bar{z}}{d\mu} < 0, \forall \bar{z}.$$

Notice that the value $\tilde{\mu}$ is the threshold between the rejection and the acceptance regions. Since $\bar{z}(\tilde{\mu}) = \frac{1}{2}$, $F(\tilde{\mu})$ does not have any discontinuities when the regions change.

Case 4. The equilibrium with $\sigma_G = \sigma_B = 1$. This equilibrium exits in the acceptance region and it requires $\bar{z} \leq \hat{z}$, where

$$\bar{z}(\mu) = \frac{1}{1 + \frac{\mu \cdot (1-\omega)}{(1-\mu) \cdot \omega}}.$$

Notice that

$$\frac{d\bar{z}}{d\mu} < 0,$$

so the maximum $\bar{z} = \hat{z}$, is obtained at μ_3,and the minimum $\bar{z} = 0$, is obtained at $\mu = 1$.

In this case, in order to differentiate the value of $F(\mu)$ in this equilibrium from the value taken by $F(\mu)$ is in the separating equilibrium, denote its value in the pooling case by $F^P(\mu)$. Thus, $F^P(\mu)$ becomes

$$F^P(\mu) = \underline{v} \cdot \left[\int_0^{\bar{z}(\mu)} (1-z) f(z|\bar{\kappa})\, dz + (1 - F(\bar{z}|\bar{\kappa}))\right] - \bar{v} \cdot \left[\int_0^{\bar{z}(\mu)} z f(z|\bar{\kappa})\, dz + (1 - F(\bar{z}|\bar{\kappa}))\right].$$

Then,

$$F^{P'}(\mu) = [-\underline{v} \cdot \bar{z} \cdot f(\bar{z}|\bar{\kappa}) + \bar{v} \cdot (1-\bar{z}) \cdot f(z|\bar{\kappa})] \frac{d\bar{z}}{d\mu}$$
$$= [\bar{v} \cdot (1-\bar{z}) - \underline{v} \cdot \bar{z}] \cdot f(z|\bar{\kappa}) \cdot \frac{d\bar{z}}{d\mu} < 0.$$

At μ_3,we have $\bar{z}(\mu_3) = \hat{z}$, so

$$F^P(\mu_3) = \underline{v} \cdot \left[\int_0^{\hat{z}} (1-z) f(z|\bar{\kappa}) dz + (1 - F(\hat{z}|\bar{\kappa}))\right] - \bar{v} \cdot \left[\int_0^{\hat{z}} z f(z|\bar{\kappa}) dz + (1 - F(\hat{z}|\bar{\kappa}))\right]$$
$$= \underline{v} - (\underline{v} + \bar{v}) \cdot \int_0^{\hat{z}} z f(z|\bar{\kappa}) dz - \bar{v} \cdot (1 - F(\hat{z}|\bar{\kappa})).$$

Denoting by $F^S(\mu_4)$ the value of $F(\mu_4)$ under the separating equilibrium, we have

$$F^P(\mu_3) = F^S(\mu_4).$$

Case 5. The equilibrium with $\sigma_G \in (0,1)$ and $\sigma_B = 1$. An equilibrium with $\sigma_B = 1$ and $\sigma_G \in (0,1)$ exists in the acceptance region, given

$$F^P(\mu_3) = F^S(\mu_4).$$

In this case, $F(\mu)$ takes the value

$$F^M(\mu) = \underline{v} \cdot \int_0^{\hat{z}} [(1-\sigma_G)(1-z) f(z|\underline{\kappa}) + \sigma_G (1-z) f(z|\bar{\kappa})] dz - \bar{v} \cdot \int_0^{\hat{z}} z f(z|\bar{\kappa}) dz$$
$$+ \underline{v} \cdot (1 - (1-\sigma_G) F(\hat{z}|\underline{\kappa}) - \sigma_G F(\hat{z}|\bar{\kappa})) - \bar{v} \cdot (1 - F(\hat{z}|\bar{\kappa}))$$
$$= \underline{v} - \underline{v} \cdot \int_0^{\hat{z}} z f(z|\bar{\kappa}) dz - \bar{v} \cdot \int_0^{\hat{z}} z f(z|\bar{\kappa}) dz - \bar{v} \cdot (1 - F(\hat{z}|\bar{\kappa}))$$

Clearly, $F^M(\mu)$ is a constant function of μ, and $F^M(\mu) = F^P(\mu_3) = F^S(\mu_4)$.

Putting the above results together, the correspondence $F(\mu)$ is single-valued for $\mu \in [0, \mu_3) \cup (\mu_4, 1]$, and it can take values $\{F^S(\mu), F^P(\mu), F^M(\mu)\}$ in the interval $\mu \in [\mu_3, \mu_4]$. Moreover, $F(0) = 0$, $F(\mu)$ is strictly increasing over the interval $[0, \mu_v)$, and strictly decreasing over the intervals $[\mu_v, \mu_3)$ and $[\mu_3, \mu_4)$. The functions $F^S(\mu)$ and $F^P(\mu)$ are strictly decreasing over the interval $[\mu_3, \mu_4)$, the function $F^M(\mu)$ is constant over this interval and takes the value $F_P(\mu_3)$. This, together with $F^P(\mu_3) = F^S(\mu_4)$ and $F(1) < 0$ imply that

1. If $F^P(\mu_3) \neq 0$, then there exists a unique $\psi \in (\mu_v, 1)$ such that $F(\psi) = 0$;
2. If $F^P(\mu_3) = 0$, then 0 is a value taken by $F(\mu)$ for $\mu \in [\mu_3, \mu_4)$.

Thus, generically, there exists a unique $\psi > 0$ such that $F(\psi) = 0$. ■

Proof of Proposition 3.

As shown in Lemma 2, the H-type agent always chooses $y = G$. Thus, $\mu \geq p$.

If $p > \mu_4$, then as shown in Lemma 3, we have $F(p) < 0$. Thus, the L-type agent would prefer choosing $y = B$ with probability $m_L = 1$, and so $\mu = p$. In this case $F(\mu) < 0$ is consistent with the L-type's choice of $y = B$, and this is the unique equilibrium on m_L.

If $p \leq \mu_3$, then $F(p) \geq 0$, and at $\mu = p$. Assume the L-type agent chooses $y = B$ with probability m_L. For this to be an equilibrium, it must be that $F(\mu) = 0$ at $\mu = p + (1-p)(1-m_L)$ with $\mu geqp$. Thus, we must have $\mu = \psi$, which implies $m_L = \frac{1-\psi}{1-p}$.

For $p \in [\mu_3, \mu_4]$, we consider the two possible cases:

1. If $\min\{F(\mu_3)\} < 0$, then

   - If $p \in [\mu_3, \psi]$, then $F^S(\mu) > 0$, which implies that in this equilibrium $m_L < 1$. For this to be an equilibrium, it must be that $\mu = \psi$, which implies $m_L = \frac{1-\psi}{1-p}$. We also have $F^P(\mu) < 0$, and $F^M(\mu) < 0$, which implies that the L-type agent derives a higher payoff from choosing $y = B$. Then, $\mu = p$, and so $m_L = 1$ is the equilibrium in these cases.
   - If $p \in [\psi, \mu_4]$, then all values of $F(\mu)$ satisfy $F(\mu) < 0$, so $m_L = 1$ and $\mu = p$ in equilibrium.

2. If $\min\{F(\mu_3)\} > 0$, then

   - If $p \in [\mu_3, \psi]$, then all values of $F(\mu)$ satisfy $F(\mu) > 0$, so it must be that $\mu = \psi$ in equilibrium, s $m_L = \frac{1-\psi}{1-p}$.
   - If $p \in [\psi, \mu_4]$, then $F^P(\mu) < 0$, which implies that in this equilibrium $m_L = 1$. We also have $F^S(\mu) > 0$, and $F^M(\mu) > 0$, which implies that the L-type agent derives a higher payoff from choosing $y = G$ at p. But, this cannot be an equilibrium since it would require $F(\mu) =$ for $\mu > p$. Thus, the unique equilibrium in this interval is $m_L = 1$.

■

Proof of Proposition 4.

Comparative Statics with respect to p. Notice also that from (24)-(27), thresholds $\mu_1 - \mu_4$ are not functions of p. As shown in the derivation of $F(\mu)$ in the proof to Lemma 3 the value of ψ does not depend on p. Also, in equilibrium,

$$\mu = p + (1-p)(1-m_L).$$

1. If $p > \psi$, then the only equilibrium is the one in which $m_L = 1$, and so $\mu = p$. Then, $\frac{d\mu}{dp} = 1$. Then, an increase in p weakly increases complexity. In particular, equilibrium complexity does not change unless in the following case: $p \in (\psi, \mu_4)$ and in the equilibrium played on complexity is $\sigma_G = 0, \sigma_B = 1$, then an increase to $p' > \mu_4$ leads to the unique equilibrium with $\sigma_G = \sigma_B = 1$

2. If $p < \mu_3$, then the unique equilibrium is the one in which $\mu = \psi$. For an increase in p to p' such that $p < p' < \mu_3$, complexity does not change. For an increase in p to p' such that $p < p' > \mu_3$, complexity weakly increases.

3. If $p \in (\mu_3, \psi)$ and $\min\{F(\mu_3) > 0\}$ the unique equilibrium, one with $\mu = p$ and the analysis is as in part 1 above. If $\min\{F(\mu_3) < 0\}$, there are two possible equilibria. If the equilibrium is the one where $m_L = \frac{1-\psi}{1-p}$, then the analysis is as in part 2 above. If $m_L = 1$, then complexity does not change with an increase in p.

From parts 1-3 above, it follows that complexity weakly increases.

Finally, for $p \to 1$, $\mu \to 1$, which by Proposition 2 implies $\sigma_G = \sigma_B = 1$ in the unique equilibrium.

Comparative Statics with respect to ω.

Notice that $\mu = p + (1-p)(1-m_L)$, where $m_L \in \{1, \frac{1-\psi}{1-p}\}$. An increase in ω affects thresholds $\mu_1 - \mu_4$ in the following way:

$$\frac{d\mu_1}{d\omega} > 0; \frac{d\mu_2}{d\omega} > 0; \frac{d\tilde{\mu}}{d\omega} > 0; \frac{d\mu_3}{d\omega} > 0; \frac{d\mu_4}{d\omega} > 0.$$

If $\min\{F(\mu_3) > 0\}$, then at $\mu = \psi$, $F(\psi) = F^P(\psi)$, so $F(\psi)$ is derived the case in which the strategies on complexity are $\sigma_G = \sigma_B = 1$. This equilibrium is in the rejection region, so

$$\bar{z} = \frac{(1-\omega)\cdot\mu}{(1-\omega)\cdot\mu + \omega\cdot(1-\mu)}.$$

Thus, $\frac{d\bar{z}}{d\omega} < 0$.

$$F^P(\psi) = (\underline{v} + \bar{v}) \cdot \int_0^{\bar{z}(\psi)} zf\left(z|\bar{\kappa}\right)dz - \bar{v}\cdot F\left(\bar{z}|\bar{\kappa}\right) - \underline{v} + \bar{v}.$$

Differentiating this with respect to $\bar{z}$ yields:

$$\left(\underline{v}\cdot\bar{z} - (1-\bar{z})\cdot\bar{v}\right) f\left(\bar{z}|\bar{\kappa}\right) < 0,$$

since $z \leq \frac{1}{2}$. So, it must be the case that the value of $\bar{z}(\psi)$ does not change in equilibrium:

$$\frac{d\bar{z}(\psi)}{d\omega} = \frac{d\bar{z}}{d\omega} + \frac{d\bar{z}}{d\psi}\cdot\frac{d\psi}{d\omega}.$$

So

$$\frac{d\psi}{d\omega} > 0.$$

Therefore, ψ and thresholds $\mu_1 - \mu_4$ all increase. So, the equilibrium complexity weakly decreases.

If $\min\{F(\mu_3) < 0\}$, then at $\mu = \psi$, $F(\psi) = F^S(\psi)$, so $F(\psi)$ is derived the case in which the strategies on complexity are $\sigma_G = 0$ and $\sigma_B = 1$. This equilibrium is in the acceptance region, so

$$\bar{z} = \frac{\omega\cdot(1-\mu)}{(1-\omega)\cdot\mu + \omega\cdot(1-\mu)}.$$

Then, $\frac{d\bar{z}}{d\omega} > 0$. Given $F^S(\mu)$, we have

$$\frac{dF(\mu)}{d\bar{z}} = -\bar{v}\cdot(1-\bar{z})\,f\left(\bar{z}|\bar{\kappa}\right) + \underline{v}\cdot\bar{z}f\left(\bar{z}|\underline{\kappa}\right) < 0,$$

since $z < \frac{1}{2}$, $\bar{z} > \hat{z}$, and so $f\left(\bar{z}|\underline{\kappa}\right) < f\left(\bar{z}|\bar{\kappa}\right)$. It must therefore be the case that the value of $\bar{z}(\psi)$ does not change in equilibrium:

$$\frac{d\bar{z}(\psi)}{d\omega} = \frac{d\bar{z}}{d\omega} + \frac{d\bar{z}}{d\psi}\cdot\frac{d\psi}{d\omega}.$$

So

$$\frac{d\psi}{d\omega} > 0.$$

Therefore, as in the previous case, ψ and thresholds $\mu_1 - \mu_4$ all increase. Thus, the equilibrium complexity weakly decreases.

Finally, for $\omega \to 0$, $\mu_4 \to 0$, which by Proposition 2 implies $\sigma_G = \sigma_B = 1$ for all μ. ■

Proof of Proposition 5. For the correspondence $F(\mu)$, we have that $\min\{F(\mu_3)\} = F^P(\mu_3)$, where

$$F^P(\mu) = \underline{v} \cdot \left[\int_0^{\bar{z}(\mu)} (1-z) f(z|\bar{\kappa})\, dz + (1 - F(\bar{z}|\bar{\kappa})) \right] - \bar{v} \cdot \left[\int_0^{\bar{z}(\mu)} z f(z|\bar{\kappa})\, dz + (1 - F(\bar{z}|\bar{\kappa})) \right]$$

As $\underline{v} \to \bar{v}$,

$$F^P(\mu) \to \bar{v} \cdot \int_0^{\bar{z}(\mu)} (1-2z) f(z|\bar{\kappa})\, dz > 0. \tag{31}$$

Also,

$$\frac{\partial F^P(\mu)}{\partial \underline{v}} = \left[\int_0^{\bar{z}(\mu)} (1-z) f(z|\bar{\kappa})\, dz + (1 - F(\bar{z}|\bar{\kappa})) \right] > 0, \tag{32}$$

Thus, given (31) and (32), there exists threshold $\underline{v}^*$ such that $\forall \underline{v} \geq \underline{v}^*$, $\min F(\mu_3) > 0$. Then, all equilibria in this case have $\sigma_G = \sigma_B = 1$. ■

A.2 Proofs for Section 4

Proof of Lemma 4.

To drive the value of $\bar{z}$, we must consider the possible equilibria in each region. Consider first the rejection region.

Case 1r: Equilibrium with $\sigma_G = \sigma_B = 0$. In this case, $\bar{z}$ is given by

$$\frac{(1-\bar{z}) \cdot \mu}{(1-\bar{z}) \cdot \mu + \bar{z} \cdot (1-\mu)} = \frac{w_0 - w(B)}{w(G) - w(B)}, \tag{33}$$

which solves to

$$\bar{z} = \frac{\mu \cdot (1-\omega)}{\mu \cdot (1-\omega) + (1-\mu) \cdot \omega}.$$

Case 2r: Equilibrium with $\sigma_G = 0$ and $\sigma_B \in (0,1)$. In this case, for the holder of the B-product to mix, it must be that $\bar{z} = \hat{z}$.

Case 3r: Equilibrium with $\sigma_G = 0$ and $\sigma_B = 1$. For this to be an equilibrium, it requires that

$$\frac{f\left(\frac{1}{2}|\underline{\kappa}\right) \cdot \mu}{f\left(\frac{1}{2}|\underline{\kappa}\right) \cdot \mu + f\left(\frac{1}{2}|\bar{\kappa}\right) \cdot (1-\mu) \cdot} < \frac{w_0 - w(B) + c}{w(G) - w(B) + c}.$$

Then, $\bar{z}$ is given by

$$\frac{(1-\bar{z}) \cdot \mu}{(1-\bar{z}) \cdot \mu + \bar{z} \cdot (1-\mu) \cdot \frac{1}{l(\bar{z})}} = \frac{w_0 - w(B) + c}{w(G) - w(B) + c}. \tag{34}$$

Consider next the acceptance region. Case 1a: Equilibrium with $\sigma_G = \sigma_B = 1$. In this case, $\bar{z}$ is given by

$$\frac{\bar{z} \cdot \mu}{\bar{z} \cdot \mu + (1 - \bar{z}) \cdot (1 - \mu)} = \frac{w_0 - w(B) + c}{w(G) - w(B)}, \tag{35}$$

which solves to

$$\bar{z} = \frac{(1-\mu) \cdot \frac{w_0 - w(B) + c}{w(G) - w(B)}}{\mu \cdot \left(1 - \frac{w_0 - w(B) + c}{w(G) - w(B)}\right) + (1-\mu) \cdot \frac{w_0 - w(B) + c}{w(G) - w(B)}}.$$

Case 2r: Equilibrium with $\sigma_B = 1$ and $\sigma_G \in (0,1)$. In this case, for the holder of the G-product to mix, it must be that $\bar{z} = \hat{z}$.

Case 3r: Equilibrium with $\sigma_G = 0$ and $\sigma_B = 1$. For this to be an equilibrium, it requires that

$$\frac{f\left(\frac{1}{2}|\underline{\kappa}\right) \cdot \mu}{f\left(\frac{1}{2}|\underline{\kappa}\right) \cdot \mu + f\left(\frac{1}{2}|\overline{\kappa}\right) \cdot (1-\mu) \cdot} \geq \frac{w_0 - w(B) + c}{w(G) - w(B) + c}.$$

Then, $\bar{z}$ is given by

$$\frac{\bar{z} \cdot \mu}{\bar{z} \cdot \mu + (1 - \bar{z}) \cdot (1 - \mu) \cdot \frac{1}{l(\bar{z})} = \frac{w_0 - w(B) + c}{w(G) - w(B) + c}}. \tag{36}$$

In both 35 and 36, the solution $\bar{z}$ is unique under Assumption 2 about the magnitude of c.

The left-hand side of equations 35 and 36 is increasing in $\bar{z}$. Thus, as c increases, $\omega(z)$ increases for all σ_G and σ_B, and thus $\bar{z}$ increases.

■

Proof of Proposition 6.

The proof is analogous to the proofs from the case with no cost. We highlight here the differences in the values of the thresholds $\mu_1 - \mu_4$ given the cost parameter. For the equilibrium with $\sigma_G = \sigma_B = 0$, the threshold for acceptance is the same as before, so μ_1 is given by (24):

$$\mu_1 = \frac{\omega}{\omega + (1-\omega)\frac{1-\hat{z}}{\hat{z}}}.$$

For the equilibrium with $\sigma_G = 0$ and $\sigma_B in\,(0,1)$, the threshold for acceptance is given by:

$$\mu(G; \hat{z}) = \frac{(1-\hat{z}) \cdot \mu}{(1-\hat{z}) \cdot \mu + \hat{z} \cdot (1-\mu) \cdot \left(\sigma_B + (1-\sigma_B) \cdot \frac{f(\hat{z}|\overline{\kappa})}{f(\hat{z}|\underline{\kappa})}\right)},$$

This leads to threshold μ_2 :

$$\mu_2 = \frac{w_c^c}{w_c^c + (1 - w_c^c)\,\frac{f(\hat{x}|\underline{\kappa})}{f(\hat{z}|\overline{\kappa})}\frac{1-\hat{z}}{\hat{z}}}, \tag{37}$$

where

$$w_c^c \equiv \frac{W_0 - W_B + c}{W_G - W_B + c}.$$

For the equilibrium with $\sigma_G = 0$ and $\sigma_B = 1$, the threshold at which the rejection region ends is given by

$$\tilde{\mu} = \frac{w_c^c}{w_c^c + (1 - w_c^c)\,\frac{f(0.5|\kappa)}{f(0.5|\bar{\kappa})}} \tag{38}$$

An equilibrium with $\sigma_G = 0$ and $\sigma_B = 1$, exists in the acceptance region for $\mu > \tilde{\mu}$ and $\mu < \mu_4$, with

$$\mu_4 = \frac{w^c}{w^c + (1 - w^c)\,\frac{\hat{z}}{1-\hat{z}}}, \tag{39}$$

where

$$w^c \equiv \frac{W_0 - W_B + c}{W_G - W_B}.$$

The equilibrium with $\sigma_B = 1$ and $\sigma_G = 0$ exists in the acceptance region between thresholds mu_3 and mu_4, with

$$\mu_3 = \frac{w_c^c}{w_c^c + (1 - w_c^c)\,\frac{f(\hat{z}|\kappa)}{f(\hat{z}|\bar{\kappa})}\frac{\hat{z}}{1-\hat{z}}}. \tag{40}$$

As shown in the proofs of the no cost case, the equilibrium with $\sigma_B = 1$ and $\sigma_G in\,(0, 1)$, exists between to thresholds μ_3 and mu_4.

From (37)-(39),

$$\frac{d\mu_2}{dw_c^c} > 0\ ,\ \frac{d\mu_3}{dw_c^c} > 0\ ,\ \frac{d\mu_4}{dw^c} > 0.$$

Also

$$\frac{dw^c}{dc} > 0\ ,\ \frac{dw^c}{dc} > 0,$$

and

$$\frac{dw_c^c}{dc} > 0.$$

Then, a marginal increase in c weakly increases thresholds μ_2-μ_4, which implies a decrease in the size of the interval (μ_2, μ_4) over which $\sigma_B > 0$. Moreover, since mu_3 increases as well, the interval $[\mu_3, 1]$ over which $\sigma_G > 0$ decreases in size. Thus, an increase in c reduces expected equilibrium complexity.

■