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Impacts of Public Information on Flexible Information Acquisition

Peter Hammond – Retirement & Beyond

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Question

Consider a policymaker (such as a central bank) who discloses public information to agents (such as firms and consumers).

Agents receive public information at no cost and flexibly acquire private information at a cost proportional to entropy reduction.

- Any distribution of private information is allowed.

When the policymaker provides more public information, agents acquire less private information, which lowers information costs.

- Does more public information really reduce uncertainty?
- In what cases should the policymaker provide more public information?
- What is the optimal disclosure of public information?

Preview of result

We address these questions in a linear-quadratic-Gaussian (LQG) game with quadratic welfare minus the information cost.

Findings:

- More public information raises uncertainty if and only if the game exhibits strategic complementarity.
- The increased uncertainty can be detrimental to welfare if strategic complementarity is strong enough.
- We give a necessary and sufficient condition for welfare to increase with public information and identify the optimal disclosure.

Related literature

Flexible information acquisition is modeled in the rational inattention framework (Sims, 2003), a benchmark information acquisition model (particularly in macroeconomics).

There are two types of flexible information acquisition in games.

- Independent information acquisition (Yang, 2015): Agents pay attention to a state.
- Correlated information acquisition (Denti, 2022): Agents pay attention to both a state and the opponents' signals.

We consider both of them in LQG games.

Hébert and Lao (2021), Denti (2015), and Rigos (2020) also study flexible information acquisition in LQG games.

Related literature

The roles of public information in LQG games

- Crémer (1990)
- Morris and Shin (2002), Angeletos and Pavan (2007), Ui and Yoshizawa (2015)
- Colombo et al. (2014), Ui (2014, 2022)

Bayesian persuasion with receivers acquiring costly information

- Lipnowski et al. (2020), Bloedel and Segal (2020)
- Bizzotto et al. (2020), Matysková and Montes (2021)

Timing

We focus on a symmetric equilibrium in a two-stage game.

1. The policymaker chooses the precision of public information and discloses a public signal.

2. Agents choose their own signal distributions, receive signals, and choose their actions.

An LQG game with a continuum of agents

A continuum of agents indexed by $i \in [0,1]$.

Agent i's action is $a_i \in \mathbb{R}$. A payoff state is $\theta \sim N(\bar{\theta}, 1/\tau_\theta)$.

We assume that τ_θ is sufficiently small.

A payoff function $u_i(a,\theta)$ is quadratic in a_i, $A \equiv \int a_j dj$, and θ.

$$u_i(a,\theta) = -a_i^2 + 2\alpha a_i A + 2\beta a_i \theta + \text{(terms independent of } a_i\text{)}$$
$$= -\left(a_i - (\alpha A + \beta\theta)\right)^2 + \text{(terms independent of } a_i\text{)}$$

Agent i's best response is the expected value of the target

$$\alpha A + \beta\theta$$

Strategic complementarity and substitutability

$$\alpha A + \beta \theta$$

$$\alpha < 1,\ \beta \neq 0$$

α is the slope of the best response

strategic complementarity $\Leftrightarrow \alpha > 0$

strategic substitutability $\Leftrightarrow \alpha < 0$

Examples

A Cournot game (Vives,1988)

$$(\theta - \delta A)\, a_i - a_i^2/2$$

A Bertrand game (Vives,1990)

$$(\theta - a_i + \delta A)\, a_i - (\theta - a_i + \delta A)^2\, /2$$

An investment game (Angeltos and Pavan, 2004)

$$(rA + (1 - r)\theta)\, a_i - a_i^2/2$$

A beauty contest game (Morris and Shin, 2002)

$$a_i = \mathrm{E}[\alpha A + (1 - \alpha)\theta | s_i]$$

Public information

The policymaker informs agents of a posterior of θ.

The posterior is assumed to be normal.

For example, after receiving public information, agents believe that θ is normally distributed with mean $\tilde{\theta}$ and precision $\tau \geq \tau_\theta$.

- $\tilde{\theta}$: a public signal
- τ: the precision of public information

The policymaker's objective function

In period 1, the policymaker chooses τ to maximize the expected value of a welfare function

$$v(a, \theta) - \int C(a_i) di$$

where $v(a, \theta)$ is a symmetric and quadratic function of (a, θ).

A typical case is the total payoff

$$v(a, \theta) = \int u_i(a, \theta) di$$

The expected welfare is represented as a linear combination of

$$\mathbf{var}[a_i], \ \mathbf{var}[A], \ \mathbf{cov}[a_i, A], \ \mathbf{cov}[a_i, \theta], \ \mathbf{cov}[A, \theta]$$

Correlated information acquisition

Agent i acquires private information s_i about his target and chooses

$$\text{the best response action} = \mathbf{E}[\alpha A + \beta\theta | s_i, \tilde{\theta}]$$

We focus on a direct signal that suggests $a_i = \mathbf{E}[\alpha A + \beta\theta | a_i, \tilde{\theta}]$.

The cost of a_i is proportional to the reduction in entropy of $\alpha A + \beta\theta$ caused by a_i, i.e., the mutual information (Shannon, 1948).

$$C(a_i) \equiv \lambda \cdot \mathbf{I}_{\tilde{\theta}}\left(\alpha A + \beta\theta; a_i\right)$$

$\mathbf{I}_{\tilde{\theta}}$ is the mutual information wrt the conditional distribution given $\tilde{\theta}$.

- The entropy of $\alpha A + \beta\theta$ is the average level of uncertainty.
- When agent i receives a_i, it decreases.
- The information cost is proportional to the reduction in uncertainty.

Correlated information acquisition

Agent i's direct signal is optimal if it is a solution to

$$\max_{\text{a random variable } a_i} \mathrm{E}[u_i(a,\theta)|\tilde{\theta}] - C(a_i)$$

In equilibrium, every agent receives an optimal direct signal.

The second-period subgame has a symmetric equilibrium such that actions and a state are jointly normally distributed.

Two types of equilibria:

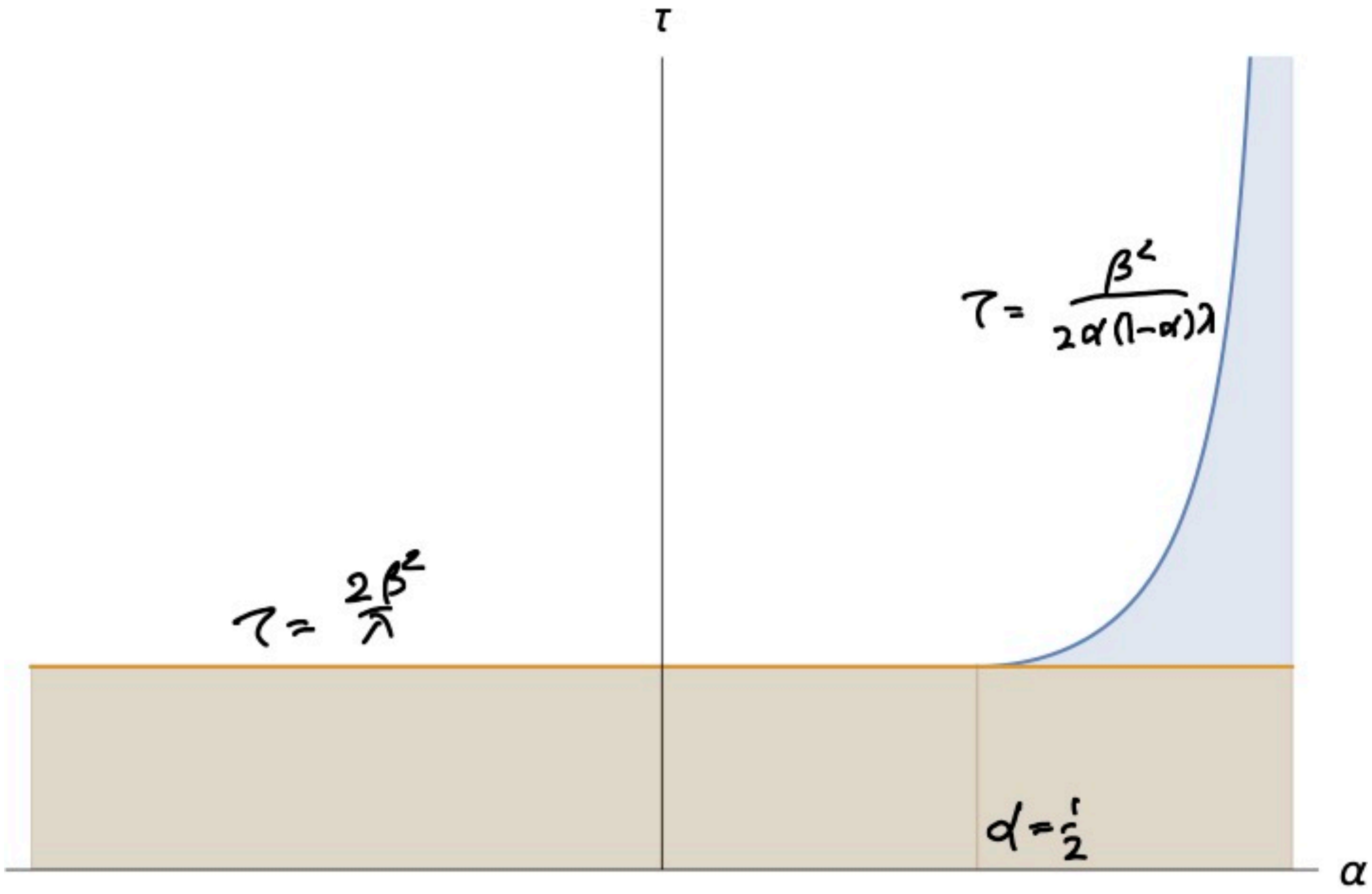
- agents acquire information
- agents do not acquire information

An equilibrium in the second-period subgame

Proposition

- $\tau < 2\beta^2/\lambda \Rightarrow$ a unique equilibrium, with info acquisition
- $\tau \geq 2\beta^2/\lambda \Rightarrow$ an equilibrium with no info acquisition
  - This is a unique equilibrium if
    - $\alpha \leq 1/2$, or $\alpha > 1/2$ and $\tau \geq \beta^2/(2\alpha(1-\alpha)\lambda)$
  - There are two other equilibria with info acquisition if
    - $\alpha > 1/2$ and $\tau < \beta^2/(2\alpha(1-\alpha)\lambda)$

The types of equilibria depend upon (α, τ)



- a unique equilibrium, with no info acquisition
- three equilibria, with large, small, and no info acquisition
- a unique equilibrium, with info acquisition

Multiple equilibria under strategic complementarity

Multiple equilibria can arise if and only if $\alpha > 1/2$.

In games with strategic complementarity, information is also a strategic complement (Hellwig and Veldkamp, 2009).

Information is more (less) valuable when the opponents acquire more (less) information.

Different types of information acquisition can be an equilibrium.

Correlated vs. independent information acquisition

Under correlated information acquisition (Denti, 2022), agents acquire information about $\alpha A + \beta\theta$ at a cost

$$\lambda \cdot \mathbf{I}_{\tilde{\theta}} \left(\alpha A + \beta\theta; a_i\right)$$

Private signals are conditionally correlated.

Under independent information acquisition (Yang, 2015), agents acquire information about θ at a cost

$$\lambda \cdot \mathbf{I}_{\tilde{\theta}} \left(\theta; a_i\right)$$

Private signals are (assumed to be) conditionally independent.

Are the equilibria under independent information acquisition different from those under correlated information acquisition?

Correlated vs. independent information acquisition

Proposition

The set of equilibria under correlated information acquisition coincides with that under independent information acquisition.

Consider independent information acquisition.

Private signals are conditionally independent, and so are actions.

By LLN, A is perfectly correlated with θ.

$$\text{information about } \theta = \text{information about } \alpha A + \beta\theta$$

The equilibrium is also that under correlated information acquisition.

Correlated vs. independent information acquisition

Consider correlated information acquisition.

Private signals are conditionally correlated.

However, the equilibrium is also that under independent information acquisition.

Thus, A is perfectly correlated with θ even under correlated information acquisition, where LLN does not work.

It is the result of a calculation, but we can explain why by using the theory of large deviations.

Total information

Agents follow the equilibrium acquiring the most precise information.

Agent i receives a public signal $\tilde{\theta}$ in period 1 and a private signal a_i in period 2. A signal vector is $(\tilde{\theta}, a_i)$.

We measure the total amount of information contained in $(\tilde{\theta}, a_i)$ about θ by the mutual information, the reduction in entropy.

$$\mathrm{I}\,(\theta; (\tilde{\theta}, a_i))$$

The entropy reduction in period 1 induced by $\tilde{\theta}$

$$I_1(\tau) \;=\; \text{(the amount of information in } \tilde{\theta}\text{)}$$

The entropy reduction in period 2 induced by a_i

$$I_2(\tau) \;=\; \text{(the amount of information in } a_i\text{)}$$

Total information

Lemma (the chain rule for mutual information)

The total amount of information is the sum of $I_1(\tau)$ and $I_2(\tau)$.

$$\mathrm{I}\,(\theta; (\tilde{\theta}, a_i)) = I_1(\tau) + I_2(\tau)$$

Total information

More public information increases the amount of information in $\tilde{\theta}$.

$\Rightarrow$ $I_1(\tau)$ is increasing in τ

More public information decreases the amount of information in a_i.

$\Rightarrow$ $I_2(\tau)$ is decreasing in τ

$\Rightarrow$ the crowding-out effect of public information

What about the total amount of information?

More public information can raise uncertainty

Proposition

More public information decreases the total amount of information (i.e., raises uncertainty) if and only if the game exhibits strategic complementarity.

$$I_1'(\tau) + I_2'(\tau) \lesseqgtr 0 \iff \alpha \gtreqless 0$$

This means that, under strategic complementarity, the crowding-out effect is so substantial that more public information raises uncertainty.

More public information can raise uncertainty

Divide the marginal change in private information by the marginal change in public information.

$$MRS \equiv \left| \frac{I_2{}'(\tau)}{I_1{}'(\tau)} \right|$$

This is MRS of public information for private information.

MRS is the decrease in private information resulting from a one-unit increase in public information.

MRS is a measure of the crowding-out effect.

Corollary

$$MRS \gtreqless 1 \;\Leftrightarrow\; \alpha \gtreqless 0$$

More public information can raise uncertainty

Consider the benchmark case of no strategic interaction ($\alpha = 0$).

Public and private signals are perfect substitutes because an agent pays no attention to the opponents' actions.

When the policymaker increases one unit of public information, an agent reduces the same amount of private information.

$MRS = 1 \Rightarrow$ the total amount of information is constant.

More public information can raise uncertainty

In the case of strategic complementarity ($\alpha > 0$), public and private signals are imperfect substitutes.

Each agent places more value on public information.

When the policymaker increases one unit of public information, an agent reduces a greater amount of private information.

$MRS > 1 \Rightarrow$ the total amount of information decreases.

More public information can raise uncertainty

We have assumed that agents follow the equilibrium acquiring the most precise information in the case of multiple equilibria with $\alpha > 1/2$.

This assumption is essential.

Proposition

Assume that $\alpha > 1/2$. More public information raises uncertainty if only if agents follow the equilibrium acquiring the most precise information.

Under the equilibrium with the second most precise information, more public information necessarily reduces uncertainty.

The crowding-in effect of public information

Consider the case of multiple equilibria with $\alpha > 1/2$.

Assume that agents follow the equilibrium acquiring the second most precise information.

More public information increases the amount of private information.

$\Rightarrow$ $I_2(\tau)$ is increasing in τ

$\Rightarrow$ the crowding-in effect of public information

Proposition

More public information increases the total amount of information.

$$I_1'(\tau) + I_2'(\tau) > 0$$

The crowding-in effect of public information

The crowding-in effect is consistent with equilibrium.

Imagine that

- the policymaker provides more precise public information,
- the opponents acquire more precise private information.

  $\Rightarrow$ private information is more valuable for an agent

$$\Downarrow$$

An agent has a strong incentive to acquire more precise private information when the agent acquires a small amount of private information, i.e., the cost of information is small.

The expected welfare

We discuss the welfare implications of the crowding-out effect under the equilibrium where agents acquire the most precise information.

We evaluate the expected welfare, a linear combination of

$$\mathbf{var}[a_i],\ \mathbf{var}[A],\ \mathbf{cov}[a_i, A],\ \mathbf{cov}[a_i, \theta],\ \mathbf{cov}[A, \theta]$$

By using FOC, we can rewrite it as a linear combination of

$$\mathbf{var}[a_i],\ \mathbf{var}[A]$$

i.e., a linear combination of

$$\mathbf{var}[A],\ \mathbf{var}[a_i] - \mathbf{var}[A]$$

The expected welfare

$\mathbf{var}[a_i] - \mathbf{var}[A]$ is called dispersion, a measure of idiosyncratic variation of actions.

$$D = \mathbf{var}[a_i] - \mathbf{var}[A]$$

$\mathbf{var}[A]$ is called volatility, a measure of common variation of actions.

$$V = \mathbf{var}[A]$$

Cf. Angeletos and Pavan (2007), Bergemann and Morris (2013).

Lemma

The expected welfare has a representation

$$W = \zeta D + \eta V - C + \text{(a term independent of } \tau\text{)}$$

Cost

$$W = \zeta D + \eta V - C$$

Lemma

$$\frac{dC}{d\tau} < 0$$

$$\frac{dC}{d\tau} \to -\infty \text{ as } \tau \to 0$$

$$\frac{dW}{d\tau} > 0 \text{ as } \tau \to 0$$

When $\tau \simeq 0$, welfare decreases with public information, so no disclosure is suboptimal.

Facing great uncertainty, agents acquire a lot of information and incur a lot of acquisition costs.

Dispersion

$$W = \zeta D + \eta V - C$$

Lemma

$$\frac{dD}{d\tau} < 0$$

$$D = \mathbf{var}[a_i] - \mathbf{var}[A]$$

More public information brings $\mathbf{var}[a_i]$ and $\mathbf{var}[A]$ closer.

When $\zeta > 0$ is very large, welfare decreases with public information.

Volatility

$$W = \zeta D + \eta V - C$$

Lemma

$$\frac{dV}{d\tau} \begin{cases} > 0 & \text{if } \alpha < 1/2 \\ < 0 & \text{if } \alpha > 1/2 \end{cases}$$

$$\frac{dV}{d\tau} \to -\infty \text{ as } \alpha \to 1$$

$\alpha < 1/2 \Rightarrow$ strategic substitutability or weak strategic complementarity
$\Rightarrow$ the crowding-out effect is weak $\Rightarrow V$ increases

$$\because V = \mathbf{var}[A] = \mathbf{cov}[a_i, a_j]$$

$\alpha > 1/2 \Rightarrow$ strong strategic complementarity
$\Rightarrow$ the crowding-out effect is strong $\Rightarrow V$ decreases

Volatility

$$W = \zeta D + \eta V - C$$

When $\eta > 0$ is very large,

- welfare increases with public information if $\alpha < 1/2$.
- welfare can decrease with public information if $\alpha > 1/2$.

When $\eta > 0$,

- welfare can decrease with public information if $\alpha \simeq 1$.

Welfare effects of public information

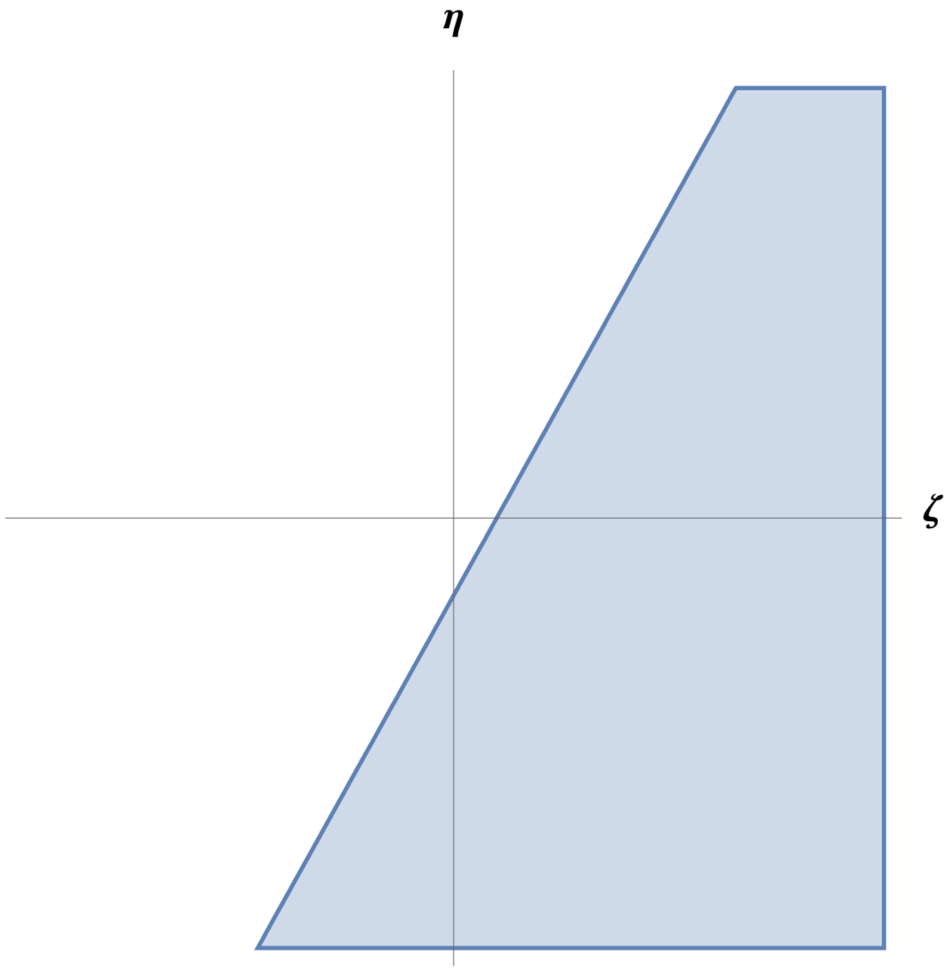
Proposition

Assume that agents follow the equilibrium acquiring the most precise information.

$$\frac{dW}{d\tau} > 0 \Leftrightarrow \zeta - \frac{(1-2\alpha)}{(1-\alpha)^2}\eta < \frac{1}{1-\gamma}$$

$\alpha < 1/2$

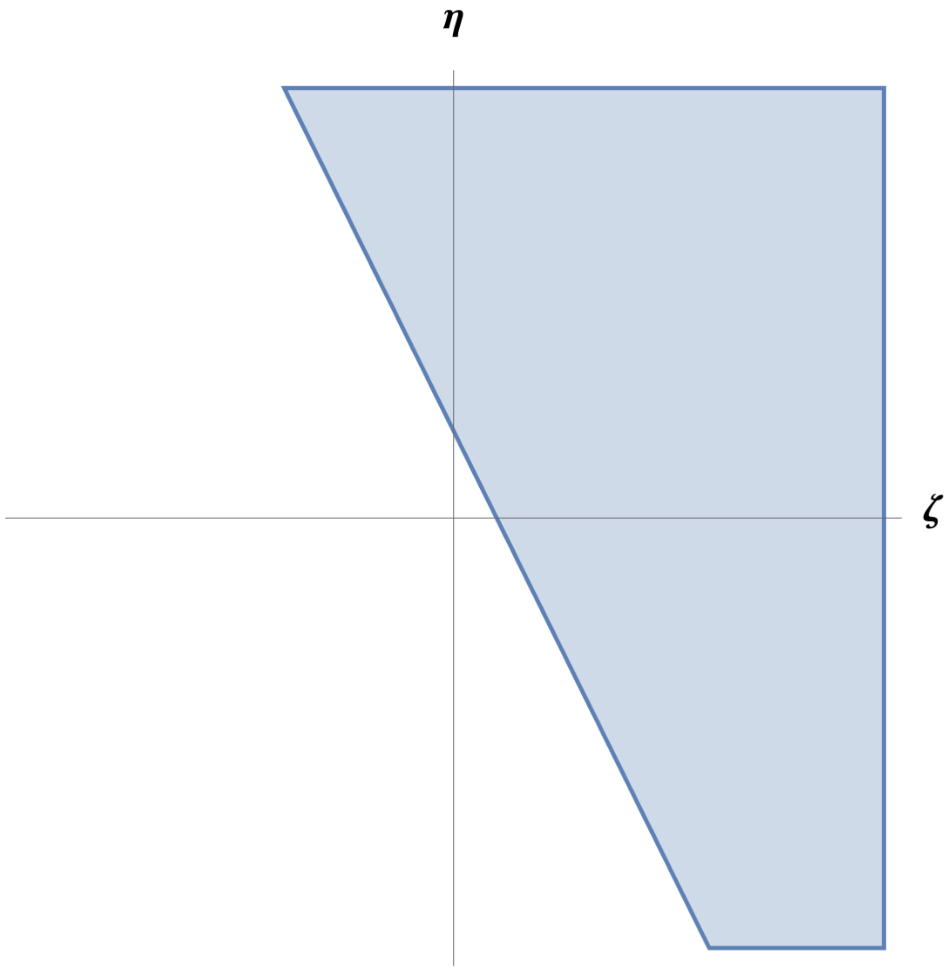
$$W = \zeta D + \eta V - C$$



In the shaded region, welfare can decrease with public information.

$\alpha > 1/2$

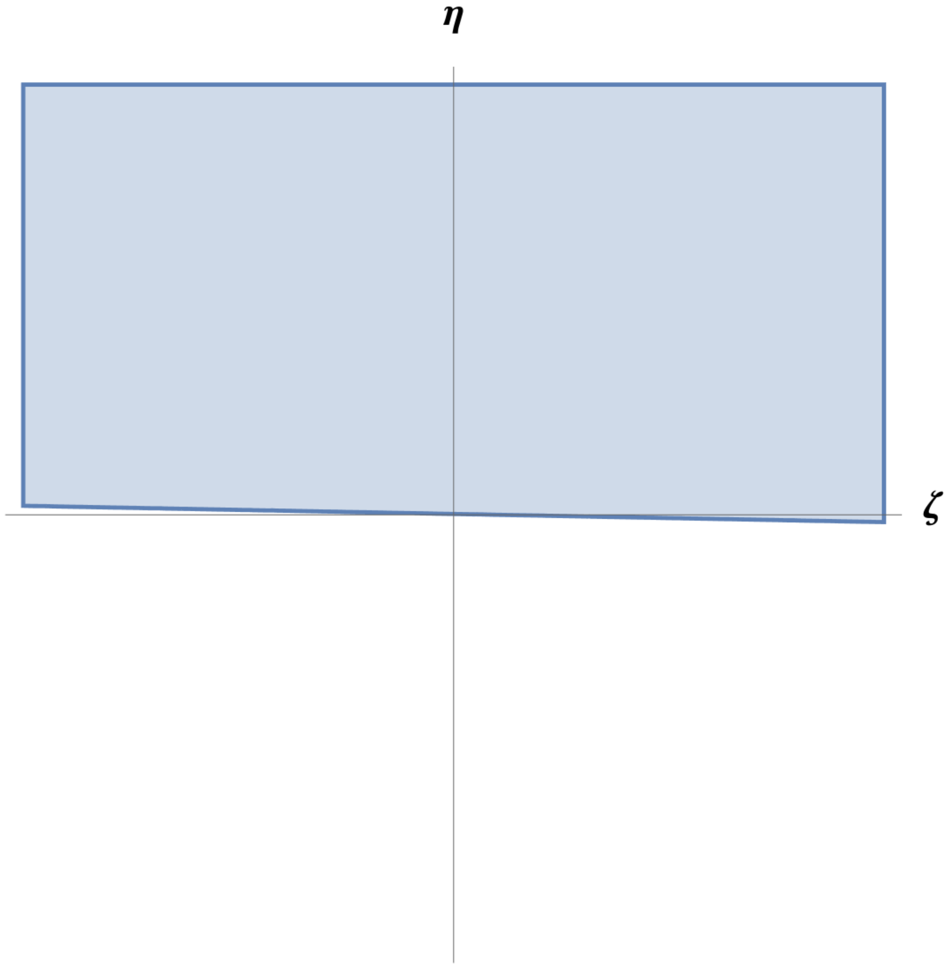
$$W = \zeta D + \eta V - C$$



In the shaded region, welfare can decrease with public information.

$\alpha \simeq 1$

$$W = \zeta D + \eta V - C$$



In the shaded region, welfare can decrease with public information.

Public information and strategic complementarity

Proposition

Fix $\zeta \in \mathbb{R}$ and $\eta > 0$, which are independent of α.

If α is sufficiently close to one, welfare can decrease with public information.

In games with strong strategic complementarity, the crowding-out effect is substantial and reduces volatility significantly.

$$\Downarrow$$

More public information can be detrimental.

Exogenous private information

Proposition (Ui and Yoshizawa, 2015)

Assume that agents receive a private signal about θ with fixed precision. Fix $\zeta \in \mathbb{R}$ and $\eta > 0$, which are independent of α.

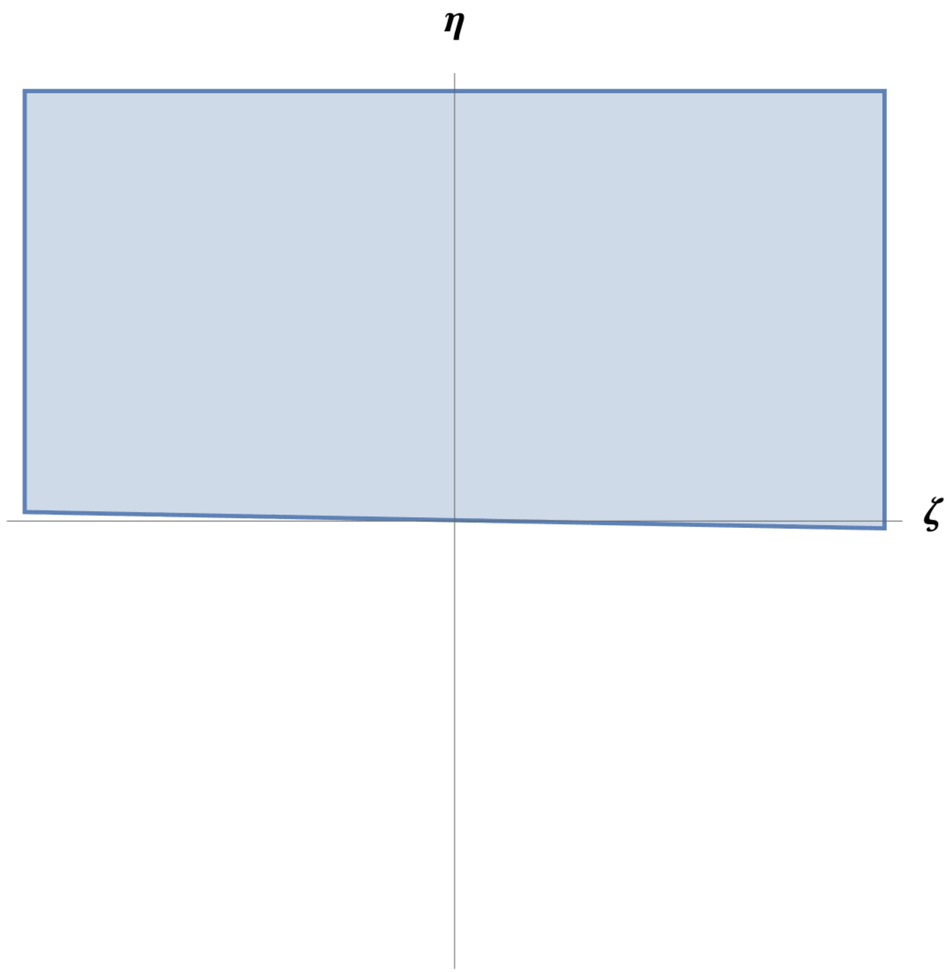
If α is sufficiently close to one, welfare necessarily increases with public information.

In games with strong strategic complementarity, public information has a higher value because of the coordination motive.

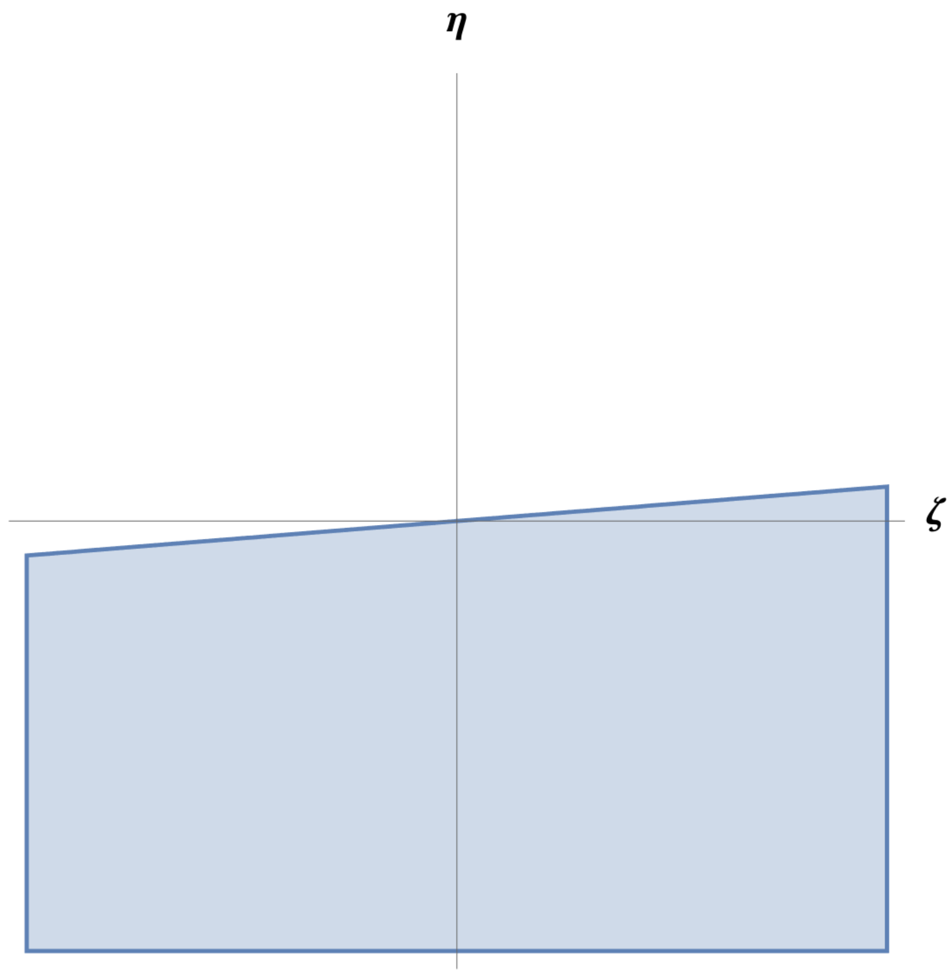
$$\Downarrow$$

More public information is always beneficial.

$\alpha \simeq 1$



flexible information acquisition



exogenous private information

An investment game and a Cournot game

Assume that agents are firms, payoffs are profits, and

$$v(a, \theta) = \int u_i(a, \theta) di$$

An investment game (Angeletos and Pavan, 2004): firms choose their investment level

$$\alpha > 0 \text{ and } \zeta = \eta = 1$$

A Cournot game (Vives, 1988): firms choose their output level

$$\alpha < 0 \text{ and } \zeta = \eta = 1$$

An investment game and a Cournot game

Assume flexible information acquisition.

Corollary

In an investment game with $\alpha > 1/2$, more public information can be detrimental to the total profit.

Corollary

In a Cournot game, more public information is always beneficial to the total profit.

In brief, more precise public information can reduce the total profit if and only if the game exhibits strong strategic complementarity.

An investment game and a Cournot game

Assume exogenous private information.

Proposition (Angeletos and Pavan, 2004)

In an investment game, more public information is always beneficial to the total profit.

Proposition (Bergemann and Morris, 2013)

In a Cournot game with $\alpha < -1/2$, more public information can be detrimental to the total profit.

In brief, more precise public information can reduce the total profit if and only if the game exhibits strong strategic substitutability.

A beauty contest game

the target $= \alpha A + (1 - \alpha)\theta,$ where $0 < \alpha < 1$

$$v(a, \theta) = -(a_i - \theta)^2 \;\Rightarrow\; \zeta = 1 + \alpha,\; \eta = 1 - \alpha$$

Proposition (Morris and Shin, 2002)

In the case of exogenous private information, more public information can be detrimental to welfare if $\alpha > 1/2$.

Corollary

In the case of flexible information acquisition, more public information can be detrimental to welfare if $\alpha > (3 - \sqrt{5})/2 \simeq 0.4$.

For public information to have detrimental effects, weaker strategic complementarity suffices in the case of flexible information acquisition.

Optimal disclosure

Assume that agents follow an equilibrium with the largest expected welfare, which is a standard assumption in Bayesian persuasion.

Definition

The precision of public information is optimal if it maximizes the expected welfare when agents follow the sender-optimal equilibrium.

Optimal disclosure

Proposition

Assume that τ_θ is sufficiently small. The optimal precision τ^* is uniquely given by

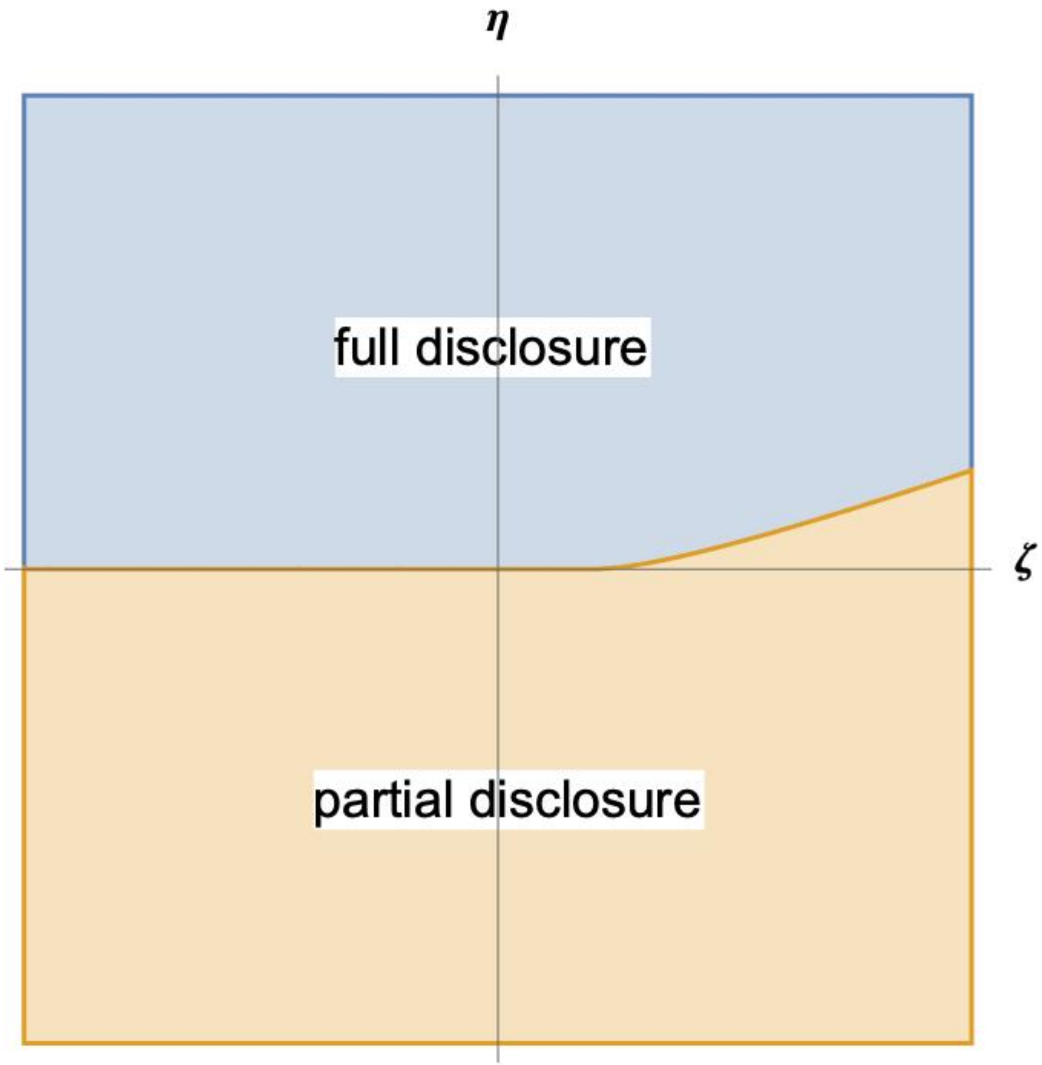
$$\tau^* = \begin{cases} \infty & \text{if } \chi < 0 \text{ and } \eta > 0, \\ \tau_+^* & \text{if } \chi > 0 \text{ or } \eta < 0, \end{cases}$$

where $\chi \equiv \zeta - 1 - 2\eta/(1-\alpha) - \log(1-\gamma_+^*)^{-1}$ and

$$\gamma_+^* \equiv \begin{cases} 1 - \dfrac{1}{\zeta - (1-2\alpha)\eta/(1-\alpha)^2} & \text{if } \zeta - (1-2\alpha)\eta/(1-\alpha)^2 > 1, \\ 0 & \text{otheriwse.} \end{cases}$$

Optimal disclosure

$$W = \zeta D + \eta V - C$$

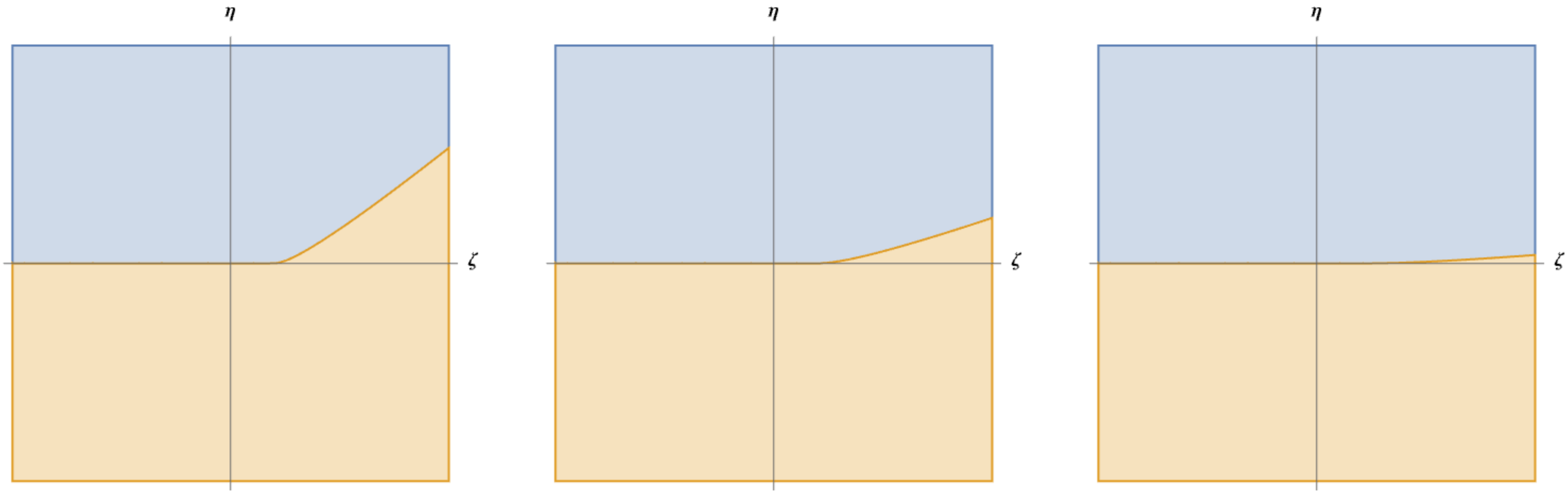


- Full disclosure is optimal in the upper region.
- Partial disclosure is optimal in the lower region.
- No disclosure is suboptimal.
  - Without public information, agents acquire a substantial amount of information

⇓

huge cost

Optimal disclosure



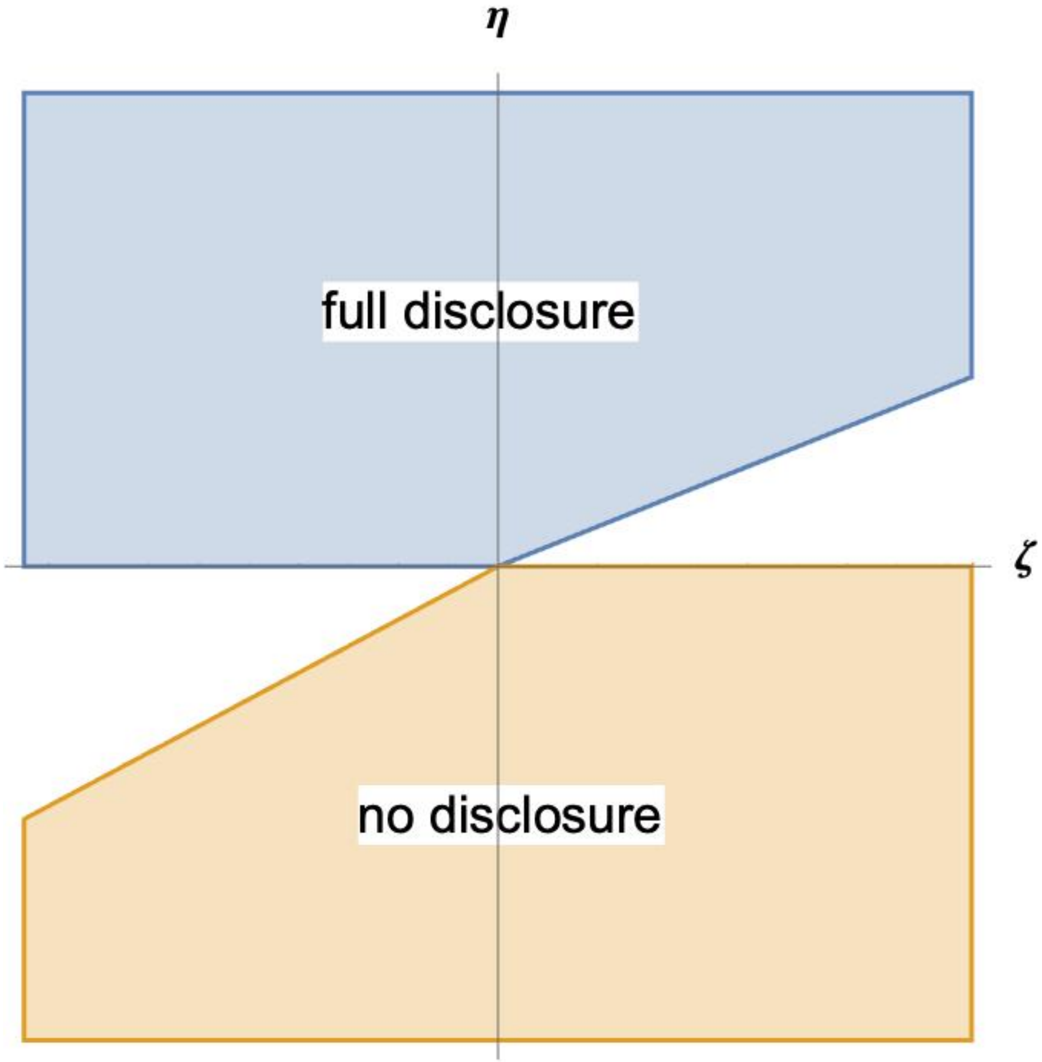
$\alpha = -0.8$ $\alpha = 0.5$ 0.8

When $\eta > 0$, full disclosure is optimal if α is sufficiently close to one.

When public information is precise enough, agents do not acquire private information, so there is no crowding-out effect.

The detrimental effect caused by the crowding-out effect vanishes.

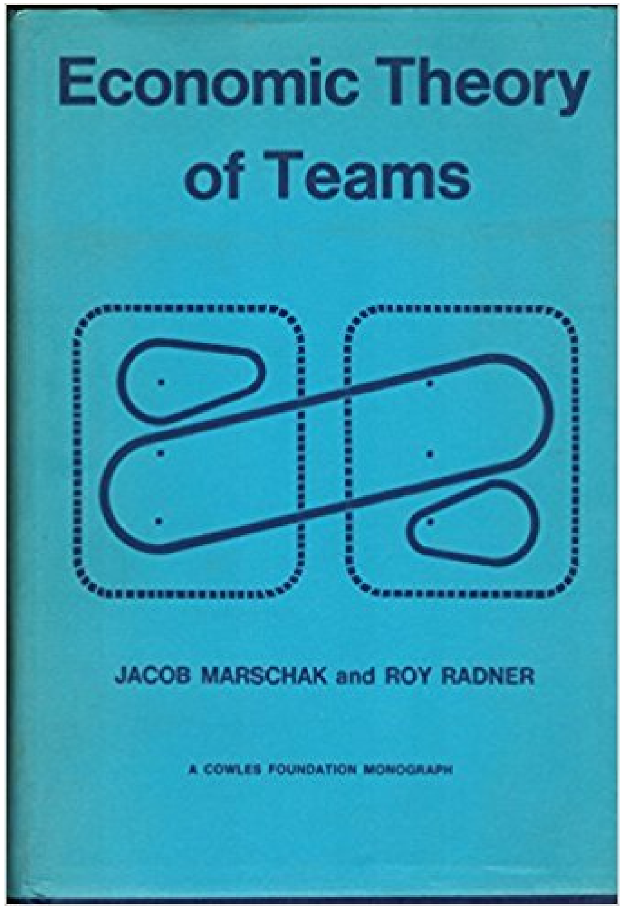
The case of exogenous private information

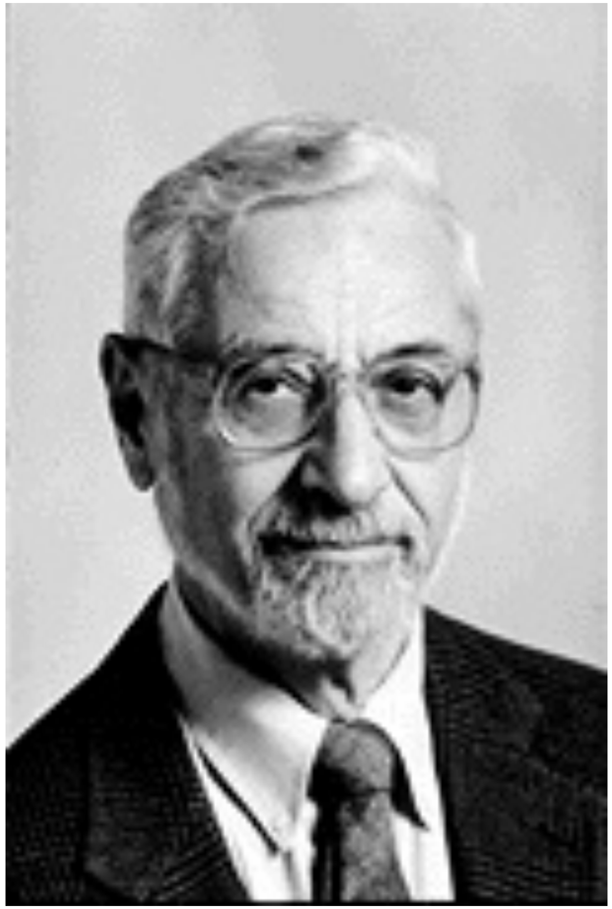


Ui and Yoshizawa (2015)

- No disclosure is optimal in the lower region.

The origin of LQG game analysis





The origin of LQG game analysis

Marschak (1955) introduces a team as a model of an organization.

- A team is a Bayesian game in which every player has an identical payoff function.

Radner (1962) introduces an LQG team and obtains a unique BNE.

- earlier than Harsanyi (1967–1968)

Marschak and Radner (1972) study what information structure maximizes the total payoff minus the total cost of information.

- Crémer (1980), Aoki (1986), Itoh (1987), Geanakoplos and Milgrom (1991), Prat (1996)...

Concluding remarks

- More public information raises uncertainty under strategic complementarity.
- The increased uncertainty can be detrimental to welfare if strategic complementarity is strong enough.
- Full disclosure is optimal even in this case.

The canonical rational inattention model with the entropic cost is said to be the “Cobb-Douglas model” of information acquisition.

Some experimental results are inconsistent with its prediction. Other information costs are proposed.

All results remain essentially the same as for the Fisher information cost (Hébert and Woodford, 2021).

Thank you!