Memorable Events in Financial Markets

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Motivation

Memorable events impact financial markets

- Individuals who experienced low stock market returns pessimistic about future stock returns (Malmendier and Nagel 2011)
- Individuals use personal experiences to form expectations about house prices and unemployment (Kuchler and Zafar 2019)
- Many more (Kaustia and Knupfer 2008, Malmendier and Nagel 2015)

Theoretical Benchmarks

- Nagel and Xu (2022) introduce **fading memory**
- Malmandier, Pouzo, and Vanasco (2020) add **heterogeneity**

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Fading memory emphasizes recent observations $\implies$ models generate return predictability

Key Implications

We assume **non-fading memory**.

$\implies$ a stationary, cross-sectional distribution over beliefs

$\implies$ pessimists who recall bear markets sell and optimists who recall bull markets buy

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We generate

1. Non-fundamental volatility
2. Realistic trade volume
3. Heavy tails

Brownian Example

- Time $t = 0, 1, 2, ...$
- Countable traders
- Single long-lived financial asset that pays no dividends
- Conjecture prices follow a discrete time Brownian motion

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Mean-variance traders correctly believe price variance is one

$$x^i(p_t) = \frac{\mathbb{E}_i[p_{t+1}|p_t] - p_t}{\rho}$$

ρ denotes risk aversion.

Memory-Constrained Bayes' Rule

Traders believe price increments have mean μ_i because certain time periods are more **memorable**. Assume

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Nagel and Xu (2022) introduce memory-constrained Bayes' Rule

$$f(\mu | x_1, ..., x_T) \propto \prod_{t=1}^{T} f(x_t | \mu)^{\alpha_t}$$

Bayes' Rule with flat prior is $\alpha_t = 1$.

Non-Fading Memory

We use weights

$$\alpha_i = \gamma T \text{ and } \sum_{t \neq i} \alpha_t = (1 - \gamma) T$$

$0 < \gamma \leq 1$ denotes idiosyncratic memory strength, and T denotes history length.

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Applying memory-constrained Bayes

$$\mu_i = \gamma(p_i - p_{i-1}) + \frac{1-\gamma}{T-1} \sum_{t \neq i} (p_t - p_{t-1}) \overset{a.s.}{\to} \gamma(p_i - p_{i-1})$$

$\implies$ non-memorable events **wash out**.

Market Maker

The cross-sectional empirical distribution of beliefs μ^i

$$F_M(t) = \frac{1}{M}\sum_{i=1}^{M} 1_{\mu^i \leq t} \overset{a.s.}{\to} N(0, \gamma^2)$$

where M is the number of traders.

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A random subset of size n trades per period (**the only r.v.**). A linear market maker from Kyle (1985) and Teeple (2022)

$$p_{t+1} = p_t + c\sum_{i=1}^{n} x^i(p_t)$$

ensures prices converge to a Brownian motion when $c = \frac{\rho}{\sqrt{n}\gamma}$. The rule

- is a linear approximation
- maps excess demand into higher prices (and vice versa)
- makes prices insensitive in liquid markets (and vice versa)

Bagehot (1971)

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The resulting market price at any point in time is not merely a consensus of the transactors in the marketplace, it is also a consensus of their mistakes. Under the heading of mistakes we may include errors in computation, errors of judgment, factual oversights and errors in the logic of analysis."

Recap

1. Model of technical trading
2. Price discovery generates volatility
3. Non-convergence because market maker “shoots at a moving target”
4. Prices volatile because they have always been volatile

Generalizations

Proposition 1 (Expanded Memory)

Say each trader remembers k disjoint periods for $k \in \{1, ..., K\}$. Then prices converge in distribution to a discrete time Brownian motion as $n \to \infty$.

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Intuition:

- Homogeneous case: good and bad days begin to cancel $\implies$ less dispersed beliefs
- Heterogeneous case: cross-section of beliefs need not be normal; apply CLT as $n \to \infty$

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Proposition 2 (Overlapping Memory)

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Intuition:

- Homogeneous case: each historical price realization equally likely to be drawn from the cross-section $\implies$ same dispersion of beliefs
- Heterogeneous case: apply argument group-by-group

Generalizations

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Say traders believe that prices are drawn from a discrete time Lévy process with stable-α increments. Then prices converge in distribution to this belief.

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Intuition:

- Heavy tailed history $\implies$ extreme beliefs $\implies$ large price movements
- Generates **power law** trade volume
- **Maximal** class of equilibria
- Caveat: demand is $x^i(p_t) = \frac{\mathbb{E}_i[p_{t+1}|p_t]-p_t}{\tau}$

Policy: Interest Rates

Formally, budget set today (a_t is riskless)

$$x_t p_t + a_t = 0$$

Budget set tomorrow (w_{t+1} is wealth and r is nominal rate)

$$w_{t+1} = x_t p_{t+1} + a_t \left(1 + \frac{r}{p_t} \operatorname{sgn}(x_t)\right)$$

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- $r = 0$ is previous case
- Consider $x_t > 0$ so $a_t < 0$. Traders borrow the riskless at rate $\frac{R}{p_t}$ and lend the asset at rate $\frac{R-r}{p_t} \implies \frac{r}{p_t}$ is the spread
- Consider $x_t < 0$ so $a_t > 0$. Traders borrow the asset at rate $\frac{R+r}{p_t}$, sell, and invest proceeds at rate $\frac{R}{p_t} \implies \frac{r}{p_t}$ is the spread

Policy: Interest Rates

Demand becomes

$$x_t^i = \begin{cases} \frac{\mathbb{E}_i[p_{t+1}|p_t]-p_t-r}{\rho\Sigma^2}, & \text{if } \mathbb{E}_i[p_{t+1}|p_t]-p_t > r \\ \frac{\mathbb{E}_i[p_{t+1}|p_t]-p_t+r}{\rho\Sigma^2}, & \text{if } \mathbb{E}_i[p_{t+1}|p_t]-p_t < -r \\ 0, & \text{otherwise} \end{cases}$$

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New variance solves

$$\Sigma^2 = \underbrace{\frac{1}{\Sigma^4}\int_{-\infty}^{0}\frac{x^2}{\Sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{x-r}{\Sigma}\right)^2\right]dx}_{\text{Left tail shifted right}} + \underbrace{\frac{1}{\Sigma^4}\int_{0}^{\infty}\frac{x^2}{\Sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{x+r}{\Sigma}\right)^2\right]dx}_{\text{Right tail shifted left}}$$

and decreases in r. ✓

Policy: Capital Gains Tax

Budget set (τ is tax)

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Intuition: Two competing forces: dampened capital gains versus dampened wealth variance; latter outweighs the former.

Policy: Borrowing Constraints

Two additional constraints ($\overline{b} > 0$ is real borrowing limit)

$$a_t \geq -p_t \overline{b}, \quad x_t \geq -\overline{b}$$

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New variance solves

$$\Sigma^2 = \frac{1}{\Sigma^4} \underbrace{\int_{-\overline{b}}^{\overline{b}} \frac{x^2}{\Sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x}{\Sigma}\right)^2\right] dx}_{\text{Unconstrained}} + \frac{2}{\Sigma^4} \underbrace{\int_{\overline{b}}^{\infty} \frac{\overline{b}^2}{\Sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x}{\Sigma}\right)^2\right] dx}_{\text{Constrained}}$$

and increases in $\overline{b}$. ✓

Conclude

Non-fundamental volatility arises from price discovery when technical traders are memory-constrained.

Heavy-tailed prices and power law trade volume are part of equilibrium; a heavy-tailed history leads to extreme belief dispersion.

Interest rate increases and borrowing limits are both effective policies, but differ in distributional consequences.