Politics Transformed? Electoral Competition under Ranked Choice Voting*

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January 31, 2024

Abstract

We compare multi-candidate elections under plurality rule versus ranked choice voting (RCV). Candidates choose whether to conduct a broad campaign that can appeal to all voters, or instead pursue a targeted campaign that favors a narrow segment of voters. We examine a widely held presumption that RCV more effectively incentivizes candidates to campaign broadly, compared to plurality rule. We identify conditions under which this presumption is true. However, we also unearth real-world relevant contexts in which the prediction reverses: when voters are divided by partisan, ethnic, geographic or cultural cleavages the possibility of winning second preferences under can intensify candidates' incentives to pursue targeted campaigns, relative to plurality.

*We thank Kathleen Bawn, Laurent Bouton, Alessandra Casella, Lindsey Cormack, Gabriele Gratton, Catherine Hafer, Navin Kartik, Adam Meirowitz, Jack Santucci, Ian Turner, and numerous seminar audiences for valuable feedback.

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1. Introduction

Ranked Choice Voting (RCV) is the most publicly debated and rapidly expanding electoral reform in the United States. Rather than voting for a single candidate, voters under RCV can rank *multiple* candidates.[1] If any candidate wins a majority of first preferences, she is elected. If no candidate wins a majority of first preferences, the candidate with the fewest first preferences is eliminated and each of her ballots transfers to the next-ranked candidate. The process continues until a single candidate wins a majority of the remaining ballots.

RCV is widely employed in elections across the world.[2] Within the United States, it is used by over 13 million voters both in general elections and in the primaries of both major political parties for local, state and federal offices.[3] Voter initiatives led to RCV's adoption in New York City (primary elections for Mayor and City Council since 2021) and Alaska (state and federal general elections since 2022).

In this paper, we introduce a new theoretical framework to analyze electoral competition under RCV. We use it to examine the widely held contention that—relative to a plurality rule—RCV better-encourages candidates to pursue a broad electoral appeal instead of focusing on their core supporters.[4]

The contention rests on the following logic: under plurality, a candidate only benefits from the support of voters that like her the most. This encourages her to focus on mobilizing the narrow segment of voters that are most likely to prefer her over all other candidates—for example, an ideological, social, or ethnic base. Under RCV, by contrast, a candidate can benefit from the support of voters that do *not* like her the most. The prospect of winning voters' second preferences is expected to raise a candidate's relative benefit from broadening her

[1] While variants of Ranked Choice Voting can also be used in multi-member districts (for a review, see Santucci 2021), we focus on the more common version with single-member districts—also called Instant Runoff. In this paper, we use RCV as a synonym of Instant Runoff, though the latter is technically a special case of the former.

[2] For example, in Australia's (House of Representatives), Canada (major party leader selection), Ireland (presidential election), and London (mayoral election).

[3] For a comprehensive list, see `https://www.fairvote.org/`.

[4] For a comprehensive summary of RCV's proposed benefits, see Cormack (2021).

platform in order to attract support from these voters, rather than focusing exclusively on a narrow segment of the electorate.

This contention is a critical component of the contemporary case for RCV as it is advanced by a diverse array of political actors.

Politicians. The Voter Choice Act—currently under consideration by Congress—proposes $40 million in federal grants to support up to 50% of costs incurred by state and local governments that choose to adopt RCV. The bill is sponsored by Senators Michael Bennett and Angus King and Congressman Dean Phillips, who argue that "by requiring the winner to receive majority support, RCV rewards candidates who appeal beyond their "base" to a broader cross-section of voters".[5]

Academics. Constitutional scholar Richard Pildes asserts that "[RCV] encourages candidates to reach out to voters who might not prefer that candidate as their first choice, but whom the candidate still wants to persuade to rank them second or third".[6]

Activists and Reform Advocates. Reform advocate organization Fairvote proposes that under RCV "[c]andidates are more likely to win when they engage a wider array of voters beyond their base"[7].

A recurrent theme of these endorsements is that RCV's benefits are particularly relevant in polarized polities characterized by loyal voting blocs. This polarization could arise from high partisanship, or from salient ethnic, geographic, of cultural cleavages (Horowitz 2004). It is in these contexts that candidates with the largest and most reliable bases of support have the greatest incentives to focus on their bases in a plurality rule election. The reason is that these candidates can win on the basis of only a plurality of votes, instead of a majority.

To evaluate the logic of these contentions we develop a model of an election between three office-seeking candidates: 1, 2, and 3, who compete for the support of an electorate divided

[5] See https://phillips.house.gov/voterchoiceact/

[6] "Can ranked-choice voting save American democracy? We ask an expert". CNN, July 12, 2022. https://www.cnn.com/2022/07/12/politics/ranked-choice-voting-ctzn/index.html

[7] See https://fairvoteaction.org/rcv-is-about-the-process-not-the-winners/.

into three groups: 1, 2, and 3. No single group is a majority, but group 1 is the largest (i.e., the plurality) group, followed by group 2, followed by smallest group 3. An example could be a district in which moderate (group 2) and conservative (group 3) Republicans are a majority of voters, but Democrats (group 1) outnumber either of the two Republican groups.

Each candidate has a group of core supporters—her *base*—with whom she shares a 'type,' i.e., a common cultural, ethnic, geographic, or partisan identity that translates into a primitive electoral affinity, in line with the approach introduced by Krasa and Polborn (2012). Our framework can capture rich patterns of mis-alignment amongst distinct groups of voters, reflecting the strength of political cleavages. For example, it includes as special cases (i) *no* mis-alignment whatsoever across groups, or (ii) the canonical *divided majority* setting in which a majority of voters (e.g., groups 2 and 3) jointly mis-align the most with voters in largest group 1 but may also mis-align to a lesser degree with one another.

Each candidate chooses whether to campaign on a broad platform that appeals to all voters, or instead pursue a campaign that targets one of the three groups (Lizzeri and Persico 2001). In a distributive politics setting, the broad policy represents an efficient spending allocation, versus a policy that exclusively directs spending towards a geographic district or ethnic group (e.g., Burgess et al. 2015). The platforms could alternatively distinguish issues that all voters care about (e.g., the economy) from wedge issues that only appeal to a narrow segment of the electorate (e.g., culture wars). Voters evaluate each candidate based on her platform, her type, and her stochastic valence—the latter reflecting developments that shape a candidate's appeal and unfold over the course of the campaign.

After candidates choose policies and valences are realized, each voter casts her ballot. We presume sincere voting throughout.[8] While our benchmark presentation also assumes full turnout, our insights hold in a formulation that incorporates two dimensions of turnout: the

[8] This approach contrasts with formulations in which voters condition their choice on the relative prospects of pivotal events. The strategic and computational burden such behavior imposes on voters leads scholars to question its plausibility in real-world plurality rule elections (Van der Straeten, Laslier, Sauger and Blais 2010). This burden intensifies under RCV: in a three-candidate contest, the set of pivotal events expands from three under plurality rule to twelve under RCV (Eggers and Nowacki 2021).

decision to vote, and the decision under RCV of how many candidates to rank.[9]

Under plurality, a candidate wins if and only if she receives more votes (first preferences) than *every* other candidate. In our three-candidate RCV setting, a candidate wins if (i) she wins a majority of first preferences, or instead (ii) no candidate wins a majority of first preferences, she is not eliminated for having won the fewest first preferences, and her combined first and second preferences exceed those of the other remaining candidate.

Two important facts about RCV follow: a candidate can win the support of voters that do not like her the most, and these second preferences can help a candidate win the election. Do these facts imply that a candidate has stronger incentives to campaign broadly, relative to plurality? Our paper shows that the answer is *no*.

In our framework, candidates favor broad campaigns when they expect voters beyond their base to reward these appeals with their support. These voters' responsiveness depends on their conflicts over ideology, class, ethnicity, religion or geography—i.e., the severity of inter-group mis-alignments. Limited inter-group conflicts raise the possibility for a candidate to win support from voters outside her base. More severe conflicts reduce voters' responsiveness to platform appeals from candidates outside of their group. This creates an obstacle for candidates trying to win the support of a majority of the electorate under both plurality and RCV.

Our key insight is that a candidate's incentives under plurality differ from her incentives under RCV only when there is a wedge between the relative competitiveness of voters' first versus their second preferences. How might this wedge exist and what are its consequences?

Second preferences more *competitive than first preferences.* Suppose that bases are very loyal to their candidates. This means that voters' first preferences are uncompetitive, making it unlikely that a majority coalesces behind any single candidate. The candidate with the largest base anticipates that the remaining two groups (i.e., the majority) will split their votes across distinct candidates. Under plurality, the candidate with the largest base is better off targeting her base rather than campaigning broadly. The reason is that when the remaining groups split, she wins the election so long as she retains her own base's support.

[9] See: https://osf.io/preprints/socarxiv/ukras.

Under RCV, the absence of a majority-preferred candidate does not ensure that the candidate with the largest base of support wins the election. Instead the candidate with the fewest votes is eliminated, making the second preferences of the group that supported her decisive in a second round of counting. If these voters' attitudes to the remaining candidates are not too imbalanced in favor of one versus the other, their second preferences may be competitive and therefore responsive to policy even when their first preferences are *not*. In these contexts, we show that the possibility of winning second preferences under RCV creates an additional margin of policy competition that encourages broad campaigning more effectively than plurality—precisely in line with the leading intuition we outlined, above.

Second preferences less *competitive than first preferences.* Consider an election in which a majority of voters—groups 2 and 3—almost always agree that candidate 1 with the largest base (group 1) is the worst of the three candidates. However, the rankings of this majority over the remaining candidates 2 and 3 is more fluid: these voters may either agree or disagree on the ranking depending on these candidates' platforms and their valence. This context corresponds to a canonical 'divided majority' setting famously studied, Borda (1781) and Condorcet (1785), but also in contemporary work by Myerson and Weber (1993); Martinelli (2002); Dewan and Myatt (2007), and Bouton and Gratton (2015). An example could be a polarized district in which moderate (group 2) and conservative (group 3) Republicans are a majority of voters, but Democrats (group 1) outnumber either of the two Republican groups.

In this setting, a voter that ranks 2 the highest is very likely to cast a second preference for candidate 3, while a voter that ranks 3 the highest is likely to cast a second preference for candidate 2. So, while the majority's first preferences are competitive between candidates 2 and 3, their second preferences are expected to be very uncompetitive.

How does candidate 2 win an election? Under plurality, she needs the support of a majority and the best way to secure it is through a broad campaign. Targeting her appeal to any single group only intensifies the risk of vote-splitting within the majority, which is fatal to her election prospects. Under RCV, by contrast, candidate 2 does not need to worry about vote-splitting. So long as candidate 2 holds on to the support of her (second largest) base group 2, eliminating candidate 3 near-guarantees her a valuable dividend of second preferences from the eliminated candidate's supporters.

In these situations, the possibility of winning second preferences changes how a candidate pursues *first* preferences. Under plurality, candidate 2 aims to win more first preferences than any other candidate. Under RCV, her priority instead shifts to winning more first preferences than candidates *whose voters are likely to rank her first*. This encourages her to pursue electoral strategies that increase the risk of diving the majority—in our setting, a campaign that targets her base.

Contribution. Electoral systems can be assessed according to their effect on voters, and their effects on candidates. Existing work—notably, the social choice tradition—almost exclusively focuses on voters under the presumption that the set of alternatives (i.e., candidates and their policies) is fixed. In that vein, existing work highlights both experimentally (Van der Straeten et al. 2010) and computationally (Eggers and Nowacki 2021) that RCV can attenuate voters' incentives to cast strategic ballots, relative to plurality rule. Dellis and Kröger (2023) also find experimentally that RCV impedes strategic voting, however they unearth a tendency for voters to employ strategic heuristics rather than resorting to fully sincere voting.

Three papers study policy outcomes in a spatial model of elections. Acharya, Cherivirala, Truax and Wahal (2023) compare the policy extremism of winning candidates under a plurality rule with primary elections versus RCV. They do this by simulating a large number of elections with randomly drawn (i.e., non-strategic) candidate platforms. They find that RCV tends to result in relatively more moderate winners. In the context of up to three potential candidates, Callander (2005) and Dellis, Gauthier-Belzile and Oak (2017) study their strategic entry and platforms. Both papers focus on the minimal degree of platform divergence that can deter entry by a third more centrist candidate.

In a citizen-candidate framework Dellis, Gauthier-Belzile and Oak (2017) show that RCV requires more platform moderation in order to deter entry. The reason is that a centrist entrant wins so long as she doesn't receive the *least* first preferences, since her centrist platform wins every voter's second preference. Under plurality, by contrast, a centrist entrant wins only if she receives the *most* first preferences. The authors conclude that RCV sustains less policy polarization than the plurality rule. Callander (2005) characterizes a continuum of equilibria in a Downsian framework with office-motivated candidates, highlighting the co-existence of equilibria with full median convergence under RCV, alongside equilibria with polarized platforms. Under plurality, by contrast, median convergence with three or more candidates cannot be supported (Cox 1987).

A few papers compare candidates' incentives across electoral systems in fully distributive contexts. Our paper's focus on voter heterogeneity differs from Myerson (1993) and Lizzeri and Persico (2005). Lizzeri and Persico (2001) compare plurality and proportional representation in two-candidate elections.

Because all voters turn out and fully utilize their ballots in both papers, and because candidates are differentiated solely by platforms, these frameworks do not address how candidates use their policy commitments to mobilize core supporters versus moderates. While our three-candidate framework abstracts from the question of how *many* candidates can be supported under RCV, both Callander (2005)'s and Dellis, Gauthier-Belzile and Oak (2017)'s analysis with endogenous candidacy highlights the stability of three-or-fewer candidate competition under RCV. This is also consistent with evidence from real-world elections documented in Jesse (2000) and Farrell and McAllister (2006).

A recent and growing body of empirical work evaluates RCV in the United States. Subject to the challenges of identifying RCV's impacts on politicians and voters from observational data, this body of work largely unearths null results. Most directly related to our work are a handful of studies evaluating RCV's moderating effect on campaigns, with small effects on negativity (Robb 2011; Donovan, Tolbert and Gracey 2016; Kropf 2021) and null effects on ideological extremism (Vishwanath 2023). McDaniel (2018) also finds that RCV increased racially polarized voting—in line with the logic of our results. RCV has not yet improved the representation of women or minorities (Vishwanath 2023; Santucci and Scott 2021; Colner 2023)—the latter being driven by higher rates of over-voting (Cormack 2023) and ballot exhaustion (McCarty 2023) by minority voters. Effects of RCV on women and minority candidacy appear to be either small and transitory (Colner 2023), or null.[10]

[10] More generally, survey experiments comparing elections under RCV and plurality show very little differences in terms of voter satisfaction and electoral outcomes (Donovan, Tolbert and Gracey 2016; Nielson 2017).

2. The Basic Model

Electorate. A unit mass of voters divide into three groups $\{1, 2, 3\} \equiv \mathcal{G}$. Group $i \in \mathcal{G}$ has mass μ_i; we assume $\mu_3 < \mu_2 < \mu_1 < .5$. Thus, no single group is a majority, group 1 is the largest (plurality) group, and group 3 is the smallest group. Each voter in group $i \in \mathcal{G}$ shares a common *type* $x_i \in \mathbb{R}$. This type could be interpreted as an ideology, or a religious or ethnic identity.

Platforms. There are two kinds of policies: a *broad* policy g and a *targeted* policy t^i. A voter in group $i \in \{1, 2, 3\}$'s payoff from policy p is

$$u_i(p) = \begin{cases} 1 & \text{if } p = t^i \\ u & \text{if } p = g \\ 0 & \text{otherwise.} \end{cases}$$

Thus, a broad campaign appeals to all voters, whereas a targeted campaign delivers a benefit only to voters that are directly targeted.

A1. $1 > u > \mu_1$.

The first restriction states that a voter receives her highest payoff from the policy targeted to her group. The second restriction states that the broad policy maximizes utilitarian welfare, which is an implicit premise of the contention our paper evaluates.[11]

Candidates. There are three office-seeking candidates: $\{1, 2, 3\} \equiv \mathcal{C}$. Each candidate i chooses a policy from the set $\{g, t^i\}$. That is: each candidate can campaign broadly, or instead deliver a targeted benefit to her own group. Each candidate $j \in \{1, 2, 3\}$ is also associated with type $x_j \in \mathbb{R}$.

We later study an unrestricted model in which any candidates can target any group—not just her own.

Voters' Payoffs. If a candidate $j \in \{1, 2, 3\}$ wins the election with policy p_j, a group-$i \in \{1, 2, 3\}$

[11] To see why, recognize that A1 implies average welfare is higher when the unit mass of voters receives payoff u than if any group i of mass $\mu_i \leq \mu_1$ receives payoff 1 and all others get zero.

voter's payoff is

$$u_i(p_j, \tau) = u_i(p_j) + \tau_j - |x_i - x_j|. \quad (1)$$

Here, τ_j is an aggregate valence shock. We assume that each shock is independently realized from a continuous distribution function $F(\cdot)$ with density $f(\cdot)$ and full support on $\mathbb{R}$. The term $|x_i - x_j|$ captures mis-alignment between groups (and thus candidates).[12] Henceforth, we use the short-hand $d_{ij} \equiv |x_i - x_j|$ to quantify the distance or conflict between groups i and j.

Candidates' Payoffs. Each candidate maximizes her probability of winning.

The timing unfolds as follows.

1. The candidates simultaneously select policies.
2. Nature realizes the valence shocks.
3. Voters observe policies and valence, and make their decisions.
4. The winning candidate implements her platform, and payoffs are realized.

Voting Rules. Under plurality, a candidate wins the election if she receives more votes (i.e., more first preferences) than any other candidate. Under RCV, a candidate wins the election if (i) she wins a majority of first preferences, or (ii) she does not win the fewest first preferences, and her combined first and second preferences exceed those of the other candidate that does not win the fewest first preferences.

All voters turn out, fully utilize their ballots, and vote sincerely. In a companion paper we show that our results also hold in settings where voters can either fully or partial abstain.[13] Our solution concept is Nash Equilibrium. We further impose indifference and tie-breaking rules that entail no loss of generality.[14]

[12] Results extend without amendment to any strictly increasing and symmetric transformation of $|x_i - x_j|$, or to higher-dimensional types.

[13] Partial abstention is often referred to as "ballot exhaustion", whereby voters turn out but do not rank all of the available candidates.

[14] If indifferent between candidate 1 and either candidate $j \in \{2, 3\}$, voters support candidate 1. If indifferent between candidates 2 and 3, voters support 2. Similarly, ties between 1 and any other candidate resolve in favor of 1, and ties between 2 and 3 resolve in favor of 2.

Discussion. We presume that voters cast their ballots sincerely. With three or more candidates a voter in a finite population (i.e., not a continuum as we assume) may have incentives to cast her ballot strategically. We view sincere voting as a reasonable starting point that facilitates our focus on strategic candidates. As we highlighted earlier, there is neither theoretical nor empirical consensus about the extent to which voters condition their strategies on pivotal events in RCV elections.[15] Later, we argue that our main insights should extend to a setting in which a share of voters cast their ballots strategically.

Our framework distinguishes between policies that target specific groups of voters versus broad policies that all voters value. Our setting differs, for example, from a one-dimensional spatial context in which targeting centrist voters with 'moderate' policies may also generate an efficiency benefit versus policies that target more extreme voters. In ongoing work we extend our insights to the spatial setting, but our goal here is to study a setting in which *any* form of targeting is inferior (from a welfare perspective) to the pursuit of broad policies that benefit all voters.

3. Results

Plurality. We first identify the set of profiles that can be supported as an equilibrium under plurality for some primitives. Recall that these primitives are the set of voter and candidate types $x = (x_1, x_2, x_3)$, the value of the broad campaign u and the distribution of valence F.

Lemma 1. *Plurality admits a (generically) unique equilibrium: either (g, g, g) or (t^1, g, g).*

For all primitives, each of candidate 2's and 3's dominant strategy is to campaign broadly. Candidate 1's best response is either to campaign broadly, herself, or instead target her base.To understand why, recognize that a voter in group $i \in \{1, 2, 3\}$ likes candidate j the most if and only if

$$\tau_j \geq \max_{k \in \mathcal{C} \setminus \{j\}} \left\{ \tau_k + \underbrace{u_i(p_k) - u_i(p_j)}_{\text{platform}} - \underbrace{(d_{ik} - d_{ij})}_{\text{misalignment}} \right\} \equiv \tau_i^j(p, \tau_{-j}, x). \quad (2)$$

[15] There are presently no theoretical results on strategic voting equilibria under RCV.

We can define a set of candidate-j critical thresholds:

$$\mathcal{T}_j(p_j, p_{-j}, \tau_{-j}, x) \equiv \left\{\tau_1^j(p, \tau_{-j}, x), \tau_2^j(p, \tau_{-j}, x), \tau_3^j(p, \tau_{-j}, x)\right\}.$$

Letting $\max_{(k)} \mathcal{T}_j$ denote the k-th largest element of $\mathcal{T}_j$, we conclude that a necessary and sufficient condition for candidate j to win the support of a majority of voters is that

$$\tau_j \geq \max_{(2)} \mathcal{T}_j(p_j, p_{-j}, \tau_{-j}, x) \equiv \overline{\tau}_j(p_j, p_{-j}, \tau_{-j}, x). \tag{3}$$

Suppose candidates 2 and 3 campaign broadly: $p_2 = p_3 = g$. Figure 1 illustrates the threshold for candidate to win the support of each group. It fixes a realization $\tau_3 \in \mathbb{R}$ of candidate 3's valence, and plots candidate 1's valence on the vertical axis and candidate 2's valence on the horizontal axis. The figure highlights that a necessary and sufficient condition for candidate 1 to win a majority is that she win votes from either group 2 or group 3. The reason is that she always wins votes from either group only if she also wins votes from her own group 1.[16] The right-panel highlights the minimum of thresholds τ_2^1 and τ_3^1, which corresponds to candidate 1 winning a majority, i.e., the satisfaction of condition (3).

Does candidate 1 have another path to victory, besides winning a majority? The answer is *yes*. The reason is that for some valence realizations *no* candidate wins a majority of votes:

$$\text{for each } j \in \mathcal{C}: \quad \max_{(3)} \mathcal{T}_j(p_j, p_{-j}, \tau_{-j}, x) \leq \tau_j \leq \overline{\tau}_1(p_1, p_{-1}, \tau_{-1}, x). \tag{4}$$

Henceforth, we define $\underline{\tau}_j \equiv \max_{(3)} \mathcal{T}_j$: candidate j wins the support of *at least* one group if and only if $\tau_j \geq \underline{\tau}_j$. She wins the support of *only* one group if, in addition, $\tau_j \leq \overline{\tau}_j$.

Condition (4) implicitly defines a set of preference shock realizations that induce a *three-way split*, whereby group 1 votes for candidate 1, group 2 votes for candidate 2, and group 3 votes for candidate 3. In the basic model, any three-way split takes this form for any primitives regardless of the candidate's strategies.[17]

[16] Note that this property is not assured for all strategy profiles in a model where every candidate can target any group.

[17] In our 'unrestricted' model that allows candidates to target any group, as well as their

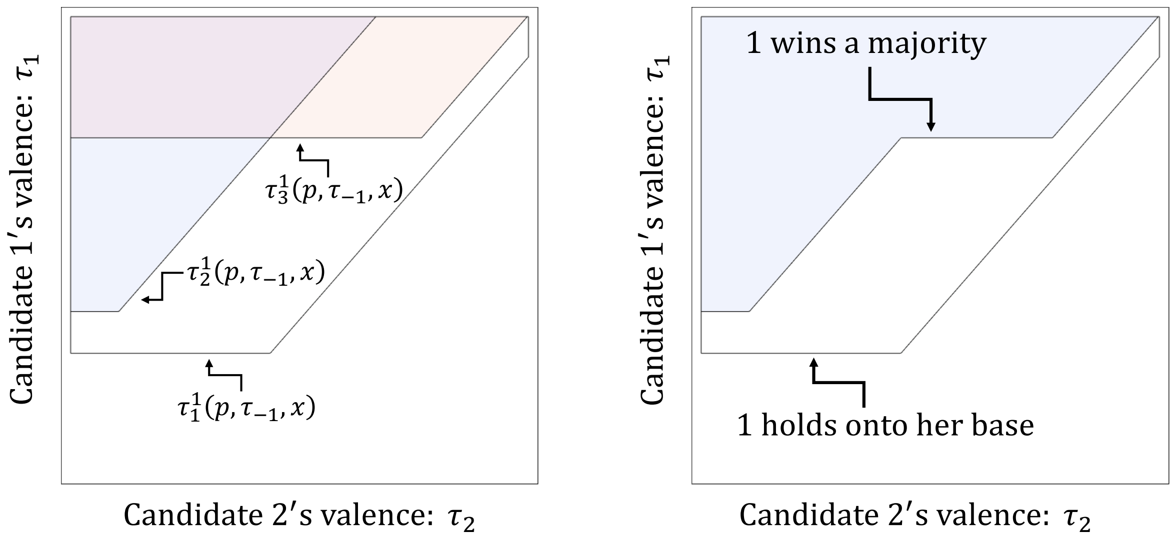

Figure 1 – The Figure identifies valence pairs for candidates 1 (vertical axis) and 2 (horizontal axis). The left-hand panel identifies critical valence thresholds under the broad strategy $p = (g, g, g)$ such that candidate i wins the support of group $j \in \{1, 2, 3\}$. The right-hand panel identifies the valence realizations such that candidate 1 wins a majority, i.e., $\tau_1 \geq \overline{\tau}_1(p, x) \equiv \min\{\tau_2^1, \tau_3^1\}$.

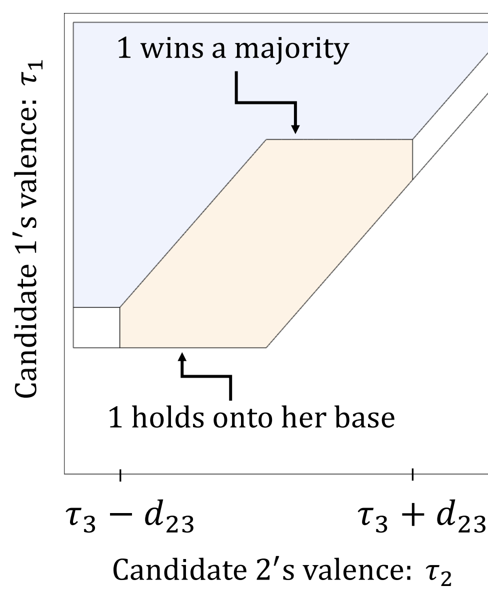

Figure 2 – The Figure identifies valence pairs for candidates 1 (vertical axis) and 2 (horizontal axis). It identifies critical valence thresholds under the broad strategy $p = (g, g, g)$ such that candidate 1 wins a majority *or* in which there is a three-way split. A three-way split occurs when every candidate i wins support from her group i.

Figure 2 augments the previous figures by identifying valence realizations that induce a

own, a candidate may win first preferences from a group other than her own in a three-way three-way split, under the presumption that the candidates pursue a broad campaign. These realizations correspond to condition (4) and are highlighted by the yellow hexagon. To understand the conditions, recognize that a three-way split at profile (g, g, g) occurs when:

$$\tau_1 \geq \max\{\tau_2 - d_{12}, \tau_3 - d_{13}\} \quad \text{(Group 1 votes for candidate 1)}$$
$$\tau_2 \geq \max\{\tau_1 - d_{12}, \tau_3 - d_{23}\} \quad \text{(Group 2 votes for candidate 2)}$$
$$\tau_3 \geq \max\{\tau_1 - d_{13}, \tau_2 - d_{23}\} \quad \text{(Group 3 votes for candidate 3)}$$

These conditions can be re-written:

$$\max\{\tau_2 - d_{12}, \tau_3 - d_{13}\} \leq \tau_1 \leq \min\{\tau_2 + d_{12}, \tau_3 + d_{13}\} \tag{5}$$
$$\tau_3 - d_{23} \leq \tau_2 \leq \tau_3 + d_{23}. \tag{6}$$

Expression (5) states that group 1—and only group 1—likes candidate 1 the most. (6) further implies that groups 2 and 3 split their votes across candidates 2 and 3, respectively. The set of valence shocks identified by (5) and (6) is traced out in Figure 3's left-hand panel. Notice that the prospect the remaining groups 2 and 3 split their votes increases with their degree of inter-group mis-alignment, captured by an increase in d_{23}. Under plurality, candidate 1 wins in a three-way split. The reason is that group 1 is the largest of three groups.

Suppose candidate 1 instead targets her group 1. While Figure 3's left-hand panel shows the thresholds for her to win at broad strategy $p = (g, g, g)$ under plurality, the right-hand panel shows how these thresholds adjust as a consequence of candidate 1's decision to target her base, which generates the action profile $p' = (t^1, g, g)$.

On the one hand, candidate 1's threshold for winning a majority *increases* by u after she deviates to t^1. This is because candidate 1's decision to focus exclusively on her base lowers her relative platform appeal to the remaining voters in groups 2 and 3. This harms candidate 1's prospects whenever remaining groups 2 and 3 do not split their votes across candidates 2 and 3. That is: whenever $|\tau_2 - \tau_3| > d_{23}$, 1's change in strategy is electorally harmful since 1 wins only with the support of at least one other group besides her own.

split depending on the strategies.

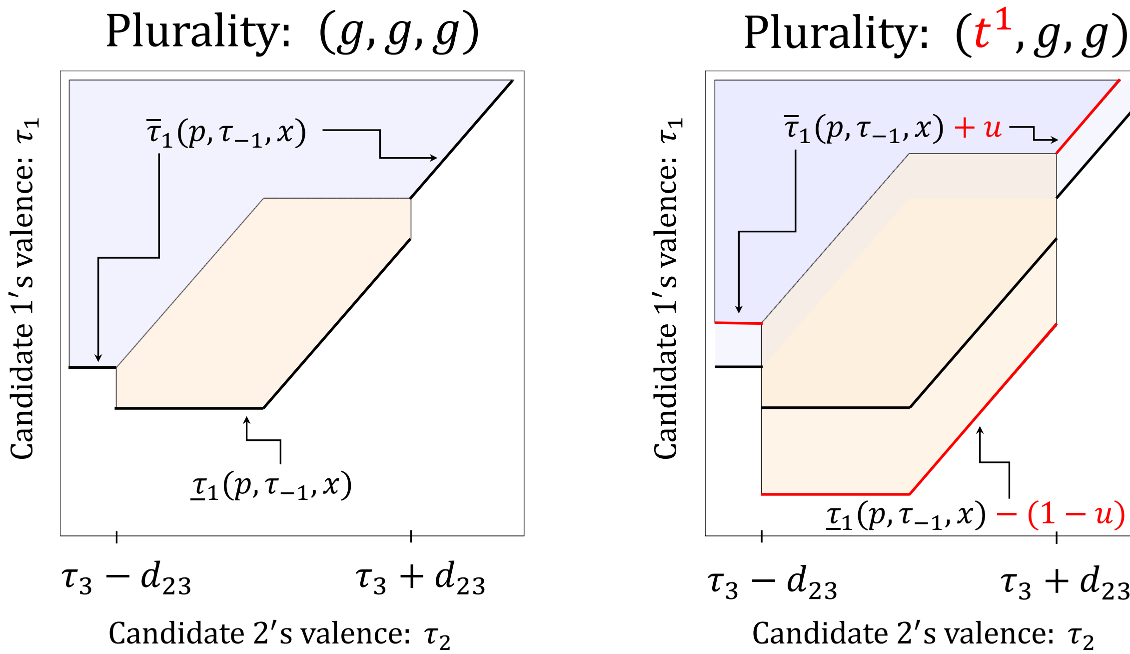

Figure 3 – The Figure identifies critical valence thresholds under plurality at profile $p = (g, g, g)$ (left-hand) or $p' = (t^1, g, g)$ (right-hand) such that candidate 1 wins a majority *or* in which there is a three-way split.

On the other hand, candidate 1′s deviation *decreases* the threshold for retaining the support of her base (group 1) since her net platform appeal to these voters over any other candidate is $1 - u$. This benefits candidate 1′s prospects whenever remaining groups 2 and 3 split their votes. That is: whenever $|\tau_2 - \tau_3| \leq d_{23}$ the change in strategy is electorally valuable for candidate 1 since she wins under plurality if and only if she wins the support of her own group.

Let $\pi_i(p_i, p_{-i}, z, x)$ denote i's probability of winning at profile (p_i, p_{-i}) under rule $z \in \{\text{plu}, \text{rcv}\}$ with types $x = (x_1, x_2, x_3)$. Putting all of this together, we conclude that candidate 1 is better off pursuing a broad campaign when candidates 2 and 3 pursue broad campaigns $p_{-1} = (g, g)$ if and only if

$$\pi_1(t^1, p_{-1}, \text{plu}, x) - \pi_1(g, p_{-1}, \text{plu}, x) \leq 0$$

$$\iff \iint_{|\tilde{\tau}_2 - \tilde{\tau}_3| > d_{23}} \Big[F(\overline{\tau}_1(g, p_{-i}, \tilde{\tau}_{-1}, x)) - F(\overline{\tau}_1(g, p_{-i}, \tilde{\tau}_{-1}, x) + u) \Big] \, dF(\tilde{\tau}_2) \, dF(\tilde{\tau}_3) \tag{7}$$

$$+ \iint_{|\tilde{\tau}_2 - \tilde{\tau}_3| \leq d_{23}} \Big[F(\underline{\tau}_1(g, p_{-1}, \tilde{\tau}_{-1}, x)) - F(\underline{\tau}_1(g, p_{-1}, \tilde{\tau}_{-1}, x) - (1 - u)) \Big] \, dF(\tilde{\tau}_2) \, dF(\tilde{\tau}_3) \tag{8}$$

$\leq 0.$

The term inside the first expression (7) is negative, reflecting 1's lost prospects for winning a majority, while the second expression (8) is positive. It is easy to verify that the sum of (7) and (8) strictly increases in u, and that there is unique a threshold $u^* < 1$ for which it is equal to zero. We now show that our insights yield a strong comparison between 1's benefit from deviating under plurality versus RCV. Notice, however, that a deviation by either candidate 2 or 3 to target her respective bases can *never* be profitable under plurality: this candidate loses in any three-way split and therefore only accounts for the fact that her deviations weakens her prospect of winning a majority.

Ranked Choice Voting. To start, we first identify the set of profiles that could be supported as an equilibrium under RCV for suitable primitives.

Lemma 2. *Under RCV, profile p is an equilibrium only if for some pair of mixed strategies $(\sigma_1, \sigma_2) \in [0,1]^2$: $p = (\sigma_1, \sigma_2, g)$.*

As under plurality, candidate 3's dominant strategy under RCV is to campaign broadly. The reason is that in a three-way split she wins the fewest first preferences from smallest group 3 and is eliminated from the contest. She therefore wins under RCV (as under plurality) only with the support of a majority of voters. Notably, the lemma does not rule out any strategies by the remaining candidates—unlike plurality, it does not state that candidate 2 must always campaign broadly.

We therefore focus on the incentives of candidates 1 and 2. If either candidate targets her base, she weakens her ability to secure a majority regardless of the remaining candidates' strategies. So, she benefits from targeting her base only in the event that the remaining groups split their votes between the remaining candidates.

Suppose candidate $i \in \{1,2\}$ chooses a broad campaign, and presume that the remaining candidates $j \in \{1,2\} \setminus i$ and 3 also campaign broadly. As under plurality, a three-way split occurs whenever each candidate retains her core supporters. In contrast with plurality, however, candidate 1 does not necessarily win the election. Under RCV, group 3's preferred candidate 3 wins the fewest first preferences and is eliminated from the contest. The second preferences

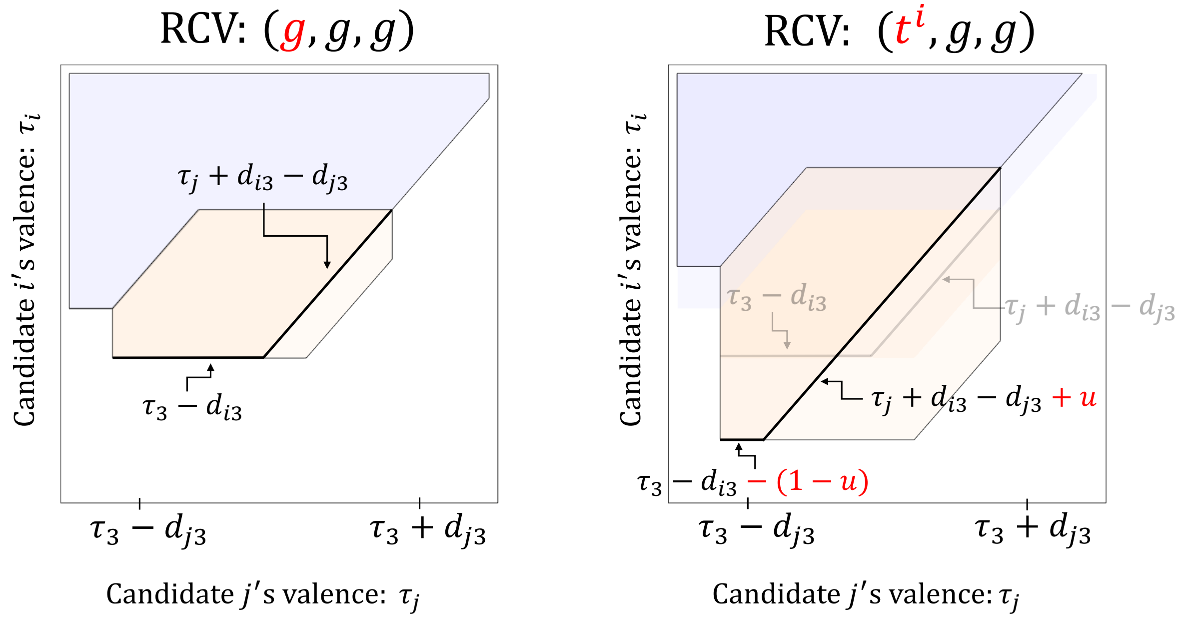

Figure 4 – The Figure identifies critical valence thresholds under RCV at profile $p = (g, g, g)$ such that candidate 1 wins a majority *or* in which there is a three-way split.

of group 3 voters are therefore decisive in the pairwise contest between the remaining two candidates. Group 3 voters prefer candidate 1 instead of candidate 2 if and only if

$$\tau_1 - d_{13} + u \geq \tau_2 - d_{23} + u \iff \tau_1 - \tau_2 \geq d_{13} - d_{23}.$$

So, when the remaining groups split their votes, candidate $i \in \{1, 2\}$ wins the election under RCV if and only if

$$\tau_i \geq \max\{\underline{\tau}_i(p, \tau_{-i}, x), \tau_j + d_{i3} - d_{j3}\}. \tag{9}$$

The first constraint $\tau_i \geq \underline{\tau}_i(p, \tau_{-i}, x)$ states that i wins first preferences from her base. The second novel constraint under RCV is $\tau_i \geq \tau_j + d_{i3} - d_{j3}$, requiring that i wins second preferences from group 3.

For a fixed realization τ_3 of candidate 3's valence, Figure 4 plots candidate $i \in \{1, 2\}$'s valence on the vertical axis, and $j \in \{1, 2\} \setminus i$'s valence on the horizontal axis. Its left-hand panel shows the threshold (9) under the conjecture that all candidates campaign broadly. Recognize that in a three-way split and when j's valence is low, candidate i's binding constraint is to maintain the support of her own group. That is: whenever she maintains the support of her own base, she (rather than candidate j) is also winning second preferences from group 3. If j's valence is sufficiently large, candidates i's binding constraint is to win group 3's second preferences. That is: if candidate i is good enough to persuade group 3 voters to rank her second instead of j, then she is *also* good enough to persuade her own core supporters to rank her ahead of both the other candidates.

Figure 4 highlights how RCV reduces the circumstances in which candidate 1 wins relative to plurality. When groups 2 and 3 do not split their votes—i.e., when $|\tau_2 - \tau_3| > d_{23}$—candidate 1's condition for winning at a broad strategy is the same under both rules. The reason is that under both rules a candidate wins whenever she commands a majority. When groups 2 and 3 instead split their votes—i.e., when $|\tau_2 - \tau_3| \leq d_{23}$—candidate 1 wins less often under RCV than under plurality because of the additional constraint that candidate 1 must win group 3 second preferences.

Conversely, RCV (weakly) expands the set of circumstances in which candidate 2 wins. Under plurality, she loses in *any* three-way split, whereas under RCV she wins a three-way split so long as she wins second preferences from group-3 voters.

Putting all of this together, we conclude that candidate $i \in \{1, 2\}$ is better off pursuing a broad campaign when candidates $j \in \{1, 2\} \setminus i$ and 3 pursue broad campaigns if and only if

$$\pi_i(t^i, p_{-i}, \text{rcv}, x) - \pi_i(g, p_{-i}, \text{rcv}, x) \leq 0$$

$$\iff \iint\limits_{|\tilde{\tau}_j - \tilde{\tau}_3| > d_{j3}} \Big[F(\overline{\tau}_i(g, p_{-i}, \tilde{\tau}_{-i}, x)) - F(\overline{\tau}_i(g, p_{-i}, \tilde{\tau}_{-i}, x) + u) \Big] dF(\tilde{\tau}_j)\, dF(\tilde{\tau}_3) \tag{10}$$

$$+ \iint\limits_{|\tilde{\tau}_j - \tilde{\tau}_3| \leq d_{j3}} \begin{bmatrix} F(\max\{\underline{\tau}_i(g, p_{-i}, \tau_{-i}, x), \tilde{\tau}_j + d_{i3} - d_{j3}\}) \\ -F(\max\{\underline{\tau}_i(g, p_{-i}, \tau_{-i}, x) - (1-u), \tilde{\tau}_j + d_{i3} - d_{j3} + u\}) \end{bmatrix} dF(\tilde{\tau}_j)\, dF(\tilde{\tau}_3) \tag{11}$$

$$\leq 0.$$

Comparing candidate 1's net value from deviating under plurality versus RCV amounts to comparing the sum of (7) and (8) with the sum of (10) and (11). This comparison yields our first insight: that candidate 1's incentive to target her base is *always* stronger under plurality than RCV.

Lemma 3. *Regardless of primitives, and for any strategy of candidates* 2 *and* 3*, candidate* 1*'s incentive to campaign broadly instead of targeting her base is strictly higher under RCV than under plurality.*

The lemma states that 1's net incentive to campaign broadly is stronger under RCV than plurality *regardless* of the remaining candidates' strategies. Here, we develop the intuition under the presumption that candidates 2 and 3 pursue broad campaigns, so that $p_{-1} = (g, g)$. Then, candidate 1's incentive to target her base under plurality versus RCV is:

$$\Delta_1(g, t_1, p_{-1}, x) \equiv \pi_1(t^1, p_{-1}, \text{plu}, x) - \pi_1(g, p_{-1}, \text{plu}, x) - \big[\pi_1(t^1, p_{-1}, \text{rcv}, x) - \pi_1(g, p_{-1}, \text{rcv}, x)\big].$$

The key observation is that 1's benefit (or loss) from targeting her base differs under plurality versus RCV *only* in the event of a three-way split. The reason is that if there is no three-way split then at least one candidate is winning a majority of votes, implying that outcomes coincide across systems. More generally, for any given strategy of the candidates RCV can *only* affect electoral outcomes when no candidate wins a majority of votes, i.e., only when there is a three-way split. This can be seen by noting that lines (7) and (10) coincide, meaning that 1's net benefit from targeting her base under plurality versus RCV is simply the difference of (8) and (11).

To see that the difference of (8) and (11) is *always* strictly positive, recognize that for any shock realizations τ_2 and τ_3 such that groups 2 and 3 disagree over the ranking of candidates 2 and 3, there are three possibilities.

Case I. For any shock realization such that $\underline{\tau}_i(g, p_{-1}, \tau_{-1}, x) - (1-u) > \tau_2 + d_{13} - d_{23} + u$, the bracketed term in (11) that describes 1's gain from targeting her base under RCV in a three-way split is

$$F(\underline{\tau}_i(g, p_{-1}, \tau_{-1}, x)) - F(\underline{\tau}_i(g, p_{-1}, \tau_{-1}, x) - (1-u)),$$

which coincides with 1's gain from targeting her base under plurality at the corresponding shock realization. In this case, candidate 1's ability to hold onto her base guarantees that she also wins group 3's second preferences regardless her platform.

Case II. For any shock realization outside of Case 1 but such that $\tau_2+d_{13}-d_{23} < \underline{\tau}_i(g, p_{-1}, \tau_{-1}, x)$, then the bracketed term inside (11) is instead

$$F(\underline{\tau}_i(g, p_{-1}, \tau_{-1}, x)) - F(\tau_2 + d_{13} - d_{23} + u),$$

which is strictly smaller than 1's gain under plurality at the corresponding shock realization. In this case, candidate 1's ability to hold onto her base guarantees that she also wins group 3's second preferences only when she campaigns broadly. When candidate 1 instead targets her base, she is no longer guaranteed to win in a three-way split.

Case III. Finally, whenever $\tau_2+d_{13}-d_{23} \geq \underline{\tau}_i(g, p_{-1}, \tau_{-1}, x)$, candidate 1's binding constraint in any three-way split under RCV is to win group 3 voters' second preferences—even when she campaigns broadly. Targeting her base only tightens this constraint, implying that for these shock realizations the bracketed term in (11) represents a net *loss* under RCV of

$$F(\tau_2 + d_{13} - d_{23}) - F(\tau_2 + d_{13} - d_{23} + u)$$

relative to 1's gain under plurality for these shock realizations.

The result captures the leading intuition for why RCV should encourage broad campaigns to a greater extent than plurality: the need to capture second preferences from group 3 weakens candidate 1's benefit from targeting her core supporters.

One might conjecture that since only candidate 1 can benefit from targeting her base under plurality, and since RCV weakens her incentive to do so, RCV can sustain convergence at the broad strategy profile for any primitives at which plurality can sustain convergence. In the next section, we show that while there are primitives under which this conjecture is true, it is generally *false*.

3.1. When RCV encourages broad campaigning

We first establish the following positive result, providing sufficient conditions for RCV to support a broad equilibrium.

Proposition 1. *There exists $\delta(u, F) > 0$ such that if $|d_{13} - d_{23}| < \delta(u, F)$, RCV supports a broad* *equilibrium.*

The proposition identifies sufficient conditions under which RCV supports a broad equilibrium. Note that these conditions are *not* sufficient to ensure a broad equilibrium under plurality. For example, if d_{13} and d_{23} coincide but are sufficiently large, 1 strictly prefers to target her base under plurality. This yields an important corollary.

Corollary 1. *If $|d_{13} - d_{23}| < \delta(u, F)$, plurality supports a broad equilibrium only if RCV supports a broad equilibrium.*

To understand why, recognize that when $|d_{13} - d_{23}|$ is small group 3's second preferences are maximally competitive between candidates 1 and 2, and therefore maximally responsive to platforms.

If $d_{13} - d_{23}$ is positive and large, then group 3 mis-aligns relatively much more with group 1 instead of group 2. For *any* strategies of the candidates, group 3 second preferences are therefore more likely to resolve in favor of candidate 2.

If $d_{13} - d_{23}$ is negative and large, then group 3 mis-aligns relatively much more with group 2 instead of group 1. For any strategies of the candidates, group 3 second preferences are therefore more likely to resolve in favor of candidate 1.

We can re-write candidate i's constraint on winning when remaining groups j and 3 disagree over their preferred candidates in expression (9):

$$\begin{aligned}\tau_i &\geq \max\{\overbrace{\tau_j - d_{ij}, \tau_3 - d_{i3}}^{=\underline{\tau}_i(p,\tau_{-i},x)}, \tau_j + d_{i3} - d_{j3}\} \\ &= \max\{\tau_3 - d_{i3}, \tau_j + d_{i3} - d_{j3}\} \end{aligned} \tag{12}$$

where the second line follows from the triangle inequality that $d_{ij} \geq |d_{i3} - d_{j3}|$. Candidate i's binding constraint is therefore to win second preferences—instead of retaining her own group i's support—if $\tau_j + d_{i3} - d_{j3} \geq \tau_3 - d_{i3}$. This is true for *all* $|\tau_j - \tau_3| \leq d_{j3}$ if $d_{i3} - d_{j3} \leq 0$. This condition is satisfied for both candidates 1 and 2 whenever $d_{13} - d_{23} = 0$.

When $d_{13} - d_{23} = 0$, candidate $i \in \{1, 2\}$'s binding constraint under RCV at the broad profile $(p_i, p_j) = (g, g)$ is winning group 3's second preferences. If candidate i instead reverts to targeting her group, she tightens this constraint since she now offers group 3 a less appealing platform relative to candidate $j \in \{1, 2\} \setminus i$. This ensures that i's victory prospects when groups j and 3 split their first preferences *weaken*. Formally, setting $d_{13} - d_{23} \approx 0$ ensures that (11) is strictly negative for each of $i \in \{1, 2\}$. Since targeting her base also strictly weakens i's prospects of winning a majority, (10) is always strictly negative. This ensures that a targeted strategy is unprofitable.

This discussion highlights the crucial logic of incentives under RCV as compared to plurality: the possibility of a wedge in the competitiveness of first versus second preferences. Large d_{j3} can be interpreted as loyalty amongst members of group 3 towards their own candidate, or antipathy towards the remaining group $j \in \{1, 2\}$. Large d_{j3} means that first preferences are relatively uncompetitive. Under plurality, whenever candidate 1 anticipates a large risk of vote-splitting between groups 2 and 3 when d_{23} is large. This vote-splitting paves the way for candidate 1 to win solely on the basis of her own base's support, and encourages her to pursue a targeted campaign. Under RCV what matters is the *difference* of d_{13} and d_{23}. Even when groups are very loyal to their own candidates, second preferences can be relatively competitive and therefore responsive to a broad platform appeal.

Note that if there is *no* wedge in the competitiveness of first and second preferences are equally competitive, RCV cannot increase the scope for broad convergence relative to plurality.

Observation 1. *If $d_{13} = d_{23} = 0$, and thus $d_{13} - d_{23} = 0$, then* both *plurality* and *RCV support a broad equilibrium.*

3.2. When RCV discourages broad campaigning

We next establish two negative results about RCV. The first serves as a converse to Proposition 1. It argues that in contexts where second preferences are relatively uncompetitive, such that either (1) plurality supports a broad equilibrium—but RCV cannot—or (2) *both* systems encourage candidate 1 to target her base.

Proposition 2. *For any (u, F) there exist $\overline{d} > \underline{d} > 0$ such that*

1. *if $d_{13} > \overline{d} > \underline{d} > d_{23}$, plurality's unique equilibrium is (g, g, g), but RCV's unique equilibrium is (g, t^2, g).*

2. *If $d_{23} > \overline{d} > \underline{d} > d_{13}$, (t^1, g, g) is an equilibrium under both plurality and RCV.*

The first part of the proposition describes a setting in which group 3 voters mis-align significantly with group 1 versus group 2. They are therefore prone to cast any second preferences for candidate 2 rather than candidate 1, regardless of the campaigns that these candidate pursue.

An electoral appeal by candidate 2 that is targeted exclusively to her base reduces her appeal to voters in group 3. This heightens the risk of vote-splitting amongst (the majority of) voters in groups 2 and 3. Under plurality vote-splitting amongst the majority is fatal to candidate 2's hopes of election. Under RCV, however, vote-splitting by the majority is *not* fatal. On the contrary, candidate 2 recognizes that so long as maintains the support of her base in a three-way split, candidate 3's elimination in a three-way split all but guarantees a dividend of second preferences with which candidate 2 defeats candidate 1.

As a consequence, plurality encourages candidate 2 to pursue a broad campaign with the goal of uniting the majority whereas RCV encourages candidate 2 to pursue a strategy that buttresses her core vote at the expense of an increased prospect of dividing the majority. We obtain the following corollary.

Corollary 2. *If $d_{13} > \overline{d} > \underline{d} > d_{23}$, RCV supports a broad equilibrium only if plurality supports a broad equilibrium.*

The second part of the proposition describes a setting in which group 3 voters mis-align significantly with group 2 versus group 1. They are therefore prone to cast second preferences for candidate 1 rather than candidate 2. While Lemma 3 states that candidate 1's incentive to target her base is weaker under RCV than plurality, Proposition 2 shows that RCV does not necessarily eliminate that incentive.

The contexts identified in Proposition 2 are real-world relevant. Notably, they describe a *divided majority* in which a majority of voters always agree that one of the available candidates is the worst, but may disagree over which of the remaining candidates is the best. Natural interpretations include two liberal (or conservative) candidates that are prone to split the votes of a majority of voters, and who compete against a conservative (or liberal) candidate whose core supporters are a plurality but not a majority of voters.

Our second negative result states that RCV can support equilibria in which *both* candidates 1 *and* 2 target their bases—a phenomenon that cannot arise under plurality.

Proposition 3. *There exist* (x, u, F) *such that* (t^1, t^2, g) *is an equilibrium under RCV.*

We make two observations about Proposition 3's scope. First: this equilibrium may be unique, or it may co-exist with a broad equilibrium. Second: this equilibrium could be sustained in the absence of *any* primitive mis-alignment between groups —i.e., when $d_{13} = d_{12} = d_{23} = 0$.

Each of candidates 1 and 2 always maximizes their chances of winning a majority by campaigning broadly. Under plurality, candidate 2 always loses a three-way split and therefore is always better off from pursuing a broad campaign. How can could RCV induce *both* candidates 1 *and* 2 to target their bases, even when they implicitly compete for second preferences?

The reason is that RCV introduces competing demands on each of candidates 1 and 2 in the contingency where three-way splits can arise. On the one hand, competition to win group 3 voters' second preferences encourages a broad campaign. On the other hand, those second preferences are only valuable to candidate $i \in \{1, 2\}$ if she *also* retains the support of her base; this calls for a targeted campaign. Which of these strategic demands dominates depends not only political primitives, but also on the strategies the candidates pursue.

If candidate 1 appeals to group 3 voters by pursuing a broad strategy, the she places greater pressure on candidate 2 to do the same in order to win second preferences. Conversely, if candidate 1 is already targeting her base—thus abandoning any attempt to appeal directly to group 3— she weakens the competitive pressure to win second preferences. She therefore indirectly encourages candidate 2 to focus primarily on the objective of retaining her core supporters. RCV therefore creates a strategic complementarity between base-oriented strategies by candidates 1 and 2 that is absent under plurality and which produces a form of inefficient coordination that is similar to the Stag Hunt problem.

To summarize: Propositions 2 and 3 highlight that the possibility of winning second preferences under RCV may change how the candidates pursue *first* preferences. Second preferences matter only when no candidate wins a majority of first preferences. This means that for fixed strategies, RCV and plurality induce different election outcomes *only* in contingencies where no candidate wins a majority of votes. Under plurality, only the candidate with the largest base of support wins in these contingencies. Under RCV, however, a candidate wins so long as she does not win the fewest votes and that she also wins second preferences from whichever candidate is eliminated at the first vote tally. This opens up a path to victory for a candidate to win despite having first preferences only from a group of supporters that are neither a majority nor even a plurality.

Under plurality, only candidate 1 faces a trade-off between buttressing her core support with a targeted campaign and appealing to voters outside her base with a broad campaign. Under RCV, both candidates 1 and 2 face this trade-off. Proposition 2 highlights that when second preferences are primitively uncompetitive, the trade-off resolves in favor of a targeted campaign. Proposition 3 highlights that even when second preferences are primitively competitive, the strategies of the candidates may render them uncompetitive. Both results show the limits of the leading intuition that the possibility of second preferences encourages broad campaigning.

4. Extensions and Future Research

What if Candidates can Target Any Group? Our benchmark model presumed that candidates can either pursue a broad campaign or target their own core supporters. The assumption that candidates can only target their own group simplifies analysis. It may also reflect real-world constraints on candidates' ability to to deliver targeted benefits to voters beyond their political, ethnic, or geographic bases. As emphasized by Dixit and Londregan (1996), differences in the candidates ability to target different groups could arise because of differences in the candidates' information, or their clientelistic networks. Candidates may also differ in their credibility when making promises to voters outside their base versus their core supporters (Robinson and Torvik 2005). Nonetheless, it is important to ask whether our insights about candidates' incentives extend to a setting in which they can freely target different groups?

In Supplemental Appendix B, we show that the answer is *yes*. We study an 'unrestricted' model in which each candidate can campaign broadly, or instead target *any* candidate's core supporters—not merely her own. Our benchmark model highlighted the possibility of multiple equilibria under RCV. This multiplicity compounds in the unrestricted model under both electoral systems, complicating comparisons. We nonetheless show that our main insights are robust.

Observation 1 stated that when group mis-alignments are small enough both plurality and RCV support a broad equilibrium. This insight extends to the unrestricted setting. This highlights the robustness of our observation that in the absence of a primitive wedge in the competitiveness of first and second preferences RCV cannot better-incentivize broad campaigns than plurality.

Corollary 1 states that when second preferences are very competitive relative to first preferences, RCV is more prone to support a broad equilibrium than plurality. This result holds in the unrestricted model only under an additional qualification: the positive wedge in the competitiveness of first and second preferences cannot be *too* large. To see why, recognize that when the first preferences of groups 1 and 2 are sufficiently loyal, candidates 1 and 2 respectively take their core supporters for granted. Instead of targeting them—or campaigning broadly—these candidates may instead compete for second preferences from decisive group 3 by targeting these voters directly.

We already emphasized that RCV and plurality generate different incentives for candidates only when there is a wedge in the competitiveness of first and second preferences. Under the restriction that candidates can only target their own bases, a take-away from our benchmark model is that more competitive second preferences *always* improve incentives to campaign broadly. The reason is that a broad campaign is the only strategy candidates 1 and 2 can use to appeal to group 3. When candidates 1 and 2 can target group 3 directly, *too much* of a wedge in the competitiveness of first and second preferences may weaken the candidates' discipline to pursue broad strategies. The unrestricted model therefore further highlights the precariousness of the conditions under which RCV improves candidates' incentives to campaign broadly, relative to plurality.

Finally, we show that Corollary 2 also extends: when the possibility of winning second preferences changes how candidate 2 pursues first preferences, RCV *weakens* the prospects for sustaining a broad equilibrium relative to plurality.

Strategic Voting. Our benchmark presumes that voters cast sincere ballots and rank all candidates. With three candidates the assumption of sincere voting is not innocuous: there *are* circumstances in which an individual voters could benefit from casting a ballot that does not reflect her sincere preferences over the candidates. Nonetheless, we expect our insights extend to a setting in which some share of voters cast their ballots strategically.

To see why, consider the divided majority setting of Proposition 2 in which $d_{23} \approx 0$ and d_{13} is large. Under plurality candidate 2 prefers not to target her base because doing so heightens the risk of of vote-splitting amongst the majority (i.e., groups 2 and 3). Under RCV, vote-splitting is mitigated by the possibility of winning second preferences, which undermines 2's incentive to campaign broadly. Strategic voting under plurality also mitigates the risk of vote-splitting and therefore is a substitute for the explicit mechanism of second preferences. As long as strategic coordination under plurality remains imperfect, however, the wedge in the candidates' incentives under the two rules should persist. We nonetheless view strategic voting as an important direction for future research.

Abstention and Ballot Exhaustion. We assume all voters turn out and fully utilize their ballots. Nonetheless there is evidence that many voters—even those that turn out—do not rank all the candidates on the ballot (for example, Burnett and Kogan 2015). In recent exit surveys both in New York and Alaska, a preponderant reason voters give for not ranking all candidates is that they simply do not like all the candidates enough to cast a preference for them. For example, a poll conducted after Alaska's August 2022 RCV election found that of those voters that ranked only a single candidate, 75% reported the reason was "that was the only candidate I liked".[18]

In ongoing work we extend our framework to study ballot exhaustion and its consequences for candidates' electoral strategies. We augment our baseline model by assuming each voter has an idiosyncratic type $t_i \in \mathbb{R}$, and that types are continuously distributed across

[18] Patinkin Research Strategies, August 30 2022, https://alaskansforbetterelections.com/wp-content/uploads/2022/08/20220830_AK_Polling_Data-combined.pdf.

the electorate. Each voter casts a ballot for any candidate j whose payoff u_j exceeds threshold t_i and she ranks any such candidate in order of preference. Higher t_i captures voters with less interest in politics or enthusiasm. The average type could vary across groups, reflecting group-level differences in engagement or familiarity with the candidates. We show that our main messages extend. To see why, consider again the divided majority setting with d_{23} small and d_{13} large. Under *both* RCV and plurality each candidate has a stronger incentive to target her base because doing so raises turnout amongst groups that are prone to support her. Under RCV, however candidate 2 is even *less* concerned about ceding group 3 voters' second preferences to group 1. In the rare circumstance that group 3 voters prefer candidate 1 over candidate 2, high polarization still leads to large amounts of ballot exhaustion in which the majority of group 3 voters simply do not cast second preferences for either candidate.

5. Conclusion

Our paper studies electoral competition under Ranked Choice Voting (RCV). We ask whether RCV necessarily provides greater incentives for candidates to pursue broad campaigns, relative to plurality.

We showed that the comparison of electoral incentives under each system turns on the relative competitiveness of first and second preferences. Platform competition for first preferences under plurality may be sufficient to incentive broad campaigns. When first preferences are relatively uncompetitive, second preferences may generate an additional margin of responsiveness to platform, and thus intensify competition among candidates. In these contexts, RCV can induce broad campaigns when plurality cannot. However, too extreme of a margin may encourage candidates to pursue targeted campaigns that pander to the decisive preference minority.

Finally, imbalances in the competitiveness of first and second preferences across different candidates may change the way some candidates pursue first preferences. Under RCV, a candidate only benefits from the second preferences of candidates that are eliminated from the contest. A candidate's electoral strategy may therefore shift to defeating other candidates whose supporters are likely to rank her second. This is particularly likely to arise when one of the leading candidates—in our setting, candidates 1 or 2—is expected to receive a disproportionate share of preference votes from voters that support the remaining candidate. A classic example that conforms to this logic is the divided majority setting. It is likely to arise in high-polarization contexts—ironically, precisely the sorts contexts in which RCV proponents argue that the reform is most urgently needed.

We close with a broader interpretation of our results, and how they relate to existing arguments that favor RCV's adoption. By allowing voters to express a preference for multiple candidates, RCV implicitly helps voters to solve a coordination problem they would otherwise face in multi-candidate elections under plurality rule. For a fixed set of alternatives, this improved implicit coordination facilitates the election of moderate policies, and in particular majority-preferred policies when they exist. However, this improved implicit coordination also changes the candidates' strategies, by opening up new pathways to electoral victory that may be absent under plurality.

Changes in electoral rules therefore have the potential to create new conflicts between candidates whose consequences can be difficult to predict. Indeed, those consequences may be opposite to the aspirations of both scholars and reformers of electoral systems.

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Supplemental Appendix A: Proofs of Results

Recall that in our basic model each candidate $i \in \{1,2,3\}$'s action set is $\{g, t^i\}$.

Proof of Lemma 1. We begin by showing that candidates 2 and 3 have a strictly dominant strategy to campaign broadly under plurality. Recall that $\tau_i^j(p_j, p_{-j}, \tau_{-j}, x)$ denotes the threshold realization of τ_j above which candidate j wins first preferences from group $i \in \{1,2,3\}$.

We first claim that candidate 3 wins the support of voters in either group 1 or 2 only if she wins the support of voters in group 3.

Claim 1. *For any $(p_1, p_2, p_3) \in \{g, p_1\} \times \{g, p_2\} \times \{g, p_3\}$, $\tau_3 \geq \tau_i^3(p_3, p_{-3}, \tau_1, \tau_2, x)$ for $i \in \{1,2\}$ only if $\tau_3 \geq \tau_3^3(p_3, p_{-3}, \tau_1, \tau_2, x)$.*

Proof. We have

$$\mathcal{T}_3(p_3, p_{-3}, \tau_{-3}, x) = \left\{ \begin{array}{c} \max\{\tau_1 + u_1(p_1) - u_1(p_3) + d_{13}, \tau_2 + u_1(p_2) - u_1(p_3) + d_{13} - d_{12}\}, \\ \max\{\tau_1 + u_2(p_1) - u_2(p_3) + d_{23} - d_{12}, \tau_2 + u_2(p_2) - u_2(p_3) + d_{23}\}, \\ \max\{\tau_1 + u_3(p_1) - u_3(p_3) - d_{13}, \tau_2 + u_3(p_2) - u_3(p_3) - d_{23}\} \end{array} \right\}$$

Recognize that under the restriction $(p_1, p_2, p_3) \in \{g, p_1\} \times \{g, p_2\} \times \{g, p_3\}$ we have that $u_3(p_i) - u_3(p_3) \leq 0 \leq \min\{u_2(p_i) - u_2(p_3), u_1(p_i) - u_1(p_3)\}$ for each of $i \in \{1,2\}$. Further, the triangle inequality yields $-d_{13} \leq \min\{d_{23} - d_{12}, d_{13}\}$ and $-d_{23} \leq \min\{d_{13} - d_{12}, d_{23}\}$. This yields the claim. □

A similar argument yields that for $i \in \{2,3\}$: $\tau_1 \geq \tau_i^1(p_1, p_{-1}, \tau_{-1}, x)$ only if $\tau_1 \geq \tau_1^1(p_1, p_{-1}, \tau_{-1}, x)$ and for $i \in \{1,3\}$: $\tau_2 \geq \tau_i^2(p_2, p_{-2}, \tau_{-2}, x)$ only if $\tau_1 \geq \tau_2^2(p_2, p_{-2}, \tau_{-2}, x)$.

These insights imply that for any strategy profile, candidate $j \in \{2,3\}$ wins the election under plurality if and only if she secures a majority of votes. We next argue that to maximize the probability of securing a majority of votes, a candidate campaigns broadly regardless of the remaining candidates' strategies.

Claim 2. *For any $p_{-i} = (p_j, p_k) \in \{g, p_j\} \times \{g, p_k\}$: candidate $i \in \{1,2,3\}$ (strictly) maximizes her probability of winning a majority by campaigning on a broad platform.*

Proof. We make the argument from the perspective of candidate 1, since the argument for candidates 2 and 3 is similar. Regardless of $(p_2, p_3) \in \{g, t^2\} \times \{g, t^3\}$ candidate 1 wins a majority if and only if she wins the first preferences of either group 2 or group 3. That is:

$$\tau_1 \geq \max_{(2)} \mathcal{T}_1(p_1, (p_2, p_3), \tau_2, \tau_3, x)$$
$$= \min \left\{ \begin{array}{c} \max\{\tau_2 + u_2(p_2) + d_{12}, \tau_3 + u_2(p_3) + d_{12} - d_{23}\} - u_2(p_1), \\ \max\{\tau_2 + u_3(p_2) + d_{13} - d_{23}, \tau_3 + u_3(p_2) + d_{13}\} - u_3(p_1) \end{array} \right\}.$$

We have
$$u_2(p_1) = u_3(p_1) = \begin{cases} u & \text{if } p_1 = g \\ 0 & \text{if } p_1 = t^1. \end{cases}$$

The claim follows. □

The previous claims imply that candidates 2 and 3 have strictly dominant strategies to campaign broadly under plurality. It follows that in any equilibrium under plurality, $p_2 = p_3 = g$. □

Proof of Lemma 2. The proof of Lemma 1 implies that candidate 3 wins under both plurality and RCV if and only if she wins a majority of first preferences, and that that the broad campaign strictly maximizes her probability of winning a majority. It follows that $p_3 = g$ is 3's strictly dominant strategy under RCV. □

Proof of Lemma 3. Define $\ell_L(p_2, p_3) = u_2(p_3) - u_2(p_2) - d_{23}$ and $\ell_H(p_2, p_3) = u_3(p_3) - u_3(p_2) + d_{23}$, and

$$\hat{\tau}(p_1, p_{-1}) \equiv \min\{\max\{\ell_L(p_2, p_3), u_1(p_3) - u_1(p_1) + u_3(p_1) - u_3(p_2) + d_{23} - 2d_{13}\}, \ell_H(p_2, p_3)\}.$$

Then for any $(p_2, p_3) \in \{g, t^2\} \times \{g, t^3\}$:

$$\Delta_1(g,t_1, p_{-1}, x) \equiv$$
$$\pi_1(t^1, p_{-1}, \text{plu}, x) - \pi_1(g, p_{-1}, \text{plu}, x) - \left[\pi_1(t^1, p_{-1}, \text{rcv}, x) - \pi_1(g, p_{-1}, \text{rcv}, x)\right]$$

$$
\begin{aligned}
&= \iint\limits_{\ell_L \le \tilde\tau_2 - \tilde\tau_3 \le \ell_H} \Big[F(\underline{\tau}_1(g, p_{-1}, \tilde\tau_{-1}, x)) - F(\underline{\tau}_1(t^1, p_{-1}, \tilde\tau_{-1}, x)) \Big] dF(\tilde\tau_2)\, dF(\tilde\tau_3) \\
&\quad - \iint\limits_{\ell_L \le \tilde\tau_2 - \tilde\tau_3 \le \ell_H} \left[\begin{array}{c} F(\max\{\underline{\tau}_1(g, p_{-1}, \tilde\tau_{-1}, x), \tilde\tau_2 + d_{13} - d_{23} + u_3(p_2) - u\}) \\ -F(\max\{\underline{\tau}_1(t^1, p_{-1}, \tilde\tau_{-1}, x), \tilde\tau_2 + d_{13} - d_{23} + u_3(p_2)\}) \end{array} \right] dF(\tilde\tau_2)\, dF(\tilde\tau_3) \\
&= \iint\limits_{\ell_L \le \tilde\tau_2 - \tilde\tau_3 \le \ell_H} \Big[F(\underline{\tau}_1(g, p_{-1}, \tilde\tau_{-1}, x)) - F(\underline{\tau}_1(g, p_{-1}, \tilde\tau_{-1}, x) - (1-u)) \Big] dF(\tilde\tau_2)\, dF(\tilde\tau_3) \\
&\quad - \iint\limits_{\ell_L \le \tilde\tau_2 - \tilde\tau_3 \le \ell_H} \left[\begin{array}{c} F(\max\{\underline{\tau}_1(g, p_{-1}, \tilde\tau_{-1}, x), \tilde\tau_2 + d_{13} - d_{23} + u_3(p_2) - u\}) \\ -F(\max\{\underline{\tau}_1(g, \tilde\tau_{-1}, x) - (1-u), \tilde\tau_2 + d_{13} - d_{23} + u_3(p_2)\}) \end{array} \right] dF(\tilde\tau_2)\, dF(\tilde\tau_3) \\
&= \iint\limits_{\ell_L \le \tilde\tau_2 - \tilde\tau_3 \le \ell_H} \Big[F(\underline{\tau}_1(g, p_{-1}, \tilde\tau_{-1}, x)) - F(\underline{\tau}_1(g, p_{-1}, \tilde\tau_{-1}, x) - (1-u)) \Big] dF(\tilde\tau_2)\, dF(\tilde\tau_3) \\
&\quad - \iint\limits_{\ell_L \le \tilde\tau_2 - \tilde\tau_3 \le \hat\tau(t^1, p_{-1})} \Big[F(\underline{\tau}_1(g, p_{-1}, \tilde\tau_{-1}, x)) - F(\underline{\tau}_1(g, \tilde\tau_{-1}, x) - (1-u)) \Big] dF(\tilde\tau_2)\, dF(\tilde\tau_3) \\
&\quad - \iint\limits_{\hat\tau(t^1, p_{-1}) \le \tilde\tau_2 - \tilde\tau_3 \le \hat\tau(g, p_{-1})} \Big[F(\underline{\tau}_1(g, p_{-1}, \tilde\tau_{-1}, x)) - F(\tilde\tau_2 + d_{13} - d_{23} + u_3(p_2)) \Big] dF(\tilde\tau_2)\, dF(\tilde\tau_3) \\
&\quad - \iint\limits_{\hat\tau(g, p_{-1}) \le \tilde\tau_2 - \tilde\tau_3 \le \ell_H} \Big[F(\tilde\tau_2 + d_{13} - d_{23} + u_3(p_2) - u) - F(\tilde\tau_2 + d_{13} - d_{23} + u_3(p_2)) \Big] dF(\tilde\tau_2)\, dF(\tilde\tau_3) \\
&> \iint\limits_{\hat\tau(t^1, p_{-1}) \le \tilde\tau_2 - \tilde\tau_3 \le \hat\tau(g, p_{-1})} \left[\begin{array}{c} F(\tilde\tau_2 + d_{13} - d_{23} + u_3(p_2)) \\ -F(\underline{\tau}_1(g, p_{-1}, \tilde\tau_{-1}, x) - (1-u)) \end{array} \right] dF(\tilde\tau_2)\, dF(\tilde\tau_3) \ge 0.
\end{aligned}
$$

The strict inequality (rather than a weak inequality) follows from the fact that for *any* $(p_1, p_{-1}) \in \{g, t^1\} \times \{g, t^2\} \times \{g, t^3\}$, $d_{13} > 0$ implies $\hat\tau(p_1, p_{-1}) < \ell_H(p_2, p_3)$, ensuring that the event $\hat\tau(g, p_{-1}) \le \tilde\tau_2 - \tilde\tau_3 \le \ell_H$ arises with strictly positive probability. □

Proof of Proposition 1. We verify that $\pi_i(t^i, p_{-i}, \text{rcv}, x) - \pi_i(g, p_{-i}, \text{rcv}, x) < 0$ for each $i \in \{1, 2\}$ when $d_{13} = d_{23}$, and then invoke continuity of winning probabilities in $x = (x_1, x_2, x_3)$ to conclude that the strict inequality also holds whenever $|d_{13} - d_{23}|$ is strictly positive but small enough. Recognize when $d_{13} = d_{23}$, for $j \in \{1, 2\} \setminus \{i\}$ we have $\tau_j + d_{i3} - d_{j3} \ge \tau_3 - d_{i3}$ whenever $|\tau_j - \tau_3| \le d_{j3}$. This implies that $\tau_j + d_{i3} - d_{j3} + u > \tau_3 - d_{i3} - (1-u)$. This implies that (11) is strictly negative, and thus the sum of (10) and (11) is strictly negative. We conclude that under RCV candidate $i \in \{1, 2\}$ strictly prefers a broad campaign, and further recall that candidate 3's strictly dominant strategy is to campaign broadly under both plurality and RCV. □

Proof of Proposition 2. We prove the first part: the second follows the same reasoning. First, recognize that for any (u, F) if d_{23} is sufficiently close to zero, candidate 1's strict best response to $p_{-1} = (g, g)$ is a broad campaign under both plurality and RCV. The reason is that when $d_{23} = 0$ a three-way split occurs with probability zero at either profile (g, g, g) or (t^1, g, g), while candidate 1's probability of winning a majority is strictly larger at (g, g, g) than at (t^1, g, g). Continuity implies that $p_1 = g$ remains a strict best response to $(p_2, p_3) = (g, g)$ for $d_{23} > 0$ sufficiently small. We conclude that if d_{23} is sufficiently close to zero, (1) plurality's unique equilibrium is (g, g, g), and (2) candidate 1 strictly prefers $p_1 = g$ when $p_2 = p_3 = g$.

We therefore focus on candidate 2's incentives to target her base under RCV. Recognize that when $d_{23} = 0$:

$$
\begin{aligned}
&\pi_2(t^2, (g, g), \text{rcv}, x) - \pi_2(g, (g, g), \text{rcv}, x) \leq 0 \\
\iff &\iint\limits_{|\tilde{\tau}_1 - \tilde{\tau}_3| > d_{13}} \Big[F(\overline{\tau}_1(g, (g, g), \tilde{\tau}_{-1}, x)) - F(\overline{\tau}_1(g, (g, g), \tilde{\tau}_{-1}, x) + u)\Big] dF(\tilde{\tau}_1)\, dF(\tilde{\tau}_3) \\
&\quad + \iint\limits_{|\tilde{\tau}_1 - \tilde{\tau}_3| \leq d_{13}} [F(\tilde{\tau}_3) - F(\max\{\tilde{\tau}_3 - (1-u), \tilde{\tau}_1 - d_{13} + u\})]\, dF(\tilde{\tau}_1)\, dF(\tilde{\tau}_3) \\
&\quad \leq 0.
\end{aligned}
$$

This is equivalent to

$$
\begin{aligned}
&\iint\limits_{|\tilde{\tau}_1 - \tilde{\tau}_3| > d_{13}} \Big[F(\overline{\tau}_1(g, (g, g), \tilde{\tau}_{-1}, x)) - F(\overline{\tau}_1(g, (g, g), \tilde{\tau}_{-1}, x) + u)\Big] dF(\tilde{\tau}_1)\, dF(\tilde{\tau}_3) \\
&+ \iint\limits_{\tilde{\tau}_3 - d_{13} < \tilde{\tau}_1 < \tilde{\tau}_3 + d_{13} - 1} [F(\tilde{\tau}_3) - F(\tilde{\tau}_3 - (1-u))]\, dF(\tilde{\tau}_2)\, dF(\tilde{\tau}_3) \\
&+ \iint\limits_{\tilde{\tau}_3 + d_{13} - 1 < \tilde{\tau}_1 < \tilde{\tau}_3 + d_{13}} [F(\tilde{\tau}_3) - F(\tilde{\tau}_1 - d_{13} + u)]\, dF(\tilde{\tau}_2)\, dF(\tilde{\tau}_3) \\
&\leq 0.
\end{aligned}
$$

As $d_{13} \to \infty$, the first and third lines vanish, while the second is always strictly positive, violating the condition. We conclude that there exists $\overline{d} > \underline{d} > 0$ such that if $d_{13} > \overline{d} > \underline{d} > d_{23}$, RCV fails to support a broad equilibrium.

Proof of Proposition 3. Set $x_1 = x_2 = x_3$, implying $d_{ij} = 0$ for all $i, j \in \{1, 2, 3\}$. Candidate 3's strictly dominant strategy is $p_3 = g$. Conjecture a strategy profile (t^1, t^2, g), and recognize that under RCV candidate 1's incentives to deviate are the same as candidate 2's when $d_{12} = d_{23} = d_{13} = 0$. We therefore focus on candidate 2's incentive to deviate from $p_2 = t^2$ to $\tilde{p}_2 = g$. We have

$$\overline{\tau}_2(t^2, (t^1, g), \tau_{-2}, x) = \min\{\max\{\tau_1 + 1, \tau_3 + u\}, \max\{\tau_1, \tau_3 + u\}\} = \max\{\tau_1, \tau_3 + u\}.$$

At profile (t^1, t^2, g), group 1 prefers candidate 1 over candidate 3 if $\tau_1 \geq \tau_3 - (1-u)$, and group 3 prefers candidate 3 over candidate 1 if $\tau_1 \leq \tau_3 + u$. In that case, candidate 2 wins if and only if

$$\tau_2 \geq \max\{\underline{\tau}_2(t^2, (t^1, g), \tau_{-2}, x), \tau_1\} = \max\{\tau_3 - (1-u), \tau_1\} = \tau_1.$$

Putting all of this together, candidate 2 wins at profile (t^1, t^2, g) if and only if

$$\tau_2 \geq \begin{cases} \tau_3 + u & \text{if } \tau_1 < \tau_3 - (1-u) \\ \tau_1 & \text{otherwise.} \end{cases}$$

Suppose candidate 2 deviates to the broad campaign. We have:

$$\overline{\tau}_2(g, (t^1, g), \tau_{-2}, x) = \min\{\max\{\tau_1 + 1 - u, \tau_3\}, \max\{\tau_1 - u, \tau_3\}\} = \max\{\tau_1 - u, \tau_3\}.$$

At profile (t^1, g, g), group 1 prefers candidate 1 over candidate 3 if $\tau_1 \geq \tau_3 - (1-u)$, and group 3 prefers candidate 3 over candidate 1 if $\tau_1 \leq \tau_3 + u$. In that case, candidate 2 wins if and only if

$$\tau_2 \geq \max\{\underline{\tau}_2(g, (t^1, g), \tau_{-2}, x), \tau_1 - u\} = \max\{\tau_3, \tau_1 - u\} = \tau_3.$$

Putting all of this together, candidate 2 wins at profile (t^1, g, g) if and only if

$$\tau_2 \geq \begin{cases} \tau_3 & \text{if } \tau_1 < \tau_3 + u \\ \tau_1 - u & \text{if } \tau_1 > \tau_3 + u. \end{cases}$$

We now construct a distribution of valence for which candidate 2's probability of winning at profile (t^1, t^2, g) is strictly larger than her probability of winning at profile (t^1, g, g). Let

$$G(x) = \begin{cases} 0 & \text{if } x < -u \\ \frac{(x-u)^2}{2u^2} & \text{if } -u \leq x \leq 0 \\ 1 - \frac{(x-u)^2}{2u^2} & \text{if } 0 < x \leq u \\ 1 & \text{if } x > u, \end{cases}$$

and let $H(x)$ denote a standard normal distribution. We let $F(x) = wG(x) + (1-w)H(x)$, where $w \in (0,1)$ is a convex weight. Using the thresholds derived above, 2s probability of winning at profile (t^1, t^2, g) when $w = 1$ is

$$\int_{-u}^{1-2u} \int_{-u}^{u} (1 - F(\tilde{\tau}_1)) dF(\tilde{\tau}_1) dF(\tilde{\tau}_3) + \int_{1-2u}^{u} \int_{-u}^{\tilde{\tau}_3-(1-u)} (1 - F(\tilde{\tau}_3 + u)) dF(\tilde{\tau}_1) dF(\tilde{\tau}_3)$$
$$+ \int_{1-2u}^{u} \int_{\tilde{\tau}_3-(1-u)}^{u} (1 - F(\tilde{\tau}_1)) dF(\tilde{\tau}_1) dF(\tilde{\tau}_3). \quad (13)$$

while her probability of winning at profile (t^1, g, g) when $w = 1$ is

$$\int_{0}^{u} \int_{-u}^{u} (1 - F(\tilde{\tau}_3)) dF(\tilde{\tau}_1) dF(\tilde{\tau}_3) + \int_{-u}^{0} \int_{-u}^{\tilde{\tau}_3+u} (1 - F(\tilde{\tau}_3)) dF(\tilde{\tau}_1) dF(\tilde{\tau}_3)$$
$$+ \int_{-u}^{0} \int_{\tilde{\tau}_3+u}^{u} (1 - F(\tilde{\tau}_1 - u)) dF(\tilde{\tau}_1) dF(\tilde{\tau}_3). \quad (14)$$

Computation verifies that (13) strictly exceeds (14) whenever $u < u^* \approx .411007$. Since the winning probabilities are continuous in the mixture parameter w, we can fix any $\mu_1 < u < u^*$ and find an arbitrarily small $w(u, \mu_1) > 0$ that ensures that the difference of (13) and (14) remains strictly positive. □

Supplemental Appendix B: What if Candidates Can Target Any Group?

We generalize the benchmark model by assuming that each candidate i's action set is $\mathcal{P} \equiv \{g, t^1, t^2, t^3\}$. That is: *any* candidate can campaign broadly or target *any* of the three voter groups.

We establish robustness of the main insights from the benchmark model by way of three results. The first extends Corollary 1.

Proposition 1A. If $\max\{d_{13}, d_{12}, d_{23}\} < \min\{u, 1/2\}$, there exists $\delta(u, F) > 0$ such that if $|d_{13} - d_{23}| < \delta(u, F)$, plurality supports a broad equilibrium only if RCV supports a broad equilibrium.

Proposition 1A imposes an additional restriction on preferences beyond Corollary 1: it requires that no individual pairwise group mis-alignment d_{ij} be 'too large'.

To understand why the additional restriction is required, suppose that groups are very loyal to their candidates—in particular, suppose groups 1 and 2 are prone to prefer candidates 1 and 2 (respectively) *regardless* of their campaigning strategies. This means that in any three-way split candidate 3 will be eliminated and group 3 second preferences will be decisive. When $|d_{13} - d_{23}|$ is close to zero, group 3 second preferences are highly competitive. As a result, candidate 1 or 2 may prefer to deviate from a broad campaign and instead target group 3 voters. This incentive is obviously absent under plurality.

Our second result verifies that Corollary 2 extends without amendment.

Proposition 2A. There exist $\overline{d}(u, F) > \underline{d}(u, F) > 0$ such that if $d_{13} > \overline{d} > \underline{d} > d_{23}$, RCV supports a broad equilibrium only if plurality supports a broad equilibrium.

Third, we extend Observation 1.

Proposition 3A. For any (u, F), if $\max_{ij} d_{ij}$ is sufficiently small, (g, g, g) is the unique pure strategy equilibrium under plurality; it is also an equilibrium under RCV.

Proof of Proposition 1A.

We begin by defining notation used throughout the proofs of Propositions 1A and 2A. $\pi_j(p, z, x)$ denotes candidate $j \in \mathcal{C}$'s probability of winning under profile $p = (p_1, p_2, p_3)$, electoral rule $z \in \{\text{plu}, \text{rcv}\}$, and types $X = \{x_1, x_2, x_3\}$. $\mathcal{L}$ denotes the set of (weak) linear orders over $\mathcal{C}$. $\succeq_i^{p,\tau,x} \in \mathcal{L}$ denotes a group i voter's preferences over the candidates at an action profile $p = (p_1, p_2, p_3)$ and shock realization $\tau = (\tau_1, \tau_2, \tau_3)$. Finally, $O(p, x, \tau, z)$ identifies the winning candidate at action profile p, shock τ, and rule $z \in \{\text{plu}, \text{rcv}\}$. Recognize that under our (wlog) tie-breaking rules the distribution over winning candidates is degenerate at every (p, x, τ, z).

Let $\mathcal{Z}(p, x)$ denote the set of τ realizations that induce a three-way split:

$$\mathcal{Z}(p,x) = \{\tau : \text{for each } j \in \mathcal{C} \text{ there exists a unique } i \in \mathcal{G} \text{ such that } j \succeq_i^{p,\tau,z} k \text{ for all } k \in \mathcal{C}\backslash\{j\}\}.$$

Our first lemma verifies that at any profile p, election outcomes across RCV and plurality differs only for realizations of aggregate preference shocks at which all three candidates win first preferences. The reason is that if two or fewer candidates win first preferences, one candidate wins a majority of first preferences. In that event, she wins the election under either plurality or RCV.

Lemma 4. *At profile $p \in \{g, t^1, t^2, t^3\}$, $O(p, x, \tau, plu) \neq O(p, x, \tau, rcv)$ only if $\tau \in \mathcal{Z}(p, x)$.*

Proof of Lemma 4. Immediate from the preceding discussion.

Next, we show that at any action profile, all shock realizations that yield a three-way split must yield the same voting patterns.

Lemma 5. *Fix any action profile p. If $\tau \in \mathcal{Z}(p, x)$ and $\tau' \in \mathcal{Z}(p, x)$ then for all $i \in \mathcal{G}$ and $j, k \in \mathcal{C}$: $j \succeq_i^{p,\tau,x} k$ implies $j \succeq_i^{p,\tau',x} k$.*

Proof. Fix a vector of parameters and a platform profile p, and assume that at shock realization $\tau \in \mathcal{Z}(p, x)$ groups 1, 2, and 3 choosing, respectively, candidates i, j, and k. Suppose that at another shock realization $\tau' \in \mathcal{Z}(p, x)$, there is a *different* three-way split. Suppose, without loss of generality, that at shock τ' group 1 votes for candidate $j \neq i$. This implies

$$\tau_j' - \tau_i' > u_1(p_i) - u_1(p_j) - d_{1i} + d_{1j} > \tau_j - \tau_i. \tag{15}$$

Combining (15) with the presumption that group 2 votes for candidate j at shock realization τ yields

$$\tau_j' - \tau_i' > \tau_j - \tau_i > u_2(p_i) - u_2(p_j) - d_{2i} + d_{2j}.$$

We conclude that group 2 also prefers candidate j to candidate i at τ'. So, $\tau' \in \mathcal{Z}(p, x)$ only if group 2 also prefers candidate k to candidate j under τ'. That is:

$$\tau_k' - \tau_j' > u_2(p_j) - u_2(p_k) - d_{2j} + d_{2k} > \tau_k - \tau_j. \tag{16}$$

Combining (16) with the presumption that group 3 votes for j at shock realization τ implies that group 3 also prefers candidate k to j at τ'. So, $\tau' \in \mathcal{Z}(p, x)$ only if group 3 prefers candidate i to candidate k at τ'. That is:

$$\tau_i' - \tau_k' > u_2(p_j) - u_2(p_k) - d_{2j} + d_{2k} > \tau_i - \tau_k.$$

Putting everything together, we obtain:

$$\begin{aligned}
\tau_j' - \tau_i' > \tau_j - \tau_i &\iff \tau_j' - \tau_j > \tau_i' - \tau_i \\
\tau_k' - \tau_j' > \tau_k - \tau_j &\iff \tau_k' - \tau_k > \tau_j' - \tau_j \\
\tau_i' - \tau_k' > \tau_i - \tau_k &\iff \tau_i' - \tau_i > \tau_k' - \tau_k.
\end{aligned}$$

This yields a contradiction. □

We now directly prove Proposition 1A by analyzing each candidate's incentive to deviate from the profile (g, g, g), starting with candidate 1.

Candidate 1's incentives. Conjecture a broad strategy $p = (g, g, g)$. We first verify that candidate 1's incentive to deviate from $p_1 = g$ to $p_1' = t^1$ given $p_{-1} = (g, g)$ is strictly lower under

RCV than plurality. We have

$$\begin{aligned}\Delta_i(p_i, p_i', p_{-i}) &\equiv \pi_i(p_i', p_{-i}, \text{plu}, x) - \pi_i(p_i, p_{-i}, \text{plu}, x) - \big[\pi_i(p_i', p_{-i}, \text{rcv}, x) - \pi_i(p_i, p_{-i}, \text{rcv}, x)\big] \\ &= \pi_i(p_i', p_{-i}, \text{plu}, x) - \pi_i(p_i', p_{-i}, \text{rcv}, x) - \big[\pi_i(p_i, p_{-i}, \text{plu}, x) - \pi_i(p_i, p_{-i}, \text{rcv}, x)\big].\end{aligned}$$

Further:

$$\mathcal{T}_1(g, g, g, \tau_2, \tau_3, x) = \left\{\begin{array}{c} \max\{\tau_2 - d_{12}, \tau_3 - d_{13}\}, \\ \max\{\tau_2 + d_{12}, \tau_3 + d_{12} - d_{23}\}, \\ \max\{\tau_2 + d_{13} - d_{23}, \tau_3 + d_{13}\} \end{array}\right\} \tag{17}$$

and

$$\mathcal{T}_1(t^1, g, g, \tau_2, \tau_3, x) = \left\{\begin{array}{l} \max\{\tau_2 - d_{12}, \tau_3 - d_{13}\} - (1-u), \\ \max\{\tau_2 + d_{12}, \tau_3 + d_{12} - d_{23}\} + u, \\ \max\{\tau_2 + d_{13} - d_{23}, \tau_3 + d_{13}\} + u. \end{array}\right\} \tag{18}$$

This implies that $\max_{(2)} \mathcal{T}_1(t^1, g, g, y, z, x) = \max_{(2)} \mathcal{T}_1(g, g, g, y, z, x) + u$ and $\max_{(3)} \mathcal{T}_1(t^1, g, g, y, z, x) = \max_{(3)} \mathcal{T}_1(g, g, g, y, z, x) - (1-u)$. Thus, in the relevant corridor $|\tau_2 - \tau_3| \le d_{23}$:

$$\begin{aligned}&\pi_1(t^1, (g,g), \text{plu}, x) - \pi_1(t^1, (g,g), \text{rcv}, x) \\ &= \iint\limits_{|\tilde\tau_2 - \tilde\tau_3| \le d_{23}} \left[\begin{array}{c} F\big[\max\{\max_{(3)} \mathcal{T}_1(g,g,g,\tilde\tau_2,\tilde\tau_3,x) - (1-u), \tilde\tau_2 + d_{13} - d_{23} + u\}\big] \\ -F\big[\max_{(3)} \mathcal{T}_1(g,g,g,\tilde\tau_2,\tilde\tau_3,x) - (1-u)\big] \end{array}\right] dF(\tilde\tau_2) dF(\tilde\tau_3).\end{aligned}$$

and

$$\begin{aligned}&\pi_1(g, (g,g), \text{plu}, x) - \pi_1(g, (g,g), \text{rcv}, x) \\ &= \iint\limits_{|\tilde\tau_2 - \tilde\tau_3| \le d_{23}} \left[\begin{array}{c} F\big[\max\big\{\max_{(3)} \mathcal{T}_1(g,g,g,\tilde\tau_2,\tilde\tau_3,x), \tilde\tau_2 + d_{13} - d_{23}\big\}\big] \\ -F\big[\max_{(3)} \mathcal{T}_1(g,g,g,\tilde\tau_2,\tilde\tau_3,x)\big] \end{array}\right] dF(\tilde\tau_2) dF(\tilde\tau_3).\end{aligned}$$

So:

$$\Delta_1(g, t^1, (g,g), x)$$

$$
\begin{aligned}
&= \iint\limits_{|\tilde{\tau}_2-\tilde{\tau}_3|\le d_{23}} \left[\begin{array}{c} F\big[\max\{\max_{(1)} \mathcal{T}_1(g,g,g,\tilde{\tau}_2,\tilde{\tau}_3,x) - (1-u), \tilde{\tau}_2 + d_{13} - d_{23} + u\}\big] \\ -F\big[\max_{(3)} \mathcal{T}_1(g,g,g,\tilde{\tau}_2,\tilde{\tau}_3,x) - (1-u)\big] \end{array} \right] dF(\tilde{\tau}_2)dF(\tilde{\tau}_3) \\
&\quad - \iint\limits_{|\tilde{\tau}_2-\tilde{\tau}_3|\le d_{23}} \left[\begin{array}{c} F\big[\max\{\max_{(3)} \mathcal{T}_1(g,g,g,\tilde{\tau}_2,\tilde{\tau}_3,x), \tilde{\tau}_2 + d_{13} - d_{23}\}\big] \\ -F\big[\max_{(3)} \mathcal{T}_1(g,g,g,\tilde{\tau}_2,\tilde{\tau}_3,x)\big] \end{array} \right] dF(\tilde{\tau}_2)dF(\tilde{\tau}_3) \\
&= \iint\limits_{|\tilde{\tau}_2-\tilde{\tau}_3|\le d_{23}} \left[\begin{array}{c} F\big[\max_{(3)} \mathcal{T}_1(g,g,g,\tilde{\tau}_2,\tilde{\tau}_3,x)\big] \\ -F\big[\max_{(3)} \mathcal{T}_1(g,g,g,\tilde{\tau}_2,\tilde{\tau}_3,x) - (1-u)\big] \end{array} \right] dF(\tilde{\tau}_2)dF(\tilde{\tau}_3) \\
&\quad - \iint\limits_{|\tilde{\tau}_2-\tilde{\tau}_3|\le d_{23}} \left[\begin{array}{c} F\big[\max\{\max_{(3)} \mathcal{T}_1(g,g,g,\tilde{\tau}_2,\tilde{\tau}_3,x), \tilde{\tau}_2 + d_{13} - d_{23}\}\big] \\ -F\big[\max\{\max_{(3)} \mathcal{T}_1(g,g,g,\tilde{\tau}_2,\tilde{\tau}_3,x) - (1-u), \tilde{\tau}_2 + d_{13} - d_{23} + u\}\big] \end{array} \right] dF(\tilde{\tau}_2)dF(\tilde{\tau}_3) \\
&\quad > 0,
\end{aligned}
$$

where the last inequality is strict whenever $\max_{(3)} \mathcal{T}_1(g,g,g,\tau_2,\tau_3,x) - 1 < \tau_2 + d_{13} - d_{23}$. □

We conclude that if candidate 1's best response to $p_{-1} = (g,g)$ is a broad campaign under plurality, then that is also her best response to that action profile under RCV.

We next consider candidate 1's incentives to target remaining groups 2 and 3 under RCV.

Lemma 6. *If $\max\{d_{13}, d_{12}, d_{23}\} < \min\{u, \frac{1}{2}\}$, there exists $\delta(u,F) > 0$ such that if $|d_{13} - d_{23}| < \delta(u,F)$, for either group $j \in \{2,3\}$:*

$$\pi_1(g,(g,g),plu,x) \ge \pi_1(t^j,(g,g),plu,x) \Rightarrow \pi_1(g,(g,g),rcv,x) \ge \pi_1(t^j,(g,g),rcv,x).$$

Proof. We prove a stronger version of the Lemma: under its assumptions we must have $\pi_1(g,(g,g),\text{rcv},x) > \pi_1(t^j,(g,g),\text{rcv},x)$. We prove the claim for candidate $j = 2$, since the argument for candidate 3 is similar. We have:

$$\mathcal{T}_1(t^2,g,g,\tau_2,\tau_3,x) = \left\{ \begin{array}{c} \max\{\tau_2 - d_{12}, \tau_3 - d_{13}\} + u, \\ \max\{\tau_2 + d_{12}, \tau_3 + d_{12} - d_{23}\} - (1-u), \\ \max\{\tau_2 + d_{13} - d_{23}, \tau_3 + d_{13}\} + u \end{array} \right\} \tag{19}$$

We first claim that for all $(\tau_2,\tau_3) \in \mathbb{R}^2$: $\max\{d_{12}, d_{13}\} < \frac{1}{2}$ implies $\max_{(3)} \mathcal{T}_1(t^2,g,g,\tau_2,\tau_3,x) = \max\{\tau_2 + d_{12}, \tau_3 + d_{12} - d_{23}\} - (1-u)$. In words: candidate 1 wins the support of group 1 (or

3) only if she wins the support of group 2 at profile (t^2, g, g), for any shock realization. To see this, notice that when $\max\{d_{12}, d_{13}\} < \frac{1}{2}$ we have

$$\max\{\tau_2 + d_{12}, \tau_3 + d_{12} - d_{23}\} - (1-u) \leq \max\{\tau_2, \tau_3 - d_{23}\} - \frac{1}{2} + u \leq \max\{\tau_2, \tau_3\} - \frac{1}{2} + u$$

and

$$\max\{\tau_2, \tau_3\} - \frac{1}{2} + u < \min\{\max\{\tau_2 - d_{12}, \tau_3 - d_{13}\} + u, \max\{\tau_2 + d_{13} - d_{23}, \tau_3 + d_{13}\} + u\}$$

We conclude by arguing that for any $\max\{d_{12}, d_{13}, d_{23}\} < u$ there exists $\delta(u, F) > 0$ such that if in addition $|d_{13} - d_{23}| < \delta(u, F)$, then $\pi_1(g, (g, g), \text{rcv}) > \pi_1(t^2, (g, g), \text{rcv})$. To verify the claim, we set $d_{13} = d_{23}$ and observe that probabilities of winning under either electoral rule are continuous in $x = (x_1, x_2, x_3)$, preserving the strict difference of winning probabilities when $|d_{13} - d_{23}|$ is strictly positive but sufficiently small. When $d_{13} = d_{23} \equiv d$, candidate 1 wins at profile (g, g, g) under RCV if

$$\tau_1 \geq \begin{cases} \tau_2 & \text{if } \tau_2 \geq \tau_3 - d \\ \tau_3 & \text{otherwise} \end{cases}$$

whereas at profile (t^2, g, g) 1 wins only if $\tau_1 \geq \max\{\tau_2, \tau_3\} + u - d$. So long as $u > d$, the latter threshold always strictly exceeds the former.

A similar argument holds for the case of $j = 3$ and we sketch the steps; recognize that at a profile (t^3, g, g), $\max\{d_{13}, d_{12}, d_{23}\} < \frac{1}{2}$ implies that in any three-way split candidate 1 wins first preferences from group-3 voters and she therefore loses with probability one. It is straightforward to verify that if $\max\{d_{13}, d_{12}, d_{23}\} < u$, candidate 1's probability of winning a majority is strictly greater at profile (g, g, g) than at profile (t^3, g, g). $\square$

Candidate 2's incentives. Recognize that for any x such that $d_{13} = d_{23}$, $\pi_1(g, (g, g), \text{rcv}, x) = \pi_2(g, (g, g), \text{rcv}, x)$ and $\pi_1(t^1, (g, g), \text{rcv}, x) = \pi_2(t^2, (g, g), \text{rcv}, x)$, and finally $\pi_1(t^2, (g, g), \text{rcv}, x) = \pi_2(t^1, (g, g), \text{rcv}, x)$. It follows that there exists $\delta(u, F)$ such that if $|d_{13} - d_{23}| < \delta(u, F)$, if $p_1 = g$ is candidate 1's strict best response to $(p_2, p_3) = (g, g)$ under RCV, then $p_2 = g$ is also candidate 2's strict best response to $(p_1, p_3) = (g, g)$ under RCV.

Candidate 3's incentives. Suppose $p_1 = p_2 = g$, and consider candidate 3's benefit from a deviation from g to another policy in $\{t^1, t^2, t^3\}$. Notice that because group 3 is the smallest group, $\pi_3(g, (g, g), \text{rcv}, x) = \pi_3(g, (g, g), \text{plu}, x)$, and $\pi_3(t^3, (g, g), \text{rcv}, x) = \pi_3(t^3, (g, g), \text{plu}, x)$. We can therefore restrict attention to deviations by candidate 3 to either t^2 or t^1, and further recognize that when $d_{13} = d_{23}$, $\pi_3(t^1, g, g, \text{rcv}, x) = \pi_3(t^2, g, g, \text{rcv}, x)$. It is therefore sufficient to verify that if $d_{13} = d_{23}$ and $\max\{d_{12}, d_{13}, d_{23}\} < \frac{1}{2}$, whenever (g, g, g) is an equilibrium under plurality candidate 3 strictly prefers to choose g instead of t^1 under RCV when the remaining candidates choose $p_1 = p_2 = g$. To verify this, recognize that if $d_{13} = d_{23}$ and $\max\{d_{12}, d_{13}, d_{23}\} < \frac{1}{2}$:

$$\begin{aligned}
&\pi_3(t^1,(g, g), \text{plu}, x) - \pi_3(t^1, (g, g), \text{rcv}, x) \\
&\quad = \iint\limits_{\tilde{\tau}_2 \leq \tilde{\tau}_1 \leq \tilde{\tau}_2 + d_{12}} \left[\begin{array}{c} F\big[\max\{\tilde{\tau}_1 + d_{13} - (1-u), \tilde{\tau}_2 + u - d_{23}\}\big] \\ -F\big[\max\{\tilde{\tau}_1 + d_{13} - (1-u), \tilde{\tau}_2 + d_{13} - d_{12} - (1-u)\}\big] \end{array} \right] dF(\tilde{\tau}_1) dF(\tilde{\tau}_2).
\end{aligned}$$

This expression is strictly positive since $\tau_2 + u - d_{23} > \tau_2 + d_{13} - d_{12} - (1-u)$ under the supposition that $d_{13} = d_{23} < \frac{1}{2}$. Recognizing that $\pi_3(g, (g, g), \text{plu}, x) - \pi_3(g, (g, g), \text{rcv}, x) = 0$ implies that:

$$\Delta_1(g, t^1, (g, g)) = \pi_3(t^1, (g, g), \text{plu}, x) - \pi_3(t^1, (g, g), \text{rcv}, x) > 0.$$

We conclude that if $p_3 = g$ is a best response to $(p_1, p_2) = (g, g)$ under plurality, then $p_3 = g$ is a strictly best response to $(p_1, p_2) = (g, g)$ under RCV when $d_{13} = d_{23}$ and $\max\{d_{12}, d_{13}, d_{23}\} \leq \frac{1}{2}$. We then apply standard continuity arguments to conclude that whenever $\max\{d_{12}, d_{13}, d_{23}\} < \frac{1}{2}$, there exists threshold $\delta(u, F) > 0$ such that if $|d_{13} - d_{23}| < \delta(u, F)$, candidate 3's best response under RCV to $(p_1, p_2) = (g, g)$ is $p_3 = g$ whenever that is also her best response to $(p_1, p_2) = (g, g)$ under plurality. □

Proof of Proposition 2A.

We consider each candidate's incentives under plurality, assuming that primitives support (g, g, g) as an equilibrium under RCV. We further impose $d_{13} - d_{23} > \frac{1}{2}$ and $d_{23} < \frac{1-u}{2}$.

Candidate 1's incentives. Consider first a deviation by candidate 1 to target group 2's base with policy t^2. We have

$$\mathcal{T}_1(g,g,g,\tau_2,\tau_3,x) = \left\{ \begin{array}{c} \max\{\tau_2 - d_{12}, \tau_3 - d_{13}\}, \\ \max\{\tau_2 + d_{12}, \tau_3 + d_{12} - d_{23}\}, \\ \max\{\tau_2 + d_{13} - d_{23}, \tau_3 + d_{13}\} \end{array} \right\}. \tag{20}$$

and

$$\mathcal{T}_1(t^2,g,g,\tau_2,\tau_3,x) = \left\{ \begin{array}{c} \max\{\tau_2 - d_{12}, \tau_3 - d_{13}\} + u, \\ \max\{\tau_2 + d_{12}, \tau_3 + d_{12} - d_{23}\} - (1-u), \\ \max\{\tau_2 + d_{13} - d_{23}, \tau_3 + d_{13}\} + u \end{array} \right\}. \tag{21}$$

If $d_{13} - d_{23} > \frac{1}{2}$, then the triangle inequality further implies $d_{12} > \frac{1}{2}$, from which it follows that $\max_{(3)} \mathcal{T}_1(t^2,g,g,\tau_2,\tau_3,x) = \max\{\tau_2 - d_{12}, \tau_3 - d_{13}\} + u$.

Recall that we are presuming (g,g,g) is an equilibrium under RCV. Conjecture that candidate 1 strictly prefers the action t^2 to g under plurality when $(p_2,p_3) = (g,g)$. This implies

$$\begin{aligned} &\iint\limits_{|\tilde{\tau}_2-\tilde{\tau}_3|>d_{23}} \Big[F(\max_{(2)} \mathcal{T}_1(g,g,g,\tilde{\tau}_2,\tilde{\tau}_3,x)) - F(\max_{(2)} \mathcal{T}(t^2,g,g,\tilde{\tau}_2,\tilde{\tau}_3,x))\Big] dF(\tilde{\tau}_2)dF(\tilde{\tau}_3) \\ &> \iint\limits_{|\tilde{\tau}_2-\tilde{\tau}_3|\le d_{23}} \Big[F(\max_{(3)} \mathcal{T}_1(g,g,g,\tilde{\tau}_2,\tilde{\tau}_3,x) + u) - F(\max_{(3)} \mathcal{T}(g,g,g,\tilde{\tau}_2,\tilde{\tau}_3,x))\Big] dF(\tilde{\tau}_2)dF(\tilde{\tau}_3). \end{aligned} \tag{22}$$

To understand why, recognize that the first line represents the change in 1's probability of winning a majority with policy t^2 instead of g, for shock realizations at which candidate 1 wins if and only if she wins a majority. The second line represents the change in 1's probability of retaining the support of group 1 when group 2 supports candidate 2 and group 3 supports candidate 3.

By presumption, the deviation to t^2 is unprofitable under RCV. We argue that $d_{13} - d_{23} > \frac{1}{2}$ implies that candidate 1 wins with probability zero under RCV for any $\tau \in \mathcal{Z}(t^2,g,g,x)$. To verify this claim, recall that since $d_{12} \ge d_{13} - d_{23} > 1/2$ group-1 voters support candidate 1 at any shock realization $\tau \in \mathcal{Z}(t^2, g, g, x)$. Then,

$$\begin{aligned}\tau \in \mathcal{Z}(t^2, g, g, x) \iff & \tau_1 \geq \max\{\tau_2 - d_{12} + u, \tau_3 - d_{13} + u\} \\ & \tau_2 \geq \max\{\tau_1 - d_{12} + 1 - u, \tau_3 - d_{23}\} \\ & \tau_3 \geq \max\{\tau_1 - d_{13} - u, \tau_2 - d_{23}\}.\end{aligned}$$

Further, candidate 1 wins at $\tau \in \mathcal{Z}(t^2, g, g, x)$ under RCV if and only if she secures second preferences from voters in group 3. This requires $\tau_1 \geq \tau_2 + d_{13} - d_{23} + u$. Putting all of this together, candidate 1 wins at $\tau \in \mathcal{Z}(t^2, g, g, x)$ under RCV if and only if

$$\max\{\tau_2 + d_{13} - d_{23}, \tau_3 - d_{13}\} + u \leq \tau_1 \leq \min\{\tau_2 - (1-u) + d_{12}, \tau_3 + d_{13} + u\}.$$

This is possible only if $d_{13} - d_{12} \leq d_{23} - 1$. Since $d_{13} - d_{12} \geq -d_{23}$, this requires $d_{23} - 1 \geq -d_{23}$, or $d_{23} \geq \frac{1}{2}$, which contradicts our restriction that $d_{23} < \frac{1-u}{2}$. We conclude that if $d_{13} - d_{23} > \frac{1}{2}$ and $\frac{1-u}{2} > d_{23}$, candidate 1 loses with probability one in any three-way split under RCV. Thus, the deviation from $p_1 = g$ to $p_1 = t^2$ is *not* profitable under RCV if and only if

$$\begin{aligned}&\iint\limits_{|\tilde{\tau}_2 - \tilde{\tau}_3| > d_{23}} \left[F(\max_{(2)} \mathcal{T}_1(g, g, g, \tilde{\tau}_2, \tilde{\tau}_3, x)) - F(\max_{(2)} \mathcal{T}(t^2, g, g, \tilde{\tau}_2, \tilde{\tau}_3, x))\right] dF(\tilde{\tau}_2) dF(\tilde{\tau}_3) \\ &\leq \iint\limits_{|\tilde{\tau}_2 - \tilde{\tau}_3| \leq d_{23}} \left[F(\max_{(2)} \mathcal{T}_1(t^2, g, g, \tilde{\tau}_2, \tilde{\tau}_3, x)) - F(\tilde{\tau}_2 + d_{13} - d_{23})\right] dF(\tilde{\tau}_2) dF(\tilde{\tau}_3). \qquad (23)\end{aligned}$$

Inequalities (22) and (23) imply:

$$\begin{aligned}&\iint\limits_{|\tilde{\tau}_2 - \tilde{\tau}_3| \leq d_{23}} \left[F(\max_{(3)} \mathcal{T}_1(g, g, g, \tilde{\tau}_2, \tilde{\tau}_3, x) + u) - F(\max_{(3)} \mathcal{T}(g, g, g, \tilde{\tau}_2, \tilde{\tau}_3, x))\right] dF(\tilde{\tau}_2) dF(\tilde{\tau}_3) \\ &< \iint\limits_{|\tilde{\tau}_2 - \tilde{\tau}_3| \leq d_{23}} \left[F(\max_{(2)} \mathcal{T}_1(t^2, g, g, \tilde{\tau}_2, \tilde{\tau}_3, x)) - F(\tilde{\tau}_2 + d_{13} - d_{23})\right] dF(\tilde{\tau}_2) dF(\tilde{\tau}_3). \qquad (24)\end{aligned}$$

Notice that when $d_{13} - d_{23} > \frac{1}{2} > d_{23}$:

$$\tau_2 - \tau_3 \in (-d_{23}, d_{23}) \Rightarrow \max_{(2)} \mathcal{T}_1(t^2, g, g, \tau_2, \tau_3, x) = \min\{\tau_2 + d_{12} - (1-u), \tau_3 + d_{13} + u\}.$$

Further, $\tau_2 + d_{12} - (1-u) < \tau_3 + d_{13} + u$ if and only if $\tau_2 < \tau_3 + d_{13} - d_{12} + 1$. Since $d_{23} < \frac{1}{2}$, a sufficient condition for $\tau_2 < \tau_3 + d_{13} - d_{12} + 1$ is that $\tau_2 < \tau_3 + d_{13} - d_{12} + 2d_{23}$. Further, the triangle inequality implies $|d_{13} - d_{12}| \leq d_{23}$, and thus $\tau_2 < \tau_3 + d_{13} - d_{12} + 2d_{23}$ is therefore satisfied for all $\tau_2 < \tau_3 + d_{23}$. We can therefore reformulate (24):

$$\iint\limits_{|\tilde{\tau}_2 - \tilde{\tau}_3| \leq d_{23}} \Big[F(\max_{(3)} \mathcal{T}_1(g,g,g,\tilde{\tau}_2,\tilde{\tau}_3,x) + u) - F(\max_{(3)} \mathcal{T}(g,g,g,\tilde{\tau}_2,\tilde{\tau}_3,x)) \Big] dF(\tilde{\tau}_2) dF(\tilde{\tau}_3)$$
$$< \iint\limits_{|\tilde{\tau}_2 - \tilde{\tau}_3| \leq d_{23}} \Big[F(\tilde{\tau}_2 + d_{12} - (1-u)) - F(\tilde{\tau}_2 + d_{13} - d_{23}) \Big] dF(\tilde{\tau}_2) dF(\tilde{\tau}_3). \tag{25}$$

The LHS of inequality (25) is strictly positive, while the RHS of the inequality is strictly positive only if $d_{23} > 1 - u + d_{13} - d_{12}$. But since $d_{23} < \frac{1-u}{2}$, we have $1 - u + d_{13} - d_{12} > 1 - u - d_{23} > d_{23}$, thereby contradicting (25).

We next consider a deviation by candidate 1 to t_3. Straightforward calculation yields that whenever $d_{13} - d_{23} > 1$:

$$\begin{aligned} &\Delta(g, t^3, (g,g)) \\ &= \iint\limits_{|\tilde{\tau}_2 - \tilde{\tau}_3| \leq d_{23}} \Big[F(\max\nolimits_{(3)} \mathcal{T}_1(g,g,g,\tilde{\tau}_2,\tilde{\tau}_3,x)) - F(\max\nolimits_{(3)} \mathcal{T}_1(g,g,g,\tilde{\tau}_2,\tilde{\tau}_3,x) + u) \Big] dF(\tilde{\tau}_2) dF(\tilde{\tau}_3) \\ &\quad - \iint\limits_{|\tilde{\tau}_2 - \tilde{\tau}_3| \leq d_{23}} \Big[F(\tilde{\tau}_2 + d_{13} - d_{23}) - F(\tilde{\tau}_2 + d_{13} - d_{23} - (1-u)) \Big] dF(\tilde{\tau}_2) dF(\tilde{\tau}_3) \\ &\quad < 0. \end{aligned}$$

Finally, a deviation by candidate 1 to t_1 is strictly unprofitable under plurality if $d_{23} = 0$ for any (u, F). Continuity implies that for any (u, F) there exists $\underline{d} > 0$ such that the deviation remains unprofitable so long as $d_{23} < \underline{d}$.

Candidate 2's and 3's incentives. We verify the claim for candidate 2, since the arguments for candidate 3 are similar. Consider a deviation by candidate 2 to targeting group 1 with campaign t^1. Recognize that

$$\mathcal{T}_2(g,g,g,\tau_1,\tau_2,x) = \left\{ \begin{array}{c} \max\{\tau_1 + d_{12}, \tau_3 + d_{12} - d_{13}\} \\ \max\{\tau_1 - d_{12}, \tau_3 - d_{23}\} \\ \max\{\tau_1 + d_{23} - d_{13}, \tau_3 + d_{23}\} \end{array} \right\}$$

and

$$\mathcal{T}_2(g,t^1,g,\tau_1,\tau_2,x) = \left\{ \begin{array}{c} \max\{\tau_1 + d_{12}, \tau_3 + d_{12} - d_{13}\} - (1-u) \\ \max\{\tau_1 - d_{12}, \tau_3 - d_{23}\} + u \\ \max\{\tau_1 + d_{23} - d_{13}, \tau_3 + d_{23}\} + u \end{array} . \right\}$$

If $d_{13} - d_{23} > \frac{1}{2}$, the triangle inequality implies $d_{12} > \frac{1}{2}$, and standard arguments yield that in any three-way split under plurality candidate 2 wins the support of group-2 voters at either profile—and therefore loses the election—in any three-way split. This implies that at either profile candidate 2 wins under plurality if and only if she wins a majority of votes.

Suppose, next, $d_{23} < \frac{u}{2}$. Then, we claim candidate 2's probability of winning a majority at profile (g,t^2,g) is strictly lower than her probability of winning a majority at profile (g,g,g). A sufficient condition for this claim to be true is that:

$$\min \left\{ \begin{array}{c} \max\{\tau_1 - d_{12}, \tau_3 - d_{23}\} + u \\ \max\{\tau_1 + d_{23} - d_{13}, \tau_3 + d_{23}\} + u \end{array} \right\} > \max \left\{ \begin{array}{c} \max\{\tau_1 - d_{12}, \tau_3 - d_{23}\} \\ \max\{\tau_1 + d_{23} - d_{13}, \tau_3 + d_{23}\} \end{array} \right\}. \quad (26)$$

Since $d_{23} - d_{13} \geq -d_{12}$, a sufficient condition for (26) is $\max\{\tau_1 - d_{12}, \tau_3 - d_{23}\} + u > \max\{\tau_1 + d_{23} - d_{13}, \tau_3 + d_{23}\}$, which is true if $d_{23} < \frac{u}{2}$. We conclude that if $d_{13} - d_{23} > \frac{1}{2}$ and $d_{23} < \frac{u}{2}$, candidate 2's deviation to t^1 is not profitable. A similar argument rules out a profitable deviation to t^3.

Finally, recognize that a deviation by candidate 2 to t^2 when $(p_1, p_3) = (g, g)$ is always unprofitable under plurality: it strictly decreases her probability of winning a majority and she loses in any three-way split both before and after the deviation. □

Proof of Proposition 3A.

We begin by narrowing down the set of action profiles that could be supported as pure strategy equilibria under either plurality or RCV when $\max_{ij} d_{ij}$ is less than $u/2$; we call this condition **A1**.

A1. $u > 2\max\{d_{12}, d_{23}, d_{13}\}$.

Lemma 7. *If **A1** holds, there is no pure strategy equilibrium under either plurality or RCV in which exactly one candidate targets group* 3*.*

For any group $I \in \{1, 2, 3\}$ and candidates $i, j \in \{1, 2, 3\}$ define $\Lambda_I^{ij} \equiv d_{Ii} - d_{Ij}$.

Proof. Suppose, to the contrary, a candidate $i \in \{1, 2, 3\}$ targets group 3 with policy $p_i = t^3$, but for the remaining candidates $j, k \in \mathcal{C} \setminus i$ we have $p_j \neq t^3$ and $p_k \neq t^3$. We first claim that at this profile, for any shock realization τ, candidate i is most-preferred by a voter in group $I \in \{1, 2\}$ only if she is most-preferred by voters in group 3. That is, we claim:

$$\max\{\tau_j + u_3(p_j) + \Lambda_3^{ij}, \tau_k + u_3(p_k) + \Lambda_3^{ik}\} - 1$$
$$< \min \left\{ \begin{array}{l} \max\{\tau_j + u_1(p_j) + \Lambda_1^{ij}, \tau_k + u_1(p_k) + \Lambda_1^{ik}\} \\ \max\{\tau_j + u_2(p_j) + \Lambda_2^{ij}, \tau_k + u_2(p_k) + \Lambda_2^{ik}\} \end{array} \right\},$$

To see that this claim is correct, recognize that it holds if the following inequalities are satisfied:

$$u_3(p_j) + \Lambda_3^{ij} - 1 < \min_{I \in \{1,2\}} \{u_I(p_j) + \Lambda_I^{ij}\} \tag{27}$$

$$u_3(p_k) + \Lambda_3^{ik} - 1 < \min_{I \in \{1,2\}} \{u_I(p_k) + \Lambda_I^{ik}\}. \tag{28}$$

Under the supposition that $p_j \neq t^3$ and $p_k \neq t^3$, we have $u_3(p_j) \in \{0, u\}$. If $u_3(p_j) = u$, then $p_j = g$, implying that $u_I(p_j) = u$ for each $I \in \{1, 2\}$. Then, (27) holds so long as $u + \Lambda_3^{ij} - 1 < u + \min_{I \in \{1,2\}} \Lambda_I^{ij}$, i.e.,

$$1 > \Lambda_3^{ij} - \min_{I \in \{1,2\}} \Lambda_I^{ij} = d_{i3} - d_{j3} - \min_{I \in \{1,2\}} \left[d_{iI} - d_{jI}\right].$$

which is true under **A1**. If, instead, $u_3(p_j) = 0$ then the RHS of (27) is nonetheless weakly greater than $0 + \min_{I\in\{1,2\}} \Lambda_I^{ij} \geq u/2$, while the LHS is $\Lambda_3^{ij} - 1 \leq u/2 - 1$. **A1** again implies (27). A similar argument implies that whenever **A1** holds, so does (28).

We conclude that at this action profile, candidate i wins the election under either plurality or RCV if and only if she wins a majority of first preferences. This implies that candidate i has a profitable deviation under either electoral rule if there exists some other action $\tilde{p}_i$ that strictly increases her probability of winning a majority of first preferences. Recognize that a deviation to $\tilde{p}_i = g$ to strictly increase i's probability of winning a majority of first preferences if:

$$\max_{(2)} \left\{ \begin{array}{c} \max\{\tau_j + u_3(p_j) + \Lambda_3^{ij}, \tau_k + u_3(p_k) + \Lambda_3^{ik}\} - u \\ \max\{\tau_j + u_1(p_j) + \Lambda_1^{ij}, \tau_k + u_1(p_k) + \Lambda_1^{ik}\} - u \\ \max\{\tau_j + u_2(p_j) + \Lambda_2^{ij}, \tau_k + u_2(p_k) + \Lambda_2^{ik}\} - u \end{array} \right\}$$
$$< \min \left\{ \begin{array}{c} \max\{\tau_j + u_1(p_j) + \Lambda_1^{ij}, \tau_k + u_1(p_k) + \Lambda_1^{ik}\} \\ \max\{\tau_j + u_2(p_j) + \Lambda_2^{ij}, \tau_k + u_2(p_k) + \Lambda_2^{ik}\} \end{array} \right\}.$$

This is satisfied whenver

$$\begin{aligned} &\max\{\tau_j + u_3(p_j) + \Lambda_3^{ij}, \tau_k + u_3(p_k) + \Lambda_3^{ik}\} - u \\ &< \min \left\{ \begin{array}{c} \max\{\tau_j + u_1(p_j) + \Lambda_1^{ij}, \tau_k + u_1(p_k) + \Lambda_1^{ik}\} \\ \max\{\tau_j + u_2(p_j) + \Lambda_2^{ij}, \tau_k + u_2(p_k) + \Lambda_2^{ik}\} \end{array} \right\}. \end{aligned} \tag{29}$$

The following inequalities are sufficient for (29):

$$u_3(p_j) + \Lambda_3^{ij} - u < \min_{I\in\{1,2\}} \{u_I(p_j) + \Lambda_I^{ij}\} \tag{30}$$
$$u_3(p_k) + \Lambda_3^{ik} - u < \min_{I\in\{1,2\}} \{u_I(p_k) + \Lambda_I^{ik}\}. \tag{31}$$

Under the supposition that $p_j \neq t^3$ and $p_k \neq t^3$, we have $u_3(p_j) \in \{0, u\}$. If $u_3(p_j) = u$, then $p_j = g$, implying that $u_I(p_j) = u$ for each $I \in \{1, 2\}$. Then, (30) holds so long as $\Lambda_3^{ij} < u + \min_{I\in\{1,2\}} \Lambda_I^{ij}$, which is true under **A1**. If $u_3(p_j) = 0$, then the RHS is weakly greater than $0 + \min_{I\in\{1,2\}} \Lambda_I^{ij}$, so that **A1** again yields that (30) holds. The same argument implies that (31) also holds. We conclude that the deviation by candidate i from $p_i = t^3$ to $\tilde{p}_i = g$ strictly increases candidate i's probability of winning, and thus she has a profitable deviation. $\square$

Lemma 8. *Under **A1**, there is no pure strategy equilibrium under plurality in which exactly one candidate targets group* 2.

Proof. Similar to the previous lemma. $\square$

Lemma 9. *Under **A1** and plurality: in a pure strategy equilibrium in which any group $I \in \{2,3\}$ is targeted by at least one candidate, then group 1 is targeted by at least one candidate.*

Proof. The previous two lemmas imply that in a pure strategy equilibrium under plurality in which a group $I \in \{2,3\}$ is targeted by at least one candidate, then that group is targeted by at least *two* candidates. The remainder of the argument proceeds by cases.

(1) Suppose group $I \in \{2,3\}$ is targeted by all three candidates. Let $J = \{2,3\} \backslash I$. Consider a candidate $i \in \{1,2,3\}$ for whom $d_{1i} \geq \max_{j \in \mathcal{C} \backslash i} d_{1j}$. This candidate does not win first preferences from voters in group 1 in any three-way split,[19] and she therefore wins the election if and only if she wins a majority of first preferences. That is: candidate i wins the election under plurality if and only if

$$\tau_i \geq \max_{(2)} \left\{ \begin{array}{c} \max\{\tau_j + \Lambda_1^{ij}, \tau_k + \Lambda_1^{ik}\} \\ \max\{\tau_j + \Lambda_I^{ij}, \tau_k + \Lambda_I^{ik}\} \\ \max\{\tau_j + \Lambda_J^{ij}, \tau_k + \Lambda_J^{ik}\} \end{array} \right\}$$

A deviation by this candidate i to $\tilde{p}_i = g$ is profitable if the deviation strictly increases her probability of winning a majority of first preferences. This condition holds if

$$\max_{(2)} \left\{ \begin{array}{c} \max\{\tau_j + \Lambda_1^{ij}, \tau_k + \Lambda_1^{ik}\} - u \\ \max\{\tau_j + \Lambda_I^{ij}, \tau_k + \Lambda_I^{ik}\} + 1 - u \\ \max\{\tau_j + \Lambda_J^{ij}, \tau_k + \Lambda_J^{ik}\} - u \end{array} \right\} < \max_{(2)} \left\{ \begin{array}{c} \max\{\tau_j + \Lambda_1^{ij}, \tau_k + \Lambda_1^{ik}\} \\ \max\{\tau_j + \Lambda_I^{ij}, \tau_k + \Lambda_I^{ik}\} \\ \max\{\tau_j + \Lambda_J^{ij}, \tau_k + \Lambda_J^{ik}\} \end{array} \right\}.$$

[19] If $0 = d_{1i} \geq \max_{j \in \mathcal{C} \backslash i} d_{1j}$, a three-way split occurs with probability zero when all three candidates choose the same action.

A sufficient condition for this inequality to hold is that

$$\max\left\{\begin{array}{c}\max\{\tau_j+\Lambda_1^{ij},\tau_k+\Lambda_1^{ik}\}\\ \max\{\tau_j+\Lambda_J^{ij},\tau_k+\Lambda_J^{ik}\}\end{array}\right\}-u<\min\left\{\begin{array}{c}\max\{\tau_j+\Lambda_1^{ij},\tau_k+\Lambda_1^{ik}\}\\ \max\{\tau_j+\Lambda_J^{ij},\tau_k+\Lambda_J^{ik}\}.\end{array}\right\}. \tag{32}$$

Notice that, since the expressions inside the $\min$ and $\max$ coincide, the inequality can only be violated if either of the two condition below holds:

$$\max\{\tau_j+\Lambda_1^{ij},\tau_k+\Lambda_1^{ik}\}>\max\{\tau_j+\Lambda_J^{ij}+u,\tau_k+\Lambda_J^{ik}+u\}$$
$$\max\{\tau_j+\Lambda_J^{ij},\tau_k+\Lambda_J^{ik}\}>\max\{\tau_j+\Lambda_1^{ij}+u,\tau_k+\Lambda_1^{ik}+u\}$$

which require $|\Lambda_1^{ij}-\Lambda_J^{ij}|>u$ or $|\Lambda_1^{ik}-\Lambda_J^{ik}|>u$, both of which contradict **A1**.

(2) Suppose, instead, group I is targeted by exactly two candidates i and j, and group 1 is targeted by no candidate. This implies that the remaining candidate $k\in\{1,2,3\}\setminus\{i,j\}$ either offers a broad policy or instead targets the remaining group $J=\{2,3\}\setminus I$.

(2i) Consider, first, $p_k=g$. Notice that at least one of i or j targeting group $I\neq 1$ fails to win support from group 1 in a three-way split (if one arises with positive probability); let j denote this candidate, who wins the election *only if*

$$\tau_j>\min\left\{\begin{array}{c}\max\{\tau_i+\Lambda_1^{ji},\tau_k+u+\Lambda_1^{jk}\}\\ \max\{\tau_i+\Lambda_J^{ji},\tau_k+u+\Lambda_J^{jk}\}.\end{array}\right\}$$

If j deviates to $\tilde{p}_j=g$, then she wins the election if

$$\tau_j>\max\left\{\begin{array}{c}\max\{\tau_i-u+\Lambda_1^{ji},\tau_k+\Lambda_1^{jk}\}\\ \max\{\tau_i-u+\Lambda_J^{ji},\tau_k+\Lambda_J^{jk}\}.\end{array}\right\}$$

It follows that this deviation is profitable if

$$\min\left\{\begin{array}{c}\max\{\tau_i+\Lambda_1^{ji},\tau_k+u+\Lambda_1^{jk}\}\\ \max\{\tau_i+\Lambda_J^{ji},\tau_k+u+\Lambda_J^{jk}\}.\end{array}\right\}>\max\left\{\begin{array}{c}\max\{\tau_i+\Lambda_1^{ji},\tau_k+u+\Lambda_1^{jk}\}\\ \max\{\tau_i+\Lambda_J^{ji},\tau_k+u+\Lambda_J^{jk}\}.\end{array}\right\}-u. \tag{33}$$

That this condition is implied by **A1** follows the same argument as verifying that (32) is implied by **A1**.

(2ii) Consider, second, $p_k = t^J$ where we recall $J = \{1,2,3\} \setminus \{I,1\}$. We claim that this candidate k has a strictly profitable deviation to $\tilde{p}_k = g$. To see why, recognize that under the conjecture $p_k = t^J$ and **A1**, candidate k wins the first preferences of group 1 voters only if she wins the first preferences of group-J voters; that is:

$$\max\{\tau_i + \Lambda_J^{ki}, \tau_j + \Lambda_J^{kj}\} - 1 < \max\{\tau_i + \Lambda_1^{ki}, \tau_j + \Lambda_1^{kj}\}.$$

It follows that candidate k wins under the conjecture $p_k = t^J$ if and only if she wins a majority of first preferences. This implies that a deviation by candidate k to $\tilde{p}_k = g$ is profitable if

$$\max_{(2)} \left\{ \begin{array}{c} \max\{\tau_i + \Lambda_1^{ki}, \tau_j + \Lambda_1^{kj}\} - u \\ \max\{\tau_i + \Lambda_I^{ki}, \tau_j + \Lambda_I^{kj}\} + 1 - u \\ \max\{\tau_i + \Lambda_J^{ki}, \tau_j + \Lambda_J^{kj}\} - u \end{array} \right\} < \max_{(2)} \left\{ \begin{array}{c} \max\{\tau_i + \Lambda_1^{ki}, \tau_j + \Lambda_1^{kj}\} \\ \max\{\tau_i + \Lambda_I^{ki}, \tau_j + \Lambda_I^{kj}\} + 1 \\ \max\{\tau_i + \Lambda_J^{ki}, \tau_j + \Lambda_J^{kj}\} - 1 \end{array} \right\}.$$

A *sufficient* condition for this inequality to hold is that

$$\max \left\{ \begin{array}{c} \max\{\tau_i + \Lambda_1^{ki}, \tau_j + \Lambda_1^{kj}\} \\ \max\{\tau_j + \Lambda_J^{ki}, \tau_k + \Lambda_J^{kj}\} \end{array} \right\} - u < \min \left\{ \begin{array}{c} \max\{\tau_i + \Lambda_1^{ki}, \tau_j + \Lambda_1^{kj}\} \\ \max\{\tau_i + \Lambda_J^{ki}, \tau_j + \Lambda_J^{ik}\} + 1 \end{array} \right\}, \tag{34}$$

which follows the same argument that verifies that (32) is implied by **A1**. □

Lemma 10. *Under **A1** and plurality: in a pure strategy equilibrium where any group 1 is targeted by at least one candidate, then group 1 is targeted by exactly one candidate.*

Proof. We conjecture, to the contrary, that candidate 1 is targeted by two or three candidates.

(1) Suppose candidate 1 is targeted by exactly two candidates $i, j \in \mathcal{C}$, so that $p_i = p_j = t^1$. Then, the previous results imply that the remaining candidate k plays a broad strategy: $p_k = g$. This implies that at least one of candidates i or j does not win first preferences from group 1

in the event of any three-way split. Call this candidate i, who therefore wins if and only if

$$\tau_i \geq \max_{(2)} \left\{ \begin{array}{c} \max\{\tau_j + \Lambda_1^{ij}, \tau_k - 1 + u + \Lambda_1^{ik}\} \\ \max\{\tau_j + \Lambda_2^{ij}, \tau_k + u + \Lambda_2^{ik}\} \\ \max\{\tau_j + \Lambda_3^{ij}, \tau_k + u + \Lambda_3^{ik}\} \end{array} \right\}. \tag{35}$$

A necessary condition for i to win is therefore

$$\tau_i \geq \min \left\{ \begin{array}{c} \max\{\tau_j + \Lambda_2^{ij}, \tau_k + u + \Lambda_2^{ik}\} \\ \max\{\tau_j + \Lambda_3^{ij}, \tau_k + u + \Lambda_3^{ik}\} \end{array} \right\}. \tag{36}$$

If candidate i deviates to $\tilde{p}_i = g$, then she wins whenever

$$\tau_i \geq \max \left\{ \begin{array}{c} \max\{\tau_j - u + \Lambda_2^{ij}, \tau_k + \Lambda_2^{ik}\} \\ \max\{\tau_j - u + \Lambda_3^{ij}, \tau_k + \Lambda_3^{ik}\} \end{array} \right\}. \tag{37}$$

The deviation is therefore profitable if

$$\max \left\{ \begin{array}{c} \max\{\tau_j + \Lambda_2^{ij}, \tau_k + u + \Lambda_2^{ik}\} \\ \max\{\tau_j + \Lambda_3^{ij}, \tau_k + u + \Lambda_3^{ik}\} \end{array} \right\} - u < \min \left\{ \begin{array}{c} \max\{\tau_j + \Lambda_2^{ij}, \tau_k + u + \Lambda_2^{ik}\} \\ \max\{\tau_j + \Lambda_3^{ij}, \tau_k + u + \Lambda_3^{ik}\}. \end{array} \right\}. \tag{38}$$

which follows from the same argument that verifies that (32) is implied by **A1**.

(2) Suppose candidate 1 is targeted by *all* three candidates. Then, candidate 2 wins the election only if

$$\tau_2 \geq \max_{(2)} \left\{ \begin{array}{c} \max\{\tau_1 + \Lambda_1^{21}, \tau_3 + \Lambda_1^{23}\} \\ \max\{\tau_1 + \Lambda_2^{21}, \tau_3 + \Lambda_2^{23}\} \\ \max\{\tau_1 + \Lambda_3^{21}, \tau_3 + \Lambda_3^{23}\} \end{array} \right\}. \tag{39}$$

If candidate 2 instead deviates to $\tilde{p}_2 = g$, she wins the election if

$$\tau_2 \geq \max_{(2)} \left\{ \begin{array}{c} \max\{\tau_1 + \Lambda_1^{21}, \tau_3 + \Lambda_1^{23}\} + 1 - u \\ \max\{\tau_1 + \Lambda_2^{21}, \tau_3 + \Lambda_2^{23}\} - u \\ \max\{\tau_1 + \Lambda_3^{21}, \tau_3 + \Lambda_3^{23}\} - u \end{array} \right\}. \tag{40}$$

A sufficient condition for the deviation to be profitable is that

$$\max \left\{ \begin{array}{c} \max\{\tau_1 + \Lambda_2^{21}, \tau_3 + \Lambda_2^{23}\} - u \\ \max\{\tau_1 + \Lambda_3^{21}, \tau_3 + \Lambda_3^{23}\} - u \end{array} \right\} < \min \left\{ \begin{array}{c} \max\{\tau_1 + \Lambda_2^{21}, \tau_3 + \Lambda_2^{23}\} \\ \max\{\tau_1 + \Lambda_3^{21}, \tau_3 + \Lambda_3^{23}\} \end{array} \right\}. \tag{41}$$

Again, this follows the same argument that verifies that (32) is implied by **A1**. □

The previous lemmas imply that whenever **A1** holds, an action p is a pure strategy equilibrium only if (1) all candidates campaign broadly, or (2) a single candidate targets group 1, and the other candidates choose the same action.

Lemma 11. *For any (u, F), there exists $\varepsilon > 0$ such that if $\max\{d_{12}, d_{23}, d_{13}\} < \varepsilon$ plurality does not support a pure strategy equilibrium in which a single candidate targets group 1, and the remaining candidates choose the same action.*

Proof. Suppose candidate $i \in \mathcal{C}$ chooses $p_i = t^1$and the remaining candidates $j, k \in \mathcal{C} \setminus i$ choose $p_j = p_k \equiv p$. Suppose first $p = g$. When $\max\{d_{12}, d_{23}, d_{13}\} = 0$, candidate i wins if and only if $\tau_i \geq \max\{\tau_j + u, \tau_k + u\}$; if i deviates to g she wins if and only if $\tau_i \geq \max\{\tau_j, \tau_k\}$. The deviation is therefore profitable, and by continuity of winning probabilities in $x = (x_1, x_2, x_3)$ it is also profitable if $\max\{d_{12}, d_{23}, d_{13}\} < \varepsilon$ so long as ε is sufficiently small. Suppose instead $p \in \{t^2, t^3\}$. If $p = t^2$, candidate i wins if and only if $\tau_i \geq \max\{\tau_j, \tau_k\}$; if i deviates to g she wins if and only if $\tau_i \geq \max\{\tau_j, \tau_k\} - u$. The argument for $p = t^3$ is the same. □

It is easy to verify that (g, g, g) is a pure strategy equilibrium under both plurality and RCV so long as $\max_{ij} d_{ij}$ is sufficiently small. The reason is that at either strategy profile (g, g, g) or after any unilateral deviation by a single candidate from this strategy profile the probability of a three-way split tends to zero as $\max_{ij} d_{ij}$ tends to zero, and any unilateral deviation strictly reduces any candidate's probability of winning a majority. □