Fifty six years of cheap talk

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FIFTY SIX YEARS OF CHEAP TALK

- Likely best-known paper on cheap talk:

  Strategic Information transmission, Crawford and Sobel (1982).

  Specific model:
  real-valued types and decisions, well-behaved utility functions.

- I will focus on the
  Aumann-Maschler (1966)'s legacy.

  Standard game-theoretic approach:
  finitely many types and decisions, arbitrary (vNM) utility functions.

FIFTY SIX YEARS OF CHEAP TALK

Aumann and Maschler 1966 (AM)
Aumann, Maschler and Stearns 1968 (AMS)

- Basic recipe in repeated games with incomplete information.
- AM and AMS's legacy in sender-receiver games and Bayesian persuasion.
- Some possible alternative approaches in sender-receiver games.

Cheap talk in the 1960's

AM and AMS: reports to the US Disarmament Agency

- motivated by concrete concerns (cold war)
- confidential (published in 1995)
- study the effects of using private information in long term relationships
- model: undiscounted infinitely repeated game between an informed player and an uninformed one.
- no discounting $\Rightarrow$
  - results hold in all sufficiently long games (robustness)
  - finitely many stages of the game can be used to transmit information at no cost (cheap talk)

Recipe to find equilibria

in a two-person infinitely repeated game with a single informed player:

1. Start by identifying what happens when the informed player does not reveal any information, for every possible prior belief of the uninformed player.

2. Make use of a cheap talk phase so that:

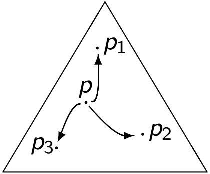
   - the informed player uses his early actions as signals;
   - given a signal s, the uninformed player updates his belief, from his prior p to p_s;
   - non-revealing solutions are followed at the posteriors p_s.

- Which posteriors p_s can be reached from the prior p?

  Those p_s's such that p can be the expectation of the p_s's (and only those).

  Straightforward consequence of Bayes formula.

  Geometrically, the prior p must lie in the convex hull of the posteriors p_s's ("splitting lemma" of AM).



Cheap talk in the 1960's

AM use their recipe to prove the existence of an equilibrium in every zero-sum infinitely repeated game with a single informed player.

1. They start by computing, for every prior p, the best ex ante expected payoff that the informed player can guarantee himself without revealing any information: $u_{NR}(p)$.

2. They prove that the value (= the unique equilibrium payoff) of the infinitely repeated game is $cavu_{NR}(p)$,
   $cavu_{NR}(p) =$ concavification of $u_{NR}(\cdot)$ at p.
   $cavu_{NR} =$ smallest concave function above u_{NR}.

EXEMPLE

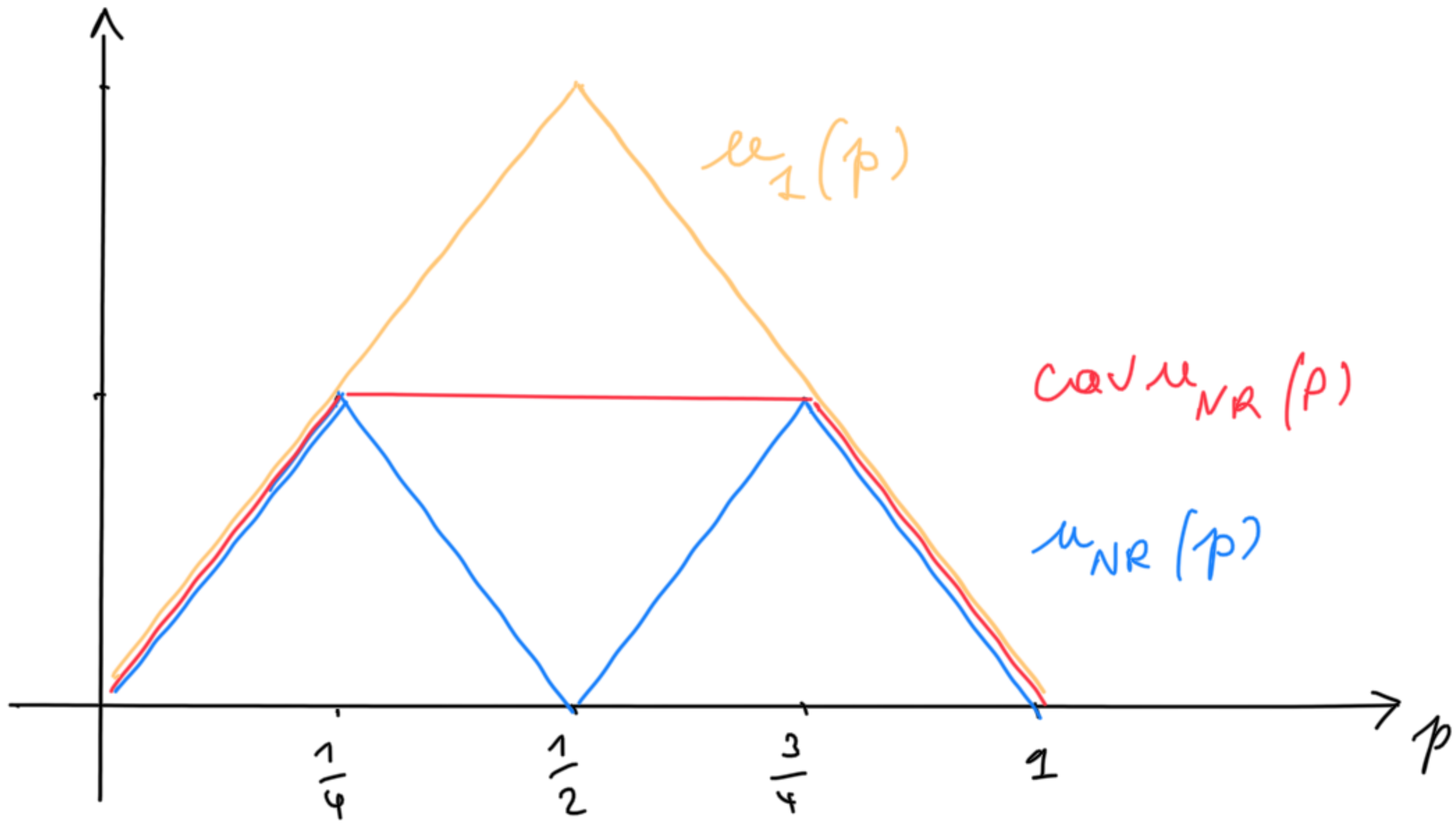
(0-SUM)

$k=1$, p

	l	c	r
H	2	0	1
L	2	0	-1

$k=2$, $1-p$

	l	c	r
H	0	2	-1
L	0	2	1



Cheap talk in the 1960's

- In AM, the game is zero-sum and the informed player's actions have an impact on the payoffs; $u_{NR}(.)$ is the value of the expected one-shot game and is continuous. The value of the infinitely repeated game is $cavu_{NR}(p)$ but *player 1 can guarantee* $> cavu_{NR}(p)$ *in the short run.*

- AMS use the recipe in non-zero-sum infinitely repeated games as a *first step* toward the characterization and the existence of equilibria.

Basic decision problem

- K: (finite) set of states of nature
- $p \in \Delta(K)$: prior belief over K.
- A_2: (finite) set of actions for the decision–maker, alias player 2.
- $(k, a_2) \to V^k(a_2)$: payoff (von Neumann utility) of player 2.

There is another individual, player 1, with payoff: $(k, a_2) \to U^k(a_2)$.

Player 2 does not know the state of nature.

Various assumptions will be made for player 1.

Bayesian persuasion

Player 1, alias the information-designer, chooses a communication device (to be used by an omniscient mediator):

- finite set of signals S (typically, $S = A_2$)
- $\{\delta(\cdot \mid k), k \in K\}$, with $\delta(\cdot \mid k) \in \Delta(S)$ for every $k \in K$.

Given $s \in S$ selected by δ, player 2 updates his belief into $p_s \in \Delta(K)$.
Which posteriors p_s can be reached from the prior p?
AM's *splitting lemma*: p_s such that p can be the expectation of p_s.

Player 2 chooses an optimal action $a_2^*(p_s)$, to maximize his expected utility given his belief p_s. In case of indifference, he maximizes player 1's expected utility.

Bayesian persuasion

Player 1's best non-revealing expected utility is

$$u_{NR}(p) = \sum_{k \in K} p^k U^k(a_2^*(p)).$$

Differences with AM:

- Omniscient mediator $\rightarrow$ **no** incentive constraints for player 1.
- $u_{NR}(.)$ is upper semi-continuous.

Kamenica and Gentzkow (2011):
The best ex ante expected utility player 1 can achieve by relying on an omniscient mediator is $cavu_{NR}(p)$.

AM's recipe applies:
Start with non-revealing analysis ($u_{NR}(.)$), then use splitting to get $cavu_{NR}(.)$.

Sender-receiver game, cheap talk

No omniscient mediator is used.
Player 1 is informed of the state and talks directly to player 2.

- Nature selects $k \in K$ according to p
- Player 1 learns the state, i.e., his type, k
- Player 1 sends a message in a (rich) set S to player 2
- Player 2 makes a decision $a_2 \in A_2$
- The payoffs are $U^k(a_2)$, $V^k(a_2)$.

Nash equilibrium, in which
player 1 sends his signal by himself
$\Rightarrow$ very demanding incentive constraints:

If player 1 randomizes over signals $s, s' \in S$,
he has to be indifferent between sending s and sending s'.

Sender-Receiver game: Example

Example

- The informed agent (player 1) has two possible types:

  "creative" (proba p) or "administrative" (proba $1 - p$).

- Player 1 sends a (costless) message to the decision-maker (player 2).

- Having received the message, player 2 assigns a task to player 1:

  "administration, local", "administration, exotic",

  "creation, exotic", "creation, local".

- Both players' payoffs are independent of the message.

Sender-Receiver game: Example

Player 2's payoffs:

Player 2 looks for a task fitting the type

	ad loc	ad exo	cr exo	cr loc
creative type:	0	3	4	5

	ad loc	ad exo	cr exo	cr loc
adminis. type:	5	4	3	0

Sender-Receiver game: Example

Player 1 and Player 2's payoffs:

Player 1 looks for a challenge

	ad loc	ad exo	cr exo	cr loc
creative type:	$7, 0$	$\cdot, 3$	$\cdot, 4$	$3, 5$

	ad loc	ad exo	cr exo	cr loc
adminis. type:	$3, 5$	$\cdot, 4$	$\cdot, 3$	$9, 0$

Sender-Receiver game: Example

Player 1 and Player 2's payoffs:

Player 1 looks for a (not too demanding) challenge

	ad loc	ad exo	cr exo	cr loc
creative type:	$7, 0$	$1, 3$	$6, 4$	$3, 5$

	ad loc	ad exo	cr exo	cr loc
adminis. type:	$3, 5$	$5, 4$	$0, 3$	$9, 0$

Sender-Receiver game: non-revealing equilibria

	ad loc	ad exo	cr exo	cr loc		ad loc	ad exo	cr exo	cr loc
p	$7, 0$	$1, 3$	$6, 4$	$3, 5$	$1 - p$	$3, 5$	$5, 4$	$0, 3$	$9, 0$

AM's recipe:

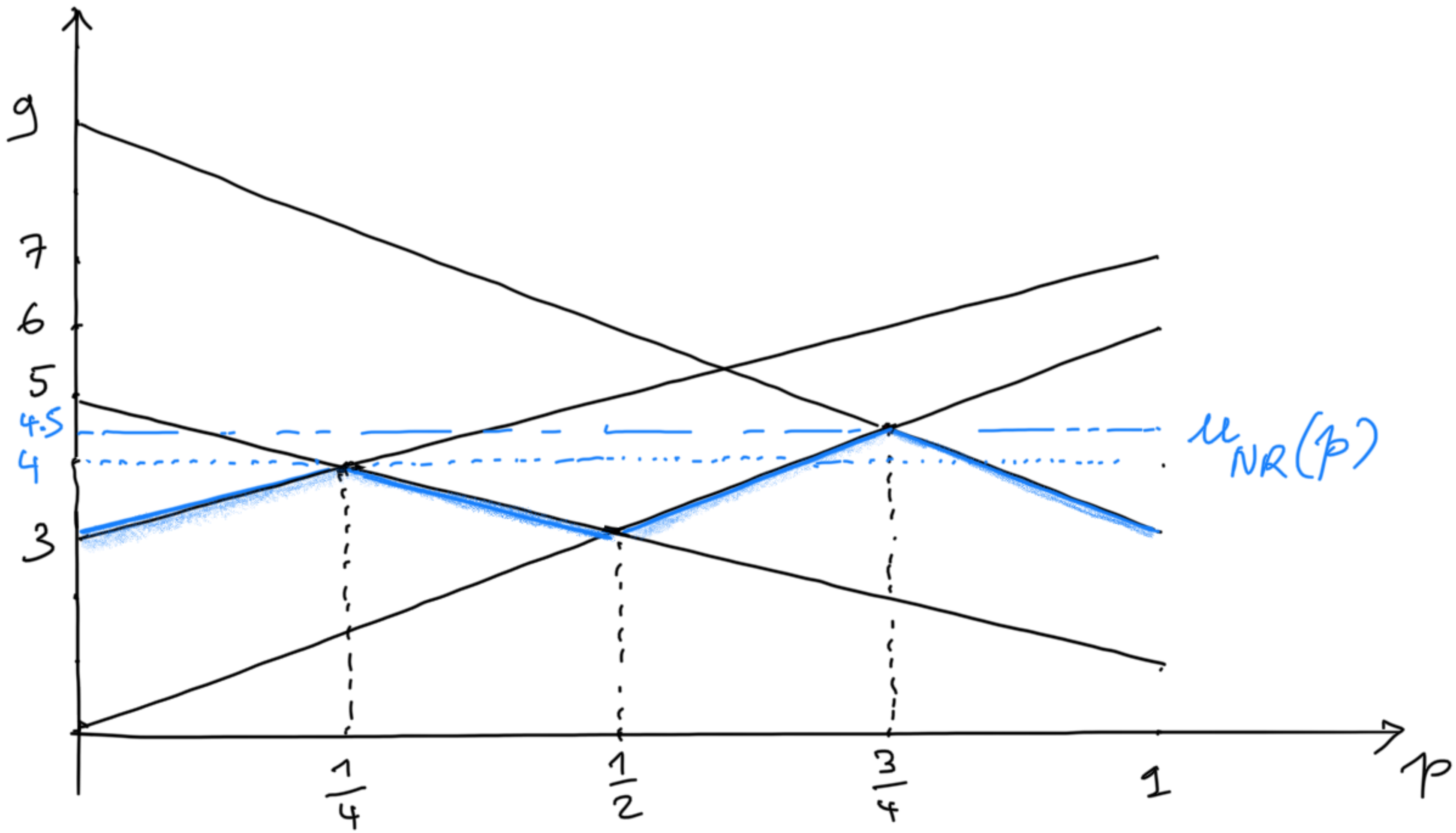
First analyze the non-revealing (NR) situation, for every prior $p \in [0, 1]$.

There is a NR ("babbling") *equilibrium* at every p.

E.g., at $p = \frac{1}{2}$, player 2's optimal decisions are
"ad exo" and "cr exo"(and he is indifferent between them).

The expected payoffs are: 3 for player 1 and 3.5 for player 2.

N.B.: There is no fully revealing equilibrium.



9
7
6
5
4.5
4
3
1/4
1/2
3/4
1
p
u_NR(p)

Sender-Receiver game: communication equilibrium

	ad loc	ad exo	cr exo	cr loc		ad loc	ad exo	cr exo	cr loc
p	$7, 0$	$1, 3$	$6, 4$	$3, 5$	$1 - p$	$3, 5$	$5, 4$	$0, 3$	$9, 0$

At $p = \frac{1}{2}$, with the help of a not omniscient mediator :

cr type: $\frac{1}{4}$: recom ad exo; $\frac{3}{4}$: recom cr exo

ad type: $\frac{3}{4}$: recom ad exo; $\frac{1}{4}$: recom cr exo

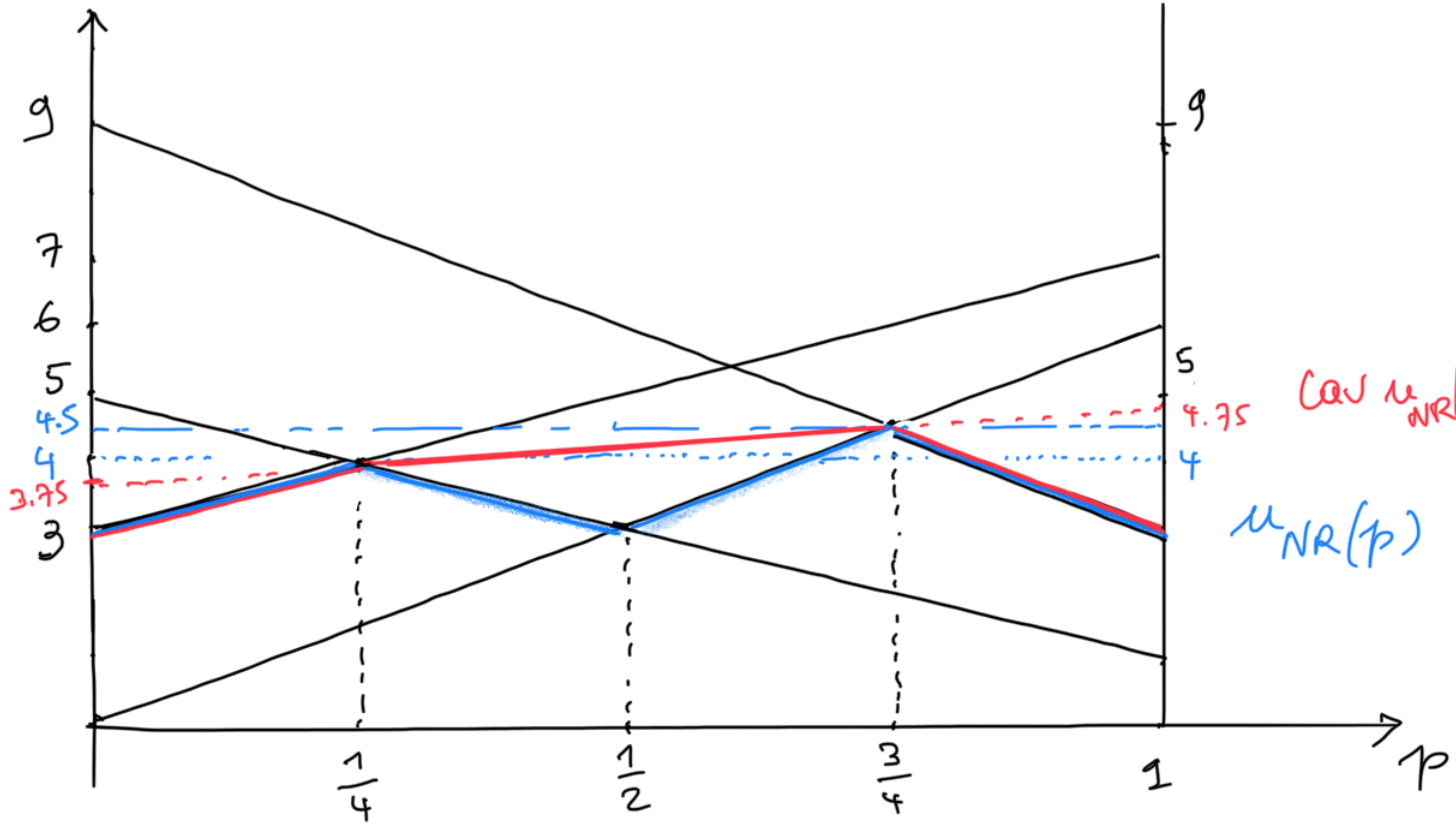
Player 1 truthfully reports his type, player 2 follows the recommendation.

Pl 2's posterior is $\frac{1}{4}$ if "ad exo" is recommended.

Pl 2's posterior is $\frac{3}{4}$ if "cr exo" is recommended.

Player 1's expected payoff is: $cavu_{NR}(\frac{1}{2}) = 4.25 > 3$
(cr type: 4.75; ad type: 3.75).
Player 2's expected payoff is $3.75 > 3.5$.

By contrast with Bayesian persuasion, no commitment is needed!
Incentive compatible but mediated - not cheap - talk!



9
7
6
5
4.5
4
3.75
3
9
5
4.75
4
cav u_NR(p)
u_NR(p)
1/4
1/2
3/4
1
p

	ad loc	ad exo	cr exo	cr loc		ad loc	ad exo	cr exo	cr loc
p	7, 0	1, 3	6, 4	3, 5	$1-p$	3, 5	5, 4	0, 3	9, 0

Player 1 can achieve $cavu_{NR}(p)$ by sending message L or R by himself (i.e., without any mediator, by cheap talk).

For instance, at $p = \frac{1}{2}$:

cr type:
- $\frac{1}{4}$: L
  - $\frac{5}{8}$: ad loc
  - $\frac{3}{8}$: ad exo
- $\frac{3}{4}$: R
  - $\frac{7}{12}$: cr exo
  - $\frac{5}{12}$: cr loc

ad type:
- $\frac{3}{4}$: L
  - $\frac{5}{8}$: ad loc
  - $\frac{3}{8}$: ad exo
- $\frac{1}{4}$: R
  - $\frac{7}{12}$: cr exo
  - $\frac{5}{12}$: cr loc

Whatever his type, player 1 is indifferent between L and R:

cr: $\frac{5}{8}7 + \frac{3}{8}1 = \frac{7}{12}6 + \frac{5}{12}3 = 4.75$

ad: $\frac{5}{8}3 + \frac{3}{8}5 = \frac{7}{12}0 + \frac{5}{12}9 = 3.75$

Player 2's posterior given L is $\frac{1}{4}$ (ad loc and ad exo are optimal), his posterior given R is $\frac{3}{4}$ (cr exo and cr loc are optimal).

Sender-receiver game

The example illustrates the following

Proposition

Consider a finite **sender-receiver** *game with prior belief p. Let $u_{NR}(p)$ be the sender's best (ex ante) expected non-revealing equilibrium payoff. If the mapping u_{NR} is* **continuous** *, there exists a perfect Bayesian* **equilibrium** *in which the sender's (best) expected payoff is $cavu_{NR}(p)$.*

- Conjecture of FF, established by E. Lipnowski (2020): "Equivalence of cheap talk and Bayesian persuasion in a finite continuous model."
- In a sender-receiver game, u_{NR} is continuous if, whenever the receiver is indifferent between optimal actions, so is the sender.
- In a general sender-receiver game, u_{NR} is upper semi-continuous, equilibria can still be constructed using AM recipe...
  see, e.g., Forges (QJE 1990).

Selection of some references.

- Application of AM and AMS's recipe: existence of Nash equilibrium in two-person (undiscounted) non-zero-sum infinitely repeated games with a single informed player: Sorin (1983), Simon et al. (1995).
- *More* Nash equilibrium payoffs (w.r.t. the ones achieved by the basic recipe, starting with AMS):
  - full characterization in two-person (undiscounted) non-zero-sum infinitely repeated games with a single informed player: Hart (1985).
  - "translation" in one-shot games with *long* cheap talk: Forges (QJE 1990), Aumann and Hart (2003).
- *Even more* equilibrium payoffs in (possibly long) cheap talk games: (Correlated) cheap talk is as powerful as a mediator Forges (1985), Vida and Forges (2013).

- *Discounted* non-zero-sum infinitely repeated games with a single informed player:
  Peski (2008, 2014), Salomon and Forges (2015).

- *Cheap talk with transparent motives*
  (i.e., in one-shot sender-receiver games in which the informed player's payoff is independent of his type):

  Lipnowski and Ravid (2020) show that the best ex ante expected utility the informed player can achieve with cheap talk is

$$qucavu_{NR}(p)$$

where $qucavu_{NR}$ is the smallest quasi-concave function above u_{NR}.

Sender-receiver games: alternative approaches

- F. Forges and J. Renault, IJGT 2021
  Strategic information transmission with sender's approval
  (finite set of types, no specific assumptions on decision set and utility functions).

- S. Sémirat and F. Forges, GEB 2022
  Strategic information transmission with sender's approval: the single crossing case
  (finite ordered set of types, real-valued decisions, well-behaved utility functions as in Crawford and Sobel (1982)).

- S. Sémirat and F. Forges (under progress)
  Forward neologism-proof equilibrium in cheap talk games.

Sender-receiver games: alternative approaches

- Pure PBE of a sender-receiver game can be characterized as incentive compatible partitions of the set of types K.
- An IC partition Π' defeats another IC partition Π if there is a cell π' of Π' such that for every $k \in \pi'$ $\quad U^k(y^{\pi'}) \geq U^k(y^{\pi(k)})$ (with at least one $>$) (Mailath, Okuno-Fujiwara and Postlewaite (1991)).

Proposition

Every sender-receiver game with finitely many types, real-valued decisions and well-behaved utility functions has an undefeated PBE.

- **Proof:** based on an algorithm.
  Given a partition Π, type k envies cell π if $U^k(y^{\pi}) > U^{\theta}(y^{\pi(k)})$.
  Start with the fully revealing partition $\Pi_0 = \{\{1\}, \{2\}, \cdots, \{K\}\}$
  Modify it gradually by moving envying types to the cell they envy (on the right).
  A unique IC partition is reached, which is undefeated.

Summary

- AM and AMS propose a recipe to find equilibria in two-person undiscounted infinitely repeated games with lack of information on one side.
- Their basic idea is to start with non-revealing strategies and then use costless information transmission.
- Their approach is insightful to study cheap talk in one-shot games, namely, in sender-receiver games (1980-2020).
- Bayesian persuasion (developed since about 2010) makes use of the same methodology (under much weaker incentive constraints).
- Under specific assumptions ($u_{NR}(.)$ continuous), cheap talk is equivalent to Bayesian persuasion.
- To select among perfect Bayesian Nash equilibria (PBE) in sender-receiver games, we (Sémirat and Forges (under progress)) follow the "opposite" approach: we start with the fully revealing strategies and propose an algorithm that gradually reaches a PBE.