

Selling Through Referrals*

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Abstract

A seller has an object for sale and can reach buyers only through intermediaries, who also have privileged information about buyers' valuations. Intermediaries can either mediate the transaction by buying the object and reselling it—the *merchant model*—or refer buyers to the seller and release information for a fee—the *agency model*. The merchant model suffers from double marginalization. The agency model suffers from adverse selection: Intermediaries would like to refer low-value buyers, but retain high-value ones and make profits from resale. We show that, in equilibrium, intermediaries specialize in agency. Seller's and intermediaries' joint profits equal the seller's profits when he has access to all buyers and all intermediaries' information. Profits' division depends on seller's and intermediaries' relative bargaining power. Our results rationalize the prevalence of the agency model in online markets.

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1 Introduction

Intermediaries often facilitate transactions between sellers and buyers.¹ For example, online travel agents (OTAs) such as Expedia, Priceline, and Orbitz connect buyers to hoteliers, airlines and car rental agencies. Amazon marketplace, Groupon, and alike, connect buyers to retailers selling a variety of goods. Amazon and Apple intermediate the E-books industry. Facebook connects potential buyers to retailers by selling targeted advertising. Google links direct users searching for a particular product to retailers. Intermediation is also prevalent in offline markets such as real estate, art, used cars, books, as well as markets for professional services. Intermediaries have two main advantages compared to producers and retailers: access to many potential customers and privileged information about their tastes.²

Intermediaries predominately operate under two business models (or hybrids of those). Under the *agency business model*, they refer buyers to sellers, who then negotiate directly on the terms of trade. In return, intermediaries receive referral fees for the creation of the match and/or commissions based on final sale. Under the more traditional *wholesaler/merchant model*, intermediaries buy the good from the suppliers and resell it to consumers.

In the online travel industry, Priceline makes most of its revenue through the agency model (almost 80 percent from agency, and the rest from acting as a merchant). Priceline's subsidiary Booking.com is an agency-based business, while its subsidiary Agoda is a merchant-based business.³ Expedia operates mainly under the merchant model—receiving roughly 75 percent of its revenue through the merchant model and some 21 percent through the agency model.⁴ Recently, Expedia has expanded its agency business by acquiring the agency-based online hotel business Venere. Orbitz's net revenue stems fairly evenly from its air and hotel businesses (34 and 29 percent respectively), with its revenue from the hotel business coming mainly from the merchant model.⁵ In the E-book industry, Apple operates under the agency model, whereas Amazon started with a merchant model and recently adopted an agency model. The agency model is widely used in the marketplace for tablets and smart-phone applications. Facebook and Google can be also thought of as operating

¹According to Spulber (1996), intermediation in its various forms accounted for about 25 percent of U.S. GDP in 1993.

²The recent advance in information technology allows intermediaries to manage large data sets on past consumers' behavior, information that may not directly available to sellers. Moreover, large intermediaries are also likely to have more-accurate information about consumers' trends compared to small producers or retailers.

³For more details, see Business Insider (2012) and priceline.com, Inc. (2011)

⁴Expedia, Inc. (2013, p.10)

⁵Orbitz Worldwide Inc. (2012, p.35). Orbitz also owns OTAs such as CheapTickets and ebookers.

under an agency model. The agency model is also used in more-traditional markets such as real estate and markets for professionals (actors, artists, sports players). The proliferation of social media has also increased the implementation of marketing strategies based on the agency model—e.g., affiliate and referral marketing.⁶ The merchant model is the standard for classic wholesalers and retailers.

The merchant model suffers from a double-marginalization problem: The seller exercises market power against intermediaries who, in turn, exercise it against buyers. As a consequence, the joint profit of seller and intermediaries is less than the seller’s profit if he had access to all buyers’ and intermediaries’ information. In contrast, once the agency model is in place, the seller and the buyers can negotiate directly, thereby overcoming this problem. There is, however, a potential adverse-selection problem: Intermediaries have incentives to adopt a hybrid model in which they refer only low-value buyers and retain high-value buyers with the hope of extracting high margins from resale. Our paper studies how these forces shape the market. We clarify the conditions under which intermediaries specialize in agency and when we should expect to observe a co-existence of the two business models. We also study the welfare consequences of the two models and consider the implications of our results for policy debates on vertical agreements.

In our model, there is a seller with one object for sale. The seller is in contact with a number of intermediaries, and each intermediary has private information about the set of buyers he has access to and about their valuations. The seller and the intermediaries have no consumption value for the object. We study the following game. At the outset, the seller and the intermediaries negotiate the level of the referral fee. The referral fee determines the payment to each intermediary per referred buyer. In the *referral phase*, intermediaries decide which buyers to refer. Once a buyer is referred, the seller deals directly with him. In the *trading phase*, the seller sells the object to the intermediaries and the referred buyers. If a buyer obtains the object, the game ends. If an intermediary obtains the object, then the intermediary sells it optimally to the buyers he has not referred.

Our first result is that when the referral fee is positive, in all equilibria, intermediaries refer all their buyers to the seller regardless of their private information about buyers (Theorem 1). Hence, even arbitrarily small referral fees lead intermediaries to specialize in agency, despite

⁶Affiliate marketing is a performance-based marketing in which a business rewards one or more affiliates for each visitor or customer brought in by the affiliate’s own marketing efforts (see Hoffman and Novak, 2000). According to a recent Forrester estimate (Forrester Research, 2012), U.S. affiliate online marketing spending will increase by a compound annual growth rate of nearly 17 percent between 2011 and 2016, growing to \$4.5 billion.

the fact that they can always choose the merchant model of buying and reselling. To see this, consider a simple example of one intermediary with one buyer; the intermediary knows the buyer’s valuation, but the seller knows only that it is either high or low. Since the seller behaves optimally when selling the good, the intermediary anticipates that the price asked by the seller is always weakly above the buyer’s lowest valuation. Then, an intermediary with a low-value buyer always prefers to refer and receive the positive referral fee. Hence, upon observing no referral, the seller infers that the intermediary has a high value-buyer and, therefore, asks a high price, leaving zero profit for the intermediary. This unraveling force implies that there is no equilibrium in which the intermediary is active with positive probability in the resale market.

Once intermediaries are specialized in agency, we show that the seller, using a small commission fee, can extract the intermediary’s information at virtually no cost.⁷ This, in combination with the previous result, implies (Theorem 2) that the aggregate equilibrium profit of the seller and the intermediaries equals the expected revenue that the seller achieves when he has access to all buyers’ and all intermediaries’ information—which we call the “integrated-industry profit.”

Theorem 1 and Theorem 2 show that the possibility of referrals leads intermediaries to adopt the agency model, and this specialization removes possible distortions arising from intermediation via resale. In particular, the presence of referrals/commissions in our model, and in trading environments more generally, may generate an outcome that is equivalent to that obtained through an explicit vertical agreement, such as a royalty contract. This is of interest to policy makers and competition authorities, who are often concerned that vertical contracts may restrict competition and decrease aggregate welfare.⁸ In Section 6, we show that, in a number of important cases, the presence of referrals/commissions unambiguously increases (ex-ante) aggregate welfare relative to a situation where there is a ban on referrals/commissions.

Our results not only speak to the prevalence of the agency model in online markets, but can also account for the large heterogeneity in the level of referral fees/commissions.

⁷In the example with one perfectly informed intermediary and one buyer, the seller asks the intermediary to report his private information about the value, commits to ask the buyer a price that equals to the reported value, and pays the intermediary a percentage of his final revenue. The interests of the seller and of the intermediary are perfectly aligned.

⁸See Rey (2003) for an extensive survey of the academic literature on vertical restraints and an account of the legal issues related to vertical restraints in the United States and the European Union. We also refer to Johnson (2013a,b), for an analysis of the agency and the merchant model in terms of consumer surplus. In contrast to Johnson (2013a,b), in our general analysis, which business model is used is endogenous.

We show that when the seller has the all bargaining power to set referral fees, then, in equilibrium, referral fees are zero (i.e., are set at marginal cost of referring), the seller obtains the integrated industry profit, and intermediaries are left with no rent (Theorem 3). However, when intermediaries have the bargaining power to set the referral fees, intermediaries can extract as much as their marginal contribution to the integrated industry profit (Theorem 4).

These findings are consistent with what we observe: An industry report on the online travel agent industry indicates that the typical OTA commissions in the three major travel sectors are as follows: Airlines \$0, Car Rental \$0, Hotels \$50-\$150 (based on Average Daily Rate of \$100 – \$300).⁹ Practitioners attribute these differences mainly to differences in the bargaining power of sellers (airlines, car rental companies, hoteliers) vis-à-vis intermediaries (the OTAs) when it comes to negotiating fees.

Our results that revenue from referrals depends on bargaining power may help improve the pricing of stocks in IPOs of companies such as Facebook, Twitter, Lore.com or Pinterest, whose business models are very different from the traditional ones. These companies start offering the service for free with the objective of maximizing the number of users. Then, they leverage their access to individuals and function as intermediaries to various sellers. On the one hand, our analysis predicts that if sellers have stronger bargaining power vis-à-vis these companies, we should expect their revenues from referrals to be low. On the other hand, when intermediaries have stronger bargaining power, they can expect higher revenues from referrals.

In the last part of the paper, we explore the limits of our result on intermediaries' specialization in agency (Section 7). To break this specialization result, we need to introduce additional frictions that prevent the seller from extracting intermediaries' revenue from resale. Such frictions include budget-constrained intermediaries; intermediaries who have positive consumption value; and strong restrictions on the selling procedures that the seller can adopt. In such environments, when referral fees are small, intermediaries may prefer not to refer high-value buyers and make positive profit in the resale market. This is possible even though the seller anticipates that high-value buyers are not referred. In equilibrium, then, intermediaries adopt a hybrid business model that involves both agency and merchant activities.

⁹See the Table on page 4 of that report Starkov (2010).

1.1 Literature Review

Although there is an extensive literature on intermediation, to the best of our knowledge, there is no systematic work that examines the interplay between the option to intermediate trade via matching the buyer and the seller (agency-model) and the option to intermediate trade by buying and reselling (merchant-model). We complement the existing literature by studying the trade-offs generated by these two business models and highlight basic mechanisms that can help explain the prevalence of the agency model in many online and offline markets.

One function of intermediaries often highlighted by the literature is to decrease the search costs of buyers and sellers (e.g., Rubinstein and Wolinsky, 1987; Yavas, 1992, 1994, 1996; Gehrig, 1993; Watanabe, 2010, 2013; Johri and Leach, 2002; Shevchenko, 2004; Smith, 2004; Shi and Siow, 2012). The basic premise of this literature is that there are frictions that may prevent buyers and sellers from transacting, and that intermediaries are endowed with a technology that reduces frictions. For example, intermediaries can hold inventories so that they improve matches between buyers and sellers. These models involve random matching and abstract away from problems of asymmetric information, focusing on the circumstances in which intermediation occurs.

Another part of the literature emphasizes the role of intermediaries in certifying the quality of goods, which is known to the sellers but not to the buyers, (e.g., Milgrom, 1981; Shavell, 1994; Lizzeri, 1999; Jovanovic, 1982; Biglaiser, 1993; Li, 1998; Inderst and Ottaviani, 2012). We share with this literature the emphasis on asymmetric information among sellers, intermediaries and buyers, as well as an explicit model on how this information can be aggregated. However, our focus is on the asymmetric information between the seller and the intermediaries with respect to buyers' valuations, and, in our model, the aggregation of information occurs via referrals.

Our work is also related to the literature on resale, (Calzolari and Pavan, 2006; Zheng, 2002; Jehiel and Moldovanu, 1999, e.g.) and the literature on intermediation in networks (e.g., Blume et al., 2007; Condorelli and Galeotti, 2012; Nava, 2012). In these papers, a seller can negotiate with a subset of buyers, who, in turn, can negotiate with other buyers. These works focus on understanding how the possibility of resale affects the seller's incentives and the possible inefficiencies that resale markets may introduce. Referrals are not possible.

Finally, our paper relates to the literature on referrals. Arbatskaya and Konishi (2012) and Park (2005) focus on horizontal referrals in models in which consumers try to find

products matched to their tastes. Park (2005) looks at the interaction between customers and experts in an infinitely repeated game, in which experts' referrals are modeled as cheap talk. Garicano and Santos (2004) focus on vertical referrals by looking at the matching between agents and opportunities. Montgomery (1991) and Galenianos (2012) study the effects of allowing firms to hire through referrals for labor-market outcomes. In these two papers, hiring through a referral means hiring a social contact of a current employee. We are interested in the role of referrals in markets where trade can be intermediated and there is asymmetric information, and so our model of referral and the questions we pose are different from these works'.

2 An Illustrative Example

We develop a simple example to illustrate the main findings of the paper. All the results hold in the context of a more general model, which is introduced in Section 3.

In this example, the seller has an object for sale and is connected to one intermediary, who is connected to one buyer. The valuation of the buyer for the object is $v_B \in \{v_L, v_H\}$, with $0 < v_L < v_H$; the seller believes that $v_B = v_H$ with probability $f > 0$ and the intermediary knows v_B . The seller and the intermediary derive no utility from consuming the object.

2.1 Merchant Model

Suppose, first, that the seller can trade only via the intermediary. If the intermediary does not acquire the object, the game ends. Otherwise, the intermediary becomes the new owner of the object and resells to the buyer.

It is optimal for the seller to post a price equal to v_L if $v_L > fv_H$ and a price equal to v_H otherwise. The intermediary accepts the offer if, and only if, the price is lower than or equal to v_B . If the intermediary acquires the object, he asks a price equal to v_B from the buyer, who accepts the offer.

Result 1 *Suppose that the seller can trade only via the intermediary. The ex-ante expected profit of the seller is $\Pi = \max\{v_L, fv_H\}$. The seller does not attain all the available surplus: if $v_L > fv_H$, the intermediary makes a profit when $v_B = v_H$; if $v_L < fv_H$, the outcome is inefficient when $v_B = v_L$.*

2.2 Agency Model: the Role of Referrals

In the agency model, the intermediary receives a referral fee $\kappa > 0$ if he refers the buyer to the seller. Given κ , the intermediary decides whether or not to refer the buyer. If the intermediary does not refer the buyer, the seller can sell only to the intermediary, and the game proceeds as in the previous section. If the intermediary refers the buyer, then the seller pays κ to the intermediary and trades directly with the buyer.

Our main result is that when the possibility of referrals is present, in equilibrium, the intermediary specializes in agency.

Result 2 (Unraveling) *In every equilibrium, the intermediary refers his buyer regardless of his type.*

The intermediary must always refer the low-valuation buyer because if the buyer is not referred, the seller will ask the intermediary for a price that is at least as high as v_L . In light of this, the intermediary must also refer the high-value buyer. Suppose that this is not the case; then, since the intermediary always refers the low-value buyer, the seller, upon non-referral, infers that the buyer has a high value and so asks the intermediary for a price v_H .

Even if the intermediary provides access to his buyer at virtually no cost, the intermediary still retains all information about his buyer's valuation. However, once the buyer has been referred, the seller can extract the intermediary's information using an arbitrarily small commission fee: The seller asks the intermediary to report the buyer's value; commits to ask the buyer for a price that equals to the reported value; and pays the intermediary a percentage of his final revenue. The interests of the seller and of the intermediary become perfectly aligned. Let $\Pi^* = fv_H + (1 - f)v_L$ be the "integrated-industry profit." Since the intermediary has complete information about buyers' valuation, the integrated-industry profit equals total surplus.

Result 3 (Integrated-Industry Profit) *The ex-ante expected aggregate profit of the seller and the intermediary equals the integrated-industry profit. The seller obtains $\Pi^* - \kappa$ and the intermediary gets κ .*

2.3 Endogenous referral fee

So far, the referral fee has been treated as positive and exogenous. Next, we consider referral fees that arise as the result of a negotiation between the seller and the intermediary.

First, consider a scenario in which the seller has all the bargaining power in setting the referral fee. In particular, suppose that at the beginning of the game, the seller announces a referral fee $\kappa \geq 0$, and then the game proceeds as in the previous section. We know from Result 3 that if the seller announces κ , then he obtains $\Pi^* - \kappa$. Since the seller can choose κ , we have:

Result 4 (Referral fees set by seller) *Consider the game in which the seller announces referral fees. In every equilibrium, the seller obtains the all integrated industry profit Π^* .*

When the seller organizes referrals, the intermediary is left with no rent. In this case, if the intermediary has the ability to commit to not referring his buyers, he will choose to do so.

Second, we now endow the intermediary with the power to propose the referral fee. In particular, the intermediary proposes a referral fee and the seller decides whether or not to accept it. If the seller rejects the referral fee, we move to the game described in Section 2.1. If the seller accepts, we move to the game described in Section 2.2. Our main result is that there is a class of equilibria where the joint profit of the seller and the intermediary equals the integrated-industry profit Π^* .

Result 5 (Referral fees set by intermediary) *Consider the game in which the intermediary announces referral fees. There is a class of equilibria where the intermediary proposes $\kappa^* \in [0, \Pi^* - \Pi]$, the seller accepts κ^* and the intermediary refers the buyer. The aggregate payoff of seller and intermediary is the integrated-industry profit Π^* , and the intermediary gets κ^* .*

2.4 Summary

The analysis of this example reveals that, in equilibrium, the agency model prevails. Moreover, the seller and the intermediary jointly achieve the highest possible profit available, given their information. Therefore, distortions typical of trading environments in which intermediaries act as merchants are absent. How the value added by referrals is shared between the seller and the intermediary depends on their bargaining power when negotiating the referral scheme.

The remainder of the paper generalizes these results, explores their limits, and discusses their policy implications.

3 Model

The seller of an indivisible object is connected to a set of intermediaries, \mathcal{I} , and each intermediary $i \in \mathcal{I}$ is linked to a set of buyers B_i . Intermediary i privately observes the set B_i that is drawn from a finite set of potential buyers \mathcal{B}_i .¹⁰ Intermediaries have exclusive access to their buyers—that is, $\mathcal{B}_i \cap \mathcal{B}_{i'} = \emptyset$ for all $i, i' \in \mathcal{I}$, $i \neq i'$. The assignment of buyers to intermediaries is independent across intermediaries.

All agents are risk-neutral. The seller and the intermediaries derive zero utility from consuming the object. All potential buyers have strictly positive and independent private consumption values. The consumption value of buyer j , denoted v_j , is drawn from a distribution with support in a bounded measurable subset of the positive real line. We assume that all values are independently drawn.

Intermediary i observes a signal s_i^j about j 's valuation, for each buyer $j \in B_i$. Only intermediary i and buyer j observe the signal s_i^j . The number of possible signals about each buyer is finite; s_i denotes the vector of signals that intermediary i observes about his buyers B_i . A type of intermediary i is $t_i = (s_i, B_i)$, and \mathcal{T}_i is the set of possible types. Anything that it is not privately observed is common knowledge.

There is a referral fee $\kappa_i \geq 0$ that determines the payment to intermediary $i \in \mathcal{I}$ per referred buyer. It is natural to think that the referral fee κ results from negotiations between the seller and the intermediaries. Initially, we take the referral fee κ_i as exogenous and derive a set of results that hold regardless of how the fee is determined. In Section 5, we endogenize the referral fee by examining alternative bargaining scenarios.

We look for perfect Bayesian equilibria of the following game:

Referral Phase. Given $\kappa_i \geq 0$ for all $i \in \mathcal{I}$, intermediaries simultaneously decide the subset of buyers they want to refer; \hat{B}_i denotes the set of buyers referred by intermediary i . Intermediary i with type $t_i = (s_i, B_i)$ can refer only a subset of his buyers in B_i . The seller observes the set of referred buyers and pays each intermediary an amount equal to the referral fee times the number of buyers he has referred. We do not impose any restriction on what intermediaries and buyers observe about the set of buyers referred by other intermediaries. However, we maintain that each intermediary always observes at least what is observed by his buyers and that it is common knowledge who observes what.

¹⁰This implies that a buyer of intermediary i does not observe who else is connected to i . This assumption is not essential.

Trading Phase. *Stage 1 (Sale):* The seller selects a selling procedure to sell the object to intermediaries and to referred buyers. The game ends if no trade takes place or if a buyer gets the good. However, if an intermediary $i \in \mathcal{I}$ acquires the object, the game proceeds to the following resale stage.

Stage 2 (Resale): If an intermediary $i \in \mathcal{I}$ acquires the object, he selects a mechanism to resell the object to the buyers that he has not referred. The game ends at the end of the resale phase, regardless of whether or not trade takes place.

We do not impose any restriction on what intermediaries and buyers observe, apart from the assumption that, in the sale phase, an intermediary always observes at least what his buyers observe and that it is common knowledge who observes what.

We assume that the seller and intermediaries propose direct incentive-compatible mechanisms that satisfy voluntary participation. Our results do not depend on the fact that the seller and the intermediary can choose *any* selling procedure: They hold for much simpler and more natural selling procedures, such as auctions with optimally-chosen reserve prices and sequential take-it-or-leave-it offers, among others. We clarify this in Section 4; in Section 7, we develop an example in which the seller and intermediaries can only post a uniform price.

4 Equilibrium Referrals: Agency Prevails

Our first result shows that, as long as referral fees are positive, in equilibrium, each intermediary refers *all* his buyers. Thus the equilibrium predicts intermediaries' specialization in agency. The driving force is that the seller will never sell the good to an intermediary at a price that allows all intermediaries' types to obtain a strictly positive profit from reselling. Thus, among types that do not refer, there must be one that obtains zero profit from resale. As long as the referral fee is positive, this type strictly prefers to refer. Hence, every equilibrium in which an intermediary refers some but not all buyers unravels.

Theorem 1 (Unraveling) *Suppose that $\kappa_i > 0$ for all $i \in \mathcal{I}$. In every equilibrium, all intermediaries refer all their buyers to the seller regardless of their types.*

Proof We prove the result in three steps: Step 1 introduces the notation required to prove Lemma 1, which is the core of Step 2. Step 3 uses Lemma 1 to conclude the proof.

Step 1: Resale Types. At any history following referrals, the seller's posterior belief about some intermediary i depends on the prior, on the set of referred buyers \hat{B}_i , and on i 's referral

strategy. Let $\mathcal{T}_i(\hat{B}_i) \subseteq \mathcal{T}_i$ denote the support of the seller's posterior.

We denote by $R(t_i, \hat{B}_i)$ the expected revenue from resale of intermediary i with type $t_i \in \mathcal{T}_i(\hat{B}_i)$. The resale value $R(t_i, \hat{B}_i)$ depends on t_i and on the set of buyers that i has referred \hat{B}_i .¹¹

We define the set $\mathcal{T}_i^*(\hat{B}_i) \equiv \{t_i \in \mathcal{T}_i(\hat{B}_i) : \hat{B}_i \subsetneq B_i \text{ and } R_i(t_i, \hat{B}_i) > 0\}$ as the subset $\mathcal{T}_i(\hat{B}_i)$ that contains all the types that have strictly positive resale values. Whenever $\mathcal{T}_i^*(\hat{B}_i)$ is not empty, we define $\underline{t}_i(\hat{B}_i)$ as the type with the lowest resale value—i.e., $\underline{t}_i(\hat{B}_i) = \arg \min_{t_i \in \mathcal{T}_i^*(\hat{B}_i)} R(t_i, \hat{B}_i)$.

Step 2: Lowest type makes zero. At the beginning of the trading stage, the seller chooses an incentive-compatible direct mechanism that satisfies participation constraints (i.e., provides payoff higher than or equal to zero to all participants). In our setup, participants are intermediaries and buyers. Buyer i 's type is his valuation v_i and, henceforth, we write $t_i \equiv v_i$ when agent i is a buyer. When the participating agent is an intermediary, his type is $t_i \equiv (s_i, B_i)$ in $\mathcal{T}_i(\hat{B}_i)$.¹²

Given a vector of reports t_i, t_{-i} from all agents, let $p_i(t_i, t_{-i})$ be the expected probability that i gets the good and let $x_i(t_i, t_{-i})$ be the expected payment.¹³ Then, under truth-telling, the expected continuation payoff of an intermediary i of type $t_i = (s_i, B_i)$ who has referred \hat{B}_i is:

$$U_i(t_i) = E_{t_{-i}} \left[p_i(t_i, t_{-i}) R_i(t_i, \hat{B}_i) - x_i(t_i, t_{-i}) \right], \quad (1)$$

where expectation is taken using the posterior belief of the intermediary, which is known to the seller and is independent of t_i .

Lemma 1 *Consider any equilibrium history of the game where intermediary i has referred a set of buyers $\hat{B}_i \subsetneq B_i$. Then, the continuation equilibrium payoff of intermediary type $\underline{t}_i(\hat{B}_i)$ is zero; i.e.,*

$$U_i(\underline{t}_i(\hat{B}_i)) = 0. \quad (2)$$

¹¹The resale value is the revenue corresponding to the revenue-maximizing mechanism chosen by the intermediary. Because intermediaries may have superior information relative to the buyers, this is a mechanism design problem by an informed principal. Theorem 6 in Skreta (2011) implies that the intermediaries' maximum expected revenue is the same whether or not the buyers know the intermediary's information. McAfee and McMillan (1987) establish a version of this result for the case when the intermediary's (seller's) information is the number of buyers. Hence, we can treat this as a standard mechanism design problem where the principal has no private information.

¹²The results in footnote 11 imply that the seller's maximum expected revenue is the same whether or not his information is common knowledge. Hence, we do not specify the private information of a buyer arising from observations made during the game, given that these observations are known to the seller.

¹³From McAfee and McMillan (1987), it follows that it is without loss of generality to assume that all participants know who participates.

Proof See Appendix A. ■

Step 3: Unraveling. Let $\kappa_i > 0$ and suppose, to derive a contradiction, that, in equilibrium, there is a type $t_i = (s_i, B_i)$ of an intermediary i who refers a subset of buyers $\hat{B}_i \subsetneq B_i$ with positive probability. This implies that there exists a history where the seller observes \hat{B}_i and the set $\mathcal{T}_i^*(\hat{B}_i)$ is not empty. In order for intermediary type $t_i(\hat{B}_i)$ to find it profitable not to refer all his buyers, it must be the case that

$$U_i(t_i(\hat{B}_i)) \geq \kappa_i \cdot |B_i \setminus \{\hat{B}_i\}| > 0,$$

where $|B_i \setminus \{\hat{B}_i\}|$ is the number of buyers that the intermediary type $t_i(\hat{B}_i)$ did not refer to the seller. This contradicts (2). ■

Theorem 1 generalizes well beyond the model described in Section 3. The result holds if the seller is forced to use simple selling procedures, as long as they leave no rent to the lowest resale value intermediary who participates in the resale market (i.e., Lemma 1 holds). This is the case if, for example, after referral, the seller offers the intermediaries a commission to report their types and commits to a sequence of take-it-or-leave-it offers, or to use auctions with individualized reserve prices. The result can also be easily extended to the case where buyers' values are statistically correlated or there are multiple objects for sale. The analysis can be also modified to incorporate fixed costs associated with referring a buyer.

The unraveling result may fail, instead, if all types of an intermediary who have positive resale value (even the lowest one) make a profit from acquiring and reselling the good. Section 7 explores a number of environments in which Theorem 1 fails and, as a consequence, intermediaries adopt a hybrid business model in equilibrium.

Theorem 1 shows that, in equilibrium, intermediaries specialize in agency. Since intermediaries are pooling in their referral strategy, the set of referred buyers does not convey any information to the seller about their valuations. Therefore, after the referral phase, intermediaries still possess private information that is valuable to the seller. Can intermediaries extract profits from this information?

The answer is no. Once the seller is connected to all buyers, the intermediaries' information becomes payoff-irrelevant to them, and the seller can acquire it at no cost. For instance, the seller can offer a very small fraction of his final revenue to intermediaries in exchange for their information, while committing to use the information to optimize his sale to the buyers. When this mechanism is used, the seller's and the intermediaries' interests are aligned, and intermediaries have an incentive to report their information correctly. The use of such

commission fees is widespread in industries in which the price between buyers and seller is negotiable, and intermediaries have relevant information about buyers' preferences (e.g., in real estate, online markets, etc.).

We call *integrated-industry profit*, denoted Π^* , the revenue that the seller achieves ex-ante when he expects to receive all signals from the intermediaries and to have access to all buyers. The unraveling result, together with the mechanism outlined above, suggest that, under the profile of referral fees $\{\kappa_i\}_{i \in \mathcal{I}}$, the seller can always obtain

$$|\mathcal{I}|(1 - \alpha)\Pi^* - \sum_i \kappa_i E[|B_i|],$$

using commission fee α , where $E[|B_i|]$ denotes the seller's prior expected number of buyers of intermediary i . Hence, given that the seller is free to pick α arbitrarily small, we obtain the following result.

Theorem 2 *Suppose that $\kappa_i > 0$ for all $i \in \mathcal{I}$. In every equilibrium, the ex-ante expected joint payoff of seller and the intermediaries is the integrated-industry profit Π^* . The ex-ante expected equilibrium payoff of the seller is $\Pi^* - \sum_i \kappa_i E[|B_i|]$, while intermediary i obtains $\kappa_i E[|B_i|]$.*

Proof See appendix A. ■

The result of Theorem 2—i.e., that all buyers are referred and information is aggregated in such a way that integrated-industry profits are achieved (jointly by the intermediaries and the seller)—is robust. For instance, we can obtain an analogous result if we write a model in which the referral fee represents a percentage κ_i of the final seller's revenue and, in the referral phase, we allow intermediaries to, in addition to referring their buyers, also send a (cheap talk) message about buyers' type. The analysis of this case can be found in an online appendix (Appendix B).

5 Endogenous referral fee

It is natural to think that the level of the referral fee κ_i is the result of a negotiation between the seller and the intermediaries. In this section, we consider two polar bargaining scenarios, one in which the seller proposes the referral fees and another in which intermediaries propose. The former case captures environments in which the seller has more bargaining power than the intermediaries, while the latter captures environments in which the bargaining power is

more concentrated on the intermediaries' side. Our main finding is that when the seller has bargaining power, referral fees will be equal to the marginal cost of referring a buyer (i.e., zero in our model), while when intermediaries have bargaining power, they can extract the value of their access to buyers and their information.

5.1 Bargaining power to the seller

Consider a variant of the model in Section 3, in which prior to the referral phase, the seller publicly proposes a referral fee $\kappa_i \geq 0$ that determines the payment to intermediary $i \in \mathcal{I}$, per referred buyer. After the announcement, the game proceeds as described in Section 3.

Theorem 2 implies that if the seller announces $\kappa_i > 0$ for all $i \in \mathcal{I}$ he obtains

$$\Pi^* - \sum_i \kappa_i E[|B_i|],$$

(recall that $E[|B_i|]$ is the expected number of buyers of intermediary i given the seller's prior, and Π^* is the integrated industry profit). Since the seller can choose κ_i arbitrarily small, we conclude that in all equilibria of this game, he must obtain Π^* .

Theorem 3 *The game in which the seller announces referral fees has a unique equilibrium outcome where the intermediaries specialize in agency and earn zero profits, and the ex-ante expected equilibrium payoff of the seller is the integrated-industry profit Π^* .*

Proof See Appendix A. ■

Suppose that we would allow seller to choose any mechanism at the beginning of the game. It's clear that the best outcome he can achieve is the integrated industry profit. One contribution of Theorem 3 is, then, to show that the seller can obtain the same outcome with a simple and natural game in which the seller offers per-buyer referral fees, and in which intermediaries can choose to be either agents (refer) or merchants (buy and resell).

While referrals maximize industry profits, when the scheme is organized by the seller, it leaves intermediaries with a zero payoff. However, when intermediaries operate as merchants, they expect an information rent. Hence, when the seller has the bargaining power to introduce referral fees, there is a conflict of interest among intermediaries and the seller with regard to the introduction of such schemes. In fact, in these cases, intermediaries would gain by being able to commit ex-ante to not referring their buyers. We show in the next section that intermediaries may also profit from referrals when they have the bargaining power to set up the scheme.

5.2 Bargaining power to intermediaries

We now consider the scenario in which intermediaries propose referral fees to the seller. The timing of the game is as follows. First, intermediaries simultaneously announce per-buyer referral fees to the seller. Let $\{\kappa_i\}_{i \in \mathcal{I}}$ be the profile of proposed referral fees. Then, the seller decides which referral fees to accept and which to reject. Intermediaries and buyers may or may not observe the fees posted by other intermediaries (and their acceptance), but who observes what is common knowledge and an intermediary observes more than his buyers. If the seller rejects all referral fees, the game moves directly to the trade phase, as described in Section 3. If at least one referral fee is accepted, the game moves to the referral phase, and intermediaries whose referral fee is accepted can refer their buyers. After referrals take place, the game moves to the trade phase, as described in Section 3.

We first observe that, in every equilibrium where the seller accepts referral fee $\kappa_i > 0$, it must be the case that intermediary i refers all his buyers to the seller, regardless of his type (Theorem 1 holds). Our second observation is that, in line with the analysis in Section 4, once intermediary i refers all his buyers, the seller is able to extract all of his information at no cost. In light of this, we can now show that, when intermediaries propose referral fees, there is a class of equilibria in which all buyers are referred (i.e., intermediaries specialize in agency); the joint profit of seller and intermediaries is the integrated-industry profit; and intermediaries can extract part of this profit.

Let Π indicate the expected payoff of the seller in the trading phase when she is connected only to intermediaries and has no additional information. For any subset of intermediaries $\hat{\mathcal{I}} \subseteq \mathcal{I}$, let $\Pi_{-\hat{\mathcal{I}}}^*$ indicate the payoff that the seller anticipates, when he expects to be connected to all buyers of intermediaries in $\mathcal{I} \setminus \{\hat{\mathcal{I}}\}$ and to know their information, but he is neither connected to nor has access to the information of intermediaries in $\hat{\mathcal{I}}$. Note that $\Pi_{-\emptyset}^* = \Pi^*$ and $\Pi_{-\mathcal{I}}^* = \Pi$. In words, $\Pi^* - \Pi_{-\hat{\mathcal{I}}}^*$ is the marginal value to the seller from being connected in the trading stage to the buyers of intermediaries in $\hat{\mathcal{I}}$ and obtaining all available information from them.

Theorem 4 *In the game where intermediaries propose referral fees, for any profile of referral fees $\kappa^* = \{\kappa_i^*\}_{i \in \mathcal{I}}$ such that*

$$\Pi^* - \Pi_{-\hat{\mathcal{I}}}^* \geq \sum_{i \in \hat{\mathcal{I}}} \kappa_i^* E[|B_i|], \text{ for all } \hat{\mathcal{I}} \subseteq \mathcal{I}, \quad (3)$$

there is an equilibrium in which each type of intermediary $i \in \mathcal{I}$ proposes κ_i^ , and the seller accepts the proposal and the intermediaries specialize in agency. In this equilibrium, the ex-*

ante expected equilibrium payoff of the seller is $\Pi^* - \sum_{i \in \hat{\mathcal{I}}} \kappa_i^* E[|B_i|]$, and the ex-ante expected equilibrium payoff of intermediary i is $\kappa_i^* E[|B_i|]$.

Proof See Appendix A. ■

Every equilibrium in this class is sustained by the seller's belief that any deviation to proposing a higher fee must come from the type of intermediary having the highest resale value.¹⁴ Given this, intermediaries never want to propose a different referral fee. The seller accepts all proposals because condition 3 guarantees that refusing any subset of them would result in a lower payoff.¹⁵

In the class of equilibria of Theorem 4, an upper bound of the referral fee that the intermediary i can ask is $\frac{\Pi^* - \Pi_{-i}^*}{E[|B_i|]}$, since condition (3) must also hold when $\hat{\mathcal{I}} = \{i\}$. Hence, an upper bound of the equilibrium payoff of intermediary i is $\Pi^* - \Pi_{-i}^*$, which is the marginal contribution of intermediary i to the integrated industry profit. It then follows that as the number of intermediaries grows larger, the maximum fee that each intermediary can ask converges to zero.

When intermediaries propose the referral fees, there also exist equilibria in which intermediaries do not specialize in agency. For example, consider an equilibrium in which intermediary i , regardless of his type, demands a referral fee above his expected per-buyer marginal value $\kappa_i^* = \frac{\Pi^* - \Pi_{-i}^*}{E[|B_i|]}$, and the seller, whenever he observes a referral fee different from κ_i^* , believes that the intermediary has the highest possible resale value. In this equilibrium, the seller refuses every proposal, including κ_i^* , and intermediaries act as merchants. Moreover, in an online appendix (Appendix C), we present an equilibrium in the same environment of Section 2, under which referral occurs with positive probability, but not with probability one, and so the two business models co-exist. In every equilibrium in which referral does not occur with probability one, the aggregate payoff of the seller and intermediaries is lower than Π^* , and the seller's payoff is always at least Π (as the seller can always ensure this payoff by rejecting all fees). When there is only one intermediary, this implies that for each of these equilibria, there is an equilibrium in which the intermediary specializes in agency (defined in Theorem 4) that results in a Pareto improvement.

¹⁴These out-of-equilibrium beliefs are natural, as the intermediaries with the highest resale value are the ones who obtain the highest payoffs in the trading phase. For these reasons, the class of equilibria defined in Theorem 4 survives to standard equilibrium refinements, such as intuitive criterion and the like.

¹⁵The condition in (3) is always satisfied for $\kappa_i^* = 0$ for all $i \in \mathcal{I}$, and so there is always an equilibrium in this class.

6 Policy implications and welfare effects

Our analysis shows that referrals generate an outcome that is equivalent to one that might be obtained through an explicit vertical agreement (e.g., royalty contracts or vertical integration). For instance, under both vertical integration and an appropriately designed royalty contract, the aggregate profit of the seller and the intermediary is the integrated industry profit Π^* . Incidentally, this raises the question of why the agency model is so widely used, instead of, say, royalty contracts. One obvious answer is that royalty contracts become very complicated in the presence of multiple intermediaries, whereas referral schemes work independently of the number of intermediaries.¹⁶

That, in trading environments, the option of referring at a positive fee and vertical agreements may have similar effects on market outcomes is of interest to policy makers and competition authorities, who are often concerned that vertical contracts may restrict competition and decrease aggregate welfare. The discussion that follows shows that, much like the classic vertical agreements that solve double marginalization in the single intermediary case, referrals are likely to have positive effects on aggregate welfare. The welfare analysis we perform compares the (ex-ante expected) aggregate welfare in the equilibrium outcome of our game to the equilibrium outcome when referrals are not possible.

If there is only one intermediary, then referrals bring, unambiguously, an increase in the aggregate welfare relative to a ban on referrals: With referrals, in equilibrium, the seller deals with the same buyers and with the same information that the intermediary would deal with after having acquired the object from the seller in a pure merchant model. So, the welfare generated in the trading phase is the same in the two situations. However, in a pure merchant model, the asymmetric information between the seller and the intermediary implies that, with some probability, the object will not reach the intermediary, thereby creating a potential loss in aggregate welfare.

When there are multiple intermediaries a new complication arises. Under the merchant model only one intermediary will obtain the object, implying that many buyers do not even have the chance to participate in the resale. Referrals eliminate this source of inefficiency but may introduce a different one. In particular, when the seller aggregates a set of asymmetric buyers coming from different intermediaries, he will distort the outcome away from the efficient allocation in an attempt to maximize revenues.

¹⁶Since intermediaries access a different subset of buyers, in order to achieve the integrated industry profit, a vertical restraint agreement would require a complicated system in which intermediaries bid after having sold the option to buy to buyers, or variants of this.

Nevertheless, there are some natural cases where we can conclude that the agency model is unambiguously more efficient than the merchant model, given that the seller will generate no extra distortions. One such case is when each intermediary has private information about the number of connected buyers, but intermediaries and the seller have the same information about buyers' valuations, which are independently and identically distributed across buyers. This may be a good approximation of industries in which the main asset of intermediaries is the large set of potential customers.

Another case is the following.¹⁷ Suppose that all consumers' valuations are drawn from the same distribution. However, while the distribution is not perfectly known to the seller, this information (i.e., the correct demand function) may be available to intermediaries. Such information asymmetries between seller and intermediaries are descriptive of the online travel industry, in which it is reasonable to assume that large intermediaries like Expedia and Bookings.com have more precise information on consumers' trends than small and localized hoteliers have.

The welfare superiority of the agency model is not limited to environments in which the seller is ultimately connected to buyers with the same expected demand. Coming to an environment with asymmetric buyers, referrals increase efficiency if there are a number of different consumer types (i.e., different distributions of valuations), and all the intermediaries are connected to sets of buyers that contain *all* the different consumer types. Under this scenario, the optimal mechanism that the seller uses in the trading phase—in which all buyers participate—cannot introduce distortions in addition to those that each intermediary would introduce in the resale phase anyway.

Finally, we note that referrals also unambiguously increase efficiency vis-à-vis intermediation when all buyers are ex-ante identical, and the seller cannot use the intermediaries' information in designing the selling procedure due to legal reasons or reputation concerns. There is, indeed, a large debate over confidentiality policies adopted by Facebook, Google, and other social networking websites. In addition, Amazon's past attempts to price-discriminate based on buyers' purchase history led to users' outrage.¹⁸

Beyond these scenarios, a derivation of a general result in the context of the model that we are using is problematic. In an online appendix (Appendix D), we develop a natural and tractable example in which such distortions are present, and we show that referrals are still,

¹⁷This environment does not explicitly fit the model but it can be easily accommodated.

¹⁸See CNN (2005).

overall, beneficial.¹⁹

7 Extensions

We now investigate a number of environments in which Theorem 1 fails. We show that the failure of the unraveling result leads to the emergence of equilibria in which referrals and intermediated trade co-exist. Often, this implies that the aggregate payoff of seller and intermediaries is below the integrated industry profit.

The unifying motive underlying the co-existence of the two business models of intermediation is the failure of Lemma 1. When extra frictions are assumed, an intermediary may sometimes expect a positive profit by not referring buyers who generate high profits from resale. When these profits exceed the referral payments, in equilibrium, intermediaries do not specialize in agency. We develop these intuitions using simple examples.

7.1 Non-specialized intermediaries

In our model, intermediaries have no private value for consuming the object. They are *specialized* in intermediation. We now consider intermediaries who have a private value for consuming the object—*non-specialized intermediaries*. We show in the context of a simple example that for low referral fees, intermediaries do not specialize in referrals. As a consequence, intermediated trade and referrals coexist, and intermediaries may make positive profits.

We assume that there is one intermediary who has no buyer with probability $q \in (0, 1)$ and has one buyer with the remaining probability.²⁰ The buyer has valuation v_H , which is known to both the intermediary and the seller. In addition, we assume that the intermediary has a known consumption value v_I and $v_H > v_I > (1 - q)v_H$. The latter assumption guarantees that, in the case where trade can be only intermediated, the seller prefers to ask v_I of the intermediary.

Proposition 1 *(i) If $\kappa \in [0, v_H - v_I)$, then, in equilibrium, the intermediary never refers the buyer. (ii) In every equilibrium of the game in which the seller announces referral fees, the seller obtains payoff v_I and the intermediary with the buyer obtains $v_H - v_I$.*

¹⁹In the example, the seller faces two intermediaries with one buyer each. The valuations of the buyers are uniformly distributed in $[0, t_1]$ and $[0, t_2]$; the intermediaries know t_1 and t_2 , but the seller does not know them.

²⁰The assumption that the seller is uncertain about which buyers are connected to the intermediary is crucial for the unraveling result to fail in this example.

Proof See Appendix A. ■

When $\kappa < v_H - v_I$, in every equilibrium the intermediary never refers his buyer. If the intermediary were to refer the buyer with positive probability, upon observing non-referral, the seller would believe that the intermediary has no buyer with a probability which is at least q and, therefore, would optimally post a price v_I . But then, when the intermediary has a buyer, he strictly prefers to hold the buyer, buying at a price v_I and then reselling at v_H , instead of referring the buyer and getting κ . The second part of the proposition shows that, since the seller cannot get access to all buyers at arbitrarily low costs, the intermediary obtains positive profits even if the seller has full bargaining power in announcing referral fees. Finally, we note that in this example, the aggregate profit of seller and intermediary is the integrated industry profit. However, it is easy to construct more-complex examples where the failure of the unraveling result prevents the seller and the intermediary from maximizing industry profits.

7.2 Posted price

We have allowed the seller to choose any incentive-compatible direct mechanism in which neither buyers nor intermediaries can be forced to participate. We now consider the simpler and common selling procedure of *posting a price*. We show that the unraveling result may fail—that, in equilibrium, referrals and intermediated trade can coexist and that the aggregate payoff of the seller and the intermediaries can be below the integrated industry profit.

We analyze the case where, in the trading stage, the seller asks the information of intermediaries who have referred buyers and then commits to post a single price that is optimal, given the information provided. The seller incentivizes intermediaries to tell the truth by offering them a commission equal to a fraction α of the final revenue. Once the price is posted, all intermediaries and buyers can buy at that price. If more than one agent wishes to purchase, each agent obtains the object with equal probability.

We use the following simple example. With probability q , the intermediary has only one buyer with value $0 < v_L < 1$ (type *A*). With probability $1 - q$, he also has a second buyer with value $v_H = 1$ (type *B*). We assume that the intermediary knows the value of the buyers and that $v_L < 1 - q$. The latter assumption implies that, absent the possibility of referrals, the seller will post price $v_H = 1$.

Proposition 2 (i) If $\kappa \in [0, (1 - v_L)/2)$, there is no equilibrium in which the intermediary always refers all buyers. (ii) In every equilibrium of the game in which the seller announces

referral fees, the seller announces $\kappa^* = 0$; there is a strictly positive probability that the high-value buyer is not referred; and the seller charges price v_H regardless of who is referred. In equilibrium, the seller obtains a payoff of $(1 - q)v_H$ and the intermediary obtains a payoff of zero. The equilibrium outcome is analogous to the one obtained if referrals are not allowed.

Proof See Appendix A. ■

When the referral fee is positive but not too high, an equilibrium with full referral is not viable. Indeed, if the intermediary always refers his buyers, whenever the intermediary refers only one buyer, the seller will post a price v_L . But this implies that when the intermediary has two buyers, he will prefer to refer only the low-value buyer and obtain a high profit from buying and reselling to the high-value buyer. The second part of the proposition shows that the impossibility of obtaining full referral with arbitrarily low referral fees implies that, in equilibrium, the intermediary always refers low-value buyers while retaining high-value buyers with positive probability. Hence, the intermediary acts both as a referral agent and as a merchant.

7.3 Budget-Constrained Intermediaries

In this section, we consider a scenario in which intermediaries are budget-constrained and have limited access to financial markets. We show that if intermediaries are budget-constrained, the unraveling result fails, and referral and intermediated trade often co-exist.

We slightly modify the example developed in Section 2. As in that example, we assume that there is only one intermediary connected to one buyer. The valuation of the buyer is $v_B \in \{v_L, v_H\}$, with $v_L < v_H$; the seller believes that $v_B = v_H$ with probability $f > 0$, and the intermediary knows v_B . In addition, we now assume that the intermediary has an amount of cash $C < v_H$, which is known to the seller.

It is clear in this case that the seller cannot get the buyer always referred with an arbitrary small referral fee. Because the maximum price that the seller can charge is C , the intermediary with a high-value buyer will never be willing to refer the buyer for a referral fee $\kappa < v_H - C$. Again, the unraveling result of Theorem 1 fails. The next Proposition shows the consequence of this failure in the case where the seller announces a referral fee.

Proposition 3 *Consider the game in which the seller announces a referral fee.*

1. *Suppose that $C < v_L$. In equilibrium, the seller announces $\kappa^* = 0$, each intermediary's type chooses not to refer the buyer, and conditional on non-referral, the seller asks*

a price C of the intermediary.²¹ Hence, the seller gets C , the intermediary with a high-value buyer gets $v_H - C$ and the intermediary with a low-value buyer gets $v_L - C$.

2. Suppose that $v_L < C < v_H$. In equilibrium, the seller announces $\kappa^* = 0$, the intermediary with a low-value buyer always refers, the intermediary with a high-value buyer never refers, and, conditional on non-referral, the seller asks a price C of the intermediary. Hence, the seller gets $(1 - f)v_L + fC$, the intermediary with a low-value buyer gets zero and the intermediary with a high-value buyer gets $v_H - C$.

Proof See Appendix A. ■

When both types are budget-constrained, referrals do not help the seller. To get the low-value buyer referred, he must offer at least $v_L - C$. Therefore, the seller is better off without using referrals.

When only the intermediary with a high-value buyer is budget-constrained things are a bit more subtle. First, for any positive referral fee, the intermediary always refers the low-value buyer because the price charged by the seller will never be below v_L . This implies that upon non-referral, the seller knows that the intermediary has the high-value buyer and can safely charge C . Given that the seller can guarantee himself a payoff arbitrarily close to $(1 - f)v_L + fC$ by setting an arbitrarily small referral fee, in the only equilibrium of the game she sets $\kappa = 0$, and the intermediary refers only the low-value buyer.²²

We conclude with a remark on the effect of competition among intermediaries. Suppose that there are two intermediaries, each connected to a buyer that has either valuation v_L or valuation v_H , with budgets $C_1 < C_2 < v_L$. The seller knows that he can get at least C_2 by selling to intermediary 2 or to his buyer. In light of this, intermediary 1 knows that he will make zero profits if he trades directly with the seller. Hence, intermediary 1 strictly prefers to refer his buyer for any arbitrarily low referral fee, regardless of the valuation of his buyer. Then, since the seller gets access to intermediary 1's buyer, he can generate at least v_L by dealing with that buyer. Therefore, the payoff of intermediary 2, if he does not refer, is zero.

²¹There is another equilibrium in which the seller announces $\kappa^* = v_L - C$, the intermediary with a low-value buyer randomizes between referring or not, and the intermediary with a high-value buyer never refers. This equilibrium is payoff-equivalent to the one described in the Proposition.

²²These results generalize to the case of a continuum of valuations, $v_B \in [v_L, v_H]$ as follows. If $C < v_L$, for a given referral fee κ , let \bar{v} be the highest-valuation intermediary who refers the buyer—i.e., $\bar{v} - C = \kappa$. Types above, strictly prefer to buy the good at C and resell it rather than receive the referral fee κ . The expected revenue the seller can extract from the referred buyers is strictly less than \bar{v} , so an upper bound (that is not reached) for his per-intermediary type payoff is $\bar{v} - \kappa$, which is exactly equal to C —the per-intermediary type payoff without referral. Hence, the seller is strictly better off by just selling the good to the intermediary at a price of C . It is easy to extend the argument for the case of $v_L < C < v_H$.

To conclude, the unraveling result is resurrected: In every equilibrium of this example where $\kappa > 0$, intermediaries always refer their buyer.

8 Conclusion

We have maintained that the seller has all the bargaining power to determine the terms of selling the good to the intermediaries. However, if intermediaries have the power to set the terms of purchase from the seller, things change. If there is only one intermediary, we conjecture that all intermediary's types will ask the same price, equal to seller's marginal cost, so there are no referrals. If there are multiple intermediaries, then competition among them for the good will dissipate rents, so we would expect referrals to co-exist with the merchant model. The full analysis of this case is interesting but is left for future work.

Another interesting direction for future work is to investigate the role of referrals and intermediated trade in a model in which there is competition among sellers. Preliminary investigation suggests that there are additional forces at play in such an environment—such as competition among multiple principals—which raises new issues that are beyond the scope of the present paper.

To conclude, we stress that Theorem 1 applies more generally to environments in which a third party mediates the relationship between the principal and the agent(s). For example, consider the following adverse-selection problem: A principal wants to hire a specialized agent to perform a given task. The outcome of the task is observable and determines the payoff of the principal. The agent may be more or less skilled, and his skill level determines the quality of his work. Suppose that the principal has no access to the agent, but is in contact with a job agency. The agency, in addition to having contact with the agent, is also better informed about his skills. In principle, the agency can either contract with the principal and then subcontract the work to the agent, or it can refer the agent to the principal in exchange for a fee. In this context (assuming that the principal makes a contract proposal to the agency in the former case), Theorem 1 predicts that any positive referral fee offered by the seller will cause the job agency to pass the agent to the principal.

More broadly, referrals play an important role in many markets and their role is bound to become more prominent with the proliferation of social media. We hope and expect this work to inspire new research on the effects of referrals on market outcomes.

Appendix A: Proofs not included in the main text

Proof of Lemma 1 Suppose, for a contradiction, that there is an incentive-compatible and revenue-maximizing mechanism where

$$U_i(t_i(\hat{B}_i)) = E_{t_{-i}} \left[p_i(t_i(\hat{B}_i), t_{-i}) R_i(t_i(\hat{B}_i), \hat{B}_i) - x_i(t_i(\hat{B}_i), t_{-i}) \right] = \underline{u}_i > 0. \quad (4)$$

Since all other types $t_i \in \mathcal{T}_i^*(\hat{B}_i)$ can mimic $t_i(\hat{B}_i)$ and have higher resale values, incentive compatibility implies that:

$$U_i(t_i) \geq E_{t_{-i}} \left[p_i(t_i(\hat{B}_i), t_{-i}) R_i(t_i, \hat{B}_i) - x_i(t_i(\hat{B}_i), t_{-i}) \right] \geq \underline{u}_i \text{ for all } t_i \in \mathcal{T}_i^*(\hat{B}_i).$$

Next, observe that in the proposed incentive-compatible mechanism, all types with zero resale value must have the same utility, say \tilde{u}_i , and $0 \leq \tilde{u}_i \leq \underline{u}_i$; otherwise, the mechanism would not be incentive-compatible. Observe, also, that since these types have zero resale value, they must obtain a transfer that is exactly equal to \tilde{u}_i .

Now, consider the following modification of the above mechanism. First, assign the object with zero probability to all types of i with zero resale value and set their payment to zero. Second, increase the interim payment of all types $t_i \in \mathcal{T}_i^*(\hat{B}_i)$ by \underline{u}_i . Clearly, if this new mechanism is incentive-compatible and satisfies participation constraints, it attains a strictly higher revenue for the seller, contradicting our initial hypothesis.

To see that the new mechanism is incentive-compatible start by noting that only the incentives of intermediary i have been affected. Next, observe that types with zero resale value are willing to participate and do not have a strict incentive to deviate to reporting other types with zero resale value or types with positive resale value. To see the latter, observe that the payment required from all types with positive resale value is strictly positive. Finally, consider the types with positive resale value. First, they are still willing to participate, as the lowest type among them is willing to do so. Second, they are not willing to deviate to types with zero resale value as these types never get the object and do not receive any positive transfer. Third, they are not willing to deviate to other types with positive resale value, as the mechanism was originally incentive-compatible and all the payments made by the positive resale types have been reduced by the same amount. ■

Proof of Theorem 2 To prove the existence of an equilibrium with the desired property, consider the following strategy profile. Intermediaries always refer all their buyers. The seller uses a revenue-maximizing direct mechanism given his posterior beliefs. In particular, along

the equilibrium path, the seller's optimal mechanism never assigns the good to intermediaries or pays them anything, but uses the truthful reports of intermediaries to sell optimally to the buyers. Intermediaries and referred buyers report their types truthfully. Whenever intermediary $i \in \mathcal{I}$ acquires the object, he commits to a revenue-maximizing direct mechanism given his posterior beliefs.

It is immediate to verify that this strategy profile generates an (ex-ante) expected payoff of $\Pi^* - \sum_{i \in \mathcal{I}} E[|B_i|]\kappa$ for the seller and of $E[|B_i|]\kappa$ for the intermediary. We now establish that this strategy profile is an equilibrium. The seller's strategy is, by definition, optimal. Instead, consider intermediary $i \in \mathcal{I}$. At the trading phase, when intermediary i has acquired the object, he employs a revenue-maximizing mechanism given his posterior beliefs, so deviating cannot increase his revenue. At the referral stage, if intermediary i deviates from his equilibrium strategy and does not refer all his buyers, there are two possibilities to consider. The first possibility is that the seller observes the deviation. In this case, we assign the seller the out-of-equilibrium belief that the intermediary has the highest resale value. Under these beliefs, in the optimal mechanism, the intermediary clearly gets a payoff of 0. Hence, this is not a profitable deviation. If the seller does not observe the deviation, then he will believe that intermediary i has referred all the buyers, so the mechanism that he employs at the trading phase never allocates the good to i and pays nothing to i , implying that intermediary i obtains a payoff of zero in the trading phase. Again, this deviation is not strictly profitable. So, it is a best reply for each intermediary to refer all his buyers. Finally, whenever buyers and intermediaries are asked to report their type, being truthful is a best reply since the mechanism is, by design, incentive-compatible.

We now show the uniqueness of equilibrium payoffs. We have already established, in Theorem 1, that in any equilibrium, all buyers are always referred. We have also pointed out a strategy of the seller that, in this case, guarantees the seller the maximum payoff that he can achieve, $\Pi^* - \sum_{i \in \mathcal{I}} E[|B_i|]\kappa$. Noting that the seller will never use a suboptimal strategy concludes the proof.²³ ■

Proof of Theorem 3 Since Theorems 1 and 2 apply when $\kappa_i > 0$ for all $i \in \mathcal{I}$, we know

²³It is evident that when the seller uses the mechanism outlined above, intermediaries do not have a strict incentive to report truthfully, and, therefore, there are multiple equilibria. However, from a formal perspective, we are taking a mechanism design approach and we are maintaining that the equilibrium selected by the seller will ensue. From a more substantial perspective, there is no equilibrium where the intermediaries fail to report valuable information to the seller. In fact, the seller can use a mechanism identical to the one above, but that uses strictly positive, but arbitrarily small, commission fees for intermediaries (i.e., offers a percentage of the final revenue), thus making truthful reporting of valuable information by the intermediaries a dominant strategy.

that the seller can always guarantee himself a payoff arbitrarily close to Π^* by setting $\kappa_i > 0$ for all $i \in \mathcal{I}$ arbitrarily small. Therefore, there is no equilibrium where the seller chooses the referral fees and obtains a payoff strictly less than Π^* .

To complete the proof, we need to show that there is, indeed, an equilibrium in which the seller sets $\kappa_i = 0$ for all $i \in \mathcal{I}$ and all intermediaries refer all buyers. To see that such an equilibrium exists, it is sufficient to refer to the same equilibrium strategy outlined in the proof of Theorem 2, complemented by the initial choice of $\kappa_i = 0$ for all $i \in \mathcal{I}$ by the seller. All the arguments of the existence part of the proof of Theorem 2 also apply when $\kappa_i = 0$ for all $i \in \mathcal{I}$, and, therefore, it is only necessary to show that setting $\kappa_i = 0$ for all $i \in \mathcal{I}$ is a best reply for the seller, which is obvious. ■

Proof of Theorem 4 For any κ^* that satisfies the conditions in the statement of the Theorem, consider the following strategy profile: For each $i \in \mathcal{I}$, each intermediary type $t_i \in \mathcal{T}_i$ proposes κ_i^* . The seller accepts $\kappa_i \leq \kappa_i^*$, and rejects every proposal $\kappa_i > \kappa_i^*$, for all $i \in \mathcal{I}$. When the seller observes a proposal $\kappa_i \leq \kappa_i^*$, his beliefs about intermediary i 's type remains equal to his prior. When the seller observes $\kappa_i > \kappa_i^*$, he believes that intermediary i is of the highest resale value type. When the seller accepts a referral fee $\kappa_i \geq 0$, the intermediary i refers all his buyers. In the trading phase, the seller sets an optimal mechanism as described in the proof of Theorem 3. We show that this profile of strategies is an equilibrium.

If the intermediary $t_i \in \mathcal{T}_i$ proposes κ_i^* , he obtains $|B_i|\kappa_i^*$. By proposing $\kappa_i < \kappa_i^*$, he obtains $|B_i|\kappa_i < |B_i|\kappa_i^*$. If he proposes $\kappa_i > \kappa_i^*$, the seller rejects the proposal and asks a price that equals the highest resale value of the intermediary. Hence, the intermediary obtains a payoff of zero regardless of whether or not he gets the object at the trading phase.

Next, consider the seller. It is clear that it is optimal to reject $\kappa > \kappa^*$ given that her posterior belief in that case is that the intermediary is of the highest resale value type. The condition in equation (3) guarantees that the seller is not willing to deviate and reject any set of proposals, when the proposed profile is lower (component-wise) than or equal to κ^* . In fact, by accepting the profile, the seller obtains $\Pi^* - \sum_{i \in \mathcal{I}} \kappa_i^* E[|B_i|]$, while by rejecting any subset of offers by $\hat{\mathcal{I}}$, the amount he saves in the referral stage is below the loss in payoff in the trading phase. ■

Proof of Proposition 1 When $\kappa < v_H - v_L$, there is no equilibrium in which the buyer is referred with some probability because, given this strategy, upon observing non-referral, the

seller will ask price v_I of the intermediary. Hence, when $\kappa < v_H - v_I$ is part of an equilibrium, the seller obtains v_I . Since the seller can always guarantee himself v_I by setting $\kappa = 0$, there is no equilibrium in which $\kappa > v_H - v_I$. When $\kappa = v_H - v_I$, the intermediary is indifferent between referring or not (the seller asks v_I of the intermediary when there is no referral, regardless of the intermediary's strategy). However, because the seller can guarantee himself v_I , there is no equilibrium of the game in which $\kappa = v_H - v_I$, and the intermediary does not refer the buyer. In light of these considerations, there is a class of equilibria where the seller proposes $\kappa < v_H - v_I$ and an equilibrium in which he proposes $\kappa = v_H - v_I$ and there is full referral. ■

Proof of Proposition 2 There are two possible types of one intermediary. With probability q , the intermediary has only one buyer with value $0 < v_L < 1$ (type *A*). With probability $1 - q$, he also has a second buyer with value 1 (type *B*). We assume that the intermediary knows the value of the buyer and that $v_L < 1 - q$. The latter assumption implies that, absent the possibility of referrals the seller would post price 1.

First, note that when $\kappa \geq (1 - v_L)/2$, both types find it weakly optimal to refer all their buyers. In fact, $(1 - v_L)/2$ is the maximum continuation payoff that type *B* can get. However, it is clear that setting $\kappa \geq (1 - v_L)/2$ is suboptimal for the seller. His expected continuation payoff becomes $q(v_L - (1 - v_L)/2) + (1 - q)(1 - (1 - v_L)) = v_L - q(1 - v_L)/2$, which is below the payoff from just asking price 1 with no referrals, which is $(1 - q)$.

Therefore, the focus on sub-games starting after $(1 - v_L)/2 > \kappa > 0$ has been posted. Type *A* always refer the low-valuation buyer, as no price below v_L is ever charged. Given this, type *B* must also always refer the low-valuation buyer, as otherwise he would identify himself. Let σ be the probability that type *B* also refers the high-valuation buyer. Let \hat{q} be the posterior probability held by the seller that the intermediary is *A*, given that only the low type has been referred. By Bayes' rule:

$$\hat{q}(\sigma) = \frac{q}{q + (1 - q)(1 - \sigma)}.$$

Upon observing referral of the low type, the seller posts price 1 when $v_L < 1 - \hat{q}$ and price v_L when $v_L > 1 - \hat{q}$. The seller is indifferent between the two prices if $\hat{q} = 1 - v_L$. Observe that \hat{q} takes value in $[q, 1]$ and is monotonically increasing in σ . Then, let $\tilde{\sigma}$ be the one that makes the seller indifferent, which is such that $\hat{q}(\tilde{\sigma}) = 1 - v_L$. We have

$$\tilde{\sigma} = \frac{1 - q + v_L}{(1 - q)(1 - v_L)}.$$

We can see immediately that there is no equilibrium with $\sigma < \tilde{\sigma}$ because the seller would ask price 1 and type B would be better off with $\sigma = 1$. We can also see that it is impossible that $\sigma > \tilde{\sigma}$, as the seller would set price v_L , and, therefore, intermediary B would be better off with $\sigma = 0$.

To sustain an equilibrium when type B plays $\tilde{\sigma}$, we need the seller to make type B indifferent between accepting the referral fee and not referring the buyer. When type B does not refer the buyer, he obtains the object with probability 1 if the seller posts price v_H and, given the competition from the low-value buyer, with probability $1/2$ when the seller posts v_L . Hence, to make B indifferent, the seller must post price v_L with probability y such that

$$y = \frac{2}{(1 - v_L)}\kappa.$$

Hence, in the unique equilibrium of the continuation game starting with $\kappa > 0$, intermediary's types A and B always refer the low-value buyer, while B refers the high-value buyer with probability $\tilde{\sigma}$. The seller posts price v_H if he observes that the high-value buyer has been referred, while he randomizes between the two prices with probability y when observing that only the low-value buyer has been referred.

In light of the above, the seller's profit for any $\kappa > 0$ is

$$(1 - q)\tilde{\sigma}(1 - 2\kappa) + (1 - q)(1 - \tilde{\sigma})[yv_L + (1 - y) - \kappa] + q(yv_L - \kappa),$$

Next, recall that $q < 1 - v_L$, and, therefore, the above expression is decreasing in κ . It follows that there is no equilibrium in which $\kappa > 0$. By taking the limit when $\kappa \rightarrow 0$, we can see that there is one equilibrium in which $\kappa = 0$, the seller always posts price 1 and the intermediary B randomizes according to $\tilde{\sigma}$. The seller obtains $1 - q$ and the intermediary zero, which is the payoff profile we would observe in the absence of referral.

Are there other equilibria when $\kappa = 0$? Yes, but in all those equilibria, the seller must always be (generically) setting a price of 1 in all cases. Suppose, in fact, that the seller sets price v_L whenever either (i) no buyer is referred or (ii) when the low buyer only is referred. Then, the intermediary of type B would always procure the object at price v_L by referring either no buyer or the low-value buyer. But this is not possible, as whenever this happens, the posteriors would be such that the seller would prefer to set $v_H = 1$. Hence, in any other equilibrium, the payoff of the seller is always $1 - q$ and the payoff of the intermediary is zero.

Because the payoff of both types of intermediaries is always zero when the seller sets price 1, we conclude that any referral strategy that guarantees posteriors such that in all states 1, is asked in part of an equilibrium in which $\kappa = 0$. For example, there is one equilibrium

where no type ever refers. If we maintain that A refers his buyer with probability 1, the only other equilibria are such that B refers with any probability $[0, \widehat{\sigma}]$ and the seller posts price 1 with probability one. ■

Proof of Proposition 3 Suppose that $C < v_L$. First, note that when the seller observes no referral, he will charge the intermediary a price $p_I = C$, regardless of his beliefs about the intermediary's type. Hence, by non-referring, the intermediary with buyer v_B obtains a payoff $v_B - C$, regardless of the strategy of the other intermediary's type. We then have that if $\kappa \in [0, v_B - C]$, the intermediary with buyer v_B prefers not to refer, and this holds strictly for $\kappa \in [0, v_B - C)$. When $\kappa > v_B - C$, the intermediary with buyer v_B strictly prefers to refer the buyer. It is immediate to check that the best referral fee that the seller can announce is $\kappa^* \in \{0, v_L - C\}$.

Suppose that $v_L < C < v_H$. First, note that in every equilibrium in which the seller announces $\kappa > 0$, the intermediary with a low-value buyer must refer with probability one. This follows because upon non-referral, the seller will always ask a price $p_I \geq v_L$. This implies that, in every equilibrium following $\kappa > 0$, when the seller observes that the intermediary has not referred his buyer, he believes that the intermediary has a high-value buyer and, therefore, asks the intermediary a price $p_I = C$. Consequently, the intermediary with a high-value buyer prefers not to refer when $\kappa \in (0, v_H - C]$, and his preferences are strict when $\kappa < v_H - C$. When $\kappa > v_H - C$, the intermediary with a high-value buyer also refers with probability one. When $\kappa = 0$, the intermediary with a low-value buyer is indifferent between referring or not, whereas the intermediary with a high-value buyer strictly prefers not to refer. Taking these continuation equilibria, it is immediate to show that in the only equilibrium of the game, the seller sets $\kappa = 0$, the intermediary with a low-value buyer must refer with probability one, and the intermediary with a high-value buyer never refers. ■

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Appendix B to “Selling Through Referrals”: Referrals with Communication

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In this Appendix, we explore a variation of the model examined in the main paper. In this variation, intermediaries communicate with the seller during the referral stage. We model communication as cheap talk. We assume that once an intermediary has referred all his buyers, such intermediary has no more interactions with the seller. If this were not the case, then this cheap talk stage would be irrelevant as the seller would be able to extract intermediaries’ information in the trading stage, as in the benchmark model. Overall, we see that the results of the main paper are essentially unchanged. The only difference is that when communication happens simultaneously with referrals, we get multiple equilibria. This is primarily because we look at very simple (but realistic) referral contracts, and communication is cheap talk.

1 Model revisited

The model is identical to the one analyzed in the paper, with the difference that now intermediaries report information about their buyers at the referral stage:

Referral Stage. Intermediaries simultaneously decide on the set of buyers to refer to the seller and on an unverifiable message to send to the seller. Intermediary i can only refer buyers in B_i . A strategy of intermediary i is denoted by σ_i , σ denotes a strategy profile and σ_{-i} a strategy profile of all intermediaries except intermediary i .

The seller observes all referred buyers and pays intermediaries according to the announced referral schedule. Since intermediaries have no consumption value, if they refer all their buyers, they leave once they have collected the referral fee.

Each intermediary may or may not observe the referral and the messages of the other intermediaries. Each buyer may or may not observe the referral and the messages of the intermediaries. As we shall see, we can allow for different possibilities.

Trade Stage. The seller updates his beliefs about the buyers’ valuations and the intermediaries’ information based on what happened during the referral state. Given these updated beliefs, the seller chooses an optimal mechanism to sell the object to the referred buyers and to the intermediaries that are still present in the market. The participants may or may not observe the outcome of the mechanism. If a buyer acquires the object, the game ends. If an intermediary acquires the object, then the intermediary chooses an optimal mechanism to sell the object to his non-referred buyers.

We look for perfect Bayesian equilibria of this game.

2 Equilibrium payoffs

Given Theorem 1 established in the main paper, the lower bound of the seller's payoff, denoted by Π , is the expected revenue of an optimal mechanism when all buyers are referred uninformatively. The best-case scenario for the seller is an equilibrium in which all buyers are referred, the truth is reported, and referral payments are zero. Denote by $\Pi^*(t)$ the expected revenue of an optimal mechanism in the case where the seller accesses every buyer and the intermediaries have truthfully reported information t .¹ Then, the maximum equilibrium payoff that the seller can extract in this environment, denoted by Π^* , is the expected value of $\Pi^*(t)$, where the expectation is with respect $t \in \mathcal{T}$.

The difference $\Delta = \Pi^* - \Pi$ is the maximum value of all the intermediaries' information. We now investigate how different referral fees affect the information that intermediaries will be induced to reveal in equilibrium. We examine whether it is possible to induce truthful communication and, if yes how the pie Π^* is split among the seller and the intermediaries. In what follows, we assume that, indeed, all the intermediaries as a group have access to relevant information for the seller, so that $\Delta > 0$. We start by analyzing equilibria in which the seller offers a flat fee per referred buyer.

2.1 Seller proposes: Non-contingent referral fees

A *non-contingent* referral schedule defines a positive payment to each intermediary for each referred buyer. Formally, a non-contingent referral schedule is $\kappa = \{\kappa_1, \dots, \kappa_I\}$, where κ_i is the referral fee that intermediary i gets for each buyer he refers to the seller.²

Equilibrium communication and payoffs

Theorem 1 implies that, under non-contingent referral fees, the equilibrium continuation payoff of an intermediary after the referral stage is independent of the communication strategy he employs: All buyers are referred and intermediary i 's payoff is equal to the number of his buyers times the referral fee, regardless of the report that he makes to the seller. Therefore, under full referral, equilibrium does not impose any restriction on the reporting strategy. This implies that for a non-contingent referral schedule, there is an equilibrium in which all intermediaries babble and not reveal any information, as well as a fully informative equilibrium where all intermediaries report their information truthfully, regardless of its realization.

Proposition 1 *There is a class of equilibria where all the buyers are referred and all the intermediaries report their information to the seller truthfully: In these equilibria, the sum of payoffs of the intermediaries is $K^* \in [0, \Delta]$ and the seller's payoff is $\Pi^* - K^*$.*

Proof. We first describe the strategy profile employed in the class of equilibria we are considering. For each $i \in \mathcal{I}$, the seller announces κ_i^* so that the expected referral payment to intermediary i equals $\kappa_i^* n_i$, where n_i is the expected number of buyers of intermediary i . Each intermediary refers all his buyers, regardless of his type and regardless of the announced κ ; if the per-consumer referral fee is $\kappa_i \geq \kappa_i^*$ for all $i \in \mathcal{I}$, then each intermediary communicates truthfully; otherwise,

¹Note that $\Pi^*(t) \geq \Pi$, because the seller can always guarantee the revenue he acquires with less information, by designing a mechanism that disregards the additional information.

²Two remarks are in order. First, the payment is independent of the message that an intermediary sends to the seller. Since messages are not verifiable, this restriction is without loss of generality: If the seller were to propose a schedule that pays an intermediary a different fee for the same set of referred buyers but different messages, the intermediary will send only messages that provide the highest fee. Second, we could have defined the non-contingent referral schedule more generally: A non-contingent referral schedule maps the set of all referred buyers into a payment to each intermediary. Formally, $\kappa = \{\kappa_1, \dots, \kappa_I\}$ where $\kappa_i : \mathcal{P}(\mathcal{B})^n \rightarrow R^+$, where $\mathcal{P}(\mathcal{B})$ denotes the power set of $\mathcal{B} = \cup_{i \in \mathcal{I}} \mathcal{B}_i$. Our results are not affected by this restriction.

each intermediary communicates uninformatively. The seller, intermediaries and buyers behave optimally in the trade stage. When the seller observes a deviation in the referral stage, he assumes that each deviator has the type with the highest resale value—i.e., if intermediary i deviates in the referral stage, then the seller believes that intermediary i is of type \bar{t}_i .

We now show that the proposed strategy profile constitutes an equilibrium. If the seller offers a referral fee κ where $\kappa_i < \kappa_i^*$ for some $i \in \mathcal{I}$, the seller will get, at most, $\Pi \leq \Pi^* - K^*$, which is the payoff to the seller if he announces κ^* and where K^* stands for the expected referral payments given κ^* .

Theorem 1 implies that for intermediary i to refer all buyers when $\kappa_i > 0$ is a best reply; when $\kappa_i = 0$, referring everyone is a best reply given the specified out-of-equilibrium belief of the seller. Once an intermediary refers all buyers, he is indifferent between any communication strategies, regardless of the seller's conjecture about his communication strategy. Hence, revealing his type when $\kappa_i \geq \kappa_i^*$ for all $i \in \mathcal{I}$, and sending uninformative messages otherwise, is a best reply. We now show that there is not an equilibrium where the payoff of the seller is lower than Π . Notice that, in view of Theorem 1, by offering a referral fee of $\kappa_i = \varepsilon$ to each intermediary i , the seller gets all buyers referred. Under full referral, the worse equilibrium is when all intermediaries babble, and this provides a gross payoff of Π and an expected referral cost that can be made negligible by setting ε arbitrarily close to 0. ■

Proposition 1 pins down the set of total equilibrium payoffs of all market participants. Note that the highest aggregate profits that intermediaries can attain in equilibrium is $\Delta = \Pi^* - \Pi$ —the value of the additional information that intermediaries have about potential buyers, as compared to the initial information that the seller has. We now provide a couple of remarks:

Remark 1 *Suppose that intermediaries are fully informed about the type of their buyers—i.e., $t_i = (s_i, B_i)$ and s_i specifies the valuation of each of the buyers $j \in B_i$. In this case, Π^* is simply the expected total surplus, and every equilibrium in Proposition 1 is ex-post efficient.*

We have assumed that each intermediary i observes $\kappa = \{\kappa_1, \dots, \kappa_I\}$. In some contexts, it is more natural to assume that intermediary i observes only the referral fee he is paid when he refers his buyers, but not the referral fee of other intermediaries. Formally, for a given κ , intermediary i observes κ_i , but does not observe $\kappa_{i'}$, $i' \in \mathcal{I} \setminus \{i\}$. We now illustrate the consequence of restricting the observability of the referral schedule.

Remark 2 *Consider that each intermediary i observes only κ_i . First, observe that Theorem 1 still holds in this environment. Second, as in the case of full observability (see Proposition 1), there is still a class of equilibria where all information is aggregated and, in each of these equilibria, the aggregate payoffs of the seller and intermediaries is Π^* . It is also the case that Π and Π^* are the minimum and maximum equilibrium payoffs attainable by the seller, respectively.*

However, in contrast to Proposition 1, we cannot guarantee that the sum of payoffs of the intermediaries is $K^ \in [0, \Delta]$ and the seller's payoff is $\Pi^* - K^*$. The reason is that, now, a seller can deviate with a subset of intermediaries by announcing a lower referral fee, and in that subgame, only the intermediaries that have observed the deviation can react to it.*

To summarize, in this section we have shown that with an extremely simple referral payment, there are equilibria in which the seller induces the intermediaries to refer all their buyers and to report their information truthfully. When intermediaries are perfectly informed about the buyers' valuations, this leads to efficient trade. When intermediaries are imperfectly informed about buyers' valuations, the asymmetries between the information of the intermediaries and the seller with regard to the information of buyers' valuations do not produce any distortion. We now examine what happens when the seller offers a referral fee that is a fraction of the revenue the seller generates. These referral contracts resemble commission fees that are often used in practice.

2.2 Seller proposes: Contingent referral fees

We now show that if the seller can offer referral fees that are fractions of the revenue generated—that is, *commissions*—then the seller can assure, at a minimum cost, not only that all the intermediaries refer all their buyers, but also that they truthfully disclose all their information. With contingent referral fees, the seller pays each intermediary i a percentage κ_i of his final revenue.³

Our first observation is that when $\kappa_i > 0$, Theorem 1 holds. Therefore, in any continuation equilibrium starting in the referral stage with $\kappa_i > 0$, all buyers are referred to the seller.⁴ We now show that, for each $\kappa_i \geq 0$, there exists a fully informative equilibrium

Proposition 2 *With contingent referral fees, there is an equilibrium in which all intermediaries refer all their buyers and report their information truthfully. The seller obtains all of the expected total surplus Π^* , whereas intermediaries obtain no rents.*

Proof. This equilibrium is sustained by the following strategy: The seller proposes a referral schedule that rewards intermediary i a fraction κ_i of the generated revenue; all intermediaries refer all their buyers and report their information truthfully. If there is non-referral by the intermediary, the seller assigns probability 1 to this coming from the type of i with the highest resale value. All players choose revenue-maximizing procedures in the trading phase, given their beliefs. We now argue that this strategy is an equilibrium.

First, note that as long as $\kappa_i > 0$, the unraveling result holds. Moreover, the referral payment is an increasing function of the seller's revenue. Hence, if the seller and the intermediaries expect all intermediaries to be truthful, then it is a best response for each intermediary i to report the truth. To see this, note that, reporting the truth, intermediary i gets $\kappa_i n_i \Pi^*(t) \geq \kappa_i n_i \Pi'$, where Π' is the optimal revenue given reports \hat{t}_i, t_{-i} with $\hat{t}_i \neq t_i$, when the true state is t_i, t_{-i} . Then, the seller's payoff is $(1 - K)\Pi^*$, where $K = \sum_{i \in I} \kappa_i n_i$, and the seller's best response is to set each κ_i arbitrarily low. ■

Unfortunately, there also exist uninformative equilibria. For example, if, after referrals take place, the seller expects all the intermediaries to babble, then it is a best response for intermediary i to babble, as his report does not affect the seller's mechanism choice.⁵ Let κ be the referral schedule employed at the babbling equilibrium; in equilibrium, this fee must be arbitrarily low since the seller's payoff is $(1 - K)\Pi$, so $K = 0$. This equilibrium is strictly Pareto dominated by the fully informative equilibrium since the intermediaries make zero in both cases, whereas for the seller, $(1 - K)\Pi^* > (1 - K)\Pi$, which follows from the fact that intermediaries have useful information, implying that $\Delta = \Pi^* - \Pi > 0$. Given that $\Delta > 0$, in addition to interim and ex-post strict Pareto domination, the informative equilibrium is the only equilibrium that survives the NITS (*no-incentive-to-separate*) criterion in Chen et al. (2008),⁶ the neologism proof criterion in Farrell (1993), or the selection criterion in Matthews et al. (1991).

An equilibrium satisfies the NITS condition if the equilibrium payoff of the worst-type intermediary is (weakly) higher than the payoff of that type had the seller known his information and reacted optimally to it. We make two observations. First, in our game, we can define the worst type as any intermediary type, so we just pick the type that strictly increases revenue by telling

³We don't need to introduce more-elaborate contracts (e.g., contracts that pay an intermediary only if the good is sold to one of the buyer referred by him) because, as we will see, focusing on this class is without loss of generality in terms of what the seller can achieve.

⁴Since the seller's revenue upon referral is always strictly greater than zero, $\kappa > 0$ implies that the expected referral payment to the intermediary is strictly positive.

⁵There can also exist partially informative equilibria.

⁶In contrast to standard cheap-talk, where there is a conflict of interest among different types of the same sender as to the level of informativeness of an equilibrium, in our case all types are weakly better off in the informative equilibrium compared to any other equilibrium.

the truth. Such a type exists because $\Delta > 0$; otherwise, we would have $\Pi^* = \Pi$. Second, for the case of non-contingent fees, the NITS criterion does not affect the equilibrium payoffs that we have characterized in Proposition 1; indeed, the intermediaries' payoffs are independent of the information they provide to the seller. However, in the contingent case, in every sub-game followed by an announcement where $\kappa > 0$, NITS uniquely selects the informative equilibrium. We obtain:

Proposition 3 *Consider contingent referral fees and suppose that $\Delta > 0$. In every equilibrium that satisfies the NITS condition, all the intermediaries refer all their buyers and report their information truthfully. The seller obtains Π^* and each intermediary obtains zero.*

Proof. We need to show that in every sub-game that follows announcement $\kappa > 0$, the only equilibrium that survives NITS is the informative equilibrium. Clearly, the informative equilibrium survives NITS. Consider the non-informative equilibrium discussed above; then, for each realized information t , the payoff of all types (including the worst type) is $\kappa_i n_i \Pi$, which is less than $\kappa_i n_i \Pi^*(t)$, and strictly less for our chosen “lowest” type. Hence, the non-informative equilibrium fails NITS. This criterion is satisfied only by the most informative equilibrium. ■

Hence, under this and the abovementioned selections, we have that when the seller can propose contingent-referral fees, the seller obtains access to the buyer and to his information at no cost, and so he extracts Π^* .

Our final observation, which also points to the robustness of selecting the informative equilibrium, is that the equilibrium outcome where the seller obtains Π^* in the sale phase is the unique outcome if there is an epsilon possibility that the seller interprets the message of the intermediaries naively—that is, if upon observing messages t , the seller selects an optimal mechanism conditional on t .⁷ In this case, given the above assumption on the relevance of the information, reporting a signal different from his own type (following full referral) is a strictly dominated action for each intermediary type.

Corollary 1 *With contingent fees, at the only robust equilibrium, the seller obtains Π^* .*

3 Alternative bargaining protocols

So far, we have assumed that the seller fully controls the referral fee: The seller moves first and commits to a referral fee schedule. A natural conjecture is that the set of equilibrium payoffs for the seller and for the intermediaries will change if we reverse the bargaining protocol and we give intermediaries the power to move first and to commit to referral schedules. We now establish that, in fact, the set of equilibrium payoffs contains all the payoffs we obtained with the alternative bargaining scenario.

To illustrate this, we consider the simple case in which there is one intermediary. The timing of the game is as follows: In the first stage, the intermediary announces a referral schedule to the seller, who, after observing the announcement, decides whether to accept it or not. If the seller accepts the proposal, the game moves to the referral stage and eventually to the trade stage, as described in our benchmark model in Section 1. If the seller rejects the proposed referral schedule, the game moves directly to the trade stage, as defined in our model of Section 1. Essentially, the only difference from the model in Section 1 is that, now, the intermediary is the one proposing the referral schedule.

For the same reason that there is multiplicity in the communication strategy of the intermediary when the seller proposes, there is also this multiplicity when the intermediary proposes. We now

⁷This is reminiscent of the model used in Section 4.3 of Chen et al. (2008).

actually show that there is a continuum of equilibria even for a fixed communication strategy. As before, we analyze non-contingent and contingent fees and focus on equilibria in which the intermediary communicates all his information truthfully.

Non-contingent fees

Here, the intermediary asks a payment per buyer referred. The highest possible fee per buyer κ that will be accepted at an equilibrium where the intermediary communicates truthfully is

$$\kappa^* = n\kappa = \Pi^* - \Pi = \Delta, \quad (1)$$

where n stands for the intermediary's expected number of buyers.

Proposition 4 *There is a class of equilibria where all the buyers are referred and all the intermediaries report their information to the seller truthfully: In these equilibria, the intermediary's total payoff comes from referrals and is $K^* \in [0, \Delta]$, and the seller's payoff is $\Pi^* - K^*$.*

The intermediary proposes κ^* , and the seller accepts κ^* and rejects all other referral proposals. Given acceptance, the intermediary always refers his buyer and sends a truthful message if $\kappa = \kappa^*$; otherwise, the message is uninformative.

Contingent fees

Here, the fee is a fraction of the revenue that the seller generates from the referred buyers. If the intermediary is proposing a referral fee that is a fraction of revenue κ , the highest possible κ that will be accepted at an equilibrium where the intermediary communicates truthfully is such that

$$\kappa^*\Pi^* = \Delta. \quad (2)$$

Proposition 5 *With contingent fees, there is an equilibrium in which all buyers are referred and the intermediary reports truthfully. The intermediary obtains a payoff of $K^* = \Delta$ and the seller a payoff of $\Pi^* - \Delta = \Pi$. This equilibrium is Pareto dominant and is the only one that survives NITS.*

Suppose that the seller accepts any fee associated with referral payments $K^* \in [0, \Delta]$ and expects the intermediary to be truthful. Then, following arguments analogous to the ones we used before, it is easy to see that it is a best response for the intermediary to refer his buyer truthfully and to propose a fee equal to Δ for all types. Given this intermediary's strategy, the seller's acceptance strategy is a best response.

Combining the case in which the intermediary proposes non-contingent referral fees, Proposition 4, with the case in which the seller has the power to propose a non-contingent referral fee, Proposition 1, we conclude that the feasible equilibrium payoffs coincide in these cases. For the case of contingent fees, we see, however, that at the Pareto dominant equilibrium, the informational rent Δ goes entirely to the party that proposes, so it goes to the seller if the seller proposes, while it goes to the intermediary when the intermediary proposes.

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Appendix C to “Selling Through Referrals”: An example with referrals and intermediated trade when the intermediary proposes

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In this appendix, we develop an example of an equilibrium in which intermediated trade and referrals co-exist. As in our illustrative example developed in Section 2, we consider the example with one seller, one intermediary and one buyer with $v_B \in \{v_L, v_H\}$. The intermediary knows the value of the buyer and proposes the referral fees. For simplicity, we let $v_H = 1$ and $f = 1/2$. Let $f(\kappa)$ be the posterior beliefs of the seller that the intermediary is of type $v_B = v_H$ given the announcement κ .

The equilibrium we construct has the feature that each intermediary type randomizes between two referral fees, and the seller accepts each of the proposed fees with positive probability, but not with probability 1. Hence, in this equilibrium, with positive probability, we do not enter the referral stage.

Consider the following strategy profile.

- The intermediary with type $v_B = v_H$ announces κ_1 with probability $\rho \in (0, 1)$ and κ_2 with probability $1 - \rho$; the intermediary with type $v_B = v_L$ announces κ_1 with probability $q \in (0, 1)$ and κ_2 with probability $1 - q$; $\kappa_1 > \kappa_2$.
- The seller rejects with probability 1 all proposals $\kappa \notin \{\kappa_1, \kappa_2\}$; the seller accepts κ_1 with probability $\eta \in (0, 1)$ and accepts κ_2 with probability $\mu \in (0, 1)$.
- Seller posteriors are: $f(\kappa_1) = \rho/(\rho + q)$, $f(\kappa_2) = (1 - \rho)/(2 - \rho - q)$ and $f(\kappa) = 1$, if $\kappa \notin \{\kappa_1, \kappa_2\}$.¹
- If we enter the referral stage, then the intermediary refers all his buyers regardless of his type.
- When we enter the trading phase, the seller chooses a revenue-maximizing mechanism. Moreover, whenever we enter the trading phase because κ_1 has been rejected and in the revenue-maximizing mechanism the seller is indifferent between asking the intermediary a price v_L and a price v_H , then he chooses a price v_L with probability $\gamma \in (0, 1)$.

We now find the equilibrium conditions for the proposed profile to be an equilibrium. These conditions determine restrictions on the primitives. We then show that these restrictions can be satisfied generically.

Step 1: Referral fee proposal. First, the intermediary, regardless of v_B , will never propose $\kappa \notin \{\kappa_1, \kappa_2\}$. Indeed, if an intermediary with type v_B proposes $\kappa \notin \{\kappa_1, \kappa_2\}$, the seller will reject this proposal and, in the trading phase, will ask a price $p = v_H$ from the intermediary because the

¹The former two expressions have been derived using Bayes' rule given intermediary's strategy, and the latter expression is set arbitrarily as Bayes' rule does not apply given intermediary's strategy.

seller's posteriors place probability 1 on the intermediary having access to a high value buyer—i.e., $f(\kappa) = 1$ for each $\kappa \notin \{\kappa_1, \kappa_2\}$.

Second, consider the intermediary with buyer $v_B = v_L$. Note that upon rejection of $\kappa \in \{\kappa_1, \kappa_2\}$, an intermediary obtains zero payoff, because the seller will ask a price that is at least v_L in the trading phase. Moreover, if the seller accepts $\kappa \in \{\kappa_1, \kappa_2\}$, then the intermediary will refer his buyer and receive the referral fee. At that point, the seller will extract his information about the buyer's type at no cost (as we saw in Section 2). So, if intermediary $v_B = v_L$ proposes κ_1 , his expected payoff is the expected referral payment $\eta\kappa_1$, whereas if he proposes κ_2 , he gets $\mu\kappa_2$. Since the intermediary with buyer $v_B = v_L$ is indifferent between the two proposals, $q \in (0, 1)$, we must have:

$$\eta\kappa_1 = \mu\kappa_2. \quad (1)$$

Third, consider now the intermediary with $v_B = v_H$. If he proposes κ_1 , he gets κ_1 if the proposal is accepted, which happens with probability η ; and if the proposal is rejected, he gets a profit from buying and reselling whenever the seller asks a price $p = v_L$. Formally, the expected profit by proposing κ_1 is

$$\eta\kappa_1 + (1 - \eta)(v_H - v_L) \Pr[p_I = v_L | \kappa_1, R],$$

where $\Pr[p_I = v_L | \kappa_1, R]$ is the probability that the seller will ask the intermediary a price v_L given that the seller has rejected κ_1 . Similarly, if the intermediary proposes κ_2 , then he gets

$$\mu\kappa_2 + (1 - \mu)(v_H - v_L) \Pr[p_I = v_L | \kappa_2, R].$$

Since the intermediary is indifferent between the two proposals, $\rho \in (0, 1)$, it must be the case that

$$\eta\kappa_1 + (1 - \eta)(v_H - v_L) \Pr[p_I = v_L | \kappa_1, R] = \mu\kappa_2 + (1 - \mu)(v_H - v_L) \Pr[p_I = v_L | \kappa_2, R],$$

and given condition 1, we obtain

$$(1 - \eta) \Pr[p_I = v_L | \kappa_1, R] = (1 - \mu) \Pr[p_I = v_L | \kappa_2, R]. \quad (2)$$

Step 2: Acceptance/Rejection of announced referral fee. First, when the seller receives proposal $\kappa \notin \{\kappa_1, \kappa_2\}$, he believes that the intermediary has access to a high-value buyer, and so by rejecting the proposal and asking a price $p = v_H$, he expects to get a payoff of v_H . This is the highest payoff he can get, and so rejecting is a best reply.

Second, if the proposal is κ_1 and the seller accepts, then the intermediary will refer his buyer for sure, and the seller will extract his information for free. Hence, the expected payoff of the seller accepting κ_1 is

$$f(\kappa_1)v_H + (1 - f(\kappa_1))v_L - \kappa_1 = \frac{\rho}{\rho + q} + \left(1 - \frac{\rho}{\rho + q}\right)v_L - \kappa_1,$$

where, recall, that $v_H = 1$ and that $f(\kappa_1) = \rho/(\rho + q)$ is the posterior of the seller that the intermediary has access to a high-value buyer given announcement κ_1 . If the seller rejects the offer κ_1 , then he will ask a price $p = v_L$ from the intermediary if $(1 - f(\kappa_1))v_L < f(\kappa_1)$; he will ask a price $p = v_H$ from the intermediary if $(1 - f(\kappa_1))v_L > f(\kappa_1)$; if $(1 - f(\kappa_1))v_L = f(\kappa_1)$, he will ask a price $p = v_L$ with probability $\gamma \in [0, 1]$; and with the remaining probability, he will ask a price $p = v_H$. Our construction here imposes that the way in which intermediaries randomize between κ_1 and κ_2 , is such that, upon rejecting κ_1 , the seller is indifferent between charging $p = v_L$ or charging $p = v_H$. Formally, we require that the probabilities ρ and q are such that,

$$(1 - f(\kappa_1))v_L = f(\kappa_1)$$

or, equivalently,

$$v_L = \frac{\rho}{\rho + q}. \quad (3)$$

Assuming that condition 3 holds, we have that the expected profit of the seller, by rejecting proposal κ_1 , is simply v_L . Since the seller must be indifferent between accepting and rejecting κ_1 , we have the following equilibrium condition

$$\frac{\rho}{\rho + q} + \left(1 - \frac{\rho}{\rho + q}\right) v_L - \kappa_1 = v_L. \quad (4)$$

Third, if the proposal is κ_2 and the seller accepts, then the intermediary will refer his buyer for certain, and the seller will extract his information for free. Hence, by accepting κ_2 , the expected payoff of the seller is

$$f(\kappa_2)v_H + (1 - f(\kappa_2))v_L - \kappa_2 = \frac{1 - \rho}{2 - \rho - q} + \left(1 - \frac{1 - \rho}{2 - \rho - q}\right) v_L - \kappa_2,$$

where, recall, that $v_H = 1$ and that $f(\kappa_2) = (1 - \rho)/(2 - \rho - q)$ is the posterior of the seller that the intermediary has access to a high-value buyer given announcement κ_2 . If the seller rejects the offer κ_2 , then he will ask a price $p = v_L$ from the intermediary if $(1 - f(\kappa_2))v_L \leq f(\kappa_2)$, and he will ask a price $p = v_H$ from the intermediary if $(1 - f(\kappa_2))v_L > f(\kappa_2)$. Our construction here imposes that the way in which intermediaries randomize between κ_1 and κ_2 , is such that, upon rejecting κ_2 , the seller strictly prefers to charge $p = v_L$. Formally, we require that the probabilities ρ and q are such that,

$$(1 - f(\kappa_2))v_L > f(\kappa_2),$$

or, equivalently,

$$v_L > \frac{1 - \rho}{2 - \rho - q}. \quad (5)$$

Assuming that condition 5 holds, we have that the expected profit of the seller, by rejecting proposal κ_2 , is simply v_L . Since the seller must be indifferent between accepting and rejecting κ_2 , we have the following equilibrium condition:

$$\frac{1 - \rho}{2 - \rho - q} + \left(1 - \frac{1 - \rho}{2 - \rho - q}\right) v_L - \kappa_2 = v_L. \quad (6)$$

In summary, for the strategy profile described above to be an equilibrium, there must exist $\kappa_1 > \kappa_2 \geq 0$, $\rho \in (0, 1)$, $q \in (0, 1)$, $\eta \in (0, 1)$, $\mu \in (0, 1)$ and $\gamma \in (0, 1)$, so that conditions 1, 2, 3, 4, 5 and 6 are mutually satisfied. We now construct such an equilibrium. We can summarize these equilibrium conditions as follows:

- Condition 1—i.e., $\eta\kappa_1 = \mu\kappa_2$
- Combining 3 and 5, we obtain

$$v_L = \frac{\rho}{\rho + q} > \frac{1 - \rho}{2 - \rho - q}.$$

- Condition 3 implies that $Pr[p_I = v_L | \kappa_1, R] = \gamma$, and condition 5 implies that $Pr[p_I = v_L | \kappa_2, R] = 1$. Therefore, we can rewrite condition 2 as follows:

$$(1 - \eta)\gamma = (1 - \mu).$$

- Using condition 3, we can rewrite condition 4 as follows:

$$\kappa_1 = v_L(1 - v_L).$$

- We can rewrite condition 6 as follows:

$$\kappa_2 = \frac{1 - \rho}{2 - \rho - q}(1 - v_L).$$

For a given v_L , note that for

$$v_L = \frac{\rho}{\rho + q} > \frac{1 - \rho}{2 - \rho - q}$$

to hold, it must be the case that $\rho = \frac{v_L}{1 - v_L}q$ and that $\rho > q$. These two conditions are satisfied if, and only if, $v_L > 1 - v_L$ or, equivalently, $v_L > 1/2$. We now show that the condition that $v_L > 1/2$ guarantees that the proposed profile is an equilibrium. Indeed, for a given $v_L > 1/2$, set $\rho = \frac{v_L}{1 - v_L}q$ and $q \in (0, (1 - v_L)/v_L)$. This assures that 3 and 5 hold and that $\rho, q \in (0, 1)$. Then, κ_1 and κ_2 are determined by the above conditions. Note that, by construction, $\kappa_1 > \kappa_2 > 0$. We can then fix $\eta \in (0, 1)$ and $\mu \in (0, 1)$ so that condition 1 holds—i.e., $\eta\kappa_1 = \mu\kappa_2$. Since $\kappa_1 > \kappa_2$, we have that $\eta < \mu$, and, therefore, we can always find a $\gamma \in (0, 1)$ so that $\gamma = \frac{1 - \mu}{1 - \eta}$ —i.e., we can satisfy condition $(1 - \eta)\gamma = (1 - \mu)$. This completes the construction of the equilibrium.²

²As an example, suppose $v_L = 3/4$, then the following constitutes an equilibrium $\kappa_1 = 3/16$, $\kappa_2 = 1/16$, $\rho = 3/4$, $q = 1/4$, and $\{\eta, \mu, \gamma\}$ such that $\eta = \mu/3$ and $\gamma = (1 - \mu)/(1 - \eta)$.

Appendix D to “Selling Through Referrals”: Efficiency of referrals in a simple example

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In this appendix, we show that the agency model is more efficient than the merchant model in the context of the following example. The seller is connected to two intermediaries, I_1 and I_2 , who have one buyer each, B_1 and B_2 . The value of B_1 is distributed uniformly in $[0, t_1]$ and the value of B_2 is distributed uniformly in $[0, t_2]$. I_1 knows t_1 and I_2 knows t_2 . From the seller’s perspective, t_1 and t_2 are identically and independently distributed with support in $(0, b]$ for some real b .

Consider the outcome of pure intermediation first. Since I_1 and I_2 are symmetric, the seller will run a symmetric standard auction with reserve price. Because the resale value of I_i is increasing in t_i , we can conclude that I_1 will get the object when $t_1 > t_2$; otherwise, the object will go to I_2 . Then, the winning intermediary I_i will set price $t_i/2$.

Next, consider the outcome under full referral. In this case, the seller acquires information on t_1 and t_2 and runs an optimal auction. The optimal auction is constructed in the standard way. The object will go to the buyer with the highest virtual value, provided that it is above zero. If no buyer has a virtual value above zero, the object remains unsold. Let v_1 and v_2 stand for the values of buyer 1 and 2. The virtual valuation of B_1 is $2v - t_1$ and the virtual valuation of B_2 is $2v - t_2$. Hence, for values above $(t_1/2, t_2/2)$, the seller will distort the auction in favor of the weakest buyer (the buyer with the smallest support of values).

Next, we now show that, for any t_1, t_2 , the equilibrium with full referral is more efficient than the outcome of pure intermediation, even if the seller were to set a reserve price of zero to the intermediaries (i.e., even if the seller were not to exploit his market power, thus increasing inefficiencies). This will be sufficient to show that the equilibrium with full referral is more efficient than the outcome of intermediation, regardless of the distribution of the intermediary types.

Henceforth, without loss of generality let’s assume that $t_1 > t_2$. The picture below depicts the allocation of the object under intermediation and under referral.

The number in the picture indicates the buyer who obtains the object. The diagonal line $v_2 = v_1 - \frac{1}{2}(t_1 - t_2)$ is given by the solution to the equation $2v_1 - t_1 = 2v_2 - t_2$. Note that the allocation under intermediation and under referral can differ only in the two upper quadrants in the picture. Hence, we can compute the welfare difference between the two outcomes by focusing on those two regions.

First, consider the total surplus in the two upper regions under intermediation, S_I :

$$S_I = \int_{t_1/2}^{t_1} \int_{t_2/2}^{t_2} \frac{v_1}{t_1 t_2} dv_1 dv_2 = \frac{3t_1}{16}.$$

Second, consider the total surplus from referrals, S_R :

$$S_R = \frac{3t_2}{16} + \int_{t_2/2}^{t_2} \int_{t_1/2}^{v_2+(t_1-t_2)/2} \frac{v_2}{t_1 t_2} dv_1 dv_2 + \left(\frac{3t_1}{16} - \int_{t_2/2}^{t_2} \int_{t_1/2}^{v_2+(t_1-t_2)/2} \frac{v_1}{t_1 t_2} dv_1 dv_2 \right) =$$

$$\frac{3t_2}{16} + \frac{5t_2^2}{48t_1} + \left(\frac{3t_1}{16} - \frac{t_2(3t_1 + t_2)}{48t_1} \right).$$

where the first term is the surplus in region A of the figure; the second term is the surplus in the B area, and the third is the surplus in the C area.

We can see that $S_R - S_I > 0$ for all $t_1 > t_2 > 0$ as:

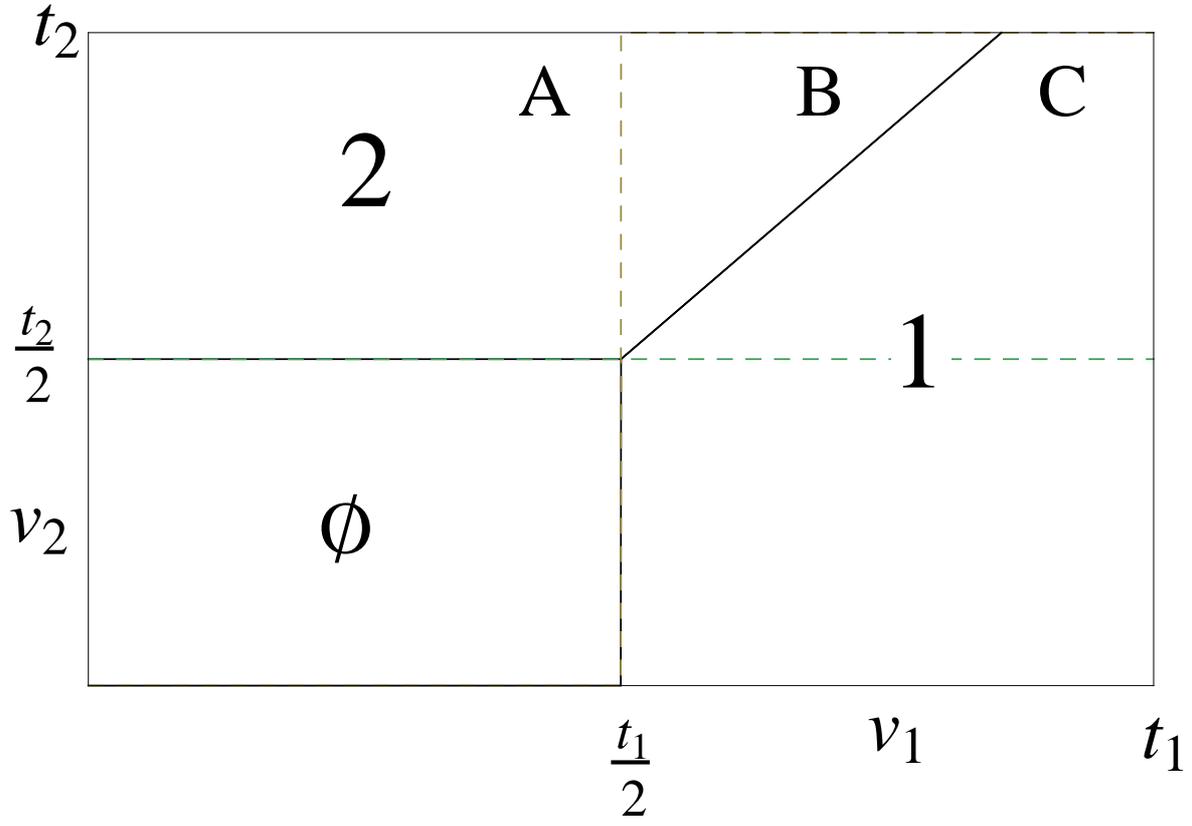
$$\frac{3t_2}{16} + \frac{5t_2^2}{48t_1} - \frac{t_2(3t_1 + t_2)}{48t_1} = \frac{t_2(3t_1 + 2t_2)}{24t_1} > 0.$$

Hence, roughly speaking, the loss in efficiency that is due to the absence of buyer 2 is always larger than the loss in efficiency generated by the distortion in favor of buyer 2 that takes place in the optimal auction. As we have argued above, this computation assumes that the seller does not introduce any further distortion under intermediation. In practice, however, the loss in efficiency under intermediation is even greater than the one we highlight above, as the seller will ultimately sell to the intermediary with the highest t_i , but will exclude some types of the intermediaries from the auction by using a reserve price.

While we have not explicitly carried out the computations, it is reasonable to assume that, if we maintain the informational structure above, the result would carry over even in cases where there are more than two intermediaries, and each intermediary has multiple buyers.

[b]0.5

Figure 1: Referral



[b]0.5

Figure 2: Intermediation

