Optimal Contract to Reward Private Experimentation*

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Abstract

This paper studies how a principal motivates an uninformed agent to learn about, and reveal, his quality through private experiments. The principal commits to a reward scheme and she aims to assign the rewards to correspond as closely as possible to the quality of the agent. To get a high reward, the agent experiments privately and discloses the results selectively. I show that the optimal reward scheme features an increasing step function: the early steps encourage a potential good type agent to continue experiments after early successes; the later steps are designed to deter a bad type agent from over-experimentation after a failure, and the scheme becomes flat when enough successes are reported. If the agent's incentives to deviate from the intended path of experimentation are weak, a one-step function is optimal: the agent receives a bonus if he reports enough successes; otherwise, he only gets a non-negative compensation. I characterise the conditions where the principal achieves the same efficiency level relative to a public information environment.

Keywords: Private Experimentation; Reward Scheme; Over-Experimentation; Commitment; Disclosure

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1 Introduction

Seeking and blundering are good, for it is only by seeking and blundering we learn.
— Johann Wolfgang von Goethe, *Faustus*

I study an environment in which a principal motivates an uninformed agent to learn and reveal his quality through costly and private experiments. The principal aims to assign the rewards to correspond as closely as possible to the quality of the agent, and she commits to a reward scheme before evidence is acquired. However, The agent, whose quality is initially unknown, only wants to get a high reward, and he experiments privately and discloses the results selectively. Thus the optimality of the principal's commitment arises as a key question.

For instance, consider a professor who wants to deliver a fair reference letter for an undergraduate student. Knowing that the student can privately take many exams and internships and selectively report the results, the professor can commit to only write a good letter if the student reports enough successes. Is this commitment optimal?

The reward scheme that is committed to by the principal is designed to reflect the agent's true quality, but it also affects the agent's incentives to acquire and reveal the evidence. A harsh standard in the commitment might deter the acquisition of the evidence due to a high cost of experiments, but a relaxed one may contaminate the informativeness of the evidence.

In designing the optimal reward scheme, the principal takes into account two options that the agent has to deviate from the intended path of experimentation: *early-stop* and *over-experimentation*. The early-stop incentive occurs when the total experimental cost is larger than the benefit from continuing experiments. In this case, a potential good type stops experimenting too early, to save experimental cost, and he doesn't learn enough and compromises on a lower reward level. The over-experimentation incentive emerges when he fails after he has acquired many successes. Now the agent has learned that he is a bad type. However, since the principal doesn't observe any experiments or results, the bad type agent can still continue to acquire successes and pretend to be a good type by hiding the unfavourable results in his later report. Thus, the bad type can still achieve a high reward level. Both incentives would cause a mismatch between types and rewards.

Building upon these two incentives, my first main result suggests that the optimal reward scheme is an increasing step function. When the motivated number of experiments is small and the agent's incentives to deviate from the intended path of experimentation are weak, a one-step function is optimal: the agent receives the conditional expected value as a bonus if he reports enough successes; otherwise, he only gets a non-negative compensation. As this number increases, in which case the conditional expected value becomes higher than the expected cost of acquiring one more success by a bad type agent, extra steps are added after many successes have been acquired, to deter the bad type's over-experimentation behaviour, who only needs a few more successes to pretend to be a good type. These steps make a bad type agent indifferent between over-experimenting and stopping immediately. Furthermore, if the failures are not verifiable, and the agent's prior expected value cannot cover the expected total cost of the experiments, the early-stop incentive distorts the reward scheme at early stage, since the agent cannot prove that an experiment has been carried out and failed. Thus the additional steps are required to encourage a potential good type to continue the remaining experiments, which make a potential good type agent indifferent between aborting experiments and continuing. As a result, the distortion results in that a bad type agent is always over-paid relative to his true value.

My second main result shows that the optimal motivated number of experiments is always weakly greater than the largest number whose expected total cost can be covered by the agent's prior expected value. On the one hand, the more successes that a potential good type agent discloses, the more accurate the principal's reward could be. This force leads to a higher motivated number of experiments. However, on the other hand, when the motivated number is too large such that the conditional expected value is higher than the cost of over-experimentation, or the agent's prior expected value cannot cover the experiments' expected total cost, the principal has to sacrifice and pay more to a bad type to motivate the agent to conduct so many experiments and deter the bad type agents from over-experimenting and contaminating the informativeness of the reported successes. This force discourages the principal from motivating a large number of experiments, and the optimal number is determined by the trade-off between these two forces.

In a public information environment where the experiments and results are publicly observed, I show that the one step function is still optimal. Moreover, when the failures are verifiable and the agent's value-cost ratio is low, the private experimentation achieves the same efficiency level as the public environment. In this case, the agent is able to prove that he has indeed conducted the experiment and failed, and the incentive for deviation from over-experimenting is weak.

I explore two extensions of the model, which show the robustness of my findings. Firstly, I introduce a small probability of bad luck in each experiment, which is privately observed and causes a failure for both types. The results suggest that this environment is equivalent to the scenario with unverifiable failures, since the failures caused by the bad luck provide no information about the agent's type but the incentive to stop at an earlier stage. Secondly, I consider a scenario with finite opportunity for experimenting, and show that the principal's optimal reward scheme is consistent with that with infinite opportunity.

The rest of the paper is organised as follows. Section 2 summarises related literature. Section 3 shows the model's setup as well as some preliminary results and the benchmark. The main analysis of the private experimentation is demonstrated in section 4. Extensions can be found in section 5.

Section 6 finally concludes. All proofs not shown in the main text are given in Appendix A.

2 Literature Review

This work relates to literature about private experimentation. Henry (2009) considers a scenario where the number of experiments is pre-determined, and the agent is not able to stop until all experiments are conducted regardless of their results. My work differs in allowing the agent to decide whether to continue after each experiment. Ispano (2015) shows the conditions such that the sender optimally reveals the unverifiable bad news. My work compares the difference when failures are verifiable and when they are not. Moreover, their work doesn't consider the receiver's optimal commitment from the perspective of mechanism design, which is the key result in my work.

The closest work is by Felgenhauer and Schulte (2014). They characterise the parameter range in which the persuasion equilibria with cut-off rule exists in costly private experimentation with symmetric information structure. In their work, the receiver makes a binary decision, and the sender applies a sanitisation strategy in which all unfavourable results are concealed. In contrast, my model considers an asymmetric information environment, and the principal offers a reward scheme according to different reported results, rather than a binary approval decision. My work follows the mechanism design approach and Delgenhauer and Schulte's does not. The cut-off function in my model is similar to their cut-off approval rule, but the interpretations are different. Also, the cut-off function is not always optimal, and the alternative step function is introduced, which is absent in their work. Other differences are that I use, and I show that, the sanitisation strategy is not optimal for the agent when failures are verifiable.

There is a large literature in strategic experimentation, such as Bergmann and Hege (2005), Halac, Kartik and Liu (2016), and Henry and Ottaviani (2014), etc. Bergman and Hege (2005) show the optimal way to finance an innovative project without full commitment, and Halac, Kartik and Liu (2016) focus on the scenario with full commitment. Henry and Ottaviani (2014) show that the principal free rides on the agent's experiments when results are public information. In most cases where results are private information, the principal or the receiver can use the timing of when they observe success to determine monetary transfer: this is a key difference from the current model, which does not include such timing.

This work also relates to literature on information disclosure and persuasion. Rayo and Segal (2010) and Kolotilin (2015) focus on the sender's optimal mechanism; Kamenica and Gentzkow (2011) find the optimal way for the sender to design the structure of the experiment, and Bergemann, Bonatti and Smolin (2015) consider a monopolist who can design the experiment and set the selling price. They all focus on public experimentation, where experimental results can be publicly observed. In contrast, this work mainly focuses on the private case, and compares the results to public case. Glazer and Rubinstein (2004, 2006) and Hart, Kremer and Perry (2016) analyse how commitment can help the principal to improve outcomes in evidence games where the agent's set of hard evidence is exogenously given and he cannot generate any other evidence. By comparison, this work considers how the optimal commitment changes if the agent can acquire hard evidence at a cost.

DeMarzo, Kremer and Skrzypacz (2017) also consider an uninformed agent who chooses one test among many different tests and strategically reveals the result to the market. In their paper, the market is competitive, and the agent has only one chance to take a test, in which the null result with positive probability is introduced and is not verifiable. By contrast, my model focuses on the softest test in which the good type always passes and the bad type passes with some probability, and the agent has infinite opportunity for experimenting. In my work, both the scenarios, when the failure is verifiable and not verifiable, are discussed. Also, my model focuses on the principal's optimal full commitment, which is absent in their paper.

This work can be compared to literature on signalling and screening, for example Spence (1973), and Rothchild and Stiglitz (1976). In their models, there is no learning process for the agent, and every type of agent can mimic the behaviour of others for a price. Here, the agent would learn his own type through experiments, and a potential good type cannot perfectly masquerade as bad type. Additionally, in this work, the agent can only be more optimistic that his type is good if no failure occurs, and full screening cannot be achieved.

Other literature that this work relates to includes Hörner and Skrzypacz (2014) and Celik (2015) who focus on gradually revealing information, and Hörner and Skrzypacz (2014) who show that sequential tests can help to mitigate the hold-up problem. Kruse and Strack (2015) focuses on mechanism design for an optimal stopping time, and a cut-off rule is proposed as optimal.

3 Model

3.1 Setup

An agent (he) wants to get reward (or evaluation) from a principal (she). Initially the agent's type[1] (value), $M_i \in \{M, 0\}$, is unknown, but a common prior is shared: type is good (G) with probability p_0, and its value is $M_G = M$, where $p_0 \in (0, 1)$ and $M \in \mathbb{R}^+$; it's bad (B) with probability $1 - p_0$, and value is $M_B = 0$.

The agent can learn his own type through the private experiments[2]. The constant cost of each

[1] This setting is equivalent to that in which the agent sells an project with unkown quality.

[2] It can also be interpreted that there are many different tests with the same level of cost and threshold for passing.

experiment is c, where $c \in \mathbb{R}^+$. In each experiment, a good type agent can always succeed. However, a bad type can only succeed with probability $1-\theta$, where $\theta \in (0,1)$. θ can be considered as the threshold to pass a test. The experiment has the property of the softest test, where a good type always passes the test and a bad type fails it with some positive probability. Since experiments are privately observed, the agent can selectively report a subset of acquired results. Give he conducts k experiments, denote the reported number of successes and failures by k^g and k^b respectively, where $k, k^g, k^b \in \mathbb{N}$.

Before experimentation, the principal can offer a reward scheme $a(\cdot)$ to the agent which specifies the different reward levels corresponding to different combinations of reported results by the agent, where $a : \mathbb{N} \times \mathbb{N} \to \mathbb{R}^+$. Specifically, $a(\cdot) = a(k^g, k^b)$. The principal cares about the precision and fairness of the reward, and her payoff function is: $-(a(k^g, k^b) - M_i)^2$.

The risk neutral agent only cares about the reward level, and his payoff function is $a(k^g, k^b) - kc$, given he has conducted k experiments and reported k^g successes and k^b failures. All parameters and payoff functions are common knowledge, and the timeline is shown below:

1. Principal offers the reward scheme, and agent chooses whether accept or not.
2. Agent begins to run experiments after acceptance.
3. Agent stops experiments and selectively reports to principal.
4. Payoffs are realised.

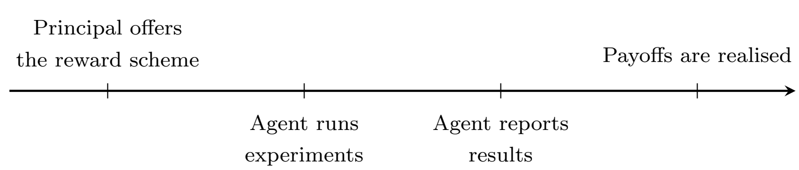

Figure 1: Timeline

3.2 Preliminaries

This section shows some preliminary results, which can help to simplify the future analysis.

First consider how the posterior belief is updated when different results are acquired. If the agent succeeds in k experiments without failure, he becomes more optimistic on that his type is good, and the posterior belief denoted by p_k is updated according to Bayes' rule:

$$Pr(\text{Good}|k, 0) = \frac{p_0}{p_0 + (1-p_0)(1-\theta)^k} = p_k \tag{3.1}$$

In this case, the agent has no reason to hide the successes. This is because, on the equilibrium path, the more successes are reported the higher the posterior belief the principal has, which leads a higher expected value. Instead, if the agent has acquired at least one failure, he learns that he is a bad type, no matter how many successes he has achieved before. This is because only the bad type can fail with positive probability in the experiment, and the posterior belief now is $Pr(G|k^b \geq 1) = 0$.

Since one failure is enough for the principal to learn the agent is bad with value $M_B = 0$, the principal needs not to provide any incentives for the bad type agent to conduct future experiments for the purpose of screening. On the one hand, more failures don't affect the belief if the principal has observed one failure, and the precision of the reward is not affected. On the other hand, If the agent continues to experiment after the first failure, he either gets more failures or more successes. Thus he can hide failures and report more successes to achieve a higher reward level, and the reward scheme is distorted. This is the agent's over-experimentation incentive, which would be discussed later in details.

As a result, the number of successes reported by the potential good type who hasn't failed is the number of experiments that the agent has conducted. In the model, this number is also called the number of experiments motivated by the principal in the reward scheme, or the number of successes motivated to report at most. Also, those who report fewer successes must have failed and the principal learns that their types are bad. The optimal reward scheme therefore must provide incentives to conduct a certain number of experiments without failure.

To determine the optimal commitment, the question can be decomposed into the following two parts: firstly, given the principal motivates the agent to conduct k experiment(s) without failures; the properties of the associated reward scheme needs to be characterised, which is denoted by $a^k(k^g, k^b)$; secondly, the optimal number k^* then can be found, which maximised the principal's *ex ante* expected payoff. Thus, the optimal commitment is the optimal reward scheme $a^{k^*}(k^g, k^b)$, and k^* experiments are motivated to run.

Since the success is verifiable, the only way for a bad type to pretend to be good is to over-experiment—continuing experiments to acquire successes by luck after his first failure arrives. To mimic the good type's behaviour, the bad type agent's selective report satisfies: $k^g + k^b \leq k$. The verifiability of failure is discussed later. Meanwhile, due to limit liability[3], all reward levels are non-negative, $a(\cdot) \geq 0$. Thus some features associated with the optimal reward scheme can be observed, and are summarised in the following Lemma 1.

Lemma 1. *Given the commitment provides incentives to run k experiments without failures, the associated reward scheme is equivalent to the reward scheme with: for $\forall k^g, k^b, k \in \mathbb{N}$,*
1) $a^k(k^g, 1) \geq a^k(k^g, k^b > 1)$*;*

[3]Intuitively, a salary or payment cannot be negative.

2) $a^k(k^g \geq k, k^b \geq 1) = 0$*;*

3) $a^k(k^g \geq k, 0) = a^k(k, 0)$.

Lemma 1.1) states that it is optimal to assign at most the same level of reward to those reporting more failures, since one failure is enough for the principal to learn that the agent is a bad type and more failures don't affect the principal's posterior belief on the agent's type. Given the reported number of successes, the principal punishes the bad type agent who has more failures by assigning a lower reward level due to his over-experimentation behaviour.

Lemma 1.2) pushes the punishment to the maximum when an agent reports more than enough successes together with failure(s). The principal learns the agent is bad when observing failures, and she also learns that the bad type has over-experimented to acquire so many successes after his first failure. This punishment is just the bad type's true value, which prevents such behaviour from occurring when a contract is accepted.

Lemma 1.3) shows that, to guarantee that an agent's ex ante optimal plan is to conduct enough experiments k given the reward scheme from principal, the marginal benefit from running one more trial is strictly less than the marginal cost after continuously obtaining k successes, and it must also be true for all numbers greater than k. If not, the agent would continue experimenting as long as no failure occurs. For any other reward scheme satisfying such criteria, it would lead to the same end as the one with assigning the same level of reward to those reporting more than enough successes without failure.

Therefore, the remaining analysis can focus on $a^k(k, 0)$, $a^k(k^g < k, 0)$ and $a^k(k^g < k, 1)$ only.

3.3 Benchmark: Public Experimentation

Before solving the main model, I discuss a benchmark to provide useful background and intuition, in which the experiments are carried out publicly and their results are observable by both parties. If a failure occurs, the principal learns immediately that the agent is a bad type, and any further experiments don't affect the principal's belief or improve the precision of the reward. The agent cannot hide any failures, or over-experiment to pretend that failure never occurs.

When the principal's commitment provides incentives to run k experiments without failure, the agent's expected payoff is:

$$U(k, p_0) = -c + [p_0 + (1 - p_0)(1 - \theta)]\, U(k - 1, p_1) + (1 - p_0)\theta a^k(0, 1) \tag{3.2}$$

Denote by $U(k - j, p_j)$ the agent's continuation value of running the remaining $k - j$ experiments with current belief p_j, where $j \leq k$. After paying the cost c in the first experiment, with probability

$p_0+(1-p_0)(1-\theta)$, the agent succeeds and becomes more optimistic with a posterior belief p_1, and his continuation value of running the remaining $k-1$ experiments is $U^A(k-1,p_1)$; with probability $(1-p_0)\theta$, the agent fails, and then he learns that his type is bad, and receives a reward level $a^k(0,1)$. The agent's *ex ante* expected payoff can also be simplified as:

$$U(k,p_0)=\mathbb{E}\left(a^k(k^g,k^b)\middle|k,p_0\right)-\tilde{k}c, \quad \text{where } \tilde{k}=\sum_{i=1}^{k}\frac{p_0}{p_{i-1}} \tag{3.3}$$

where $\tilde{k}$ is the expected number of experiments that the agent would run after accepting the contract, and it equals to the summation of the likelihood ratio of prior to posterior beliefs. Thus $\tilde{k}c$ is the *ex ante* expected total cost[4]. Following this plan, the agent's *ex ante* expected gain[5] is $\mathbb{E}\left(a^k(k^g,k^b)\middle|k,p_0\right)$. The agent would accept the contract and run experiments as long as Individual Rationality (IR) is satisfied, and his *ex-ante* expected payoff is non-negative. Thus the principal maximises her expected payoff conditional on the prior belief and the number of experiments provided incentives to run:

$$\begin{aligned}&\underset{a(\cdot)\in R^+}{Max} && V(k,p_0)=\mathbb{E}\left(-\left(a^k(\cdot)-M_i\right)^2\middle|k,p_0\right)\\ &\text{s.t}: && \text{IR}: \ U(k,p_0)\geq 0\end{aligned} \tag{3.4}$$

Proposition 1. *In public experimentation:*

1) Given the commitment provides incentives to run k experiments without failures, the associated optimal reward scheme is a cut-off function at k (CF), specifically:

$$CF=\begin{cases}a^k(j<k,1)=\max\left\{0,\tilde{k}c-p_0M\right\}\\ a^k(k,0)=p_kM+\max\left\{0,\tilde{k}c-p_0M\right\}\end{cases} \tag{3.5}$$

2) The optimal number k^P satisfies $\overline{k}\leq k^P<\infty$, where

$$\overline{k}=\begin{cases}\max\{k\in\mathbb{N}:p_0M\geq\tilde{k}c\} & p_0M\geq c\\ 0 & p_0M<c\end{cases}$$

Given the principal provides incentives to run any positive number of experiments without

[4]In this plan, the agent only needs to continue experimenting when no failure occurs, thus it's easily to see the probability that no failure occurs in k experiment is $p_0+(1-p_0)(1-\theta)^k$, and the probability that first failure occurs in j_{th} experiments is $(1-p_0)(1-\theta)^{j-1}\theta$. Therefore, the expected total cost is: $\left[p_0+(1-p_0)(1-\theta)^k\right]kc+\sum_{i=1}^{k}(1-p_0)(1-\theta)^{i-1}\theta(i-1)c=\sum_{i=1}^{k}\left[p_0+(1-p_0)(1-\theta)^{i-1}\right]c=\sum_{i=1}^{k}\frac{p_0}{p_{i-1}}c$.

[5]$\mathbb{E}\left(a^k(k^g,k^b)\middle|k,p_0\right)=[p_0+(1-p_0)(1-\theta)^k]a^k(k,0)+\sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta a^k(i,1)$.

failures, the optimal reward scheme is an one-step function, or cut-off function (CF), and it can be interpreted as follows: the principal commits that the agent gets a high reward level if reporting weakly more than k successes without failures, and he is treated as a bad type if reporting strictly less successes with one failure. The agent then runs experiments after accepting the contract, and reports all results he acquires. Thus when any less than k successes are reported, there is a failure associated.

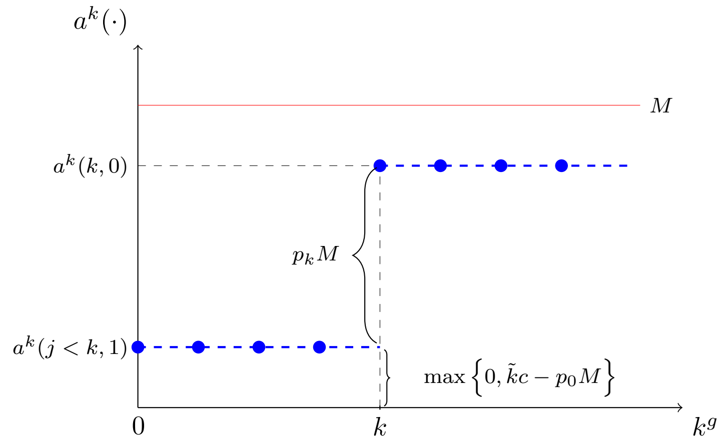

Figure 2: CF in public experimentation

The participation threshold $\overline{k}$ is now introduced, which is the largest number of experiments whose expected total cost can be covered by the agent's prior expected value. When the prior expected value is very low, $p_0 M < c$, it can cover the cost for at least one experiment, in which case $\overline{k} = 0$. When the prior expected value is high, $p_0 M \geq c$, the threshold equals to the round down of the number of experiments such that the prior expectation equals to the expected total cost[6], $p_0 M = \tilde{k}c$. When the incentivised number in the contract is smaller than this threshold, $k < \overline{k}$, the agent is willing to carry out the principal's desired plan since the prior expectation of agent's value is higher than the *ex ante* expected total cost, $p_0 M \geq \tilde{k}c$. Then the principal can simply assign the bad type's true value to those facing early failures, and the posterior expectation, $p_k M$, to those who have achieved the desired number of successes without failure. These reward levels are the same as the agent's self evaluation after learning through experiments.

If the principal incentivises more than the participation threshold, $k > \overline{k}$, she needs to take care of the agent's individual rationality, IR, to guarantee the agent's acceptance of her contract in the first place. *Ex ante*, it's too costly to achieve the desired number of successes without failures, $\tilde{k}c > p_0 M$. Thus it becomes optimal to assign a positive reward level to a bad type reporting one

[6]If k is continuous, the threshold number would be the one such that the prior expectation just equals to the total expected cost. Due to the discreteness of experiment, the threshold needs to be rounded down.

failure, which is just equal to the excess cost—the difference between the expected total cost and the prior expectation. This can be treated as a reward for audacity, or compensation for fear of failure, even if the agent's type turns out to be bad. If the exact k successes are acquired without failure, the agent gains a bonus which is equal to the posterior expectation, $p_k M$. It captures the feature that the principal is risk averse, and she optimally reduces the risk of making a mistake—assigning weakly a higher level of reward to every *ex post* scenario. This is equivalent to the principal paying the excess cost up front when agent accepts the contract, and incentivise him to collect the bonus if the desired number of successes are obtained without failure.

In the public experimentation, the principal optimally motivates k^P experiments without failures, and it's always weakly greater than the first threshold $\overline{k}$. On the one hand, a higher reward is closer to the true valuation of good type, which is the gain in the *ex post* scenario where the agent's type is good; on the other hand, the loss also increases in *ex post* scenarios where the agent's type is bad. Therefore, the principal is only willing to push k^P as large as possible if the gain can cover the loss. This k^P must be finite, since the good type's value is finite and increasing required successes without failure would raise losses in every *ex post* scenario beyond the number whose associated reward is closest to the good type's value. A numeric example is shown below.

Example 3.1. When $\theta = 0.6$, $p = 0.4$, $M = 3$ and $c = 1$, then the participation threshold number is $\overline{k} = 1$. In public experimentation, the optimal number is $k^P = 2$ and its associated reward scheme is $\text{CF} = \begin{cases} a^2(2,0) = 2.86 \\ a^2(j < 2, 1) = 0.44 \end{cases}$ and the principal's expected payoff is $V^P(2, p_0) = -0.89$.

4 Analysis in Private Experimentation

In private experimentation, experiments and results are privately observed by the agent, thus extra incentives must be provided in the principal's commitment. Given $j < k$ successes are acquired without failure in the first j experiments, the agent only cares about whether the future experiments can generate a net benefit since the cost of previous experiments is sunk, and it illustrates the agent's first type of incentive constraints, IC_j^S, where the agent would only continue and fulfil the initial plan if and only if:

$$U(k - j, p_j) \geq a^k(j, 0) \tag{4.1}$$

If the agent stops and reports j successes, he gains less on the one hand, but saves experiment costs on the other hand. If it's beneficial to do so, the agent would pretend to he has failed and conceal the failure, and earn $a^k(j, 0)$. This is called the agent's early-stop incentive. Thus the inequality (4.1) can guarantee that the agent sticks to the initial plan and continues the remaining experiments after early successes.

Now consider the following scenario where the first failure occurs at the $j+1_{th}$ trial. The agent then privately learns his type is bad, which the principal doesn't observe. Obviously he can stop and report results to date. If he conceals the failure and only reports successes, the reward level would be $a^k(j,0)$; if he also reports the failure, he can get $a^k(j,1)$. Besides, the agent has another alternative choice compared to the public experimentation scenario: he can over-experiment until he acquires enough successes by luck and conceal the failures, in which case he can achieve a higher reward level by masquerading that he never fails or that failure occurs at later trials. This is called over-experimentation incentive. If the bad type is doing so, the report is less informative. For a bad type, the expected cost to guarantee one more success is $\frac{c}{1-\theta}$, and the agent would over-experiment if the extra gain from over-experimentation can cover this cost. To prevent such behaviour on the equilibrium path, incentives constraints for over-experimentation in the principal's commitment, IC_j^F, needs to be provided. Thus the following constraints can guarantee that the agent shall not over-experiment following the first failure[7]:

$$\max\left\{a^k(j+1,1), a^k(j+1,0)\right\} - \max\left\{a^k(j,1), a^k(j,0)\right\} \leq \frac{c}{1-\theta} \tag{4.2}$$

Given k experiments without failures are motivated, there are $k-1$ early-stop incentives constraints IC^S, and $k-1$ over-experimentation incentive constraints IC^F. Together with IR, the principal faces $2k-1$ constraints.

4.1 Verifiable failures

The verifiability of failures plays a crucial role in determining the optimal contract offered by the principal. When it's verifiable, it's the hard evidence to prove that the agent indeed has paid the cost, conducted the experiment and failed. When it's not verifiable, the such conducted experiment cannot be proved, and the principal cannot distinguish the agent who fails from those haven't undertaken it. This section focuses on the scenario in which failures are verifiable. The scenario with unverifiable failures is discussed later in Section 4.2.

Lemma 2. *In private experimentation with verifiable failures, for $k^g < k$,*

$$a^k(k^g,1) \geq a^k(k^g,0) = 0 \tag{4.3}$$

When the failures are verifiable, the agent's must be a bad type if less than the required number of successes are reported without failures since the principal provides enough incentives to the

[7]The same constraint is satisfied if the agent only plans to try one more trial after failure.

potential good type to continue experimenting after early success. A reward for honesty can be created by assigning a weakly higher reward level to those who report a failure. As the largest punishment to those who pretend to face a failure, the principal then can simply assign the bad type's true value, $a^k(j < k, 0) = 0$, if "no failure presented" when fewer successes are reported. Thus IC^S and IC^F can be simplified as:

$$\begin{aligned}
&\text{IC}^{S,V}_{0\le j\le k-1}: && U(k-j, p_j) \ge 0 \\
&\text{IC}^{F,V}_{0\le j< k-1}: && a^k(j+1,1) - a^k(j,1) \le \frac{c}{1-\theta} \\
&\text{IC}^{F,V}_{k-1}: && a^k(k,0) - a^k(k-1,1) \le \frac{c}{1-\theta}
\end{aligned} \tag{4.4}$$

Thus, this simplification shows that IR is the same as $\text{IC}^{S,V}_0$. After the success in the first trial, the agent is more optimistic, and he is willing to carry out the remaining experiments. Thus, as long as IR is satisfied, $\text{IC}^{S,V}_{1\le j\le k-1}$ must be slack, in which the early-stop incentive can be discarded.

These constraints demonstrate that the associated optimal reward scheme must share the property of screening: a bad type would not blend into a potential good type, and the potential good type would not pretend to be bad. However, full screening cannot be achieved here. This is because the agent can only be more optimistic after more successes are achieved without failure and treat himself as a potential good type, but he cannot be sure whether he is a good type or a lucky enough bad type who hasn't failed yet.

Proposition 2. *In private experimentation with verifiable failures,*
1) Given the commitment provides incentives to run k experiments without failures, the associated optimal reward scheme $a^k(\cdot)$ is:

a) CF when $k \le \hat{k}$, where $\hat{k} = \begin{cases} \max\{k \in \mathbb{N} : p_k M \le \frac{c}{1-\theta}\} & p_1 M \le \frac{c}{1-\theta} \\ 0 & p_1 M > \frac{c}{1-\theta} \end{cases}$;

b) Type I multi-step function (MF-I) when $k > \hat{k}$;

2) The optimal number k^*_V satisfies $\overline{k} \le k^*_V < \infty$. Especially, $k^*_V = k^P$ if $\hat{k} \ge k^P$.

Here the over-experimentation threshold $\hat{k}$ is introduced, which measures the largest number of reported successes where the agent's conditional expected value is weakly smaller than the expected cost of a bad type achieving a success. Proposition 2.1.a) shows CF, the optimal cut-off reward scheme in public experimentation, is still optimal when the motivated number of experiments is low, and the over-experimentation threshold $\hat{k}$ determines the scope of CF in private experimentation with verifiable failures.

The demonstration of the optimal reward scheme at different value-cost ratio can be summarised in Figure 4. When the agent's value-cost ratio is low, $\frac{M}{c} \in \left[0, \frac{1}{1-\theta}\right]$, $\hat{k} \to \infty$, the over-experimentation threshold $\hat{k}$ doesn't affect the optimality of CF. Now optimal commitment is exactly the same as that in public experimentation. Intuitively, this happens when the good type is not superior enough or the experimental cost is relatively high. Also, since the failures are verifiable, once the commitment is made and experiments are carried out, all reports which have less than the required level of successes must contain a failure, and the agent would disclose all of the information he acquires.

When the agent's value-cost ratio is medium, $\frac{M}{c} \in \left(\frac{1}{1-\theta}, \frac{1}{(1-\theta)p_1}\right]$, $0 < \hat{k} < \infty$, in which case the difference between the good and bad type's values are not too large, or the cost of a single experiment is relatively low. Now the over-experimentation threshold equals to the rounded down number of reported successes where the agent's conditional expected value equals to the expected cost of a bad type achieving a success, $p_k M = \frac{c}{1-\theta}$. When the motivated number of experiments is lower than the over-experimentation threshold, $k \leq \hat{k}$, and the optimal commitment in public experimentation could still be optimal in private with verifiable failures, when the over-experimentation threshold is sufficiently high, $\hat{k} \geq k^P$. However, if the motivated number of experiments is too large, $k > \hat{k}$, CF cannot be applied. Consider the following situation when the first failure occurs at the k_{th} experiment. If the principal still sticks to CF, the agent would definitely over-experiment since such behaviour would lead to an extra gain $p_k M$ which can cover the expected cost of acquiring one more success for the bad type. As a result, the incentive constraint $\text{IC}_{k-1}^{F,V}$ is violated and report becomes less informative. Therefore, alternative reward schemes need to be considered, and the optimal one among them is a type-I multi-step function (MF-I), which is proposed in Proposition 2.1.b).

Definition 1. Type I multi-step function (MF-I) is a reward scheme such that $\text{IC}_{l \leq j \leq k-1}^{F,V}$ are all binding, where $l = \max\{l \in \mathbb{N} : p_0 M \leq \sum_{i=l+1}^{k} \frac{p_0}{p_i} \frac{c}{1-\theta} \ \&\ 0 \leq l \leq k-1\}$:

$$\text{MF-I} = \begin{cases} a^k(j < l, 1) = \max\left\{0, \tilde{k}c - p_0 M\right\} \\ a^k(l, 1) = p_l M - \displaystyle\sum_{i=l+1}^{k} \frac{p_l}{p_i} \frac{c}{1-\theta} + \max\left\{0, \tilde{k}c - p_0 M\right\} \\ a^k(l < j < k, 1) = (j-l)\frac{c}{1-\theta} + a^k(l, 1) \\ a^k(k, 0) = (k-l)\frac{c}{1-\theta} + a^k(l, 1) \end{cases} \tag{4.5}$$

The structure of MF-I demonstrates the feature of "setting blocks at the end": the difference of rewards between rewards among a neighboured number of reported successes cannot exceed $\frac{c}{1-\theta}$, which is the cost level of acquiring a success for a bad type. For those failures which occurs early enough, that is before l_{th} experiments, it's still too costly to over-experiment: the agent is then happy to stop and report what he acquires, and the reward level could be same as those in CF at the same level since failures are verifiable. Notice that, the later the first failure occurs, the stronger the incentive for the agent to over-experiment. Thus it's optimal to have $IC^{F,V}_{l\leq j\leq k-1}$ be all binding. As a result, not only are bad types overpaid, but also they are treated differently when the first failure occurs in different experiments: the later the failure, the higher is the reward associated. Meanwhile, it's easy to see that when the required number of successes k are reported, the associated reward level for a potential good type in MF-I is strictly lower than that in CF[8]. One example of MF-I's structure can be found in Figure 3.

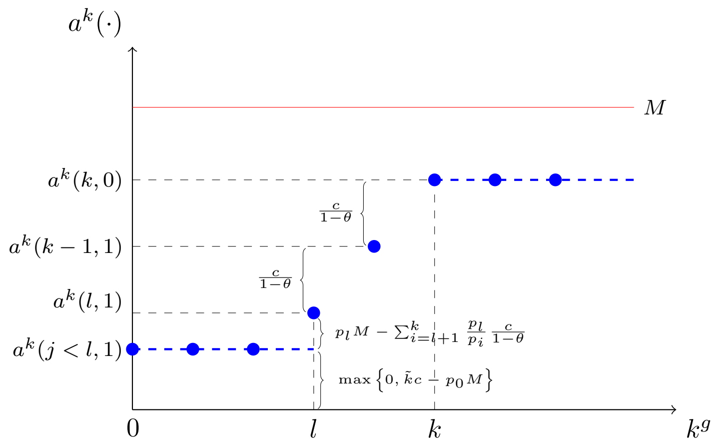

Figure 3: MF-I in private experimentation (Example: $k > 2$ and $l = k-2$)

If the agent's value-cost ratio is sufficiently high, $\frac{M}{c} \in \left(\frac{1}{(1-\theta)p_1}, \infty\right)$, in which case the good type is much better than the bad type or the cost of a single experiment is too low, the agent would over-experiment at any positive number of reportable successes, where $\hat{k} = 0$. This leads to all $IC^{F,V}$ are binding. As a result, the potential good type is still underpaid relative to the agent's self-evaluation, but the bad type is overpaid.

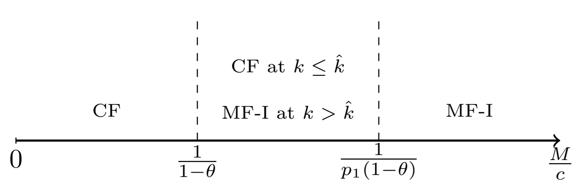

Figure 4: Optimal reward scheme with verifiable failures at different value-cost ratio

Under the optimal reward scheme, the principal still optimally motivate the agent to run a

[8]When comparing the reward level to those reporting required k successes without failure in CF and MF-I, it has $p_k M - (k-l)\frac{c}{1-\theta} - \left(p_l M - \sum_{i=l+1}^{k} \frac{p_l}{p_i} \frac{c}{1-\theta}\right) > 0$.

number of experiments that is higher than the participation threshold $\overline{k}$, even if it exceeds the over-experimentation threshold $\hat{k}$. This is because the benefit of making the reward level for a potential good type closer to the good type's true valuation M is sufficiently large to cover the expected increased loss of overpayment in other *ex post* scenarios where the type is bad. The lower bound of optimal amount k_V^* is the same as that in public experimentation, and the exact number depends on the parameter range, and CF and MF-I are the only possible candidates of reward scheme. A numeric example is shown below.

Example 4.1. When $\theta = 0.6$, $p = 0.4$, $M = 3$ and $c = 1$, then the participation threshold is $\overline{k} = 1$, and the over-experimentation threshold is $\hat{k} = 2$. In private experimentation with verifiable failures, at $k = 2$, the associated optimal reward scheme is $\text{CF} = \begin{cases} a^2(2,0) = 2.86 \\ a^2(j < 2, 1) = 0.44 \end{cases}$, and the principal's expected payoff is $V_V(2, p_0) = -0.89$; at $k = 3$, the associated optimal reward scheme is $\text{MF-I} = \begin{cases} a^3(3,0) = 3.65 \\ a^3(2,1) = 1.15 \\ a^3(j < 2, 0) = 0.94 \end{cases}$, and the principal's expected payoff is $V_V(3, p_0) = -1.19$. In the optimal contract, implicitly determined number of experiments is $k_V^* = 2$ and its associated reward scheme is CF.

4.2 Unverifiable failures

The situation becomes more complicated when failures are not verifiable. This implies that the agent can easily or cheaply lie when reporting failures, which occurs when the hard evidence of failure is hard to find or stored or cost of fake evidence is cheap. Thus the principal cannot tell whether the experiment associated with the failure has been indeed carried out. If it is easy for the agent to conceal a failure, the idea of "rewarding honesty" in public and private experimentation with verifiable failures cannot be applied, and Lemma 2 doesn't hold. If the principal assigns a strictly higher reward to those reporting failures, the potential good type agent would pretend to be a bad type if it's beneficial to do so. On the one hand, when pretending to be those who face an early failure, the gain is smaller than continuing to undertake the remaining experiments; but, on the other hand, the agent can save experimental costs. Therefore, the best that the principal can do is to assign the same level to those reporting less than required number of successes with and without failures, and $\mathrm{IC}^{F,NV}$ and $\mathrm{IC}^{S,NV}$ can now be written as:

$$\begin{aligned} &\mathrm{IC}^{S,NV}_{0\le j\le k-1}: && U(k-j,p_j)\ge a^k(j,0)=a^k(j,1)\\ &\mathrm{IC}^{F,NV}_{0\le j<k-1}: && a^k(j+1,1)-a^k(j,1)\le \frac{c}{1-\theta}\\ &\mathrm{IC}^{F,NV}_{k-1}: && a^k(k,0)-a^k(k-1,1)\le \frac{c}{1-\theta} \end{aligned} \tag{4.6}$$

With the help of incentive constraints above, the agent would disclose all of the information acquired on the equilibrium path, and "masquerading" behaviour is deterred: $\mathrm{IC}^{F,NV}$ prevent from "pretending to be good" and $\mathrm{IC}^{S,NV}$ deter "pretending to be bad". Together with IR, there are still $2k-1$ constraints, whose number is the same as under private experimentation with verifiable failures, but they are more strict. Thus it's natural to check whether CF and MF-I proposed before are feasible, and whether they would still be optimal if all constraints were satisfied; otherwise, other reward schemes need to be considered.

Proposition 3. *In private experimentation with unverifiable failures,*
1) Given the commitment provides incentives to run k experiments without failures, the associated optimal reward scheme is:

a) CF when $k\le \min\left\{\hat{k},\overline{k}\right\}$;
b) MF-I when $\hat{k}<k\le\overline{k}$;
c) Type II multi-step function (MF-II) when $\overline{k}<k\le\hat{k}$;
d) Type III multi-step function (MF-III) when $k>\max\left\{\hat{k},\overline{k}\right\}$.

2) The optimal number k^*_{NV} satisfies $\overline{k}\le k^*_{NV}<\infty$.

Proposition 3.1) shows that the structure of the optimal reward scheme is determined mutually by the participation and over-experimentation threshold. When the motivated number of experiments is smaller than both, $k<\min\left\{\hat{k},\overline{k}\right\}$, the agent finds that, in CF, the benefit from early-stop is too small, and the cost of over-experimentation after failure is too high. This happens when the value-cost ratio stays at a medium level, $\frac{M}{c}\in\left[\frac{1}{p_0},\frac{1}{(1-\theta)p_1}\right]$, where $0<\overline{k}<\infty$ and $\hat{k}>0$. Instead, when the ratio is sufficiently high or too low, $\frac{M}{c}\in\left[0,\frac{1}{p_0}\right)\cup\left(\frac{1}{(1-\theta)p_1},\infty\right)$ where now $\hat{k}=0$ or $\overline{k}=0$, if the principal still sticks to a CF scheme, at least one incentive constraint is violated, and the agent then would either stop earlier without failure or over-experiment after early failure. As a result, CF is no longer optimal in this case. The discussion of the optimal reward scheme at different value-cost ratio when failures are not verifiable is summarised in Figure 6.

Since there is no straightforward way of knowing which threshold number is larger, different scenarios need to be discussed. If the participation threshold is relatively larger, $\hat{k}<\overline{k}$, MF-I is optimal when the number of desired experiments is between the two threshold numbers, $\hat{k}<k\le\overline{k}$.

Since the desired number of experiments is still smaller than the participation threshold, the agent doesn't gain from pretending to be bad by stopping early without failure. On the other hand, however, now $k > \hat{k}$ is large enough to generate a gain from over-experimentation, specially if a failure occurs when the required number of successes is almost achieved. Then only $\text{IC}^{F,NV}$ need to be attended to, and MF-I scheme is optimal.

If the over-experimentation threshold is relatively larger, $\overline{k} < \hat{k}$, when the incentivised number k is between two thresholds, the conclusions are different. When $k < \hat{k}$, the agent is not willing to pretend to be a good type by over-experimentation since the benefit from such behaviour cannot cover the cost. However, when $k > \overline{k}$, in which the prior expectation cannot cover the *ex-ante* expected total cost, the agent find that it's better to pretend to be a bad type and receive a relatively smaller reward after accepting the principal's contract, among which the worst case is that the agent reports failure immediately without any experiments. To deal with this, the type II step function (MF-II) is introduced.

Definition 2. The type II multi-step function (MF-II) is a reward scheme such that $\text{IC}^{S,NV}_{0\leq j\leq m}$ are all binding, where $m = \max\{m \in \mathbb{N} : p_0 M \leq \sum_{i=m+1}^{k} \frac{p_0}{p_{i-1}} c \ \&\ 0 \leq m \leq k-1\}$:
1) When $0 \leq m < k-1$,

$$\text{MF-II} = \begin{cases} a^k(0,0) = 0 \\ a^k(0 < j \leq m, 1) = \sum_{i=1}^{j} \frac{p_i}{p_{i-1}} c \\ a^k(m < j < k, 1) = \displaystyle\sum_{i=m+1}^{k} \frac{p_{m+1}}{p_{i-1}} c - p_{m+1} M + a^k(m,1) \\ a^k(k,0) = p_k M + a^k(m+1,1) \end{cases} \tag{4.7}$$

2) When $m = k-1$,

$$\text{MF-II} = \begin{cases} a^k(0,0) = 0 \\ a^k(0 < j < k, 1) = \displaystyle\sum_{i=1}^{j} \frac{p_i}{p_{i-1}} c \\ a^k(k,0) = \displaystyle\sum_{i=1}^{k} \frac{p_i}{p_{i-1}} c \end{cases} \tag{4.8}$$

Definition 3. The type III multi-step function (MF-III) is a reward scheme such that $IC^{S,NV}_{0\leq j\leq m}$ and $\text{IC}^{F,NV}_{l\leq j\leq k-1}$ are all binding, $0 \leq m < l \leq k-1$:

$$
\text{MF-III} = \begin{cases}
a^k(0,1) = 0 \\
a^k(0 < j \leq m, 1) = \sum_{i=1}^{j} \frac{p_i}{p_{i-1}} c \\
a^k(m < j < l, 1) = \displaystyle\sum_{i=m+1}^{k} \frac{p_{m+1}}{p_{i-1}} c - p_{m+1} M + a^k(m,1) \\
a^k(l,1) = p_l M - \displaystyle\sum_{i=l+1}^{k} \frac{p_l}{p_i} \frac{c}{1-\theta} + \sum_{i=m+1}^{k} \frac{p_{m+1}}{p_{i-1}} c - p_{m+1} M + a^k(m,1) \\
a^k(l < j < k, 1) = (j-l) \frac{c}{1-\theta} + a^k(l,1) \\
a^k(k,0) = (k-l) \frac{c}{1-\theta} + a^k(l,1)
\end{cases} \tag{4.9}
$$

MF-II captures the feature of "building stairs at beginning": the principal raises the reward level for those whose first failure occurs at some early stage to compensate for the high expected total cost, until the agent is optimistic enough to carry out the remaining experiments if no failure occurs. This implies that this reward scheme makes only the first $\text{IC}^{S,NV}_{0 \leq j \leq m}$ constraints binding. Then the remaining reward levels in MF-II share the same feature as those in CF: if the motivated number of successes k is reported, the agent can claim a bonus $p_k M$. If the value-cost ratio is too small, all $\text{IC}^{S,NV}$ are binding, and steps are built until the very end. These stairs in MF-II have a different effect on an agent's behaviour compared to those blocks at the end of MF-I where the bad type is deterred from over-experimentation.

If the motivated number k is larger than both of the thresholds, $k > \max\left\{\hat{k}, \overline{k}\right\}$, the agent has an incentive to stop earlier without failures at beginning and to over-experiment at the end. On the one hand, due to $k > \overline{k}$, the agent finds that the *ex ante* expected total cost is too high to follow the planned commitment, and he would be better off by stopping earlier without failure. On the other hand, since $k > \hat{k}$, even if previous incentives are solved, the agent would over-experiment since pretending to be good type is more attractive than ceasing to experiment. Therefore, both types of incentives need to be addressed, and the type III step function (MF-III) is optimal among all feasible alternatives.

MF-III then is the mixture of MF-I and MF-II: stairs at beginning and blocks at the end. In this reward scheme, the first $\text{IC}^{S,NV}_{0 \leq j \leq m}$ and the last $\text{IC}^{F,NV}_{l \leq j \leq k-1}$ are binding. At the beginning, the principal raises the reward level for those facing early failure to guarantee experiments are conducted; when enough experiments have been carried out, she sets blocks by fixing the neighboured number of reported good successes at $\frac{c}{1-\theta}$ to deter over-experimentation. Examples of MF-II's and MF-III's structure are shown in Figure 5.

Proposition 3.2) shows that the lowest possible optimal number of experiments motivated by

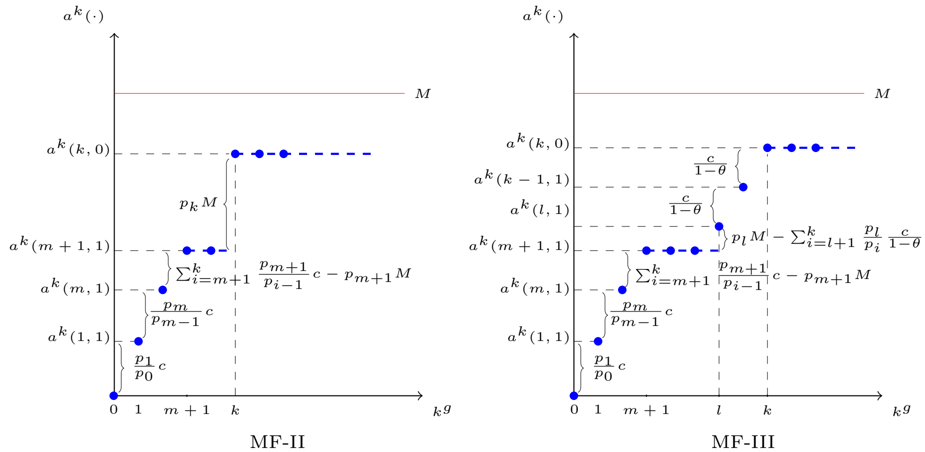

Figure 5: MF-II and MF-III in private experimentation (Example: $m = 2$, $l = k - 2$)

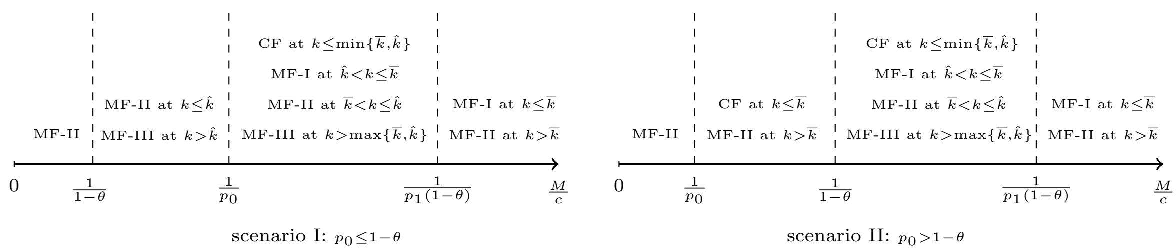

Figure 6: Optimal reward scheme with unverifiable failures at different value-cost ratio

the principal equals to the participation threshold, $k^*_{NV} \geq \overline{k}$, which is consistent with public and private experimentation with verifiable failures. One numeric example is shown below.

Example 4.2. When $\theta = 0.6$, $p = 0.4$, $M = 3$ and $c = 1$, then the participation threshold is $\overline{k} = 1$, and the over-experimentation threshold is $\hat{k} = 2$. In private experimentation with unverifiable failures, at $k = 1$, the associated optimal reward scheme is , CF $= \begin{cases} a^1(1,0) = 1.88 \\ a^2(0,1) = 0 \end{cases}$, and the principal's expected payoff is $V_{NV}(1, p_0) = -1.35$; at $k = 2$, the associated optimal reward scheme is MF-II $= \begin{cases} a^2(2,0) = 3.11 \\ a^2(1,1) = 0.69 \\ a^2(0,1) = 0 \end{cases}$, and the principal's expected payoff is $V_{NV}(2, p_0) = -0.9993$; at

$k = 3$, the associated optimal reward scheme is MF-III = $\begin{cases} a^3(3,0) = 4.17 \\ a^3(2,1) = 1.67 \\ a^3(1,1) = 1.46 \\ a^3(0,1) = 0 \end{cases}$, and the principal's expected payoff is $U^P_{NV}(3, p_0) = -1.68$. In the ptimal contract, $k^*_{NV} = 2$ and its associated reward scheme is MF-II.

4.3 Comparisons between Public and Private Experimentation

Previous results have characterised the properties of the optimal reward scheme given different motivated numbers of experiments, and showed that the optimal number must be weakly larger than the participation threshold $\overline{k}$, which is the largest number of experiments whose expected total cost can be covered by the agent's prior expected value. However, these results don't suggest that it's always optimal to motivate a positive number of experiments.

Corollary 1. *There exist parameter ranges such that the principal is better off conducting no experiments and assigning a single reward level* p_0M *to both types.*

Imagine the case when the cost of an experiment is very high, which implies the excess cost is also prohibitively high. Thus, the principal finds it's too costly to incentivise a single experiment, even if a success can help improve posterior beliefs. Following similar reasoning, when the value-cost ratio, $\frac{M}{c}$, is too low, the principal would not motivate the agent to run any experiments, since the benefit from improving the precision of reward is not enough to cover the cost of doing so.

Corollary 2. *When* $\frac{M}{c} \leq \frac{1}{1-\theta}$*, private experimentation with verifiable failures is equivalent to public experimentation.*

Corollary 2 suggests that, when the value-cost level is low, the principal's optimal contract and the efficiency level in private experimentation with verifiable failures are the same as the public one. When $\frac{M}{c} < \frac{1}{1-\theta}$, $\hat{k} \to \infty$, and the over-experimentation threshold doesn't play a role since the extra gain from over-experimentation is too low. As a result, in private experimentation with verifiable failures, the principal can optimal motivate the same number of experiments by CF as that in public information environment.

When the value-cost ratio is increasing, it shows that a good type agent becomes relatively more valuable, or the experimental cost is relatively cheaper. Thus the prior expectation now can cover larger numbers of experiments, meaning the participation threshold $\overline{k}$ becomes larger. In the public case, the excess cost is also getting smaller, implying the reward levels for both types become more precise and closer to the posterior beliefs. As a result, the principal is willing to weakly increase the optimal number of experiments, which is the positive effect.

Proposition 4. *1) as $\frac{M}{c}$ increases, $\overline{k}$ and k^P are increasing and $\hat{k}$ is decreasing;*
2) k_V^* and k_{NV}^* are increasing as M increases.

When the value-cost ratio is getting larger, the over-experimentation threshold $\hat{k}$ becomes smaller, which leads that it becomes more attractive to deviate for a bad type who fails when only one more success is needed to prove he is a potential good type. Thus the bad type in such situation has the strongest incentive to over-experiment and it's easier to violate the original incentive constraint. As a result, the principal needs to distort the reward scheme at an earlier point of failure to prevent such behaviour, so that the imprecision occurs at an earlier stage and the principal is willing to reduce the number of experiments. This is the negative effect, but it is not the only effect. Since the participation threshold also increases in the private case, it's not clear which effect dominates. If the increasing of the value-cost is solely from the increasing of the good type's value M, then the positive effect dominates, and the principal is willing to raise the optimal number of experiments; if the value-cost ratio increases only due to increase in the cost of experiments, the result is ambiguous.

Now consider the reward levels that different types received under the optimal reward scheme. For a bad type whose value is zero, he can achieve a (weakly) positive reward level as long as he accepts the contract offered by the principal. In the public case, the principal pays the excess cost to ensure that the agent does not deviate from conducting experiments. In the private case, bad types who fail early would receive different reward levels, which are increasing as the early failure occurs later. As a result, even if the agent learns his type is bad after observing an early failure, he can still receive something from the principal, which is weakly higher than his true value. Now both the principal and the agent have the posterior belief p_k, and the agent's a posterior expectation is $p_k M$. The agent can now receive the highest reward level in the associated optimal reward scheme. In the public case, the potential good type can receive the amount of the posterior expectation as a bonus, on top of the excess cost. The reward level is now higher than the posterior expectation. However, in the private case, due to the distortion from the over-experimentation threshold, the highest reward level should cooperate with other reward levels for the bad type to guarantee the informativeness of reported successes, and result is ambiguous when comparing to the posterior expectation level.

Proposition 5. *1) A bad type is always overpaid in both the public and private cases;*
2) A potential good type is overpaid in the public case, but this doesn't always hold in the private case.

5 Extensions

5.1 Bad Luck

Now a small probability of bad luck is introduced. In each experiment, the bad luck occurs with a small but strictly positive probability $\sigma \in (0, 1)$, in which both types fail. It's privately observed by the agent when bad luck occurs. This can be considered as an exogenous negative shock which causes the failure of both types, for instance, a bad health condition causes a capable candidate to fail a CFA test.

When the bad luck occurs in an experiment, a failure occurs. This negative shock provides no information about the agent's quality, and the principal may still want the agent to continue if not enough successes are acquired. Thus, the principal can tolerate more failures which are caused only by bad luck. Also, since the bad luck may happen in every experiment, the number of failures are uncertain before experiments are carried out. Thus the principal optimally assigns them the same reward level,

$$a_B^k(k^g, k^b > 1) = a_B^k(k^g, 1) \quad \text{and} \quad a_B^k(k, 0) = a_B^k(k^g, 1) \tag{5.1}$$

Now the term "k experiments incentivised to run" in $a_B^k(k^g, k^b)$ means k experiments which are not affected by bad luck. Due to the presence of the failure caused by the bad luck, a potential good type now can easily masquerade as a bad type to save future cost. Thus, when bad luck arrives, the principal still needs the agent's expected benefit from continuing initial plan to be higher than stopping immediately and reporting results,

$$\text{IC}^{S,B}_{0\le j\le k-1}: \quad U_B(k-j, p_j) \ge a_B^k(j, 1) \ge a_B^k(j, 0) \tag{5.2}$$

If failures are verifiable, Lemma 2 holds then $a_B^k(j < k, 0) = 0$; if they are not, then $a_B^k(j, 1) = a_B^k(j, 0)$. Meanwhile, those who have already learned that they are a bad type should be deterred from further experimentation, and the incentives are given such that the benefit from over-experimentation to acquire one more success is lower than its cost, where

$$\begin{aligned} &\text{IC}^{F,B}_{0\le j<k-1}: \quad a_B^k(j+1, 1) - a_B^k(j, 1) \le \frac{c}{(1-\theta)(1-\sigma)} \\ &\text{IC}^{F,B}_{k-1}: \quad a_B^k(k, 0) - a_B^k(k-1, 1) \le \frac{c}{(1-\theta)(1-\sigma)} \end{aligned} \tag{5.3}$$

Then it's easily seen that the incentives provided are the same as conditions (4.6) in private experimentation with unverifiable failures and no bad luck, in which the cost level is $\frac{c}{1-\sigma}$. Therefore, the associated reward scheme would be the same as that in proposition 3 at a different cost level. Also, since the cost is higher, the *ex ante* expected total cost is higher, leading the threshold number $\overline{k}$, the number for which the prior expectation can cover the *ex ante* expected total cost, to be lower. Under the optimal commitment, the agent continues to experiment until either the first failure, which is not caused by bad luck, occurs, or the required number of successes are achieved. Also, the agent is indifferent between reporting one failure and not. Results are summarised in the following proposition, where all types may obtain more than one failure.

Proposition 6. *When a bad luck exists with probability $\sigma \in (0,1)$ in each experiment and is privately observed by the agent, the associated optimal reward scheme is the same as that in private experimentation with unverifiable failures and no bad luck at cost level $\frac{c}{1-\sigma}$.*

5.2 Finite Opportunities for Experimentation

Up to this point, it has been assumed that the agent can conduct an infinite number of experiments. Imagine now a scenario where opportunity T is finite, $T < \infty$. Firstly, consider the situation in which the number of experiments provided incentives to run is $k < T$. The participation incentive constraints should be the same as IC^S shown previously, since these motivate the agent to run a sufficiently number of experiments. Suppose the agent fails at the k_{th} experiment. To prevent the agent from pretending to be a potential good types by over-experimentation, the following condition must be satisfied:

$$\begin{aligned} & \frac{(1-\theta^{T-k})}{1-\theta}\left(-c+(1-\theta)a_F^k(k,0)\right)+\theta^{T-k}a_F^k(k-1,1) \leq a_F^k(k-1,1) \\ \Longrightarrow \quad & -\frac{c}{1-\theta}+a_F^k(k,0) \leq a_F^k(k-1,1) \end{aligned} \tag{5.4}$$

Similar constraints can be obtained for $j < k-1$. Notice that these are exactly the same as IC^F shown previously, so the optimal reward scheme should be the same as before.

If the incentive is to run T experiments, the constraints are slightly different. Imagine that the agent fails for the first time in T_{th} experiments. Now he has no further opportunity to continue experimenting, and the only remaining option is to disclose results selectively. Thus the principal only needs to motivate the agent to disclose information as her desire. For an early first failure in j_{th} experiments where $j < T$, the agent may still continue experimenting to get a higher reward level, thus $\mathrm{IC}^F_{0\leq j<T-1}$ must be satisfied. On the side of IC^S constraints, stopping early without failure can be interpreted as revealing that the agent is bad type when failures are verifiable, and $a_F^k(k^g < k, 0) = 0$ would be optimal and CF scheme is optimal; when failures are not verifiable, the principal still needs to provide incentives to prevent the agent from pretending to be a bad type, which would be the same as $IC^{S,NV}_{0\leq j\leq T-1}$ in (4.6).

Proposition 7. *When the number of private experiments T is finite, given the incentive to run $k \leq T$ experiments,*

1) If $k < T$, the associated optimal reward scheme is the same as that proposed in $T \to \infty$;

2) If $k = T$, the associated optimal reward scheme is CF when failures are verifiable; when failures are not verifiable, CF is optimal if $T \leq \overline{k}$, and MF-II is optimal otherwise;

3) The optimal number is the same as that in the public case with $T \to \infty$ when T is sufficiently large.

Proposition 7 shows that the reward schemes proposed in section 4 are still optimal at every positive number of experiments, even if the agent cannot conduct experiments infinitely. Pushing motivated number of experiments to the boundary T can mitigate incentives for misbehaving, especially when failures are verifiable the same reward scheme in public experimentation, CF, can be applied. However, the proposition above also shows it is not always optimal to do so. When the boundary is sufficiently loosen, in which case T is sufficiently large, it doesn't help to improve the principal's benefit, and the optimal amount stays the same as that in the scenario with infinite opportunities, $T \to \infty$.

6 Conclusion

This paper characterises the properties of the principal's optimal commitment when the agent can privately run costly experiments and selectively report favourable results.

The single cut-off function, or one-step function, is the optimal reward scheme in public experimentation, and it still optimal in the private scenario if the motivated number of experiments is small and the agent's incentives to deviate from the intended path of experimentation are weak. When this number raises, a multi-step function is introduced, where a bad type agent receives a different level of rewards when reporting different numbers of successes. These different levels feature two potential types of agent's incentives of deviation in a private environment, which encourage a potential good type agent to continue experimenting after early successes of block a bad type agent from over-experimentation after an early failure. Moreover, the principal faces a trade-off when determining the optimal motivated number of experiments, which is at least the largest number whose expected total cost can just be covered by the prior expectation of the good type agent's value. These results are robust when introducing finite opportunity for experiments or privately observed bad luck.

There is still room to improve upon the current work. The current work only considers the scenario of learning from bad news. In future research, a more general setting on information structure could be considered, and it would also be interesting to introduce the strategic third party that designs experiments.

Appendix A Proofs

A.1 Proofs for Preliminaries and Public Experimentation

Proof of Lemma 1

Lemma 1.1): When the agent reports more than one failure, the principal learns that the agent's type is bad and he must have over-experimented after his first failure. Notice that the principal wants to deter such behaviour on the equilibrium path, thus $a^k(k^g, 1) \geq a^k(k^g, k^b > 1)$ is a plausible candidate to achieve such goal. If there exists a contract with $a^k(k^g, 1) < a^k(k^g, k^b > 1)$ which can achieve the same goal, the bad type's equilibrium path behaviour in such contract would be the same as that with $a^k(k^g, 1) \geq a^k(k^g, k^b > 1)$.

Lemma 1.2): Suppose the principal motivates a potential good type agent to report k successes without failures. When the agent reports $k^g \geq k$ successes with one failure, the principal learns that the agent is a bad type and he has over-experimented. To deter such deviation from the bad type agent, the principal would assign $a^k(k^g \geq k, 0) \geq a^k(k^g \geq k, 1) \geq 0$. Thus $a^k(k^g \geq k, 1) = 0$ is a candidate which can achieve such goal, and it's equal to bad type's true value. For any other contracts with $a^k(k^g \geq k, 1) > 0$ which can achieve the same goal, the bad type's equilibrium path behaviour would be the same as the contract with $a^k(k^g \geq k, 1) = 0$. Moreover, together with Lemma 1.1), $a^k(k^g \geq k, k^b \geq 1) = 0$.

Lemma 1.3): After achieving k successes without failures, the agent would stop experimenting if:

$$[1 - \theta(1 - p_j)]\triangle a^k(j + 1, 0) < c, \;\; j > k \tag{A.1}$$

where $\triangle a^k(j+1, 0) = a^k(j+1, 0) - a^k(j, 0)$. Thus $\triangle a^k(j+1, 0)$ must be bounded, $\triangle a^k(j+1, 0) \in \left[0, \frac{c}{1-\theta(1-p_j)}\right)$. For $\forall k, j \in \mathbb{N}$, $\triangle a^k(j + 1, 0) \in [0, c)$ since $1 - \theta(1 - p_j)$ is increasing as j increases, the same incentive can be achieved by setting $\triangle a^k(j + 1, 0) = 0$.

Proof of Proposition 1

Given motivating the agent to run k experiments, the principal's *ex ante* expected payoff can be represented as following:

$$V_P(k, p_0) = -p_0 \left(a^k(k, 0) - M\right)^2 - (1-p_0)(1-\theta)^k \left(a^k(k, 0)\right)^2 - \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta \left(a^k(i, 1)\right)^2 \tag{A.2}$$

Given $a^k(\cdot)$ in proposition 1.1) is committed, the expected payoff above can be simplified as:

$$V_P(a^k(\cdot)) = -p_0(1-p_k)M^2 - \left(\max\left\{0, \tilde{k}c - p_0M\right\}\right)^2 \tag{A.3}$$

Proof by contradiction then can be applied in order to achieve the conclusion that $a^k(\cdot)$ in Proposition 1.1) is optimal and unique on the equilibrium path. Suppose not, then, given Properties in Lemma 1 are satisfied, there must exist another reward scheme $\tilde{a}^k(\cdot)$:

$$\begin{cases} \tilde{a}^k(j<k,1) = \max\left\{0, \tilde{k}c - p_0M\right\} + \epsilon_j \\ \tilde{a}^k(k,0) = p_kM + \max\left\{0, \tilde{k}c - p_0M\right\} + \epsilon_k \end{cases} \tag{A.4}$$

where $(\epsilon_k, ..., \epsilon_0) \in \mathbb{R}^k$ and $(\epsilon_k, \epsilon_{k-1}, ..., \epsilon_0) \neq \mathbf{0}$, such that $V_P(\tilde{a}^k(\cdot)) \geq V_P(a^k(\cdot))$ and IR constraint is still satisfied. Now the principal's expected payoff can be represented as:

$$\begin{aligned} V_P(\tilde{a}^k(\cdot)) =& - p_0\left(p_kM + \max\left\{0, \tilde{k}c - p_0M\right\} + \epsilon_k - M\right)^2 \\ &- (1-p_0)(1-\theta)^k\left(p_kM + \max\left\{0, \tilde{k}c - p_0M\right\} + \epsilon_k\right)^2 \\ &- \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\left(\max\left\{0, \tilde{k}c - p_0M\right\} + \epsilon_i\right)^2 \\ =& - p_0(1-p_k)M^2 - \left(\max\left\{0, \tilde{k}c - p_0M\right\}\right)^2 - 2\max\left\{0, \tilde{k}c - p_0M\right\}\bar{\epsilon} \\ &- \left\{\left[p_0 + (1-p_0)(1-\theta)^k\right]\epsilon_k^2 + \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\epsilon_i^2\right\} \end{aligned} \tag{A.5}$$

Where $\bar{\epsilon} = \left[p_0 + (1-p_0)(1-\theta)^k\right]\epsilon_k + \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\epsilon_i$. Since $\tilde{a}^k(\cdot)$ must satisfy IR constraint:

$$\begin{aligned} &\mathbb{E}(\tilde{a}^k(k^g, k^b)|k, p_0) - \tilde{k}c \geq 0 \\ \Longrightarrow\quad & \bar{\epsilon} \geq -\max\left\{0, \tilde{k}c - p_0M\right\} + \tilde{k}c - p_0M = \min\left\{\tilde{k}c - p_0M, 0\right\} \\ \Longrightarrow\quad & \max\left\{0, \tilde{k}c - p_0M\right\}\bar{\epsilon} \geq \max\left\{0, \tilde{k}c - p_0M\right\}\min\left\{\tilde{k}c - p_0M, 0\right\} \geq 0 \end{aligned} \tag{A.6}$$

Notice $(\epsilon_k, \epsilon_{k-1}, ..., \epsilon_0) \neq \mathbf{0}$:

$$\left[p_0 + (1-p_0)(1-\theta)^k\right]\epsilon_k^2 + \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\epsilon_i^2 > 0 \tag{A.7}$$

Combining from (A.5) to (A.7), the following result can be achieved:

$$\begin{aligned} V_P(\tilde{a}^k(\cdot)) - V_P(a^k(\cdot)) = &- \left\{ \left[p_0 + (1-p_0)(1-\theta)^k\right] \epsilon_k^2 + \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\epsilon_i^2 \right\} \\ &- 2\max\left\{0, \tilde{k}c - p_0M\right\}\bar{\epsilon} \quad < 0 \end{aligned} \tag{A.8}$$

which contradicts to $V_P(\tilde{a}^k(\cdot)) \geq V_P(a^k(\cdot))$. Therefore, it can concluded that $a^k(\cdot)$ is uniquely optimal on the equilibrium path.

The following part focuses on the optimal motivated number of experiments, k^P in public experimentation. When $k \leq \overline{k}$, $\max\left\{0, \tilde{k}c - p_0M\right\} = 0$, from (A.3):

$$V_P(a^k(\cdot)) = -p_0(1-p_k)M^2 \tag{A.9}$$

and the principal's expected payoff is increasing as k increases, and this implies $k^P = \overline{k}$ in this scenario.

When $k > \overline{k}$, $\max\left\{0, \tilde{k}c - p_0M\right\} = \tilde{k}c - p_0M$, (A.3) becomes:

$$V_P(a^k(\cdot)) = -p_0(1-p_k)M^2 - \left(\tilde{k}c - p_0M\right)^2 \tag{A.10}$$

the second term $\left(\tilde{k}c - p_0M\right)^2$ is increasing as k increases, and it would undermine the benefit of exploration through experiment. Notice that when $k \to \infty$, $V_P(a^k(\cdot)) \to -\infty$, the optimal number k^P must be finite, $k^P < \infty$.

A.2 Proofs for Private Experimentation

Given motivating the agent to run k experiments if no failure occurs, the principal solves the following utility maximisation problem in private experimentation:

$$\begin{aligned} \underset{a^k(\cdot)\geq 0}{Max} \quad & V(k, p_0) = \mathbb{E}\left(-\left(a^k(\cdot) - M_i\right)^2 \Big| k, p_0\right) \\ \text{s.t :} \quad & \text{IR : } U(k, p_0) \geq 0 \\ & \text{IC}^S\text{ : } U(k-j, p_j) \geq a^k(j, 0),\ 0 \leq j < k \\ & \text{IC}^F\text{ : } \max\left\{a^k(j+1, 1), a^k(j+1, 0)\right\} \\ & \qquad - \max\left\{a^k(j, 1), a^k(j, 0)\right\} \leq \frac{c}{1-\theta} \end{aligned} \tag{A.11}$$

The Karush-Kuhn-Tucker condtion (KKT) can be applied to solve the constrained maximisation problem above, since the feasible set under the constraints is convex and utility function is continuous and quasi-concave. However, it's too tedious and not convenient to follow the logic if the details of KKT are shown. Thus an alternative way could be adopted—proof by contradiction, which is similar as the proof of Proposition 1.1).

Proof of Lemma 2

Suppose $0 \le a^k(j<k,1) < a^k(j,0)$, the agent would never disclose failures if any. Thus, on the equilibrium path, the principal can only observe successes are reported, and constraints become:

$$\begin{aligned}
&\text{IR}': && \sum_{i=1}^{k} \frac{p_0}{p_i}\left[a^k(i,0)-a^k(i-1,0)\right] + a^k(0,0) \ge \sum_{i=1}^{k} \frac{p_0}{p_{i-1}} c \\
&\text{IC}^{S'}: && \sum_{i=j+1}^{k} \frac{p_0}{p_i}\left[a^k(i,0)-a^k(i-1,0)\right] \ge \sum_{i=j+1}^{k} \frac{p_0}{p_{i-1}} c \\
&\text{IC}^{F'}: && a^k(j+1,0)-a^k(j,0) \le \frac{c}{1-\theta},\ 0 \le j < k
\end{aligned} \tag{A.12}$$

The structure of constraints is the same as that in private experimentation with unverifiable failures. Instead, if assigning $a^k(j<k,1) \ge a^k(j,0)$, the principal gives the incentive to the agent to disclose all acquired realisations. She can do so because failures are verifiable and the agent can prove himself that he indeed ran experiments but failed. Constrains now are:

$$\begin{aligned}
&\text{IR}: && \sum_{i=1}^{k} \frac{p_0}{p_i}\left[a^k(i,1)-a^k(i-1,1)\right] + a^k(0,1) \ge \sum_{i=1}^{k} \frac{p_0}{p_{i-1}} c \\
&\text{IC}^{S}: && \sum_{i=j+1}^{k} \frac{p_0}{p_i}\left[a^k(i,1)-a^k(i-1,1)\right] + \frac{p_0}{p_j}\left[a^k(j,1)-a^k(j,0)\right] \ge \sum_{i=j+1}^{k} \frac{p_0}{p_{i-1}} c \\
&\text{IC}^{F}: && a^k(j+1,1)-a^k(j,1) \le \frac{c}{1-\theta} \ \ \&\ \ a^k(k,0)-a^k(k-1,1) \le \frac{c}{1-\theta}
\end{aligned} \tag{A.13}$$

Notice that the feasible set in (A.12) is weakly smaller than that in (A.13), and the latter reaches the largest when $a^k(j<k,0)=0$. Without solving original maximisation problem, it can be concluded that the solution in scenario "$a^k(j<k,1) \ge a^k(j,0)=0$" is weakly better than that in scenario "$0 \le a^k(j<k,1) < a^k(j,0)$". This conclusion can also be confirmed later when the optimal reward schemes with verifiable and unverifiable failures are compared, and this is because that the structure of solution to maximisation problem when "$0 \le a^k(j<k,1) < a^k(j,0)$" is the same as that with unverifiable failures.

Proof of Proposition 2

Consider the optimal reward scheme CF in public experimentation. When $k \le \hat{k}$, $p^k M \le \frac{c}{1-\theta}$ and this implies that IC^F and IC^S are always satisfied in CF. Moreover, since CF is the optimal reward scheme in public experimentation which contains least constraints, CF must be the optimal reward scheme in this current scenario as well. However, when $k > \hat{k}$, $p_k M > \frac{c}{1-\theta}$ and this leads the last IC^F, $a^k(k,0) - a^k(k-1,1) \leq \frac{c}{1-\theta}$, to be violated.

Now proof by contradiction can be applied to check if the Type-I step function (MF-I) proposed in Proposition 2.1.b) is optimal when $k > \hat{k}$. Suppose not, then, there must exist another reward scheme $b^k(\cdot)$:

$$\begin{cases} b^k(j<k,1) = a^k(j,1) + \eta_j \\ b^k(k,0) = a^k(k,0) + \eta_k \end{cases} \tag{A.14}$$

where $(\eta_k, ..., \eta_0) \in \mathbb{R}^k$ and $(\eta_k, ..., \eta_0) \neq \mathbf{0}$, such that $V_V\left(b^k(\cdot)\right) \geq V_V\left(a^k(\cdot)\right)$ and all constraints are satisfied, where $V_V(\cdot)$ is the principal's expected payoff in private experimentation with verifiable failures. The principal's expected payoff with $b^k(\cdot)$ then can be represented as:

$$\begin{aligned} V_V(b^k(\cdot)) =& -p_0\left(a^k(k,0)+\eta_k - M\right)^2 - (1-p_0)(1-\theta)^k\left(a^k(k,0)+\eta_k\right)^2 \\ & - \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\left(a^k(i,1)+\eta_i\right)^2 \\ =& V_V(a^k(\cdot)) - \left\{\frac{p_0}{p_k}\eta_k^2 + \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\eta_i^2\right\} - 2\max\left\{0, \tilde{k}c - p_0 M\right\}\overline{\eta} \\ & + 2p_0 M\eta_k - 2(k-l)\frac{p_0}{p_k}\frac{c}{1-\theta}\eta_k - 2\frac{c}{1-\theta}\sum_{i=l+1}^{k-1}(1-p_0)(1-\theta)^i\theta(i-l)\eta_i \\ & - 2\left(p_l M - \sum_{i=l+1}^{k}\frac{p_l}{p_i}\frac{c}{1-\theta}\right)\left[\frac{p_0}{p_k}\eta_k + \sum_{i=l}^{k-1}(1-p_0)(1-\theta)^i\theta\eta_i\right] \end{aligned} \tag{A.15}$$

Where $\overline{\eta} = \frac{p_0}{p_k}\eta_k + \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\eta_i$. Notice $b^k(\cdot)$ must satisfy IR, and it's similar to (A.6):

$$\begin{aligned} & \mathbb{E}(b^k(\cdot)|k,p_0) - \tilde{k}c \geq 0 \\ \implies \quad & \overline{\eta} \geq -\max\left\{0, \tilde{k}c - p_0 M\right\} + \tilde{k}c - p_0 M = \min\left\{\tilde{k}c - p_0 M, 0\right\} \\ \implies \quad & \max\left\{0, \tilde{k}c - p_0 M\right\}\overline{\eta} \geq \max\left\{0, \tilde{k}c - p_0 M\right\}\min\left\{\tilde{k}c - p_0 M, 0\right\} \geq 0 \end{aligned} \tag{A.16}$$

Meanwhile, $\text{IC}^{F,V}$ must be satisfied:

$$\begin{cases} b^k(k,0) - b^k(k-1,1) \leq \frac{c}{1-\theta} \\ ... \\ b^k(l+1,1) - b^k(l,1) \leq \frac{c}{1-\theta} \\ ... \\ b^k(1,1) - b^k(0,1) \leq \frac{c}{1-\theta} \end{cases} \implies \begin{cases} \eta_k - \eta_{k-1} \leq 0 \\ ... \\ \eta_{l+1} - \eta_l \leq 0 \end{cases} \implies \eta_{l\leq j<k} \leq -\eta_k \tag{A.17}$$

Now apply this result to Equation (A.15):

$$
\begin{aligned}
V_V(b^k(\cdot)) \leq & V_V(a^k(\cdot)) - \left\{ \frac{p_0}{p_k}\eta_k^2 + \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\eta_i^2 \right\} - 2\max\left\{0, \tilde{k}c - p_0 M\right\}\overline{\eta} \\
& + 2p_0 M\eta_k - 2\frac{c}{1-\theta}\eta_k\left[(k-l)\frac{p_0}{p_k} + \sum_{i=l+1}^{k-1}(1-p_0)(1-\theta)^i\theta(i-l)\right] \\
& - 2\left(p_l M - \sum_{i=l+1}^{k}\frac{p_l}{p_i}\frac{c}{1-\theta}\right)\left[\frac{p_0}{p_k} + \sum_{i=l}^{k-1}(1-p_0)(1-\theta)^i\theta\right]\eta_k \\
= & V_V(a^k(\cdot)) - \left\{ \frac{p_0}{p_k}\eta_k^2 + \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\eta_i^2 \right\} - 2\max\left\{0, \tilde{k}c - p_0 M\right\}\overline{\eta} \\
& + 2\left(p_0 M - \sum_{i=l+1}^{k}\frac{p_0}{p_i}\frac{c}{1-\theta}\right)\eta_k - 2\left(p_0 M - \sum_{i=l+1}^{k}\frac{p_0}{p_i}\frac{c}{1-\theta}\right)\eta_k \\
= & V_V(a^k(\cdot)) - \left\{ \frac{p_0}{p_k}\eta_k^2 + \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\eta_i^2 \right\} - 2\max\left\{0, \tilde{k}c - p_0 M\right\}\overline{\eta}
\end{aligned}
\tag{A.18}
$$

Since $(\eta_k, ..., \eta_0) \neq \mathbf{0}$:

$$
\frac{p_0}{p_k}\eta_k^2 + \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\eta_i^2 > 0 \tag{A.19}
$$

Combing (A.16), (A.18) and (A.19), it can be concluded that:

$$
\begin{aligned}
V_V(b^k(\cdot)) - V_V(a^k(\cdot)) \leq & - \left\{ \frac{p_0}{p_k}\eta_k^2 + \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\eta_i^2 \right\} \\
& - 2\max\left\{0, \tilde{k}c - p_0 M\right\}\overline{\eta} < 0
\end{aligned}
\tag{A.20}
$$

This result contradicts to $V_V(b^k(\cdot)) \geq V_V(a^k(\cdot))$. Therefore, the proposed reward scheme in Proposition 2.1) is optimal.

When $\hat{k} \geq k^P$, CF is still feasible at level k^P, thus k^P must be the number of experiment which gives the principal least expected loss for $k \leq \hat{k}$. On the other hand, for $k > \hat{k}$, compared to CF, MF-I is the optimal reward scheme with more constraints and it implies that the principal's expected loss is higher under MF-I scheme than that under CF scheme give the same number of experiments k. Therefore, k is dominated by k^P for $\forall k > \hat{k}$ in this case.

When $\hat{k} < k^P$, CF is no longer feasible at level k^P, and MF-I is optimal and the proof is the same as above. To show that the optimal number of experiments in private environment is still higher than the first threshold number, $k_V^* \geq \overline{k}$, it needs to be proved that principal is better off as k increases when $k \leq \overline{k}$. In the case where $k \leq \overline{k} \leq \hat{k}$ or $k < \hat{k} < \overline{k}$, the optimal reward scheme is CF and rest of proof would be the same as that in in proof of proposition 2.2).

Notice that $V_V(k)$ is the principal's expected payoff in the optimal reward scheme given the incentive to run k experiments if no failure occurs, In the case where $k = \hat{k} < \overline{k}$, it needs to be shown that $V_V(\hat{k}+1) \geq V_V(\hat{k})$. The difference between $V_V(\hat{k}+1)$ and $V_V(\hat{k})$ is:

$$\begin{aligned}
&V_V(\hat{k}+1) - V_V(\hat{k}) \\
&= -p_0\left[p_{\hat{k}}M + \left(1-\frac{p_{\hat{k}}}{p_{\hat{k}+1}}\right)\frac{c}{1-\theta}\right]^2 - (1-p_0)(1-\theta)^{\hat{k}+1}\left[p_{\hat{k}}M + \left(1-\frac{p_{\hat{k}}}{p_{\hat{k}+1}}\right)\frac{c}{1-\theta}\right]^2 \\
&\quad - (1-p_0)(1-\theta)^{\hat{k}}\theta\left(p_{\hat{k}}M - \frac{p_{\hat{k}}}{p_{\hat{k}+1}}\frac{c}{1-\theta}\right)^2 + p_0\left(1-p_{\hat{k}}\right)M^2 \\
&= \underbrace{\left(2p_0M - \frac{p_0}{p_{\hat{k}+1}}\frac{c}{1-\theta} + \frac{p_0p_{\hat{k}}}{p^2_{\hat{k}+1}}\frac{c}{1-\theta}\right)}_{\text{“}>0\text{” as } p_0M > \frac{p_0}{p_{\hat{k}+1}}\frac{c}{1-\theta}}\left(1-\frac{p_{\hat{k}}}{p_{\hat{k}+1}}\right)\frac{c}{1-\theta} + (1-p_0)(1-\theta)^{\hat{k}}\theta\left(\frac{p_{\hat{k}}}{p_{\hat{k}+1}}\frac{c}{1-\theta}\right)^2 \\
&> 0
\end{aligned} \tag{A.21}$$

This result suggests that the principal would strictly prefer $\hat{k}+1$ experiments are conduct rather than $\hat{k}$.

Consider the other case where $\hat{k} < k \leq \overline{k}$. It needs to be proved that the principal's expected payoff is an increasing function of k in this region. Thus the problem is equivalent to show that $V_V(k+1) - V_V(k) > 0$ in this region. At $k+1$, the "l" in Definition 1 would be $l(k+1) = l$ or $l(k+1) = l+1$, depending on the parameters. If $l(k+1) = l$, then the extra expected payoff that the principal can gain from increasing one more experiment is:

$$\begin{aligned}
V_V(k+1) - V_V(k) =& 2p_0M\frac{c}{1-\theta} - \frac{p_0}{p_{k+1}}(2k-2l+1)\left(\frac{c}{1-\theta}\right)^2 \\
&+ \frac{p_0}{p_l}\left[\left(p_lM - \sum_{i=l+1}^{k+1}\frac{p_l}{p_i}\frac{c}{1-\theta}\right)^2 - \left((p_lM - \sum_{i=l+1}^{k}\frac{p_l}{p_i}\frac{c}{1-\theta}\right)^2\right] \\
=& \frac{p_0}{p_{k+1}}\frac{c}{1-\theta}\left[2p_{k+1}M - (2k-2l+1)\frac{c}{1-\theta}\right] \\
&- \frac{p_l}{p_{k+1}}\frac{c}{1-\theta}\left(2p_0M - \sum_{i=l+1}^{k+1}\frac{p_0}{p_i}\frac{c}{1-\theta} - \sum_{i=l+1}^{k}\frac{p_0}{p_i}\frac{c}{1-\theta}\right)
\end{aligned} \tag{A.22}$$

Thus,

$$\begin{aligned}
Sign\left(V_V(k+1) - V_V(k)\right) =& \frac{p_{k+1}}{p_0}\left[2p_0M - \frac{p_0}{p_{k+1}}(2k-2l+1)\frac{c}{1-\theta}\right] \\
&- \frac{p_l}{p_0}\left(2p_0M - \sum_{i=l+1}^{k+1}\frac{p_0}{p_i}\frac{c}{1-\theta} - \sum_{i=l+1}^{k}\frac{p_0}{p_i}\frac{c}{1-\theta}\right)
\end{aligned} \tag{A.23}$$

Notice that

$$(k-l)\frac{p_0}{p_{k+1}} < \sum_{i=l+1}^{k} \frac{p_0}{p_i} \quad \text{and} \quad p_{k+1} > p_l$$

Together with(A.23), it can be achieved that (A.22) is strictly positive.

If $l(k+1) = l$, the principal's gain from one more experiment is:

$$\begin{aligned} V_V(k+1) - V_V(k) =& \frac{p_{l+1}}{p_0}\left(p_0 M - \sum_{i=l+2}^{k+1} \frac{p_0}{p_i}\frac{c}{1-\theta}\right)^2 - \frac{p_l}{p_0}\left(p_0 M - \sum_{i=l+1}^{k} \frac{p_0}{p_i}\frac{c}{1-\theta}\right)^2 \\ &+ \sum_{i=l+2}^{k+1} \frac{p_0}{p_i}(\frac{c}{1-\theta})^2 - \sum_{i=l+1}^{k} \frac{p_0}{p_i}(\frac{c}{1-\theta})^2 - 2\frac{p_0}{p_{k+1}}(k-l)(\frac{c}{1-\theta})^2 \end{aligned} \tag{A.24}$$

Notice that

$$(k-l)\frac{p_0}{p_{k+1}} < \sum_{i=l+2}^{k+1} \frac{p_0}{p_i} < \sum_{i=l+1}^{k} \frac{p_0}{p_i} \quad \text{and} \quad p_{l+1} > p_l$$

thus (A.24) is strictly positive. Therefore, it's always true that $V_V(k+1) > V_V(k)$ when $\hat{k} < k \leq \overline{k}$. As a result, if $k \leq \overline{k}$, the principal is always better off by increasing the motivated number of experiments in the commitment, and it implies that $k_V^* \geq \overline{k}$. To prove $k_V^* < \infty$, the argument is the same as that in the proof of proposition 1.2).

Proof of Proposition 3

The arguments and steps to prove the optimality of reward scheme proposed in Propostion 3.1) are similar to those in proof of Proposition 1.1) and 2.1), and proof by contradiction is applied.
Proposition 3.1.a): When $k \leq \min\left\{\hat{k}, \overline{k}\right\}$, it can be seen that $\tilde{k}c \leq p_0 M \leq \frac{p_0}{p_k}\frac{c}{1-\theta}$, which implies that $\text{IC}^{S,NV}$ and $\text{IC}^{F,NV}$ are satisfied under CF scheme. Thus CF must be the optimal reward scheme in this scenario.

Proposition 3.1.b): When $\hat{k} < k \leq \overline{k}$, it becomes that $p_k M > \frac{c}{1-\theta}$ and $p_0 M \geq \tilde{k}c$. Now CF scheme leads the last $\text{IC}^{F,NV}$, $p_k M < \frac{c}{1-\theta}$, to be violated. MF-I scheme satisfies all $\text{IC}^{F,NV}$ and $\text{IC}^{S,NV}$ in this scenario, so it must be optimal.

Proposition 3.1.c): When $\overline{k} < k \leq \hat{k}$, $p_0 M < \tilde{k}c$ and $p_k M \leq \frac{c}{1-\theta}$, it's easy to check that both CF scheme and MF-I scheme violate at least one $\text{IC}^{S,NV}$. Then proof by contradiction can be applied to check the optimality of MF-II in this scenario. Suppose MF-II is not optimal, then there exists another feasible reward scheme $d^k(\cdot)$:

$$\begin{cases} d^k(j<k,1) = a^k(j,1) + \tau_j \\ d^k(k,0) = a^k(k,0) + \tau_k \end{cases} \tag{A.25}$$

where $(\tau_k, ..., \tau_0) \in \mathbb{R}^k$ and $(\tau_k, ..., \tau_0) \neq \mathbf{0}$, such that $V_{NV}\left(d^k(\cdot)\right) \geq V_{NV}\left(a^k(\cdot)\right)$ and all constraints are satisfied. When $0 \leq m < k-1$, the principal's expected payoff with $d^k(\cdot)$ is:

$$
\begin{aligned}
V_{NV}(d^k(\cdot)) = & - p_0 \left(a^k(k,0) + \tau_k - M\right)^2 - (1-p_0)(1-\theta)^k \left(a^k(k,0) + \tau_k\right)^2 \\
& - \sum_{i=0}^{k-1} (1-p_0)(1-\theta)^i \theta \left(a^k(i,1) + \omega_i\right)^2 \\
= & V_{NV}(a^k(\cdot)) - \left\{ \frac{p_0}{p_k} \eta_k^2 + \sum_{i=0}^{k-1} (1-p_0)(1-\theta)^i \theta \eta_i^2 \right\} \\
& - 2 \sum_{n=1}^{m} \frac{p_n}{p_{n-1}} c \left[\frac{p_0}{p_k} \tau_k + \sum_{i=n}^{k-1} (1-p_0)(1-\theta)^i \theta \tau_i \right] \\
& - 2 \left(\sum_{i=m+1}^{k} \frac{p_{m+1}}{p_{i-1}} c - p_{m+1} M \right) \left[\frac{p_0}{p_k} \tau_k + \sum_{i=m+1}^{k-1} (1-p_0)(1-\theta)^i \theta \tau_i \right]
\end{aligned} \tag{A.26}
$$

Since IR and all $\text{IC}^{S,NV}$ are satisfied, the following inequalities must be true:

$$
\begin{cases}
\frac{p_0}{p_k} \tau_k + \sum_{i=0}^{k-1} (1-p_0)(1-\theta)\theta \tau_i \geq 0 \\
\frac{p_0}{p_k} \tau_k + \sum_{i=1}^{k-1} (1-p_0)(1-\theta)\theta \tau_i \geq (1-p_0)(1-\theta)\theta \tau_0 \\
\frac{p_0}{p_k} \tau_k + \sum_{i=2}^{k-1} (1-p_0)(1-\theta)\theta \tau_i \geq (1-p_0)(1-\theta)^2\theta \tau_1 \\
... \\
\frac{p_0}{p_k} \tau_k + \sum_{i=m+1}^{k-1} (1-p_0)(1-\theta)\theta \tau_i \geq (1-p_0)(1-\theta)^{m+1}\theta \tau_m
\end{cases} \tag{A.27}
$$

If $\tau_0 \geq 0$, it's true that $\frac{p_0}{p_k}\tau_k + \sum_{i=1}^{k-1}(1-p_0)(1-\theta)\theta\tau_i \geq 0$ from the second inequality in (A.27); if $\tau_0 < 0$, it also states that $\frac{p_0}{p_k}\tau_k + \sum_{i=1}^{k-1}(1-p_0)(1-\theta)\theta\tau_i \geq 0$ from the first inequality in (A.27). Thus it always holds that $\frac{p_0}{p_k}\tau_k + \sum_{i=1}^{k-1}(1-p_0)(1-\theta)\theta\tau_i \geq 0$. Similarly, if $\tau_1 \geq 0$, $\frac{p_0}{p_k}\tau_k + \sum_{i=2}^{k-1}(1-p_0)(1-\theta)\theta\tau_i \geq 0$ and this is achieved from the third equality in (A.27); if $\tau_1 \geq 0$, it's still obtained that $\frac{p_0}{p_k}\tau_k + \sum_{i=2}^{k-1}(1-p_0)(1-\theta)\theta\tau_i \geq 0$ from $\frac{p_0}{p_k}\tau_k + \sum_{i=1}^{k-1}(1-p_0)(1-\theta)\theta\tau_i \geq 0$. Thus it can always hold that $\frac{p_0}{p_k}\tau_k + \sum_{i=2}^{k-1}(1-p_0)(1-\theta)\theta\tau_i \geq 0$. Together with the same logic and (A.27), it's concluded that:

$$
\frac{p_0}{p_k}\tau_k + \sum_{i=j}^{k-1}(1-p_0)(1-\theta)\theta\tau_i \geq 0, \text{ where } 1 \leq j \leq m+1 \tag{A.28}
$$

Meanwhile, notice that $(\tau_k, ..., \tau_0) \neq \mathbf{0}$ and $\sum_{i=m+1}^{k} \frac{p_{m+1}}{p_{i-1}} c - p_{m+1} M \geq 0$, in (A.26):

$$\begin{aligned} V_{NV}(d^k(\cdot)) \leq & V_{NV}(a^k(\cdot)) - \left\{ \frac{p_0}{p_k}\eta_k^2 + \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\eta_i^2 \right\} \\ < & V_{NV}(a^k(\cdot)) \end{aligned} \tag{A.29}$$

This result contradicts to $V_{NV}(d^k(\cdot)) \geq V_{NV}(a^k(\cdot))$, therefore, MF-II scheme is optimal in this scenario when $0 \leq m < k-1$. When $m = k-1$, similarly, the principal's expected payoff now is:

$$\begin{aligned} V_{NV}(d^k(\cdot)) = & V_{NV}(a^k(\cdot)) - \left\{ \frac{p_0}{p_k}\eta_k^2 + \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\eta_i^2 \right\} \\ & - 2\sum_{n=1}^{k-1} \frac{p_n}{p_{n-1}} c \left[\frac{p_0}{p_k}\tau_k + \sum_{i=n}^{k-1}(1-p_0)(1-\theta)^i\theta\tau_i \right] - 2\frac{p_0}{p_{k-1}} c\tau_k \end{aligned} \tag{A.30}$$

Together with (A.27), it shows that the last two terms in (A.29) are both negative. Notice that the second term in (A.29) is strictly negative, therefore MF-II scheme is optimal when $m = k-1$. To sum up, it can be concluded that MF-II is optimal when $\overline{k} < k \leq \hat{k}$.

Proposition 3.1.d): When $k > \max\left\{\hat{k}, \overline{k}\right\}$, it states that $\frac{p_0}{p_k}\frac{c}{1-\theta} < p_0 M < \tilde{k}c$. Now CF and MF-I violate at least one IC^{S,NV}, and MF-II violates at least one IC^{F,NV}. Thus consider the optimality of MF-III. Suppose MF-III is not optimal in this scenario, then there exists another feasible reward scheme $e^k(\cdot)$:

$$\begin{cases} e^k(j < k, 1) = a^k(j, 1) + \omega_j \\ e^k(k, 0) = a^k(k, 0) + \omega_k \end{cases} \tag{A.31}$$

where $(\omega_k, ..., \omega_0) \in \mathbb{R}^k$ and $(\omega_k, ..., \omega_0) \neq \mathbf{0}$, such that $V_{NV}\left(e^k(\cdot)\right) \geq V_{NV}\left(a^k(\cdot)\right)$ and all constraints are satisfied. The principal's expected payoff with $e^k(\cdot)$ then can be represented as:

$$
\begin{aligned}
V_{NV}(e^k(\cdot)) =& - p_0\left(a^k(k,0)+\omega_k - M\right)^2 - (1-p_0)(1-\theta)^k\left(a^k(k,0)+\omega_k\right)^2 \\
& - \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\left(a^k(i,1)+\omega_i\right)^2 \\
=& V_{NV}(a^k(\cdot)) - \left\{\frac{p_0}{p_k}\omega_k^2 + \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\omega_i^2\right\} + 2p_0M\omega_k \\
& - 2(k-l)\frac{p_0}{p_k}\frac{c}{1-\theta}\omega_k - 2\frac{c}{1-\theta}\sum_{i=l+1}^{k-1}(1-p_0)(1-\theta)^i\theta(i-l)\omega_i \\
& - 2\left(p_lM - \sum_{i=l+1}^{k}\frac{p_l}{p_i}\frac{c}{1-\theta}\right)\left[\frac{p_0}{p_k}\omega_k + \sum_{i=l}^{k-1}(1-p_0)(1-\theta)^i\theta\omega_i\right] \\
& - 2\sum_{n=1}^{m}\frac{p_n}{p_{n-1}}c\left[\frac{p_0}{p_k}\omega_k + \sum_{i=n}^{k-1}(1-p_0)(1-\theta)^i\theta\omega_i\right] \\
& - 2\left(\sum_{i=m+1}^{k}\frac{p_{m+1}}{p_{i-1}}c - p_{m+1}M\right)\left[\frac{p_0}{p_k}\omega_k + \sum_{i=m+1}^{k-1}(1-p_0)(1-\theta)^i\theta\omega_i\right]
\end{aligned}
\tag{A.32}
$$

The rest of proof is similar to the proofs of MF-I and MF-II. Since IR and IC^{S,NV} are satisfied, the following inequalities can be achieved, which are similar to (A.27):

$$
\begin{cases}
\frac{p_0}{p_k}\omega_k + \sum_{i=0}^{k-1}(1-p_0)(1-\theta)\theta\omega_i \geq 0 \\
\frac{p_0}{p_k}\omega_k + \sum_{i=1}^{k-1}(1-p_0)(1-\theta)\theta\omega_i \geq (1-p_0)(1-\theta)\theta\omega_0 \\
\frac{p_0}{p_k}\omega_k + \sum_{i=2}^{k-1}(1-p_0)(1-\theta)\theta\omega_i \geq (1-p_0)(1-\theta)^2\theta\omega_1 \\
\dots \\
\frac{p_0}{p_k}\omega_k + \sum_{i=m+1}^{k-1}(1-p_0)(1-\theta)\theta\omega_i \geq (1-p_0)(1-\theta)^{m+1}\theta\omega_m
\end{cases}
\tag{A.33}
$$

Similar to (A.28), the same the logic in proof of MF-II can be applied and the following inequality can be achieved:

$$
\frac{p_0}{p_k}\tau_k + \sum_{i=j}^{k-1}(1-p_0)(1-\theta)\theta\omega_i \geq 0, \text{ where } 1 \leq j \leq m+1 \tag{A.34}
$$

This implies that the last two terms in (A.32) are negative, noticing that $\frac{p_0}{p_k}\omega_k + \sum_{i=2}^{k-1}(1-p_0)(1-\theta)\theta\omega_i \geq 0$.

Also, since $\text{IC}^{F,NV}$ are satisfied, similar to (A.17), the following inequalities are true:

$$\begin{cases} \omega_k - \omega_{k-1} \leq 0 \\ ... \\ \omega_{l+1} - \omega_l \leq 0 \end{cases} \implies \begin{cases} -\omega_{k-1} \leq -\omega_k \\ ... \\ -\omega_l \leq -\omega_{l+1} \leq ... \leq -\omega_k \end{cases} \tag{A.35}$$

Then (A.32) becomes:

$$\begin{aligned} V_{NV}(e^k(\cdot)) \leq & V_{NV}(a^k(\cdot)) - \left\{ \frac{p_0}{p_k}\omega_k^2 + \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\omega_i^2 \right\} \\ & + 2\left(p_0 M - \sum_{i=l+1}^{k} \frac{p_0}{p_i}\frac{c}{1-\theta}\right)\omega_k - 2\left(p_0 M - \sum_{i=l+1}^{k} \frac{p_0}{p_i}\frac{c}{1-\theta}\right)\omega_k \\ = & V_{NV}(a^k(\cdot)) - \left\{ \frac{p_0}{p_k}\omega_k^2 + \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\omega_i^2 \right\} \end{aligned} \tag{A.36}$$

Again, notice $(\omega_k, ..., \omega_0) \neq \mathbf{0}$:

$$\begin{aligned} V_{NV}(e^k(\cdot)) \leq & V_{NV}(a^k(\cdot)) - \left\{ \frac{p_0}{p_k}\omega_k^2 + \sum_{i=0}^{k-1}(1-p_0)(1-\theta)^i\theta\omega_i^2 \right\} \\ < & V_{NV}(a^k(\cdot)) \end{aligned} \tag{A.37}$$

This result contradicts to $V_{NV}(e^k(\cdot)) \geq V_{NV}(a^k(\cdot))$, therefore, MF-III scheme must be optimal in this scenario.

Proposition 3.2): if $\overline{k} \leq \hat{k}$, for $\forall k \leq \overline{k}$, CF is optimal and then principal is strictly better off as k increasing, which can be obtained from the proof of proposition 1.2); if $\hat{k} < \overline{k}$, for $\forall k \leq \overline{k}$, optimal reward scheme is either CF or MF-I, and the optimal amount in this region is $\overline{k}$, which is the same proof as that in proposition 2.2). Therefore, $k^*_{NV} \geq \overline{k}$. To prove $k^*_{NV} < \infty$, it's the same argument as that in proposition 1.2).

Proof of Corollary 1

If the principal motivates the agent not to run any experiments, a single reward scheme should be determined by the prior belief p_0, regardless of public and private experimentation. Thus the principal's expected payoff is $V_{NO}(p_0) = -p_0(1-p_0)M^2$. In stead, given the incentive to run any positive number of experiments k, the principal can achieve $V_P(a^k(\cdot))$, $V_V(a^k(\cdot))$ and $V_{NV}(a^k(\cdot))$ in public and private cases respectively, which are shown in previous proofs. It's clear that the set of parameter ranges is not empty, which satisfies:

$$V_{NO}(p_0) \geq \left\{V_P(a^k(\cdot)), V_V(a^k(\cdot)), V_{NV}(a^k(\cdot))\right\}$$

Proof of Corollary 2

When $\frac{M}{c} \leq \frac{1}{1-\theta}$, $\hat{k} \to \infty$. From Proposition 2, CF is always optimal for $\forall k \in \mathbb{N}^+$ in private experimentation with verifiable failures. Thus the optimal reward scheme is always the same as that in public experimentation. As a result, $k_V^* = k^P$ and $V_p^{CF}\left(k^P, p_0\right) = V_V^{CF}\left(k_V^*, p_0\right)$.

Proof of Proposition 4

For $p_0 M \geq c$, the participation threshold can be rewritten into $\overline{k} = \max\{k \in \mathbb{N} : p_0\frac{M}{c} \geq \tilde{k}\}$. When $\frac{M}{c}$ increases, the left hand side of the inequality constraint is increasing, and it implies that this condition can hold for a larger number of experiments. As a result, the first threshold $\overline{k}$ becomes larger. For $p_0 M < c$, when $\frac{M}{c}$ increases, this inequality is easier to be violated, thus $\overline{k}$ tends to become larger. To sum up, $\overline{k}$ is increasing as $\frac{M}{c}$ increases.

For $p_1 M \leq \frac{c}{1-\theta}$, the over-experimentation threshold can be rewritten into $\hat{k} = \max\{k \in \mathbb{N} : \frac{M}{c} \leq \frac{1}{(1-\theta)p_k}\}$. When $\frac{M}{c}$ increases, the left hand side of the inequality constraint is increasing, and it implies that this condition would be violated at a lower level of experiment. As a result, the second threshold $\hat{k}$ shrinks. For $p_1 M > \frac{c}{1-\theta}$, $\hat{k}$ stays at zero when $\frac{M}{c}$ increases. To sum up, the participation threshold $\hat{k}$ is decreasing as $\frac{M}{c}$ increases.

In the public case, $k^P \geq \overline{k}$, which implies that the lower bound of the potential optimal number of experiments is increasing. Now it can focus on the number which satisfied $k > \overline{k}$. Firstly, take the first different between $k+1$ and k:

$$V_P(k+1) - V_P(k) = \left[p_0(p_{k+1} - p_k)(\frac{M}{c})^2 - \left(\frac{p_0}{p_k} + 2\sum_{i=1}^{k}\frac{p_0}{p_{i-1}} - 2p_0\frac{M}{c}\right)\frac{p_0}{p_k}\right]c^2 \tag{A.38}$$

Then the first derivate with respect to $\frac{M}{c}$ can be achieved:

$$\frac{\partial(V_P(k+1) - V_P(k))}{\partial\frac{M}{c}} = 2\left(\sum_{i=1}^{k+1}\frac{p_0}{p_{i-1}}c + \sum_{i=1}^{k}\frac{p_0}{p_{i-1}}c - p_0 M\right)\frac{p_0}{p_k}\frac{c^2}{M} > 0 \tag{A.39}$$

This strictly positive first difference for $\forall k > \overline{k}$ suggests that the (local and global) maximum point is getting larger as $\frac{M}{c}$ increases. Together with that $\overline{k}$ is increasing as $\frac{M}{c}$ increases, it can be concluded that k^P is increasing as $\frac{M}{c}$ is increases.

Similar arguments can be applied in the private experimentation scenario. In private with verifiable failures, if $\overline{k} < k_V^* \leq \hat{k}$, the conclusion is the same as (A.38). If $k_V^* > \max\left\{\overline{k}, \hat{k}\right\}$, it can be focus on the difference of the principal's expected payoff at $k+1$ and k, for $\forall k > \max\left\{\overline{k}, \hat{k}\right\}$. Similar to (A.22) and (A.24), if $l(k+1) = l$, it's first derivative with respect to M would be

$$\frac{\partial(V_V(k+1) - V_V(k))}{\partial M} = p_0 \frac{c}{1-\theta}\left(2 - \frac{p_l}{p_{k+1}}\right) + 2\frac{p_0^2}{p_k}c > 0 \tag{A.40}$$

Instead, if $l(k+1) = l+1$, the first derivative becomes

$$\frac{\partial(V_V(k+1) - V_V(k))}{\partial M} = 2p_{l+1}\left(p_0 M - \sum_{i=l+2}^{k+1} \frac{p_i}{p_0}\frac{c}{1-\theta}\right) - 2p_l\left(p_0 M - \sum_{i=l+1}^{k} \frac{p_i}{p_0}\frac{c}{1-\theta}\right) + 2\frac{p_0^2}{p_k}c > 0 \tag{A.41}$$

The positive signs in (A.40) and (A.41) imply the first difference is increasing as agent's value M increases, and it leads the maximum point k_V^* to increase together with $\overline{k}$ increasing.

In private with unverifiable failures, if $\hat{k} \leq \overline{k}$, for $\forall k > \overline{k}$, then associated optimal reward scheme is MF-III, then the first difference of principal's expected could be obtained. From Definition 3, if $m(k+1) = m+1$, the first derivative would be

$$\begin{aligned}\frac{\partial(V_{NV}(k+1) - V_{NV}(k))}{\partial M} = &\frac{\partial(V_V(k+1) - V_V(k))}{\partial M} + 2(p_{m+2} - p_{m+1})\left(p_0 M + \sum_{i=m+2}^{k} \frac{p_0}{p_{i-1}}c\right) \\ &+ 2\frac{p_0}{p_k}(p_{m+2} - p_0)c > 0\end{aligned} \tag{A.42}$$

Instead, if $m(k+1) = m = m(k)$, the first derivative becomes:

$$\frac{\partial(V_{NV}(k+1) - V_{NV}(k))}{\partial M} = \frac{\partial(V_V(k+1) - V_V(k))}{\partial M} + 2\frac{p_0}{p_k}(p_{m+2} - p_0)c > 0 \tag{A.43}$$

The positive signs in (A.42) and (A.43) suggest that the first difference is increasing as M increases for $\forall k > \overline{k} \geq \hat{k}$. As a result, k_{NV}^* increases due to the same reason in private with verifiable failures.

If $\hat{k} > \overline{k}$, for $\forall k > \hat{k}$, MF-III is still optimal and the conclusions are the same as (A.42) and (A.43). For $\overline{k} < k < \hat{k}$, MF-II is optimal, and it can focus on the first derivative of the first difference of the principal's expected payoff with respect to M. From Definition 2, if $m(k) = m < k-1$ and $m(k+1) = m+1$

$$\begin{aligned}\frac{\partial(V_{NV}(k+1) - V_{NV}(k))}{\partial M} = &2p_0(p_{k+1} - p_k)M + 2(p_{m+2} - p_{m+1})\left(p_0 M + \sum_{i=m+2}^{k} \frac{p_0}{p_{i-1}}c\right) \\ &+ 2\frac{p_{m+2}}{p_k}p_0 c > 0\end{aligned} \tag{A.44}$$

if $m(k) = m < k-1$ and $m(k+1) = m$, the first derivative is

$$\frac{\partial(V_{NV}(k+1) - V_{NV}(k))}{\partial M} = 2p_0(p_{k+1} - p_k)M + 2\frac{p_{m+2}}{p_k}p_0 c > 0 \tag{A.45}$$

if $m(k) = k-1$, the first derivative becomes

$$\frac{\partial(V_{NV}(k+1) - V_{NV}(k))}{\partial M} = 2\frac{p_{k+1}}{p_k}p_0 c > 0 \tag{A.46}$$

From the positive signs in (A.44), (A.45) and (A.46), it shows that the first difference is increasing as M increases in this case. For $\forall k > \hat{k} > \overline{k}$, the conclusions would be the same as (A.42) and (A.43). To sum up, it concludes that k^*_{NV} is increasing as M increases.

Proof of Proposition 5

The bad type agent's value is zerio if early failure occurs and he would learn it. For the potential good type, his posterior value is $p_k M$ given he successfully collected k successes in k experiments without failure, and he has the posterior belief p_k that his type is good. The proofs below are comparing these values to the rewards that different types of agents can received in the optimal contracts of public and private experimentation respectively.

In the public case, the optimal contract would deliver the bad type agent an reward level $a^{k^P}(k < k^P) = \max\left\{0, (\sum_{i=1}^{k^P} \frac{p_0}{p_{i-1}}c - p_0 M)^2\right\} \geq 0$, and it implies that the bad type is overpaid. For the potential good type, he would receive the reward level $a^{k^P}(k = k^P) = p_{k^P}M + a^{k^P}(k < k^P) \geq p_{k^P}M$, and it's clear to see that he is also overpaid.

In the private case with verifiable failures, the bad types who face a later failure would receive a weakly higher reward. Comparing the lowest reward among them, the bad type would receive $\max\left\{0, (\sum_{i=1}^{k^*_V} \frac{p_0}{p_{i-1}}c - p_0 M)^2\right\} \geq 0$, so it shows that all bad types are overpaid at differently level of early failure. However, for the potential good type, he would receive

$$(k^V - l(k^V))\frac{c}{1-\theta} + p_{l(k^*_V)}M - \sum_{i=l(k^*_V)+1}^{k^*_V} \frac{p_0}{p_i}\frac{c}{1-\theta} + \max\left\{0, (\sum_{i=1}^{k^*_V}\frac{p_0}{p_{i-1}}c - p_0 M)^2\right\}$$

and it's not clear whether it's higher than $p_{k^*_V}M$, and it concludes that the potential good type is not necessarily overpaid.

In the private case with unverifiable failures, the lowest possible reward that a bad type agent receives under the optimal contract is 0, which is the same as his true valuation. Therefore the bad type is weakly overpaid. For the potential good type, with similar argument in private with verifiable bad ones, the conclusion is still not clear whether he is overpaid or not, when comparing the reward that the potential good type receives to $p_{k^*_{NV}}M$.

Proof of Proposition 6

Given the agent has acquired j successes without failures, the benefit from fulfilling the remaining experiments is:

$$\begin{cases} U_B(k-j,p_j) = -c + (1-\sigma)U_B^A(k-j-1,p_{j+1}) + \sigma U_B^A(k-j-1,p_j) \\ U_B(1,p_{k-1}) = -c + (1-\sigma)a_B^k(k,0) + \sigma U_B^A(1,p_{k-1}) \end{cases} \tag{A.47}$$

Where $0 \leq j < k-1$. It can be simplified as:

$$U_B(k-j,p_j) = \frac{p_0}{p_k}a^k(k,0) + \sum_{i=j}^{k}(1-p_0)(1-\theta)^j\theta a^k(j,1) - \sum_{i=j}^{k}\frac{p_0}{p_{i-1}}\frac{c}{1-\sigma} \tag{A.48}$$

Then condtions (5.2) now become

$$\text{IC}^{S,B}_{0\leq j\leq k-1}: \quad \frac{p_0}{p_k}a^k(k,0) + \sum_{i=j}^{k}(1-p_0)(1-\theta)^j\theta a^k(j,1) \geq a^k(j,1) + \sum_{i=j}^{k}\frac{p_0}{p_{i-1}}\frac{c}{1-\sigma} \tag{A.49}$$

Conditions (A.49) then are the same as those in private experimentation with unverifiable failures as well as IR constraint, and the cost level of a single experiment is $\frac{c}{1-\sigma}$. Also, $\text{IC}^{F,B}$ in (5.3) are the same as those in conditions (4.6). Additionally, when failures are verifiable, Lemma 2 can be applied. Therefore, the principal is maximising the expected payoff under the same constraints in the scenario with unverifiable failures, and the optimal solution should be the same.

Proof of Proposition 7

1) When $k < T$, the agent still has further opportunity for over-experimenting even if the first failure occurs in the k_{th} experiment. Thus the condition (5.4) must be satisfied. Similarly, when the first failure occurs in the $j+1_{th}$ experiment, where $j < k$, to prevent agent from pretending to be those whose have more successes, the following incentive constraints need to be satisfied:

$$\begin{aligned} &\frac{(1-\theta^{T-j-1})}{1-\theta}\left[-c + (1-\theta)a_F^k(j+1,1)\right] + \theta^{T-j-1}a_F^k(j,1) \leq a_F^k(j,1) \\ \Longrightarrow \quad &-\frac{c}{1-\theta} + a_F^k(j+1,0) \leq a_F^k(j,1) \end{aligned} \tag{A.50}$$

These constraints together with (5.4) are the same as IC^F in (4.4) and (4.6). Meanwhile, to prevent agent from stopping experimenting earlier without a failure, the following $\text{IC}^{S,F}$ constraints need to be satisfied:

$$\text{IC}^{S,F}_{0\leq j\leq k-1}: \quad U_F(k-j,p_j) \geq a_F^k(j,0) \tag{A.51}$$

These conditions are exactly the same as those IC^S when failures are verifiable and not verifiable

respectively. Therefore, the principal is solving the same maximisation problem as that in $T \to \infty$, and the optimal solution should be the same.
2) When $k = T$, the constraint (5.4) can be removed since the first failure occurs in the last experiment and the agent has no chance to over-experiment. But other constraints in (A.50) and (A.51) are still the same as those in section 4.1 and 4.2 when failures are verifiable and unverifiable respectively. When failures are verifiable, it can be easily show that CF scheme satisfies all these constraints, so it has be optimal. In the other scenario where failures are not verifiable, it can be shown that all constraints are satisfied under CF and MF-II when $T \leq \overline{k}$ and $T > \overline{k}$ respectively. This is still true even if $T > \hat{k}$.

3) Denote by $V_F(k)$ the principal's expected payoff given the incentive to run k experiments in finite opportunity case. In public experimentation, CF is optimal from Proposition 7.1), thus $V_{F,P}(k) = V_P(k)$ for $\forall k > 0$. Therefore, if $T \geq k^P$, the principal can just provide the incentive to run k^P experiments if no failure occurs, and achieve the same expected payoff as that in the public case with infinite opportunities.

If experiments are private and failures are verifiable, since more constraints are binding and the feasible set shrinks, the principal is worse off relative to the private case, $V_V(k) \leq V_P(k)$ for $\forall k > 0$. Notice that $V_V(k)$ and $V_P(k)$ are decreasing functions when $k > k^P$ and $k > k_V^*$ respectively, it's must be true that $\exists k_V = \max\{k \in \mathbb{N} : V_P(k) \leq V_V(k_V^*)\}$ and $V_P(k) \leq V_P(k_V)$ for $\forall k \geq k_V$. From Proposition 7.2), CF is still optimal at $k = T$ in the case with finite opportunities, which implies that it's still true that $V_{F,P}(T) = V_P(T)$. Notice that $k_V^* \leq k_V$, the principal would optimally motivate to agent to run $k_V^* < T$ experiments When $T > k_V$.

If experiments are private and failures are not verifiable, the expected payoff under MF-II scheme is weakly higher than that under MF-III given the same number of experiments is motivated, $V_{NV}(k) \leq V_V(k)$ for $\forall k > \overline{k}$, according to Definition 3 and Proposition 3.1). Notice that $V_{NV}(k)$ and $V_V(k)$ are decreasing function when $k > k_V^*$ and $k > k_{NV}^*$ respectively, thus it's must be true that $\exists k_{NV} = \max\{k \in \mathbb{N} : V_V(k) \leq V_{NV}(k_{NV}^*)\}$ and $V_V(k) \leq V_V(k_{NV})$ for $\forall k \geq k_{NV}$. From Proposition 7.2), MF-II is optimal at $k = T > \overline{k}$ in the case with finite opportunities, and $V_{F,NV}(T) = V_V(T)$. Notice that$k_{NV}^* \leq k_{NV}$, as a result, the principal would optimally motivate the agent to run $k_{NV}^* < T$ experiments in the optimal contract when $T > k_{NV}$.

References

Bergemann, D., A. Bonatti, and A. Smolin (2015): "Selling Experiments," *Working paper.*

Bergemann, D. and U. Hege (2005): "the Financing of Innovation: Learning and Stopping," *RAND Journal of Economics*, 719–752.

Bolton, P. and C. Harris (1999): "Strategic Experimentation," *Econometrica*, 67, 349–374.

Brocas, I. and J. D. Carrillo (2007): "Influence through Ignorance," *The RAND Journal of Economics*, 38, 931–947.

Celik, G. (2015): "Implementation by Gradual Revelation," *The Rand Journal of Economics*, 46, 271–296.

DeMarzo, P. M., I. Kremer, and A. Skrzypacz (2017): "Test Design and Disclosure," *Working paper.*

Deneckere, R. and S. Severinov (2007): "Optimal Screening with Costly Misrepresentation," *Working paper.*

Dziuda, W. (2011): "Strategic Argumentation," *Journal of Economic Theory*, 146, 1362–1397.

Felgenhauer, M. and E. Schulte (2014): "Strategic Private Experimentation," *American Economic Journal: Microeconomics*, 6, 74–105.

Gill, D. and D. Sgroi (2012): "the Optimal Choice of Pre-Launch Reviewer," *Journal of Economic Theory*, 147, 1247–1260.

Glazer, J. and A. Rubinstein (2004): "On Optimal Rules of Persuasion," *Econometrica*, 72, 1715–1736.

Halac, M., N. Kartik, and Q. Liu (2016): "Optimal Contracts for Experimentation," *The Review of Economic Studies*, 83, 1040–1091.

Hart, S., I. Kremer, and M. Perry (2017): "Evidence Games: Truth and Commitment," *The American Economic Review*, 107, 690–713.

Henry, E. (2009): "Strategic Disclosure of Research Results: the Cost of Proving Your Honesty," *The Economic Journal*, 119, 1036–1064.

Henry, E. and M. Ottaviani (2014): "Research and the Approval Process," *Working paper.*

Hörner, J. and A. Skrzypacz (2016): "Selling Information," *Journal of Political Economy*, 124, 1515–1562.

Ispano, A. (2013): "Information Acquisition and the Value of Bad News," *Working paper*.

Kamenica, E. and M. Gentzkow (2011): "Bayesian Persuasion," *The American Economic Review*, 101, 2590–2615.

Kolotilin, A. (2015): "Experimental Design to Persuade," *Games and Economic Behavior*, 90, 215–226.

Kruse, T. and P. Strack (2015): "Optimal Stopping with Private Information," *Journal of Economic Theory*, 159, 702–727.

Milgrom, P. (2008): "What the Seller Won't Tell You: Persuasion and Disclosure in Markets," *The Journal of Economic Perspectives*, 22, 115–131.

Rayo, L. and I. Segal (2010): "Optimal Information Disclosure," *Journal of political Economy*, 118, 949–987.

Rosar, F. and E. Schulte (2010): "Imperfect Private Information and the Design of Information–Generating Mechanisms," *Working paper*.

Rothschild, M. and J. Stiglitz (1976): "Equilibrium in Competitive Insurance Markets: an Essay on the Economics of Imperfect Information," *The quarterly journal of economics*, 629–649.

Rubinstein, A. and J. Glazer (2006): "A Study in the Pragmatics of Persuasion: a Game Theoretical Approach," *Theoretical Economics*, 1, 395–410.

Spence, M. (1973): "Job Market Signaling," *The quarterly journal of Economics*, 87, 355–374.