Foundations for Research in Collusive Conduct:

Bidding Rings and Cartels

Part 1: Bidding Rings

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Part I: Bidding Rings

- Equilibrium at first and second price auctions and revenue equivalence.
- Each student should familiarize himself or herself with a bid rigging case.

Examples:

1. Addyston Pipe (sealed-bid auction)
2. John Asker's stamp cartel (NY et al. v. Feldman et al.): deposition of Ed Younger: `http://www.johnasker.com/deposition.pdf` (ascending-bid auction with limited observability of bidders' identity at the auction)

3. disk drives (optical disk drives complaint)
4. antiques (U.S. v. Ronald Pook)
5. machinery (U.S. v. Seville Industrial Machinery Corp.)
6. real estate (District of Columbia, ex rel. John Payton, Corporation Counsel v. George Basiliko, et al.) (3-5 are examples of an ascending-bid auction with full observability of bidder identities)
7. U.S. v. Inryco (1981)
8. U.S. v. Lyons (1982)
9. U.S. v. Metropolitan (1984)
10. U.S. v. A-A-A Electrical (1986)
11. U.S. v. Brinkley & Son (1986)
12. Finnegan v. Campeau (1989).

Auction Theory

The standard symmetric, independent, private value model:

- N bidders for one object.
- Bidders are (ex-ante) symmetric: Each bidder's type v is independently drawn from the increasing, cumulative distribution $F(v)$ with support $[\underline{v}, \overline{v}]$.
- F has a continuous density function f and $E[v] < \infty$.
- Bidders are risk neutral, and values are private: a type v bidder's payoff when winning the object at price p is:

$$v - p$$

- Each bidder knows his own type, but not the types of the other bidders.

Standard Auctions

- In a **first-price sealed bid auction**, bidders simultaneously submit bids; the highest bidder wins and pays his own bid.
- In a **second-price sealed bid auction** or **Vickrey auction**, bidders simultaneously submit bids; the highest bidder wins and pays the second highest bid.
- In an **open ascending auction**, or **English auction**, the price starts low and it is raised continuously by the auctioneer; bidders decide when to drop out; once a bidder has dropped out, he cannot re-enter (this is the theoretically simplest way of modelling ascending open auctions).
- In an **open descending auction**, or Dutch auction, the price starts high and it is lowered continuously by the auctioneer; the first bidder to take the current price wins the auction at the current price.

Bayesian Equilibrium

A **Bayesian equilibrium** in a sealed bid auction is a bidding function for each player, specifying what each type of each player bids.

It must be optimal for each type of each bidder to use the bid prescribed by his bidding function, given that all types of all other players use their equilibrium bids.

- It is common to study symmetric auctions and focus on symmetric equilibria, in which all bidders use the same bidding function.
- Asymmetric (first-price) auctions are harder, but we need to understand them if we want to understand collusion in practice.

Dutch and First Price

Proposition. The Dutch auction is strategically equivalent to the sealed bid first-price auction.

English and Second Price

Proposition. Under private values, (not under interdependent values), the English auction is equivalent to the sealed bid, second-price auction, in the sense that the optimal strategies in the two auctions are the same.

Proposition. Both in a second-price and in an English auction, it is a weakly dominant strategy for each bidder to bid $b(v_i) = v_i$ (hence, it is also a Bayesian equilibrium).

Proof. Consider a bidder i of type v.

Bidding $y < v$ leads the same outcome for i as bidding v if (i) the highest bid of the opponents is a price $p < y$. (winning at price p); (ii) the highest bid of the opponents is a price $p > v$ (losing the auction). If the highest bid of the opponents is a price $p = v$, then bidding v or $y < v$ leads to the same zero payoff for i. If, instead, the highest bid of the opponents is $p \in [y, v)$, then bidding v is strictly better than bidding y as it leads to a sure win and a positive payoff when winning.

Similarly, Bidding $y > v$ leads the same outcome for i as bidding v if (i) the highest bid of the opponents is a price $p < v$. (winning at price p); (ii) the highest bid of the opponents is a price $p > y$ (losing the auction). If the highest bid of opponents is a price $p = v$, then bidding v or $y > v$ leads to the same zero payoff for i. If, instead, the highest bid of the opponents is $p \in (v, y]$, then bidding v is strictly better than bidding y as it leads to a sure loss and a zero payoff rather than a negative payoff.

$\square$

- The second price (and the English) auction has (collusive) Bayesian equilibria in which players use weakly dominated strategy.
- For example: $b(v_1) = \overline{v}$ for all $v_1 \in [\underline{v}, \overline{v}]$ and $b(v_i) = \underline{v}$ for all $i \neq 1$ and all $v_i \in [\underline{v}, \overline{v}]$.
- It is common to disregard these collusive equilibria, but...

First Price Auction

- Let the order statistic $v_{(k)}$ be the k-th highest of the N valuations (random draws from the distribution $F(v)$, with symmetric bidders) and $v_{(-i,k)}$ be the k-th highest of the $N-1$ valuations with the exclusion of v_i
- The distribution of $v_{(-i,1)}$ is $F^{N-1}(v)$ (with symmetric bidders); its density is $(N-1)f(v)F^{N-2}(v)$.

Proposition. Under private values, a symmetric equilibrium bid of a first-price auction is:

$$\begin{aligned}\beta(v) &= E[v_{(-i,1)} | v_{(-i,1)} < v] \\ &= \int_{\underline{v}}^{v} \frac{y(N-1)f(y)F^{N-2}(y)}{F^{N-1}(v)} dy \\ &= v - \int_{\underline{v}}^{v} \frac{F^{N-1}(y)}{F^{N-1}(v)} dy\end{aligned}$$

Proof. Suppose all other bidders use the increasing bidding function $\beta(v)$. The payoff that a bidder i of type $\widetilde{v}$ makes from bidding b is:

$$[\widetilde{v} - b]F^{N-1}(\beta^{-1}(b))$$

The FOC is...

$$[\widetilde{v} - b]\left.\frac{dF^{N-1}(y)}{dy}\right|_{y=\beta^{-1}(b)} \frac{d\beta^{-1}(b)}{db} - F^{N-1}(\beta^{-1}(b)) = 0$$

or

$$[\widetilde{v} - b]\left.\frac{dF^{N-1}(y)}{dy}\right|_{y=\beta^{-1}(b)} - F^{N-1}(\beta^{-1}(b))\left.\frac{d\beta(y)}{dy}\right|_{y=\beta^{-1}(b)} = 0 \qquad (1)$$

In equilibrium, it must be optimal for the bidder type $\widetilde{v}$ to bid $b = \beta(\widetilde{v})$ (hence $y = \widetilde{v}$):

$$[\tilde{v} - \beta(\tilde{v})] \frac{dF^{N-1}(\tilde{v})}{d\tilde{v}} - F^{N-1}(\tilde{v}) \frac{d\beta(\tilde{v})}{d\tilde{v}} = 0$$

or...

$$\frac{d(\beta(\tilde{v}) F^{N-1}(\tilde{v}))}{d\tilde{v}} = \tilde{v} \frac{dF^{N-1}(\tilde{v})}{d\tilde{v}}$$

Integrating between $\underline{v}$ and v yields:

$$\begin{aligned} \beta(v) F^{N-1}(v) &= \int_{\underline{v}}^{v} \tilde{v} \frac{dF^{N-1}(\tilde{v})}{d\tilde{v}} d\tilde{v} \\ &= vF^{N-1}(v) - \int_{\underline{v}}^{v} F^{N-1}(y) dy. \end{aligned}$$

Hence

$$\beta(v) = v - \int_{\underline{v}}^{v} \frac{F^{N-1}(y)}{F^{N-1}(v)} dy.$$

Note that $\beta(v) = v - \int_{\underline{v}}^{v} \frac{F^{N-1}(y)}{F^{N-1}(v)} dy$ is increasing in v, as

$$\beta'(v) = \frac{dF^{N-1}(v)}{dv} \int_{\underline{v}}^{v} \frac{F^{N-1}(y)}{\left(F^{N-1}(v)\right)^2} dy = (v - \beta(v)) \frac{dF^{N-1}(v)}{dv} \frac{1}{F^{N-1}(v)}.$$

Replacing the formula above into the LHS of (1) for $y = \beta^{-1}(b)$ gives that an increase in the bid b has an effect on bidder i's payoff equal to

$$[v - \beta(y)] \frac{dF^{N-1}(y)}{dy} - F^{N-1}(y) \frac{d\beta(y)}{dy} = [v - y] \frac{dF^{N-1}(y)}{dy}$$

which is increasing for $y < v$, or equivalently $b < \beta(v)$ and decreasing for $y > v$, or equivalently $b > \beta(v)$. Hence bidding according to $\beta(v)$ is optimal for bidder i of type v.

$\square$

Uniform Example

Suppose $F = \frac{v}{\overline{v}}$ is the uniform distribution on $[0, \overline{v}]$ (the density is $\frac{1}{\overline{v}}$). In such a case:

$$\beta(v) = v - \int_{\underline{v}}^{v} \frac{F^{N-1}(y)}{F^{N-1}(v)} dy$$

$$= v - \int_{\underline{v}}^{v} \frac{y^{N-1}}{v^{N-1}} dy$$

$$= v - \frac{v^N}{Nv^{N-1}}$$

$$= \frac{(N-1)}{N} v$$

Mechanism Design

- An auction can be viewed as a mechanism in which bidders send messages to the designer (their bids) and the designer commits to an outcome decision (e.g., who gets the item) and transfer function (who pays what). A mechanism can be used for any general allocation, or social decision, problem.
- Formally, a mechanism is a triple $\langle M, D, T \rangle$ where:
  - $M = \times_{i=\{1,\ldots,N\}} M_i$ and M_i is the set of messages that player i can send to the designer;
  - $D : M \rightarrow A$, the decision function, maps messages into the set of possible allocations A (in the auction of a single object, A is the set of vectors of probabilities $(q_1, \ldots, q_N)$, where q_i is the probability that i wins the object);
  - $T : M \rightarrow \mathbb{R}$, the transfer function, maps messages into a transfer payment from each bidder to the designer.

The Revelation Principle

A **direct mechanism** is a mechanism in which the message space M_i of each player is his set of types (in the auction described before, the set $[\underline{v}, \overline{v}]$. It is standard to denote a direct mechanism by the pair $\langle D, T \rangle$.

Theorem. (**The Revelation Principle**) Given any mechanism and a (Bayesian) equilibrium for that mechanism, there exists a direct mechanism in which (1) it is an equilibrium for each buyer to report her true type, and (2) the equilibrium outcomes (decision and transfers) are the same as in the given equilibrium of the original mechanism.

Bidder-Payoff Equivalence

Theorem. With independent, private values, bidders expected payoffs are the same in any mechanism $\langle q, p \rangle$ having the same outcome function q and yielding the same payoffs to the lowest type. Bidder $i's$ expected payoff is given by

$$U_i(v_i) = \underline{u} + \int_{\underline{v}}^{v_i} \int_{\underline{v}}^{\overline{v}} \dots \int_{\underline{v}}^{\overline{v}} q_i(v, v_{-i}) f_{-i}(v_{-i}) dv_{-i} dv$$

Where $f_{-i}(v_{-i}) = \Pi_{j \neq i} f(v_j)$.
In particular, if $\underline{u} = 0$ and the outcome function q is efficient, (so that $q_i = 1$ if $v_i > v_j$ for all $j \neq i$ and $q_i = 0$ if $v_i < v_j$ for some $j \neq i$), then the above expression reduces to

$$U_i(v_i) = \int_{\underline{v}}^{v_i} F^{N-1}(y) dy$$

Proof. Let $< q, p >$ be a direct mechanism in the case of allocating a single item to bidders with private values; $q_i(r_i, r_{-i})$ is the probability that bidder i wins the object and $p_i(r_i, r_{-i})$ is i's payment when i reports type r_i and the other bidders report r_{-i}.

Let $f_{-i}(v_{-i}) = \Pi_{j \neq i} f(v_j)$. Bidder i's expected payoff when his type is $\widetilde{v}_i$, but he reports r_i and all other bidders report truthfully $r_{-i} = v_{-i}$ is:

$$U_i(r_i; \widetilde{v}_i) = \int_{\underline{v}}^{\overline{v}} ... \int_{\underline{v}}^{\overline{v}} \left[\widetilde{v}_i q_i(r_i, v_{-i}) - p_i(r_i, v_{-i}) \right] f_{-i}(v_{-i}) dv_{-i}$$

By the revelation principle we can restrict attention to mechanisms in which it is optimal for agent i to bid truthfully $r_i = \widetilde{x}_i$. Define the indirect utility function $U_i(\widetilde{v}_i) = U_i(r_i; \widetilde{v}_i)|_{r_i = \widetilde{v}_i}$. By the envelope theorem:

$$\frac{dU_i(\widetilde{v}_i)}{d\widetilde{v}_i} = \int_{\underline{v}}^{\overline{v}} ... \int_{\underline{v}}^{\overline{v}} q_i(\widetilde{v}_i, v_{-i}) f_{-i}(v_{-i}) dv_{-i}$$

$$\frac{dU_i(\widetilde{v}_i)}{d\widetilde{v}_i} = \int_{\underline{v}}^{\overline{v}} \dots \int_{\underline{v}}^{\overline{v}} q_i(\widetilde{v}_i, v_{-i}) f_{-i}(v_{-i}) dv_{-i}$$

Integrating both sides with respect to $\widetilde{v}_i$ from $\underline{v}$ to v_i yields:

$$U_i(v_i) = \underline{u} + \int_{\underline{v}}^{v_i} \int_{\underline{v}}^{\overline{v}} \dots \int_{\underline{v}}^{\overline{v}} q_i(v, v_{-i}) f_{-i}(v_{-i}) dv_{-i} dv$$

In particular, if $\underline{u} = 0$, the outcome function q is efficient then the above expression reduces to:

$$U_i(v_i) = \int_{\underline{v}}^{v_i} F^{N-1}(y) dy$$

□

Revenue Equivalence

As a simple consequence of bidder's payoff equivalence, we have the *revenue equivalence theorem*

Theorem. With independent private values, the seller's expected revenue is the same in any mechanism $\langle q, p \rangle$ having the same outcome function q and yielding the same payoff to the lowest type. In particular, in all efficient auctions in which the lowest type expected payoff is zero (these include the first-price, second-price and English auction), the seller's expected revenue is the same.

- Revenue equivalence does not hold if signals are not independent.
- If signals are affiliated (a strong form of positive correlation) the English auction yields more revenue than the second price auction, which yields more revenue than the first price auction (Milgrom and Weber 1982).

Optimal Auctions

What is the optimal auction for the seller (i.e. the auction that maximizes revenue)?

- Allow bidders to be asymmetric so that bidder i's type is drawn from the distribution F_i.
- Assume independent private values.

Monotonicity Assumption
All the virtual valuations $v_i - \frac{1-F_i(v_i)}{f_i(v_i)}$ are increasing in v_i.

- An increasing hazard rate $\frac{f_i(v_i)}{1-F_i(v_i)}$ is sufficiently for the monotonicity assumption to hold.

Theorem. Under the monotonicity assumption, the optimal auction for the seller assigns the object with probability one to the bidder with the highest virtual valuation $v_i - \frac{1-F_i(v_i)}{f_i(v_i)}$ if such a virtual valuation is above the seller's value v_0. If the highest virtual valuation is below v_0, then the seller keeps the object.

- The monotonicity assumption is needed to satisfy the soc for the IC constraint which is used to derive $U_i(v_i)$. Without it, there must be some bunching of types to make sure that the soc holds.
- The lowest type of bidder i that will win the object with positive probability is determined by the condition

$$v_i - \frac{1-F_i(v_i)}{f_i(v_i)} = v_0$$

Corollary. If bidders are symmetric, $F_i = F$ for all i, then the optimal auction is any efficient auction (e.g., any standard auction: first-price, second-price, english, dutch) with either an entry fee or a reserve price that restrict entry to types above the value R that solves $R - \frac{1-F(R)}{f(R)} = v_0$.

- Note: The optimal cut-off type R, and hence the optimal entry fee or reserve price do not depend on the number of bidders.
- For example, if F is uniform on $[0, 1]$, and $v_0 = 0$, then the optimal reserve price is $R = 1/2$.
- In the optimal auction bidders obtain an information rent.

First-Price Auctions with Asymmetric Bidders

- Assume all distribution functions F_i have the same support and have a continuously differentiable density f_i bounded away from zero. Then:

Lemma. An equilibrium at a first-price auction exists in pure strategies, the bid functions are strictly increasing and differentiable, and the equilibrium is unique.

Risk Aversion

Bidder i of type v_i has payoff $U(v_i - p)$ when he wins and pays a price p, where U is a concave function with $U(0) = 0$. (v_i is the monetary value of the object.)

- If bidders are risk-averse, then revenue equivalence breaks down. The first price auction yields higher revenue than the second-price auction.
- In a second-price auction, it is optimal for bidder i of type v_i to bid $\beta(v_i) = v_i$. This is the same bidding function as in the risk-neutral case. Hence the seller makes the same revenue he would make if bidders were risk-neutral.
- In a first-price auction, on the other hand, the bid is on average higher. The bidder insures himself against the risk of losing the object.

Theory of collusion at auctions

- First price collusion: McAfee and McMillan (AER, 1992)
- Second price collusion: Mailath and Zemsky (GEB, 1991)
- Contrasting first and second price collusion: Marshall and Marx (JET, 2007).

Collusion

- Case law is replete with examples of Section 1 violations of the Sherman Act for bid rigging - these are just the bidders who were apprehended.
- Commonly thought that oral ascending bid auctions, and second-price sealed-bid auctions, are more susceptible to collusion than first-price sealed-bid auctions.
- Heterogeneous independent private values model, we look at bidder collusion at first-price and second-price auctions, allowing for within-cartel transfers.
- Rings may contain all or a strict subset of the bidders.
- Focus on "pre-auction mechanisms," i.e., collusive mechanisms that rely on pre-auction communication and that do not rely on information from the auction itself, such as the identity of the winner or the amount paid.

- Antitrust cases involving pre-auction mechanisms include US v. Inryco,; US v. Lyons; US v. Metropolitan; US v. A-A-A; US v. Brinkley; Finnegan v. Campeau.

- In the landmark antitrust case *US v. Addyston Pipe and Steel Co. et al.*, colluding cast-iron pipe manufacturers met prior to the auction, determined which one of the colluding firms would participate in the auction, and agreed on transfer payments.

- In a “bid coordination mechanism,” or BCM, the cartel can arrange transfers and recommend bids to the ring members, but has no power to control the bids of the ring members.
- In a “bid submission mechanism,” or BSM, the cartel does have the power to control the bids of the ring members. E.g., the cartel actually submits a bid on behalf of each ring member (as in US v. Brinkley: Brinkley turned in the bid form for at least one of his competitors.), or the center selects one ring member to attend the auction and can prevent all other ring members from bidding.
- The auction format (first price versus second price) may lead to different results in terms of the viability and profitability of collusion.
- Several antitrust cases involve repeated interaction among the colluding firms, (e.g., US v. Addyston, US v. Inryco, and US v. Lyons.).

- For repeated auctions, collusion by an all-inclusive ring can be sustained in some environments:
  - Fudenberg Levine and Maskin (1994) prove a folk theorem for the case in which bidders can communicate prior to each auction and can observe all ring bids but cannot make transfers.
  - Aoyagy (2003) considers a repeated auction environment with no transfers. In equilibrium the repeated play provides the opportunity for intertemporal payoff transfers through a bid rotation scheme.
  - Without communication or the ability to observe bids, Skrzypacz and Hopenhayn (2004) show that for discount factors sufficiently large, an all-inclusive ring can do better than non-cooperative play or a bid rotation scheme by using implicit transfers of equilibrium continuation payoffs.
  - Hörner and Jamison (2007) shows that an all-inclusive ring can approximate first-best profits when the discount factor is close to one using review strategies.

- Several other cases involves only a single auction or procurement (e.g., US v. Metropolitan, US v. A-A-A, US v. Brinkley, and Finnegan v. Campeau).
- We focus on collusion for single auctions.

Intuition:

- A ring at a second-price auction:
  - must suppress the bids of all members except the bidder with highest value who bids as he would were he acting non-cooperatively;
  - any ring member breaking ranks and competing at the auction faces the highest ring bidder and the highest nonring bidder, each submitting same as without ring;
  - there is no gain to deviading from the ring-prescribed behavior.

- A ring at a first-price auction:
  - ring member with the highest value must lower his bid below what the non-cooperative bid and other ring members must suppress their bids;
  - opportunity for a non-highest-valuing cartel member to enter a bid at the auction, either on his own or through a shill, and secure the item
  - harder to collude

- For the case of an all-inclusive cartel composed of homogeneous bidders operating a BSM satisfying ex post budget balance, McAfee-McMillan (1992) and Mailath-Zemsky (1991) show that, regardless of the auction format, the cartel can suppress all ring competition, sending only the highest-valuing ring member to the auction, where he wins the object for a price equal to the auctioneer's reserve.
- Marshall-Marx (2007) consider BCM as well as BSM, and consider cartels that are not all inclusive and bidders that are heterogeneous. They show that the result that all ring competition can be suppressed at both first-price and second-price auctions extends to this environment when the cartel operates a BSM.

- Assuming ex ante budget balance, M&M also show that a ring can suppress all ring competition using a BCM at a second-price auction, but not at a first-price auction. Thus, they provide a formalization of the intuition that collusion is more difficult at first-price than at second price-auction.
- Marshall and Marx (2007) show that for first-price auctions, with a BCM multiple ring members submit bids that are close together. This provides a potential empirical test for collusion.

- Let the order statistic $v_{(k)}$ be the k-th highest of the N valuations (random draws from the distributions $F_i(x)$, bidders are no necessarily symmetric) and $v_{(-i,k)}$ be the k-th highest of the $N-1$ valuations with the exclusion of v_i
- The distribution of $v_{(1)}$ is $G(x) = \Pi_i F_i(x)$; its density is denoted $g(x)$.
- The distribution of $v_{(2)}$ is $H(x) = \sum_i \left(\Pi_{j \neq i} F_j(x)\right)(1 - F_i(x)) + \Pi_i F_i(x)$; its density is denoted $h(x)$.
- The distributions of $v_{(-i,1)}$ and $v_{(-i,2)}$ are denoted as $G_{-i}(x)$ and $H_{-i}(x)$; their densities $g_{-i}(x)$ and $h_{-i}(x)$.
- Consider a ring including bidders $1, ..., K \leq N$. Let $\theta_i(x) = \Pi_{\ell \in \{1,...,K\} \setminus \{i\}} F_\ell(x)$ be the distribution of highest valuing ring member other than i.

Second Price Auction with Reserve Price R

Proposition. In a second-price auction with reserve price R, it is a weakly dominant strategy for a bidder to bid her valuations if it exceeds R, and not to bid if her valuation is below R.

Proposition. Type v_i of bidder i's interim utility $u_i\left(v_i; R\right)$ (expected payoff from participating) in a second price auction with reserve price R is zero if $v_i \leq R$. If $v_i > R$ the interim utility is

$$u_i\left(v_i; R\right) = \int_R^{v_i} G_{-i}(v)dv$$

Collusion in a Second-Price Auction: Mailath and Zemsky (GEB, 1991)

- Bidders in the ring (with members $1, \ldots, K \leq N$) run a pre-auction mechanism in which each bidder i in the ring:
  - makes a report r_i about her valuation,
  - is selected with probability $\rho_i(r_1, \ldots, r_K)$ to be the only member of the ring to bid at the auction,
  - makes a payment $p_i(r_1, \ldots, r_K)$ to the ring (p_i will be negative for some bidders; in practive often the mechanism requires that bidders first put "money in the hat" and then divide the money in the hat once the winner of the pre-auction mechanism is chosen; think of p_i as the net payment)

- The ring allows only one ring member to bid at auction (BSM locked door mechanism); the selected ring member is asked to bid her valuation at the auction (this is IC)
- The pre-auction mechanism satisfies ex-post budget balance and interim individual rationality

Definition. Ex-post budget balance requires: for all $v_1, ..., v_K$

$$\sum_{i=1}^{K} p_i(v_1, ..., v_K) \geq 0$$

Let $U_i\left(v_i, R\right)$ be the expected utility from participating in the pre-auction mechanism of ring member i of type v_i.

Definition. Interim individual rationality requires: for all v_i

$$U_i\left(v_i, R\right) \geq u_i\left(v_i, R\right)$$

Theorem. (Mailath-Zemski) In an ex-post efficient pre-auction mechanism, the highest reporting bidder is selected with probability one to be the only ring member to bid at the auction and asked to bid her valuation (if above the reserve price). The pre-auction mechanims is also ex-post budget balanced and interim individually rational with the net payments to the ring given by:

$$p_i(r_1, \ldots, r_K) \equiv \int_{\underline{v}}^{r_i} \pi(v)\theta_i'(v)dv - \frac{1}{K-1} \sum_{\ell \in \{1,\ldots,K\} \setminus \{i\}} \int_{\underline{v}}^{r_\ell} \pi(v)\theta_\ell'(v)dv,$$

where

$$\pi(v) \equiv \text{a bidder's expected payoff from being the only ring member at auction when her value is } v \text{ (it is independent of } i\text{)}$$

and

$$\theta_i(v) \equiv \text{distribution of highest valuing ring member other than } i.$$

- Ex-post Budget Balance follows by noting that

$$
\begin{aligned}
& \sum_{i=1}^{K} p_i(r_1, \ldots, r_K) \\
= \; & \sum_{i=1}^{K} \left(\int_{\underline{v}}^{r_i} \pi(v)\theta_i'(v)dv - \frac{1}{K-1} \sum_{\ell \in \{1,\ldots,K\}\setminus\{i\}} \int_{\underline{v}}^{r_\ell} \pi(v)\theta_\ell'(v)dv \right) \\
= \; & 0.
\end{aligned}
$$

- It is immediate that the highest-valuing ring member does not want to deviate from bidding her value at the legitimate second-price auction

- To see that incentive compatibility in the pre-auction mechanism holds (bidders want to report thruthfully), note that, assuming all other ring member report truthfully their types, bidder i solves:

$$\max_{r_i} \pi(v_i)\theta_i(r_i)$$
$$-E_{v_{-i}}\left(\int_{\underline{v}}^{r_i} \pi(v_i)\theta_i'(v)dv - \frac{1}{K-1}\sum_{\ell\in\{1,\dots,K\}\setminus\{i\}}\int_{\underline{v}}^{v_\ell}\pi(v)\theta_\ell'(v)dv\right)$$

and differentiating wrt to r_i gives:

$$\pi(v_i)\theta_i'(r_i) - \pi(r_i)\theta_i'(r_i)$$

or

$$\left[\pi(v_i) - \pi(r_i)\right]\theta_i'(r_i)$$

- Since $\theta_i' > 0$ and π is an increasing function, this expression is positive for $r_i < v_i$, negative for $r_i > v_i$ and zero for $r_i = v_i$. Hence reporting her true type is optimal for bidder i.

- To see that it is individually rational to participate in the pre-auction mechanism:
  - Let
  $$\psi(v) \equiv \Pi_{j \in \{k+1,\dots,n\}} F_j(v_j)$$
  be the distribution of the first order statistic of non-ring members.
  - Then
  $$\pi(v) = \int_R^v (v - x)\psi'(x)dx + (v - R)\,\psi(R) \text{ for } v > R$$
  and
  $$\pi\,(v) = 0 \text{ for } v < R.$$

- Individual Rationality requires:

$$U_i\left(v_i, R\right) \geq u_i\left(v_i, R\right) = \int_R^{v_i} G_{-i}(v)dv = \int_R^{v_i} \psi(v)\theta_i(v)dv$$

and it holds since, letting

$$X = E_{v_{-i}}\left(\frac{1}{K-1}\sum_{\ell\in\{1,\ldots,K\}\setminus\{i\}}\int_{\underline{v}}^{v_\ell}\pi(v)\theta'_\ell(v)dv\right) > 0$$

we have:

$$\begin{aligned}
U_i\left(v_i, R\right) &= E_{v_{-i}}\left(\pi(v_i)\theta_i(v_i) - p_i(v_i, v_{-i})\right) \\
&= \pi(v_i)\theta_i(v_i) - \int_{\underline{v}}^{v_i}\pi(v)\theta'_i(v)dv + X \\
&= \int_{\underline{v}}^{v_i}\pi'(v)\theta_i(v)dv + X \\
&= \int_R^{v_i}\psi(v)\theta_i(v)dv + X
\end{aligned}$$

Symmetric (or ex-ante identical) bidders with ring of size N

When all bidders collude and they are ex-ante identical, so that $F_i = F$ for all i, the net payments in the pre-auction mechanism become:

$$p_i(r_1, \ldots, r_K) \equiv \int_R^{r_i} v g_{-i}(v) dv - \frac{1}{N-1} \sum_{\ell \in \{1, \ldots, K\} \setminus \{i\}} \int_R^{r_\ell} v g_{-\ell}(v) dv,$$

- The bidders can implement the pre-auction mechanism by holding a first-price (pre-)auction before participating in the legitimate second-price auction.
- If the highest bid in the pre-auction is strictly positive, then the pre-auction winner is the sole bidder participating in the legitimate second-price auction, where she bids the reserve price R.
- The pre-auction winner pays each of the other bidders (ring members) an equal share of her bid in the pre-auction.

Other Properties and Questions

- To defend himself against the presence of a bidding ring, the seller could adjust the reserve price.
- The profit maximizing reserve price is an increasing function of the size of the ring. (True even with ex-ante symmetric bidders; contrast with the optimal mechanism with no collusion.)
- Open questions (for both second- and first-price):
  - how do different levels of the reserve and more generally strategic behavior of the auctioneer affect the size of the ring?
  - The seller's optimal mechanism in the presence of collusion is not known. It is likely to require that different bidders be treated differently and some degree of ex-post inefficiency - as in Myerson's optimal auction)

Drawbacks of the Mailath-Zemski pre-auction mechanism

- Without locked door, for some distributions, bidders have an incentive to underreport in the pre-auction mechanism and then bid at the auction.
- In the asymmetric case, in some situations, a bidder that is not selected as the sole winner must make payments to the other ring members. Hence the mechanism is not ex-post individually rational. It is also not immune to deviations by sub-coalitions (e.g., two high value bidders may want to collectively deviate).

Collusion in a First-Price Auction: McAfee and McMillan (AER, 1992)

- Because of the difficulties of studying asymmetric first-price auctions, they look at:
  - ex-ante symmetric bidders, $F_i = F$ for all i
  - all inclusive ring $K = N$
- Like in Mailath and Zemsky, bidders in the ring run a BSM pre-auction mechanism in which each bidder i in the ring:
  - makes a report r_i about her valuation,
  - is selected with probability $\rho_i(r_1, ..., r_N)$ to be the only member of the ring to bid at the auction,
  - makes a payment $p_i(r_1, ..., r_K)$ to the ring
- the highest reporting ring member is selected and asked to bid the reserve price at the auction; others prevented from bidding
- The pre-auction mechanism satisfies ex-post budget balance and interim individual rationality

Theorem. (McAfee-McMillan) In an all-inclusive ex-post efficient pre-auction mechanism, the highest reporting bidder is selected with probability one to be the only ring member to bid at the auction and asked to bid the reserve price (if valuation is above it). The pre-auction mechanism is also ex-post budget balanced and interim individually rational; the total payment of the winner, say bidder i, is

$$
\begin{aligned}
p_i(r_1, \dots, r_N) &\equiv R + \frac{1}{F(r_i)\theta_i(r_i)} \int_R^{r_i} (x - R)\, \theta_i'(x) F(x) dx \\
&= R + (N-1) \int_R^{r_i} (x - R) \frac{F^{N-1}(x)}{F^N(r_i)} f(x) dx
\end{aligned}
$$

where

$\theta_i(v) \equiv F^{N-1}(v) =$ distribution of highest valuing ring member other than i.

Each loser receives $\frac{1}{N-1}$ of the winner's payment to the ring $p_i(r_1, \dots, r_N) - R$.

- It is immediate that the pre-auction mechanism is ex-post budget balanced and that it is IC for highest-valuing ring member to bid the reserve price at the legitimate auction
- Pre-auction mechanism can be implemented by a first-price auction: it relies on an all-inclusive ring and symmetric bidders. (A second-price or ascending auction would not work: incentive to overbid when low value in order to raise money collected from winner.)

Incentive Compatibility in the Pre-Auction Mechanism

Assuming all other ring members report truthfully their types, bidder i of type v_i solves:

$$\max_{r_i}\left[v_i - R - (N-1)\int_R^{r_i}(x-R)\frac{F^{N-1}(x)}{F^N(r_i)}f(x)dx\right]F^{N-1}(r_i)$$

$$+\int_{r_i}^{\overline{v}}\left(\int_R^{v}(x-R)\frac{F^{N-1}(x)}{F^N(v)}f(x)dx\right)dF^{N-1}(v)$$

differentiating wrt to r_i gives:

$$(N-1)(v_i-R)F^{N-2}(r_i)f(r_i) - (N-1)(r_i-R)F^{N-2}(r_i)f(r_i)$$

$$+\frac{(N-1)f(r_i)}{F^2(r_i)}\int_R^{r_i}(x-R)F^{N-1}(x)f(x)dx$$

$$-\frac{(N-1)f(r_i)}{F^2(r_i)}\int_R^{r_i}(x-R)F^{N-1}(x)f(x)dx$$

$$= (N-1)(v_i - r_i)\, F^{N-2}(r_i) f(r)$$

- this expression is positive for $r_i < v_i$, negative for $r_i > v_i$ and zero for $r_i = v_i$. Hence reporting her true type is optimal for bidder i.

- To see that it is individually rational to participate (indeed ex-post IR) in the pre-auction mechanism:
  - Note that all losing bidders in the pre-auction receive a positive transfer (including bidders whose value is below the reserve price, as long as highest value is above R)
  - The winning bidder with value $v_i > R$ obtains:

$$
\begin{aligned}
& v_i - R - (N-1)\int_R^{v_i} (x-R)\,\frac{F^{N-1}(x)}{F^N(v_i)} f(x)dx \\
= \quad & v_i - R - \frac{(N-1)}{NF^N(v_i)}\int_R^{v_i} (x-R)\, dF^N(x) \\
= \quad & v_i - R - \frac{(N-1)}{NF^N(v_i)}\left[(v_i - R)\,F^N(v_i) - \int_R^{v_i} F^N(x)dx\right] \\
= \quad & \frac{v_i - R}{N} + \frac{N-1}{N}\int_R^{v_i} \frac{F^N(x)}{F^N(v_i)} dx > 0
\end{aligned}
$$

Contrast Collusion at First- and Second-Price Auction

- For all-inclusive ring with symmetric bidders:
  - Highest-valuing ring member wins the pre-auction
  - Pays the reserve price at the legitimate auction
- Profitability of the cartel is the same.
- What happened to intuition that it is easier to collude at second-price?

Marshall and Marx (2007)

- Unlock the door, look at BCM
- Pre-auction mechanism is only ex-ante budget balanced
- Non-all-inclusive cartel
- Allow for asymmetric bidders
- For simplicity, no reserve price
- At a second-price auction, cartel can get all surplus.
- At a first-price auction, cannot suppress all competition among ring members.

BCM Pre-auction Mechanism in Second-Price Auction

- Tell highest-reporting ring member to bid value and others to bid zero.
- If $r^{(2)}$ is the second-highest report, the highest-reporting ring member pays $\pi(r^{(2)})$ to the center (recall, $\pi(v)$ is a type v bidder's expected payoff from being the only ring member at auction)
- The center pays each of the K ring members an amount

$$\frac{1}{K} E_{v_1,\ldots,v_K} \left(\pi(v^{(2)}) \right)$$

BCM Second-Price Pre-auction Mechanism

- It is clearly ex-ante budget balanced.
- It is clearly IC to truthfully report: Report does not affect amount of payment, only whether you pay. And truthful reporting makes you win when you benefit from it (second-price logic)
- It is clearly IC to bid as recommended in the legitimate auction as it is second-price (losing bidders have no incentive to bid) (Note: in the legitimate auction the highest-valuing bidder wins. If that bidder is a ring member, then he pays price equal to maximum value among outside bidders.)
- It is IR, since losing ring members get paid and winning ring member is the highest reporting and expects a payoff $\pi(r^{(1)})$ which is greater than the amount $\pi(r^{(2)})$ he has to pay to the ring.

Other Properties

- Nice mechanism
  - meet only before auction
  - does not depend on information provided by the auctioneer
  - does not require enforcement of payments or bids
  - shills are not a problem
  - ok if there is a reserve price
  - also works for English auctions

BCM Pre-auction Mechanism in First-Price Auction

- Cannot suppress all ring competition at legitimate first-price auction
- What are the problems?
  - If highest-valuing ring member is asked to bid optimally against the outside bidders (as if he were the only ring member to bid), then non-highest-valuing ring members may cheat.
  - If highest-valuing ring member is asked to raise his bid, then highest-valuing ring member may deviate.
- Solution:
  - Ring must send at least two ring members to bid.
  - Loosely speaking, to prevent deviations from non-highest-valuing ring members, the ring must recommend that the highest-valuing ring member bid sufficiently high, but then to prevent deviations from that ring member, the center must recommend that some other ring member submit a bid just below his.

Theorem. (Marshall-Marx) The BCM pre-auction mechanism for a first-price auction prescribes that, for a positive-measure set of value realizations, at least two ring members submit bids at the legitimate first-price auction that are greater than or equal to the optimal bid for a sole ring member bidding against the outside bidders, with one strictly greater. Furthermore, for any $\varepsilon > 0$, there is a positive-measure set of value realizations such that the highest two ring bids are within ε of each other.

Policy Implications

- Possibility of a completely non-parametric test for collusion
  - Look for changes in the distribution of bids
  - Look for too many close bids
  - Test based on statistical results for spacings

Suggestive Data

- Timber data – Forest Service, late 1980s, Region 1 (MT,ND,ID)
- U.S. Senate hearings (1977) identified collusion as a potential problem
- Baldwin, et al. (1997) found evidence of collusion at English Forest Service auctions

Bids from Three First-Price Forest Service Timber Sales

Reserve	High Bid	2nd Bid	3rd Bid	4th Bid	5th Bid	6th Bid
$21,272	**$62,270**	**$62,072**	$51,770	$42,080	$38,504	$28,310
$37,523	$85,365	**$77,213**	**$76,860**	$65,847	**$57,960**	**$57,683**
$27,208	$129,240	$70,000	**$46,800**	**$46,480**	-	-

- Region 1, 1983–1992, 434 auctions
  - In 43 auctions, two bids within 1% and 20% greater than the reserve
  - 54 bidding pairs with bids within 1% and 20% greater than the reserve

Extensions

- Ability to use a shill not relevant in a non-cooperative setting
- Second-Price: feasibility of shill bidding does not affect the ring's expected payoff.
- First-Price: when shill bidding is feasible, ability to lock the door does not help at all.
- Shills can affect the profitability of collusive mechanisms at first-price auctions.
- In a First-Price auction: If ring can condition payments on the ID of winner (and no shills), then it is the same as if he can lock the door.

- Repeated interaction has no effect for collusion in second-price auctions in environments we considered.
- Repeated interaction helps collusion in first-price auction; e.g., in absence of transfers, Skrzypacz and Hopenhayn (2004) show ring can do better than a bid rotation scheme, but worse than first best.

Examples

In the following cases, colluding bidders met prior to the auctions to discuss their bids and determine transfer payments:

- **U.S. v. A-A-A Elec. Co., Inc. (1986)**
  Contractors bidding for work at the Raleigh-Durham Airport discussed their bids before submitting them and designated A-A-A as the one who would submit the lowest bid. After receiving final payment for the work, A-A-A made payments to its co-conspirators.
- **U.S. v. W.F. Brinkley & Son Construction Company, Inc. (1986)**
  Brinkley's competitors for a pumping station and pipeline contract discussed their bids prior to the procurement, agreeing that Brinkley would submit the winning bid. One of the competitors even filled out his bid and gave it to Brinkley to turn in for him.
- **U.S. v. Raymond J. Lyons (1982)** (sheet metal)

- **U.S. v. Addyston Pipe & Steel Co. et al. (1897)**
  Six colluding cast-iron pipe manufacturers met prior to the auction (a first-price sealed bid). To determine which one of the colluding firms would participate in the auction they used a knockout in which each ring member submitted a bid. The high bidder was selected to be the only member to bid at the auction and the aumount bid by that ring member in the knockout was redistributed to all ring members. To facilitate the knockout they set a "representative board" who would take all the bids. Losing knockout members were told to submit deliberately losing bids.
- Example of collusive gains: Howard-Harrison won a procurement auction in St, Louis by bidding $24, while Addyston and Dennis Long protected it with bids of $24.37 and $24.57. The Chattanooga foundry could have provided the pipes at $17 or $18 and still made a profit.

- "... the executive committee determines the price at which the bid is to be put in by some company in the association, and the question to which company this bid shall go is settled by the highest bonus which any one of the companies ... will agree to pay or bid for the order. ... the company to whom the right to bid upon the work is assigned sends in its estimate or bid to the city or company desiring pipe, and the amount thus bid is "protected" by bids from such of the other members of the association as are invited to bid, and by the bidding in all instances being slightly above the one put in by the company to whom the contract is to go. ... Settlements are made at stated times of the bonus account debited against each company, where these largely offset each other, so that small sums are in fact paid by any company in balancing accounts." (U.S. v. Addyston Pipe & Steel Co. et al. (1897) at p.3).

Wholesale Stamp Dealers

Ascending-bid auction without observability of bidder identities at the auction. (Marshall and Marx (book, 2012, chapter 9.6.2), Asker (2010), Graham, Marshall, and Richard (1990))

- In the 1990's: ring of 11 wholesale stamp dealers at ascending price auctions (primarily New York auctions)
- members reported values to ring center; ring center bid at auction for the highest-value ring member, other did not bid
- if ring won, after the auction the highest-value ring member made payments to other ring members based on reports to ring center:
- reports ranked, say $r_1 > r_2 > \ldots r_k > p > r_{k+1} \ldots > r_K$ where $p =$ price at legitimate auction

- Ring members transfer payments:

$$
\begin{aligned}
\text{Member } 1 \quad &: \quad -\frac{r_k - p}{2} - \frac{r_{k-1} - r_k}{2} - \ldots - \frac{r_2 - r_3}{2} \\
\text{Member } 2 \quad &: \quad \frac{r_k - p}{2}\frac{1}{k-1} + \frac{r_{k-1} - r_k}{2}\frac{1}{k-2} + \ldots + \frac{r_2 - r_3}{2} \\
&\vdots \\
\text{Member } k \quad &: \quad \frac{r_k - p}{2}\frac{1}{k-1} \\
\text{Member } k+1 \quad &: \quad 0
\end{aligned}
$$

- mechanism is a variant of the mechanism described by Graham, Marshall and Richard (1990); designed to give each ring member a measure of its contribution to the ring
- ring members have an incentive to overreport to inflate payoffs from the knockout

Ascending-bid auctions with observability of bidder identities

- Antiques, machinery, and real estate (Marshall and Marx (book, 2012, chapter 9.6.3) and Graham and Marshall (1987))
- Possible for rings to operate without pre-auction communication
- Follow rule of not raising bid if the current winner is a ring member
- After the auction, any objects won by a ring member are reallocated and gains distributed among ring members
- Only need to know who the ring members are

Post-auction Mechanisms

- Collusion at ascending-bid auctions
  - U.S. v. Seville Industrial Machinery
  - U.S. v. Ronald Pook (antiques)
  - D.C. v. George Basiliko, et al. (real estate)
- Limited paper trail
- No pre-auction communication, post-auction mechanism (inefficient)

- "When a dealer pool was in operation at a public auction of consigned antiques, those dealers who wished to participate in the pool would agree not to bid against the other members of the pool. If a pool member succeeded in purchasing an item at the public auction, pool members interested in that item could bid on it by secret ballot at a subsequent private auction ("knock out") .... The pool member bidding the highest at the private auction claimed the item by paying each pool member bidding a share of the difference between the public auction price and the successful private bid. The amount paid to each pool member ("pool split") was calculated according to the amount the pool member bid in the knock out." (U.S. v. Ronald Pook)

Nested Knockouts

- Used by the antique and machinery rings
- Items won by the ring go to a first knockout auction where a subset of the ring members have formed a sub-ring and bid as one bidder
- Item won by the sub-ring go to a second knockout auction where an inner subset of the sub-ring members have formed an inner ring and bid as one ...
- The court's decision noted with astonishment the potential inefficiency and viewed the subcartels as a kind of collusive cannibalism:
  "If the evidence presented in this case is indicative of the ethics of this or any segment of the business community, then we should weep for its existence and fear for its future." (U.S. v. Seville Industrial Machinery, p. 993.)

Questions

1. Can a post-auction mechanism can be constructed that does not interfere with efficiency at the auction?
2. Do the expected gains from collusion in terms of reducing the price paid at the auction more than offset any expected losses associated with the post-auction mechanism?

- Lopomo, Marshall, and Marx, "Inefficiency of Collusion at English Auctions," Contributions in Theoretical Economics 5 (1), Article 4 (2005).

A Simple Model

- Single-object, non-strategic seller, second-price auction
- Risk neutral bidders, heterogeneous IPV: v_i from distribution F_i with continuous density on $[0, \bar{v}]$
- Bidders 1 and 2 in a ring (RM1, RM2)
- Bidder 3 is outside the ring
- Collusive mechanism:
  - non-binding recommendation for bidding behavior at the auction
  - post-auction mechanism

Proposition. There does not exist an efficient, ex-post BB mechanism that increases the expected payoff to the ring member relative to noncooperative play.

- *Proof sketch.*
  - In the legitimate auction a ring member with the lowest possible value bids her value
  - If a ring member wins the legitimate auction and the other ring member bids the lowest possible value, then the post-auction mechanism leaves the object to the legitimate auction winner and gives no transfers to the loser
  - by the payoff equivalence logic, any mechanism that is ex-post efficient (like the legitimate second-price auction under noncooperative play) and gives zero payoff to the lowest type, must give all bidders the same expected payoff.

Source of Inefficiency

- Question:
  - Can the inefficiency can be attributed to behavior by RMs at the auction or to behavior by RMs at the post-auction mechanism?
- Answer:
  - It is neither the auction nor the post-auction mechanism that is necessarily inefficient. When the two are combined, IC for one prevents IC at the other

Profitable, ex-Post Inefficient, Collusion: an Example

- Outside bidder with value $\frac{1}{4}$.
- Two ring bidders with values uniformly distributed in $[0, 1]$.
- Collusive mechanism:

  - recommend both RMs exit at $\frac{1}{4}$
  - report $r_i \in \left\{0, \frac{1}{8}\right\}$
  - higher report wins the object (allocate randomly if tied) and pays the amount of his report to the other RM (and pays for object)

- In the equilibrium of the post-auction mechanism, each RM reports zero if his value is less than or equal to $\frac{1}{2}$, and reports $\frac{1}{8}$ otherwise.
  - Proof: Given one RM follows such a strategy, the payoff for the other RM with value v_i from reporting 0 is

$$\frac{1}{2}\frac{1}{2}\left(v_i - \frac{1}{4}\right) + \frac{1}{2}\left(\frac{1}{8}\right) = \frac{v_i}{4}$$

    the payoff from reporting $\frac{1}{8}$ is:

$$\left(\frac{1}{2} + \frac{1}{2}\frac{1}{2}\right)\left(v_i - \frac{1}{8} - \frac{1}{4}\right) + \frac{1}{2}\frac{1}{2}\left(\frac{1}{8}\right) = \frac{3v_i}{4} - \frac{1}{4}$$

- Joint expected payoff increases from $\frac{9}{32}$ under noncooperative play to $\frac{12}{32}$ under collusion (increase of 33%)
- Auctioneer's revenue falls from $\frac{37}{96}$ to $\frac{1}{4}$ (decrease of 35%).
- Inefficient allocation among the RMs and inefficiency in overall allocation

- Open Question: Optimal Collusion in General?
- Antitrust enforcement—inefficiency introduced by collusion may result in an "innocent" non-colluding bidder not receiving an object that he would have in the absence of collusion.
- Auctioneer strategies: "quick knock," "protecting bidder". Intent: to destabilize the cartel with adverse allocations. But, they introduce inefficiencies.
  - The quick knock involves awarding the item to an outside bidder while cartel bidders are still willing to bid higher
  - A protecting bidders is a shill bid for the auctioneer.

Multi-Unit Auctions

- Brusco and Lopomo (RES, 2002) study multi-unit ascending auctions in which bidders collude without forming a collusive ring:
  - The presence of multiple objects facilitates collusion by allowing the bidders to signal their willingness to abstain from competing over certain objects, provided they are not challenged on others. In this way, the bidders can allocate the objects among themselves without paying much.
  - As the ratio between the number of bidders and the number of objects increases, the room for collusive schemes such as the ones indicated above becomes smaller.
  - Large variance in the complementarities in the bidders' utility functions tend to hinder collusion, because each bidder is less satisfied with owning only a subset of the objects on sale; she has therefore an incentive to break the collusion and compete for all the objects in order to fully realize the synergies.

Some Current Issues: Collusion in two-sided auctions

- Discussed in Loertscher, Marx, and Wilkening (WP, 2013) and Milgrom (book, 2004)
- In thin two-sided settings, collusion across buyer and seller pairs may be lucrative to the pair and costly to the market maker (or designer).
  - E.g. see Hobbs, Rothkopf, and O'Neill (2000);
  - a practical example: the LIBOR scandal, where reports by potential buyer and seller trading pairs guided the prices paid in transactions with a third party
- Collusion between buyers and sellers and its effect on market efficiency depend on the mechanism used
  - E.g., in McAfee's double auction (1992) collusion between buyers and sellers *appears* to increase efficiency.

- Connection with multi-unit auctions (see Milgrom, 2004, Section 7.2)
  - Uniform price auctions with multi-unit demand (and fixed supply) often have low-price equilibria
  - In a model with elastic supply the lowest equilibrium prices are the Cournot prices (with the N bidders on the demand side as players)
  - this *suggests* that collusion in double auctions may be harder than in one-side auctions.

Conclusions: Some Lessons for Auctioneer - Antitrust

- When bidder collusion is a potential concern, use first-price sealed bidding.
- Do not reveal losing bids when using sealed bidding (both during and after): ring would like to monitor compliance.
- All information of relevance known to the auctioneer/procurer about the item for sale/procurement should be revealed ex ante to the entire bidding public: Concealing information about the object being sold or desired for purchase is pro-collusive: "winner's curse" is a strong motivation for collusion (on common value auctions, see Hendricks, Porter and Tan, Rand 2008.)
- Allow bidders to submit multiple bids, with some under disguised identities: The threat of shill bidders can be disruptive (more so with sealed bidding than open bidding).

- If possible, bid solicitations should prohibit subcontracting by the awardee: Subcontracting can be pro-collusive; just a transfer (cash transfers are often too transparent); inter-conspirator transactions at non-market prices.
- Beware of split awards. "Insurance" of having second supplier can be expensive.
  - Example. Procurement, two potential suppliers, each firm can make 2 units and each have the same cost structure – first unit costs 5 and second unit costs 100 to make.
  - Sole award – buyer pays 105
  - Split award possible – buyer pays 200
  - The bidding is non-cooperative, but potential suppliers restrict output ex ante to realize benefits of split awards

- When conducting a private or public antitrust investigation, analyze the communications used to implement these price increases. Investigate if supply and demand factors can explain the observed price increases or if instead time elapsed between price announcements better explains the observed price increases: Many cartels have used price announcements to seek "acceptance" of a price increase.
- Whenever possible, every aspect of the auction/procurement should be documented, and the records should be retained for a long period. The recording and documentation should include, but not be limited to, announcement of the auction/procurement, who was invited to bid, who actually bid, all discussions and conversations, and all bids. All bidders should be notified ex ante that the entire record of all auctions/procurements will be made available to public enforcement authorities and/or private litigants should an investigation of collusive bidding occur: Non-standard losing bids often reflect collusion, not costs. Adjacent bids might be "too close"