

Growth or stagnation in pre-industrial Britain? A revealed income growth approach.

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In recent years two ambitious historical national account reconstructions of British economic development over the very long run have been published (Broadberry et al 2015, Clark 2010).¹ Broadberry et al estimates national income and per capita income growth using output data while Clark is using income data, that is wage, capital income and land rent data. In principle results from these reconstructions should be similar but, alas, they differ profoundly. Clark suggests that per capita income is stationary before the 19th century but Broadberry finds a modest but positive long run growth, which accelerates in the mid 17th century.

This controversy relates to a discussion of the nature of medieval and Early Modern Britain in which M. M. Postan (1966) set the tone arguing that the British medieval economy was trapped in Malthusian stagnation. That interpretation soon developed into a consensus view. Meghnad Desai (1991) criticized the underlying quantitative analysis and suggested that a closer econometric scrutiny of the data did not support a Malthusian interpretation. Recently Greg Clark (2007) reiterated the Postan thesis although without mentioning Postan. Focusing on Early Modern Britain Møller and Sharp (2014) applied co-integration analysis to test whether real wages and population were endogenous, as they should be in a Malthusian framework. That is, wage increases above 'subsistence' should only be transitory triggering off an increase in population which ultimately brings down wages. This Malthusian thesis is rejected for that period. While Clark's reconstruction of British national income supports his Malthusian conviction, Broadberry et al argue explicitly that their results do not.

Historical reconstructions of national income are based on the same methodology as contemporary national accounting. However, the quality and representativeness of different historical data series differ which probably explains part of the differing results. For example, the results from Clark, depend crucially on the real wage series he is using.

¹ We would like to thank Leigh Shaw-Taylor for advice and a helpful discussion on the evolution of the occupational structure in Britain, and to participants at the MEHR seminar, University of Copenhagen.

The motivation for this paper is to offer an alternative method of measuring income growth which needs little data, in fact just four parameters combined with occupational and wage premium data, which are comparatively robust. The paper contributes to the longstanding debate regarding the nature of pre-industrial Britain: Growth or stagnation.

What we propose is a *revealed income growth* approach. *Revealed* indicates that we derive income changes from observed changes in occupational structure which in turn are linked to changes in consumption patterns. The logic builds on Engel's law. This is the empirically supported claim that as income increases, a falling share of income is used for consumption of food. A general income increase will thus imply a shift in the production and price structure leading to an increasing share of industry and services in production. There are a number of earlier attempts to measure income growth by looking at changes in occupational structure and consumption patterns. E. A. Wrigley (1967) analyzed pre-industrial England in a pioneering article. K. G. Persson (1991) formalized Wrigley's intuition analyzing Medieval 'Low Countries' and Tuscany and R. A. Allen (2000) used a similar method and applied it to pre-industrial Europe. However, none of these attempts was built on a comprehensive and consistent economic model. One purpose of this paper is to fill that void.

The paper is organized as follows. In section 1 the basics of the model is exposed. Section 2 develops the analytical properties of the model. In section 3 the data used in the calibration of the model is presented. Section 4 presents a convenient formula for estimates of income growth and presents the results. And section 5 concludes.

1. The model

There are two production sectors, Sector 1, interpreted as "agriculture", and Sector 2, interpreted as "industry and services" or the "urban sector".

1.1 Production

Sector 1 produces basic food as well as intermediates (say wool) demanded by Sector 2. Output in Sector 1 is produced by a Cobb-Douglas technology using labour, land, and intermediate goods delivered from Sector 2 as inputs. Sector 2 produces more refined consumption goods, including luxury goods, as well as intermediates demanded by Sector 1. Output in Sector 2 is also produced by a Cobb-Douglas technology, but here only labour and intermediate goods delivered from Sector 1 are inputs. What we call "output" of a sector is really net output of the sector, i.e. the output remaining after subtracting input of raw materials produced by the sector itself. We ignore capital, i.e. all intermediate goods are non-durable. The public sector as well as foreign trade are likewise ignored.

With Q_{1t} and Q_{2t} denoting output of sector 1 and sector 2 goods, respectively, in year t , the aggregate production functions are thus given by

$$Q_{1t} = A_{1t} X_{2t}^\alpha L_{1t}^\beta Z^{1-\alpha-\beta}, \quad 0 < \alpha < \alpha + \beta < 1, \quad (1.1)$$

and

$$Q_{2t} = A_{2t} X_{1t}^\gamma L_{2t}^{1-\gamma}, \quad 0 < \gamma < 1, \quad (1.2)$$

where X_{it} is input of intermediates *from* sector i , L_{it} is input of labor (standardized man-years) *in* sector i , Z is land (a constant) and A_{it} is total factor productivity in sector i , $i=1, 2$ and $t = 0, 1, 2, \dots, T$. Let the time unit be one year.

With $p_t \equiv p_{2t} / p_{1t}$ being the relative price of sector 2 goods in terms of sector 1 goods, value added in the two sectors are

$$\begin{aligned} Q_{1t} - p_t X_{2t} &= w_{1t} L_{1t} + \hat{r}_t Z, \\ p_t Q_{2t} - X_{1t} &= w_{2t} L_{2t}, \end{aligned} \quad (1.3)$$

respectively, where \hat{r}_t is land rent and w_{1t} and w_{2t} are the agrarian and urban wage rates, respectively.

The data seems to indicate a persistent “urban premium” or “skill premium” ($w_{2t} / w_{1t} > 1$).² Even if the premium reflects a higher skill level in urban production, this skill need not reflect costly craft’s apprenticeship and similar. The background may be that fast and costless learning by doing and learning by watching is an appendage to being an urban citizen. Should we then not expect the urban premium to be eliminated through migration? No, strong barriers to mobility of labour were prevalent. Hence, we consider L_{1t} and L_{2t} as essentially state variables, at any time determined by previous history.³ In the longer run, however, the pulling force from a higher wage level in urban areas tends to partly erode the barriers to migration. But at the same time total factor productivity may rise faster in urban than in agrarian technology whereby an urban premium tends to be maintained.

Economically motivated migration in the longer run from agrarian areas to towns and formation of new towns are thus consistent with the model. But other explanations of the empirically observed growth in L_{2t} / L_{1t} in pre-industrial Great Britain (along with a rising total

² Clark (2010) Table 1.

³ This means that from an overall production point of view, at any given point in time, subject to the proviso that there is “full employment”, the only “choice variables” are X_{1t} and X_{2t} .

population at least since the early 16th century) may also be possible and consistent with the model. At least theoretically, it is for instance conceivable that the higher urban standard of living enabled by the urban premium might lead to higher fertility and/or lower mortality in urban areas, thus resulting in growth in L_{2t} / L_{1t} even in the absence of migration. Empirically, however, the disease burden seems to have been larger in cities, so that this particular alternative explanation does not seem plausible. Nevertheless we underline the susceptibility to alternative explanations, because the point of the simple bookkeeping to be carried out below is that, whatever the explanation, the actual evolution of L_{2t} / L_{1t} is tantamount to a rising general wage level and, under reasonable parameter values, a rising per capita income.

Neither in the agrarian nor in the urban sector should surplus labor (idle labor, disguised unemployment) be ruled out. There are many indications that even in times of considerable surplus labor, employed workers' wages do not and did not drop to zero. To accommodate this feature we assume the real wage w_{it} in sector i is determined as the maximum of the competitive wage level w_{it}^c and the wage corresponding to "subsistence minimum" which we denote \bar{w}_i . So

$$w_{it} = \max(w_{it}^c, \bar{w}_i), \quad i = 1, 2. \quad (1.4)$$

From now on we omit the explicit dating of the variables unless needed for clarity. We assume that market forces tend to come through sooner or later so as to allow us to describe prices, regional wages, and allocation of goods as being essentially competitively determined. Firms choose inputs with the aim of maximizing profits, taking total factor productivity and wages and prices as given. Profit maximization in the agrarian sector thus implies

$$\begin{aligned} \partial Q_1 / \partial X_2 &= \alpha A_1 X_2^{\alpha-1} L_1^\beta Z^{1-\alpha-\beta} = \alpha Q_1 / X_2 = p, \\ \partial Q_1 / \partial L_1 &= \beta A_1 X_2^\alpha L_1^{\beta-1} Z^{1-\alpha-\beta} = \beta Q_1 / L_1 = w_1, \\ \partial Q_1 / \partial Z &= (1-\alpha-\beta) A_1 X_2^\alpha L_1^\beta Z^{-\alpha-\beta} = (1-\alpha-\beta) Q_1 / Z = \hat{r}. \end{aligned} \quad (1.5)$$

And profit maximization in the urban sector implies

$$\begin{aligned} p \partial Q_2 / \partial X_1 &= p \gamma A_2 X_1^{\gamma-1} L_2^{1-\gamma} = p \gamma Q_2 / X_1 = 1, \\ p \partial Q_2 / \partial L_2 &= p(1-\gamma) A_2 X_1^\gamma L_2^{-\gamma} = p(1-\gamma) Q_2 / L_2 = w_2. \end{aligned} \quad (1.6)$$

So far these equations describe input demands at firm level for given output and factor prices.⁴ On the other hand, with quasi-competitive forces operating within sectors and free mobility of goods across sectors, the equations can be interpreted as describing the equilibrium factor and output prices and the traded quantities of intermediates, X_1 and X_2 , that at a given

⁴ This includes that whatever the maximum in (1.4), the firm is a price taker vis-à-vis labor.

point in time are consistent with total factor productivities, A_1 and A_2 , the given amount of land and the observed amounts of agrarian and urban labour, respectively. For simplicity as well as historical reasons we consider land as not traded but inherited from father to son.

National income as seen from the production side and the income side, respectively, is

$$\begin{aligned} Y &= Q_1 - pX_2 + pQ_2 - X_1 = (1-\alpha)Q_1 + (1-\gamma)pQ_2 \\ &= w_1L_1 + w_2L_2 + \hat{r}Z, \end{aligned} \quad (1.7)$$

where the second equality is due to (1.5) and (1.6) while the last comes from (1.3) and indicates national income as the sum of labour income and land rent. Output in each sector is used partly as raw material in the other sector and partly for consumption:

$$\begin{aligned} Q_1 &= X_1 + C_1, \\ Q_2 &= X_2 + C_2. \end{aligned} \quad (1.8)$$

Substitution of (1.8) into the first part of (1.7) gives

$$Y = C_1 + pC_2, \quad (1.9)$$

which reflects that in our closed economy without capital, all income is consumed.

1.2 Households

The key element of the model is that annual consumption of agrarian goods by a household with annual income y follows the rule

$$c_1 = \begin{cases} y & \text{if } y \leq b & (b > 0), \\ b + m(y-b) & \text{if } y > b & (0 \leq m < 1), \end{cases} \quad (1.10)$$

where b is necessary basic food (subsistence minimum), a constant, and m is a constant marginal propensity to consume agrarian goods out of "residual income", $y - b$, when positive. This behavior reflects Engel's law claiming that the expenditure on basic food falls as a share of income when income rises (above b). Given the budget constraint $c_1 + pc_2 = y$, by (1.10) follows that spending on urban goods will be

$$pc_2 = \begin{cases} 0 & \text{if } y \leq b, \\ (1-m)(y-b) & \text{if } y > b, \end{cases} \quad (1.11)$$

where $1 - m$ is the marginal propensity to spend on urban goods when $y > b$.⁵

⁵ How the consumption function (1.10) can be derived from household preferences is shown in Appendix A.

The model thus assumes that households have the same consumption *functions*, (1.10) and (1.11), in spite of household incomes varying across land owners, agrarian workers and urban workers. We consider the labour supply of a household to be inelastic. To save notation, we imagine that a household, whether in the agrarian or urban sector, consists of just one adult. Then we can identify the number of households with the size, N , of population. With L denoting total employment (number of standardised man-years), we have

$$L_1 + L_2 = L. \quad (1.12)$$

The labour participation rate (more precisely the employment rate) L/N is assumed to be the same in agrarian and urban areas. We assume that society can at least “reproduce” itself. As a substantial part of the population receives neither an urban premium nor land rent, this reproducibility essentially requires that

$$w_2 L/N \geq w_1 L/N \geq b, \quad (1.13)$$

which we assume satisfied. This condition ensures that no social class is below subsistence minimum, and so aggregate consumption satisfies

$$\begin{aligned} C_1 &= bN + m(Y - bN), \\ pC_2 &= (1 - m)(Y - bN). \end{aligned} \quad (1.14)$$

2. Analysis

For fixed t , total factor productivities, A_1 and A_2 , and available labour, L_1 and L_2 , are predetermined. The following eleven variables are endogenous: Q_1 , Q_2 , X_1 , X_2 , p , w_1 , w_2 , \hat{r} , Y , C_1 and C_2 .⁶ By appropriate substitutions we may concentrate on the variables p , w_1 , w_2 and C_1 . Let us first consider the equilibrium relations between these variables as seen from the supply side. We choose measurement units such that $Z = 1$.

2.1 The supply side

By the first line in (1.5), $X_2 = \alpha Q_1 / p$. Substituting this into (1.1) and isolating Q_1 yields

$$Q_1 = (\alpha^\alpha A_1 L_1^\beta)^{1/(1-\alpha)} p^{-\alpha/(1-\alpha)}. \quad (2.1)$$

⁶ Strictly speaking, allowing for less than “full employment”, L_i and w_i , $i = 1, 2$, may switch place in this “predetermined-endogenous” dichotomy. This happens when the “subsistence” wage \bar{w}_i in (1.4) is binding. The previous and subsequent equations, as well as our application of them, remain valid also in this case.

The second line in (1.5) gives $w_1 = \beta Q_1 / L_1$. By substituting (2.1) into this, we have

$$w_1 = \beta(\alpha^\alpha A_1)^{1/(1-\alpha)} L_1^{-(1-\alpha-\beta)/(1-\alpha)} p^{-\alpha/(1-\alpha)}. \quad (2.2)$$

The second line of (1.6) gives $w_2 = (1-\gamma)pQ_2 / L_2 = (1-\gamma)pA_2(X_1 / L_2)^\gamma$, where the last equality comes from (1.2). Substituting into this that $X_1 / L_2 = \gamma w_2 / (1-\gamma)$ (from combining the two lines of (1.6)) and isolating w_2 yields

$$w_2 = (1-\gamma)(\gamma^\gamma A_2)^{1/(1-\gamma)} p^{1/(1-\gamma)}. \quad (2.3)$$

The asymmetry exhibited by the appearance of L_1 in (2.2) but neither L_1 nor L_2 in (2.3) is due to the fixed factor, land, in the agrarian sector, resulting in diminishing returns to labor and intermediates in that sector in contrast to constant returns to scale in the urban sector. A higher price p of urban goods reduces use of these goods as input in the agrarian sector, and due to technological complementarity this reduces the marginal product of labor in that sector. This explains the negative association between p and w_1 exhibited in (2.2) for given L_1 . On the other hand, a higher p means higher marginal value products of the inputs in the urban sector which explains the positive association between p and w_2 appearing in (2.3).

In view of (2.2) and (2.3), the relationship between the urban premium and p is given by

$$\frac{w_2}{w_1} = \frac{(1-\gamma)(\gamma^\gamma A_2)^{1/(1-\gamma)} p^{(1-\alpha\gamma)/[(1-\alpha)(1-\gamma)]}}{\beta(\alpha^\alpha A_1)^{1/(1-\alpha)} L_1^{-(1-\alpha-\beta)/(1-\alpha)}}. \quad (2.4)$$

Isolating p in (2.2) and inserting into (2.4) gives

$$\frac{w_2}{w_1} = \frac{(1-\gamma)(\gamma^\gamma A_2)^{1/(1-\gamma)} \left(\beta^{(1-\alpha)/[\alpha(1-\gamma)]} \right) (\alpha^\alpha A_1)^{1/[\alpha(1-\gamma)]}}{L_1^{(1-\alpha-\beta)/[\alpha(1-\gamma)]}} w_1^{-(1-\alpha\gamma)/[\alpha(1-\gamma)]}. \quad (2.5)$$

For fixed A_1 , A_2 and L_1 , we thus have a negative relationship between the two endogenous variables, the urban premium w_2 / w_1 and the agrarian wage rate w_1 . The intuition comes from, first, the technological complementarity mentioned above, implying that a higher w_1 goes hand in hand with a lower price of urban goods stimulating the use of these in the agrarian sector and raising marginal productivity of labor in that sector for given L_1 . Second, the lower price of urban goods amounts to a lower marginal value product of labor in the urban sector, hence lower w_2 and, a fortiori, lower w_2 / w_1 . It is also noteworthy that for given w_1 and L_1 , the urban premium is an increasing function of total factor productivity in *both* sectors.

The net supply of agrarian goods is linked to w_2 / w_1 and w_1 the following way:

$$C_1 = Q_1 - X_1 = \frac{w_1}{\beta} L_1 - \frac{\gamma}{1-\gamma} w_2 L_2 = \left[\frac{1}{\beta} (1-\ell) - \frac{\gamma w_2 / w_1}{1-\gamma} \ell \right] w_1 L, \quad (2.6)$$

where we have used (1.8), (1.5) and (1.6), and where ℓ is the urban employment share, $\ell \equiv L_2 / L$.

2.2 The final demand side

We now consider C_1 from the point of view of final demand. In view of (1.14), (1.7), (1.5) and (1.6), we have

$$\begin{aligned} C_1 &= (1-m)bN + mY = (1-m)bN + m[(1-\alpha)Q_1 + (1-\gamma)pQ_2] \\ &= (1-m)bN + m \left[(1-\alpha) \frac{w_1 L_1}{\beta} + w_2 L_2 \right] = (1-m)bN + m \left[\frac{1-\alpha}{\beta} (1-\ell) + w_2 \ell / w_1 \right] w_1 L. \end{aligned} \quad (2.7)$$

By market clearing, the right-hand sides of (2.6) and (2.7) are equal. Hence, after rearranging,

$$w_1 \frac{L}{N} = \frac{\beta(1-m)b}{1 - (1-\alpha)m - \left[1 - (1-\alpha)m + \beta \left(m + \frac{\gamma}{1-\gamma} \right) w_2 / w_1 \right] \ell}. \quad (2.8)$$

This is our second equation linking the two endogenous variables, the urban premium w_2 / w_1 and the agrarian wage rate w_1 , in equilibrium. For given participation rate L / N and given urban employment share ℓ , this equation shows that from a demand side point of view there is a positive relationship between w_2 / w_1 and w_1 in equilibrium.

2.3 General equilibrium

Let us now combine supply and demand. For fixed A_1 , A_2 and L_1 , the solid downward-sloping curve in Figure 1 represents the supply side relationship (2.5). And for L / N and ℓ given, the solid upward-sloping curve represents the demand side relationship (2.8). Given the general inequalities stated above for the parameters, the two curves will always cross, and do so only at one point. This point, with coordinates $(w_1^*, w_2^* / w_1^*)$, is denoted E in the figure and represents the general equilibrium for the period considered.

The figure also shows the position of the relationship (2.5) for a larger L_1 , but unchanged total factor productivities A_1 and A_2 , and the position of the relationship (2.8) for a larger ℓ , but unchanged L / N . The corresponding new equilibrium point is denoted E' in the figure and has $w_1 = w_1^*$ which may be smaller or larger than w_1^* (the figure exhibits the latter case). Considering, in addition, a larger A_1 or a larger A_2 (or both), the final new equilibrium point

E'' will necessarily have higher value of w_1 than the point E and higher value of w_2 / w_1 than the point E' . A position as indicated by the point E'' in the figure is for instance conceivable.

Over the pre-industrial era from the sixteenth to the late eighteenth century the agricultural labour force has been rising in absolute terms, but declining in relative terms, i.e. ℓ rising, while the urban premium w_2 / w_1 has been roughly constant (Broadberry et als 2015, Table 9.10, Clark 2010, Table 1). This evolution corresponds to a shift from E to precisely a point like E'' in the figure. Given our model, behind this evolution must be sufficient rises in total factor productivities so as to support not only a rising population in the agrarian sector with diminishing returns to labour but also rising real wages w_1 and w_2 .

Figure 1 about here

It remains to check the parameter conditions needed for this “story” to be internally consistent. Since the story relies on the aggregate consumption function (1.14), it requires the subsistence condition (1.13) satisfied. In turn, in view of (2.8), (1.13) holds if and only if

$$\frac{\beta(1-m)}{1-(1-\alpha)m - \left[1-(1-\alpha)m + \beta\left(m + \frac{\gamma}{1-\gamma}\right)w_2/w_1\right]\ell} \geq 1. \quad (2.9)$$

Other than the endogenous urban premium w_2 / w_1 , this inequality involves only parameters and one pre-determined, but time-dependent variable, the urban employment share ℓ . We assume the inequality is satisfied throughout the pre-industrial era even in case of no urban premium, i.e. $w_2 / w_1 = 1$. This assumption is equivalent to requiring that

$$\frac{(1-\beta)(1-m) + \alpha m}{1-(1-\alpha)m + \beta\left(m + \frac{\gamma}{1-\gamma}\right)} \leq \ell < \frac{1-(1-\alpha)m}{1-(1-\alpha)m + \beta\left(m + \frac{\gamma}{1-\gamma}\right)} \quad (2.10)$$

holds for all t in the considered time span. In our calibrations below we shall ensure this parameter condition holds throughout.

The denominator of the two fractions in (2.10) is the same and is automatically positive by the general inequalities stated above for the parameters. What the inequalities in (2.10) rule out is both a “too small” and a “too large” ℓ . This could make the denominator in (2.9) either negative or nil and would then indicate the economy were in a “below-subsistence” regime, which is inconsistent with the assumed aggregate consumption function (1.14), given that b is

subsistence minimum. Indeed, the double inequality in (2.10) is needed because both sectors require labor as well as input produced in the *other* sector. If one of the inequalities were violated, the economy would not be productive enough to feed its population.

2.4 Analytical results

By per capita “basic” wage income we mean $w_1 L / N$ (as if the agrarian wage rate were the general wage rate). Until further notice we assume a constant urban premium $w_2 / w_1 \geq 1$. Then:

Result 1. By equation (2.8) above follows that a rising fraction of the labor force being employed in the urban sector reveals a rising per capita “basic” wage income $w_1 L / N$. Moreover, if the labor participation rate is constant or only rising “modestly”, a rising agrarian wage rate w_u is revealed.

The economy-wide per capita labour income is

$$\frac{w_1 L_1 + w_2 L_2}{N} = \frac{L_1 + (w_2 / w_1) L_2}{L} \frac{w_1 L}{N} = \left(1 + \left(\frac{w_2}{w_1} - 1 \right) \ell \right) \frac{w_1 L}{N}. \quad (2.11)$$

Result 2. By equation (2.11) and Result 1 follows that a rising fraction of the labor force being employed in the urban sector reveals a rising per capita labour income. Indeed, when $w_2 / w_1 > 1$, a rising ℓ reveals that per capita labour income is rising *faster* than $w_1 L / N$.

Conclusions regarding growth in *total* income (i.e. including land rent) per capita and total income per unit of labor (“labour productivity”) depend in a less clearcut way on parameter values and are therefore postponed to section 4.

3. Data

We need to assign values to the following parameters and variables on the basis of empirical information:

The elasticity of output in the agricultural sector with regard to intermediate goods from the urban sector (industry and services), α .

The elasticity of output with regard to labour in the agricultural sector, β .

The elasticity of a output in the urban sector with regard to intermediate goods from the agricultural sector, γ .

The marginal propensity to consume food, m .

The ratio of urban (industry and service) employment to total employment, L_2 / L .

The labour participation rate, L/N , interpreted as the number of standardized man-years per adult person.

The urban premium, that is the urban to agrarian wage ratio, seems to be stable over time (Clark 2010, Table 1), w_2/w_1 .

Rough estimates of the mentioned elasticities are based on the input-output table for 1688, constructed by Dodgson (2013) on the basis of Gregory King's social tables from the 1690s. In table 1 we have aggregated the original 17-sector table to a three-sector table.

Table 1. A three-sector input-output table for 1688. £millions. England.

	Agriculture	Industry	Services	Consumption	Investment	Exports	Total
Agriculture	4.45	14.14	1.0	6.48	0.25	0.19	26.51
Industry	0	17.56	0.84	30.94	3.04	2.72	55.1
Services	1.51	4.56	0	9.61	0.1	0	15.8
Imports	0.16	2.14	0				2.3
Indirect taxes	0.05	1.48	0				1.53
Value added	20.34	15.59	13.96				49.5
Total	26.52	55.47	15.8	47.03	3.39	2.91	

Source: Based on Dodgson (2013).

Aggregating services and industry to 'urban sector' and transforming the table into an input-net output table, we arrive at 0.08 for α and 0.37 for γ . We make a slight rounding and use $\alpha = 0.1$ and $\gamma = 1/3$. The upward adjustment of the intermediates parameter in agriculture, α , is motivated by the suspicious total absence of industrial inputs in the agricultural sector and the downward adjustment of the intermediates parameter in urban production, γ , is motivated by our presumption that this gives a plausible average value for the whole period 1500-1759.

Share-cropping contracts often stipulate a half and a half divide and we use $\beta = 0.5$ which also satisfies the double inequality (2.10).

The results are sensitive to the magnitude of m , the marginal propensity to consume food. An overestimated value, in combination with fixed elasticity parameters, will inflate the revealed income growth estimates. As our baseline case we have chosen the value $m = 0.05$, which is in the lower end of a likely interval, to get a conservative estimate of income growth. It might seem too low a figure compared to estimates in underdeveloped economies. However these contemporary estimates include food in a much broader sense than discussed in this paper, that is processed goods from the alimentary industry. It is worth considering that such a low value of m does not support the view that the British economy was at the brink of a Malthusian subsistence trap.

Labour force participation rates are not a well researched area, but there is an emerging consensus that number of working days increased in the Early Modern period up to the Industrial Revolution. The magnitude of that increase is, however, still a matter of debate. An increase in the number of constant-hours working days will result in a difference between growth of real income per capita and growth of the real wage per unit of labour. Using the various estimates (see Broadberry et al., Table 6) we have settled for an increase in constant hours working days from 230 in 1522 to 290 in 1801. There is very little precise data about hours worked per workday. However, daylight was the limit and hours varied seasonally. Hence, the annual average number of hours per workday is assumed constant over the whole period.

The determination of the urban wage premium, w_2/w_1 , takes its point of departure in the ratio between skilled and unskilled wages, which has been fairly constant over long stretches of time, at around 1.5 (Clark 2010, Table 1). However, urban workers were not exclusively skilled so we have opted for a somewhat lower ratio, at $w_2/w_1 = 1.25$.

A variety of sources are available for the determination of the occupational distribution of the labour force. Robust census data, however, arrive comparatively late, not until the 19th century. Before that poll tax returns, muster rolls and parish registers have been used as well as the so called social tables edited by Gregory King (1688), Joseph Massie (1759) and Patrick Colquhoun (1801/03).

Female labour force participation and occupational distribution are more difficult to determine than male occupational distribution. There is a consensus that a smaller fraction of the female labour force was active in agriculture than the corresponding fraction for men and that a higher share of the female labour force was found in services.

The first robust female labour force distribution data are from the 1851 census. L. Shaw-Taylor and E. A. Wrigley (2015) argue that the comparatively low agricultural share of the female labour force, 16.8 percent, can plausibly be used also for the early 19th century. However further back in history the female share is higher. They suggest a doubling by the early 18th century.

S. N. Broadberry et als (2015), as a contrast, assumes that the low agricultural female labour share prevails throughout the Early Modern period. We are inclined to follow Shaw-Taylor and Wrigley, however.

Table 2 presents our ‘baseline’ as well as alternative estimates of the occupational share of industry and services in the total labour force, men and women, from 1522 to 1817. We use the ‘baseline’ estimate in our calibration. The table indicates a dramatic change in the occupational structure over the three centuries, although the ‘baseline’ estimate and Broadberry et als differ regarding the share of industry and services in 1522 and 1688. That difference has to do with different assumptions regarding the female labour force in agriculture, where we have opted for a higher share of the female labour force in agriculture in 1522 to 1759. The percentage point change in the industry and service share is almost identical in our and the Broadberry et als estimate between 1522 and 1688.

In the calibration we use the data between 1522 and 1759. The reason we do not include the later period is that for Britain the assumption of zero net imports of agricultural goods does not hold in that period. By around 1780 Britain becomes a net importer of grain (Sharp 2010) and there is a significant increase in the imports of colonial goods such as sugar, coffee and tea. Furthermore the drift towards a more industrial occupational structure does not solely depend on changes in domestic income but because of foreign demand for British industrial output. Not controlling for net imports of food would lead to an overestimation of the revealed income growth.

Table 2. Employments in industry and services as a share of total employment, 1522-1817. Percent.

Year	1522	1688	1710	1759	1801	1817
Baseline	40.5	57.6		62.4	68.3	
S-T&W			54.2			67.4
B et als	44.4	61.1		63.2	68.3	

Note and sources: The Baseline estimate row is based on male occupational shares from Broadberry et als (2015), Tables 9.03-9.06. In c.1801 we follow Shaw-Taylor and Wrigley and assume the female share of agricultural occupations in the female labour force is 17 percent while it is 25 and 34 percent in 1759 and 1688 respectively. We assume that the ratio of female agricultural labour to male agricultural labour in 1688 also prevails in 1522. The Shaw-Taylor and Wrigley (S-T&W) row is based on male occupational share in Table 2.2 (2015) and female agricultural share as in Baseline estimate. The Broadberry et als row (B et als) from table 9.01 in (2015). Throughout the whole period women are assumed to constitute 30 per cent of the labour force.

The results reported in the next section are based on the parameter values and empirical data summarized in Table 3.

Table 3. Parameter values used in Baseline case.

Elasticity of output in the agrarian sector with regard to inputs from the industry and service sector	$\alpha = 0.1$
Elasticity of output in the agrarian sector with regard to labour	$\beta = 0.5$
Elasticity of output in the industry and service sector with regard to inputs from the agrarian sector	$\gamma = 1/3$
The marginal propensity to consume agricultural goods	$m = 0.05$
The urban to rural wage premium	$w_2/w_1 = 1.25$
Change in the labour force participation rate, L/N	From 230 days to 290 days of constant hours days per year from 1522 to 1801

4. Results

A convenient formula for the numerical analysis is shown in equation (4.1). Let time 0 represent for instance our initial year 1522 A.C. and let t represent year 1688. Considering per capita “basic” wage income, $w_1 L / N$, in year t relative to that in year 0, we get from (2.8) with

$$\frac{w_{1t} L_t / N_t}{w_{10} L_0 / N_0} = \frac{1 - (1 - \alpha)m - \left[1 - (1 - \alpha)m + \beta \left(m + \frac{\gamma}{1 - \gamma} \right) w_{20} / w_{10} \right] \ell_0}{1 - (1 - \alpha)m - \left[1 - (1 - \alpha)m + \beta \left(m + \frac{\gamma}{1 - \gamma} \right) w_{2t} / w_{1t} \right] \ell_t}. \quad (4.1)$$

This ratio is seen to be independent of the subsistence minimum b .

Until further notice we report numbers based on our ‘baseline’ calibration, including $m = 0.05$ and $w_2 / w_1 = 1.25$ (constant). By (4.1) we find an average (= annual compound) growth rate in the agrarian per capita labour income over the period 1522-1688 of 0.44% per year, in total a doubling, and over the period 1688-1759 of 0.5 per year, in total more than a doubling, see

Table 4. By correcting for a slight average annual rise in the participation rate L/N ,⁷ we find the corresponding numbers for the growth rate of the “basic” real wage per unit of work to be on average 0.35% per year in the first period and 0.43% per year in the second. The robustness checks carried out indicate that the results are somewhat sensitive to the values of the output elasticities with respect to inputs. (TO BE ELABORATED)

Table 4. Summary of results: Compound annual growth. Per cent per year.

Period	1522-1688	1688-1759
Agrarian real per capita labour income	0.44	0.50
Agrarian real wage per unit of work	0.35	0.43
Economy-wide real per capita labour income	0.46	0.52
Economy-wide real wage per unit of work	0.38	0.44
Real income per capita (Y/N)	0.40	0.48
Real income per unit of work (Y/L)	0.32	0.40

Sources: see text.

For the economy-wide per capita labour income $(w_1L_1 + w_2L_2)/N$ in (2.11) we find an average growth rate of 0.46% per year over the period 1522-1688 and 0.52% per year over the period 1688-1759. The corresponding numbers for the growth rate of labour income per unit of work $(w_1L_1 + w_2L_2)/L$ are 0.38% and 0.44% per year, respectively.

Now, consider *total* income per capita (average “standard of living”). In view of (1.7), total income (including land rent) per capita is

$$\frac{Y}{N} = \frac{w_1L_1 + w_2L_2 + \hat{r}Z}{N} = \left(\frac{L_1 + (w_2/w_1)L_2}{L} + \frac{\hat{r}Z}{w_1L_1} \frac{L_1}{L} \right) \frac{w_1L}{N} = \left(\frac{1-\alpha}{\beta} + \left(\frac{w_2}{w_1} - \frac{1-\alpha}{\beta} \right) \ell \right) \frac{w_1L}{N}. \quad (4.2)$$

With our baseline parameter values, the growth rate of Y/N is 0.40% per year over 1522- 1688 and 0.48% per year over 1688-1759. The corresponding numbers for the growth rate of total income per unit of work Y/L are 0.32% and 0.40% per year, respectively. This suggests that economy-wide *labor productivity* has almost doubled over the last period.

⁷ This is based on an estimated increase in days worked from 230 days per year in 1522 to 290 days per year in 1801 (see Clark 2010, Table 1). There is very little precise data about hours worked per workday but daylight was the limit and hours varied seasonally. Hence, the annual average number of hours per workday is assumed constant over the whole period.

Are these notable growth rates driven by a high marginal propensity to consume food, m ? No, if we change our value for m from 0.05 to 0.00, the growth rates are only reduced moderately, cf. Table 5. The growth rate of for instance Y / N becomes 0.36% per year over 1522-1688 and 0.40% per year over the period 1688-1759.

Or are our growth rates perhaps driven by a high urban premium? No, if we replace our value for w_2 / w_1 from 1.25 to 1.00, the growth rates are again only reduced moderately. The growth rate of for instance Y / N becomes 0.31% per year over 1522- 1688 and 0.34% per year over the period 1688-1759.

Table 5. Sensitivity analysis and comparisons. Compound annual growth, per cent.

Period	1522-1688	1688-1759
Baseline Y/N	0.40	0.48
Baseline Y/N except $w_2/w_1 = 1$	0.31	0.34
Baseline Y/N except $m = 0$	0.36	0.40
Broadberry et als GDP per capita	0.04 (1470-1650)	0.44 (1650-1770)

Sources: See text and Broadberry et als 2015, Tables 5.07 and 11.01. Broadberry results cover slightly different periods indicated in the cells.

Real income growth has two sources, increased productivity and more hours per worker, that is an increase in L/N . The increase in hours worked is usually associated with the so-called *industrious revolution* and is measured by the difference between real per capita income growth and growth in real income per unit of work. Table 5 indicates that the *industrious revolution effect* amounts to about one fifth of the real per capita income growth in both periods.

The comparison with the new estimates by Broadberry et als is both reassuring and puzzling. The periods covered differ slightly but that does not affect the general trends. It is not the fact that the Broadberry period starts in 1470 that drags down the results for the first period relative to our results. The break in the GDP per capita growth comes in the mid 17th century in the Broadberry analysis (see Broadberry et als 2015 ,Table 5.7) , while our results suggest a healthy growth also in the first period. What is puzzling from our perspective is that the drift towards an increase in the share of industry and service employment is as strong, in percentage points, in the Broadberry data as in ours in the 1522-1688 period, see our Table 2 above, even if the levels differ slightly. Furthermore the Broadberry data (Table 5.7 in particular) for the 1500-1650 period indicates a small fall in GDP per capita with a sharp fall in per capita agrarian output, about 20 per cent, and a 10 percent *increase* in industrial per capita output. Further inquiry into the nature of growth in this period is called for, given the contradictory results.

5. Conclusion/summary

A first calibration of the model is carried out, taking into account a limited number of empirical observations, primarily changes in the occupational structure, which are uncontroversial or not seriously contended. The 'baseline' calibration, based on $m = 0.05$ and on a slight annual rise in the participation rate, suggests an annual compound growth rate in the average real wage (per unit of labour) over the period 1522- 1688 of 0.38 % per year and over the period 1688 - 1759 of 0.44 %. The robustness checks indicate that the results are somewhat sensitive to parameter changes. However, even with downward adjustments of $w_2/w_1 = 1$ and $m = 0$ will the hypothesis of stationary real wages be rejected.

Our results indicate slow but significant growth in real wages and in total income per head and therefore lend support to the conclusion reached by Broadberry et al (2015) of a positive GDP per head growth in pre-industrial Britain well before the Industrial Revolution. We argue, more controversially, that the period from the early 16th century to the mid 17th century experienced robust growth, not found by Broadberry et al, despite the fact that we agree on the magnitude and direction of changes in the occupational structure.

Our results have wider repercussions because it challenges a longstanding Malthusian tradition in the interpretation of European and British economic history, known as 'l'histoire immobile', the stationary history. Today this view is associated with Greg Clark (2007) but it goes back to economic historians like M. M. Postan (1966) and E. Le Roy Ladurie (1974). More specifically our results challenge the prevailing view that states real wages were stationary well into the Industrial Revolution (Phelps Brown and Hopkins 1955, 1956; Clark 2010). Can we explain why our results differ from received opinion?

There are four major reasons why our results differ. First we refer to real wages and real income of the entire labour force while the much cited and used British real wage series are based on nominal day wages of a small fraction of the labour force. Furthermore the size and potentially the representativeness of that fraction can vary over time. We do not rely on these series at all in our estimates. Second, traditional historical national accounts arrive at GDP per capita estimates by controlling for population growth. However, population levels before the mid 16th century are still not fully researched. As a rule population level estimates at early dates are reached by backward interpolation from some robust benchmark estimate. An assumed too low population level at some initial year will generate a too high population growth which will affect income per capita growth numbers negatively. In our per capita estimates we do not use or need population level numbers. The third reason is that the often used real wage deflator with constant commodity composition or fixed weights might exaggerate inflation because it does not adjust for changes in the consumption pattern over time as a response to changes in relative prices. Finally,

the real wage deflator exaggerates inflation because it does not as a rule control for quality improvements in goods and services. Fixed expenditure weights and neglect of quality improvements are major problems in modern real national income accounting since these factors generate spurious inflation and there is no reason to believe it was not a problem in the past (Nordhaus 1999). The revealed income growth approach does not use real wage or GDP deflators. The real wage change is instead detected by the behavioral response of the economic agents: when real wages increase then there is a change in the consumption pattern and subsequently in the occupational distribution. We derive the change in real wages from that occupational shift.

Among the limitations of the simple model used here, we believe that not including capital (for instance cattle in agriculture and buildings in the urban sector) is the most serious one. With capital as a necessary production factor, part of the rise in incomes is absorbed by saving and investment. Although this may imply an upward bias in our growth estimates, in view of the modest quantitative role of capital in the pre-industrial period, we believe the bias is modest. Another limitation lies probably in the assumed constancy in both the agrarian and the urban sector of the output elasticities with respect to the different inputs. These problems are considered more closely in ongoing research.

6. Appendix

A. Behavior of a single household

Let each household inelastically supply a units of labor (hours) per year to non-homework activities. In a given year aggregate labour supply is $\bar{L} = aN$ (all three variables may be time dependent), where N is the number of households. To save notation, we identify this number with the size of population. The focus in the text is on the employment level $L \leq \bar{L}$, which is what matters for production.

Let y_i denote annual income of household no. i . Since, by assumption, there are no tradeable assets in which to save, the household faces a simple static optimization problem,

$$\max u(c_{1i}, c_{2i}) = \begin{cases} c_{1i} - b & \text{if } c_{1i} \leq b, \\ (c_{1i} - b)^m c_{2i}^{1-m} & \text{if } c_{1i} > b, \end{cases}$$

$$\text{s.t. } c_{1i} + pc_{2i} \leq y_i.$$

Case 1: $y_i \leq b$. As in this case $c_{1i} \leq y_i \leq b$, the household maximises $c_{1i} - b$ s.t. $c_{1i} \leq y_i$, which has the solution $c_{1i} = y_i$.

Case 2: $y_{li} > b$. In this case, if $c_{li} \leq b$ or $c_{2i} = 0$, then $u(c_{li}, c_{2i}) \leq 0$, whereas $u(c_{li}, c_{2i}) > 0$ is obtained by letting $c_{li} > b$ and $c_{2i} > 0$, which belongs to the budget set when $y_{li} > b$. To find the optimal composition of $\tilde{c}_{li} \equiv c_{li} - b$ and c_{2i} , it is convenient to take a logarithmic transformation of u and solve

$$\begin{aligned} \max \tilde{u}(\tilde{c}_{li}, c_{2i}) &= m \ln \tilde{c}_{li} + (1-m) \ln c_{2i}, \\ \text{s.t. } \tilde{c}_{li} + p c_{2i} &= y_i - b. \end{aligned}$$

Inserting the constraint and differentiating by \tilde{c}_{li} gives the first-order condition

$$\frac{d\tilde{u}}{d\tilde{c}_{li}} = m \frac{1}{c_{li} - b} + (1-m) \frac{p}{y_{li} - c_{li}} = 0.$$

In combination with the budget constraint this gives

$$c_{li} = b + m(y_i - b) \text{ and } c_{2i} = (1-m)(y_i - b) / p.$$

Given the reproducibility assumption (1.13), for any distribution of household incomes in the economy, aggregation over i gives the aggregate consumption functions (1.14).

B. The relative price

Given the more or less constant urban premium, w_2 / w_1 , the rising w_1 reflected by the rising urban share of labor implies a rising urban wage rate, w_2 . How does the relative price p evolve? The answer lies in (2.3). Solving for p gives

$$p = \frac{w_2^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma} A_2}.$$

We see that the relative price p will be rising or falling depending on whether TFP in the urban sector grows slower or faster than $w_2^{1-\gamma}$. Indeed, letting g_x denote the growth rate of an arbitrary positively-valued variable x , we have

$$g_p \begin{cases} \geq 0 \\ < 0 \end{cases} \text{ for } g_{A_2} \begin{cases} \leq \\ > \end{cases} (1-\gamma)g_{w_2},$$

respectively, where $g_{w_2} = g_{w_1}$ when w_2 / w_1 constant. DATA ABOUT g_p ?

C. Migration

As suggested in Section 1.1, a part of the dynamic “story” behind these numbers could be migration from agrarian to urban regions, induced by the urban premium. If M_{lt} denotes the net inflow per time unit from agrarian to urban regions, and the biological population growth rate in the agrarian area is n_{lt} , the time derivative of agrarian population N_{lt} is

$$dN_{1t} / dt = n_{1t}N_{1t} - M_{1t}.$$

The migration process may take the following form

$$M_t = \begin{cases} \lambda_1(w_{2t} / w_{1t} - 1)N_{1t} & \text{if } w_{2t} / w_{1t} \geq 1 \quad (\lambda_1 > 0), \\ \lambda_2(w_{2t} / w_{1t} - 1)N_{2t} & \text{if } w_{2t} / w_{1t} < 1 \quad (\lambda_2 > 0), \end{cases} \quad (4.3)$$

where N_{2t} is urban population and λ_1 and λ_2 are adjustment speed parameters, possibly time-dependent although not visible here (only the upper case is relevant in the present context). At the same time, total factor productivities may be rising sufficiently to maintain a more or less constant urban premium $w_2 / w_1 > 1$, along with a slow but persistent rise in w_{1t} , cf. (2.5), as well as a rising urban employment share ℓ .

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Figure 1 next page.

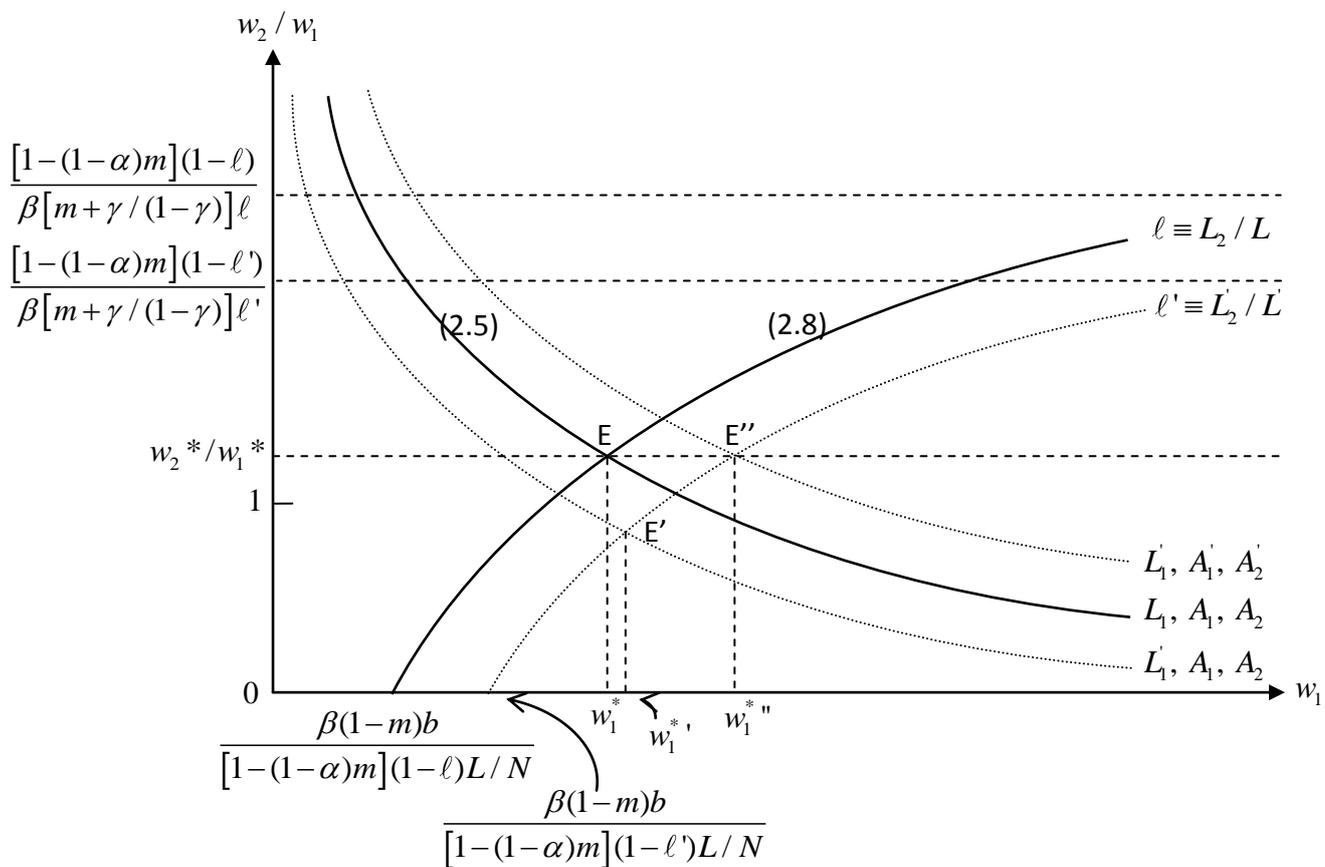


Figure 1. General equilibrium shifts rightward along with a rising $\ell \equiv L_2/L$ (from E to E'' when urban premium is constant). Note: $\ell' > \ell$, $L'_1 > L_1$, $A'_1 > A_1$ or $A'_2 > A_2$ or both.