

# Dynamic Selection and the New Gains from Trade with Heterogeneous Firms\*

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## Abstract

This paper develops an open economy growth model in which firm heterogeneity increases the gains from trade. Technology spillovers from incumbent firms to entrants cause the productivity threshold for firm survival to grow over time as competition becomes tougher. By raising the profits of exporters, trade increases the entry rate and generates a dynamic selection effect that leads to higher growth. The paper shows that the gains from trade can be decomposed into: static gains that equal the total gains from trade in an economy without technology spillovers, and; dynamic gains that are strictly positive. Since trade raises growth through selection, not scale effects, the positive growth effect of trade vanishes when firms are homogeneous. Thus, firm heterogeneity creates a new source of dynamic gains from trade. Calibrating the model to the U.S. economy implies that dynamic selection approximately triples the gains from trade.

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# 1 Introduction

Does firm level heterogeneity matter for the aggregate gains from trade? In models with cross-firm productivity differences that follow in the tradition of Melitz (2003) trade liberalization causes the least productive firms to exit and leads to a reallocation of resources towards more productive firms. By increasing average firm productivity, this selection effect generates a new source of gains from trade that is absent from both neoclassical trade theory and Helpman and Krugman (1985) models of intra-industry trade with homogeneous firms.

However, recent work by Atkeson and Burstein (2010) and Arkolakis, Costinot and Rodríguez-Clare (2012) (henceforth ACRC) finds that in general equilibrium the existence of firm heterogeneity makes little difference to the aggregate gains from trade because the welfare gains from selection are offset by changes in entry and innovation. In particular, ACRC show that in both Krugman (1980) and a version of Melitz (2003) with a Pareto productivity distribution, the gains from trade are given by the same function of two observables: the import penetration ratio and the elasticity of imports with respect to variable trade costs (the trade elasticity). These findings lead the authors to conclude that firm heterogeneity is not important for estimating the gains from trade.

This paper argues that if we move beyond static steady state economies<sup>1</sup> and incorporate cross-firm productivity differences into a dynamic growth model, firm heterogeneity leads to a new dynamic source of gains from trade. Suppose that when new firms are created, innovators can learn from incumbent firms. Then selection on firm productivity not only increases the average productivity of existing firms, but causes knowledge spillovers to new entrants. By strengthening the selection effect, trade generates knowledge spillovers and raises the growth rate. The paper shows that these dynamic gains from trade are not offset by countervailing general equilibrium effects and increase the aggregate gains relative to those found in either an equivalent dynamic model with homogeneous firms or the static steady state economies considered by ACRC.

To formalize this argument I extend Melitz (2003) by allowing for productivity spillovers from incumbent firms to new entrants. In particular, I assume that the distribution from which entrants draw their productivity is endogenous to the productivity distribution of existing firms.<sup>2</sup> Consequently, selection on firm

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<sup>1</sup>In this paper I use the term “static steady state economies” to refer to both static models and papers such as Melitz (2003) and Atkeson and Burstein (2010) that incorporate dynamics, but do not allow for growth and, consequently, have a steady state that is constant over time.

<sup>2</sup>Luttmer (2007) uses an equivalent assumption to develop a closed economy growth model with heterogeneous firms. Unlike

productivity leads to spillovers that raise the average productivity of entrants and this is sufficient to generate endogenous growth. On the balanced growth path, entry causes the exit cut-off below which firms do not produce to shift upwards over time and, as the productivity distribution evolves, average firm productivity grows at a constant rate.

In an open economy, only high productivity firms export and the resulting reallocation of resources across firms generates a selection effect that raises the exit cut-off as in Melitz (2003). However, in addition to the usual static selection effect that raises the level of the exit cut-off, there is also a dynamic selection effect whereby trade raises the growth rate of the exit cut-off and, consequently, of average productivity and consumption per capita. The key to understanding why trade generates the dynamic selection effect is the free entry condition. For a given exit cut-off, trade increases average profits. In a static steady state economy this induces a rise in the exit cut-off, which lowers the probability of successful entry and ensures the free entry condition is satisfied. However, with productivity spillovers a higher exit cut-off does not affect the probability of successful entry. Therefore, in order to satisfy free entry, the exit cut-off growth rate must rise, increasing the rate of creative destruction and decreasing firms' expected lifespan.<sup>3</sup>

The dynamic selection effect is a new channel through which trade can lead to welfare gains when firms are heterogeneous. However, given the findings of Atkeson and Burstein (2010) and ACRC it is natural to ask whether the gains from dynamic selection are offset by other general equilibrium effects. To rule out this possibility, the paper shows that the welfare effects of trade can be decomposed into two terms. First, a static term that is identical to the gains from trade in Melitz (2003) (assuming a Pareto productivity distribution) and can be expressed as the same function of the import penetration ratio and the trade elasticity that gives the gains from trade in ACRC. Second, a dynamic term in which the growth rate of per capita consumption is the only endogenous variable. The dynamic term is strictly increasing in the growth rate.

When firms are homogeneous there is no dynamic selection and trade does not affect the growth rate. Therefore, it follows from the welfare decomposition that, in the dynamic growth model developed in this paper, the gains from trade are greater when firms are heterogeneous. In addition, the gains from trade in this paper are strictly higher than in the Melitz (2003) model with a Pareto productivity distribution and, conditional on the observed import penetration ratio and trade elasticity, the gains from trade are strictly

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Luttmer (2007) I abstract from post-entry productivity dynamics and focus instead on the implications of trade.

<sup>3</sup>Atkeson and Burstein (2010) also stress the importance of the free entry condition in determining the general equilibrium gains from trade. However, while in a static steady state economy the free entry condition limits the gains from static selection, in this paper free entry is critical for dynamic selection.

higher than in the class of static steady state economies analyzed by ACRC.<sup>4</sup>

To assess the magnitude of the gains from dynamic selection I calibrate the model to the U.S. economy. With a dynamic economy more information is required to calibrate the model than is used by ACRC, but the welfare effects of trade can still be calculated in terms of a small number of observables and parameters. In addition to the import penetration ratio and the trade elasticity, the calibration uses the rate at which new firms are created, the population growth rate, the intertemporal elasticity of substitution, the discount rate and the elasticity of substitution between goods. The baseline calibration implies that due to trade U.S. growth is 10 percent higher than it would be under autarky. More importantly, by increasing growth dynamic selection triples the gains from trade relative to an economy with homogeneous firms. The finding that dynamic selection is quantitatively important for estimating the gains from trade, and at least doubles the gains from trade compared to static steady state economies, is extremely robust to plausible parameter variations.

In addition to contributing to the debate over the gains from trade, this paper is closely related to the endogenous growth literature. It develops a tractable growth model with heterogeneous firms in which the productivity distribution evolves over time. Of particular note is that the equilibrium growth rate does not depend on population size – there are no scale effects. Thus, productivity spillovers based growth implies neither the counterfactual prediction that larger economies grow faster (Jones 1995a) nor the semi-endogenous growth prediction that population growth is the only source of long-run growth (Jones 1995b). Scale effects are absent from this paper because both the productivity distribution and the mass of varieties produced are endogenous. In equilibrium a larger population leads to a proportional increase in the mass of varieties produced (unlike in quality ladders growth models), but since the creation of new goods does not reduce the cost of future innovations (unlike in expanding varieties growth models) the growth rate is unaffected.

The lack of scale effects is related to the logic of Young (1998) who develops an endogenous growth model without scale effects by merging the quality ladders and expanding varieties frameworks while only allowing knowledge spillovers along the vertical dimension. However, in Young (1998) trade does not affect growth because there is no selection on productivity and trade is simply equivalent to an increase in scale. By contrast, in this paper trade raises growth because the fixed cost of exporting strengthens the selection

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<sup>4</sup>The important distinction to note here is that the predicted import penetration ratio and trade elasticity in this paper depend on the same underlying parameters as in the Pareto productivity version of Melitz (2003), but differ from the predictions made by some of the models considered by ACRC.

effect. Similarly, the paper shows that growth is increasing in the fixed cost of production because a higher fixed cost leads to tougher selection and generates productivity spillovers.

By arguing that trade affects growth because of selection not scale effects, this paper stands in stark contrast to the previous open economy endogenous growth literature which finds that the implications of trade for growth in a single sector economy depend on scale effects and international knowledge spillovers.<sup>5</sup> For example, Baldwin and Robert-Nicoud (2008) develop an expanding varieties version of Melitz (2003), but do not allow for productivity spillovers. Consequently, the firm productivity distribution is constant on the balanced growth path, the relationship between trade and growth is mediated through a scale effect and trade only increases growth if international knowledge spillovers are sufficiently strong. In this paper the effects of trade do not depend on whether productivity spillovers are national or international in scope.

The remainder of the paper is organized as follows. Section 2 introduces the model, while Section 3 solves for the balanced growth path equilibrium and discusses the effects of trade on growth. In Section 4 I characterize household welfare on the balanced growth path and then Section 5 calibrates the model and quantifies the gains from trade. Finally, Section 6 demonstrates the robustness of the paper's results to two extensions of the baseline model, before Section 7 concludes.

## 2 Productivity spillovers model

Consider a world comprised of  $J + 1$  symmetric economies. When  $J = 0$  there is a single autarkic economy, while for  $J > 0$  we have an open economy model. Time  $t$  is continuous and the preferences and technological possibilities of each economy are as follows.

### 2.1 Preferences

Each economy consists of a set of identical households with dynastic preferences and discount rate  $\rho$ . The population  $L_t$  at time  $t$  grows at rate  $n \geq 0$  where  $n$  is assumed to be constant and exogenously fixed. Each household has constant intertemporal elasticity of substitution preferences and seeks to maximize:

$$U = \int_{t=0}^{\infty} e^{-\rho t} e^{nt} \frac{c_t^{1-\frac{1}{\gamma}} - 1}{1 - \frac{1}{\gamma}} dt, \quad (1)$$

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<sup>5</sup>Rivera-Batiz and Romer (1991) and Grossman and Helpman (1991) consider the implications of trade in first generation endogenous growth models. More recent work incorporates heterogeneous firms into expanding varieties (Baldwin and Robert-Nicoud 2008) and quality ladders (Haruyama and Zhao 2008) models.

where  $c_t$  denotes consumption per capita and  $\gamma > 0$  is the intertemporal elasticity of substitution. The numeraire is chosen so that the price of the consumption good is unity. Households can lend or borrow at interest rate  $r_t$  and  $a_t$  denotes assets per capita. Consequently, the household's budget constraint expressed in per capita terms is:

$$\dot{a}_t = w_t + r_t a_t - c_t - n a_t, \quad (2)$$

where  $w_t$  denotes the wage. Note that households do not face any uncertainty.

Under these assumptions and a no Ponzi game condition the household's utility maximization problem is standard<sup>6</sup> and solving gives the Euler equation:

$$\frac{\dot{c}_t}{c_t} = \gamma(r_t - \rho), \quad (3)$$

together with the transversality condition:

$$\lim_{t \rightarrow \infty} \left\{ a_t \exp \left[ - \int_0^t (r_s - n) ds \right] \right\} = 0. \quad (4)$$

## 2.2 Production and trade

Output is produced by monopolistically competitive firms each of which produces a differentiated good. Labor is the only factor of production and all workers are homogeneous and supply one unit of labor per period. There is heterogeneity across firms in labor productivity  $\theta$ . A firm with productivity  $\theta$  at time  $t$  has marginal cost of production  $\frac{w_t}{\theta}$  and must also pay a fixed cost  $f$  per period in order to produce. The fixed cost is denominated in units of labor. The firm does not face an investment decision and firm productivity remains constant over time. The final consumption good is produced under perfect competition as a constant elasticity of substitution aggregate of all available goods with elasticity of substitution  $\sigma > 1$  and is non-tradable.<sup>7</sup>

Differentiated good producers can sell their output both at home and abroad. However, as in Melitz (2003) firms that select into exporting face both fixed and variable costs of trade. Exporters incur a fixed cost  $f_x$ , denominated in units of domestic labor, per export market per period, while variable trade costs take

<sup>6</sup>See, for example, Chapter 2 of Barro and Sala-i-Martin (2004).

<sup>7</sup>This is equivalent to assuming households have constant elasticity of substitution preferences over available goods.

the iceberg form. In order to deliver one unit of its product to a foreign market a firm must ship  $\tau$  units. I assume  $\tau^{\sigma-1} f_x > f$  which is a necessary and sufficient condition to ensure that in equilibrium not all firms export. Since I consider a symmetric equilibrium, all parameters and endogenous variables are constant across countries.

For a given productivity distribution and a fixed time  $t$  the structure of output production and demand in this economy is equivalent to that in Melitz (2003) and characterizing firms' static profit maximization decisions is straightforward. Firms that produce face isoelastic demand and set factory gate prices as a constant mark-up over marginal costs. Firms only choose to produce if their total variable profits from domestic and foreign markets are sufficient to cover their fixed production costs and firms only export to a given market if their variable profits in that market are sufficient to cover the fixed export cost. Variable profits in each market are strictly increasing in productivity and since  $\tau^{\sigma-1} f_x > f$  the productivity above which firms export exceeds the minimum productivity for entering the domestic market. In particular, there is a productivity cut-off  $\theta_t^*$  such that firms choose to produce at time  $t$  if and only if their productivity is at least  $\theta_t^*$ . This exit cut-off is given by:

$$\theta_t^* = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma-1} \left( \frac{f w_t^\sigma}{c_t L_t} \right)^{\frac{1}{\sigma-1}}. \quad (5)$$

In addition, there is a threshold  $\tilde{\theta}_t > \theta_t^*$  such that firms choose to export at time  $t$  if and only if their productivity is at least  $\tilde{\theta}_t$ . The export threshold is:

$$\tilde{\theta}_t = \left( \frac{f_x}{f} \right)^{\frac{1}{\sigma-1}} \tau \theta_t^*. \quad (6)$$

Firms can lend or borrow at interest rate  $r_t$  and the market value  $V_t(\theta)$  of a firm with productivity  $\theta$  is given by the present discounted value of future profits:

$$V_t(\theta) = \int_t^\infty \pi_\tau(\theta) \exp\left(-\int_t^\tau r_s ds\right) d\tau, \quad (7)$$

where  $\pi_t$  denotes the profit flow net of fixed costs at time  $t$  from both domestic and export sales and  $\pi_t(\theta) = 0$  if the firm does not produce.

In what follows, it will often be convenient to use the change of variables  $\phi_t \equiv \frac{\theta}{\theta_t^*}$ , where  $\phi_t$  is firm productivity relative to the exit cut-off. I will refer to  $\phi_t$  as a firm's relative productivity. Let  $W_t(\phi_t)$  be

the value of a firm with relative productivity  $\phi_t$  at time  $t$ . Obviously, only firms with  $\phi_t \geq 1$  will choose to produce and only firms with  $\phi_t \geq \tilde{\phi} \equiv \left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}} \tau$  will choose to export. For these firms prices, employment and profits in the domestic and export markets are given by:

$$p_t^d(\phi_t) = \frac{\sigma}{\sigma-1} \frac{w_t}{\phi_t \theta_t^*}, \quad p_t^x(\phi_t) = \tau p_t^d(\phi_t),$$

$$l^d(\phi_t) = f [(\sigma-1)\phi_t^{\sigma-1} + 1], \quad l^x(\phi_t) = f\tau^{1-\sigma} [(\sigma-1)\phi_t^{\sigma-1} + \tilde{\phi}^{\sigma-1}], \quad (8)$$

$$\pi_t^d(\phi_t) = fw_t [\phi_t^{\sigma-1} - 1], \quad \pi_t^x(\phi_t) = f\tau^{1-\sigma} w_t [\phi_t^{\sigma-1} - \tilde{\phi}^{\sigma-1}], \quad (9)$$

where I have used  $d$  and  $x$  superscripts to denote the domestic and export markets, respectively. Observe that employment is a stationary function of relative productivity and that, conditional on relative productivity  $\phi_t$ , domestic profits are proportional to the fixed cost of production. Since there are  $J$  export markets, total firm employment is given by  $l(\phi_t) = l^d(\phi_t) + Jl^x(\phi_t)$  and total firm profits are  $\pi_t(\phi_t) = \pi_t^d(\phi_t) + J\pi_t^x(\phi_t)$ .

### 2.3 Entry

To invent a new good, entrants must hire workers to perform research and development (R&D). The R&D technology is such that employing  $R_t f_e$  workers to undertake R&D generates a flow  $R_t$  of innovations. Each innovation creates both an idea for a new good (product innovation) together with a production technology for transforming the idea into output (process innovation). The efficiency of the technology determines the new firm's productivity. I assume that innovators learn from the techniques and processes used by existing firms and, consequently, that their productivity depends on the distribution of  $\theta$  at the time of innovation. In particular, I assume that the productivity distribution of new entrants is a scaled version of the productivity distribution of existing producers where the scaling parameter  $\lambda$  measures the strength of spillovers from incumbents to new entrants. Thus, if  $G_t(\theta)$  is the cumulative distribution function for productivity of firms that produce at time  $t$ , then new entrants receive a productivity draw from a distribution with cumulative distribution function  $\tilde{G}_t$  defined by  $\tilde{G}_t(\theta) = G_t(\theta/\lambda)$  where  $\lambda \in (0, 1]$ .<sup>8</sup>

<sup>8</sup>Given symmetry across countries, the productivity distribution is the same in all countries and it is irrelevant whether spillovers are national or international in scope. Consequently, in this model the effects of trade on growth and welfare do not depend on the extent of international knowledge spillovers.



There is free entry into R&D, implying that in equilibrium the expected cost of innovating equals the expected value of creating a new firm:

$$f_e w_t = \int_{\theta} V_t(\theta) d\tilde{G}_t(\theta). \quad (10)$$

Entry is financed by a competitive and costless financial intermediation sector which owns the firms and, thereby, enables investors to pool the risk faced by innovators. Consequently, each household effectively owns a balanced portfolio of all firms and R&D projects.<sup>9</sup>

In Melitz (2003) and most of the subsequent literature on firm heterogeneity new entrants receive a productivity draw from an exogenously fixed distribution and there is no long run growth. By contrast, the model developed in this paper introduces productivity spillovers from existing producers to innovators and endogenizes the productivity distribution of new entrants. These spillovers are sufficient to generate steady state growth in this economy. Previous work on trade and growth with heterogeneous firms has either allowed knowledge spillovers to reduce the fixed costs of entry, production and export generating an expanding varieties growth model (Baldwin and Robert-Nicoud 2008), or included product quality spillovers from incumbents to entrants in a quality ladders framework with a constant number of sectors (Haruyama and Zhao 2008). By allowing for both the creation of new varieties and the evolution of the productivity distribution, the R&D technology in this paper draws on elements from both the expanding variety and quality ladder growth paradigms.

The particular structure of knowledge spillovers assumed above could be rationalized by assuming that each innovator is matched with a randomly chosen existing firm and imperfectly adopts its process technology (but not its product). Luttmer (2007) also assumes that the productivity distribution of new entrants is a scaled version of the productivity distribution of existing firms, while Alvarez, Buera and Lucas (2008, 2011) study the evolution of the production cost distribution when there is no entry, but producers can learn from their lower cost peers.

How does the relative productivity distribution evolve over time? Let  $H_t$  and  $\tilde{H}_t$  be the cumulative distribution functions of relative productivity  $\phi$  for existing firms and new entrants, respectively. Given the structure of productivity spillovers we must have  $\tilde{H}_t(\phi) = H_t(\phi/\lambda)$ . Note also that since  $\theta_t^*$  is the exit cut-off,  $\tilde{H}_t(\lambda) = H_t(1) = 0 \forall t$ . To characterize the intertemporal evolution of  $H_t$  I will first formulate a

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<sup>9</sup>Again, since countries are symmetric it is irrelevant whether asset markets operate at the national or global level.

law of motion for  $H_t(\phi)$  between  $t$  and  $t + \Delta$  and then take the continuous time limit. Let  $M_t$  be the mass of producers in the economy at time  $t$  and assume the exit cut-off is strictly increasing over time.<sup>10</sup> Then the mass of firms with relative productivity no greater than  $\phi$  at time  $t + \Delta$  is:

$$M_{t+\Delta}H_{t+\Delta}(\phi) = M_t \left[ H_t \left( \frac{\theta_{t+\Delta}^*}{\theta_t^*} \phi \right) - H_t \left( \frac{\theta_{t+\Delta}^*}{\theta_t^*} \right) \right] + \Delta R_t \left[ H_t \left( \frac{\phi}{\lambda} \right) - H_t \left( \frac{1}{\lambda} \right) \right]. \quad (11)$$

Since  $\phi_t \leq \frac{\theta_{t+\Delta}^*}{\theta_t^*} \phi \Leftrightarrow \phi_{t+\Delta} \leq \phi$  the first term on the right hand side is the mass of time  $t$  producers that have relative productivity less than  $\phi$ , but greater than one at time  $t + \Delta$ .  $M_t H_t \left( \frac{\theta_{t+\Delta}^*}{\theta_t^*} \right)$  is the mass of time  $t$  firms that exit between  $t$  and  $t + \Delta$  because their productivity falls below the exit cut-off. The second term on the right hand side gives the mass of entrants between  $t$  and  $t + \Delta$  whose relative productivity falls between one and  $\phi$ .

Letting  $\phi \rightarrow \infty$  in (11) implies:

$$M_{t+\Delta} = M_t \left[ 1 - H_t \left( \frac{\theta_{t+\Delta}^*}{\theta_t^*} \right) \right] + \Delta R_t \left[ 1 - H_t \left( \frac{1}{\lambda} \right) \right], \quad (12)$$

and taking the limit as  $\Delta \rightarrow 0$  gives:<sup>11</sup>

$$\frac{\dot{M}_t}{M_t} = -H_t'(1) \frac{\dot{\theta}_t^*}{\theta_t^*} + \left[ 1 - H_t \left( \frac{1}{\lambda} \right) \right] \frac{R_t}{M_t}. \quad (13)$$

Now using (12) to substitute for  $M_{t+\Delta}$  in (11), rearranging and taking the limit as  $\Delta \rightarrow 0$  we obtain the following law of motion for  $H_t(\phi)$ :

$$\begin{aligned} \dot{H}_t(\phi) = & \left\{ \phi H_t'(\phi) - H_t'(1) [1 - H_t(\phi)] \right\} \frac{\dot{\theta}_t^*}{\theta_t^*} \\ & + \left\{ H_t \left( \frac{\phi}{\lambda} \right) - H_t \left( \frac{1}{\lambda} \right) - H_t(\phi) \left[ 1 - H_t \left( \frac{1}{\lambda} \right) \right] \right\} \frac{R_t}{M_t}. \end{aligned} \quad (14)$$

<sup>10</sup>When solving the model I will restrict attention to balanced growth paths on which  $\theta_t^*$  is strictly increasing in  $t$  meaning firms will never choose to temporarily cease production. In an economy with a declining exit cut-off, equilibrium would depend on whether exit from production was temporary or irreversible. I abstract from these issues in this paper.

<sup>11</sup>In obtaining both this expression and equation (14) I assume that  $\theta_t^*$  is differentiable with respect to  $t$  and  $H_t(\phi)$  is differentiable with respect to  $\phi$ . Both these conditions will hold on the balanced growth path considered below.

## 2.4 Equilibrium

In addition to consumer and producer optimization, equilibrium requires the labor and asset markets to clear in each economy in all periods. Labor market clearing requires:

$$L_t = M_t \int_{\phi} l(\phi) dH_t(\phi) + R_t f_e, \quad (15)$$

while asset market clearing implies that aggregate household assets equal the combined worth of all firms:

$$a_t L_t = M_t \int_{\phi} W_t(\phi) dH_t(\phi). \quad (16)$$

Finally, as an initial condition I assume that at time zero there exists in each economy a mass  $\hat{M}_0$  of potential producers whose productivity  $\theta$  has cumulative distribution function  $\hat{G}(\theta)$  where  $\hat{M}_0$  and  $\hat{G}$  are such that in equilibrium some potential producers will choose to exit at time zero. We are now ready to define the equilibrium.

An equilibrium of the world economy is defined by time paths for  $t \in [0, \infty)$  of consumption per capita  $c_t$ , assets per capita  $a_t$ , wages  $w_t$ , the interest rate  $r_t$ , the exit cut-off  $\theta_t^*$ , the export threshold  $\tilde{\theta}_t$ , firm values  $W_t(\phi)$ , the mass of producers per economy  $M_t$ , the flow of innovations per economy  $R_t$  and the relative productivity distribution  $H_t(\phi)$  such that: (i) households choose  $c_t$  to maximize utility subject to the budget constraint (2) implying the Euler equation (3) and the transversality condition (4); (ii) producers maximize profits implying the exit cut-off satisfies (5), the export threshold satisfies (6) and firm value is given by (7); (iii) free entry into R&D implies (10); (iv) the exit cut-off is strictly increasing over time and the evolution of  $M_t$  and  $H_t(\phi)$  are governed by (13) and (14); (v) labor and asset market clearing imply (15) and (16), respectively, and; (vi) at time zero there are  $\hat{M}_0$  potential producers in each economy with productivity distribution  $\hat{G}(\theta)$ .

## 3 Balanced growth path

I will solve for a balanced growth path equilibrium on which  $c_t, a_t, w_t, \theta_t^*, \tilde{\theta}_t, W_t(\phi), M_t$  and  $R_t$  grow at constant rates,  $r_t$  is constant and the distribution of relative productivity  $\phi$  is stationary, meaning  $\dot{H}_t(\phi) = 0 \forall t, \phi$ . First, observe that if  $\phi$  has a Pareto distribution at time  $t$  then  $\dot{H}_t(\phi) = 0$ . Thus, given the structure

of productivity spillovers in this economy the Pareto distribution is self-replicating.<sup>12</sup> Therefore, to obtain a balanced growth path with no transition dynamics I assume that the initial productivity distribution is Pareto.

**Assumption 1.** *The productivity distribution of potential producers at time zero is Pareto:  $\hat{G}(\theta) = 1 - \theta^{-k}$  for  $\theta \geq 1$  with  $k > \max\{1, \sigma - 1\}$ .*

Note that since some potential producers will choose to exit immediately there is no loss of generality in assuming the distribution has a scale parameter equal to one. Given Assumption 1, equation (14) implies that the distribution of relative productivity is Pareto with scale parameter one and shape parameter  $k$  for all  $t$ . Thus,  $H(\phi) = 1 - \phi^{-k} \forall t, \phi \geq 1$ . In addition, it immediately follows that the distribution of productivity  $\theta$  is Pareto, the employment, revenue and profit distributions converge asymptotically to Pareto distributions in the right tail and the employment distribution is stationary.<sup>13</sup>

Now, let  $\frac{\dot{c}_t}{c_t} = q$  be the growth rate of consumption per capita. Then the household budget constraint (2) implies that assets per capita and wages grow at the same rate as consumption per capita:

$$\frac{\dot{a}_t}{a_t} = \frac{\dot{w}_t}{w_t} = \frac{\dot{c}_t}{c_t} = q,$$

while the Euler equation (3) gives:

$$q = \gamma(r - \rho), \tag{17}$$

and the transversality condition (4) requires:

$$r > n + q \Leftrightarrow \frac{1 - \gamma}{\gamma}q + \rho > n, \tag{18}$$

where the equivalence follows from (17). This inequality is also sufficient to ensure that household utility is

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<sup>12</sup>More generally, solving (14) with  $\dot{H}_t(\phi) = 0$  implies:

$$H(\phi) = 1 - \phi^{-k} + \phi^{-k} \int_1^\phi F(s) s^{k-1} ds,$$

where  $k > 0$  and  $F(\phi)$  satisfies:

$$F'(\phi) \phi \frac{\dot{\theta}_t^*}{\theta_t^*} = F(\phi) \frac{\dot{M}_t}{M_t} - F\left(\frac{\phi}{\lambda}\right) \frac{R_t}{M_t},$$

with  $F(1) = 0$ . Obviously,  $F(\phi) = 0$  solves this equation and implies  $\phi$  has a Pareto distribution, but it is not known whether other solutions exist.

<sup>13</sup>It is well known that the upper tails of the distributions of firm sales and employment are well approximated by Pareto distributions (Luttmer 2007). Axtell (2001) argues that Pareto distributions are a good fit for the entire distributions in the U.S.

well-defined. Since all output is consumed in each period, output per capita is always equal to consumption per capita.

Next, differentiating equation (5) which defines the exit cut-off implies:

$$q = g + \frac{n}{\sigma - 1}. \quad (19)$$

where  $g = \frac{\dot{\theta}_t^*}{\theta_t^*}$  is the rate of growth of the exit cut-off and, therefore, the rate at which the productivity distribution shifts to the right. From equation (6) the export threshold is proportional to the exit cut-off meaning that  $g$  is also the growth rate of the export threshold and since each firm's productivity  $\theta$  remains constant over time  $g$  is the rate at which a firm's relative productivity  $\phi_t$  decreases. Equation (19) makes clear that there are two sources of consumption per capita growth in this economy. First, population growth which I will discuss below and second, productivity growth resulting from a dynamic selection effect. As the exit cut-off increases, the least productive firms are forced to exit and this leads to a reallocation of resources to more productive firms raising average labor productivity and output per capita. This effect is the dynamic analogue of the static selection effect identified in Melitz (2003) that results from changes in the level of the exit cut-off. Understanding what determines the size of the dynamic selection effect is the central concern of this paper.

Now, we can use (9) and  $\phi_t = \frac{\theta}{\theta_t^*}$  to substitute the profit function into (7) and solve for the firm value function obtaining:

$$\begin{aligned} V_t(\theta) &= W_t(\phi_t), \\ &= fw_t \left[ \frac{\phi_t^{\sigma-1}}{(\sigma-1)g + r - q} \left( 1 + I[\phi_t \geq \tilde{\phi}] \frac{Jf_x}{f} \tilde{\phi}^{1-\sigma} \right) \right. \\ &\quad \left. + \frac{(\sigma-1)g}{r-q} \frac{\phi_t^{\frac{q-r}{g}}}{(\sigma-1)g + r - q} \left( 1 + I[\phi_t \geq \tilde{\phi}] \frac{Jf_x}{f} \tilde{\phi}^{\frac{q-r}{g}} \right) \right. \\ &\quad \left. - \frac{1}{r-q} \left( 1 + I[\phi_t \geq \tilde{\phi}] \frac{Jf_x}{f} \right) \right]. \end{aligned} \quad (20)$$

where  $I[\phi_t \geq \tilde{\phi}]$  is an indicator function that takes value one if a firm's relative productivity is greater than or equal to the export threshold and zero otherwise. Thus, the value of a firm with relative productivity  $\phi$  grows at rate  $q$ . Substituting (20) into the free entry condition (10), using  $\tilde{G}_t(\theta) = \tilde{H}(\phi) = H\left(\frac{\phi}{\lambda}\right)$  and

integrating to obtain the expected value of an innovation implies:

$$q = kg + r - \frac{\sigma - 1}{k + 1 - \sigma} \frac{\lambda^k}{f_e} \left( f + J f_x \tilde{\phi}^{-k} \right). \quad (21)$$

Since (17), (19) and (21) are three equations for the three unknowns  $q$ ,  $g$  and  $r$  they can be solved to obtain:

$$\begin{aligned} q &= \frac{\gamma}{1 + \gamma(k - 1)} \left[ \frac{\sigma - 1}{k + 1 - \sigma} \frac{\lambda^k f}{f_e} \left( 1 + J \tau^{-k} \left( \frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \right) + \frac{kn}{\sigma - 1} - \rho \right], \\ g &= \frac{\gamma}{1 + \gamma(k - 1)} \left[ \frac{\sigma - 1}{k + 1 - \sigma} \frac{\lambda^k f}{f_e} \left( 1 + J \tau^{-k} \left( \frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \right) - \frac{1 - \gamma}{\gamma} \frac{n}{\sigma - 1} - \rho \right], \\ r &= \frac{\gamma}{1 + \gamma(k - 1)} \left[ \frac{1}{\gamma} \frac{\sigma - 1}{k + 1 - \sigma} \frac{\lambda^k f}{f_e} \left( 1 + J \tau^{-k} \left( \frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \right) + \frac{1}{\gamma} \frac{kn}{\sigma - 1} + (k - 1)\rho \right]. \end{aligned} \quad (22)$$

When characterizing the evolution of the relative productivity distribution in Section 2.3 I assumed  $g > 0$ . To ensure this condition is satisfied and the transversality condition (18) holds I impose the following parameter restrictions.

**Assumption 2.** *The parameters of the world economy satisfy:*

$$\begin{aligned} \frac{\sigma - 1}{k + 1 - \sigma} \frac{\lambda^k f}{f_e} &> \rho + \frac{1 - \gamma}{\gamma} \frac{n}{\sigma - 1}, \\ \frac{(1 - \gamma)(\sigma - 1)}{k + 1 - \sigma} \frac{\lambda^k f}{f_e} \left[ 1 + J \tau^{-k} \left( \frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \right] &> \gamma k(n - \rho) - (1 - \gamma) \frac{k + 1 - \sigma}{\sigma - 1} n. \end{aligned}$$

The first expression ensures that  $g > 0$  holds for any  $J \geq 0$ , while the second expression is implied by the transversality condition.

All that now remains to prove the existence of a balanced growth path is to show that  $M_t$  and  $R_t$  grow at constant rates. Using the employment function (8), the labor market clearing condition (15) simplifies to:

$$L_t = \frac{k\sigma + 1 - \sigma}{k + 1 - \sigma} M_t f \left[ 1 + J \tau^{-k} \left( \frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \right] + R_t f_e. \quad (23)$$

Consequently, on a balanced growth path we must have that the mass of producers and the flow of innovations grow at the same rate as population:

$$\frac{\dot{L}_t}{L_t} = \frac{\dot{M}_t}{M_t} = \frac{\dot{R}_t}{R_t} = n.$$

Thus, the link between population growth and consumption per capita growth shown in (19) arises because when the population increases the number of varieties produced grows and, since the final good production technology exhibits love of varieties, this raises consumption per capita. This completes the proof that the world economy has a unique balanced growth path. Note that the proof holds for any non-negative value of  $J$  including the closed economy case where  $J = 0$ .

**Proposition 1.** *When Assumptions 1 and 2 hold the world economy has a unique balanced growth path on which consumption per capita grows at rate:*

$$q = \frac{\gamma}{1 + \gamma(k - 1)} \left[ \frac{\sigma - 1}{k + 1 - \sigma} \frac{\lambda^k f}{f_e} \left( 1 + J\tau^{-k} \left( \frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \right) + \frac{kn}{\sigma - 1} - \rho \right].$$

Remembering that Assumption 1 ensures  $k > \max\{1, \sigma - 1\}$ , we immediately obtain a corollary of Proposition 1 characterizing the determinants of the growth rate.

**Corollary 1.** *On the balanced growth path, the growth rate of consumption per capita is strictly increasing in the fixed production cost  $f$ , the strength of productivity spillovers  $\lambda$ , the intertemporal elasticity of substitution  $\gamma$ , the population growth rate  $n$  and the number of trading partners  $J$ , but is strictly decreasing in the R&D labor requirement  $f_e$ , the fixed export cost  $f_x$ , the variable trade cost  $\tau$  and the discount rate  $\rho$ .*

To understand Proposition 1 and Corollary 1 it is useful to start by setting  $J = 0$  and considering a closed economy. Two features of the autarky equilibrium are particularly noteworthy relative to previous endogenous growth models. First, growth is increasing in the fixed production cost<sup>14</sup> and second, growth is independent of population size meaning there are no scale effects. Let us consider each of these findings in turn.

To see why a higher fixed cost of production increases the growth rate, start by observing from (9) that holding  $\phi$  constant profits are proportional to  $f$ . Since on the balanced growth path innovators draw  $\phi$  from a stationary distribution this means that the expected initial profits of a new entrant, relative to the wage, are increasing in  $f$ . However, the free entry condition (10) requires the expected value of innovating,

<sup>14</sup>Luttmer (2007) also finds that the consumption growth rate is increasing in  $\frac{f}{f_e}$  when there are productivity spillovers from incumbents to new entrants.

relative to the wage, to be independent of  $f$ , meaning that higher initial profits for given  $\phi$  must be offset by  $\phi$  declining more quickly over time and the firm's expected lifespan falling. Thus, higher  $f$  leads to an increase in the growth rate  $g$  of the exit cut-off strengthening the dynamic selection effect and raising  $q$ . When new entrants receive a productivity draw from an exogenously fixed distribution and there are no productivity spillovers as in Melitz (2003), an increase in  $f$  still raises profits conditional on  $\phi$ , but it also lowers entrants' expected  $\phi$  by raising the level of the exit cut-off. By contrast, in the growth model considered in this paper variation in the level of the exit cut-off does not change the expected value of R&D because the productivity spillovers are such that entrants draw  $\phi$  from a stationary distribution. Consequently, variation in  $f$  must have implications for growth.

The channel through which increasing the fixed production cost raises growth can be isolated by considering the allocation of resources between production and R&D. Since  $M_t$  grows at rate  $n$ , the exit cut-off  $\theta_t^*$  grows at rate  $g$ ,  $H_t'(1) = k$  and  $H_t(\frac{1}{\lambda}) = \lambda^k$ , equation (13) implies that on a balanced growth path:

$$\frac{R_t}{M_t} = \frac{n + gk}{\lambda^k}, \quad (24)$$

and substituting this expression back into the labor market clearing condition we obtain:

$$M_t = \left[ \frac{k\sigma + 1 - \sigma}{k + 1 - \sigma} f \left( 1 + J\tau^{-k} \left( \frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \right) + (n + gk) \frac{f_e}{\lambda^k} \right]^{-1} L_t. \quad (25)$$

From these two expressions we see that raising  $f$  reduces the mass of goods produced and it is this reduction in competition that leads to higher profits conditional on  $\phi$ . In addition, higher  $f$  raises the flow of innovations relative to the mass of producers and the increased competition from new entrants pushes the exit cut-off up more quickly strengthening the dynamic selection effect and leading to higher growth.

Now let us consider the absence of scale effects. Scale effects are a ubiquitous feature of the first generation of endogenous growth models (Romer 1990; Grossman and Helpman 1991; Aghion and Howitt 1992) where growth depends on the size of the R&D sector which, on a balanced growth path, is proportional to population. However, Jones (1995a) documents that despite continuous growth in both population and the R&D labor force, growth rates in developed countries have been remarkably stable since the second world war.<sup>15</sup> Such concerns prompted Jones (1995b) to pioneer the development of semi-endogenous growth models in which the allocation of resources to R&D remains endogenous, but there are no scale effects

<sup>15</sup>Although, see Kremer (1993) for evidence that scale effects may be present in the very long run.



because diminishing returns to knowledge creation mean that population growth is the only source of long-run growth. Semi-endogenous growth models have in turn been criticized for attributing long-run growth to a purely exogenous factor and understating the role of incentives to perform R&D in driving growth.

To understand why there are no scale effects in this paper observe first that the productivity spillovers from incumbents to new entrants in this model are roughly analogous to the knowledge spillovers found in quality ladders growth models (Grossman and Helpman 1991; Aghion and Howitt 1992) where innovators improve the output quality of incumbents by some fixed proportion. In quality ladders models the number of goods produced is constant and, consequently, the profit flow received by innovators is increasing in population, which generates the scale effect. However, in this model the number of goods is endogenous and grows at the same rate as population. Thus, in larger economies producers face more competitors and the incentive to innovate does not depend on market size. Moreover, unlike in expanding varieties growth models, the creation of new goods does not reduce the cost of R&D for future innovators implying that there are no knowledge spillovers along the horizontal dimension of the model. As equation (24) makes clear, the equilibrium growth rate depends not on the innovation rate which is proportional to population, but on the innovation rate relative to the mass of producers which is scale independent. A related model that features endogenous growth without scale effects is developed by Young (1998) who allows for R&D to raise both the quality and the number of goods produced, but assumes that knowledge spillovers only occur in the vertical dimension of production. However, in Young (1998) there is no selection on productivity, implying that the dynamic selection effect analyzed in this paper is missing and trade does not affect growth because it is equivalent to an increase in scale.

Setting aside the fixed production cost, the relationships between other parameters and the autarky growth rate are unsurprising. Increasing the cost of innovating by raising  $f_e$  must, in equilibrium, lead to an increase in the expected value of innovating and this is achieved through lower growth which increases firms' expected lifespan. Similarly, growth is strictly increasing in the learning parameter  $\lambda$ , which measures the extent of knowledge spillovers, because when knowledge spillovers are stronger new entrants' expected initial relative productivity and profits are higher and to ensure the free entry condition (10) holds this must be offset by faster growth in the exit cut-off. A higher intertemporal elasticity of substitution or a lower discount rate raise growth by making households more willing to invest now and consume later, while, as already mentioned, population growth affects consumption per capita growth through its impact on the growth rate of the mass of producers  $M_t$ .

Now let us return to an open economy setting with  $J > 0$  and analyze how trade integration affects growth. Relative to autarky, trade is equivalent to increasing  $f$  by a factor  $1 + J\tau^{-k} \left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}$  and, consequently, the equilibrium growth rate is higher in the open economy than in autarky. Moreover, either increasing the number of countries  $J$  in the world economy, reducing the variable trade cost  $\tau$  or reducing the fixed export cost  $f_x$  raises growth. The effect of trade on growth operates through the same mechanism as an increase in the fixed production cost. To understand why, start by considering the domestic and export net profit functions given in (9). Conditional on a firm's relative productivity and the wage level, domestic profits are independent of the extent of trade integration, while trade increases the profits of firms whose productivity exceeds the export threshold. Thus, since entrants draw relative productivity from a stationary distribution, trade liberalization increases the expected initial profits of a new entrant relative to the wage. The free entry condition (10) then implies that following trade integration the exit cut-off must grow more quickly in order to offset the increase in entrants' expected initial profits. From (24) and (25) we see that the exit cut-off grows more quickly because trade reduces the mass of domestic producers and increases the ratio of new entrants to existing producers. Thus, trade raises growth through a dynamic selection effect.<sup>16</sup> This dynamic selection effect of trade is analogous to the static selection effect found in heterogeneous firm models without long run growth such as Melitz (2003). In both cases export profits mandate an increase in the exit cut-off to satisfy free entry. However, while static selection generates a one-off increase in the productivity level, when there are productivity spillovers free entry induces growth effects.

Early work on the effects of trade in endogenous growth models found that global integration increases growth via the scale effect, although in some variants of the expanding varieties model it is the extent of international knowledge spillovers, rather than international trade, that matters for growth (Rivera-Batiz and Romer 1991; Grossman and Helpman 1991).<sup>17</sup> More recent papers have shown that if firm heterogeneity is included in standard expanding variety (Baldwin and Robert-Nicoud 2008) and quality ladder (Haruyama and Zhao 2008) models the relationship between trade and growth is still mediated through the scale effect. It is unsurprising then that in models without scale effects such as Young (1998) and the semi-endogenous growth model of Dinopoulos and Segerstrom (1999) the long run growth rate is independent an economy's trade status. However, in contrast to the previous literature, the dynamic selection mechanism identified this

<sup>16</sup>Note that this analysis holds both for comparisons of the open economy with autarky and for the consequences of a partial trade liberalization resulting from an increase in  $J$  or a reduction in either  $\tau$  or  $f_x$ .

<sup>17</sup>A complementary line of research examines how trade integration affects the incentives of asymmetric countries with multiple production sectors to undertake R&D (Grossman and Helpman 1991).

paper establishes a channel through which trade affects growth that does not require the existence of scale effects and can only occur when firms are heterogeneous.

Both the static and dynamic selection effects are new sources of gains from trade that do not exist when firms are homogeneous. However, as pointed out by Atkeson and Burstein (2010) and Arkolakis, Costinot and Rodríguez-Clare (2012), in general equilibrium the welfare gains generated by the static selection effect are offset by lower entry implying that, conditional on the import penetration ratio and the trade cost elasticity of imports, the gains from trade in Melitz (2003) are the same as in the homogeneous firms model of Krugman (1980). Therefore, incorporating Melitz style firm heterogeneity into static trade models does not increase the calibrated gains from trade. Can the same reasoning be applied to the dynamic selection effect? To answer this question we must move beyond simply considering the equilibrium growth rate to solving for the welfare implications of trade. This is the goal of the next section.

## 4 Welfare

Proposition 1 gives the consumption growth rate, but household welfare also depends on the level of consumption. This section solves for the consumption level, analyzes the efficiency properties of the decentralized equilibrium and considers how trade affects welfare.

Substituting  $c_t = c_0 e^{qt}$  into the household welfare function (1) and integrating implies:

$$U = \frac{\gamma}{\gamma - 1} \left[ \frac{\gamma c_0^{\frac{\gamma-1}{\gamma}}}{(1 - \gamma)q + \gamma(\rho - n)} - \frac{1}{\rho - n} \right]. \quad (26)$$

From the household budget constraint (2), the Euler equation (3) and the transversality condition (18) we can write the initial level of consumption per capita  $c_0$  in terms of initial wages and assets as:<sup>18</sup>

$$c_0 = w_0 + \left( \frac{1 - \gamma}{\gamma} q + \rho - n \right) a_0, \quad (27)$$

where  $\frac{1 - \gamma}{\gamma} q + \rho - n$  is the marginal propensity to consume out of wealth, which is positive by the transversality condition.

Now using (20) to substitute for  $W_t(\phi)$  in the asset market clearing condition (16), integrating the right hand side to obtain average firm value and using (21) gives:

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<sup>18</sup>This is a textbook derivation. See, for example, Barro and Sala-i-Martin (2004), pp.93-94.

$$a_t L_t = \frac{f_e}{\lambda^k} w_t M_t, \quad (28)$$

which has the intuitive interpretation that the value of the economy's assets at any given time equals the expected R&D cost of replacing all active firms.

Next, to obtain the initial value of the exit cut-off  $\theta_0^*$  apply Assumption 1, which states that at time zero there are  $\hat{M}_0$  potential producers whose productivity is distributed Pareto with shape parameter  $k$  and scale parameter one. Therefore, it follows that:

$$\theta_0^* = \left( \frac{\hat{M}_0}{M_0} \right)^{\frac{1}{k}}. \quad (29)$$

We can now solve for initial consumption per capita by combining this expression with equations (5), (19), (22), (25), (27) and (28) to give:

$$c_0 = A_1 f^{-\frac{k+1-\sigma}{k(\sigma-1)}} \left[ 1 + J \tau^{-k} \left( \frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \right]^{\frac{1}{k}} \left[ 1 + \frac{\sigma-1}{k\sigma+1-\sigma} \frac{n+gk}{n+gk+\frac{1-\gamma}{\gamma}q+\rho-n} \right]^{-\frac{k\sigma+1-\sigma}{k(\sigma-1)}}, \quad (30)$$

with:

$$A_1 \equiv (\sigma-1) \left( \frac{k}{k+1-\sigma} \right)^{\frac{\sigma}{\sigma-1}} \left( \frac{k+1-\sigma}{k\sigma+1-\sigma} \right)^{\frac{k\sigma+1-\sigma}{k(\sigma-1)}} \hat{M}_0^{\frac{1}{k}} L_0^{\frac{k+1-\sigma}{k(\sigma-1)}} > 0.$$

Remember that Assumption 2 ensures  $g > 0$  and  $\frac{1-\gamma}{\gamma}q + \rho - n > 0$ . Thus, both the numerator and the denominator of the final term in (30) are positive.

Armed with the equilibrium growth rate (22) and the initial consumption level (30) we can now analyze the welfare implications of trade integration. Since there are no transition dynamics, we can compare welfare under different equilibria by considering household welfare on the balanced growth path. Observe that trade affects both growth and the consumption level only through the value of  $T \equiv J \tau^{-k} \left( \frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}}$ .  $T$  measures the extent of trade integration between countries and I show when calibrating the model in Section 5 that  $T$  is a monotonically increasing function of the import penetration ratio.  $T$  is strictly increasing in the number of countries  $J$  in the world economy and the fixed production cost  $f$ , but strictly decreasing in the variable trade cost  $\tau$  and the fixed export cost  $f_x$ .

Trade affects welfare through two channels. First, trade raises the growth rate through the dynamic selection effect. I will refer to the change in welfare caused by trade induced variation in the growth rate as the dynamic gains from trade. From (26) we see that increased growth has a direct positive effect on welfare, but (30) shows that it also affects the level of consumption. The level effect is made up of two components. First, there is the increase in  $n+gk$  which from (24) occurs because trade raises the innovation rate relative to the mass of producers. This requires a reallocation of labor out of production and into R&D which decreases the consumption level. Second, variation in  $q$  changes households' marginal propensity to consume out of wealth  $\frac{1-\gamma}{\gamma}q + \rho - n$ . The sign of this effect depends on the intertemporal elasticity of substitution  $\gamma$ , but it is positive when  $\gamma < 1$ . In general, the net effect of higher growth on the consumption level can be either positive or negative and substituting  $g = q - \frac{n}{\sigma-1}$  into (30) and differentiating with respect to  $q$  shows that higher growth increases  $c_0$  if and only if:

$$n \left( 1 - \frac{1}{k} \frac{1-\gamma}{\gamma} \frac{k+1-\sigma}{\sigma-1} \right) > \rho.$$

However, regardless of the sign of the level effect, substituting for  $c_0$  using (30) and then differentiating (26) with respect to growth shows that the dynamic gains from trade are positive.<sup>19</sup> Thus, the direct positive effect of growth on welfare always outweighs any indirect negative effect resulting from a decline in  $c_0$ .

Second, trade affects welfare by increasing the level of consumption for a constant growth rate. These static gains from trade  $z^s$  are given by the term:

$$z^s = \left[ 1 + J_{\mathcal{T}}^{-k} \left( \frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \right]^{\frac{1}{k}},$$

in (30). The static gains from trade result from the combined effects of increased access to imported goods, a reduction in the number of goods produced domestically and reallocation gains caused by an increase in the level of the exit cut-off. Most importantly, the static gains equal the total gains from trade in the absence of dynamic selection. Thus, in static steady state economies such as the variant of the Melitz (2003) model where entrants draw productivity from a Pareto distribution (Arkolakis, Costinot and Rodríguez-Clare 2012) and a version of the model above where innovators draw productivity from a time invariant Pareto distribution (in this case the exit cut-off is constant on the balanced growth path and trade does not affect the consumption growth rate) the gains from trade equal  $z^s$ . Proposition 2 summarizes the welfare effects

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<sup>19</sup>See the proof of Proposition 2 for details.

of trade. The proposition is proved in Appendix A.

**Proposition 2.** *Trade integration resulting from an increase in the number of trading partners  $J$ , a reduction in the fixed export cost  $f_x$  or a reduction in the variable trade cost  $\tau$  increases welfare through two channels: (i) by raising the growth rate of consumption per capita (dynamic gains), and; (ii) by raising the level of consumption for any given growth rate (static gains). The static gains equal the total gains from trade in a static steady state version of the model.*

Two observations follow immediately from Proposition 2. First, since both the static and dynamic gains from trade are positive, trade is welfare improving. Second, by raising the growth rate, the dynamic selection effect strictly increases the gains from trade relative to static steady state versions of the model. Since firm heterogeneity is a necessary condition for the existence of the dynamic selection effect, this shows that including heterogeneous firms in a dynamic model with productivity spillovers leads to a new source of gains from trade that is not offset by other general equilibrium effects. In contrast to the findings of Atkeson and Burstein (2010) and Arkolakis, Costinot and Rodríguez-Clare (2012), in this paper firm heterogeneity matters for the gains from trade. Quantifying the magnitude of the dynamic gains from trade is the goal of Section 5.

To understand why the higher growth resulting from trade liberalization is welfare improving consider the efficiency properties of the decentralized equilibrium. There are two sources of inefficiencies. First, monopoly pricing by differentiated goods producers, which is unaffected by trade. Second, as entry forces the exit cut-off upwards knowledge spillovers cause the productivity distribution of new entrants to shift rightwards, but innovators cannot appropriate the social value of these spillovers. By equation (24) the exit cut-off growth rate is increasing in  $\frac{R_t}{M_t}$ . Thus, there is a positive externality from R&D investment and the flow of innovations relative to the mass of existing producers is inefficiently low in the decentralized equilibrium. In autarky, a benevolent government can raise welfare by introducing either a R&D subsidy or a tax on fixed production costs since both policies incentivize R&D relative to production and raise  $\frac{R_t}{M_t}$ . Similarly, since the effect of trade on  $\frac{R_t}{M_t}$  is equivalent to an increase in the fixed production cost as discussed in Section 3, trade exploits the productivity spillovers externality to generate dynamic welfare gains.

## 5 Quantifying the gains from trade

Define the gains from trade  $z$  in equivalent variation terms as the proportional increase in the autarky level of consumption required to obtain the open economy welfare level. Thus,  $z$  satisfies  $U(zc_0^A, q^A) = U(c_0, q)$  where  $U$ ,  $q$  and  $c_0$  are defined by (26), (22) and (30), respectively, and  $A$  superscripts denote autarky values.<sup>20</sup> From (26) we have:

$$z = \frac{c_0}{c_0^A} \left[ \frac{(1-\gamma)q^A + \gamma(\rho-n)}{(1-\gamma)q + \gamma(\rho-n)} \right]^{\frac{\gamma}{\gamma-1}}.$$

Observe that if  $q = q^A$  the gains from trade are given by the increase in the initial consumption level, which from (30) equals the static gains from trade  $z^s$ . The dynamic gains from trade  $z^d$  are defined by  $z^d = \frac{z}{z^s}$ .

I will start by calibrating the gains from trade for the U.S. and then perform robustness checks against this baseline. The key to the calibration is showing that  $z$  can be expressed in terms of observable quantities and a small number of commonly used parameters. In particular, it is not necessary to specify values of  $J$ ,  $f$ ,  $f_x$ ,  $f_e$  or  $\lambda$  to obtain  $z$ .<sup>21</sup>

The static gains from trade depend on the import penetration ratio (IPR) and the trade elasticity (TE) in the same manner as in Arkolakis, Costinot and Rodríguez-Clare (2012). To see this first calculate total expenditure on imports in each country (IMP) which is given by:

$$IMP_t = \frac{k\sigma}{k+1-\sigma} M_t w_t f J \tau^{-k} \left( \frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}}. \quad (31)$$

Equation (31) shows that  $k$  equals the trade elasticity (the elasticity of imports with respect to variable trade costs). Anderson and Van Wincoop (2004) conclude based on available estimates that this elasticity is likely to be in the range five to ten. I set  $k = 7.5$  for the baseline calibration. Now divide (31) by total domestic sales  $c_t L_t$  to obtain:

$$z^s = \left( \frac{1}{1-IPR} \right)^{\frac{1}{TE}}.$$

This is identical to the formula for calibrating the gains from trade obtained by Arkolakis, Costinot and

<sup>20</sup>The goal of this section is to compare welfare at observed levels of trade with autarky welfare. However, the same techniques could be used to quantify the welfare effects of variation in the extent of trade integration.

<sup>21</sup>This exercise is similar in spirit to the calibration of the gains from trade in Arkolakis, Costinot and Rodríguez-Clare (2012), although in the dynamic setting considered here more information is required than just the import penetration ratio and the trade elasticity.

Rodríguez-Clare (2012). It also gives the calibrated gains from trade in the model above if there are no productivity differences across firms.<sup>22</sup> The U.S. import penetration ratio for 2000, defined as imports of goods and services divided by gross output, was 0.081.<sup>23</sup>

Next, we can express  $\frac{\lambda^k f}{f_e}$  which enters the expression for the growth rate  $q$  as a function of  $n$ ,  $k$ ,  $\sigma$ ,  $\gamma$ ,  $\rho$ ,  $IPR$  and the rate  $NF$  at which new firms are created relative to the mass of existing firms. Since a fraction  $\lambda^k$  of innovations lead to the creation of new firms we have  $NF = \lambda^k \frac{R_t}{M_t}$  and using (19), (22) and (24) gives:

$$\frac{\lambda^k f}{f_e} = \frac{k+1-\sigma}{\gamma k(\sigma-1)}(1-IPR) \left\{ [1+\gamma(k-1)](NF-n) + \frac{k(1-\gamma)}{\sigma-1}n + \gamma k \rho \right\}.$$

Luttmer (2007) reports that U.S. Small Business Administration data shows an entry rate of 11.6% per annum in 2002. Therefore, I set  $NF = 0.116$ . The last observable needed for the calibration is the population growth rate. According to the World Development Indicators average annual U.S. population growth from 1980-2000 was 1.1%, so I let  $n = 0.011$ .

Finally, there are three parameters to calibrate:  $\sigma$ ,  $\gamma$  and  $\rho$ . To calibrate  $\sigma$  observe that the firm employment distribution converges asymptotically to a power function with index  $\frac{-k}{\sigma-1}$ . Luttmer (2007) shows that for U.S. firms in 2002 the right tail index of the employment distribution equals  $-1.06$ . Therefore, I let the elasticity of substitution  $\sigma = k/1.06 + 1$  implying  $\sigma = 8.1$ . Note that  $k > \max\{1, \sigma - 1\}$  as required by Assumption 1. Helpman, Melitz and Yeaple (2004) use European firm sales data to estimate  $k + 1 - \sigma$  at the industry level, obtaining estimates that mostly lie in the interval between 0.5 and 1 implying  $\sigma \in [k, k + 1/2]$ . In the robustness checks I allow  $\sigma$  to vary over a range that includes this interval. Although controversy exists over the value of the intertemporal elasticity of substitution, estimates typically lie in the range  $(1/4, 1)$ . Following García-Peñalosa and Turnovsky (2005) I let  $\gamma = 1/3$  in the baseline calibration. A low intertemporal elasticity of substitution will tend to reduce the dynamic gains from trade by making consumers less willing to substitute consumption over time. I also follow García-Peñalosa and Turnovsky (2005) in choosing the discount rate and set  $\rho = 0.04$ . Taking into account the uncertainty that exists regarding the appropriate parameter values to use I analyze below the robustness of the calibration

<sup>22</sup>To see this assume all firms have unit productivity and there are no fixed costs of exporting. Then it is straightforward to show that  $q = \frac{n}{\sigma-1}$ , meaning there are no dynamic gains from trade, and that the static gains from trade equal  $(1 + J\tau^{1-\sigma})^{\frac{1}{\sigma-1}} = \left(\frac{1}{1-IPR}\right)^{\frac{1}{\sigma-1}}$ .

<sup>23</sup>Imports of goods and services are from the World Development Indicators (Edition: April 2012) and gross output is from the OECD STAN Database for Structural Analysis (Vol. 2009).



results to variation in each of the parameters, but Table 1 summarizes the data and parameter values used for the baseline calibration. Assumption 2 is satisfied both for the baseline calibration and in all the robustness checks.

Table 2 shows the calibration results. The model predicts that consumption per capita growth is 10.7% higher at observed U.S. trade levels than in a counterfactual autarkic U.S. economy. Due to the dynamic welfare gains resulting from higher growth, the total calibrated gains from trade are 3.2 times higher than the static gains. Thus, dynamic selection has a quantitatively important effect on the gains from trade.

Next, I consider the robustness of these results to variation in the parameters used in the calibration. First, the import penetration ratio. Unsurprisingly, the gains from trade are higher when trade integration is greater (Figure 1). Increasing the import penetration ratio from 0.051 (Japan) to 0.36 (Belgium) raises welfare gains from 2.2% to 19.2%. More importantly, the ratio of the total gains to the static gains, which measures the proportional increase in the gains from trade due to dynamic selection, remains approximately constant as the import penetration ratio varies. Figure 2 plots the growth rate under trade relative to the autarky growth rate on the left hand axis and the total gains from trade relative to the static gains from trade on the right hand axis. The total gains are a little over three times larger than the static gains for all levels of the import penetration ratio between zero and 0.5.

Finally, Figure 3 shows the consequences of allowing the remaining calibration parameters to vary. I plot how the growth increase due to trade and the ratio of the total gains to the static gains depend on each parameter in turn, while holding the other parameters fixed at their baseline values.<sup>24</sup> In all cases the dynamic gains from trade are quantitatively important and the results suggest that dynamic selection at least doubles the gains from trade. For example, either lowering the intertemporal elasticity of substitution or raising the discount rate reduces the dynamic gains from trade because it lowers the value of future consumption growth. However, even if the intertemporal elasticity of substitution is reduced to one quarter, the gains from trade are 2.7 times higher with dynamic selection, while the discount rate must exceed 14% before the total gains from trade are less than double the static gains.

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<sup>24</sup>The sole exception is Figure 3c, where I adjust  $\sigma$  to ensure  $\sigma = k/1.06 + 1$  always holds as the trade elasticity varies.

## 6 Extensions – market structure and R&D technology

This section considers two variants of the baseline model that relax particular assumptions made in Section 2. First, I show that monopolistic competition is not necessary for dynamic selection by developing a version of the model in which there is perfect competition between firms producing a single homogeneous output good. In this extension I also relax the assumption that all economies are symmetric by analyzing a small open economy. Second, I show that the predictions of the baseline model are robust to allowing for decreasing returns to scale in R&D.

### 6.1 Competitive output markets

Firm heterogeneity, productivity spillovers and fixed costs of exporting are necessary for trade to induce dynamic selection, but monopolistic competition is not. To see this point consider the following variant of the model in which output is sold in competitive markets. Assume there is a single homogeneous output good produced by heterogeneous firms that face decreasing returns to scale in production. A firm with productivity  $\theta$  that employs  $l$  workers produces  $\theta l^\beta$  units of output with  $\beta \in (0, 1)$ . In order to produce all firms must also pay a fixed cost  $f$  per period, denominated in units of labor. Consider a small home economy that takes world prices as given and faces segmented export markets. The output good can be freely traded at a price of one or firms can choose to pay a fixed export cost  $f_x$ , denominated in units of domestic labor, in order to access export markets in which the output price is  $p \geq 1$ .<sup>25</sup> Otherwise the model is as described in Section 2 above.

Under these assumptions the model can be solved using similar reasoning to that applied in Section 3 and the basic structure of the home economy's balanced growth path equilibrium is unchanged.<sup>26</sup> There is an exit cut-off  $\theta_t^* = \beta^{-\beta}(1-\beta)^{-(1-\beta)}f^{1-\beta}w_t$  below which firms do not produce and an export threshold  $\tilde{\theta}_t = \left[ \left( p^{\frac{1}{1-\beta}} - 1 \right) \frac{f}{f_x} \right]^{-(1-\beta)} \theta_t^*$  above which firms pay the fixed export cost. However, unlike in the baseline model, firms whose productivity exceeds the export threshold export all their output to take advantage of the higher output price. The entry process is unchanged from the baseline model and firm productivity continues to have a Pareto distribution with shape parameter  $k$ . I assume  $k > \frac{1}{1-\beta}$ . Since there is no love of variety both the exit cut-off and consumption per capita grow at the same rate  $q$ , which is given by:

<sup>25</sup> Autarky is equivalent to setting  $p = 1$  since in this case there is no incentive to trade.

<sup>26</sup> A detailed characterization of the balanced growth path equilibrium is given in Appendix B.

$$q = \frac{\gamma}{1 + \gamma(k-1)} \left[ \frac{1}{k(1-\beta) - 1} \frac{\lambda^k f}{f_e} \left( 1 + \left( p^{\frac{1}{1-\beta}} - 1 \right)^{k(1-\beta)} \left( \frac{f}{f_x} \right)^{k(1-\beta)-1} \right) - \rho \right]. \quad (32)$$

Thus, growth is higher with trade than in autarky and  $q$  is increasing in  $p$  and decreasing in  $f_x$ . In autarky growth is strictly increasing in  $\frac{f}{f_e}$  as in the baseline model. The dynamic selection mechanism that drives these results is the same as that identified in the baseline model.

Solving for the initial consumption level shows that:

$$c_0 \propto f^{\frac{1}{k} - (1-\beta)} \left[ 1 + \left( p^{\frac{1}{1-\beta}} - 1 \right)^{k(1-\beta)} \left( \frac{f}{f_x} \right)^{k(1-\beta)-1} \right]^{\frac{1}{k}} \left[ 1 + \frac{1}{k-1} \frac{n + qk}{n + qk + \frac{1-\gamma}{\gamma} q + \rho - n} \right]^{-\frac{k-1}{k}}. \quad (33)$$

Since household welfare is still given by (26), equations (32) and (33) imply that trade affects welfare through the term  $T = \left( p^{\frac{1}{1-\beta}} - 1 \right)^{k(1-\beta)} \left( \frac{f}{f_x} \right)^{k(1-\beta)-1}$  which measures the extent of trade integration.<sup>27</sup> Moreover, the gains from trade can be decomposed into a static term and a dynamic term which depends on the growth rate  $q$ . Differentiating welfare with respect to  $q$  shows that the dynamic gains from trade are positive implying that, as in the baseline model, dynamic selection generates a new source of gains from trade. Proposition 3 summarizes these results.

**Proposition 3.** *When output markets are competitive and firms that pay a fixed export cost can sell their output for a higher price abroad, the growth rate of consumption per capita is higher under trade than in autarky. The positive growth effect of trade raises the welfare gains from trade relative to a static steady state version of the model.*

Finally, it is worth noting that the economy considered in this section is formally equivalent to an autarky economy with a competitive output market, where firms can increase their productivity by a factor  $p$  by paying a fixed cost  $f_x$ . Thus, trade has the same implications for growth and welfare as introducing a non-convex investment technology. While the focus of this paper is on using the productivity spillovers model to understand the welfare effects of trade, this observation highlights how the framework developed here may

<sup>27</sup> Assuming balanced trade and that only firms which pay the fixed export cost are exporters (i.e. gross and net imports are equal)  $T$  can be written in terms of observables as:

$$1 + T = \left( 1 - PX^{\frac{k(1-\beta)-1}{k}} \right) \frac{1}{1 - IPR},$$

where  $PX$  denotes the share of firms that export.

prove useful in other contexts.

## 6.2 R&D technology

In the baseline model there are constant returns to scale in R&D. Suppose now we generalize the R&D technology by allowing for the possibility of congestion in innovation. Assume that when  $R_t f_e$  workers are employed in R&D the flow of new innovations is  $\Psi(R_t, M_t)$  where  $\Psi$  is homogeneous of degree one, strictly increasing in  $R_t$ , weakly increasing in  $M_t$  and  $\Psi(0, 0) = 0$ .<sup>28</sup>  $\Psi$  can be interpreted as a matching function that gives the number of potential innovators who are successfully matched with firms and (imperfectly) adopt the firms' process technologies. As discussed in Section 2.3 this interpretation also holds in the baseline model, but allowing  $\Psi$  to depend on  $M_t$  introduces decreasing returns to scale in R&D investment and means that R&D is more productive when there are fewer potential innovators relative to the mass of existing firms.

Given this R&D technology we can solve for the balanced growth path equilibrium following the same reasoning applied above. Modifying the R&D technology does not affect households' welfare maximization problem or firms' static profit maximization problem meaning that (17) and (19) continue to hold. However, the free entry condition now implies:

$$q = kg + r - \frac{\sigma - 1}{k + 1 - \sigma} \frac{\lambda^k f}{f_e} \left[ 1 + J\tau^{-k} \left( \frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \right] \psi \left( \frac{M_t}{R_t} \right),$$

where  $\psi \left( \frac{M_t}{R_t} \right) \equiv \Psi \left( 1, \frac{M_t}{R_t} \right) = \frac{1}{R_t} \Psi(R_t, M_t)$ . Combining this expression with (17) and (19) gives:

$$q = \frac{\gamma}{1 + \gamma(k - 1)} \left[ \frac{\sigma - 1}{k + 1 - \sigma} \frac{\lambda^k f}{f_e} \left( 1 + J\tau^{-k} \left( \frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \right) \psi \left( \frac{M_t}{R_t} \right) + \frac{kn}{\sigma - 1} - \rho \right]. \quad (34)$$

Comparing (34) with the baseline economy growth rate given by equation (22), the only difference is the inclusion of  $\psi \left( \frac{M_t}{R_t} \right)$ . To obtain the equilibrium value of  $\frac{R_t}{M_t}$  note that in this version of the model equation (13), which gives the rate at which new firms are created, becomes:

$$\frac{R_t}{M_t} \psi \left( \frac{M_t}{R_t} \right) = \frac{1}{\lambda^k} \left( kq - \frac{k + 1 - \sigma}{\sigma - 1} n \right). \quad (35)$$

<sup>28</sup>The baseline model corresponds to the case  $\Psi(R_t, M_t) = R_t$ .

Equations (34) and (35) define a system of two equations in the two unknowns  $q$  and  $\frac{R_t}{M_t}$ . It is not possible to solve for  $q$  explicitly, but it is straightforward to show that  $q$  is higher under trade than in autarky and is strictly increasing in  $T = J\tau^{-k} \left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}$ .<sup>29</sup> Thus, trade raises growth by strengthening the dynamic selection effect. Moreover, solving for the initial consumption level shows that  $c_0$  is given by (30) as in the baseline model. Therefore, it follows that even if there is congestion in innovation the dynamic gains from trade resulting from higher growth increase the total gains from trade relative to a static steady state version of the model. Proposition 4 summarizes these results.

**Proposition 4.** *When there is congestion in innovation, the growth rate of consumption per capita is higher under trade than in autarky. The positive growth effect of trade raises the welfare gains from trade relative to a static steady state version of the model.*

## 7 Conclusions

A central insight of the firm heterogeneity literature inspired by Melitz (2003) is that trade induces selection on firm productivity. However, existing work has focused exclusively on the static selection effect of trade on productivity levels. This paper shows that, if the productivity of new entrants is endogenous to the productivity distribution of incumbent firms, then trade also has a dynamic selection effect that raises the growth rate of average firm productivity. The dynamic selection effect is important for two reasons. First, it identifies a new channel linking trade and growth that does not rely on scale effects. Second, it generates a new source of gains from trade that can only exist when firms are heterogeneous and is additional to the gains from trade in static steady state models. Thus, dynamic selection provides a mechanism through which firm heterogeneity matters for the size of the gains from trade. It is hoped that future work will assess the robustness, both qualitatively and quantitatively, of this mechanism to alternative modeling strategies.

On the empirical side, this paper suggests that firm level studies of trade liberalization episodes should attempt to test for growth effects in addition to level changes. Moreover, comparing this paper with the findings of Baldwin and Robert-Nicoud (2008) demonstrates that including firm heterogeneity in an endogenous growth framework is not sufficient to generate dynamic selection. Dynamic selection also requires that knowledge spillovers affect the productivity distribution of new entrants, rather than the cost of R&D. More empirical work on the nature of knowledge spillovers would be useful to assess the relative importance of

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<sup>29</sup>See the proof of Proposition 4 for details.

these two spillover channels and guide future theoretical work.

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## Appendix A – Proofs

### Proof of Proposition 2

To show that the dynamic gains from trade are positive substitute (30) and (19) into (26) and differentiate with respect to  $q$  to obtain:

$$\begin{aligned} \frac{dU}{dq} &\propto -(k\sigma + 1 - \sigma)\gamma D_1 \left( kD_1 - \frac{1-\gamma}{\gamma} D_2 \right) + k\gamma(D_1 + D_2) [k\sigma(D_1 + D_2) - (\sigma - 1)D_1], \\ &= k^2\gamma\sigma D_2^2 + D_1 D_2 [k^2\gamma\sigma + (k\sigma + 1 - \sigma)(1 + \gamma(k - 1))], \\ &> 0 \end{aligned}$$

where  $D_1 \equiv \frac{1-\gamma}{\gamma}q + \rho - n$  and  $D_2 \equiv n + gk$ . In the first line of the above expression, the first term on the right hand side captures the indirect effect of higher growth on welfare through changes in  $c_0$ , while the second term captures the direct effect. The final inequality comes from observing that Assumption 2 implies both  $D_1 > 0$  and  $D_2 > 0$ .

To obtain a static steady state version of the model developed in this paper assume that new entrants receive a productivity draw from a Pareto distribution with scale parameter one and shape parameter  $k$ . Thus, there are no productivity spillovers and  $\tilde{G}(\theta) = 1 - \theta^{-k}$  independent of  $t$ . Assuming the baseline model is otherwise unchanged, the same reasoning used in Section 2.3 above implies:

$$\frac{\dot{M}_t}{M_t} = -k \frac{\dot{\theta}_t^*}{\theta_t^*} + \frac{R_t}{M_t} \theta_t^{*-k}.$$

It immediately follows that on a balanced growth path the exit cut-off must be constant implying  $g = 0$ . Consumer optimization and the solution for the exit cut-off (5) then give  $q = \frac{n}{\sigma-1}$  meaning that the growth rate is independent of trade integration. With this result in hand it is straightforward to solve the remainder of the model and show  $c_0 \propto z^s$ .

### Proof of Proposition 4

To prove the proposition I need to show that  $q$  is strictly increasing in  $T$ . The result can be proved by taking the total derivatives of (34) and (35) and rearranging to obtain  $\frac{dq}{dT}$ , but here is a simpler argument. Suppose

$T$  increases, but  $q$  does not. Then (34) implies that  $\psi\left(\frac{M_t}{R_t}\right)$  must decrease which requires a fall in  $\frac{M_t}{R_t}$ . From the definition of  $\psi$  we have that  $\frac{R_t}{M_t}\psi\left(\frac{M_t}{R_t}\right) = \Psi\left(\frac{R_t}{M_t}, 1\right)$  which increases when  $\frac{M_t}{R_t}$  falls. Therefore, we must have that the left hand side of (35) increases, while the right hand side does not giving a contradiction. It follows that an increase in  $T$  must lead to an increase in  $q$ .

## Appendix B – Competitive output markets

This Appendix characterizes in greater detail the balanced growth path equilibrium of the competitive output markets model considered in Section 6.1. First, observe that preferences and the R&D technology are unchanged from the baseline model implying that the free entry condition (10), the Euler equation (17), the transversality condition (18), the entry rate condition (24), the welfare expression (26), the initial consumption level equation (27), the asset value equation (28) and the initial exit cut-off condition (29) continue to hold.

Now consider production and trade. Profit maximization by a firm with productivity  $\theta$  implies:

$$\begin{aligned} l_t(\theta) &= p^{\frac{\iota_t}{1-\beta}} \left( \frac{\beta\theta}{w_t} \right)^{\frac{1}{1-\beta}} + f + \iota_t f_x, \\ \pi_t(\theta) &= p^{\frac{\iota_t}{1-\beta}} (1-\beta) \beta^{\frac{\beta}{1-\beta}} \theta^{\frac{1}{1-\beta}} w_t^{\frac{-\beta}{1-\beta}} - f w_t - \iota_t f_x w_t, \end{aligned} \quad (36)$$

where  $\iota_t$  is an indicator variable that takes value one if the firm chooses to pay the fixed export cost at time  $t$  and zero otherwise. Since the firm will choose to pay the fixed export cost only if profits are higher when  $\iota_t = 1$ , it follows from (36) that there exists an exit cut-off  $\theta_t^* = \beta^{-\beta} (1-\beta)^{-(1-\beta)} f^{1-\beta} w_t$  and an export threshold  $\tilde{\theta}_t = \left[ \left( p^{\frac{1}{1-\beta}} - 1 \right) \frac{f}{f_x} \right]^{-(1-\beta)} \theta_t^*$  such that at time  $t$  only firms with productivity below  $\theta_t^*$  exit and only firms with productivity above  $\tilde{\theta}_t$  pay the fixed export cost. To ensure that some, but not all, firms pay the fixed export cost I assume  $p > 1$  and  $\left[ \left( p^{\frac{1}{1-\beta}} - 1 \right) \frac{f}{f_x} \right]^{-(1-\beta)} > 1$ . As  $p > 1$  all firms that pay the fixed export cost choose to export their entire output.

As in the baseline model, the relative productivity distribution is Pareto with scale parameter one and shape parameter  $k$ . Moreover, differentiating the definition of the exit cut-off gives:

$$g = q.$$

Next, we can make the change of variables  $\phi_t = \frac{\theta}{\theta_t^*}$  in the profit function (36) and substitute into the free entry condition (10) to obtain:

$$q = kg + r - \frac{1}{k(1-\beta) - 1} \frac{\lambda^k f}{f_e} \left[ 1 + \frac{f_x}{f} \left( \frac{\tilde{\theta}_t}{\theta_t^*} \right)^{-k} \right].$$

Combining this expression with  $g = q$ , the Euler equation (17) and the definition of the export threshold gives the equilibrium growth rate (32) given in the main text. Having solved for  $q$  it is then straightforward to check that the initial consumption level satisfies (33).

Finally, to ensure that the assumption  $g > 0$  and the transversality condition are satisfied for all  $p \geq 1$  Assumption 2 should be replaced by:

$$\begin{aligned} \frac{1}{k(1-\beta)-1} \frac{\lambda^k f}{f_e} &> \rho, \\ (1-\gamma) \frac{1}{k(1-\beta)-1} \frac{\lambda^k f}{f_e} &> \gamma k(n-\rho) + (1-\gamma)n. \end{aligned}$$

**Table 1: Calibration observables and parameters**

Observable/parameter		Value	Source
Import penetration ratio	IPR	0.081	U.S. import penetration ratio in 2000
Firm creation rate	NF	0.116	U.S. Small Business Administration 2002
Population growth rate	n	0.011	U.S. average 1980-2000
Trade elasticity	k	7.5	Anderson and Van Wincoop (2004)
Elasticity of substitution across goods	$\sigma$	8.1	$\sigma = k/1.06 + 1$ to match right tail index of employment distribution
Intertemporal elasticity of substitution	$\gamma$	0.33	García-Peñalosa and Turnovsky (2005)
Discount rate	$\rho$	0.04	García-Peñalosa and Turnovsky (2005)

**Table 2: Calibration results**

Outcome		Value
Growth rate - trade	q	0.0156
Growth rate - autarky	$q^A$	0.0141
<b>Growth (trade vs. autarky)</b>	<b><math>q/q^A</math></b>	<b>1.107</b>
Consumption level (trade vs. autarky)	$c_0/c_0^A$	1.010
Static gains from trade	$z^s$	1.011
Dynamic gains from trade	$z^d$	1.025
Total gains from trade	z	1.036
<b>Gains from trade (total vs. static)</b>	<b><math>(z-1)/(z^s-1)</math></b>	<b>3.2</b>

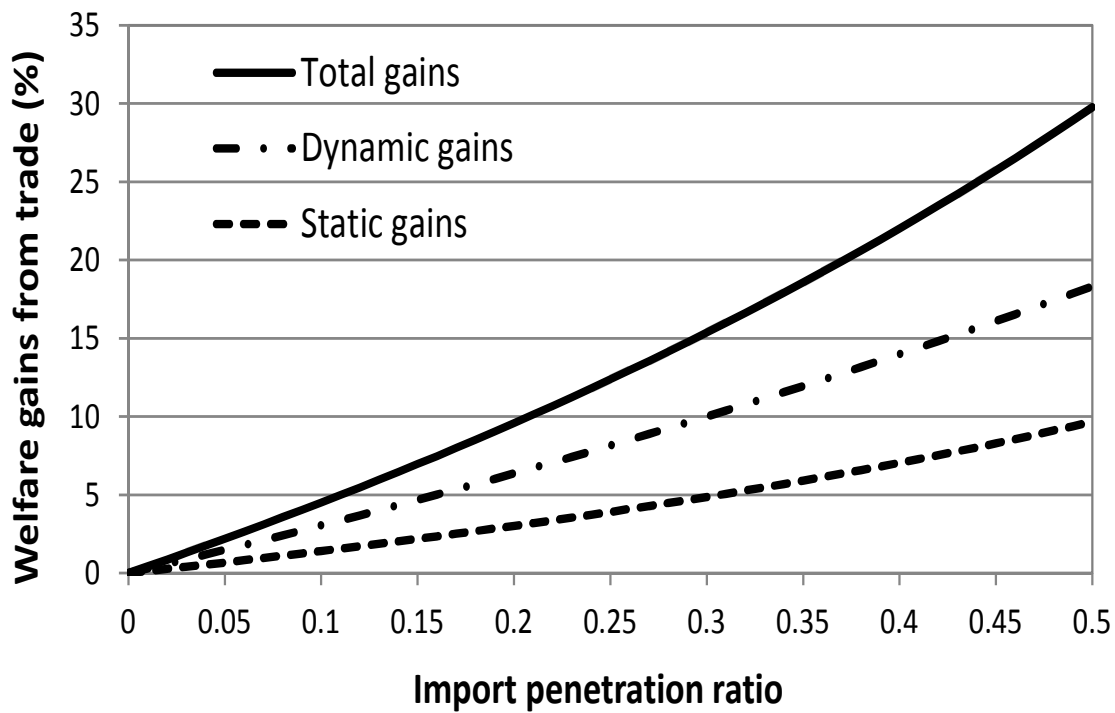


Figure 1: Import Penetration Ratio and the Gains from Trade

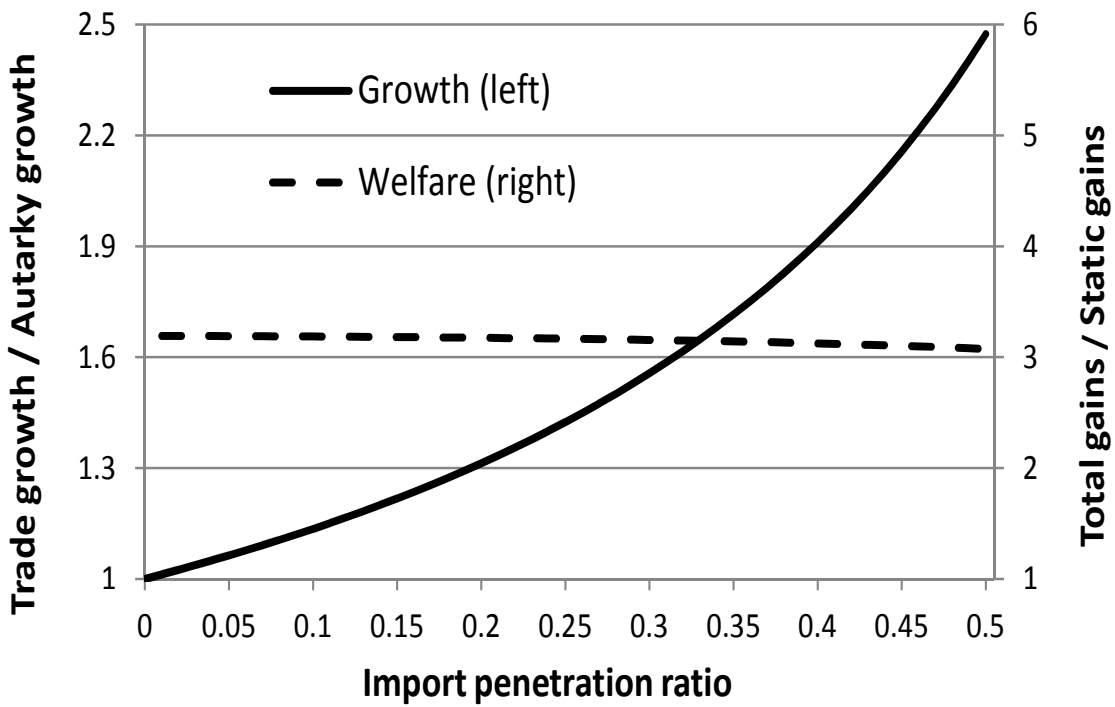


Figure 2: Import Penetration Ratio and the Dynamic Gains from Trade

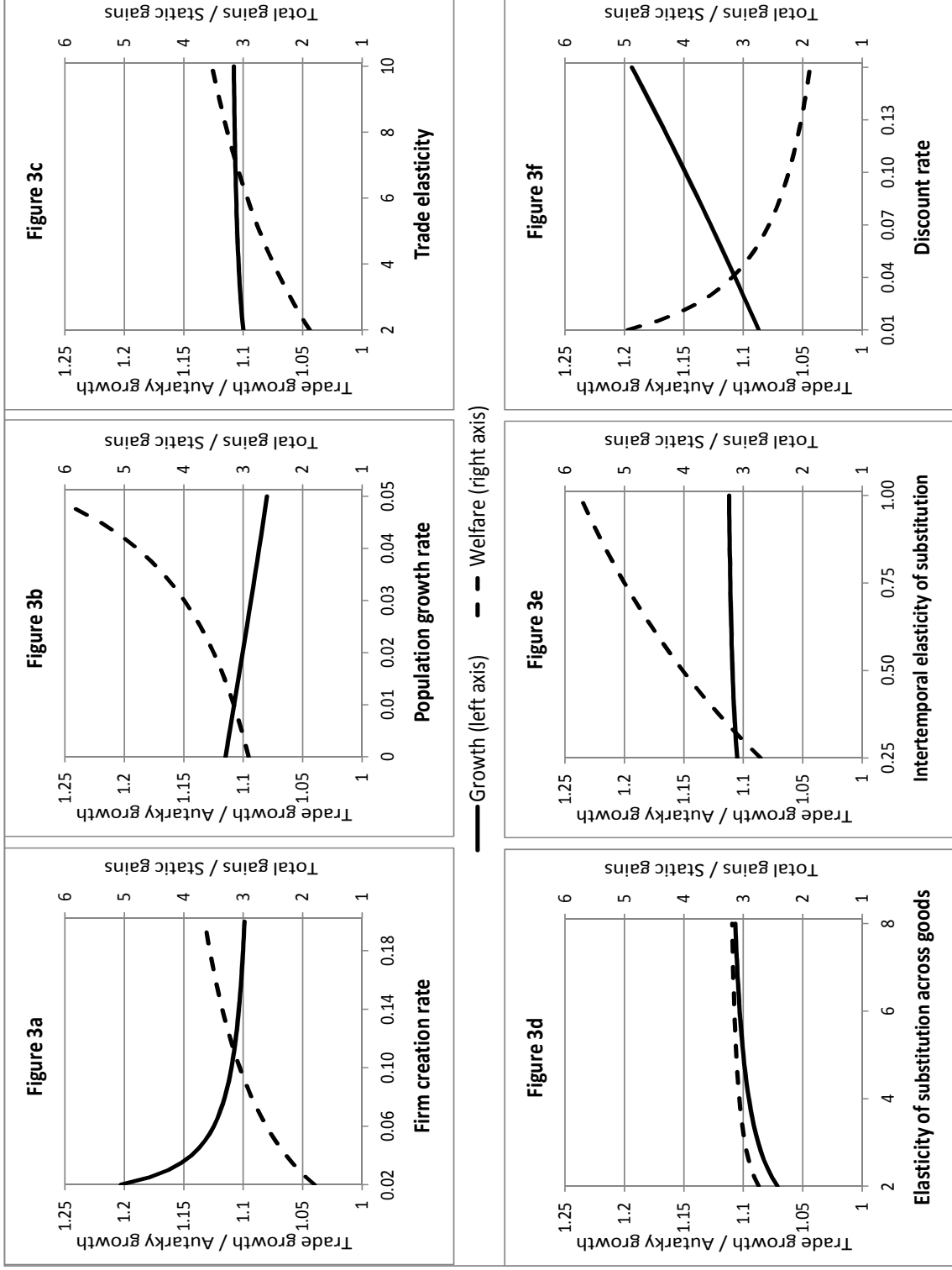


Figure 3: Dynamic Gains from Trade