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Characterizing Behavioral Decisions with Choice Data[*]

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Abstract

This paper provides an axiomatic characterization of choices in a setting where a decision-maker may not fully internalize all the consequences of her choices on herself. Such a departure from rationality, it turns out, is common across a variety of positive behavioral models and admits the standard rational choice model as a special case. We show that choice data satisfying (a) Sen's axioms α and γ fully characterize behavioral decisions, and (b) Sen's axiom α and β fully characterize standard decision-making. In addition, we show that (a) it is possible to identify a minimal and a maximal set of psychological states using choice data alone, and (b) under specific choice scenarios, "revealed mistakes" can be inferred directly from choice data.

JEL: D03, D60, I30.

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1 Introduction

There is considerable evidence that certain intrinsic features of decision-making environments, assumed to be normatively irrelevant in a conventional account of rationality, do affect choices in systematic ways. Deadlines, default options, frames, reference points, aspirations, goals, states of mind, emotions and mood are some examples of such features.

Evidence from psychology and economics shows instances in which such features, far from being exogenous parameters of the environment, are in fact endogenous. For example, people can control their emotions, state of mind and mood (e.g. Baron, 2008), can self-impose deadlines (e.g. Ariely and Wertenbroch, 2002), can limit their focus as a self-control device (e.g. Carrillo and Mariotti, 2000) or set a goal to increase their own performance (e.g. Heath et al., 1999). For expositional convenience, in what follows, we will label such features of the decision-making environment as psychological states i.e. as any normatively relevant feature of the environment that the DM may (mistakenly) not internalize at the moment the decision is made. We broadly interpret psychological states to include reference points, emotions, temptations, mood, states of mind, goals, aspirations, etc.

A relevant question for positive and normative economics is whether or not people internalize the impact of their choices on psychological states. We distinguish between two types of decision-makers (hereafter DMs). Rational DMs are those who internalize these features and hence choose optimally, and behavioral DMs who systematically, and mistakenly, behave as if these (normatively relevant) psychological states were exogenous to their choices.

The evidence in favour of behavioral (boundedly rational) DMs is vast. For example, we know that people systematically tend to stay at a default option even when they report it is not in their best interest;[1] take actions in the "heat of the moment" that they would not have otherwise intended to take (Loewenstein, 1996); fail to quit smoking even though they report that it is what they should do;[2] mispredict the utility derived from future

[1] Two-thirds of survey respondents at one company reported that their current savings rate was too low relative to their ideal savings rate. A third of these undersavers said they were planning to increase their savings plan contribution rate in the next two months, but almost none of them actually did so (Choi et al., 2006)

[2] 70% of smokers in the U.S. report that they want to quit. Moreover, 41% temporarily stopped smoking

consumption (e.g. Loewenstein et al., 2003), overestimate the utility of future income (e.g. Easterlin, 2001) or underestimate the effect of the price of add-ons (e.g. ink of a printer) when buying a base good (e.g. printer) (e.g. Gabaix and Laibson, 2006). An array of different models in the literature on behavioral economics have been constructed to account for this evidence and predict systematic suboptimal behavior in specific decision-making environments.[3]

While each of these behavioral models accounts for a specific departure from rationality over a fixed domain of fundamentals (preferences, action sets), little is known about the general properties of the choices which are consistent with these models. Do behavioral models, as a class, have some general choice-theoretic structure? Do they impose (also as a class) any restrictions on choice data and if so, which ones? Can we extract any relevant normative information about psychological states from choices? Can we use choices to identify mistakes?

The purpose of this paper is to take a first step in addressing the above questions. We begin by proposing a general framework of individual decision-making that is a reduced form representation of a number of different models of decision-making. Crucially, in our framework, we allow for standard decision-making (corresponding to rational choice) and behavioral decision-making (corresponding to boundedly rational choice). We provide an axiomatic characterization of choice data consistent with behavioral and standard decision-making. Finally, we construct a scenario where it is possible to identify mistakes from choice data alone i.e. infer "revealed mistakes".

In our framework, the DM chooses among mutually exclusive actions. Each action has an effect on payoffs both directly and indirectly through its effect on a psychological state, via a feedback function. The DM's preferences rank both actions and psychological states which are in turn, determined by actions: in effect, following Harsanyi (1954), we assume that there is intrapersonal comparability of utility.

for a day or more during the previous twelve months in an unsuccessful attempt to quit (Beshears et al., 2008). Smokers according to their own evaluation, consider themselves to be better off if smoking was restricted by a tax (Gruber and Mullainathan, 2005),

[3]Some examples are models of projection bias (Loewenstein et al., 2003), cognitive dissonance (Akerlof and Dickens, 1982), emotions (Bracha and Brown, 2007) or self-control problems (Loewenstein, 1996). Other relevant papers are referred to in Section 3.3 below.

We consider two types of decision procedures: a Standard Decision Procedure (SDP) and Behavioral Decision Procedure (BDP). In a SDP, the DM fully internalizes the feedback from actions to psychological states, and chooses an action that maximizes his welfare: this is equivalent to rational decision-making. In a BDP, in contrast, a (behavioral) DM fails to internalize the effect of his action on his psychological state, and chooses an action taking as given his psychological state (although psychological states and actions are required to be mutually consistent at a BDP outcome): this is a form of boundedly rational decision-making.

Despite its simplicity, our framework is general enough to unify seemingly disconnected models in the literature, from more recent positive behavioral economics models to older ones. In addition, it encompasses the standard rational model as a special case (SDP).

Next, we provide an axiomatic characterization of choice data compatible with BDPs and SDPs. It turns out, surprisingly, that three axioms in Sen (1971), axioms α, β and γ, are all that is needed to characterize choice data compatible with a SDP and a BDP respectively. Axiom α, which was also introduced by Chernoff (1954), states that the choice correspondence is (weakly) increasing as the choice set shrinks when all alternatives chosen in the larger set are also present in the smaller set. Axiom β states that when two actions are both chosen in a given set, and one of them is chosen in a larger set that includes the first set, then both are chosen in the larger set. Axiom γ states that if an action is chosen in each set in a class of sets, it must be also chosen in their union.[4] We show that: (a) choice data is compatible with a BDP if and only if such data satisfies both axioms α and γ, and (b) choice data is compatible with an admissible[5] SDP if and only if such data satisfies both axioms α and β. Heuristically, axioms α and γ imply that choice data are representable by a binary relation whether or not that relation is transitive (Sen, 1971). To ensure choice data satisfy transitivity requires that such data satisfies axioms α and β.[6] Evidently, whenever

[4] In Section 4, Remark 1, we point out that there is a canonical decision scenario in which any decision scenario in our framework can be embedded. Namely, one where the set of psychological states is the set of actions and the feedback effect is the identity function. The canonical scenario is, thus, trivially identifiable from choice data.

[5] We provide a formal definition of an admissible decision problem in Section 4. In short, admissibility requires the preference relation over consistent pairs of actions and psychological states to be transitive.

[6] We make this point precise in Section 4.

axiom α is satisfied, axiom γ implies axiom β but the reverse implication does not always hold.

Rational choice theory is falsifiable if *Arrow's axiom* holds.[7] Sen (1971) shows (in Theorem 3 and 7) that Arrow (1959)'s axiom (and hence, WARP and menu independence) is satisfied if and only if both Sen's axioms α and β are satisfied. Sen's axioms α and β, taken together, are equivalent to Arrow's (1959) axiom, which in turn is equivalent to Samuelson's weak axiom of revealed preferences (WARP) and menu independence. Axioms α and γ imply that choice data compatible with a BDP rules out pairwise cycles although such data need not satisfy menu independence or WARP.[8] The violation of Arrow's axiom in a BDP comes from the fact that alternatives may not be irrelevant even when they are never chosen if the DM doesn't fully internalize the endogeneity of psychological states.

Then we ask what sort of information about psychological states can choice reveal in the presence of behavioral DMs. We show that choice data can be used to identify a "minimal" and a "maximal" set of psychological states. It is relatively straightforward to note that when choice data satisfies axioms α and β, no more than one psychological state is required to rationalize such data as the outcome of a SDP. Under the additional requirement that for any fixed psychological state the (implied) ranking over actions be transitive, we show that with three or more actions, there exists choice data that satisfies axioms α and γ (but not axiom β) which requires at least two psychological states in order to be rationalized as the outcome of a BDP. Note also, that there exists also a set of maximal psychological states- namely, the set of actions with feedback from actions to psychological states as the identity map- that can be directly identified from choice data. In this way, we obtain a lower bound and an upper bound for the set of psychological states that can be identified from choice data.

Finally, we propose a way to infer "revealed mistakes" in specific decision-scenarios. Interestingly, our result holds even in scenarios where choice data satisfy both Sen's axiom

[7] Arrows axiom: the choice correspondence remains the same as the choice set shrinks when all alternatives chosen in the larger set are also present in the smaller set.

[8] It turns out that the fact that choice data compatible with a BDP rules out pairwise cycles distinguishes the specific form of bounded rationality studied here from other axiomatic characterizations of decision-making models with some specific behavioral flavor (see Section 4, Remark 2).

α and β (and hence, Arrow's axiom), thus potentially qualifying the welfare analysis based on choices alone (e.g. Bernheim and Rangel, 2009).

The remainder of the paper is organized as follows. Section 2 introduces our framework with the aid of a simple example. Section 3 develops the general framework together with a dynamic interpretation and states the existence result. Section 4 provides the axiomatic characterization of our theory and studies the identification of psychological states and mistakes from choice data. The last section concludes. The details of the existence proof and the dynamic interpretation of our framework are contained in the appendix.

2 Example (Addiction)

Consider a DM who is considering whether to drink alcohol. The psychological state will either be sober (if he does not drink) or inebriated (if he does). The payoff table below provides a quick summary of the decision problem:

	inebriated	sober
alcohol	$1-2$	$1+0$
no alcohol	$0-2$	$0+0$

In this example, the payoffs are an additive function of the action-based payoff and the psychological state-based payoff. Alcohol generates utility of 1; no alcohol generates utility of 0. Sobriety generates utility of 0; inebriation generates utility of -2.

An DM who uses a SDP to solve this problem recognizes that he has to choose between the on-diagonal elements. Alcohol goes together with the psychological state of inebriation. No alcohol goes together with the psychological state of sobriety. Hence, the off-diagonal paths are not options.

However, the behavioral DM mistakenly believes that (or at least acts as if) he can change his alcohol consumption without changing his psychological state. Consequently, the behavioral DM decides to consume alcohol (since alcohol is always better, conditional on a fixed psychological state) and ends up inebriated (with net payoff -1). This is a mistake in the sense that the DM would be better off if he chose to drink no alcohol and ended up sober (with net payoff 0). In this sense, by using a BDP the DM imposes an externality on himself. Thus, the outcomes of a BDP can (although not necessarily) be welfare dominated.

The rest of the paper works out the implications of modelling and characterizing boundedly rational decision-making where a DM chooses actions without internalizing their impact on psychological states.

3 The General Framework

3.1 The Model

The primitives of the model consist of set A of actions, a set P of psychological states and a function $\pi : A \rightarrow P$ modelling the feedback effect from actions to psychological states. It is assumed that $\pi(a)$ is non-empty and single-valued for each $a \in A$. A decision state is a pair of an action and psychological state (a, p) where $a \in A$ and $p \in P$. A *consistent* decision state is a decision state (a, p) such that $p = \pi(a)$.

Following Harsanyi (1954), we assume intra-personal comparability of utility. That is, the DM is not only able to rank different elements in A for a given p, but he is also able to assess the subjective satisfaction he derives from an action when the psychological state is p with the subjective satisfaction he derives from another action when the psychological state is p'. In other words, we assume that the DM is able to rank elements in $A \times P$. This formulation is critical in order to make meaningful welfare comparisons.

Accordingly, the preferences of the DM are denoted by $\succeq$, a binary relation ranking pairs of decision states in $(A \times P) \times (A \times P)$.

A decision scenario is, thus, a collection $D = (A, P, \pi, \succeq)$.

We study two decision procedures:

1. Given a non-empty *feasible* set of actions $A' \subseteq A$, a *standard decision procedure* (SDP) is one where the DM chooses a consistent decision state (a, p), $a \in A'$ and $p = \pi(a)$. The outcomes of a SDP, denoted by S, are

$$S = \left\{(a, p) : (a, p) \succeq \left(a', p'\right) \text{ for all } \left(a', \pi(a')\right), a' \in A', p = \pi(a)\right\}.$$

2. Given a non-empty *feasible* set of actions $A' \subseteq A$, a *behavioral decision procedure*

(BDP) is one where the DM takes as given the psychological state p when choosing $a \in A'$. Define a preference relation $\succeq_p$ over A as follows:

$$a \succeq_p a' \Leftrightarrow (a,p) \succeq \big(a',p\big) \text{ for } p \in P.$$

The outcomes of a BDP, denoted by B, are

$$B = \big\{(a,p) : a \succeq_p a' \text{ for all } a' \in A',\, p = \pi(a)\big\}.$$

In both, SDPs and BDPs, a decision outcome must be a consistent decision state where the action is chosen from some feasible set of actions. In a SDP, the DM internalizes that his psychological state is determined by his action via the feedback effect when choosing an action from the set of feasible actions. In a BDP, the DM takes the psychological state as given when he chooses an action from the set of feasible actions.

Our framework assumes that psychological states are normatively relevant. Therefore, the preferences of a standard DM provide the relevant normative benchmark. These normative preferences $\succeq$ *over the set of consistent decision states* directly induce a unique ranking of actions, $(a, \pi(a)) \succeq (a', \pi(a'))$.

3.2 A Dynamic Interpretation

We interpret the outcomes of a SDP and a BDP as corresponding to distinct steady-states associated with an adaptive preference mechanism where the DM's preferences over actions at any t, denoted by $\succeq_{p_{t-1}}$, depends on his past psychological state where p_t is the psychological state for period t. The statement $a \succeq_{p_{t-1}} a'$ means that the DM finds a at least as good as a', given the psychological state p_{t-1}. The DM takes as given the psychological state from the preceding period.

Note that an outcome of a BDP corresponds to the steady state of an adjustment dynamics where the DM is myopic (i.e. does not anticipate that the psychological state at $t+1$ is affected by the action chosen at t).

Let $h(p) = \{a \in A : a \succeq_p a',\, a' \in A\}$. For ease of exposition, assume that $h(p)$ is unique. Fix a $p_0 \in P$. A sequence of *short-run* outcomes is determined by the relations $a_t \in h(p_{t-1})$

and $p_t = \pi(a_t)$, $t = 1, 2, ...$: at each step, the DM chooses a myopic best-response.[9] *Long-run* outcomes are denoted by a pair (a, p) with $p = \pi(a)$ where a is defined to be the steady-state solution to the short-run outcome functions i.e. $a = h(\pi(a))$. In other words, long-run behavior corresponds to the outcome of a BDP.[10] In contrast, in a SDP, the DM is farsighted (i.e. anticipates that the psychological state at $t + 1$ is affected by the action chosen at t). The outcome of a SDP is one where a is defined to be the steady state solution to $a \in \{a \in A : a \succeq_{\pi(a)} a', a' \in A\}$ and $p = \pi(a)$. In this case, the DM anticipates that p adjusts to a according to $\pi(\cdot)$ and taking this into account, chooses a. Note that in this simple framework, in a SDP the DM instantaneously adjusts to the steady-state outcome so that p_0, the initial psychological state, has no impact on the steady state solution with farsightedness.[11] [12]

3.3 Reduced Form Representation

Various interpretations can be given to a psychological state p. It can be a reference point, an expectation, an emotion, mood, aspiration or, more generally, any normatively relevant feature of the environment that the DM may (mistakenly) not internalize at the moment the decision is made. Are all of these interpretations consistent with our general theoretical framework? We argue that the answer is yes.

Our analysis assumes that DMs' preferences depend on both current action and psychological state. In some cases, the action causes the psychological state. This is the case of a reference point or an emotional state like fear, anxiety or stress that quickly adjusts to

[9] Under the assumptions required to prove Proposition 1 below (existence), as shown in the appendix, $h(\cdot)$ is increasing map of p so that the sequence of short-run outcomes is an (component-wise) increasing sequence (as by assumption contained in a compact set) and therefore, converges to its supremum which is necessarily a BDP. So the existence result covers not only cases where a solution to a BDP (equivalently, a steady-state solution to the myopic preference adjustment mechanism) exists but also ensures that short-run outcomes converge to a BDP.

[10] Von Weizsacker (1971), Hammond (1976), Pollak (1978) made a similar point for the case of adaptive preferences defined over consumption.

[11] Non-trivial dynamics would be associated with farsighted behavior if underlying preferences or action sets were time variant.

[12] In the appendix we extend our framework to situations that allow DMs (i) to anticipate short-run psychological states that arise from their actions but not the long-run psychological states, and (ii) to make partial prediction of changes in psychological states as a function of their chosen actions (i.e. projection bias introduced by Loewenstein et al., 2003).

current actions. But in other situations, the psychological state precedes the action, and in this sense, our definition of “consistent decision state” is an equilibrium concept. This is the case where the psychological state concerns expectations, endowments or beliefs.[13]

For example, in Tversky and Kahneman (1991)’s theory of reference-dependent preferences over consumption, a could be a consumption bundle and p is a reference point (another commodity bundle). If the DM chooses a when the pre-decision reference point is p, the post-decision reference point shifts to a. In this sense, the model of decision-making studied here corresponds to a situation where "the reference state usually corresponds to the decision-maker’s current state." (Tversky and Kahneman, 1991, pp. 1046). Shalev (2000), Köszegi and Rabin (2006, 2007) and Köszegi (2010) also consider models of endogenous reference-dependent preferences.[14] Caplin and Leahy (2001) analysis of anticipatory feelings is also related to our paper as these can be interpreted as a specific example of a psychological state.[15]

By using similar reasoning, it follows that our general framework, unifies seemingly disconnected models in the literature, from situations where the psychological state corresponds to beliefs (Geanakoplos et al., 1989; Akerlof and Dickens, 1982), emotions (Bracha and Brown, 2007) and aspirations (Dalton et al., 2010; Heifetz and Minelli, 2006).

Given our interpretation of the outcomes of a SDP and a BDP as corresponding to distinct steady-states associated with an adaptive preference mechanism, as already argued our model can be seen as a reduced form representation of adaptive preferences over consumption (Von Weizsacker, 1971; Hammond, 1976 and Pollak, 1978, already referred to above), the theory of melioration where consumers fail to take into account the effect of current choices on future tastes (Herrnstein and Prelec, 1991) and projection bias (Loewenstein et al., 2003) where a DM tends to exaggerate the degree to which their future tastes will resemble their current state.[16]

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[13] A similar notion of equilibrium is proposed by Koszegi (2010) and Geanakopolos et al. (1989).

[14] Our paper complements this literature by studying the situations in which the DM doesn’t internalize the endogeneity of the reference points and by providing testable restrictions in which actual choice data can in principle be compared.

[15] Caplin and Leahy (2001) provide a set of axioms so that the representation of underlying preferences with anticipatory feeling is possible in an expected utility setting. Given our emphasis on testable restrictions our axiomatic characterization complements their work.

[16] Projection bias provides a possible explanation of why DMs may use a BDP instead of a SDP in

Below we present two further examples that illustrate how our framework encompasses models of status-quo bias and dynamic inconsistency.

Example 1: Status-quo Bias

Consider a DM who is considering whether to switch to a different service provider (e.g. gas and electricity) from his current one. The psychological state (in this case the reference point) will either be current supplier (if he sticks with the current supplier) or the alternative supplier (if he makes the change). There are two payoff relevant dimensions of choice with outcome denoted x_1 and x_2 and preferences $u(x) = x_1 + v(x_1 - r_1) + x_2 + v(x_2 - r_2)$ where $v(\cdot)$ is a Kahneman-Tversky value function with $v(z) = z$ if $z \geq 0$, $v(z) = \alpha z$, $\alpha > 2.5$ if $z < 0$ and $v(0) = 0$. The cost of switching is equal to 0.5. The status-quo option is defined by $q = (0, 1)$ and the alternative option is $a = (2, 0)$. The payoff table below provides a quick summary of the decision problem:

	status quo	alternative
current supplier	1	$2 - 2\alpha$
alternative supplier	$3.5 - \alpha$	1.5

In this example, again, the payoffs are an additive function of the action-based payoff and the psychological state-based payoff.

A DM who uses a SDP recognizes that he has to choose between the on-diagonal elements. Sticking with the current supplier goes with the reference point status quo. Choosing the alternative supplier goes together with the reference point of the alternative. Hence, the off-diagonal paths are not options and the outcome of a SDP will be to switch to the alternative supplier.

However, the behavioral DM mistakenly believes that (or at least acts as if) he can choose between the two suppliers without changing his psychological state. Consequently, there are two payoff ranked outcomes: one where the behavioral DM sticks with the current supplier and the reference point is status-quo and the other where he switches suppliers and the reference point is the alternative. The former choice is a mistake in the sense that

some particular situations. For example, projection bias can explain why behavioral DMs get trapped in addiction or overconsumption of durable goods. However, projection bias cannot account for all the models encompassed in BDPs. This is the case, for instance, for models of cognitive dissonance.

the DM would be better off if he chose to switch and ended up with the alternative as the reference point.

Example 2: Dynamic Inconsistency

Consider a three period problem $t = 0, 1, 2$ where a DM has preferences defined over a single consumption good $c_t, t = 0, 1, 2$. The DM is endowed with a single unit of the consumption good at $t = 0$ but has no endowment of the consumption good in either of the subsequent two periods. The DM obtains no utility from consumption at $t = 0$ but obtains utility from consumption at $t = 1, 2$ with an instantaneous linear utility function c. Assume that the DM quasi-hyperbolically discounts the future with $0 < \beta < 1$ and $\delta = 1$.

There are two assets: (i) an illiquid asset I where one unit invested yields nothing at $t = 1$ and $R > 1$ units of the consumption good at $t = 2$, (ii) a liquid asset where one unit invested at $t = 0$ yields 1 unit of the consumption good if liquidated at $t = 1$ and nothing at $t = 2$ or if not liquidated at $t = 1$ yields $R' > R$ units of the consumption good at $t = 2$. We assume that $\beta < \frac{1}{R'}$. The DM at $t = 0$ will choose which asset to invest in in order to maximize $\beta(c_1 + c_2)$. At $t = 1$ the current self of the DM will maximize $c_1 + \beta c_2$.

To represent this decision problem in our framework we proceed as follows. The psychological states of the DM at $t = 0$ are p_1 ="tempted to liquidate at $t = 1$" and p_2 ="not tempted to liquidate at $t = 1$" (corresponding to not liquidate). Note that at $t = 1$, if L was chosen at $t = 0$, the current self of the DM would be tempted and liquidate if $\beta R' < 1$ i.e. $\beta < \frac{1}{R'}$. Clearly, the current self of the DM cannot be tempted to liquidate if at $t = 0$ the DM has invested in the illiquid asset.

Therefore, the action "invest in the illiquid asset" goes with the psychological state p_2 ="not tempted to liquidate at $t = 1$" while the action "invest in the liquid asset" goes with the psychological state p_1 ="tempted to liquidate at $t = 1$". The DM at $t = 0$ has to decide whether to invest in the liquid or the illiquid asset. A quick summary of his decision problem at $t = 0$ is:

	tempted	not tempted
liquid	1	R'
illiquid	R	R

If the DM follows a SDP, he will correctly anticipate that the asset chosen today will affect his psychological state at $t = 1$ and will choose to invest in the illiquid asset and obtain a payoff of $R > 1$. In a SDP the DM exhibits self-control by using the illiquid asset as a pre-commitment device. If the DM follows a BDP, he will believe that (or act as if) the asset chosen today will not affect his psychological state at $t = 1$. Interestingly, there is no pure action solution to a BDP. If the psychological state is "tempted", he will choose to invest in the illiquid but if the psychological state is "not tempted" he will invest in the liquid asset. There is, however, a random solution where the behavioral DM chooses to invest in the liquid asset with probability $p = \frac{R'-R}{R'-1}$: if a behavioral DM believes that the distribution over psychological states is $\left\{\frac{R'-R}{R'-1}, \frac{R-1}{R'-1}\right\}$, he is indifferent between investing in either the liquid or the illiquid asset and is willing to randomize between the two actions. By computation, it is easily checked that the expected payoff from such a random action is less than R, the payoff of a standard DM.

3.4 Stackelberg vs. Nash in an Intra-self Game

In a formal sense, we could also interpret the distinction between a SDP and a BDP as corresponding to the *Stackelberg* and, respectively, the *Nash* equilibrium of dual self intra-personal game where one self chooses actions a and the other self chooses the psychological state p and $\pi(a)$ describes the best-response of the latter for each $a \in A$.

In a Stackelberg equilibrium, the self choosing actions anticipates that the other self chooses a psychological state according to the function $\pi(\cdot)$. In a Nash equilibrium, both selves take the choices of the other self as given when making its own choices.

Consistent with the dynamic interpretation of the general framework, in the definition of a SDP, internalization (i.e. rationally anticipating the actual effects of one's actions) also encompasses the DM anticipating that equilibrium (e.g. one's own actions is what one expected it to be, or what others expected it to be) and behaving accordingly.

Given this interpretation, it follows that in the welfare analysis reported below, only the preferences of the self that chooses actions is taken into account.

3.5 Existence

So far we have implicitly assumed that both SDP and BDP are well-defined i.e. lead to well defined outcomes. In what follows, we check for the existence of solutions to an SDP and a BDP in situations where the underlying preferences are not necessarily complete or transitive and underlying action sets are not necessarily convex.[17] So we allow preferences to be incomplete, non-convex and acyclic (and not necessarily transitive) and we show existence of a solution to a BDP extending Ghosal's (2011) result for normal form games to behavioral decision problems.[18]

Proposition 1. *Suppose $A \times P \subseteq \Re^n \times \Re^m$ and the function $\pi : A \to P$ is continuous and increasing. Under assumptions of single-crossing, quasi-supermodularity and monotone closure,*[19] *a solution to a BDP exists.*

Proof. See appendix. ■

The preceding existence result doesn't cover situations with payoffs as in Example 2 (dynamic inconsistency). In such cases, where there are no pure action solutions to a BDP, what are the possible outcomes?

Given that the outcome of a BDP can be interpreted as a Nash equilibrium of a two person game (as shown in Section 3.4), as long as A and P are finite, a behavioral decision outcome involving randomization always exists. A different possibility, referring back to the dynamic interpretation of model, is that in such situations, the sequence of short-run outcomes will cycle.

Moreover, under the assumptions required to prove Proposition 1, as shown in the appendix, $h(.)$ is an increasing map of p so that the sequence of short-run outcomes is a (component-wise) increasing sequence (as by assumption is contained in a compact set and therefore, converges to its supremum which is necessarily a BDP). So the existence result

[17] Mandler (2005) shows that incomplete preferences and intransitivity is required for "status quo maintenance" (encompassing endowment effects, loss aversion and willingness to pay-willingness to accept diversity) to be outcome rational. Tversky and Kahneman (1991) argue that reference dependent preferences may not be convex.

[18] The seminal proof for existence of equilibria with incomplete preferences in Shafer and Sonnenschein (1975) requires convexity both for showing the existence of an optimal choice and using Kakutani's fix-point theorem.

[19] These terms are all defined in the appendix.

covers not only cases where a solution to a BDP (equivalently, a steady-state solution to the myopic preference adjustment mechanism) exists, but also ensures that short-run outcomes converge to a BDP.

4 Axiomatic Characterization of SDP and BDP

Under what conditions can choice data be rationalized as the outcome of a standard or a behavioral decision procedure? In what follows, we show that both decision procedures are fully characterized by three observable properties of choice.

Fix $\succeq$, $\pi : A \to P$ and a family $\mathcal{A}$ of non-empty subsets of A. Define two correspondences, $\mathfrak{S}$ and $\mathfrak{B}$, from $\mathcal{A}$ to A as

$$\mathfrak{S}(A') = \left\{a : (a,p) \succeq \left(a',p'\right) \text{ for all } a' \in A',\, p' = \pi(a') \text{ and } p = \pi(a)\right\}$$

and

$$\mathfrak{B}(A') = \{a : a \succeq_{\pi(a)} a' \text{ for all } a' \in A'\},$$

as the choices corresponding to a standard and behavioral decision procedure, respectively.

We say that $\mathfrak{S}(.)$ is *admissible* if the preference relation $\succeq$ is transitive over the set of consistent decision states. We say that $\mathfrak{B}(.)$ is *admissible* if for each $p \in P$, $\succeq_p$ over actions pairs in A.

Suppose we observe a non-empty correspondence C from $\mathcal{A}$ to A such that $C(A') \subseteq A'$. We say that SDP (respectively, BDP) rationalizes C if there exist P, π and $\succeq$ such that $C(A') = \mathfrak{S}(A')$ (respectively, $C(A') = \mathfrak{B}(A')$).

Next, consider the following axioms introduced by Sen (1971).

Sen's axiom α*.* For all $A', A'' \subseteq A$, if $A'' \subseteq A'$ and $C(A') \cap A''$ is non-empty, then $C(A') \cap A'' \subseteq C(A'')$. In words, the choice correspondence is (weakly) increasing as the choice set shrinks when all alternatives chosen in the larger set are also present in the smaller set.

Sen's axiom β*.* For all $A', A'' \subseteq A$, if $A'' \subseteq A'$ and $a, a' \in C(A'')$, then $a \in C(A')$ if and only if $a' \in C(A')$. In words, when two actions are both chosen in a given set, and one of them is chosen in a larger set that includes the first set, then both are chosen in the larger set.

Sen's axiom γ. Let M be any class of sets $\{A'_k \subseteq A : k \geq 1\}$ and let V be the union of all sets in M. Then any a that belongs to $C(A')$ for all A' in M must belong to $C(V)$. In words, if an action is chosen in each set in a class of sets, it it must be also be chosen in their union.

We are now in a position to fully characterize choice data compatible with a SDP and BDP. We begin by characterizing choice data compatible with an admissible SDP.

Proposition 2. *Choice data are rationalizable as the outcome of an admissible SDP if and only if both Sen's axioms α and β are satisfied.*

Proof. (i) We show that if choice data are rationalizable as the outcome of an admissible SDP, then, both Sen's axiom α and β hold. Fix $\succeq$, $\pi : A \to P$. For $A'' \subseteq A' \subseteq A$, if

$$a \in \mathfrak{S}(A') = \left\{ \begin{array}{c} a : (a,p) \succeq (a',p') \text{ for all } a' \in A',\, p' = \pi(a') \\ \text{and } p = \pi(a) \end{array} \right\}$$

then

$$a \in \mathfrak{S}(A'') = \left\{ \begin{array}{c} a : (a,p) \succeq (a',p') \text{ for all } a' \in A'',\, p' = \pi(a') \\ \text{and } p = \pi(a) \end{array} \right\}.$$

Therefore, $\mathfrak{S}(A') = C(A') \cap A'' \subseteq C(A'') = \mathfrak{S}(A'')$ so that Sen's axiom α is satisfied. Next, given $A'' \subseteq A'$, suppose $a', a'' \in C(A'') = \mathfrak{S}(A'')$ but $a' \in \mathfrak{S}(A')$ and $a'' \notin \mathfrak{S}(A')$. By construction, both $(a',p') \succeq (a'',p'')$ and $(a',p') \preceq (a'',p'')$ for $p' = \pi(a')$ and $p'' = \pi(a'')$. Therefore, by transitivity of $\succeq$ over consistent decision states, $a'' \in \mathfrak{S}(A')$, a contradiction so that Sen's axiom β is satisfied.

(ii) We show that if choice data satisfy Sen's axioms α and β, they are rationalizable as the outcome of an admissible SDP. To this end, we specify $\pi : A \to P$, $\#P \geq 1$ so that π is onto. Next we specify preferences $\succeq$: for each non-empty $A' \subseteq A$ and $a \in C(A')$, $\succeq$ satisfies the condition that $(a,p) \succeq (a',p')$ for all $a' \in A'$, $p = \pi(a)$ and $p' = \pi(a')$, $p, p' \in P$. Consider $C(A')$ for some non-empty $A' \subseteq A$. By construction if $a \in C(A') \Rightarrow \mathfrak{S}(A')$ and therefore, $C(A') \subseteq \mathfrak{S}(A')$. We need to check that for the above specification of $\succeq$, $\pi : A \to P$, $\mathfrak{S}(A') \subseteq C(A')$. Suppose to the contrary, there exists $a' \in \mathfrak{S}(A')$ but $a' \notin C(A')$. It follows that $(a', \pi(a')) \succeq (b, \pi(b))$ for all $b \in A'$. Since $a' \notin C(A')$, by construction this is only possible if for each $b \in A'$, $a' \in C(A''_b)$ with $\{a,b\} \subseteq A''_b$. By Sen's axiom α, as $a' \in C(\{a,b\})$ and as $\{a,b\} \subseteq A'$, again by Sen's axiom α, $b \in C(\{a,b\})$ for $b \in C(A')$. Now, by construction, $A' = \cup_{b \in A'} \{a,b\}$. By Sen's axiom β, $a' \in C(A')$. Therefore, $\mathfrak{S}(A') = C(A')$. Finally, note that when choice data satisfy axioms α and β, $\succeq$ is transitive (Theorem 1, Sen (1971)) and therefore, $\mathfrak{S}(A')$ is admissible. ■

Rational choice theory is falsifiable if *Arrow's axiom* holds. Sen (1971) shows (in Theorem 3 and 7) that Arrow (1959)'s axiom[20] (and hence, WARP and menu independence) is satisfied if and only if both Sen's axioms α and β are satisfied. So, Proposition 2 implies that choice data are compatible with SDP if and only if they are also compatible with rational choice theory.[21]

Note that we can set $\#P = 1$ in part (ii) of the proof of Proposition 2 so that the set of minimal psychological states that can be identified from choice data is equal to one if such data can be rationalized as the outcome of an admissible SDP. Therefore, it is without loss of generality to assume that psychological states are exogenous to the actions of the DM if choice data can be rationalized as the outcome of an admissible SDP.

Next, we characterize choice data compatible with a BDP.

Proposition 3. *Choice data are rationalizable as the outcome of a BDP if and only if both Sen's axioms α and γ are satisfied.*

Proof. (i) We show that if choice data are rationalizable as the outcome of a BDP, then both *Sen's* α and γ hold. Fix $\succeq$, $\pi : A \to P$. For $A'' \subseteq A' \subseteq A$, if

$$a \in \mathfrak{B}(A') = \left\{a : a \succeq_{\pi(a)} a' \text{ for all } a' \in A'\right\}$$

then

$$a \in \mathfrak{B}(A'') = \left\{a : a \succeq_{\pi(a)} a' \text{ for all } a' \in A''\right\}.$$

Therefore, $C(A') \cap A'' \subseteq C(A'')$ as required so that Sen's axiom α is satisfied. Next, let M

[20] Arrow (1959)'s axiom: If $A' \subseteq A$ and $C(A) \cap A'$ is non-empty, then $C(A') = C(A) \cap A'$.When the set of feasible alternatives shrinks, the choice from the smaller set *consists precisely* of those alternatives chosen in the larger set and remain feasible, if there is any.

[21] Masatlioglu and Ok (2005)'s axiomatic characterization of rational choice with status quo bias (exogenous to the actions chosen by the DM) satisfies *Arrow's axiom* among other axioms.

denote a class of sets $\{A'_k \subseteq A : k \geq 1\}$. If

$$a \in \mathfrak{B}(A'_k) = \{a : a \succeq_{\pi(a)} a' \text{ for all } a' \in A'_k\}$$

and $V = \cup_{k \geq 1} A'_k$, it follows that

$$a \in \mathfrak{B}(V) = \{a : a \succeq_{\pi(a)} a' \text{ for all } a' \in V\}$$

so that Sen's axiom γ is satisfied.

(ii) We show that if choice data satisfy both Sen's α and γ, they are rationalizable as the outcome of a BDP. To this end, we specify $\pi : A \to P$ so that $\#P \geq 1$ and π is onto. Next we specify preferences $\succeq$: for each non-empty $A' \subseteq A$ and $a \in C(A')$, $\succeq$ satisfies the condition that $a \succeq_p a'$ for all $a' \in A'$ and $p = \pi(a)$. Consider $C(A')$ for some non-empty $A' \subseteq A$. By construction if $a \in C(A')$, then $a \in \mathfrak{B}(A')$ and therefore, $C(A') \subseteq \mathfrak{B}(A')$. We need to check that for the above specification of $\succeq$, $\pi : A \to P$, $\mathfrak{B}(A') \subseteq C(A')$. Suppose to the contrary, there exists $a' \in \mathfrak{B}(A')$ but $a' \notin C(A')$. It follows that $a' \succeq_{p'} b$ for all $b \in A'$ and $p' = \pi(a')$. Since $a' \notin C(A')$, by construction this is only possible if $a' \in C(A''_b)$ for some A''_b with $\{a', b\} \subseteq A''_b$. Let $A'' = \cup_{b \in A'} A''_b$. It follows that $a' \in A''$ and by Sen's axiom γ, $a' \in C(A'')$. As $A' \subseteq A''$ and $a' \in C(A'')$, by Sen's axiom α, $a' \in C(A')$ a contradiction. Therefore, $\mathfrak{B}(A') = C(A')$. ■

The violation of Arrow's axiom in a BDP comes from the fact that alternatives may not be irrelevant even when they are never chosen if the DM doesn't fully internalize the endogeneity of psychological states.

Note that in contrast to Proposition 2, Proposition 3 provides an axiomatic characterization of choice data compatible with any BDP whether admissible or not. Evidently, choice data generated by an admissible BDP will satisfy axioms α and γ. The following corollary characterizes a key requirement when choice data compatible with axioms α and γ are rationalized as the outcome of an admissible BDP.

Corollary 1. *Choice data satisfying Sen's axioms α and γ can be rationalized as the outcome of an admissible BDP if $\#A \geq 3$, $\#P \geq 2$.*

Proof. Assume that $\#P = 1$ with $P = \{p\}$. Consider the preference relation defined over actions $\succeq_p$ where $P = \{p\}$ and for each non-empty $A' \subseteq A$ and $a \in C(A')$, $\succeq_p$ satisfies the condition that $a \succeq_p a'$ for all $a' \in A'$ and $p = \pi(a)$. Suppose we require that choice data satisfying axioms α and γ have to be rationalized as the outcome of a BDP where for each $p \in P$, $\succeq_p$ is required to be transitive. Consider $A = \{a, b, c\}$, $C(\{a, b, c\}) = \{a\}$, $C(\{a, b\}) = \{a, b\}$, $C(\{a, c\}) = \{a\}$, $C(\{b, c\}) = \{c\}$ which satisfies axioms α and γ (but not β). Suppose it is required that this choice data be rationalized as the outcome of a BDP (i.e. $\mathfrak{B}(A') = C(A')$, $A' \subseteq A$) with $\#P = 1$, with $P = \{p\}$ where $\succeq_p$ is transitive. Then, $a \succeq_p b$, $a \succeq_p c$, $b \succeq_p a$, $c \succeq_p b$ so that as $\succeq_p$ is transitive, $c \succeq_p a$ and therefore, $C(\{a, b, c\}) = \{a, c\}$, a contradiction. It follows that $\#P > 1$. It also follows that in the proof part (ii) of Proposition 3, we must have that $\#P \geq \#A - 1$ so that whenever $\#A \geq 3$, $\#P \geq 2$. ■

Therefore, if choice data satisfies axioms α and γ and such data is rationalized as the outcome of a BDP where $\succeq_p$ is required to be transitive for each $p \in P$, then as long as $\#A \geq 3$, $\#P \geq 2$ so that the assumption that psychological states are endogenous to the actions of the DM is essential and can be inferred directly from choice data. Without the additional requirement that choice data satisfying axioms α and γ be rationalized as the outcome of an *admissible* BDP, it is without loss of generality to set $\#P = 1$ in part (ii) of the proof of Proposition 3.

Remark 1. (Maximal Psychological States) Is there a maximal collection of psychological states that can be identified from choice data? Two decision scenarios are equivalent if and only if (i) the unique ranking over actions induced by the ranking over consistent decision states in the two different decision scenarios is identical (so that these two rankings are normatively equivalent over actions), and (ii) the ranking over actions, relevant for the computation of BDP outcomes, is the same in the two decision scenarios (so that the two rankings are equivalent from a behavioral perspective over actions). Consider a fixed decision scenario $D = (A, P, \pi, \succeq)$. Consider also the decision scenario $D_{Id.} = (A, P = A, Id., \tilde{\succeq})$ ($Id.$ denotes the identity function from A to itself) where: (i) $(a, a) \tilde{\succeq} (a', a') \Leftrightarrow (a, \pi(a)) \succeq (a', \pi(a'))$ for all $a, a' \in A$, (ii) $(a, a) \tilde{\succeq} (a', a) \Leftrightarrow (a, \pi(a)) \succeq (a', \pi(a))$ for all $a, a' \in A$, with $\tilde{\succeq}$ arbitrarily defined otherwise. Then, $D_{Id.} = (A, P = A, Id., \tilde{\succeq})$ is, by construction, equivalent to $D = (A, P, \pi, \succeq)$. Given any decision scenario, there is an equivalent (both from a normative and behavioral perspective) decision scenario where the set of psychological states is the set of actions and the function π is the identity function. Label such a decision scenario as a maximal decision scenario and $P = A$ as the set of maximal psychological states. Taken together with Corollary 1, we obtain that whenever choice data (with $\#A \geq 3$) satisfies axioms α and γ and such data is required to be rationalized as the outcome of an admissible BDP, the number of psychological states that can be identified from choice data must lie between 2 and $\#A$.

Remark 2. (Related literature) There is an emerging literature that provides axiomatic characterizations of decision-making models with some specific behavioral flavor. Relevant contributions to this literature are Manzini and Mariotti (2007, 2012), Cherepanov et al. (2008) and Masatlioglu et al. (2012). We argue that a BDP is observationally distinguishable from each of these models on the basis of choice data alone. To start with, choice data consistent with the different procedures of choice proposed by each of these papers can account for pairwise cycles, while choice data consistent with BDP cannot: pairwise cycles of choice are simply inconsistent with Sen's axiom α and γ. For example, suppose $A = \{a, b, c\}$ and $C(A) = \{a\}$, $C(\{a, b\}) = \{a\}$, $C(\{b, c\}) = \{b\}$ but $C(\{c, a\}) = \{c\}$. This choice function can be rationalized, for example, by Manzini and Mariotti's (2012) Categorize then Choose (CTC) procedure of choice, but is not consistent with a BDP. The choice data would be consistent with BDP if, for example, $C(\{c, a\}) = \{c, a\}$. Moreover, the Rationalized Shortlist Method (RSM) proposed by Manzini and Mariotti (2007) cannot accommodate menu dependence, whereas a BDP can.

Like us, Masatlioglu et al. (2012) model of Limited Attention allows for violations of menu independence, but in a form very different from (and incompatible with) our characterization of BDP. They define a consideration set (a subset of the set of feasible alternatives) and assume that the DM only pays attention to elements in the consideration set. In their paper revealed preferences are defined as follows: an alternative x is revealed preferred to y if x is chosen whenever y is present and x is not chosen when y is deleted. That is, the choice of an alternative from a set should be unaffected if an element which is not in the consideration set is deleted. If choice changes when an alternative is deleted, then the latter alternative was in the consideration set and clearly the chosen alternative was revealed preferred to it. This is a violation of independence of irrelevant alternatives, but in a form that is incompatible with Sen's axiom α. Such data cannot be rationalized as an outcome of a BDP, precisely because in a BDP (and also in a SDP), if x is chosen whenever y is present, x must be chosen when y is deleted.

Remark 3. **(Revealed mistakes)** Is it possible to infer a conflict between choice and welfare using choice data alone? Here we propose a possible way to do this. Fix A the set of alternatives and a family $\mathcal{A}$ of non-empty subsets of A. Suppose, as before, that we observe a non-empty correspondence C from $\mathcal{A}$ to A such that $C(A') \subseteq A'$. Consider the following two scenarios:

Choice Scenario 1, the DM ranks each pair of non-empty subsets $A', A'' \subseteq A$;

Choice Scenario 2, which reveals a ranking over sets of actions in $\mathcal{A}$ as follows: for any $A' \subseteq A$ and non-empty $C(A')$ such that $C(A') \subset A'$, the set $C(A')$ is said to be weakly preferred to the set $A'/C(A')$.

Let $\mathcal{R}_1$ denote the binary ranking of pairs of non-empty subsets $A', A'' \subseteq A$ revealed in *Choice Scenario 1* and let $\mathcal{R}_2$ denote the binary ranking of pairs of non-empty subsets $A', A'' \subseteq A$ revealed in *Choice Scenario 2*. Clearly $\mathcal{R}_2$ is incomplete as might (though, obviously, not necessarily) $\mathcal{R}_1$.

The following proposition examines the conditions under which $\mathcal{R}_1$ and $\mathcal{R}_2$ coincide where both are defined and states the welfare implications when the two rankings do not coincide.

Proposition 4. *Suppose $\mathcal{R}_1$ and $\mathcal{R}_2$ do not necessarily coincide where both are defined. Then, the DM cannot be choosing in his best interests in one of the two choice scenarios.*

Proof. For each $A' \subseteq A$ and $A'' \subseteq A'$, we say that $A'' \in K(A')$ iff $A''\mathcal{R}_1 A'$ with $A'' \subseteq A'$. Clearly, if the DM is using a SDP in both choice scenarios (respectively, BDP), $C(A') \in K(A')$ so that $\mathcal{R}_1$ and $\mathcal{R}_2$ must coincide where both are defined. So suppose $\mathcal{R}_1$ and $\mathcal{R}_2$ do not necessarily coincide where both are defined. It follows that there exists $A' \subseteq A$ such that $C(A') \notin K(A')$ i.e. $(A'/C(A'))\,\mathcal{R}_1 A'$ but $A'\mathcal{R}_2\,(A'/C(A'))$ and $\sim (A'/C(A'))\,\mathcal{R}_2 A'$. As the DM cannot be using a SDP (respectively, BDP) in both choice scenarios assume that the DM is using a SDP in Choice Scenario 1. Then, $C(A') = \mathfrak{B}(A')$ for all A' and $C(A') \notin K(A')$ for some A'. Therefore, for some $A' \subseteq A$: (i) there exists a pair of actions $a, a' \in A'$ with $a' \in C(A)$ but $a \notin C(A')$, and (ii) $\pi : A \to P$ such that $(a, \pi(a)) \succ (a', \pi(a'))$

but both $(a', \pi(a)) \succ (a, \pi(a))$ and $(a', \pi(a')) \succeq (a, \pi(a'))$ so that a welfare dominates a' even though in Choice Scenario 2 the DM chooses a'. Conversely, suppose that the DM is solving a BDP in Choice Scenario 1. Then, $C(A') = \mathfrak{S}(A')$ for all A' and $C(A') \notin K(A')$ for some A'. Therefore, for some $A' \subseteq A$: (i) there exists a pair of actions $a, a' \in A'$ with $a' \in C(A')$ but $a \notin C(A')$, and (ii) $\pi : A \to P$ such that $(a', \pi(a')) \succ (a, \pi(a))$ but both $(a, \pi(a')) \succ (a', \pi(a'))$ and $(a, \pi(a)) \succeq (a', \pi(a))$ but so that a' welfare dominates a even though in Choice Scenario 1 the DM chooses a. ■

The following example clarifies the intuition behind Proposition 4. If a is smoking and a' is not-smoking, $\{a'\}$ is a situation in which the option of smoking is not available, and the only available option is "not-smoking" (i.e. go for dinner to a non-smoking restaurant) whereas $\{a\}$ is a situation in which the option of "not-smoking" is not available and the only available option is to smoke (i.e. go for dinner to a restaurant that only admits smokers).[22] In *Choice Scenario 1,* the DM is asked to choose between a situation where only action a is available and another one where only action a' is available i.e. between $\{a\}$ and $\{a'\}$. In *Choice Scenario 2,* the DM is asked to choose between the two actions used in the preceding pairwise comparison when both actions are available, i.e. actions in $\{a, a'\}$ for each such pair of actions. For example, choose between smoking and not smoking over dinner in a restaurant where both choices are available.[23] Proposition 4 says that whenever observed choices in the two scenarios are inconsistent, then the DM's observed choice in one of the two choice scenarios is welfare dominated.[24]

It is of interest to note that if DM is behavioral, Proposition 4 could hold even in those cases where choice data satisfies both Sen's axiom α and β (and hence, Arrow's axiom). This, at least, potentially qualifies the limits of a welfare analysis that is based solely on choice as in Bernheim and Rangel (2009).

[22] This type of scenarios could exist, for example, if there is a law that gives the option to owners of restaurants to decide whether to have "smoke free restaurants", i.e. $\{a'\}$ or "smoke friendly restaurants", i.e. $\{a\}$.

[23] This type of scenario can exist before the implementation of the law we referred to in the previous footnote.

[24] For a related but complementary approach to the construction of a measure of rationality and welfare see Apesteguia and Ballester (2011).

5 Concluding Remarks

Unlike much existing work that focuses on a specific behavioral procedure of choice to predict some specific behavior, this paper focuses on what choices can tell us when we allow for behavioral decision-making in a general framework. We provide an axiomatic characterization of the choice theoretical structure of a large set of seemingly disconnected behavioral procedures. We show that it is possible to identify a minimal and a maximal set of psychological states using choice data alone, and that under specific choice scenarios, "revealed mistakes" can be inferred directly from choice data.

Although it is not our main focus here, this paper sheds some light on the understanding of the normative implications of behavioral economics. Our axiomatic characterization has indeed normative implications. If choice data satisfy Sen's axioms α and γ but violate Arrow's axiom, then typically these data are generated by a DM who is not necessarily choosing in his best interest. Moreover, the DM can be systematically choosing against his best interest even if choice data satisfy Arrow's axiom (as the example of addiction in Section 2 shows). We also show that, in principle, it is possible to infer the divergence of choice and welfare based on choice data alone, although this can be done in very limited settings.

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5.1 Appendix 1: the dynamic interpretation

Predicting short-run, but not long-run, psychological states

So far we have assumed that DMs fail to anticipate that their future psychological state depend on their current choices including the immediate future. We will now extend our framework to situations where DMs may anticipate short-run psychological states that arise from their actions but not the long-run psychological states.

Let $h^2(p) = h(\pi(h(p)))$ and define $h^t(p) = h\left(\pi(h^{t-1}(p))\right)$ iteratively $t = 1, 2, \ldots$. Fix a $p_0 \in P$. A sequence of *short-run* outcomes compatible with T-period (for some fixed, finite $T \geq 1$) forecasting is determined by the relations $a_t \in h^T(p_{t-1})$ and $p_t = \pi(a_t)$, $t = 1, 2, \ldots$: at each step, the DM chooses a best-response that anticipates the short-run psychological states within a T-period horizon.

Long-run outcomes compatible with T-period forecasting are denoted by a pair a', p' with $p' = \pi(a')$ and a' is defined to be the steady-state solution to the short-run outcome function i.e. $a' = h^T(\pi(a'))$.

It follows that long-run behavior corresponds to the outcome of a BDP where the feedback effect is defined to be $\pi'(a) = \pi(h^{T-1}(a))$.

Partial prediction

Next, we extend our framework to situations where DMs may make partial prediction of changes in psychological states as a function of their chosen actions. There are many different ways of modelling partial prediction. We adopt a simple approach: we will assume that each decision maker predicts that the psychological state will respond to their chosen actions with probability q, $0 \leq q \leq 1$. It will be convenient at this point to assume that the binary relation $\succeq$ has a (expected) utility representation $u : A \times P \rightarrow \Re$. Let $v(a) = u(a, \pi(a))$.

Let $h(p; q) = \{a \in A : a \in \arg\max_{a \in A} qv(a) + (1 - q)u(a, p)\}$. In what follows, we will assume that that $h(p; q)$ is unique.

Fix a $p_0 \in P$. A sequence of *short-run* outcomes is determined by the relations $a_t \in h(p_{t-1}; q)$ and $p_t = \pi(a_t)$, $t = 1, 2, ...$: at each step, the DM chooses a myopic best-response.

Long-run outcomes are denoted by a pair a, p with $p = \pi(a)$ and a is defined to be the steady-state solution to the short-run outcome functions i.e. $a = h(\pi(a); q)$.

It follows that long-run behavior corresponds to the outcome of a BDP where the preferences are represented by a utility function $w(a, p) = qv(a) + (1 - q)u(a, p)$. This formulation is formally equivalent to the modelling of projection bias in Loewenstein et al. (2003).

Note that the above representation is consistent with incomplete learning: as long as the DM doesn't fully learn to internalize the feedback effect from actions to psychological states, there is a way of relabelling variables so that the steady-state preferences corresponding to an adaptive preference mechanism are the outcomes of a BDP.

6 Appendix 2: Proof of Proposition 1[25]

Recall that the preferences of the DM is denoted by $\succeq$ a binary relation ranking pairs of decision states in $(A \times P) \times (A \times P)$. As the focus is on incomplete preferences, in this section, instead of working with $\succeq$, we find convenient to specify two other preference relations, $\succ$ and $\sim$. The expression $\{(a, p), (a', p')\} \in \succ$ is written as $(a, p) \succ (a', p')$ and is to be read as "(a, p) is strictly preferred to (a', p') by the DM". The expression $\{(a, p), (a', p')\} \in \sim$ is written as $(a, p) \sim (a', p')$ and is to be read as "(a, p) is indifferent to (a', p') by the DM". Define

$$(a, p) \succeq (a', p') \Leftrightarrow \textit{either } (a, p) \succ (a', p') \textit{ or } (a, p) \sim (a', p').$$

Once $\succeq$ is defined in this way, the results obtained in the preceding sections continue to apply. In what follows, we do not require either $\succeq$ or $\succ$ or $\sim$ to be transitive.

[25] The seminal proof for existence of equilibria with incomplete preferences in Shafer and Sonnenschein (1975) requires convexity both for showing the existence of an optimal choice and using Kakutani's fix-point theorem.

Suppose $\succ$ is

(i) acyclic i.e. there is no finite set $\{(a^1, p^1), ..., (a^T, p^T)\}$ such that $(a^{t-1}, p^{t-1}) \succ (a^t, p^t)$, $t = 2, ..., T$, and $(a^T, p^T) \succ (a^1, p^1)$, and

(ii) $\succ^{-1}(a, p) = \{(a', p') \in A \times P : (a, p) \succ (a', p')\}$ is open relative to $A \times P$ i.e. $\succ$ has an open lower section[26].

Suppose both A and P are compact. Then, by Bergstrom (1975), it follows that S is non-empty.

Define

$$a \succ_p a' \Leftrightarrow (a, p) \succ (a', p).$$

The preference relation $\succ_p$is a map, $\succ: P \to A \times A$. If $\succ$ is acyclic, then for $p \in P$, $\succ_p$is also acyclic. If $\succ$ has an open lower section, then $\succ_p^{-1}(a) = \{a' \in A : a \succ a'\}$ is also open relative to A i.e. $\succ_p$ has an open lower section. In what follows, we write $a' \notin \succ_p (a)$ as $a \not\succ_p a'$ and $a' \in \succ_p (a)$ as $a' \succ_p a$.

Define a map $\Psi : P \to A$, where $\Psi(p) = \{a' \in A : \succ_p (a') = \varnothing\}$: for each $p \in P$, $\Psi(p)$ is the set of maximal elements of the preference relation $\succ_p$.

We make the following additional assumptions:

(A1) A is a compact lattice;

(A2) For each p, and a, a', (i) if $\inf(a, a') \not\succ_p a$, then $a' \not\succ_p \sup(a, a')$ and (ii) if $\sup(a, a') \not\succ_p a$ then $a' \not\succ_p \inf(a, a')$ (quasi-supermodularity);

(A3) For each $a \geq a'$ and $p \geq p'$, (i) if $a' \not\succ_{p'} a$ then $a' \not\succ_p a$ and (ii) if $a \not\succ_p a'$ then $a \not\succ_{p'} a'$ (single-crossing property)[27]

(A4) For each p and $a \geq a'$, (i) if $\succ_p (a') = \varnothing$ and $a' \not\succ_p a$, then $\succ_p (a) = \varnothing$ and (ii) if $\succ_p (a) = \varnothing$ and $a \not\succ_p a'$, $\succ_p (a') = \varnothing$ (monotone closure).

Assumptions (A2)-(A3) are quasi-supermodularity and single-crossing property defined by Milgrom and Shannon (1994).

[26] The continuity assumption, that $\succ$ has an open lower section, is weaker than assuming that that preferences have both open upper and lower sections (Debreu (1959)), which in turn is weaker than the assumption that preferences have open graphs. Note that assuming $\succ$ has an open lower section is consistent with $\succ$ being a lexicographic preference ordering over $A \times P$.

[27] For any two vectors $x, y \in \Re^K$, the ussual component-wise vector ordering is defined as follows: $x \geq y$ if and only if $x_i \geq y_i$ for each $i = 1, .., K$, and $x > y$ if and only if both $x \geq y$ and $x \neq y$, and $x \gg y$ if and only if $x_i > y_i$ for each $i = 1, .., K$.

Assumption (A4) is new. Consider a pair of actions such that the first action is greater (in the usual vector ordering) than the second action. For a fixed p, suppose the two actions are unranked by $\succ_p$. Then, assumption (A4) requires that either both actions are maximal elements for $\succ_p$or neither is.

The role played by assumption (A4) in obtaining the monotone comparative statics with incomplete preferences is clarified in Ghosal (2011) who also shows that assumptions (A1)-(A4), taken together, are sufficient to ensure that $\Psi(p)$ is non-empty and compact and monotone in p i.e. for $p \geq p'$ if $a \in \Psi_1(p)$ and $a' \in \Psi_1(p')$, then $\sup(a, a') \in \Psi_1(p)$ and $\inf(a, a') \in \Psi_1(p')$.

To complete the proof of Proposition 1, define a map $\Psi : A \times P \to A \times P$, $\Psi(a, p) = (\Psi_1(p), \Psi_2(a))$ as follows: for each (a, p), $\Psi_1(p) = \{a' \in A : \succ_p (a') = \phi\}$ and $\Psi_2(a) = \pi(a)$. It follows that $\Psi_1(p)$ is a compact (and consequently, complete) sublattice of A and has a maximal and minimal element (in the usual component wise vector ordering) denoted by $\bar{a}(p)$ and $\underline{a}(p)$ respectively. By assumption 1, it also follows that for each a, $\pi(a)$ has a maximal and minimal element (in the usual component wise vector ordering) denoted by $\bar{\pi}(a)$ and $\underline{\pi}(a)$ respectively. Therefore, the map $(\bar{a}(p), \bar{\pi}(a))$ is an increasing function from $A \times P$ to itself and as $A \times P$ is a compact (and hence, complete) lattice, by applying Tarski's fix-point theorem, it follows that $(\bar{a}, \bar{p}) = (\bar{a}(\bar{p}), \bar{\pi}(\bar{a}))$ is a fix-point of Ψ and by a symmetric argument, $(\underline{a}(p), \underline{\pi}(a))$ is an increasing function from $A \times P$ to itself and $\left(\underline{a}, \underline{p}\right) = \left(\underline{a}(\underline{p}), \underline{\pi}(\underline{a})\right)$ is also a fix-point of Ψ; moreover, $(\bar{a}, \bar{p})$ and $\left(\underline{a}, \underline{p}\right)$ are respectively the largest and smallest fix-points of Ψ.