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Abstract

Combining an intensive labor supply margin with an extensive, productivity-enhancing migration margin, we determine how regional inequality and labor mobility shape optimal redistribution. We propose the use of delayed optimal-control techniques to obtain optimal tax formulae with location-dependent productivity and two-dimensional heterogeneity. Our baseline simulations using the productivity differences between large metropolitan and other regions in the US indicate that productivity-increasing internal migration can constitute a quantitatively important constraint on redistribution. Allowing for regionally differentiated taxation with location-dependent productivity, we find that marginal tax rates in high- (low-)productivity regions should be corrected downwards (upwards) relative to a no-migration benchmark.

JEL classification: H11, J45, R12

Keywords: Optimal taxation, redistribution, regional inequality, migration, multidimensional screening, delayed optimal control

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1 Introduction

Regional productivity differences are large in many countries. Real per capita GDP in the New England Region was 40% higher than in the Southeast Region in the US in 2013 (BEA, 2014), for example. In Italy, the 2011 real per capita GDP of the Northern and Central Regions was even 71% higher than in the Southern and Islands Region (ISTAT 2013). The spatial dispersion of wages and incomes is well documented, and the underlying causes are still subject to debate (Barro and Sala-i-Martin 1991, Ciccone and Hall 1996, Kanbur and Venables 2005, Acemoglu and Dell 2010, and Young 2013, among others). Given such productivity differences, the efficiency-enhancing potential of interregional mobility is substantial, and increases in personal income are key drivers of this mobility; see Kennan and Walker (2011).

Centralized redistribution schemes such as a federal income tax or federal social transfers reduce interregional migration incentives, since an individual who migrates from a low- to a high-productivity region has to share the realized productivity gains with the government through higher taxes or lower transfers. This generates a trade-off for an inequality-averse policy maker between redistribution and productivity-enhancing interregional migration. Interestingly, this role of internal migration for optimal tax policy has been neglected in the literature, in stark contrast to the role of emigration of high-income earners to low-tax countries or the immigration of welfare recipients from less generous jurisdictions.[1] We develop a conceptual framework to analyze the implications of internal migration for an optimal tax-transfer policy and assess its quantitative importance. While our focus is on productivity-enhancing migration between regions with permanent productivity differences, our approach may also be used to address the related optimal-taxation problem that arises with respect to migration responses to idiosyncratic shocks to regional labor markets, as discussed by Blanchard and Katz (1992) and Yagan (2014).

[1] Studies addressing external migration include Mirrlees (1982), Wildasin (1991), Wilson (1992), Lehmann et al. (2014), and others.

We propose a two-dimensional optimal-taxation model which combines an extensive interregional migration decision with an intensive labor supply decision. Our key innovation is the productivity-enhancing nature of the migration margin. The actual or realized productivity of individuals of any given innate productivity is location-dependent, so that individuals can increase their productivity by migrating from a low- to a high-productivity region. Thus, the extensive migration margin also affects the intensive labor supply decision, since productivity and, unless the marginal tax rate is constant, the marginal tax rate change whenever an individual decides to migrate, even if the same tax schedule applies nationwide.

This framework allows us to determine the optimal federal tax schedule as a function of the government's redistributive preferences, the observed regional earnings distributions, the earnings elasticity, and the distribution of migration costs. Our analysis shows that optimal marginal tax rates tend to be below the benchmark without interregional migration, since the decision to migrate to an area with higher productivity bears a fiscal externality. Its size depends on migration costs, on the income distribution, and on the tax differential, which is itself a function of regional productivity differences and the tax schedule. If marginal tax rates are positive, the fiscal externality is positive, so that optimal marginal tax rates are lower than in a situation without migration. However, for some productivity distribution, the migration opportunity makes negative marginal tax rates optimal. The latter result is similar to other studies that have analyzed the optimal tax-transfer schedule with an extensive participation margin (see, e.g., Saez 2002 and Jacquet et al. 2013).

Our approach provides a methodological contribution to the theory of optimal taxation, in that we endogenize individuals' type along the productivity dimension through the extensive margin. This is a useful extension to the class of multidimensional screening models, originally discussed by Rochet and Choné (1998) and Armstrong (1996), and further developed to study the taxation of couples by Kleven et al. (2009) in the optimal-taxation context. We argue that variants of these models with endogenous productivity can be fruitfully studied using the delayed optimal-control approach as recently formally analyzed by Göllmann et al. (2008) in its entire generality. Variations of our framework are suitable to address a range of two-dimensional screening problems, where an extensive margin directly affects agents' productivity (their type), and thus the intensive margin. The decision to participate in the labor market, for example, affects productivity, because nonparticipation tends to depreciate human capital. Similarly, the decision to switch to a better job also affects actual individual productivity. The same holds true for discrete education decisions such as the decision to attend college. All these examples represent empirically relevant dimensions of the labor supply decision. Our framework and the proposed delayed optimal-control solution can be applied to the corresponding optimal-taxation problems.

We also study regionally differentiated tax-transfer schemes. To the extent that such schemes are explicit, they are often difficult to enforce in practice in view of the challenge of monitoring the actual place of residence of individuals, and may also be challenged on the grounds of the violation of horizontal equity. Despite these caveats, regional differentiation of labor income taxation can be an element of real-world tax systems. From 1971 to 1994, the German tax system, for example, treated residents in West Berlin differently from people in the rest of the country. Another example is the current path towards a more fiscally integrated Europe. As the EU is moving towards deeper fiscal integration, the option of a regionally differentiated (albeit coordinated) tax transfer remains an alternative to uniform EU-wide taxation (e.g., Lipatov and Weichenrieder 2015a). This decision requires understanding of the advantages and the challenges of a differentiated system vis-à-vis an integrated system.[2] Finally, nominally nondifferentiated federal income taxation amounts to regionally differentiated taxation in real terms, due to cost-of-living differences; see Albouy (2009). We show that within our optimal-taxation framework with productivity-increasing migration, the migration effect

[2] To the extent that the Member States are unrestricted by the center in deciding on their own tax-transfer schemes, additional considerations of tax competition have to be taken into account. See Lehmann et al. (2014) for the analysis of such considerations in the optimal-taxation framework. Bargain et al. (2013) have contrasted a Member-States-based redistribution scheme with an integrated scheme in Europe. However, they address the implications for macroeconomic stabilization, whereas we study the efficiency of redistribution.

exerts additional downward (upward) pressure on differentiated marginal tax rates in the high- (low-)productivity region. This is the opposite of the implied differentiation resulting from cost-of-living differences and nominally undifferentiated taxation.

Conceptually, we add to the debate on tagging in optimal taxation by considering the region of residence as an endogenous tag. Moreover, the tag can not only be used by the government to relax its information constraint, but differentiated taxation can also be used to encourage productivity-enhancing migration.

The next section discusses the related literature. Section 3 introduces the theoretical framework, and we derive our theoretical results for unified and differentiated taxation in Sections 4 and 5, respectively. We present a numerically calibrated illustrative simulation based on US micro data in Section 6 and leave the proofs to Appendices. A further Supplement is also available separately.

2 Related literature

The normative implications of productivity-enhancing internal migration for optimal redistribution have, to the best of our knowledge, not been studied to date. The constraint of interjurisdictional or international mobility for the redistribution policy of a single jurisdiction or country, however, has received considerable attention within the optimal-taxation literature and beyond; see, in particular, Mirrlees (1982), Wildasin (1991), Wilson (1992), Simula and Trannoy (2011), Lehmann et al. (2014), and Lipatov and Weichenrieder (2015b), among others. We show that labor mobility within a sufficiently large jurisdiction or between regions within a country can also be important for redistribution.

Conceptually, our analysis belongs to a class of two-dimensional screening models that have been recently used to analyze a range of tax policy questions. Lehmann et al. (2014) combine the intensive labor supply margin with an extensive migration margin. Their focus is on independent governments competing for internationally mobile high-productivity individuals, and it is therefore complementary to our analysis of optimal taxation by a single government. Moreover, individual productivity is not location-dependent in their analysis, and they only focus on the threat of migration, whereas actual, productivity-enhancing migration is at the heart of our approach. Gordon and Cullen (2012) also use an optimal-taxation approach to study interregional migration in a model with several states. However, they focus on the assignment problem of whether redistribution should be carried out at the national or the subnational level and do not consider productivity differences. Jacquet et al. (2013) also study a two-dimensional optimal-taxation model but focus on participation.

The structure of our approach owes much to Kleven et al. (2006, 2009), who study the optimal taxation of couples with cooperative households. Their analysis combines the intensive labor supply decision with the household's choice to become a single- or a double-earner household. However, our analysis differs in several important ways from their framework. First, we consider individuals and not households consisting of two persons whose incomes may be taxed separately. Secondly, in our approach, individuals originally reside in different regions, so they differ not only in costs of changing the location, but also in the group they originally belonged to. Finally and most importantly, we introduce a link between endogenous individual productivity and the decision along the extensive margin, and we show how models featuring this link can be solved using delayed optimal control.

Rothschild and Scheuer (2014), Rothschild and Scheuer (2013), and Gomes et al. (2014) also study optimal taxation of rent-seeking activities and optimal taxation in the Roy model, using two-dimensional screening approaches. Wages are endogenously determined in their work, either by the total labor supply in a given sector, or by total rent-seeking activities. Similarly, Scheuer (2014) studies entrepreneurial taxation with an endogenous decision of whether to become an entrepreneur or a worker, where these decisions determine relative compensation in the aggregate. In our study, individual productivity and thus market compensation depend directly on the discrete decision of individuals and not on aggregate outcomes. Accordingly, our argument for optimally adjusting marginal tax rates is not based on the attempt to manipulate relative wages, but on the wish to encourage productivity-enhancing regional mobility.

Our analysis of regionally differentiated taxation relates to the increased interest in tagging in the design of tax-transfer schemes. The idea that the government's information problem can be relaxed by using additional observable characteristics (*tags*) that are correlated with individual productivity goes back to Akerlof (1978) and has recently been discussed intensively in the optimal-taxation literature; see Immonen et al. (1998), Weinzierl (2012), Mankiw and Weinzierl (2011), Boadway and Pestieau (2005), Cremer et al. (2010), and Best and Kleven (2013). We add to this literature in several ways. First, we consider the region of residence as a potential tag. Secondly, we explicitly study a tag that is endogenous and can be adjusted by individuals subject to some cost. Moreover, changing the tag directly affects productivity. In this respect, our paper is related to the literature that studies the interplay between human capital formation and optimal taxation, where the former shapes the productivity distribution and the latter influences incentives for human capital formation; see Stantcheva (2015) and the references therein. The endogeneity of productivity also relates our work to that of Best and Kleven (2013), who consider a dynamic setting where individual productivity depends on the previous intensive labor supply decisions.

Albouy (2009) has argued that nondifferentiated nominal federal taxation effectively implies de facto regionally differentiated taxation due to cost-of-living differences. He reasons that such implicit differential taxation distorts the spatial allocation in the economy, and he analyzes the associated efficiency costs and the corresponding interregional redistribution, but his analysis does not consider the question of optimal redistribution between heterogeneous individuals. Our normative approach to regionally differentiated taxation can be regarded as complementary to his work, since we ask whether, and to what extent, federal taxes should be regionally differentiated for redistribution purposes, if such differentiation is possible. Finally, Eeckhout and Guner (2015) also study the effects of a progressive federal income tax on the spatial allocation of economic activity with a heterogeneous population, and also consider regionally differentiated taxation, but they do not use a Mirrleesian optimal-taxation framework and do not consider the interaction of the intensive labor supply decision and the interregional migration decision.

3 The framework

We consider two sources of heterogeneity across workers: *innate* productivity n and migration costs q. These original individual characteristics are distributed over $[n_{\min}, n_{\max}] \times [0, +\infty)$, and the government can observe neither of them. There are two regions, $i = A, B$, with total population normalized to two. Originally, half of the population resides in each region, but the endogenous migration decisions of individuals change these population shares. Our key assumption is that the regions differ in their productivity. An individual's *actual* or *realized* productivity n_i is a function of her innate productivity and her region of residence: $n_i = \omega(n, i) = \omega_i(n)$, where ω_i is strictly increasing in n. We normalize $n_A = \omega_A(n) = n$. Accordingly, the function $n_B = \omega_B(n) = \omega(n)$ not only assigns the actual productivity to all original residents of region B, but also indicates the transformation of productivity for individuals who migrate from A to B. Without loss of generality we assume that region B is the more productive region, so that $\omega(n) > n$. Innate productivity is distributed in each region i according to the unconditional probability distribution $f(n)$ on $[n_{\min}, n_{\max}]$.[3] As in most of the optimal-taxation literature, we treat wages as exogenous and independent of individual labor supply and aggregate migration decisions. Accordingly, the analysis applies to a situation where the effect of migration flows on wages is negligible. The empirical evidence supports the view that, for sufficiently large regions, the effects of internal migration on wages are rather small; see, for the US, Boustan et

[3] It is straightforward to extend the analysis to the case in which regions also differ in their distribution of innate productivity. Similarly, we could allow for negative migration costs for some subset of individuals at each innate productivity level without affecting the results qualitatively. The latter can generate migration in both directions. For clarity, we abstract from these further aspects.

al. (2010), and for evidence from the German reunification, D'Amuri et al. (2010) and Frank (2009).[4]

Following Diamond (1998), we use preferences that are separable in consumption and labor. The utility function of a worker of type (n, q) is similar to the formulation in Kleven et al. (2009), but depends on the region of residence:

$$u(c, z, l) = c_i - n_i h\left(\frac{z_i}{n_i}\right) - q^c l + q^h (1 - l), \tag{1}$$

where l is an indicator variable that takes the value 1 in case of migration. The function $h(\cdot)$ is increasing, convex, and twice differentiable. It is normalized so that $h'(1) = 1$ and $h(0) = 0$. The other variables have standard interpretations. Consumption c_i equals gross income z_i minus taxes T_i, the latter depending on gross income: $c_i = z_i - T_i(z_i)$. Total migration costs are potentially made up of two components: $q = q^c + q^h$, where q^c is the cost of moving (the need to adapt to new conditions, to learn a new language on moving between regions where different languages are spoken, the transaction costs of selling your old house and buying a new one, etc.), and q^h is the utility derived from being at home and benefiting from the existing social networks. To isolate the effects of the two types of heterogeneity, it is useful to consider them separately. The pure *cost-of-moving* model sets $q = q^c$ and $q^h = 0$; the pure *home-attachment* model uses $q = q^h$ and $q^c = 0$. Ex post (i.e., after migration has taken place), heterogeneity in q^c reflects the differences between individuals who migrate, whereas heterogeneity in q^h reflects the differences between individuals who stay in their home region. In what follows we focus on the cost-of-moving case, but, with some minor modifications, the home-attachment case is quite analogous. However, our optimal tax schedules and their derivations are sufficiently general to encompass both cases.

Each individual chooses l and z_i to maximize (1) for a given tax schedule, i.e., she decides whether to move or not and determines her gross earnings, given that

[4] For similar findings in the case of immigration of foreigners see Borjas (1994) and Ottaviano and Peri (2007, 2008).

she resides in region i. The first-order condition for gross earnings is

$$h'\left(\frac{z_i}{n_i}\right)=1-\tau_i\left(z_i\right), \tag{2}$$

where τ_i is the marginal tax rate. Accordingly, n_i can be interpreted as potential income, in that individuals facing a marginal tax rate of zero would realize this level of gross earnings. The elasticity of gross earnings with respect to net-of tax-rate is defined as

$$\varepsilon_i\equiv\frac{1-\tau_i}{z_i}\frac{\partial z_i}{\partial\left(1-\tau_i\right)}=\frac{n_ih'\left(\frac{z_i}{n_i}\right)}{z_ih''\left(\frac{z_i}{n_i}\right)},$$

and is a function of gross earnings and the region of residence. Finally, we require the following property:

Assumption The function $x\rightarrow\frac{1-h'(x)}{xh''(x)}$ is decreasing.

Consider now the migration decision. We denote by $p\left(q|\,n\right)$ the density of q conditional on n, and by $P\left(q|\,n\right)$ the cumulated distribution of q conditional on n. Conditional on residing in region i, the individuals' choice of gross earnings is determined by (2), which allows us to define indirect utility conditional on the place of residence and net of the migration costs as

$$V_i\left(n_i\right)=z_i-T_i\left(z_i\right)-n_ih\left(\frac{z_i}{n_i}\right).$$

Individuals will move from i to j, $j=A,B$, $i\neq j$, whenever their migration costs are below the net gain from moving, so that $\bar{q}_i\equiv\max\left\{V_j\left(n_j\right)-V_i\left(n_i\right),0\right\}$ is the critical level of migration costs that determines the actual number of migrants for any innate productivity level.

3.1 The government's optimal-tax problem

The government wants to maximize the social welfare function

$$\sum_i \int_{n_{\min}}^{n_{\max}} \int_0^{+\infty} \Psi\left(V_i\left(n\right) - q^c l + q^h\left(1-l\right)\right) p\left(q,n\right) f(n) dq dn, \tag{3}$$

where $\Psi(.)$ is a concave and increasing transformation of individual utilities. Denoting by E the exogenous expenditure requirement, it needs to respect the budget constraint

$$\sum_i \int_{n_{\min}}^{n_{\max}} \int_0^{+\infty} T_i(z_i) p\left(q,n\right) f(n) dq dn \geq E. \tag{4}$$

Moreover, the government's tax schedule needs to be incentive-compatible. Denoting by a dot above a variable its derivative with respect to n, this implies

$$\dot{V}\left(n\right) = \left[-h\left(\frac{z_i}{n_i}\right) + \frac{z_i}{n_i} h'\left(\frac{z_i}{n_i}\right)\right] \omega_i'\left(n\right) \geq 0. \tag{5}$$

Moreover, in case of nondifferentiated taxation, $T_A\left(z\right) = T_B\left(z\right)$. We show in the Supplement that a path for z_A and z_B can be truthfully implemented by the government using a nonlinear tax schedule.

Let $\lambda > 0$ be the multiplier associated with the budget constraint (4). The government's redistributive tastes may be represented by region-dependent social marginal welfare weights. In terms of income, our welfare weights will take the form

$$g_i(z) = \frac{\Psi'\left(V_i\left(z\right)\right)\left(1 - P(\bar{q}_i|z)\right) + \int_0^{\bar{q}_j} \Psi'\left(V_i\left(z\right) - q^c\right) p(q|z) dq}{\lambda\left(1 + P(\bar{q}_j|z) - P(\bar{q}_i|z)\right)}$$

for the cost-of-moving model, where $\bar{q}_i\left(z\right) \equiv \max\left\{V_j\left(z\right) - V_i\left(z\right), 0\right\}$.

4 Optimal unified taxation

We first investigate the optimal nondifferentiated tax-transfer system. The government maximizes (3) subject to (4) and (5) through its choice of $T(z)$. This problem formally amounts to a delayed optimal-control problem such as has been analyzed by Göllmann et al. (2008) in its entire generality. In our model, though, the delay is a nonfixed lag, given that we do not require the productivity gain from moving to be constant. The necessary conditions for optimal control in such a setting are presented in Abdeljawad et al. (2009). We describe in Appendix A how the delayed optimal-control approach can be applied to solve the optimal-taxation problem, and we derive all our results rigorously there. Below, however, we follow the intuitive perturbation approach pioneered by Piketty (1997) and Saez (2001) to derive the optimal tax scheme. This heuristic derivation disentangles the economic forces that determine the shape of the optimal marginal tax rate schedule, including the effects generated by productivity-enhancing migration.

We denote the endogenous distribution of gross incomes in both regions by $v_i\left(z_i\right)$, and we denote by k the endogenously defined function that maps gross income in the low-productivity region to the gross income this individual would earn in the high-productivity region, given his innate productivity and the respective tax treatment, i.e., $z_B = k(z_A)$.[5] We consider an increase in taxes for all individuals above gross income z. The increase is engineered through an increase in the marginal tax rate $d\tau$ in the small band $(z, z + dz)$, such that for all individuals with gross earnings above z the tax payments increase by $dzd\tau$. This tax increase gives rise to three different effects.

Revenue effect All taxpayers in either region with gross incomes above z pay additional taxes of $dzd\tau$. The net welfare effect of this tax payment for an affected individual in region i with gross earnings z' is given by $dzd\tau\left(1 - g_i\left(z'\right)\right)$, and the total effect is then

$$R = dzd\tau \int_z^{\infty} \left\{\left[1 - g_A\left(z'\right)\right] v_A\left(z'\right) s_A\left(z'\right) + \left[1 - g_B\left(z'\right)\right] v_B\left(z'\right) s_B\left(z'\right)\right\} dz',$$

where $s_A\left(z\right) \equiv 1 - P\left(\left.\bar{q}_A\right| z\right)$ and $s_B\left(z\right) \equiv 1 + P\left(\left.\bar{q}_A\right| k^{-1}(z)\right)$.

Behavioral effect Individuals in the band $(z, z + dz)$ will change their labor supply in response to the increase in the marginal tax rate. Given that $\varepsilon \equiv \frac{1-\tau}{z}\frac{dz}{d(1-\tau)}$, each individual in the band will reduce its income by $-d\tau\varepsilon\frac{z}{1-\tau}$. There

[5] In terms of our previous formulation, an individual of ability n receives gross income $z_A = z(n)$ in region A and gross income $z_B = z(\omega(n))$ in region B, where this notation abstracts from the fact that the gross income also depends on the tax schedule.

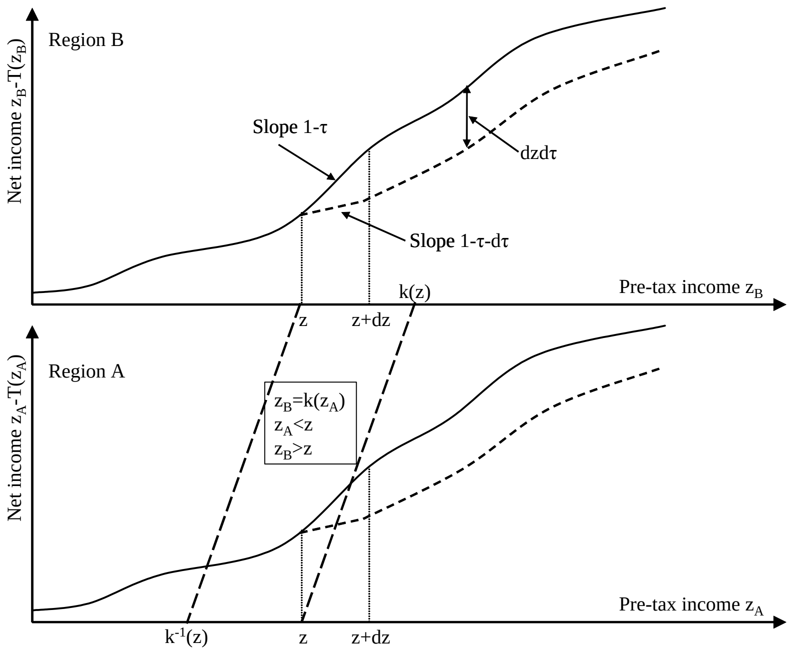

Figure 1: The migration effect comes into play for individuals for which $z'_A < z$ and $z'_B \geq z$.

are approximately $dz\left[v_A(z)s_A(z) + v_B(z)s_B(z)\right]$ of these individuals. The total effect on tax revenue is

$$L = -d\tau dz \frac{\tau z \varepsilon}{1-\tau}\left[v_A(z)s_A(z) + v_B(z)s_B(z)\right].$$

Migration effect An increase in taxes for all individuals above gross income z does not affect the migration decision of individuals with gross income $z'_A \geq z$, and accordingly also $z'_B > z$, such that the tax increase affects them in both regions alike. The same holds true for all individuals for which $z'_B = k(z'_A) < z$ and accordingly $z'_A = k^{-1}(z'_B) < z$. However, as illustrated in Figure 1, for all individuals for which $z'_A < z$ and $z'_B \geq z$ the migration decision is negatively affected. In this range, all individuals whose cost of moving is between $\bar{q} - dzd\tau$ and $\bar{q}$ will now decide not to migrate. There are $p\left(\bar{q}\middle|\, z\right) v_A(z) dzd\tau$ affected individuals at any concerned level of income, with a resulting tax effect of $T_A(z) - T_B(k(z))$

for each of them. The total migration effect is thus

$$M = d\tau dz \int_{\tilde{z}}^{z} \left[T(z') - T(k(z'))\right] p\left(\bar{q}|\, z'\right) v_A(z')dz',$$

where $\tilde{z} \equiv k^{-1}(z)$. Note that there is an endogenous effect on the income distribution in each region. This effect does not come into play explicitly here, since we express the effects in terms of the posterior distribution. The three effects must balance out in the optimum: $R + L + M = 0$. From this we have our first result.

Proposition 1 *The optimal unified tax schedule is characterized by*

$$\begin{aligned}
\frac{\tau}{1-\tau} &= \mathfrak{A}(z)\mathfrak{B}(z)\left[\mathfrak{C}(z) + \mathfrak{D}(z)\right], \ \textit{where} \qquad (6)\\
\mathfrak{A}(z) &\equiv \frac{1}{\varepsilon}, \qquad \mathfrak{B}(z) \equiv \frac{1}{z\left(v_A(z)s_A(z) + v_B(z)s_B(z)\right)},\\
\mathfrak{C}(z) &\equiv \int_z^{\infty} \left\{\left[1 - g_A\left(z'\right)\right] v_A\left(z'\right) s_A + \left[1 - g_B\left(z'\right)\right] v_B\left(z'\right) s_B\right\} dz',\\
\mathfrak{D}(z) &\equiv \int_{\tilde{z}}^{z} \left[T(z') - T(k(z'))\right] p\left(\bar{q}|\, z'\right) v_A(z')dz'.
\end{aligned}$$

Proof. This follows from the exposition above. The equivalence to the optimal, tax formula formally derived by using the delayed optimal-control technique is presented in Appendix A. ■

It is straightforward to compare the result with the alternative benchmark without migration. The optimal tax schedule then follows the usual Diamond (1998) and Saez (2001) results for the earnings distribution in the entire country without a migration effect. In this case, optimal marginal tax rates are determined by

$$\frac{\tau}{1-\tau} = \mathfrak{A}(z)\mathfrak{B}(z)\mathfrak{C}(z). \qquad (7)$$

With $\mathfrak{D}(z) < 0$, the disincentive effects of higher tax rates on productivity-increasing mobility tend to reduce marginal tax rates, but note that $\mathfrak{B}(z)$ and $\mathfrak{C}(z)$ are endogenously determined by the migration flows, so that (6) and (7) cannot be directly compared in general. To make the result more formal, we consider the benchmark in which the government faces the same distribution of realized productivity v and of population shares s as in the posterior situation generated by the optimal tax schedule with migration. Given this posterior distribution, assume that there is, or the government believes so, no reaction with regard to location choice from the tax system, i.e., that the posterior distribution is fixed and individuals react to the taxation through their intensive labor supply margin only. In this case the optimal tax follows the formula (7) with terms $\mathfrak{A}(z) > 0, \mathfrak{B}(z) > 0, \mathfrak{C}(z) > 0$ identical to the ones in (6). In this case, for $\mathfrak{D}(z) < 0$, we have $\tau_m(z) < \tau_o(z)$, where the subscripts m and o indicate the migration and the no-migration case, respectively. This allows us to formulate the following proposition:

Proposition 2 *If marginal tax rates are positive, a government taking the effect of taxes on the migration decision into account sets lower marginal tax rates than a government which disregards the migration decision, but faces the same posterior income distribution generated by migration.*

Proof. See appendix. ■

Note that positive marginal tax rates are a sufficient but not a necessary condition for this result. Whenever $\mathfrak{D} < 0$ for any given level of gross income z, marginal tax rates are lower with migration than for the no-migration benchmark with the posterior distribution. Thus, whenever productivity-enhancing migration implies a positive fiscal externality at a given innate productivity level, marginal tax rates should be reduced to take the fiscal externality of interregional migration appropriately into account. This constrains optimal redistribution beyond the classic adverse labor supply responses.

Another direct implication of the optimal unified taxation formula (6) is stated in the following proposition.

Proposition 3 *Optimal marginal tax rates can be negative.*

Proof. For $\mathfrak{D}(z) < 0$, it is possible that $\mathfrak{C}(z) + \mathfrak{D}(z) < 0$, and thus $\tau < 0$.[6] ■

[6] By simulative example (available upon request) it can be shown that this is the case for certain parameter values.

Similarly to the findings of other studies that combine an extensive participation decision with the intensive labor supply decision, our setting potentially gives rise to negative marginal tax rates.

The posterior distribution (as in Proposition 2) is our preferred benchmark, as it allows switching migration on and off while keeping the productivity distribution fixed. This benchmark also corresponds directly to the empirically observed spatial distribution of individuals and productivity at a given point in time. Accordingly, we also focus on it in our simulations in Section 6. However, for completeness, another benchmark to compare our optimal solution with is an economy with the ex ante distribution of productivity and without internal migration. As we show in the Appendix, the comparison of optimal marginal tax rates is less clear-cut in this case. The endogeneity of the posterior productivity distribution when allowing for migration also affects $\mathfrak{B}(z)$ and $\mathfrak{C}(z)$, and these effects may drive optimal marginal tax rates in the opposite direction. Formally, we provide a sufficient condition for mobility to decrease the marginal tax rates for this alternative benchmark in Appendix AA.

Finally, we make the following remark about the welfare comparison in the unified taxation case, highlighting the desirability of productivity-enhancing migration.

Remark The welfare achieved with unified taxation in the no-migration case is not higher than the welfare achieved with migration.

Proof. Consider the tax schedule that maximizes welfare if migration is not allowed. Migration brings a Pareto improvement, because individuals move only if they find themselves better off. Furthermore, with migration to the richer region only, the government budget constraint will not be violated, since the tax schedule is nondecreasing in income without migration. Thus, under the same tax schedule the welfare cannot decrease with the introduction of a migration possibility. Finally, the government will change the tax schedule only if it brings a further increase in welfare. Thus, the welfare with migration cannot be lower than welfare with no migration. ■

5 Optimal differentiated taxation

We now consider the possibility that the central government can choose differentiated tax schedules for both regions. If there were regional productivity differences but no migration, this setting would correspond to the analysis of an optimal tax scheme with tagging on the region of residence. However, we continue to assume that migration between the regions is possible, that productivity is location-dependent, and that individuals are heterogeneous with respect to their migration costs, which are unobservable by the government. Again we employ the perturbation approach and delegate the formal proofs to Appendix A.

We first study the optimal tax schedule in the low-productivity region. Consider an increase of taxes in region A for all individuals above gross income z_A. The increase is engineered through an increase in the marginal tax rate $d\tau_A$ in the small band $(z_A, z_A + dz_A)$, such that all individuals with gross earnings above z_A increase their tax payments by $dz_A d\tau_A$. This generates three effects.

Revenue effect: All taxpayers in A with incomes above z_A pay additional taxes of $dz_A d\tau_A$. The net welfare effect of this tax payment for an individual with gross earnings z'_A is given by $dz_A d\tau_A \left(1 - g_A\left(z'_A\right)\right)$, and the total effect is

$$R_A = dz_A d\tau_A \int_{z_A}^{\infty} \left[1 - g_A\left(z'_A\right)\right] v_A\left(z'_A\right) s_A\left(z'_A\right) dz'_A.$$

Behavioral effect: Individuals in the band $(z_A, z_A + dz_A)$ will change their labor supply in response to the increase in the marginal tax rate. Given that $\varepsilon \equiv \frac{1-\tau_i}{z_i} \frac{dz_i}{d(1-\tau_i)}$, each individual in the band will reduce its income by $-d\tau_A \varepsilon \frac{z_A}{1-\tau_A}$. There are approximately $dz_A v_A(z_A) s_A(z_A)$ of these individuals, so that the total effect on tax revenue is

$$L_A = -d\tau_A dz_A \varepsilon \frac{\tau_A}{1-\tau_A} z_A v_A(z_A) s_A(z_A).$$

Migration effect: An increase in taxes for all individuals above gross income z_A affects the migration decision of individuals with gross income in region A above this level. At any income level $z \geq z_A$ individuals whose cost of moving is between

$\bar{q}$ and $\bar{q} + dz_A d\tau_A$ will now decide to migrate. There are $p(\bar{q}|\, z_A)\, v_A(z_A) dz_A d\tau_A$ affected individuals, with a resulting tax effect of $T_B(k(z_A)) - T_A(z_A)$ for each of them. If the schedule results in migration from region B for people of income z, the argument is analogous, as we show formally in the Appendix. The total effect is thus

$$M_A = dz_A d\tau_A \int_{z_A}^{\infty} \left[T_B(k(z'_A)) - T_A(z'_A)\right] p(\bar{q}|\, z'_A)\, v_A(z'_A) dz'_A.$$

In the optimum, these effects should cancel out, so that optimal marginal tax rates can be characterized by

$$\begin{aligned} \frac{\tau_A}{1-\tau_A} &= \frac{1}{\varepsilon}\frac{1}{z_A v_A(z_A) s_A(z_A)} \\ &\quad \times \int_{z_A}^{\infty} \left\{\left[1 - g_A(z'_A)\right] s_A(z'_A) + \left[T_B(k(z'_A)) - T_A(z'_A)\right] p(\bar{q}|\, z'_A)\right\} v_A(z'_A) dz'_A. \end{aligned} \tag{8}$$

We turn now to the optimal tax schedule in the high-productivity region. We consider a small increase in taxes by $dz_B d\tau_B$ for all individuals above z_B in region B. This again generates three effects, which must balance out along the optimal tax schedule, so that

$$\begin{aligned} \frac{\tau_B}{1-\tau_B} &= \frac{1}{\varepsilon}\frac{1}{z_B v_B(z_B) s_B(z_B)} \\ &\quad \times \int_{z_B}^{+\infty} \left\{\left[1 - g_B(z'_B)\right] s_B(z'_B) - \left[T_B(z'_B) - T_A(k^{-1}(z'_B)\right] p(\bar{q}|\, z'_B)\right\} v_B(z'_B)\, dz'_B. \end{aligned} \tag{9}$$

Both optimal tax schedules are derived rigorously in Appendix A. The optimal tax formulae not only differ in the average welfare weights and in the corresponding productivity distributions above the relevant gross income level, but they also take the fiscal migration externality into account. Typically, this externality will be negative for the high-productivity region and positive for the low-productivity region. Accordingly, from these optimal tax schedules (8) and (9) we have the following result.

Proposition 4 *For all levels of innate productivity and the corresponding gross* *incomes, the marginal tax rate τ_A in the low-productivity region is increasing in the difference in total tax liability between the high- and the low-productivity regions, and the marginal tax rate τ_B in the high-productivity region is decreasing in this difference in total tax liability.*

Proof. The result follows directly from (8) and (9). ■

Intuitively, the larger the potential fiscal gains are from working in the high-productivity region instead of working in the low-productivity region, the more the government distorts labor supply in the low-productivity region and the less it distorts labor supply in the high-productivity region. In the Supplement we additionally rearrange the formulae (8) and (9) to show how the migration semielasticities act as a correction factor to the region-dependent marginal social welfare weights to determine optimal marginal tax rates.

Next we consider the asymptotic properties of the differentiated tax schedule. Suppose the distribution of innate ability, $f(n)$, has an infinite tail ($n_{\max} = \infty$). As is standard in the literature, we assume that $f(n)$ has a Pareto tail with parameter $a > 1$ ($f(n) = C/n^{1+a}$). Moreover, we assume that $P(q|n)$, $T_B - T_A, \tau_A, \tau_B$, $\bar{q}_A, \bar{q}_B$ converge to $P^\infty(q)$, $\Delta T^\infty, \tau_A^\infty < 1, \tau_B^\infty < 1$, $\bar{q}_A^\infty, \bar{q}_B^\infty$ as $n \to \infty$. Finally, we concentrate on the case where, for sufficiently large n, we have $\omega(n) = n + c$, where $c \geq 0$ is a finite constant. In this case, the following proposition arises:

Proposition 5 *Under the assumptions on convergence formulated above, (i) the average marginal social welfare weights in the two regions converge to the same value $\bar{\psi}/\lambda \geq 0$; (ii) the difference between the taxes in the two regions converges to zero, $\Delta T^\infty = 0$; and (iii) the marginal tax rate in both regions converges to τ^∞ with*

$$\frac{\tau^\infty}{1 - \tau^\infty} = \frac{1}{a\varepsilon^\infty}\left(1 - \frac{\bar{\psi}}{\lambda}\right). \tag{10}$$

Proof. *See Appendix AA.* ■

The intuition for zero difference of top taxes is similar to that in Kleven et al. (2009). Starting from a wedge between T_B and T_A, say, with $T_B > T_A$, welfare could be increased by marginally reducing this wedge due to the migration effect.

In particular, consider a reform that increases T_A and decreases T_B above some n, while keeping the tax revenue constant in the absence of migration. The direct welfare effects of such a reform cancel out, because g_A and g_B have converged to g^∞. The fiscal effects due to the earnings responses cancel out as well, because ΔT^∞ is constant and thus the labor supply elasticities are identical. The fiscal effect due to migration is however positive, because some people would move to region B and pay higher taxes. If the starting wedge is characterized by $T_B < T_A$, the welfare can be improved by the opposite reform. Thus, though there are substantial differences between a differentiated and a unified tax schedule, they disappear in the limit at the top of the distribution.

Finally, we consider how tax rates should be differentiated if a full differentiation of the entire tax schedule is not feasible. First, we study a tax system that is separable in the sense that the same income faces the same marginal tax in both regions, but allowing for region-dependent total tax liabilities. The maximization problem of the government is as before, except that instead of the restriction that $\Delta T = 0$ we have $\Delta T = C$, where C is constant and $\Delta T := T_B - T_A$. For this setting, we find:

Proposition 6 *Starting from a unified taxation schedule in the two regions, if the government is allowed to make a lump-sum transfer between regions, it will choose to make a transfer from the less productive to the more productive region.*
Proof. *See Appendix AA.* ■

While this result may appear surprising, there is a clear economic intuition behind it. The tax on the poor region has to be higher in order to induce extra migration, which is productivity-enhancing. The extensive margin is used to increase efficiency via increased labor mobility, whereas redistribution is engineered through the intensive margin. This result is independent of the interpretation of migration costs.

Moreover, consider a tax schedule that is separable in the sense that $\tau_A = \tau_B$. Starting from this schedule, the following proposition shows that in the cost-of-moving model decreasing the marginal tax in region B and increasing it in region

A would be desirable:

Proposition 7 *If Ψ' is convex, q and n are independently distributed, and $\omega'(n) \geq 1$, then it is optimal to introduce some wedge in marginal taxes into the system of separable taxation of the two regions. In particular, in the cost-of-moving model it is optimal to decrease the marginal tax in the high-productivity region and increase it in the low-productivity region.*

Proof. *See Appendix AA.* ■

The proof is based on the fact that in the cost-of-moving model the difference in marginal welfare weights of residents of regions A and B is decreasing in the productivity, if the social welfare exhibits prudence (marginal social welfare is convex). Thus, it makes sense to make the lower part of the productivity distribution in region A marginally happier than in region B, while making the upper part of productivity distribution in region B marginally happier than in region A. Hence, lower marginal tax rates in region B are optimal. The productivity transformation function $\omega(n)$ may however reverse this finding, if migration in the lower part of the distribution is subject to substantially larger productivity gains than migration in the upper part of the distribution, i.e., $\omega'(n) < 1$; hence the condition on this function.

6 Simulation and Calibration

In this section we provide numerical simulations for the US, to gain insights into the quantitative importance of productivity-enhancing migration for the design of tax policy and optimal redistribution.[7] We first focus on the difference between an optimal unified tax schedule with and without migration for a given posterior productivity distribution as in Propositions 1 and 2. To apply our framework empirically, we divide the US into a high-productivity region (large metropolitan) and a low-productivity region (other). We use the observable income distribution

[7] Our simulations use a modified and extended version of the code developed by Henrik Kleven, Claus Thustrup Kreiner, and Emmanuel Saez applied in Kleven et al. (2009). We would like to express our gratitude to them for providing their original code to us.

to recover the underlying productivity distributions in both regions, as well as the implied migration gains for workers of different innate productivity. We then simulate the unified optimal tax formulae with and without productivity-enhancing migration for the posterior productivity distribution to gauge the difference between them. Finally, we also simulate the optimal differentiated tax schedules for the high- and low-productivity regions. In what follows, we first specify functional forms and parameters used in the simulations, and then describe the calibration.

6.1 Simulation specification

For simulations we use isoelastic utility $h(\frac{z}{n}) = (\frac{z}{n})^{1+\epsilon}/(1+\epsilon)$ with a constant earnings elasticity $\varepsilon = \frac{1}{\epsilon}$ as in Saez (2001). Paralleling our theoretical derivations, we concentrate on the cost-of-moving model; hence $q = q^c$. Moreover, we follow Kleven et al. (2009) by assuming a power law distribution for the costs at the extensive margin on the interval $[0, q_{\max}]$ with $P(q) = (q/q_{\max})^{\eta}$ and $p(q) = (\eta/q_{\max}) \cdot (q/q_{\max})^{\eta-1}$. This distribution of q is the same in each region and independent of n, that is, $\partial q_{\max}/\partial n = 0$. The parameter η may be interpreted as a migration elasticity of the form $\eta = \frac{\bar{q}}{P(\bar{q}|n)} \frac{\partial P(\bar{q}|n)}{\partial \bar{q}} = \frac{\bar{q} p(\bar{q}|n)}{P(\bar{q}|n)}$. We use $\eta = 1.5$, and additionally consider lower and higher values in a range between $\eta = 0.2$ and $\eta = 2$ to assess the sensitivity of the results to this parameter, since there are no reliable estimates for the migration elasticity between more and less urbanized regions. As the social objective function, we use the constant rate of risk aversion (CRRA) function $\Psi(V) = V^{1-\gamma}/(1-\gamma)$, where the parameter γ measures the government's preference for equity. We choose $\gamma = 1$; hence $\Psi(V) = \log(V)$ in line with Chetty (2006).

6.2 Calibration to the US

We proceed with the calibration of our economy to the US in four steps. First, we choose regions by focusing on the considerable productivity discrepancy between regions with different levels of urbanization in the US. To do this, we draw on the Rural Urban Continuum Code (RUCC, also known as the Beale code) that is

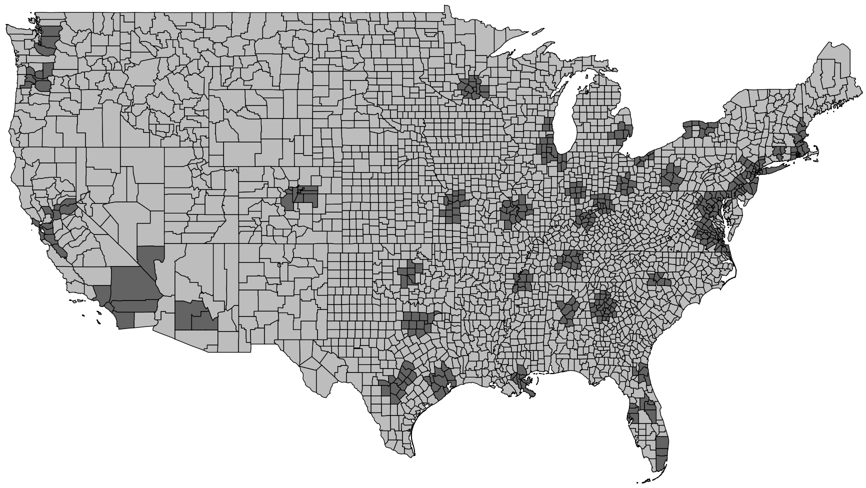

Figure 2: Split of the mainland US districts into two regions: region B consists of the more metropolitan counties (dark), region A of all others (light). Boundaries are taken from US Census Bureau (census.gov: Cartographic Boundary Shapefiles).

provided by the US Department of Agriculture. The RUCC assigns each county to one of nine classes. Starting with highly urban counties central in a metropolitan area and with a population of more than 1 million (class 1), the code goes up to 9 for completely rural counties that are not adjacent to a metropolitan area and/or exhibit a population of less than 2,500. The Panel Study of Income Dynamics (PSID) provides the RUCC for each individual's county of residence. We define all counties belonging to class 1 as the large metropolitan areas (region B), and counties of classes 2 through 9 as other areas (region A), as illustrated by Figure 2.

Second, we recover the ability distributions for these regions using individuals' maximization as given by Equation (2) with earnings elasticity $\varepsilon = 0.25$ as suggested by Saez (2001). Specifically, we combine the 2008 individual gross labor income data from the 2009 PSID for working head of the householdes with the corresponding marginal tax rate from the NBER TAXSIM model.[8] This procedure is similar to that of Best and Kleven (2013), who differentiate individuals by age, whereas we use a regional distinction. As suggested by Diamond (1998) and Saez (2001), very high incomes are well approximated by a Pareto distribution. There-

[8]The NBER TAXSIM model v9 is applied, see http://www.nber.org/taxsim/.

fore, the skill distributions are modified by assuming a Paretian shape for each top 5% of the respective distribution corresponding to abilities above \$184,717 (\$130,271) in the large metropolitan (other) areas, and we estimate the specific Pareto parameter for both regions.[9]

The procedure to recover the productivity distribution is as follows: Each individual's innate ability is computed from the individual utility maximization, using her income data from the PSID, and the actual marginal tax rate corresponding to this income level from TAXSIM together with the earnings elasticity ε and the functional assumption for $h(\frac{z}{n})$. For all potential incomes above \$184,717 (\$130,271), the computed ability is then replaced by the respective Pareto value. Figure 3A depicts the computed skill distributions in regions A and B. The resulting descriptive statistics for both areas exhibit a median ability difference of 17.1% and a mean difference of 38.5% between the large metropolitan and other areas.

Third, we estimate the transformation function $\omega(n)$ from the regional ability distributions. We compute the difference in the mean ability in each per-mille of the distribution. This difference is assumed to be the productivity increase for the mean person (sampling point) of each per-mille. The lag function $\kappa(n) = \omega(n) - n$ is then estimated by linear interpolation using the sampling points, and it is presented in Figure 3B.[10] We find a substantial productivity increase from migration to a large metropolitan district from elsewhere, in particular for the types with potential annual incomes of around \$100,000–\$250,000 at their origin. For high ability levels (above \$250,000) we make the additional assumption that $\kappa(n)$ is constant, because of the lack of sampling points in this range. The cutoff productivity level of \$250,000 implies that we use more than 99% of the total mass of the productivity distribution.

Fourth, the migration cost distribution is calibrated using the migration elas-

[9]Denoting by n_m the average ability above, or equal to, ability n, the estimation of the respective Pareto parameter is conducted by regressing n_m/n on a constant in the interval of ability levels between the top 10% and the top 5%. This interval corresponds to ability levels between \$135,699 (\$93,487) and \$184,717 (\$130,271).

[10]The Supplement provides a detailed justification of our empirical construction of ω.

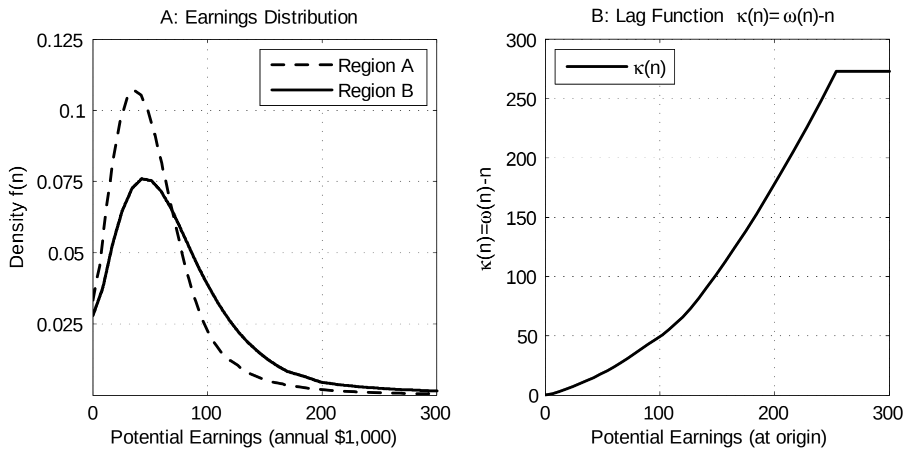

Figure 3: Revealed true abilities (plot A) and revealed lag function $\kappa(n)$ (plot B) for the two chosen regions of the US, based on data from PSID 2008/09 and TAXSIM. Revealed abilities are continously smoothed.

ticity η and the parameter $q_{\max}$. We choose $q_{\max}$ such that the average moving costs across all productivity levels amount to approximately $8,500.[11] Finally, the simulation is done in a way such that, with the optimal tax rates obtained, the ratio of exogenous budget expenditures E to aggregate production is 0.25 as in Saez (2001).

6.3 Results for nondifferentiated taxation

Figure 4 illustrates the simulation outcomes for potential earnings up to $500,000. In the no-migration case (dashed line in Figure 4A), which uses the posterior productivity distribution, the government chooses a higher marginal tax rate than in the migration case (solid lines), in line with Proposition 2. For low income levels, there is no difference in the marginal tax rate schedules. Migration-induced productivity increases are small in this income range, and tax liabilities are rather low as well, so that the migration effect is negligible. However, this is no longer true for higher levels of income, on account of the higher productivity gains and progressive taxation.

[11] Our analysis considers migration gains on a yearly basis. Interpreting migration costs as a perpetually accruing disutility and applying a reasonable discount rate, this cost level corresponds to the range of migration-cost estimates obtained by Kennan and Walker (2011) or Bayer and Juessen (2012).

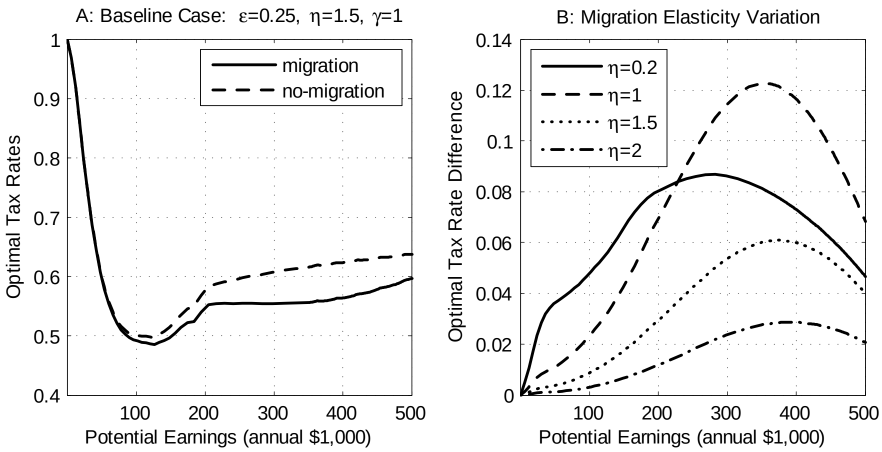

Figure 4: Optimal uniform tax simulations for the US, based on data from PSID 2008/09 and TAXSIM (panel A), and tax rate differences of the optimal schedules with and without migration for different values of the migration elasticity (panel B).

Moreover, differences in the regional productivity distributions also matter for the difference between the two schedules, since they also determine the size of the migration effect and the corresponding fiscal externality. As Figure 4A (baseline case) shows, the migration effect reduces marginal tax rates by up to six percentage points in the higher income range.

In Figure 4B we plot the marginal tax rate differences between the migration and the no-migration case for alternative values of the migration elasticity. This illustrates that the relationship between the tax rate reduction due to productivity-enhancing migration relative to the no migration case is quite sensitive to the migration elasticity, and that, overall, this relationship is not monotone. Consistently with the theory, the differences in marginal tax rates are reduced for sufficiently low values of η, as can be seen from the comparison of the tax rate differences for $\eta = 0.2$ and $\eta = 1$ in 4B, respectively.[12] Note also from 4B that the marginal tax rate differences may be substantially more pronounced than the six percentage points in our baseline case, exceeding twelve percentage points for $\eta = 1$ for some ability levels.

Finally, further simulations show that, as usual, a higher labor supply elasticity lowers marginal tax rates throughout the ability distribution, and that this also

[12] Trivially, the differences in marginal tax rates completely disappear for $\eta \to 0$.

reduces the gap between the migration and the no-migration tax schedule (not shown). Intuitively, lower progressivity tends to reduce the interregional difference in tax payments between individuals of the same innate productivity, so that the fiscal migration externality is reduced.

6.4 Results for differentiated taxation

The results for differentiated taxation are depicted in Figures 5 and 6. We use the same baseline parameter settings for migration costs and the exogenous government expenditure requirement E as before. Figure 5A illustrates the case of differentiated taxation without migration (fixed-residence case). This is a useful benchmark, since it corresponds to the standard tagging case with an exogenous tag. The marginal tax rates in the high-productivity region are very similar to those in the low-productivity region for low levels of potential income. However, in the range of potential incomes between approximately \$85,000 and \$350,000 the mass of individuals in region B is substantially larger than in region A, so that the larger potential for behavioral adjustments reduces optimal marginal tax rates in the more productive region below the level of marginal tax rates in the low productivity region. We compare this fixed residence benchmark with the other three graphs of Figure 5, where we allow for productivity-increasing migration and display the optimal differentiated marginal tax rate schedules for different values of the migration elasticity.

Figure 5B is based on a migration elasticity $\eta = 1.5$. In comparison to the fixed-residence case, marginal tax rates are higher for intermediate abilities and for individuals with high potential earnings in region A. The opposite is true in region B, where marginal tax rates are substantially reduced. Additionally, we display the optimal marginal tax rates for a lower and a higher migration elasticity, $\eta = 2$ and $\eta = 0.5$, respectively, in Figure 5C and 5D. In all cases U-shaped marginal tax rate patterns reappear for both regions, but the regional tax rates are substantially different. For all analyzed values of the migration elasticity, regional marginal tax rates are relatively closely together at low potential income levels, but they begin

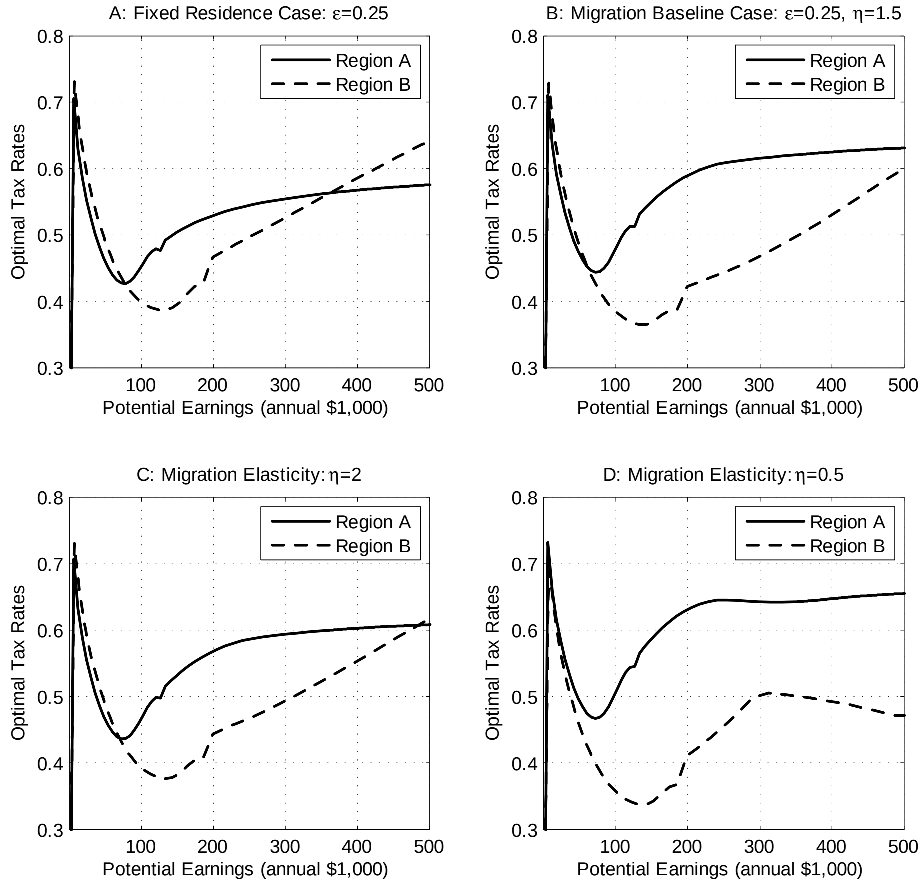

Figure 5: Optimal differentiated tax simulations for the US based on data from PSID 2008/09 and TAXSIM.

to diverge sharply for potential earnings above $50,000. Above this level, marginal tax rates in the less productive region are substantially higher than those in the more productive region, and only converge again for very high levels of potential income. This differentiation illustrates the downward pressure of the migration effect on marginal tax rates in the high-productivity region and the reverse pressure in the low-productivity region as suggested by Proposition 4. Thus, only for very low or high abilities do the marginal tax rates in region B approximate those in region A, whereas for a very substantial intermediate income range lower tax rates in region B are optimal.

With a lower migration elasticity of $\eta = 0.5$, the difference in marginal tax

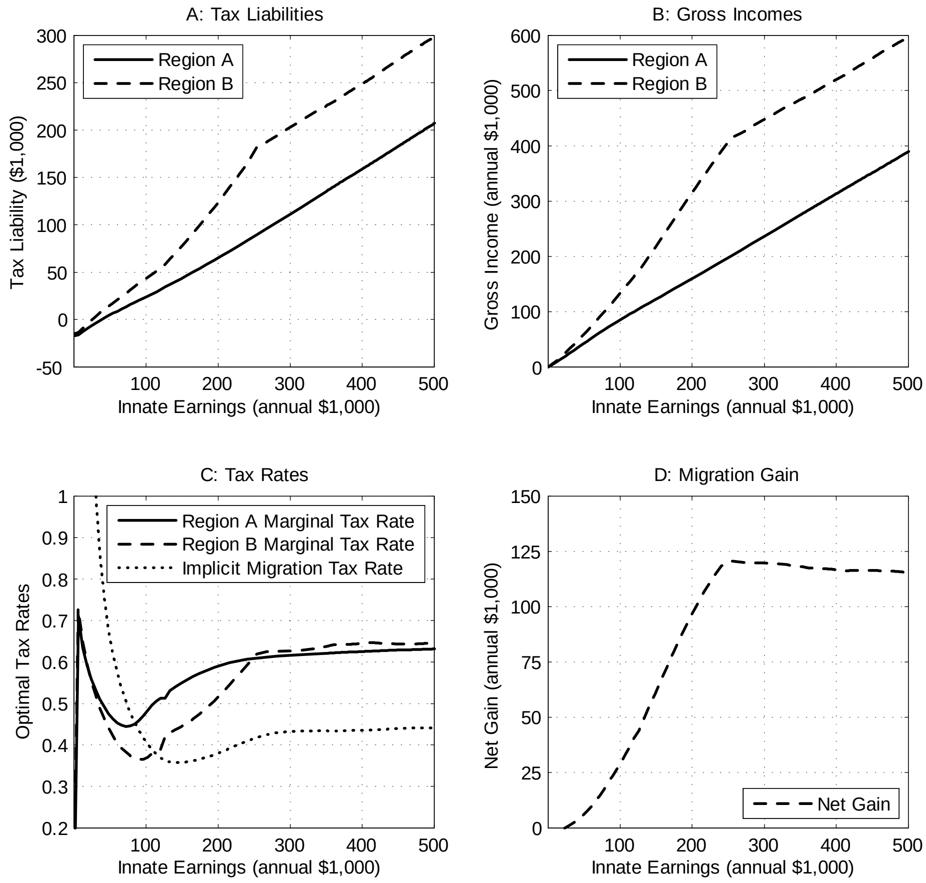

Figure 6: Further simulation results for the migration baseline case of differentiated taxation, based on PSID and TAXSIM. Parameter values for all illustrations: $\varepsilon = 0.25$, $\eta = 1.5$, $\gamma = 1$.

rates is even more pronounced and increases to more than 20 percentage points (Figure 5D) whereas tax rate convergence occurs only at substantially higher ability levels (not shown). For a higher migration elasticity, as displayed in Figure 5C, the regional marginal tax rate spread is reduced, but still larger than in the fixed residence case. To sum up the comparison to the fixed residence case, productivity-enhancing migration substantially alters the shape of optimal differentiated taxation relative to the benchmark of a purely exogenous tag of fixed productivity differences.

Figure 6 uses *innate earnings* – defined as potential income with innate productivity, which corresponds to potential income in the low-productivity region

– to compare the optimal tax treatment of individuals of the same innate ability in the two regions. This takes into account that individuals will have a different realized productivity and correspondingly a different potential income in the high-productivity region. We depict optimal absolute tax liabilities in Figure 6A and gross incomes in Figure 6B. We observe a basic income support of roughly \$15,000 (\$17,000) in region B (A), which is taxed away quickly in both regions as earnings increase. Gross incomes begin to considerably diverge from about \$50,000 innate earnings and show a roughly constant surplus in the metropolitan region that occurs around \$250,000 innate earnings.

Figure 6C shows the same simulation results as Figure 5B, but considers the optimal marginal tax rate schedules as functions of innate earnings. This allows us to compare the optimal marginal tax rates an individual of a given innate productivity should face in both regions. In addition, we show the ratio of tax liability differences to realized income differences between the two regions, which is the implicit migration tax rate. The implicit migration tax is greater than one for low ability levels, but falls quickly over the range of \$30,000–\$100,000 in innate earnings. It is roughly constant at about 44% above these levels of innate earnings. This corresponds to the evidence from Figure 6A and B, which indicate almost constant differences of the tax liability schedules and the gross incomes for high earnings levels.

Finally, we display the absolute migration gain net of taxes (net gain) in Figure 6D. The displayed pattern corresponds to the lag function $\kappa(n)$ (Figure 3B), but additionally takes the effects of differentiated taxation into account. It is evident that, except for the lowest innate earnings levels, there is a positive and increasing net gain from migration, until the constant level of κ is reached.

It is interesting to compare our optimally differentiated tax schedule with the actual regional differentiation of income taxation due to cost-of-living differences. Albouy (2009) argues that undifferentiated federal income taxation taxes inhabitants of urban agglomerations effectively more heavily than those of rural areas, resulting in a spatial distortion of economic activity. Our analysis of optimal redistribution with location-dependent productivity suggests that optimally differentiated taxation should bear more heavily on low-productivity regions, with the exception of the very low and very high income earners. Thus, the de facto differentiation in favor of low-productivity regions appears to be the opposite of an optimally differentiated tax system. The current implicit regional tax differentiation in the US may therefore additionally be criticized from our redistribution perspective with location-specific productivity.

7 Concluding remarks

Regional inequality and the corresponding possibility of productivity-enhancing migration can be an important determinant of the optimal redistributive tax-transfer scheme. The migration possibility implies a fiscal externality that modifies the equity-efficiency trade-off beyond the intensive labor supply margin, and our simulations indicate that this additional constraint to redistribution can be quantitatively important. However, our analysis abstracts from a number of potentially important aspects. First, we restrict the central government to the use of a unified or regionally differentiated tax scheme; it is not allowed to use targeted subsidies to migrants only. If such targeted transfers were available to the government, they could potentially loosen the trade-off between redistribution and internal migration. However, a fixed migration subsidy typically cannot eliminate the problem completely, since the fiscal migration externality differs by earnings level. Even if the migration subsidy could be adjusted by earnings level, the adjustments would have to be incentive-compatible, implying an additional constraint for tax policy. Secondly, regional productivity differences are partly reflected in local prices of nontradable goods, rents, and house prices, which also reduces migration incentives. While this is important, it does not challenge the main economic intuition: The fiscal migration externality still exists and should be taken into account. Finally, we have used a static framework. One may argue that migration also contains an inherently dynamic aspect. At the same time, to the extent that the migration costs are recurring costs, say, because the disutility of being in a less preferred region accrues every period, our approach easily maps into a dynamic framework. However, further research may explore how the potentially stochastic evolution of regional productivity differences and the option of repeated migration may qualify our results.

Our results have immediate policy implications. Tax policy should not only worry about external migration, but also needs to consider the potential role of productivity-increasing internal migration for a progressive tax-transfer schedule. This constraint is less important for countries that are characterized by low regional inequality. However, in countries where regional inequality is substantial, policy makers should carefully assess how tax progressivity may hurt productivity-enhancing interregional migration and design their tax-transfer schemes accordingly.

8 Appendix A: derivation of the optimal tax formulae

We now show that the optimal tax formulae (6) and (8) and (9) can also be rigorously derived by standard optimal control techniques. The equivalence of the expression in terms of n and z is shown in Supplement D. We start with the differentiated case, since the unified case can be interpreted as the same problem with the additional constraint of the tax schedules to be identical in both regions.

8.1 Regionally differentiated taxation

The government maximizes

$$\begin{aligned} W &= \int_{n_{\min}}^{n_{\max}} [\int_{\bar{q}_B}^{+\infty} \Psi\left(V_B\left(\omega(n)\right) + q^h\right) p(q|n)dq + \int_0^{\bar{q}_A} \Psi\left(V_B\left(\omega(n)\right) - q^c\right) p(q|n)dq \\ &\quad + \int_{\bar{q}_A}^{+\infty} \Psi\left(V_A\left(n\right) + q^h\right) p(q|n)dq + \int_0^{\bar{q}_B} \Psi\left(V_A\left(n\right) - q^c\right) p(q|n)dq] f(n)dn, \end{aligned}$$

where $\bar{q}_A = \max\left\{V_B\left(\omega(n)\right) - V_A\left(n\right), 0\right\}$, $\bar{q}_B = \max\left\{V_A\left(n\right) - V_B\left(\omega(n)\right), 0\right\}$, $q = q^c + q^h$, and either $q^c = 0$ or $q^h = 0$. The first term in this expression stands for the social welfare from the population of region B who did not move, the second term stands for that of the population moved from A to B, the third term is for those who stayed in A, and the fourth term is for those who moved from B to A. Note that either the second or the fourth term is equal to zero, because migration in both direction at the same ability level is not possible. The maximization is subject to

$$\int_{n_{\min}}^{n_{\max}}[\left(z_B-\omega(n)h\left(\frac{z_B}{\omega(n)}\right)-V_B(\omega(n))\right)(1+P(\bar{q}_A|n)-P(\bar{q}_B|n))$$
$$+\left(z_A-nh\left(\frac{z_A}{n}\right)-V_A\right)(1+P(\bar{q}_B|n)-P(\bar{q}_A|n))]f(n)dn\geq E$$

and the corresponding incentive compatibility constraints. Note that either $P(\bar{q}_A|n)$ or $P(\bar{q}_B|n)$ is zero for the same reason as discussed above.

Let the Hamiltonian be $H(z_A,z_B,V_A,V_B,\lambda,\mu_A,\mu_B,n)$. The necessary conditions are

1. There exist absolutely continuous multipliers $\mu_A(n),\mu_B(n)$ such that on $(n_{\min},n_{\max})$ $\dot{\mu}_B(n)=-\frac{\partial H(n)}{\partial V_B(n)}$, $\dot{\mu}_A(n)=-\frac{\partial H(n)}{\partial V_A(n)}$ almost everywhere with $\mu_i(n_{\min})=\mu_i(n_{\max})=0$.

2. We have $H(z_i(n),V_i,\lambda,\mu_i,n)>H(z_i,V_i,\lambda,\mu_i,n)$ almost everywhere in n for all z. The first order conditions are $\frac{\partial H}{\partial z_A}=0$, $\frac{\partial H}{\partial z_B}=0$.

Uniqueness of z_A and z_B that solve the equations above can be established in a similar way to Kleven et al. (2009), using the assumption that $\varphi(x)=(1-h'(x))/(xh''(x))$ is decreasing in x. The FOCs for Region A and B are presented in Supplement A, where we also lead the reader through the derivation steps and provide additional representations of the optimal tax formulae:

$$\frac{\tau_A}{1-\tau_A}=\frac{1}{nf(n)\varepsilon_A(1+P(\bar{q}_B|n)-P(\bar{q}_A|n))}\int_n^{n_{\max}}[\left(1-g_A(n')\right)\left(1+P(\bar{q}_B|n')-P(\bar{q}_A|n')\right)$$
$$-(T_A-T_B)\left(p(\bar{q}_B|n')+p(\bar{q}_A|n')\right)]f(n')dn',$$

$$\frac{\tau_B}{1-\tau_B}=\frac{1}{\omega(n)f(n)\varepsilon_B(1+P(\bar{q}_A|n)-P(\bar{q}_B|n))}\int_n^{n_{\max}}[\left(1-g_B(n')\right)\left(1+P(\bar{q}_A|n')-P(\bar{q}_B|n')\right)$$
$$-(T_B-T_A)\left(p(\bar{q}_B|n')+p(\bar{q}_A|n')\right)]f(n')dn'$$

for the marginal rates in A and B, respectively. The formulae are similar to Kleven et al (2009) except that two terms (rather than one) reflect the possibility of either immigration to or emigration from the given region. Note that for each n, there are two mutually exclusive scenarios: either there is migration from A to B (and $V_B(\omega(n))>V_A(n)$).

8.2 Delayed optimal control

For the next section, we will need some results in delayed optimal control theory. In particular, we adapt Göllmann et al. (2008) setting that adresses delayed arguments of constant size. We consider variable "delays" of the size $\omega(n)-n$ in both the state variable $x(n)$ (in our case this is $V(n)$) and in the control variable $u(n)$ (in our case this is $z(n)$). Consider a welfare maximization problem. Because social welfare depends on the indirect utilities only, our objective function will not depend on control variables.

We have the following retarded optimal control problem (ROCP):

$$J(u,x) = \int_a^b L\left(n, x(n), x(\omega(n))\right) dn,$$

subject to the differential equation (ICC in our context, can be formulated without delayed variables and without states on the rhs) and inequality constraint (government budget)

$$\begin{aligned} \dot{x}(n) &= f\left(n, u(n)\right), n \in [a,b] \quad &(11a)\\ \int_a^b C\left(n, u(n), u(\omega(n)), x(n), x(\omega(n))\right) dn &\geq 0. &(11b)\end{aligned}$$

For convenience, the functions

$$\begin{aligned} L &: [a,b] \times \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R} \\ f &: [a,b] \times \mathbb{R}^n \to \mathbb{R} \\ C &: [a,b] \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R} \end{aligned}$$

are assumed to be twice continuously differentiable wrt all arguments. A pair of functions (u,x) is called an admissible pair for the problem (ROCP) if the state x and control u satisfy (11a) and (11b). An admissible pair $(\hat{u},\hat{x})$ is called locally optimal pair or weak maximum for (ROCP), if

$$J(u,x) \leq J\left(\hat{u},\hat{x}\right)$$

hold for all (u,x) admissible in a neighbourhood of $(\hat{u},\hat{x})$. Define the Hamiltonian for (ROCP) as

$$H\left(n, x, x^w, u, u^w, \mu, \lambda\right) := L\left(n, x, x^w\right) + \lambda C\left(n, u, u^w, x, x^w\right) + \mu f\left(n, u\right).$$

Göllmann et al (2008) show that a necessary condition for (u,x) to be locally optimal is existence of a costate absolutely continuous function $\mu : [a,b] \to R$, a multiplier function $\lambda : [a,b] \to R$ such that the following conditions hold for all $t \in [a,b]$:

(i) adjoint differential equation

$$\begin{aligned} \dot{\mu}(n) &= -\frac{\partial H\left(n\right)}{\partial x} - \mathbb{I}_{[\omega(a),\omega(b)]}\frac{\partial H\left(\omega^{-1}(n)\right)}{\partial x^w} \\ &= -\frac{\partial H\left(n, x(n), x\left(\omega(n)\right), u(n), u\left(\omega(n)\right), \mu(n), \lambda(n)\right)}{\partial x} \\ &-\mathbb{I}_{[\omega(a),\omega(b)]}\frac{\partial H\left(n, x(\omega^{-1}(n)), x\left(n\right), u(\omega^{-1}(n)), u(n), \mu(\omega^{-1}(n)), \lambda(\omega^{-1}(n))\right)}{\partial x^w}, \end{aligned}$$

(ii) transversality condition

$$\mu(a) = \mu(b) = 0,$$

(iii) local maximum condition

$$\frac{\partial H(n)}{\partial u} + I_{[\omega(a),\omega(b)]} \frac{\partial H\left(\omega^{-1}(n)\right)}{\partial u^w} = 0,$$

(iv) nonnegativity of multiplier and complementarity slackness.

8.3 Non-differentiated tax schedule

In this case the tax schedules in two regions must be identical, and hence the indirect utilities also are (there are no differences in preferences). The government problem is to maximize

$$\begin{aligned} W &= \int_{n_{\min}}^{n_{\max}} [\int_0^{+\infty} \Psi\left(V\left(\omega(n)\right) + q^h\right) p(q|n)dq + \int_0^{\bar{q}} \Psi\left(V\left(\omega(n)\right) - q^c\right) p(q|n)dq \\ &+ \int_{\bar{q}}^{+\infty} \Psi\left(V\left(n\right) + q^h\right) p(q|n)dq] f(n)dn, \end{aligned}$$

where $\bar{q} = V\left(\omega(n)\right) - V\left(n\right)$, and either q^h or q^c is equal to zero. We have also dropped the subscript B from the omega function for more parsimonious notation. The maximization is subject to

$$\begin{aligned} &\int_{n_{\min}}^{n_{\max}} [\left(z\left(\omega(n)\right) - \omega(n)h\left(\frac{z\left(\omega(n)\right)}{\omega(n)}\right) - V\left(\omega(n)\right)\right)\left(1 + P(\bar{q}|n)\right) \\ &+ \left(z - nh\left(\frac{z}{n}\right) - V\right)\left(1 - P(\bar{q}|n)\right)] f(n)dn \geq E, \end{aligned}$$

where the superscript w stands for the individuals with productivity $\omega(n)$. Note that in the uniform case there cannot be migration from B to A, as this would imply $V\left(\omega(n)\right) < V\left(n\right)$ that contradicts incentive compatibility (the productivity type $\omega(n)$ can pretend to have productivity n without any costs).

Let the Hamiltonian be $H(z, z^w, V, V^w, \lambda, \mu, n)$. This is a delayed optimal control problem analogous to the one formally analyzed by Göllmann et al. (2008) in its entire generality. The difference is that whereas Göllmann et al. have a lag of fixed size over the whole domain of their functions, our lag is a smooth increasing function of n, namely $\omega(n) - n$. The necessary conditions for optimal control in such a setting is presented in Abdeljawad et al (2009). Namely, in our context the necessary conditions for the maximum are:

1. There exist absolutely continuous multipliers $\mu(n)$ such that on $(n_{\min}, n_{\max})$ $\dot{\mu}(n) = -\frac{\partial H(n)}{\partial V(n)} - \mathbb{I}_{[\omega(n_{\min}),\omega(n_{\max})]} \frac{\partial H\left(\omega^{-1}(n)\right)}{\partial V^w(n)}$ almost everywhere with $\mu(n_{\min}) = \mu(n_{\max}) = 0$.

2. We have $H(z(n), z^w(n), V, V^w, \lambda, \mu, n) > H(z, z^w, V, V^w, \lambda, \mu, n)$ almost everywhere in n for all z. The first order condition is

$$\frac{\partial H}{\partial z} + I_{[\omega(n_{\min}),\omega(n_{\max})]} \frac{\partial H\left(\omega^{-1}(n)\right)}{\partial z_B^w} = 0$$

The fact that this condition describes a global maximum can be established similar to Kleven et al. (2009), using the assumption that $\varphi\left(x\right) = \left(1 - h'\left(x\right)\right) / \left(xh''\left(x\right)\right)$ is decreasing in x.

$$\frac{\mu}{n}\frac{z}{n}h''\left(\frac{z}{n}\right)+\lambda\left(1-h'\left(\frac{z}{n}\right)\right)\left(\left(1-P(\bar{q}|n)\right)f(n)+\left(1+P(\bar{q}_1|\omega^{-1}(n))\right)f\left(\omega^{-1}(n)\right)\right)=0$$

$$\varphi\left(\frac{z}{n}\right)=-\frac{\mu(n)}{\lambda n\left(\left(1-P(\bar{q}|n)\right)f(n)+\left(1+P(\bar{q}_1|\omega^{-1}(n))\right)f\left(\omega^{-1}(n)\right)\right)}$$

LHS is decreasing in z_i/n whereas RHS is constant, which implies that $z_i(n)$ is a unique solution and a global maximum indeed. Continuity can be then established in a way similar to Kleven et al (2009). Further derivation is presented in the Supplement A, whereby we arrive at

$$\int_{\omega^{-1}(n)}^{n_{\max}}\left(1-g_B\left(n'\right)\right)\left(1+P(\bar{q}|n')\right)f\left(n'\right)dn'+\int_{n}^{n_{\max}}\left(1-g_A(n')\right)\left(1-P(\bar{q}|n')\right)f(n')dn'$$
$$-\int_{\omega^{-1}(n)}^{n}\left(T\left(\omega(n')\right)-T(n')\right)p(\bar{q}|n')f\left(n'\right)dn'=\frac{\tau}{1-\tau}\times$$
$$\times n\varepsilon\left(\left(1-P(\bar{q}|n)\right)f(n)+\left(1+P(\bar{q}_1|\omega^{-1}(n))\right)f\left(\omega^{-1}(n)\right)\right).$$

Additional interpretation of this formula is also left to the Supplement A.

9 Appendix AA: Further propositions and proofs

9.1 Proof of Proposition 2

Proof. Going through the derivation of the optimal tax formula in the Appendix A under the assumption that the effect of tax on migration decision is neglected, i.e. $\frac{\partial \bar{q}}{\partial z}=\frac{\partial \bar{q}}{\partial V}=0$, we arrive at the optimal tax formula (**??**) short of the term

$$-\int_{\omega^{-1}(n)}^{n}\left(T\left(\omega(n')\right)-T(n')\right)p(\bar{q}|n')f\left(n'\right)dn'.$$

If this is non-positive (that is equivalent to $\mathfrak{D}(z)\leq 0$), the result immediately follows. If marginal tax rates are positive, this condition is always fulfilled. ■

9.2 Proof of Proposition 5

Proof. Under the assumptions formulated in the text, $V_A(n)$ and $V_B(n)$ are increasing in n without bound, because $\tau_A^{\infty}<1,\tau_B^{\infty}<1$. As $\Psi'>0$ is decreasing, it converges to some $\bar{\psi}\geq 0$. Then, we have

$$g_A(n)=\frac{\int_{\bar{q}_A}^{+\infty}\Psi'\left(V_A\left(n\right)+q^h\right)p(q|n')dq+\int_0^{\bar{q}_B}\Psi'\left(V_A\left(n\right)-q^c\right)p(q|n)dq}{\lambda\left(1+P(\bar{q}_B|n)-P(\bar{q}_A|n)\right)},$$
$$g_B\left(n\right)=\frac{\int_{\bar{q}_B}^{+\infty}\Psi'\left(V_B\left(\omega(n')\right)+q^h\right)p(q|n')dq+\int_0^{\bar{q}_A}\Psi'\left(V_B\left(\omega(n)\right)-q^c\right)p(q|n)dq}{\lambda\left(1+P(\bar{q}_A|n)-P(\bar{q}_B|n)\right)},$$

which converge to

$$g_A^{\infty} = g_B^{\infty} = \frac{\bar{\psi}}{\lambda}.$$

If $T_B - T_A$ converges, it must be that $\tau_A^{\infty} = \tau_B^{\infty} = \tau^{\infty}$. But since

$$h'\left(\frac{z_i}{n_i}\right) = 1 - \tau_i\left(z_i\right),$$

z_i/n_i converges and hence elasticities converge to the same limit ε^{∞}. Moreover, $\lim_{n\to\infty} \frac{z_A}{n} = \lim_{n\to\infty} \frac{z_B}{\omega(n)}$. Because $P(q|n)$ and $\bar{q}_A, \bar{q}_B$ converge, $P(\bar{q}_i|n)$ and $p(\bar{q}_i|n)$ converge to $P^{\infty}(\bar{q}_i^{\infty})$ and $p^{\infty}(\bar{q}_i^{\infty})$. The Pareto distribution implies that $(1 - F(n))/(nf(n)) = 1/a$ in the tail. Take the limit of our optimal tax formulae to get

$$\begin{aligned}
\frac{1}{a\varepsilon^{\infty}}[1 - \frac{\bar{\psi}}{\lambda} + \frac{\Delta T^{\infty}\left(p^{\infty}(\bar{q}_B^{\infty}) + p^{\infty}(\bar{q}_A^{\infty})\right)}{1 + P^{\infty}(\bar{q}_B^{\infty}) - P^{\infty}(\bar{q}_A^{\infty})}] &= \frac{\tau^{\infty}}{1-\tau^{\infty}}, \\
\frac{1}{a\varepsilon^{\infty}}[1 - \frac{\bar{\psi}}{\lambda} - \frac{\Delta T^{\infty}\left(p^{\infty}(\bar{q}_B^{\infty}) + p^{\infty}(\bar{q}_A^{\infty})\right)}{1 + P^{\infty}(\bar{q}_A^{\infty}) - P^{\infty}(\bar{q}_B^{\infty})}] &= \frac{\tau^{\infty}}{1-\tau^{\infty}}
\end{aligned}$$

for the marginal rates in Region A and B, respectively. The right hand sides are equal, so we need $\Delta T^{\infty} = 0$ for the left hand sides to be equal as well.[13] ■

9.3 Proof of Proposition 6

Proof. The maximization problem of the government with the restriction that $\Delta T = C$ is constant in n is

$$\begin{aligned}
W &= \int_{n_{\min}}^{n_{\max}} [\int_0^{+\infty} \Psi\left(V\left(\omega(n)\right) + q^h\right) p(q|n)dq + \int_0^{\bar{q}} \Psi\left(V\left(\omega(n)\right) - q^c\right) p(q|n)dq \\
&\quad + \int_{\bar{q}}^{+\infty} \Psi\left(V\left(n\right) + q^h + C\right) p(q|n)dq] f(n)dn,
\end{aligned}$$

where $\bar{q} = V\left(\omega(n)\right) - V\left(n\right) - C$, and either q^h or q^c is equal to zero, and we assume C is small enough not to induce "reverse" migration (to the low productivity region). The maximization is subject to

$$\begin{aligned}
&\int_{n_{\min}}^{n_{\max}} [\left(z\left(\omega(n)\right) - \omega(n)h\left(\frac{z\left(\omega(n)\right)}{\omega(n)}\right) - V\left(\omega(n)\right)\right)(1 + P(\bar{q}|n)) \\
&+ \left(z - nh\left(\frac{z}{n}\right) - V - C\right)(1 - P(\bar{q}|n))] f(n)dn \geq E.
\end{aligned}$$

[13] Note that the optimal tax formula for the uniform case simplifies to

$$2\varepsilon^{\infty}\frac{\tau^{\infty}}{1-\tau^{\infty}} = \frac{1-F(n)}{nf(n)}\left(2 - \frac{\bar{\psi}}{\lambda}\right),$$

which is identical to (10) under the Pareto distribution and proper rescaling of the Lagrange multiplier.

Note that we express everything here in terms of Region B taxes - that is why C appears in the expressions for Region A as a correction term to increase indirect utility (in the objective function) or to reduce the tax revenue (in the government budget constraint). By the envelope theorem,

$$
\begin{aligned}
\frac{\partial W^*}{\partial C} &= \int_{n_{\min}}^{n_{\max}} [\left[\Psi\left(V(n) + \bar{q}^h + C\right) - \Psi\left(V(\omega(n)) - \bar{q}^c\right)\right] p(\bar{q}|n) \\
&\quad + \int_{\bar{q}}^{+\infty} \Psi'\left(V(n) + q^h\right) p(q|n)dq - \lambda\left(1 - P(\bar{q}|n)\right) \\
&\quad - \lambda\left(T(\omega(n)) - T(n) + C\right) p(\bar{q}|n)] f(n)dn.
\end{aligned}
$$

$$
\frac{\partial W^*}{\partial C}|_{C=0} = \int_{n_{\min}}^{n_{\max}} [\int_{\bar{q}}^{+\infty} \left(\Psi'\left(V(n) + q^h\right) - \lambda\right) p(q|n)dq - \lambda\left(T(\omega(n)) - T(n)\right) p(\bar{q}|n)] f(n)dn,
$$

which is negative, if $\Psi'\left(V(n) + q^h\right)/\lambda = g_A \leq 1$ and $T(\omega(n)) > T(n)$ (a sufficient condition is that the marginal tax rate is positive everywhere). ■

9.4 Proof of Proposition 7

Proof. A separable tax schedule implies that $T_B - T_A$ is constant. Since $z_A/n = z_B/\omega(n)$, we have

$$
\bar{q}_A = V_B - V_A = (\omega(n) - n)\left(\frac{z}{n} - h\left(\frac{z}{n}\right)\right) - (T_B - T_A),
$$

so we can write

$$
\dot{q}_A = \left(\omega'(n) - 1\right)\left(\frac{z}{n} - h\left(\frac{z}{n}\right)\right) - \left(T'_B - T'_A\right).
$$

In particular, under separable taxation and $\omega'(n) = 1$, we have $\dot{q}_A = 0$. At that point, for the cost-of-moving model

$$
\frac{d(g_A - g_B)}{dn} = \left[\frac{\Psi''(V_A(n))}{\lambda} - \frac{\Psi''(V_A + \bar{q}_A) + \int_0^{\bar{q}_A} \Psi''(V_A + \bar{q}_A - q^c) p(q)dq}{\lambda(1 + P(\bar{q}_A))}\right] \dot{V}_A < 0
$$

iff Ψ' is convex. ■

Similar to Kleven et al. (2006, 2009) we can consider a tax reform introducing a little bit of "negative jointness" (a lower marginal tax for the higher productivity region). This reform has two components. Above ability level n, we increase the tax in Region A and decrease the tax in Region B. Below ability level n, we decrease the tax in Region A and increase the tax in Region B. These tax burden changes are associated with changes in the marginal tax rates on earners around n. The direct welfare effect created by redistribution across regions at each income level:

$$
\begin{aligned}
dW &= \frac{dT}{F(n)} \int_{n_{\min}}^{n} \left(g_A(n') - g_B(n')\right) f(n')dn' \\
&\quad - \frac{dT}{1 - F(n)} \int_{n}^{n_{\max}} \left(g_A(n') - g_B(n')\right) f(n')dn'.
\end{aligned}
$$

Because $g_A - g_B$ is decreasing, $dW > 0$. Second, there are fiscal effects associated with earnings responses induced by the changes in τ_A and τ_B around n. Since the reform increases the marginal tax rate in Region A around n and reduces it in Region B, the earnings responses are opposite. As we start from $\tau_A = \tau_B$, and hence identical elasticities, $\varepsilon_A = \varepsilon_B$, the fiscal effects of earning responses cancel out exactly. Finally, the reform creates migration responses. Above n, migration to B will be induced. Below n, migration to B will be inhibited. The fiscal implications of these responses cancel out exactly only if $\omega'(n) = 1$. The elasticity η is constant in this case and since initially $T_A - T_B$ is constant, the revenue gain from migrants above n will be compensated by the revenue loss from migrants below n. By the same logic, for $\omega'(n) > 1$ the gain from migration will be stronger then the loss from it, so we will have another positive effect. With $\omega'(n) < 1$ the revenue gain from migration is smaller than the loss, so a bit of negative jointness is not necessarily optimal. To complete the proof, our reasoning needs to hold for $\omega'(n) > 1$, i.e. $\dot{g}_A - \dot{g}_B < 0$ also for this case. Differentiating $g_A - g_B$ in this case, we have

$$\begin{aligned}\dot{g}_A - \dot{g}_B &= \dot{V}_A \frac{\Psi''(V_A)}{\lambda} - \omega'(n)\dot{V}_A \frac{\Psi''(V_A + \bar{q}) + \int_0^{\bar{q}} \Psi''(V_A + \bar{q} - q^c)\, p(q) dq}{\lambda(1 + P(\bar{q}))} \\ &\quad - \frac{g_A - g_B}{1 + P(\bar{q})} p(\bar{q}) \left(\omega'(n) - 1\right) \dot{V}_A < 0,\end{aligned}$$

The first two terms are negative, because Ψ' is convex by assumption. The third term is negative, because $g_A > g_B$ (which follows from concavity of Ψ).

9.5 Two propositions using the alternative benchmark of the ex ante productivity distribution

Proposition 8 *In a country with regional productivity differences and internal migration, the optimal non-differentiated marginal tax rates may be higher or lower relative to a benchmark without internal migration and the ex ante productivity distribution. Assuming exogenous marginal welfare weights, they are lower if*

$$\begin{aligned}&\int_{\omega^{-1}(n)}^{n} (1 - g_B(n'))\, P(\bar{q}|n') f(n')\, dn' - \int_{\omega^{-1}(n)}^{n} \left(T(\omega(n')) - T(n')\right) p(\bar{q}|n') f(n')\, dn' \\ &\quad < \frac{P(\bar{q}_1|\omega^{-1}(n)) f(\omega^{-1}(n)) - P(\bar{q}|n) f(n)}{f(n) + f(\omega^{-1}(n))} \int_n^{n_{\max}} [1 - g_A(n')] f(n') dn' \\ &\quad\quad + \int_n^{n_{\max}} [g_B(n') - g_A(n')]\, P(\bar{q}|n') f(n') dn'\end{aligned} \tag{14}$$

Proof. *The optimal tax formula in case of unified taxation is presented by (??). For a government that does not take into account the possibility of migration, optimal marginal tax rates are implicitly defined by*

$$\begin{aligned}&\int_n^{n_{\max}} [2 - g_A(n') - g_B(n')] f(n') dn' + \int_{\omega^{-1}(n)}^{n} (1 - g_B(n'))\, f(n')\, dn' \\ = \ &\varepsilon \frac{\tau}{1-\tau} \left(f(n) + f\left(\omega^{-1}(n)\right)\right).\end{aligned}$$

Comparing the two expressions, we arrive at the condition (14). ∎

Proposition 9 *Starting from two identical regions, introducing a marginal difference in productivity distribution lowers optimal marginal tax if and only if*

$$\frac{1-g_B(n)}{2}+\frac{\int_n^{n_{\max}}\left[2-g_A(n')-g_B(n')\right]f(n')dn'}{4\left(f\left(n\right)\right)^2}\left(\frac{P(\bar{q}|n)}{\partial n}f\left(n\right)+f'\left(n\right)\right)<0. \tag{15}$$

Proof. *We express the marginal tax rate $\frac{\tau}{1-\tau}$ from the optimal tax formula (**??**) under the assumption that $\omega(n) = n+\Delta$, take a derivative of it with respect to Δ, and evaluate it at $\Delta = 0$, keeping in mind that there is no migration at this point. The resulting expression is proportional to the left hand side of (15). Correspondingly, the marginal tax rate decreases with the introduction of marginal productivity differences if this expression is negative and it increases in case it is positive.* ∎

We can see that the terms in (15) related to the revenue effect are always positive. Thus, a sufficient condition for an increase in marginal tax is that $\frac{P(\bar{q}|n)}{\partial n}f\left(n\right)+f'\left(n\right)\geq 0$. This is satisfied for independent distribution of costs ($\frac{P(\bar{q}|n)}{\partial n}=0$) and a uniform distribution of ability. On the other hand, if the distribution of ability is sufficiently "decreasing", like the Pareto distribution, for example, then introducing marginal productivity differences puts downward pressure on marginal taxes.

For a linear approximation, $T''\left(z\right)=0$, so we get $n\dot{z}\left(n\right)=z(n)$ indeed. This completes the proof of equivalence for the case of non-differentiated taxation. The derivation for differentiated taxation is analogous.

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Supplement to “Optimal Taxation under Regional Inequality”

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1 Overview

Section A provides the details of derivations of optimal tax formulae in differentiated and uniform cases. Section B shows that a given path of earnings $(z_A(n), z_B(\omega(n)))$ is implementable. Section C provides the conditions for the existence of a solution to the maximization problem. Section D establishes the equivalence of representation via earnings or via productivity. Section E explains further aspects of the simulation procedure.

2 Supplement A: detailed derivation of optimal tax formulae

2.1 Regionally differentiated taxation

We have presented the maximization problem in Appendix A. The FOCs for Region A and B, respectively, can be rewritten as

$$\frac{\mu_A(n)}{n}\frac{z_A}{n}h''\left(\frac{z_A}{n}\right)+\lambda\left(1-h'\left(\frac{z_A}{n}\right)\right)\left(1+P(\bar{q}_B|n)-P(\bar{q}_A|n)\right)f(n)=0,$$
$$\varphi\left(\frac{z_A}{n}\right)=-\frac{\mu_A(n)}{\lambda n f(n)\left(1+P(\bar{q}_B|n)-P(\bar{q}_A|n)\right)}$$

$$\frac{\mu_B(n)}{\omega(n)}\frac{z_B}{\omega(n)}h''\left(\frac{z_B}{\omega(n)}\right)+\lambda\left(1-h'\left(\frac{z_B}{\omega(n)}\right)\right)\left(1+P(\bar{q}_A|n)-P(\bar{q}_B|n)\right)f(n)=0$$
$$\varphi\left(\frac{z_B}{\omega(n)}\right)=-\frac{\mu_B(n)}{\lambda\omega(n)f(n)\left(1+P(\bar{q}_A|n)-P(\bar{q}_B|n)\right)}$$

for Region B. In both cases, LHS is decreasing in z_A/n ($z_B/\omega(n)$) whereas RHS is constant, which implies that $z_i(n)$ is a unique solution and a global maximum. Continuity can be then established in a way similar to Kleven et al (2009). The conditions for $\dot{\mu}_i(n)$ imply

$$\begin{aligned}
-\dot{\mu}_A(n) &= f(n)[\int_{\bar{q}_A}^{+\infty} \Psi'\left(V_A(n) + q^h\right) p(q|n)dq + \int_0^{\bar{q}_B} \Psi'\left(V_A(n) - q^c\right) p(q|n)dq \\
&\quad -\lambda\left(1 + P(\bar{q}_B|n) - P(\bar{q}_A|n)\right) \\
&\quad +\lambda\left(T_A - T_B\right)\left(p(\bar{q}_B|n) + p(\bar{q}_A|n)\right)],
\end{aligned}$$

$$\begin{aligned}
-\dot{\mu}_B(n) &= f(n)[\int_{\bar{q}_B}^{+\infty} \Psi'\left(V_B(\omega(n)) + q^h\right) p(q|n)dq + \int_0^{\bar{q}_A} \Psi'\left(V_B(\omega(n)) - q^c\right) p(q|n)dq \\
&\quad -\lambda\left(1 + P(\bar{q}_A|n) - P(\bar{q}_B|n)\right) \\
&\quad +\lambda\left(T_B - T_A\right)\left(p(\bar{q}_B|n) + p(\bar{q}_A|n)\right)],
\end{aligned}$$

Integrating this, we get for Regions A and B, respectively,

$$\begin{aligned}
-\frac{\mu_A(n)}{\lambda} &= \int_n^{n_{\max}} [-\frac{1}{\lambda}\left(\int_{\bar{q}_A}^{+\infty} \Psi'\left(V_A(n) + q^h\right) p(q|n')dq + \int_0^{\bar{q}_B} \Psi'\left(V_B\left(\omega(n')\right) - q^c\right) p(q|n')dq\right) \\
&\quad +1 + P(\bar{q}_B|n') - P(\bar{q}_A|n') \\
&\quad -\left(T_A - T_B\right)\left(p(\bar{q}_B|n') + p(\bar{q}_A|n')\right)]f(n')dn',
\end{aligned}$$

$$\begin{aligned}
-\frac{\mu_B(n)}{\lambda} &= \int_n^{n_{\max}} [-\frac{1}{\lambda}\left(\int_{\bar{q}_B}^{+\infty} \Psi'\left(V_B\left(\omega(n')\right) + q^h\right) p(q|n')dq + \int_0^{\bar{q}_A} \Psi'\left(V_B\left(\omega(n')\right) - q^c\right) p(q|n')dq\right) \\
&\quad +\left(1 + P(\bar{q}_A|n') - P(\bar{q}_B|n')\right) \\
&\quad -\left(T_B - T_A\right)\left(p(\bar{q}_B|n') + p(\bar{q}_A|n')\right)]f(n')dn'.
\end{aligned}$$

Defining by $g_A(n)$ the average marginal social welfare weight of the region A residents with inborn ability n, by $g_B(n)$ the average marginal social welfare weight of the region B initial residents with inborn ability n, we have

$$\begin{aligned}
g_A(n) &= \frac{\int_{\bar{q}_A}^{+\infty} \Psi'\left(V_A(n) + q^h\right) p(q|n)dq + \int_0^{\bar{q}_B} \Psi'\left(V_A(n) - q^c\right) p(q|n)dq}{\lambda\left(1 + P(\bar{q}_B|n) - P(\bar{q}_A|n)\right)}, \\
g_B(n) &= \frac{\int_{\bar{q}_B}^{+\infty} \Psi'\left(V_B(\omega(n)) + q^h\right) p(q|n)dq + \int_0^{\bar{q}_A} \Psi'\left(V_B(\omega(n)) - q^c\right) p(q|n)dq}{\lambda\left(1 + P(\bar{q}_A|n) - P(\bar{q}_B|n)\right)}.
\end{aligned}$$

Using these, we can rewrite the optimality conditions as

$$\begin{aligned}
-\frac{\mu_A(n)}{\lambda} &= \int_n^{n_{\max}} [\left(1 - g_A(n')\right)\left(1 + P(\bar{q}_B|n') - P(\bar{q}_A|n')\right) \\
&\quad -\left(T_A - T_B\right)\left(p(\bar{q}_B|n') + p(\bar{q}_A|n')\right)]f(n')dn',
\end{aligned}$$

$$
\begin{aligned}
-\frac{\mu_B(n)}{\lambda} \quad = \quad & \int_n^{n_{\max}} [\left(1-g_B(n')\right)\left(1+P(\bar{q}_A|n')-P(\bar{q}_B|n')\right) \\
& -(T_B-T_A)\left(p(\bar{q}_B|n')+p(\bar{q}_A|n')\right)]f(n')dn'.
\end{aligned}
$$

Inserting into the FOCs, we get

$$
\begin{aligned}
\frac{\tau_A}{1-\tau_A} = \frac{1}{nf(n)\varepsilon_A\left(1+P(\bar{q}_B|n)-P(\bar{q}_A|n)\right)} & \int_n^{n_{\max}} [\left(1-g_A(n')\right)\left(1+P(\bar{q}_B|n')-P(\bar{q}_A|n')\right) \\
& -(T_A-T_B)\left(p(\bar{q}_B|n')+p(\bar{q}_A|n')\right)]f(n')dn',
\end{aligned}
$$

$$
\begin{aligned}
\frac{\tau_B}{1-\tau_B} = \frac{1}{\omega(n)f(n)\varepsilon_B\left(1+P(\bar{q}_A|n)-P(\bar{q}_B|n)\right)} & \int_n^{n_{\max}} [\left(1-g_B(n')\right)\left(1+P(\bar{q}_A|n')-P(\bar{q}_B|n')\right) \\
& -(T_B-T_A)\left(p(\bar{q}_B|n')+p(\bar{q}_A|n')\right)]f(n')dn'
\end{aligned}
$$

for the marginal rates in A and B, respectively. The formulae are similar to Kleven et al (2009) except that two terms (rather than one) reflect the possibility of either immigration to or emigration from the given region. Note that for each n, there are two mutually exclusive scenarios: either there is migration from A to B (and $V_B(\omega(n)) > V_A(n)$) so that the formulae take the form

$$
\begin{aligned}
\frac{\tau_A}{1-\tau_A} = \frac{1}{nf(n)\varepsilon_A\left(1-P(\bar{q}_A|n)\right)} & \int_n^{n_{\max}} [\left(1-g_A(n')\right)\left(1-P(\bar{q}_A|n')\right) \\
& -(T_A-T_B)\,p(\bar{q}_A|n')]f(n')dn',
\end{aligned}
$$

$$
\begin{aligned}
\frac{\tau_B}{1-\tau_B} = \frac{1}{\omega(n)f(n)\varepsilon_B\left(1+P(\bar{q}_A|n)\right)} & \int_n^{n_{\max}} [\left(1-g_B(n')\right)\left(1+P(\bar{q}_A|n')\right) \\
& -(T_B-T_A)\,p(\bar{q}_A|n')]f(n')dn',
\end{aligned}
$$

or there is migration from B to A (and $V_B(\omega(n)) < V_A(n)$) so that the formulae turn to

$$
\begin{aligned}
\frac{\tau_A}{1-\tau_A} = \frac{1}{nf(n)\varepsilon_A\left(1+P(\bar{q}_B|n)\right)} & \int_n^{n_{\max}} [\left(1-g_A(n')\right)\left(1+P(\bar{q}_B|n')\right) \\
& -(T_A-T_B)\,p(\bar{q}_B|n')]f(n')dn',
\end{aligned}
$$

$$
\begin{aligned}
\frac{\tau_B}{1-\tau_B} = \frac{1}{\omega(n)f(n)\varepsilon_B\left(1-P(\bar{q}_B|n)\right)} & \int_n^{n_{\max}} [\left(1-g_B(n')\right)\left(1-P(\bar{q}_B|n')\right) \\
& -(T_B-T_A)\,p(\bar{q}_B|n')]f(n')dn'.
\end{aligned}
$$

The average marginal tax rate is then

$$f(n)[n\frac{\tau_A}{1-\tau_A}\varepsilon_A\left(1+P(\bar{q}_B|n)-P(\bar{q}_A|n)\right)$$
$$+\omega(n)\frac{\tau_B}{1-\tau_B}\varepsilon_B\left(1+P(\bar{q}_A|n)-P(\bar{q}_B|n)\right)]=\int_n^{n_{\max}}\left(2-\bar{g}\left(n'\right)\right)f(n')dn',\text{ where}$$

$$\bar{g}\left(n\right):=g_A(n)\left(1+P(\bar{q}_B|n)-P(\bar{q}_A|n)\right)+g_B(n)\left(1+P(\bar{q}_A|n)-P(\bar{q}_B|n)\right)$$

is the average social marginal welfare weight of the individuals with innate ability n and we have 2 instead of 1 because total population is of measure 2. Clearly, $\tau_A\left(n_{\max}\right)=0=\tau_B\left(\omega\left(n_{\max}\right)\right)$ and $\tau_A\left(n_{\min}\right)=0=\tau_B\left(\omega\left(n_{\min}\right)\right)$ from the transversality conditions.

Define migration semi-elasticities $\mu_i^+\left(n\right):=\frac{1}{1+P(\bar{q}_i|n)}\frac{\partial P(\bar{q}_i|n)}{\partial\bar{q}}=\frac{p(\bar{q}_i|n)}{1+P(\bar{q}_i|n)}$ for the region with inflow of population and $\mu_i^-\left(n\right):=\frac{1}{1-P(\bar{q}_i|n)}\frac{\partial P(\bar{q}_i|n)}{\partial\bar{q}}=\frac{p(\bar{q}_i|n)}{1-P(\bar{q}_i|n)}$ for the region with an outflow. Define the migration elasticity as $\nu_i:=\mu_i\left(T_A-T_B\right)$, whereby normalizing in terms of the tax rather than the utility differential V_B-V_A is for notational conviniency. We have

$$\begin{aligned}
\frac{\tau_A}{1-\tau_A} &= \frac{1}{nf(n)\varepsilon_A\left(1-P(\bar{q}_A|n)\right)}\int_n^{n_{\max}}\left[1-g_A(n')-\nu_A^-\left(n'\right)\right]\left(1-P(\bar{q}_A|n')\right)f(n')dn',\text{ and}\\
\frac{\tau_B}{1-\tau_B} &= \frac{1}{\omega(n)f(n)\varepsilon_B\left(1+P(\bar{q}_A|n)\right)}\int_n^{n_{\max}}\left[1-g_B(n')+\nu_A^+\left(n'\right)\right]\left(1+P(\bar{q}_A|n')\right)f(n')dn'.
\end{aligned}$$

The effect of the migration elasticity as a top-up to the marginal social welfare weight is evident from the resulting formulae. Indeed, the marginal tax rate in region A (source region) is reduced by the migration elasticity in the same way it is reduced by the welfare weight of region A citizens. Conversely, the marginal tax rate in region B (recepient region) is increased by migration elasticity in the same way it is reduced by the welfare weight of region B citizens. Intuitively, marginal increase of tax for all skill levels above n in region A will lead to outflow of people resulting in the loss of revenue differential T_A-T_B between two regions, properly accounted for by the term $\nu_A^-\left(n'\right)$ at each skill level n'. In region B, the same mechanism is in action, only the loss of revenue differential is properly accounted for by the term $-\nu_A^+\left(n'\right)$. From the formulae above we can also see that more elastic migration response leads to higher marginal tax rates in region A and lower marginal tax rates in region B (migration elasticity is negative whenever $T_A<T_B$).

2.2 Non-differentiated tax schedule

We have presented the maximization problem in Appendix A. Multiplier evolution can be written as

$$
\begin{aligned}
-\dot{\mu}(n) \quad = \quad & f(\omega^{-1}(n)) \int_{0}^{+\infty} \Psi'\left(V(n)+q^{h}\right) p(q|\omega^{-1}(n))dq \\
& +f\left(\omega^{-1}(n)\right) \int_{0}^{\bar{q}_1} \Psi'\left(V(n)-q^{c}\right) p(q|\omega^{-1}(n))dq \\
& +f(n) \int_{\bar{q}}^{+\infty} \Psi'\left(V(n)+q^{h}\right) p(q|n)dq \\
& +\lambda[-\left(1-P(\bar{q}|n)\right) f(n)-\left(1+P(\bar{q}_1|\omega^{-1}(n))\right) f(\omega^{-1}(n)) \\
& -\left(z^{w}-n^{w} h\left(\frac{z^{w}}{n^{w}}\right)-V^{w}-\left(z-nh\left(\frac{z}{n}\right)-V\right)\right) p(\bar{q}|n) f(n) \\
& +\left(z-nh\left(\frac{z}{n}\right)-V-\left(z^{-w}-n^{-w} h\left(\frac{z^{-w}}{n^{-w}}\right)-V^{-w}\right)\right) p(\bar{q}_1|\omega^{-1}(n)) f(\omega^{-1}(n))],
\end{aligned}
$$

where superscript $-w$ stands for the argument $\omega^{-1}(n)$ of corresponding functions. Also, $\bar{q}_1 = V(n) - V\left(\omega^{-1}(n)\right)$ and $\bar{q} = V(\omega(n)) - V(n)$. Rewriting in terms of taxes, we have

$$
\begin{aligned}
-\dot{\mu}(n) \quad = \quad & f(\omega^{-1}(n)) \left(\int_{0}^{+\infty} \Psi'\left(V(n)+q^{h}\right) p(q|\omega^{-1}(n))dq + \int_{0}^{\bar{q}_1} \Psi'\left(V(n)-q^{c}\right) p(q|\omega^{-1}(n))dq \right) \\
& +f(n) \int_{\bar{q}}^{+\infty} \Psi'\left(V(n)+q^{h}\right) p(q|n)dq \\
& +\lambda[-\left(1-P(\bar{q}|n)\right) f(n)-\left(1+P(\bar{q}_1|\omega^{-1}(n))\right) f(\omega^{-1}(n)) \\
& -\left(T(\omega(n))-T(n)\right) p(\bar{q}|n) f(n) \\
& +\left(T(n)-T(\omega^{-1}(n))\right) p(\bar{q}_1|\omega^{-1}(n)) f(\omega^{-1}(n))]
\end{aligned}
$$

Defining by $g_i(n)$ the average marginal social welfare weight of the region i residents with inborn ability n, we have

$$
\begin{aligned}
g_A(n) \quad &= \quad \frac{1}{\lambda} \frac{\int_{\bar{q}}^{+\infty} \Psi'\left(V(n)+q^{h}\right) p(q|n)dq}{1-P(\bar{q}|n)}, \\
g_B\left(\omega^{-1}(n)\right) \quad &= \quad \frac{\frac{1}{\lambda}\left(\int_{0}^{\bar{q}_1} \Psi'\left(V(n)-q^{c}\right) p(q|\omega^{-1}(n))dq + \int_{0}^{+\infty} \Psi'\left(V(n)+q^{h}\right) p(q|\omega^{-1}(n))dq\right)}{1+P(\bar{q}_1|\omega^{-1}(n))}.
\end{aligned}
$$

Thus, we can write

$$
\begin{aligned}
-\frac{\dot{\mu}(n)}{\lambda} \quad = \quad & \left(g_B\left(\omega^{-1}(n)\right)-1\right)\left(1+P(\bar{q}_1|\omega^{-1}(n))\right) f\left(\omega^{-1}(n)\right) \\
& +\left(g_A(n)-1\right)\left(1-P(\bar{q}|n)\right) f(n) \\
& -\left(T(\omega(n))-T(n)\right) p(\bar{q}|n) f(n) \\
& +\left(T(n)-T(\omega^{-1}(n))\right) p(\bar{q}_1|\omega^{-1}(n)) f(\omega^{-1}(n))
\end{aligned}
$$

and integrating

$$
\begin{aligned}
-\frac{\mu(n)}{\lambda} \quad = \quad & \int_n^{n_{\max}} [\left(1-g_B\left(\omega^{-1}(n')\right)\right)\left(1+P(\bar{q}_1|\omega^{-1}(n'))\right) f\left(\omega^{-1}(n')\right) \\
& +\left(1-g_A(n')\right)\left(1-P(\bar{q}|n')\right) f(n') \\
& +\left(T\left(\omega(n')\right)-T(n')\right) p(\bar{q}|n') f(n') \\
& -\left(T\left(n'\right)-T(\omega^{-1}(n'))\right) p(\bar{q}_1|\omega^{-1}(n')) f(\omega^{-1}(n'))] dn'
\end{aligned}
$$

and substituting into the FOC (using the definition of elasticity $\varepsilon = nh'/zh''$),

$$
\begin{gathered}
\frac{1}{n\varepsilon}\frac{1}{(1-P(\bar{q}|n)) f(n)+(1+P(\bar{q}_1|\omega^{-1}(n))) f\left(\omega^{-1}(n)\right)} \times \\
\int_n^{n_{\max}} [\left(1-g_B\left(\omega^{-1}(n')\right)\right)\left(1+P(\bar{q}_1|\omega^{-1}(n'))\right) f\left(\omega^{-1}(n')\right)+\left(1-g_A(n')\right)\left(1-P(\bar{q}|n')\right) f(n') \\
+\left(T\left(\omega(n')\right)-T(n')\right) p(\bar{q}|n') f(n')-\left(T\left(n'\right)-T(\omega^{-1}(n'))\right) p(\bar{q}_1|\omega^{-1}(n')) f(\omega^{-1}(n'))] dn' = \frac{\tau}{1-\tau}.
\end{gathered}
$$

Simplifying the integral expression, we get[1]

$$
\begin{gathered}
\int_{\omega^{-1}(n)}^{n_{\max}}\left(1-g_B\left(n'\right)\right)\left(1+P(\bar{q}|n')\right) f\left(n'\right) dn' + \int_n^{n_{\max}}\left(1-g_A(n')\right)\left(1-P(\bar{q}|n')\right) f(n') dn' \\
-\int_{\omega^{-1}(n)}^{n}\left(T\left(\omega(n')\right)-T(n')\right) p(\bar{q}|n') f\left(n'\right) dn' = \frac{\tau}{1-\tau} \times \\
\times n\varepsilon\left((1-P(\bar{q}|n)) f(n)+\left(1+P(\bar{q}_1|\omega^{-1}(n))\right) f\left(\omega^{-1}(n)\right)\right)
\end{gathered}
$$

Defining by $\bar{g}\,(n)$ the average marginal social welfare weight of the people with observed productivity n as

$$
\bar{g}\,(n) := g_A(n)\,(1-P(\bar{q}|n)) + g_B(n)\,(1+P(\bar{q}|n))\,,
$$

we can rewrite the optimal tax formula as

$$
\begin{aligned}
&\int_n^{n_{\max}}\left(2-\bar{g}(n')\right) f(n') dn' + \int_{\omega^{-1}(n)}^{n}\left(\left(1-g_B(n')\right)\left(1+P(\bar{q}|n')\right)-\left(T\left(\omega(n')\right)-T(n')\right) p(\bar{q}|n')\right) f\left(n'\right) dn' \\
&= \frac{\tau}{1-\tau} n\varepsilon\left((1-P(\bar{q}|n)) f(n)+\left(1+P(\bar{q}_1|\omega^{-1}(n))\right) f\left(\omega^{-1}(n)\right)\right)
\end{aligned}
\tag{1}
$$

which is analogous to the celebrated Mirrlees formula apart from the integral from $\omega^{-1}(n)$ to n that takes care of the revenue effect ($(1-g_B(n))\,(1+P(\bar{q}|n))$ term) and the migration effect ($(T(\omega(n'))-T\,(n'))\,p(\bar{q}|n')$). If $n \in [n_{\min}, \omega(n_{\min})]$, only non-migrated region A inhabitants

[1]Note that we have assumed $\omega(n_{\max}) = n_{\max}$, that is why $\omega^{-1}(n_{\max}) = n_{\max}$ and we get the expressions below. In a more general setting, we would have upper limit of the first integral term equal to $\omega^{-1}(n_{\max})$, and we would also have an additional integral term $\int_{\omega^{-1}(n_{\max})}^{n_{\max}}\left(T\left(\omega(n')\right)-T(n')\right) p(\bar{q}|n') f\left(n'\right) dn'$.

have this productivity, so the formula becomes

$$\int_{n}^{n_{\max}}[1-g_A\left(n'\right)]f(n')dn'=\varepsilon\frac{\tau}{1-\tau}nf(n),$$

which is exactly the Mirrleesean formula. Note that the additional terms admit straightforward interpretation: $\int_{\omega^{-1}(n)}^{n}\left(\left(T\left(\omega(n')\right)-T(n')\right)p(\bar{q}|n')\right)f\left(n'\right)dn'$ is the tax paid by all migrants with skill from $\omega^{-1}(n)$ to n over and above the tax they would have paid if remaining in their home region. This characterizes a distortion that the government creates on extensive margin, stimulating $\left(T\left(\omega(n)\right)<T(n)\right)$ or discouraging $\left(T\left(\omega(n)\right)<T(n)\right)$ migration. The need for distortion comes from differences in social marginal welfare weights; its magnitude is determined, among other things, by the shape of the transformation function $\omega\left(n\right)$.

The other additional term, $\int_{\omega^{-1}(n)}^{n}\left(1-g_B(n')\right)\left(1+P(\bar{q}|n')\right)f\left(n'\right)dn'$, stands for the welfare effect of marginally increasing the tax for all productivity levels between n and $\omega\left(n\right)$ who migrate from region B to region A becasue of this increase (and thus realize productivity from $\omega^{-1}(n)$ to n). Using the elasticity defined as $\nu(n):=\left(T\left(\omega(n)\right)-T(n)\right)\frac{p(\bar{q}|n)}{1+P(\bar{q}|n)}$, we can rewrite the optimal tax formula as

$$\int_{\omega^{-1}(n)}^{n_{\max}}\left(1-g_B\left(n'\right)\right)\left(1+P(\bar{q}|n)\right)f\left(n'\right)dn'+\int_{n}^{n_{\max}}\left(1-g_A(n')\right)\left(1-P(\bar{q}|n')\right)f(n')dn'$$
$$-\int_{\omega^{-1}(n)}^{n}\nu(n')\left(1+P(\bar{q}|n')\right)f\left(n'\right)dn'=\frac{\tau}{1-\tau}\times$$
$$\times n\varepsilon\left(\left(1-P(\bar{q}|n)\right)f(n)+\left(1+P(\bar{q}_1|\omega^{-1}(n))\right)f\left(\omega^{-1}(n)\right)\right).$$

More elastic migration puts downward pressure on marginal tax rates whenever the migration elasticity is positive on $[\omega^{-1}(n),n]$.

3 Supplement B: Implementability

We follow the supplementary material to Kleven et al. (2009). The same reasoning applies. In particular, an action profile $\left(z_A\left(n\right),z_B\left(\omega(n)\right)\right)_{n\in(n_{\min},n_{\max})}$ is implementable if and only if there exist transfer functions $\left(c_A(n),c_B(\omega(n))\right)_{n\in(n_{\min},n_{\max})}$ such that $\left(z_i\left(n_i\right),c_i\left(n_i\right)\right)_{i\in\{A,B\},n\in(n_{\min},n_{\max})}$ is a truthful mechanism. A mechanism is called truthful if there is a $\{\bar{q}_A(n),\bar{q}_B(n)\}$ such that (i) for $q<\max_{i\in\{A,B\}}\bar{q}_i(n)$, the set $(i'=\arg\min_{i\in\{A,B\}}\bar{q}_i(n),n'=n)$ maximizes $u(z_{i'}(n'),i',c_{i'}(n'),(n,q))$ over all (i',n'); (ii) for $q\geq\max_{i\in\{A,B\}}\bar{q}_i(n)$, the set $(i'=\arg\max_{i\in\{A,B\}}\bar{q}_i(n),n'=n)$ maximizes $u(z_{i'}(n'),i',c_{i'}(n'),(n,q))$ over all (i',n'). The usual implementability theorem for the one-dimensional case also applies in our setting.

Lemma 1 (Kleven-Kreiner-Saez) *An action profile $(z_A(n),z_B(\omega(n)))_{n\in(n_{\min},n_{\max})}$ is implementable if and only if $z_A(n)$ and $z_B(\omega(n))$ are both nondecreasing in n.*

Proof. *The utility function $c - nh(z/n)$ (and also $c - \omega(n)h(z/\omega(n))$) satisfies the usual single crossing condition. Indeed, it is equivalent to $xh''(x) > 0$ for all $x > 0$, which is ensured by convexity of h. That is why from the one-dimensional case we know that $z(n)$ (and $z(\omega(n))$) is implementable, which means that there is some $c(n)$ such that $c(n) - nh(z(n)/n) \geq c(n') - nh(z(n')/n)$ for all n, n' if and only if $z(n)$ is nondecreasing.*[2]
Suppose $(z_A(n), z_B(\omega(n)))$ is implementable, so that there exists $(c_A(n), c_B(\omega(n)))$ such that $(z_i(n_i), c_i(n_i))_{i\in\{A,B\}, n\in(n_{\min}, n_{\max})}$ is a truthful mechanism. This implies that $c_A(n) - nh(z_A(n)/n) \geq c_A(n') - nh(z_A(n')/n)$ and $c_B(\omega(n)) - \omega(n)h(z_B(\omega(n))/\omega(n)) \geq c_B(\omega(n')) - \omega(n)h(z_B(\omega(n'))/\omega(n))$ for all n, n', and so by one-dimensional result $z_A(n)$ and $z_B(\omega(n))$ are nondecreasing. Conversely, suppose $z_A(n)$ and $z_B(\omega(n))$ are nondecreasing. One-dimensional result implies then that there exist such $c_A(n)$ and $c_B(\omega(n))$ that $c_A(n) - nh(z_A(n)/n) \geq c_A(n') - nh(z_A(n')/n)$ and $c_B(\omega(n)) - \omega(n)h(z_B(\omega(n))/\omega(n)) \geq c_B(\omega(n')) - \omega(n)h(z_B(\omega(n'))/\omega(n))$ correspondingly. We have to show that the mechanism $(z_i(n_i), c_i(n_i))_{i\in\{A,B\}, n\in(n_{\min}, n_{\max})}$ is actually truthful . We have

$$\begin{aligned}\bar{q}_A &= \max\{V_B(\omega(n)) - V_A(n), 0\}, \\ \bar{q}_B &= \max\{V_A(n) - V_B(\omega(n)), 0\},\end{aligned}$$

where we define $V_B(\omega(n)) := c_B(\omega(n)) - \omega(n)h(z_B(\omega(n))/\omega(n))$ and $V_A(n) := c_A(n) - nh(z_A(n)/n)$.
In case $\bar{q}_A > 0$, for all $n, n', q \geq \bar{q}_A(n)$ we have

$$u_A(z_A(n), c_A(n), 0, (n,q)) = V_A(n) \geq V_B(\omega(n)) - q \geq u_B(z_B(n'), c_B(n'), 1, (n,q));$$

for all $n, n', q \leq \bar{q}_A(n)$ we have

$$u_B(z_B(\omega(n)), c_B(\omega(n)), 1, (n,q)) = V_B(\omega(n)) - q \geq V_A(n) \geq u_A(z_A(n'), c_A(n'), 0, (n,q)).$$

In case $\bar{q}_B > 0$, for all $n, n', q \geq \bar{q}_B(n)$ we have

$$u_B(z_B(\omega(n)), c_B(\omega(n)), 0, (n,q)) = V_B(\omega(n)) \geq V_A(n) - q \geq u_A(z_A(n'), c_A(n'), 1, (n,q));$$

for all $n, n', q \leq \bar{q}_B(n)$ we have

$$u_A(z_A(n), c_A(n), 1, (n,q)) = V_A(n) - q \geq V_B(\omega(n)) \geq u_B(z_B(\omega(n')), c_B(\omega(n')), 0, (n,q)).$$

■

As in Kleven et al (2009), the separability of q in the utility specification allows to get these simple results. The proof for the uniform tax is analogous, with the restriction $z_A(n) -$

[2] Clearly, the same is true if $\omega(n)$ rather than n is the appropriate argument.

$c_A(n) \equiv z_B(n) - c_B(n).$

4 Supplement C: Existence

We follow Kleven et al (2009) to establish similar conditions for our problem with differential taxation. Formally, our maximization problem is the optimal control problem $\dot{V} = b(n, V, z)$ with the objective $B^0 = \int_{n_{\min}}^{n_{\max}} b^0(n, V(n))\, dn$ and constraint $\int_{n_{\min}}^{n_{\max}} b^1(n, z(n), V(n)) \geq 0$, where

$$
\begin{aligned}
b(n, V, z) &= \left(-h\left(\frac{z_A}{n}\right) + \frac{z_A}{n}h'\left(\frac{z_A}{n}\right), -h\left(\frac{z_B}{\omega(n)}\right) + \frac{z_B}{\omega(n)}h'\left(\frac{z_B}{\omega(n)}\right)\right), \\
b^0(n, V) &= [\int_{\max\{V_A - V_B, 0\}}^{+\infty} \Psi\left(V_B + q^h\right) p(q|n)dq + \int_0^{\max\{V_B - V_A, 0\}} \Psi\left(V_B - q^c\right) p(q|n)dq \\
&\quad + \int_{\max\{V_B - V_A, 0\}}^{+\infty} \Psi\left(V_A + q^h\right) p(q|n)dq \\
&\quad + \int_0^{\max\{V_A - V_B, 0\}} \Psi\left(V_A - q^c\right) p(q|n)dq] f(n),
\end{aligned}
$$

$$
\begin{aligned}
&b^1(n, z(n), V(n)) = [\left(z_B - \omega(n)h\left(\frac{z_B}{\omega(n)}\right) - V_B\right) \\
&\times (1 + P(\max\{V_B - V_A, 0\}|n) - P(\max\{V_A - V_B, 0\}|n)) \\
&+ \left(z_A - nh\left(\frac{z_A}{n}\right) - V_A\right) \\
&\times (1 + P(\max\{V_A - V_B, 0\}|n) - P(\max\{V_B - V_A, 0\}|n))] f(n) - E
\end{aligned}
$$

The functions b, b^0, b^1 are continuous and continuously differentiable in (z, V) by construction. Analogously to Kleven et al (2009), if we assume that there is an a priori bound on the path of admissible z, we need to show that the sets $B(n, V, \lambda) = \{(y, b(n, V, z)|z_0 \geq 0, z_1 \geq 0, y \geq -b^0 - \lambda b^1\}$ are convex for all n, V and $\lambda \geq 0$, then there exists an optimal control z measurable on $(n_{\min}, n_{\max})$. We have

$$
\begin{aligned}
B(n, V, \lambda) &= \{(y, -h\left(\frac{z_A}{n}\right) + \frac{z_A}{n}h'\left(\frac{z_A}{n}\right), \\
-h\left(\frac{z_B}{\omega(n)}\right) + \frac{z_B}{\omega(n)}h'\left(\frac{z_B}{\omega(n)}\right))|z_A &\geq 0, z_B \geq 0, \\
y &\geq -b^0(n, V) \\
&\quad -\lambda f(n)[\left(z_B - \omega(n)h\left(\frac{z_B}{\omega(n)}\right) - V_B\right)(1 + P_B - P_A) \\
&\quad + \left(z_A - nh\left(\frac{z_A}{n}\right) - V_A\right)(1 + P_A - P_B)]
\end{aligned}
$$

We denote by $K(.)$.the inverse of the strictly increasing function $x \rightarrow -h(x) + xh'(x)$ with $K(0) = 0$, so we can write

$$\begin{aligned}
B(n, V, \lambda) &= \{(y, x_A, x_B | x_A \geq 0, x_B \geq 0, \\
y + b^0(n, V) &\geq \lambda f(n)[\omega(n)(h(K(x_B)) - K(x_B) + V_B)(1 + P_B - P_A) \\
&\quad + n(h(K(x_A)) - K(x_A) + V_A)(1 + P_A - P_B)].
\end{aligned}$$

Therefore, $B(n, V, \lambda)$ is convex when $x \rightarrow h(K(x)) - K(x) \equiv \phi(x)$ is convex, which it is by the same reasoning as in Kleven et al (2009). In particular, by definition of $K(.)$, $-h(K(x)) + K(x)h'(K(x)) \equiv x$, so that differentiation gives us $K(x)h''(K(x))K'(x) \equiv 1$. Consider $\phi'(x) = (h'(K(x)) - 1)K'(x)$. By our previous result, $K'(x) = 1/[K(x)h''(K(x))]$. Therefore, we can write $\phi'(x) = -(1 - h'(K(x)))/[K(x)h''(K(x))]$. By assumption 1, $\phi'(x)$ is an increasing function of $K(x)$. Since $x \rightarrow K(x)$ is strictly increasing, $\phi'(x)$ is increasing and thus $\phi(x)$ is convex.

5 Supplement D: Equivalence of representations via income and via ability

Here we show that the optimal tax formulae obtained in the text are equivalent to those in the appendix. Consider the formula for non-differentiated taxation in the text:

$$\begin{aligned}
\frac{\tau}{1-\tau} &= \frac{1}{z\varepsilon(v_A(z)s_A(z) + v_B(z)s_B(z))} \times \\
&\quad [\int_z^\infty \{[1 - g_A(z')] v_A(z') s_A + [1 - g_B(z')] v_B(z') s_B\} dz' \\
&\quad + \int_{\tilde{z}}^z [T(z') - T(k(z'))] p(\bar{q}| z') v_A(z') dz'],
\end{aligned}$$

where $s_A(z) \equiv 1 - P(\bar{q}| z(n)) = 1 - P(\bar{q}| n)$ and $s_B \equiv 1 + P(\bar{q}| z') = 1 + P(\bar{q}_1| \omega^{-1}(n))$ and $v_A(z(n)) = f(n)/\dot{z}(n)$, $v_B(z(n)) = f(\omega^{-1}(n))/\dot{z}(n)$. Further, $T(z(n)) = T(n)$, $T(k(z(n))) = T(\omega(n))$ and $\tilde{z} = k^{-1}(z(n)) = z(\omega^{-1}(n))$; $g_i(z(n)) = g_i(n)$. Plugging into the expression above, we get

$$\begin{aligned}
\frac{\tau}{1-\tau} &= \frac{1}{\frac{z(n)}{\dot{z}(n)}\varepsilon((1 - P(\bar{q}|n)) f(n) + (1 + P(\bar{q}_1|\omega^{-1}(n))) f(\omega^{-1}(n)))} \\
&\quad [\int_n^{n_{\max}} \{[1 - g_A(n')] \frac{f(n')}{\dot{z}(n')} (1 - P(\bar{q}| n')) \\
&\quad + [1 - g_B(n')] \frac{f(\omega^{-1}(n))}{\dot{z}(n')} (1 + P(\bar{q}_1| \omega^{-1}(n)))\} \dot{z}(n') dn' \\
&\quad + \int_{\omega^{-1}(n)}^n [T(n') - T(\omega(n'))] p(\bar{q}| n') \frac{f(n')}{\dot{z}(n')} \dot{z}(n') dn'],
\end{aligned}$$

compared to

$$\frac{1}{n\varepsilon}\frac{1}{\left(1-P(\bar{q}|n)\right)f(n)+\left(1+P(\bar{q}_1|\omega^{-1}(n))\right)f\left(\omega^{-1}(n)\right)}\times$$
$$\int_n^{n_{\max}}[\left(1-g_B\left(\omega^{-1}(n')\right)\right)\left(1+P(\bar{q}_1|\omega^{-1}(n'))\right)f\left(\omega^{-1}(n')\right)+\left(1-g_A(n')\right)\left(1-P(\bar{q}|n')\right)f(n')]dn'$$
$$-\int_{\omega^{-1}(n)}^{n}\left(T\left(\omega(n')\right)-T(n')\right)p(\bar{q}|n')f\left(n'\right)dn'=\frac{\tau}{1-\tau}.$$

The expressions are identical, if $\frac{z(n)}{\dot{z}(n)}=n$. To prove that this is indeed the case for the tax schedule linearized around the optimum in our model, simply totally differentiate the first order condition (2):

$$h''\left(\frac{z}{n}\right)\frac{n\dot{z}\left(n\right)-z}{n^2}=-T''\left(z\right)\dot{z}\left(n\right).$$

For a linear approximation, $T''\left(z\right)=0$, so we get $n\dot{z}\left(n\right)=z(n)$ indeed. This completes the proof of equivalence for the case of non-differentiated taxation. The derivation for differentiated taxation is analogous.

6 Supplement E: Simulation notes

Simulations are done with the Matlab software package and a modified and extended version of the program of Kleven et al (2009) which is documented in their corresponding supplementary material. Hence, their iterative procedure applies in general. In a similar way, we use a two dimensional grid of 1000 elements from $n_{\min}$ to $n_{\max}$. The second dimension carries abilities in Region B. Integration of the variable n and $\omega(n)$ is done by means of the Matlab trapezoidal approximation routine, integration along the variable q is implemented using the incomplete beta function as described in the supplementary material to Kleven et al (2009). Given the ability distribution, each routine starts with a given vector of T_A', T_B' in the differentiated taxation case and computes the vectors z_A, z_B, V_A, V_B, $\overline{q}$, g_A, g_B, T_A, T_B, and so on, according to the given restrictions on budget and transversality. Using this outcome, new marginal tax rate vectors are computed by applying the optimal tax equations. The next loop of the routine starts with marginal tax rate vectors T_A', T_B' that are the adaptive weighted average of the current vectors and the new vectors of each region, respectively. This looping procedure is done until it has converged. The weighting is adaptive in the sense that new marginal tax rate vectors have more weight if the procedure converges and vice versa. In contrast, in the uniform case only one marginal tax rate vector T' together with only one tax schedule T is calculated, and all vectors are computed satisfying the additional constraint of equal tax liability for the same income level in both regions. The fixed residence case is simulated by prohibiting migration flows.

Our empirical construction of ω may be justified as follows. By the definition of the transformation function ω, the cumulative distribution functions of ability in two regions are related as $F_B(n) = F_A(\omega(n))\ \forall n \in [n_{\min}, n_{\max}]$. Then, at each α-percentile it is true that $F_B(n_\alpha) = F_A(\omega(n_\alpha)) = \alpha$. From the assumed properties of the cdfs (strictly increasing) it follows that the function ω can be reconstructed from F_A and F_B at any point $n_\alpha\ \forall \alpha \in [0, 1]$ or, equivalently, $\forall n \in [n_{\min}, n_{\max}]$. In our simulation, we do not observe the true cdfs, but only their empirical counterparts, $\hat{F}_A$ and $\hat{F}_B$. The proof of the statistical properties of our approach is beyond the scope of this paper. Note, however, that under the assumption that we actually observe the true cdfs at a limited number of data points m, a smooth interpolation is the best way to fill in the missing values in the estimates of F_A and F_B, because the cdfs are smooth by continuity of the pdfs. Once we have the estimates of cdfs defined over the whole domain, we can recover the function ω for any point in the domain. We apply linear interpolation of the sampling points employing the Matlab routine "interp1". The only remaining problem then are the corners. Whereas theoretically we should observe abilities starting from $n_{\min}$ in one region and $\omega(n_{\min})$ in the other region, empirically we observe only the lowest income category and hence $n_A(z_{\min}) = n_B(z_{\min})$. For the upper corner, a fixed migration gain is assumed. For the lower corner, we assume that the lowest ability in Region A receives the median migration gain which is 17.1%, hence $\omega(n_{\min}) = 1.171 n_{\min}$.

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