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# Decisions with Endogenous Frames\*

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## Abstract

This paper contrasts the normative implications of a model of decision-making with endogenous frames to those of choice theoretic models of Bernheim and Rangel (2007, 2009) and Rubinstein and Salant (2008) in which observed choices are determined by exogenous frames or ancillary conditions. We argue that frames, though exogenous to the individual at the point when choices are made, matter for welfare purposes.

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# 1 Introduction

In recent prominent contributions, Bernheim and Rangel (2007, 2009) (hereafter BR) and Rubinstein and Salant (2008) (hereafter RS) model choice problems where observed choices are determined by frames (RS) or ancillary conditions (BR). A frame is defined as an "observable information that is irrelevant in the rational assessment of the alternatives, but nonetheless affects choice" (RS, abstract). An ancillary condition is "an exogenous feature of the choice environment that may affect behavior, but is not taken as relevant to a social planner's evaluation" (Bernheim and Rangel, 2008, pp. 4). Examples of frames or ancillary conditions include the order in which candidates are listed on a ballot, default alternatives, salience of the alternative, deadline for making a choice or list of alternatives with an aspiration threshold (RS), the point in time at which a choice is made, the manner in which alternatives are presented, the labeling of a particular option as the "status-quo" (BR), etc.

A key dilemma raised by behavioral economics is whether welfare assessments can rely on observed choice alone. When choice is affected by frames or ancillary conditions, the issue is whether such frames or ancillary conditions matter from a welfare viewpoint. There are two diametrically opposite views: (i) frames and ancillary conditions, via their impact on choices, do not matter for making welfare assessments, and (ii) frames and ancillary conditions, though exogenous to the individual at the point when choices are made, matter for welfare purposes. BR and RS endorse the first view. Both papers construct binary relations solely from observed choice and show that such derived binary relations can be used to rank actions available to the decision-maker from a welfare viewpoint: frames or ancillary conditions do not matter in the welfare ranking.

Arguably, there are instances in which frames could be viewed as endogenous. For example, RS consider a deadline as an exogenous frame, although there is considerable empirical evidence that people self-impose deadlines to overcome procrastination (see for example Ariely and Wertenbroch, 2002). The number of alternatives that the decision-maker actually considers (limited focus) is also viewed as an exogenous frame. However, limited focus can be used strategically as a self-control device preventing the decision-maker from embarking in a hazardous activity which may

later regret (Carrillo and Mariotti, 2000). Finally, RS consider the case in which decision-makers encounter the alternatives in the form of a list and choose given an aspirations threshold which is exogenous. There is also vast evidence, however, that aspirations adapt to actions (see for example Easterlin, 2001).

Given the above evidence, this paper studies a class of decision problems with endogenous frames. We compare, and contrast, the relationship between choice and welfare in our set-up with those in BR's and RS's choice theoretic models.

In this paper, a frame is broadly interpreted to include different psychological states such as reference points, beliefs, emotions, temptations, moods, aspirations, etc. There is considerable work from social psychology and economics that psychological states affect behavior<sup>1</sup>. In addition, there is also a great deal of evidence which suggests that what a person does (or expect to do) determines her psychological states. Baron (2008, pp. 68) argues that emotions are partly under our control: individuals can "induce or suppress emotions in themselves almost on cue." Some people may reshape their character, so that their emotional responses change. Albert Bandura (1986) defines *reciprocal determinism* to the view of human functioning as the product of a dynamic interplay of personal, behavioral, and environmental influence: the way in which people interpret the results of their own behavior informs and alters their environments and personal factors which, in turn, inform and alter subsequent behavior through an "environmental feedback effect."

Section 2 takes into account these insights from psychology and introduces a decision-making model with endogenous frames. We endogenize frames in the following way. We distinguish a pre-decision frame from a post-decision frame and assume that the post-decision frame depends on a single-valued map modelling the feedback effect from actions and the pre-decision frame. The individual faces a decision problem which include a given pre-decision frame, a post-decision frame, a feedback map and a set of feasible actions. The decision problem consists in choosing one action in that setting. Since post-decision frames depend on the action chosen and the pre-decision frame, we define a *consistent* decision state as a profile of action and frame where the post-decision frame coincides with the pre-

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<sup>1</sup>Elster (1998) provides a review on how individual choices are affected by emotions; Sen (1977) discusses how personal values shape choices; Appadurai (2004) studies the relationship between aspirations and behavior.

decision frame. A *standard* decision problem (where the decision-maker is rational) is one where the decision-maker chooses a consistent decision state that maximizes ex post (experienced) utility i.e. the decision-maker understands, and internalizes, the feedback mechanism. A *behavioral* decision problem (where the decision-maker is boundedly rational) is one where the decision-maker takes the pre-decision frame (mistakenly) as fixed when choosing actions although an outcome of a behavioral decision problem is required to be consistent. In that sense, the boundedly rational choice is also stable.<sup>2</sup> We show the link between our model and models of reference-dependent preferences where the frames (i.e. reference points) are also actions (Kahneman-Tversky, 1979; Tversky-Kahneman, 1991). We, then, explore the relationship between the standard and behavioral decision problems.

Section 3 summarizes the choice-theoretic framework in BR and RS and section 4 clarifies the relationship between their frameworks and our model. We contrast our decision model to the choice frameworks of BR and RS as follows. Even in scenarios where the "rational" ranking of decision states directly induces a unique ranking of actions, we show that the weak welfare optima derived from observed choice may have an empty intersection with, or exclude elements of, the set of actions associated with the decision outcomes of a standard decision problem. In other scenarios, in order to compare the "rational" ranking over decision states with the ranking of BR and RS, we work with the choice correspondences (projection of decision outcomes on actions) associated with standard and behavioral decision problems respectively. We ask whether the "rational" ranking over decision states, via their projection on the set of actions implies the BR (and RS) ranking assuming that actual choices are described by a behavioral decision-maker. We find that this is indeed so in the special case when frames are exogenous. In general, when the outcomes of a standard decision problem are a subset of the of the outcomes of a behavioral decision problem, the "rational" ranking of consistent decision states

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<sup>2</sup>In a companion paper "Behavioral Decisions and Welfare" (Dalton and Ghosal, 2010) we provide the axiomatic characterization, via choice correspondences, of standard vs behavioral decisions in a class of models where frames depend solely on actions via a feedback effect. In this paper, we generalize this decision model by distinguishing between a pre-decision and a post-decision frame. Moreover, we focus on comparing decision-making with endogenous frames to choice with exogenous frames or ancillary conditions.

implies, via their projection on the set of actions, the BR's ranking. Therefore, the weak welfare optima derived from the choice correspondence associated with a standard decision problem will be included in the set of weak welfare optima derived from the observed choice correspondence.

In section 5, we analyze the policy implications of our argument. Clearly, when the outcomes of both standard and behavioral decision problems coincide, there is no case for any sort of intervention by a social planner. In contrast, in scenarios where there are multiple welfare ranked outcomes, the "libertarian paternalism" approach advocated by Thaler and Sunstein (2003) that only seeks to alter the frames or ancillary conditions could work. In general, however, we argue that the scope for such policy interventions is limited and if "hard paternalism" (i.e. directly constraining the choices of individuals) is to be avoided, interventions should aim to ensure that decision-makers internalize, with high probability, the feedback from actions to frames.

The last section concludes and discusses directions for further research.

## 2 Decisions with Frames

In our framework, the objects of preferences are ordered pairs  $(a, q) \in A \times Q$  where  $a$  is an "action" and  $q$  is a "frame". A decision state is a pair of actions and preference parameters  $(a, q)$  where  $a \in A$  and  $q \in Q$ . We distinguish between the pre-decision frame  $q_0$  and the post-decision frame  $q$ . There is a map  $\pi : A \times Q \rightarrow Q$  modelling the feedback effect from actions and the pre-decision frame to the post-decision frame and it is assumed that  $\pi(a, q)$  is non-empty and single-valued for each  $(a, q) \in A \times Q$ . We define a consistent state as a decision state  $(a, q)$  such that  $q = \pi(a, q)$ .

The preferences of the decision-maker are denoted by  $\succeq$ , a binary relation ranking pairs of decision states in  $(A \times Q) \times (A \times Q)$ . The expression  $\{(a, q), (a', q')\} \in \succeq$  is written as  $(a, q) \succeq (a', q')$  and is to be read as " $(a, q)$  is weakly preferred to  $(a', q')$  by the decision-maker".

We interpret the preferences of the decision-maker as reflecting some form of ex-post utility (interpreted as experienced utility) which depends on the chosen action and the post-decision frame. In effect, following Harsanyi (1954), we go beyond the assumptions of the usual ordinal utility theory and assume the intra-

personal comparability of utility. We assume not only that the DM is able to rank different elements in  $A$  for a given  $q$  but also that she is able to assess the subjective satisfaction she derives from an action when the psychological state is  $q$  with the subjective satisfaction she derives from another action when the psychological state is  $q'$ . In other words, we assume that the individual is able to rank elements in  $A \times Q$ . This formulation is critical in order to make meaningful welfare comparisons.

We study two distinct decision problems:

1. A *standard* decision problem (interpreted as rational decision-making) is one where the decision-maker chooses a pair  $(a, q)$  from within the set of consistent decision states i.e. the decision-maker understands, and fully internalizes, the feedback mechanism. A consistent pair  $(a, q)$  is the outcome of a standard decision problem (i.e. a rational choice) if and only if

$$(a, q) \succeq (a', q') \text{ for all } (a', q') \text{ with } q' = \pi(a', q').$$

Let  $M$  denote the set of all outcomes of a standard decision problem.

2. A *behavioral* decision problem (interpreted as boundedly rational decision-making) is one where the decision-maker takes the pre-decision frame  $q$  (mistakenly) as fixed when choosing  $a$ . A consistent pair  $(a, q)$  is the outcome of a behavioral decision problem (if and only if

$$(a, q) \succeq (a', q) \text{ for all } a' \in A \text{ with } q = \pi(a, q).$$

Note that a boundedly rational choice is also in some sense stable.

Let  $E$  denote the set of outcomes of a behavioral decision problem.

In our model, it is rational to maximize ex-post (experienced) utility (SDP). A behavioral decision-maker, however, mistakenly chooses an action as if  $q$  were fixed, thus typically failing to maximize her ex-post utility<sup>3</sup>.

*Remarks:*

1. It is convenient to relate the model of decision-making studied here to the case of reference-dependent preferences where the frames (i.e. reference points) are also

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<sup>3</sup>We acknowledge that this view is subject to criticism (see for example, Bernheim 2009). However, other competing normative criteria proposed in the literature of behavioral welfare economics have also important drawbacks. Kahneman and Sugden (2005), while acknowledging the criticisms of the normative position adopted here, suggest that it could represent a useful starting point for welfare analysis.

actions: in this case, we require that  $Q = A$  and  $a = \pi(a, q)$  for all  $q$ . For example, in Tversky and Kahneman (1991)'s theory of reference-dependent preferences over consumption,  $a$  could be a consumption bundle and  $q$  is a reference point (another commodity bundle). If the decision-maker chooses  $a$  when the pre-decision reference point is  $q$ , the post-decision reference point shifts to  $a$ . In this sense, the model of decision-making studied here corresponds to situation where "the reference state usually corresponds to the decision-maker's current state." (Tversky and Kahneman, 1991, pp. 1046).

2. Consider the special case of our model where  $Q = \{q_1, q_2\}$ ,  $A = \{a_1, a_2\}$  and  $q_i = \pi(a_i, q)$  for all  $q$ . We show that even in this special case,  $M \neq E$  and behavioral decision outcomes have properties normally associated with the Nash equilibria of two-person normal form games. Assume that the preferences  $\succeq$  are represented by an utility function  $u : A \times Q \rightarrow \mathfrak{R}$ . Let  $\alpha(q) = \arg \max_{a \in A} u(a, q)$ . A pure action outcome of a behavioral decision is an action profile  $\tilde{a}$  such that  $\tilde{a} \in \alpha(\tilde{q})$  and  $\tilde{q} = \pi(\tilde{a}, \tilde{q})$ . A pure action outcome of a standard decision problem is one where  $(\hat{a}, \hat{q}) \in \arg \max_{(a, q) \in A \times Q} u(a, \pi(a, q))$ . In each example, the decision problem is represented by a payoff table where rows are actions and columns are the utility parameters. Under the assumptions made so far, the payoffs associated with consistent decision states are on the diagonal of these payoff tables.

**Example 1.** *A unique inefficient behavioral decision in dominant actions: addiction*

Consider the following payoff table:

	$q_1$	$q_2$
$a_1$	1	-1
$a_2$	2	0

(Table 1)

We interpret these payoffs as an example of addiction where  $a_2$  corresponds to *smoking* and  $a_1$  corresponds to *not smoking* and  $q_i$  to different health states of the individual ( $q_2$  is less healthy than  $q_1$ ). In this case, in a behavioral decision problem, the decision-maker always chooses  $a_2$  as  $a_2$  is the dominant action for each value of  $q$ : if the individual takes her health state  $q$  as given she always prefers to smoke. The unique behavioral decision outcome is  $(a_2, q_2)$  with a payoff of 0. However, note that the consistent decision state  $(a_1, q_1)$  with a payoff of 1 is the only element of  $M$ :

once the individual takes the feedback from actions to health states into account, she always chooses not to smoke.

**Example 2.** *No pure action behavioral decision: the grass is always greener on the other side*

	$q_1$	$q_2$
$a_1$	0	1
$a_2$	1	0

(Table 2)

We interpret these payoffs as an example of a situation where the individual makes a choice between two different lifestyle so that  $q_i$  denotes a specific lifestyle and  $a_i$  denotes the action that chooses location  $q_i$ . Starting from  $q_1$ , the decision-maker prefers  $a_2$  to  $a_1$  while starting from  $q_2$ , the decision-maker prefers  $a_1$  to  $a_2$ : the individual always believes that the grass is greener on the other side. There is no behavioral decision in pure strategies. The decision-maker is, however, indifferent between both the two consistent decision-states  $(a_1, q_1)$  and  $(a_2, q_2)$ .

**Example 3.** *Multiple welfare ranked equilibria: aspirations*

	$q_1$	$q_2$
$a_1$	1	0
$a_2$	0	2

(Table 3)

We interpret these payoffs as an example of an aspiration failure. Let  $a_1$  = *undertaking an action that perpetuates the status quo* and  $a_2$  = *undertaking an action that changes the status quo*, with  $q_2$  = *"high aspirations"* and  $q_1$  = *"low aspirations"* being the consistent psychological states associated with  $a_1$  and  $a_2$  respectively. In this example, there are two strict behavioral decision outcomes  $(a_1, q_1)$  and  $(a_2, q_2)$ . Note that the pure action equilibrium  $(a_1, q_1)$  is dominated by the pure action equilibrium  $(a_2, q_2)$ . When decision-maker's aspirations are high,  $(a_2, q_2) \succ (a_1, q_2)$ , while when her aspirations are low,  $(a_2, q_1) \succ (a_1, q_1)$ . Thus, the behavioral decision outcome  $(a_1, q_1)$  is an instance of an aspirations failure.

3. In the case of reference-dependent preferences, frames are also actions (i.e.  $Q = A$ ) and a chosen action becomes a post-decision frame  $\pi(a, q) = a$  for all  $(a, q) \in A \times Q$ . In this case, the ranking of consistent decision states  $(a, a)$  is a frame-independent ranking of actions as "viewed from themselves". Note, in addition, that the payoffs in example 2 can be interpreted as reference dependence without loss

aversion while the payoffs in example 3 can be interpreted as reference dependence with loss aversion (hypothesis supported by most experimental evidence).

4. In a purely formal sense, a standard (respectively, behavioral) decision problem with endogenous frames can be viewed as the Stackelberg (respectively, Nash) equilibrium of a dual self intra-personal game where one self chooses actions and the other self chooses frames. This equivalence, though purely formal, makes the point that in a behavioral decision problem (in contrast to standard decision problem), the individual imposes an externality on herself that she doesn't fully internalize.

5. Example 2 demonstrates that, in general,  $E$  may be empty even when  $M$  isn't. However, given that a behavioral decision outcome can be interpreted as a Nash equilibrium of a two person game so that as long as  $A$  and  $Q$  are finite, a mixed strategy behavioral decision outcome always exists.

6. Consider the following condition on preferences:

$\hat{C}$ : For any consistent decision states  $(a, q)$  and  $(a', q')$  such that  $(a, q) \succeq (a', q')$ ,  $(a, q) \succeq (a', q)$ .

Fix the consistent states  $(a, q)$ ,  $(a', q')$ . Condition  $(\hat{C})$  states that if the consistent state  $(a, q)$  is preferred to the consistent state  $(a', q')$ , then the action  $a$  weakly dominates the action  $a'$  at the psychological state  $q$ . Note that preferences in Example 3 satisfies  $(\hat{C})$  while the preferences in Example 1 violate  $(\hat{C})$ . Clearly, under  $(\hat{C})$ ,  $M \subseteq E$  and if  $M \subseteq E$   $(\hat{C})$  has to hold is also immediate (from negating  $(\hat{C})$  and the definition of  $B$  and  $S$ ). It follows that  $(\hat{C})$  is necessary and sufficient conditions for  $M \subseteq E$ :

**Result 1.** *Suppose that both  $M$  is non-empty. Then,  $M \subseteq E$  if and only if  $(\hat{C})$  holds.*

### 3 Choice with Frames or Ancillary Conditions

In this section, we present the key features of the analysis of choice with frames or ancillary conditions studied by BR and RS. We assume that both  $A$  and  $Q$  are non-empty finite sets containing at least two elements each.

Both BR and RS study generalized (or extended) choice problems  $(A, q)$  where  $q$  is a frame or an ancillary condition. Both BR and RS make the point that, in practice, it is difficult to draw a distinction between characteristics of elements in  $A$

and variables in  $Q$  which could also be viewed as characteristics of elements in  $A$ .

An individual's choices are described by a correspondence  $c(A', q) \subseteq A'$  where  $A' \in \tilde{\mathbf{A}}$  the set of all non-empty subsets of  $A$  and further, that  $c(A', q)$  is non-empty for all pairs  $(A', q)$ .<sup>4</sup> Define  $aP^*b$ <sup>5</sup> iff for all admissible  $(A', q)$  with  $a, b \in A'$ ,  $b \notin c(A', q)$ : when  $aP^*b$ , following BR,  $a$  is strictly unambiguously chosen over  $b$ .<sup>6</sup> Define  $aR^*b$  iff  $\sim aP^*b$ : there is some generalized choice problem where both  $a$  and  $b$  are present and  $a$  is chosen. Define  $xI^*y$  iff  $xR^*y$  and  $yR^*x$ : there is some generalized choice problem where both  $a$  and  $b$  are present and  $a$  is chosen and some other generalized choice problem where both  $a$  and  $b$  are present and  $b$  is chosen. BR show that  $R^*$  is necessarily complete: for any  $a$  and  $b$  the individual must necessarily choose either  $a$  or  $b$  from any  $(\{a, b\}, q)$ . Moreover, they also show that none of the binary relations need be transitive although they do show that  $P^*$  is acyclic i.e. for any  $a_1, \dots, a_K$ , if  $a_k P^* a_{k+1}$  then  $\sim a_K P^* a_1$ . BR, then, go on to make the following definition:

**Definition** (*Weak Welfare Optimum, Bernheim and Rangel, 2009*): It is possible to strictly improve on a choice  $a \in A$  if there is  $b \in A$  such that  $bP^*a$ . When a strict improvement is impossible,  $a$  is defined as a Weak Welfare Optimum.

In using a Weak Welfare Optimum to rank actions, BR construct a frame-independent ranking of actions. In effect, BR take the position that all frames are normatively irrelevant.

BR show the following result that underpins their welfare analysis:

**Result 2** (*FACT 1, Bernheim and Rangel, 2007*): If  $a \in c(A', q)$  for some  $(A', q)$ , then  $a$  is a weak welfare optimum.

Notwithstanding the fact that some frames in the sense of BR and RS are exogenous, we close this section by pointing out that, in some instances, frames could be viewed as endogenous and may have normative consequences. We take two examples of shuck frames introduced by RS to illustrate our point.

#### **Example 4 Deadlines**

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<sup>4</sup>RS study choice functions while BR allow for choice correspondences.

<sup>5</sup>For ease of exposition, since we allow for choice correspondences and focus on normative implications of choice, we follow BR although we note that RS also derive a preference relation similar to  $P^*$ .

<sup>6</sup>In words, the statement " $aP^*b$ " means that whenever  $a$  and  $b$  are available,  $b$  is never chosen.

Consider the case where the frame is a deadline. Then,  $Q = T$  where the pair  $(x, t)$  is interpreted as consisting of an alternative  $x$  that has to be chosen by the deadline  $t$ . An extended (or generalized) choice problem is one with a value function  $v : X \rightarrow \mathfrak{R}$  and a processing time function  $d(x)$  which assigns to each  $x$  the time it takes the decision-maker to understand its meaning. Given a frame  $t$ , the decision-maker chooses from the  $v$ -maximal alternative with  $d(x) \leq t$ . As pointed out in the introduction of this paper, agents routinely self-impose deadlines, so they are not exogenous to the decision. More importantly, this will have normative significance. In this case, we would require that an action have two components  $a = (x, t)$  and  $t = \pi(a, t')$  for all  $t'$ .

**Example 5** *Lists with an aspirations threshold*

Consider the case where the set of alternatives is presented to the decision-maker as a list. For simplicity, assume that  $X$  is a finite set of alternatives and  $>_X$  denote the set of all ordering of  $X$  where  $x > y$  is interpreted as " $x$  precedes  $y$ ",  $x, y \in X$ . The decision-maker has in mind a value function  $v : X \rightarrow \mathfrak{R}$ . Further, let  $v^*$  denote an aspirations threshold and let  $V^* \subset \mathfrak{R}$  denote the set of aspirations threshold. Then,  $Q = (>_X \times V^*)^7$ . An extended (or generalized) choice problem is one where for each  $(>, v^*)$  the decision-maker chooses the first  $>$ -element that is  $v$ -maximal; if there are none the last element is chosen. If aspirations adapt, then clearly at least one component of the frame is endogenous and normatively significant. In this case, we would require  $A = X$  and  $\pi(a, q) = (\pi_1(a, >), \pi_2(a, v^*))$  such that  $> = \pi_1(a, >)$  and  $v(a) = \pi_2(a, v^*)$ .

## 4 Choice and Welfare

In this section, we compare and contrast the normative implications of the decision model introduced in Section 2 with that of BR and RS described in Section 3.

The framework of decision making studied in Section 2 takes the position that post-decision frames are normatively relevant while BR's position, given their definition of a Weak Welfare Optimum, is that frames are normatively irrelevant. The former normative position implies that a rational decision-maker construes his choice

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<sup>7</sup>We differ in the interpretation of a frame from RS: for them only the ordering over actions is a frame while for us both the ordering and the aspirations threshold is a frame.

as a choice over consistent decision states while BR's normative position implies that a rational decision-maker construes his choice as a frame-independent choice among actions.

In general, for RS and BR, what matters for welfare is the binary relation  $P^*$  constructed solely on actions. In contrast, what matters for welfare purposes in our model is the ranking of consistent decision states where both actions and frames matter. The question then is whether the ranking over actions using the binary relation  $P^*$  constructed solely on the basis of observed choices coincides with the fixed underlying preference relation  $\succsim$  over the set of consistent decision states. Observe that the ranking of the preference relation  $\succsim$  over the set of consistent decision states corresponds to that of a standard decision-maker and we refer to this as the "rational" ranking of decision states.

To begin with, consider scenarios where the "rational" ranking of decision states directly induces a unique ranking of actions. For example, assume, to begin with, that there is a unique  $q(a)$  with the property that  $q(a) = \pi(a, q)$  for each  $a \in A$  so that the "rational" ranking of consistent decision states  $(a, q(a))$  induces a unique ranking of actions. We can then ask whether this makes sense as a BR ranking i.e. corresponds to  $P^*$ . Clearly, one necessary condition for this to make sense is that there are no  $a$  and  $a'$  such that (i)  $(a, q(a)) \succ (a', q(a'))$  and (ii) for all  $q$ ,  $(a', q) \succ (a, q)$  (by Result 2, for BR  $a'P^*a$ ). Further, under the assumption that there is a unique  $q(a)$  with the property that  $q(a) = \pi(a, q)$  for each  $a \in A$ , notice that the conjunction of (i) and (ii) is ruled out whenever  $(\hat{C})$  (studied in Result 1, Remark 6, Section 2) holds: therefore, whenever  $M \subseteq E$ , the unique ranking over actions also makes sense as a BR's ranking of actions. As already pointed out, Example 1 shows how this condition may fail.

The "rational" ranking of consistent decision-states may also make sense as a BR's ranking in the case of reference dependent preferences: the ranking of consistent decision states  $(a, a)$  is a frame-independent ranking of actions. Munro and Sugden (2003), in their reformulation Tversky and Kahneman, define a concept of reference-neutral preference. Their definition of loss aversion implies that if  $a'$  is preferred to  $a$  in the reference neutral sense, then  $a'$  is preferred to  $a$  when the reference point is  $a'$ : this rules out the conjunction of the two conditions (i) and (ii) above. It could be argued that this is also the ranking of actions given by  $P^*$  in BR.

What happens when the "rational" ranking over consistent decision states doesn't directly induce a unique ranking over actions? In such a case, in order to compare the rational ranking over decision states with BR's ranking we will work with the choice correspondences (projection of decision outcomes on actions) associated with standard and behavioral decision problems respectively.

We introduce some notation. For any  $A' \in \tilde{\mathbf{A}}$ , let  $\hat{c}(A') \subseteq A'$  be defined as the choice correspondence of a standard decision-maker i.e. rational choice:

$$\hat{c}(A') = \left\{ \begin{array}{l} a \in A' : \text{there exists } q = \pi(a, q) \text{ and } q' \in \pi(a', q') \\ \text{such that } (a, q) \succeq (a', q') \text{ for all } a' \in A'. \end{array} \right\}$$

and let  $\tilde{c}(A') \subseteq A'$  be defined as the choice correspondence of a behavioral decision-maker i.e. bounded rational choice:

$$\tilde{c}(A') = \left\{ \begin{array}{l} a \in A' : \text{there exists } q = \pi(a, q) \text{ such that} \\ (a, q) \succeq (a', q) \text{ for all } a' \in A'. \end{array} \right\}.$$

Define the *standard* binary relation  $a\hat{P}^*b$  iff for all admissible  $(A', q)$  with  $a, b \in A'$ ,  $b \notin \hat{c}(A')$  and define the *behavioral* binary relation  $a\tilde{P}^*b$  iff for all admissible  $(A', q)$  with  $a, b \in A'$ ,  $b \notin \tilde{c}(A')$  with  $\hat{R}^*, \hat{I}^*$  and  $\tilde{R}^*, \tilde{I}^*$  defined analogously.

We assume that the decision-maker always solves a behavioral decision problem so that observed choice is described by  $\tilde{c}(A')$  for each  $A' \subseteq A$ . Let  $\hat{W}$  denote the weak welfare optima corresponding to  $\hat{P}^*$  and let  $\tilde{W}$  denote the weak welfare optima corresponding to  $\tilde{P}^*$  (and  $\tilde{P}^*$ ).

To begin with, consider the case of *exogenous* frames i.e. scenarios where  $\pi(a, q) = q$  for all  $(a, q) \in A \times Q$ . Observe that the "rational" ranking of decision states in this case does not induce a ranking solely over actions.

We begin by stating the following result as an immediate consequence of the analysis presented so far:

**Proposition 1.** Suppose  $\pi(a, q) = q$  for all  $(a, q)$ . Then,  $\tilde{c}(A') = \cup_{q \in Q} c(A', q)$  and  $\tilde{P}^* = P^*$ .

**Proof:** Since  $\pi(a, q) = q$  for all  $(a, q)$ , any pair  $(a, q)$  is a consistent decision state. Thus, for each  $q$  if the decision-maker solves the decision problem in a behavioral way, for each  $A' \subseteq A$ , each chosen action will be an element of  $c(A', q)$  and therefore, weak individual welfare optimum in  $A$ . ■

The following proposition examines the link between  $\hat{W}$  and  $\tilde{W}$  in the case with exogenous frames.

**Proposition 2.** Suppose  $\pi(a, q) = q$  for all  $(a, q)$ . Then,  $M \subseteq E$ ,  $\hat{c}(A') \subseteq \tilde{c}(A')$  and  $\hat{W} \subseteq \tilde{W}$ .<sup>8</sup>

**Proof:** If  $\pi(a, q) = q$  for all  $(a, q)$ , then  $(a, q) \in M$  implies that  $(a, q) \succeq (a', q)$  for all  $a' \in A$  and therefore,  $(a, q) \in E$ . That  $\hat{c}(A') \subseteq \tilde{c}(A')$  is a direct consequence of  $M \subseteq E$ . Finally, as  $\hat{c}(A') \subseteq \tilde{c}(A')$ ,  $b \notin \tilde{c}(A')$  implies  $b \notin \hat{c}(A')$ . It follows that if  $a \in \hat{W}$  there is no  $b \in A$  such that  $b \hat{P}^* a$ . Therefore, for every  $b \in A$ , there exists a non-empty  $A' \subseteq A$  with  $a, b \in A'$  such that  $a \in \hat{c}(A')$ . As  $\hat{c}(A') \subseteq \tilde{c}(A')$ , it also follows that there is no  $b \in A$  such that  $b \tilde{P}^* a$ . Therefore,  $a \in \tilde{W}$  and  $\hat{W} \subseteq \tilde{W}$ . ■

Proposition 2 shows that with exogenous frames, the set of weak welfare optima derived solely from observed choice contain all the actions corresponding to elements in  $M$  so that the "rational" ranking of consistent decision states implies, via their projection on the set of actions, BR's ranking. However, the inclusion of the weak welfare optima induced by the "rational" ranking of consistent decision states in the set of weak welfare optima consistent with observed choice may be strict. For instance, in example 3,  $\hat{W} = \{a_1\} \subset \tilde{W} = \{a_1, a_2\}$ . Therefore,  $a_2 \in \tilde{W}$  but  $a_2 \notin \hat{W}$ .

What would happen in the general case when frames aren't necessarily exogenous? The following proposition states that a necessary and sufficient condition for obtaining the desired conclusion for a general  $\pi(\cdot)$  is  $M \subseteq E$ .

**Proposition 3.** In the general case when frames aren't necessarily exogenous,  $\hat{W} \subseteq \tilde{W}$  iff  $M \subseteq E$  (equivalently,  $\hat{W} \subseteq \tilde{W}$  iff  $\hat{C}$ ).

**Proof:** Clearly, if  $M \subseteq E$ ,  $\hat{c}(A') \subseteq \tilde{c}(A')$  and by using argument similar to those in Proposition 1,  $\hat{W} \subseteq \tilde{W}$ . Next, suppose that  $\hat{W} \subseteq \tilde{W}$  but  $M \not\subseteq E$ . As  $M \not\subseteq E$ , by Result 1, it is equivalent to assume that condition  $(\hat{C})$  doesn't hold. As  $(\hat{C})$  doesn't hold, there exists  $a \in A$  such that  $(a, q) \succeq (a', q')$  for some  $q \in \pi(a)$  and  $q' \in \pi(a')$  but  $(a, q) \prec (a', q)$  for all  $q \in \pi(a)$ . It follows that there exists some non-empty  $A' \subseteq A$ , such that  $a \in \hat{c}(A')$  but  $a \notin \tilde{c}(A')$  and therefore,  $\hat{c}(A') \not\subseteq \tilde{c}(A')$ . But, then, there exists  $b \in A$  (take  $b = a'$ ) and a non-empty subset of  $A$ ,  $A'$ , such that  $a \in \hat{c}(A')$  but  $a \notin \tilde{c}(A')$ . Therefore,  $a \in \hat{W}$  but  $a \notin \tilde{W}$ , a contradiction. ■

Assuming that actual choices are described by a behavioral decision-maker,

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<sup>8</sup>Recall that  $M$  is the set of all outcomes of a standard decision problem and  $E$  is the set of all outcomes of a behavioral decision problem.

Proposition 3 shows that when the outcomes of a standard decision problem are a subset of the of the outcomes of a behavioral decision problem, the "rational" ranking of consistent decision states implies, via their projection on the set of actions, the BR's ranking. Therefore, the weak welfare optima derived from the choice correspondence associated with a standard decision problem will be included in the set of weak welfare optima derived from the observed choice correspondence.

## 5 Policy Implications

What are the policy implications of our model?

The "libertarian paternalism" approach to policy interventions recently advocated by Thaler and Sunstein (2003) proposes manipulation of frames as a means of nudging individuals towards welfare-maximizing choices. This idea fits with example 3 with multiple welfare ranked decision outcomes since a frame manipulation can move the decision-maker to a welfare dominating decision outcome. In general, manipulation of frames would fit with situations where  $M \subseteq E$ .

However, in cases such as example 1 where the intersection of the decision outcomes of a standard decision problem and those of a behavioral decision problem (i.e.  $M \cap E = \phi$ ), frame manipulations would not necessarily result in welfare improvements or have no effect on behavior. On the face of it, an intervention directed at welfare maximization would have to paternalistic in a stronger sense of constraining individual choice.

In general, however, with incomplete information about individual preferences and feedback effects, direct policy intervention along the lines of "soft" paternalism (changing frames) or "hard" paternalism (constraining individual choice) could make matters worse.

One possible policy recommendation in scenarios with incomplete information about an individual's preferences is to directly act on the way in which a person internalizes the feedback effect from actions to frames. Examples of such policies could be psychotherapy sessions, projects aiming to foster people's emotional intelligence and empowerment, etc.

To fix ideas, consider example 1. In this example, if the individual doesn't take the feedback effect from actions to psychological states into account, she always

chooses  $a_2$  (*smoking*) over  $a_1$  (*not smoking*); however, the reverse would be true, if she took the feedback into account. Let  $\alpha$ ,  $0 \leq \alpha \leq 1$ , denote the probability with which the individual does take the feedback effect into account. A straightforward computation shows that as long as  $\alpha > \frac{1}{2}$ , the individual will choose  $a_1$  over  $a_2$ : as long as the individual takes the feedback effect into account with a high enough probability, she will choose not to smoke so that interventions that increase the probability with which an individual internalizes the feedback effect could result in welfare improvements.

Likewise, in example 3, a policy intervention that might work may be an "empowerment" policy, that would help the individual to become aware of her "internal constraints" and thus "gaining control over her own life"<sup>9</sup>.

## 6 Final Remarks

The results reported here have some empirical caveats. Both, the endogenous frames and the feedback-map are key variables for policy considerations, though they are not directly observable or even inferred from choice behavior. One possible approach to identify these "unobservable" may be to use evidence from neuroscience and psychology on the neural processes driving decision making.

Extending the one-person model studied here to  $n$ -players, dynamic and sequential decision scenarios are topics for future research.

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<sup>9</sup>See for instance Appadurai (2004) on the "capacity to aspire" or World Bank (2002) and Stern (2004) on Empowerment.

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